GEAR-WHEEL DRIVEN GENEVA WHEELS

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The subject treated in this paper concerns a geared Geneva wheel mechanism that was initially proposed by Kraus [2] in 1963. The mechanism seems to be important enough to give it another approach and to add some graphs that might prove useful to the designer. Apart from the examples given in the paper, detailed instructions are presented to design the mechanism for any given number of stations and any given ratio of times moving/cycle that is typical for intermittent motion mechanisms of the kind.

1. INTRODUCTION

There are many ways to drive Geneva wheels. Normally, this is done by a single crank bearing a pin with a roller that intermittently interlocks with the Geneva wheel, as is shown in figure 1. The center of the pin then traces a circle that touches the central-lines of two successive slots that are engraved in the wheel. However, the mechanism that so generates intermittent motion shows a number of disadvantages. For instance

i) The in- and output axes are not co-axial;

ii) Values of 360° or even 180° for the output-angle of the wheel are not possible for this type of mechanism;

iii) The ratio of times $v$, which is the ratio of times between the motion period of the wheel, and the cyclic period, which is the time-lapse

\[ n = s = g \]

\[ v = \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \]

Fig. 1. — External Geneva wheel mechanism driven by a single crank.

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for motion and dwell together, is dependent on the number of stations \( n \). It is therefore not possible to choose \( v \) and \( n \) independently from one another. This limits the mechanisms' usefulness to the designer. For an *external* Geneva wheel, for instance, the ratio \( v \) equals the value represented by the equation:

\[
v_{ex} = \frac{1}{2} - \frac{1}{n},
\]

For an *internal* Geneva wheel the ratio is represented by

\[
v_{ia} = \frac{1}{2} + \frac{1}{n}.
\]

\( iv \) Caused by the regular circle motion of the pin, an *angular jerk* appears at the start and at the end of the motion of the wheel.

When the driving part of the mechanism is replaced by a gear-wheel mechanism, however the in- and output axis may be made coaxial. The dependency of \( v \) and \( n \) may be relieved and the angular jerk of the Geneva wheel may be reduced to a lesser value.

The curve traced by the driving pin is then a *cycloidal* curve instead of a circle.

Here, the pin, that produces the curve, is attached to a gear-wheel that rolls about another wheel which is fixed. The fixed wheel comprises a pitch circle which is the *fixed poleide* \((\pi_f)\) of the motion. Similarly, the moving wheel is comprised by the *moving poleide* \((\pi_m)\).

If the moving poleide rolls *inside* the fixed poleide, a *hypo-cycloid* motion is at hand. If it rolls over and about the fixed poleide an *epi-cycloid* motion occurs. Finally, if the moving poleide embraces the fixed one, a so-called *peri-cycloid* motion may be generated. In figure 2 all

![Diagram](image)

the mentioned possibilities are shown: by gradually changing the size of either the fixed or the moving poleide, it is demonstrated in the figure how the occurring possibilities may blend into each other.
Not counting the particular motions in which one of the polodes is a straight-line, only the hypo-cycloid, the epi-cycloid and the pericycloid motion remain to be considered.

A (coupler-) point attached to the moving polode in these cases, will trace a hypo-cycloid, an epi-cycloid or a pericycloid respectively. Also, if the center of the pin lies inside the moving polode, the point traces a contracted cycloid, otherwise it is a protracted cycloid. Only if the curves traced by these centers are "suitable", the pin may be used to drive a Geneva wheel (See for instance the curves shown in the figures 3 and 4).

As it, has already been pointed out by G. Bellerman [1] in 1867, however there are always two gear-wheel pairs that will produce the same curve. Therefore, the two gear-wheel mechanisms that are linked in this way are curve-cognates of one another.

So, it may be proved that protracted hypocycloids and contracted hypocycloids are identical. In the same way one may prove that a protracted epicycloid and a contracted pericycloid are identical, and reversed.

This fact is of some importance for the designer of these mechanisms: once the curve is chosen, he still has the choice of the mechanism. Naturally, the two curve-cognates do not occupy the same space as can easily be verified. So, if a compact mechanism is wanted, it is possible to meet such a demand.

2. PROOF OF THE TWO-FOLD GENERATION (See figures 3 and 4)

Suppose the source mechanism produces a protracted hypocycloid. (See figure 3). The curve may then be traced by any point $C$ that lies outside the moving polode. Thus, during the rolling motion, the center $M$ of this polode and the point $C$, once it is chosen, are at a constant distance from each other. The same applies for the centers $M$ and $M_0$. Therefore, $M$ and $M_0$ may be joined by a single bar $MM_0$.

Next, we adjoin the dyad $M_0M'C$ such that a linkage parallelogram $M_0MCM'$ is created.

We then investigate the motion of the adjoined bar $M'C$ with respect to the frame. We shall prove then that the motion of this bar may be directed or reproduced by another pair of wheels. In this case we shall prove that the bar $M'C$ can be attached to a circle that rolls inside another circle, which is the fixed polode for that motion.

To do this, we first determine the location of the velocity-pole $P'$ of the observed bar. This is easily done, using the Kennedy-Aronhold theorem about the three relative poles that join a straightline.

According to this theorem we find that the pole $P'$ of the bar observed has to join the Kennedy-line $PC$ as well as the Kennedy-line $M_0M'$. Thus, $P' = (M_0M', PC)$. Here, $P$ resembles the pole of the bar $MC$ which is attached to the moving polode of the source mechanism. Further, we see that

$$P'M_0 = M'M_0 \cdot \frac{PM_0}{PM} = M'M_0 \cdot \frac{R_0}{R} = \text{constant} = R_0'$$
Fig. 3. — Identical hypocycloids traced by cognate gear-wheel mechanisms.
Fig. 4. — Identical curves traced by cognate gear-wheel mechanisms.
Throughout the motion, therefore, the locus of poles $P'$ in the frame resembles a circle about $M_0$ with radius $R'_0$. Thus, the fixed polode ($\pi'_0$) of the motion observed is a circle about $M_0$ that joins the pole $P'$. Similarly, we find that

$$P'M' = P'M_0 - M'M_0 = M'M_0 \left( \frac{R_0}{R} - 1 \right) = \text{constant} = R'$$

Hence, the locus of poles $P'$ in the moving plane of the observed bar $M'C$ is a circle too. Therefore, the moving polode ($\pi'_0$) which is the fixed polode of the inverse motion, is a circle about $M$ with radius $R'$. Conclusion: the motion of the bar $M'C$ may be generated by the rolling motion of a circle inside another one.

Since the point $C$ of the bars $M'C$ and $MC$ is now a common point that takes part in two different motions, the curve traced by that point is to be generated by two different wheel-pairs. These pairs are represented by the source mechanism and by the curve-cognate of the source mechanism.

Both mechanisms produce hypocycloid motions, since for the two mechanisms the rolling motion occurs inside the fixed wheel. Moreover, since $C$ lies inside the moving polode of the curve-cognate, the point $C$ traces a contracted hypocycloid if it is attached to that mechanism. If $C$ is attached to the source mechanism the point traces a protracted hypocycloid. Thus, the contracted hypocycloid is identical to the protracted hypocycloid.

### 3. RELATED DIMENSIONS OF THE CURVE-COGNATES

It is further seen, that

$$\frac{R'}{R'_0} = 1 - \frac{R}{R_0} \quad (1)$$

Thus, if by definition

$$k = \frac{R_0}{R} \quad \text{and} \quad k' = \frac{R'_0}{R'}$$

we have found that

$$\frac{1}{k} + \frac{1}{k'} = 1 \quad (2)$$

By further introducing the angular velocities $\omega$ and $\omega'$ of the respective moving polodes of the source mechanism and of the curve-cognate, we find that

$$\frac{\omega}{\omega'} = \frac{CP'}{CP} = \frac{CM'}{MP} = \frac{MM_0}{MP} = \frac{PM_0 - PM}{MP} = 1 - \frac{R'_0}{R} = 1 - k.$$  

Thus

$$\omega/k = -\omega'/k' \quad (4)$$

Through figure 4 it may be similarly shown that a protracted epicycloid is identical to a contracted peri-cycloid.
The derived formulas too, are valid for such a case. However, one has to keep in mind that the gear-ratio of the wheels that generate an epicycloid motion, always obtains a negative value. Thus, $k < 0$ for the epicycloid motion.

For the pericycloid motion however, $0 < k < 1$. Also for the hypocycloid motion we note that $k > 1$. We further note that

$$\frac{M' \cdot C}{M \cdot C} = \frac{M' \cdot M}{M \cdot M'} = \frac{R_0 - R}{R_0' - R_0'} = \frac{(k - 1)}{(k' - 1)} \cdot \frac{R}{R'} = \left(\frac{k}{k'}\right)^2 \frac{R}{R'}$$  \hspace{1cm} (5a)

This relation shows how the location of the common point $C$ is transformed from its location with respect to the source mechanism into that with respect to the corresponding curve-cognate.

From the figures 3 and 4, we further see that

$$\frac{R'}{M \cdot C} = \frac{k}{k'}$$  \hspace{1cm} (5.b)

and

$$\frac{M' \cdot C}{R_0} = \frac{M_0 \cdot C}{R_0} \cdot \frac{R}{M \cdot C},$$  \hspace{1cm} (5c)

where $M'_0 = M_0$.

4. DIMENSIONS OF THE MECHANISM

As pointed out earlier, the input and output axes of the mechanism may be made coaxial, which means that the input-crank $M_0 \cdot M$ and the Geneva wheel must rotate about the same fixed center $M_0$. Besides, the slots of the Geneva wheel are radial. So, in order to obtain smooth output motions, that is to say motions without a jump in the angular velocity of the wheel, the driving pin of the wheel may only enter or leave a slot if the tangent to the curve is directed on the center $M_0$ (See again the figures 3 and 4). This fact relates some dimensions of the mechanism. In order to investigate this relationship, we shall introduce the design-position of the mechanism, which is the position where the driving pin just enters or just leaves the Geneva wheel. In the figures 3 and 4 for instance, such design positions are drawn.

As in the design-position the tangent to the curve at $C$ is directed on the center $M_0$, the point $C$ of such a position has to join a locus that resembles the circle with diameter $M_0 \cdot P$ (Point $P$ being the velocity-pole of $\pi_m$ with respect to $\pi_f$ in the design-position). Generally, therefore, the mechanism considered allows for two degrees of freedom in design, viz. the choice of the point $C$ on the mentioned locus in addition to the chosen ratio $R/R_0$ of the polodes. Thus, in comparison to the Geneva wheel that is driven by a single crank, we have obtained an additional degree of freedom in design. This fact actually gives us the freedom to choose the ratio of times $v_0$ and the number of stations $n$, independently from one another.
We now define $\gamma$ as the angle enclosing two wheel-diameters touching a lobe of the curve on both sides. We further confine ourselves to angles $\gamma$ for which

$$\gamma = \frac{2\pi}{n} \quad n = 4, 5, \ldots$$

(6)

where $n$ resembles a positive integer.

In addition, we define the angle $\alpha$ as the angle needed for the input-axis to turn from the position in which the driving pin just enters the wheel to the position in which the pin is at the point of leaving the wheel.

For the hypocycloid motion this angle $\alpha$ may be calculated as follows: Since in the design-position, at $C$, the curve-tangent joins $M_0$, the tangent $M_0C$ must be perpendicular to the path normal $PC$. And so

$$M_0C = R_0 \cdot \cos \frac{1}{2} (\alpha + \gamma). \quad \text{(See figure 5)}$$

(7)

According to the rule of Sines for $\Delta M_0MC$, we additionally have

$$MC \cdot \sin \frac{1}{2} \beta = M_0C \cdot \sin \frac{1}{2} (\alpha + \gamma)$$

(8)

in which $\frac{1}{2} \beta \cdot R = \frac{1}{2} \alpha \cdot R_0$

Thus

$$\beta = k_2$$

(9)
Further

\[ R_o - R = M_o M = M_o C \cdot \cos \frac{1}{2} (\alpha + \gamma) - M C \cdot \cos \frac{1}{2} \beta. \]  \hspace{1cm} (10)

From eq. (7), (8) and (9) we so derive that

\[ \frac{M C}{R_o} = \frac{\sin (\alpha + \gamma)}{2 \sin \frac{1}{2} k \alpha}. \]  \hspace{1cm} (11)

Similarly, if we combine eqs. (7), (9) and (10) we find

\[ \frac{M C}{R_o} = \frac{k^{-1} - \sin^2 \frac{1}{2} (\alpha + \gamma)}{\cos \frac{1}{2} k \alpha}. \]  \hspace{1cm} (12)

Thus, equating the right-hand sides of the last two equations, we get

\[ \sin(\alpha + \gamma) = 2 \left\{ k^{-1} - \sin^2 \frac{1}{2} (\alpha + \gamma) \right\} \cdot \tan \frac{1}{2} k \alpha, \]

whence we find, after some calculation

\[ (2k^{-1} - 1) \sin \frac{1}{2} k \alpha - \sin \left\{ \alpha \left(1 - \frac{1}{2} k \right) + \gamma \right\} = 0 \]  \hspace{1cm} (13)

This formula is derived only for the hypo-cycloid motion for which \( k > 2 \). The derived formula determines the value \( \alpha \) if the gear-ratio \( k \) and the number of stations \( n \) are known. However, since in the design position the coupler-point \( C \) joins the circle with diameter \( P M_o \), only those values for \( \alpha \) are permissible for which \( \frac{\alpha + \gamma}{2} \leq \frac{\pi}{2} \). Thus any value for \( \alpha \) that is derived through eq. (13) has to meet the condition \( \alpha \leq \pi - \gamma \). In case the derived \( \alpha \)-values do not meet this condition no mechanism corresponds to the given gear-ratio and given number of stations.

Between the curve-cognate and the source mechanism there exist the relations

\[ \omega = \frac{d\alpha'}{dt} \text{ and } \omega' = \frac{d\alpha}{dt} \]  \hspace{1cm} (14)

So, with eq. (3) and (4) we arrive at the relation

\[ \alpha' = \alpha (1 - k) \text{ or } k' \alpha' = - k \alpha. \]  \hspace{1cm} (15)

Substituting, therefore, the expressions for \( \alpha \) and \( k \) from

\[ \alpha = \alpha' (1 - k') \text{ and } k = k' / (k' - 1) \]

into (13), we find that the left-hand side of eq. (13) remains invariable. Thus, eq. (13) is valid also for the curve-cognate mechanism.
Hence, eq. (13) is valid for all the hypo-cycloid driven Geneva wheel mechanisms. That is to say: eq. (13) holds true for $k > 1$.

For negative values of $k$ such as for the epicycloid driven Geneva wheel mechanisms, we similarly arrive at eq. (13).

Since, in addition, the equation remains unchanged if we transform the driving mechanism into its curve-cognate, eq. (13) must be valid also for the peri-cycloid driven Geneva wheel mechanisms. Thus, finally, eq. (13) holds true for any value of $k$.

5. THE RATIO OF TIMES $v_0$

According to its definition the ratio of times $v_0$, answers the equation

$$v_0 = \frac{\beta}{2\pi},$$

whence, according to eq. (9),

$$v_0 = k\alpha/2\pi$$

(16)

Thus, according to eq. (15), we see that

$$v_0 = -v_0$$

(17)

For the designer, this means that, apart from the sign, the two curve-cognates always spend the same time for the motion period in relation to the time needed for the full cycle.

Eliminating $\alpha$ from the eq. (13) and (16) we obtain eq.

$$(2k^{-1} - 1) \sin \pi v_0 - \sin \left\{ \pi v_0(2k^{-1} - 1) + \frac{2\pi}{n} \right\} = 0$$

(18)

which is still valid for all values of $k = R_o/R$, providing that eq. (16) holds.

The equation shows that unlike as for the single crank driven Geneva wheel, the ratio $v_0$ is not dependent on the number of stations alone, but may be varied instead by choosing other values for $k$. For the designer, this is very practical.

6. THE GEAR-RATIO AND THE ACTUAL NUMBER OF KNOTS

Generally, however, not all the real values of $k$ are allowed. They are restricted to rational numbers only. In order to give the reader more insight in this respect, we shall define a new number $m$ by making $m$ equal to the number of lobes that could be placed between two successive lobes of the curve. Thus, $m$ equals the number of unreal lobes that just fit between two successive, real ones, that actually appear in the curve.

If $m$ is a rational positive number, instead of a positive integer, such as $m_1/m_2$, we say that $m_1$ unreal lobes just fit between the first and the $(m_2 + 1)$th lobe that are really there.
So, we define $m$ by
\[ m = \frac{\text{max. possible number of lobes that would fit for any number of cycles minus the actual number of lobes that appear for that number of cycles}}{\text{actual number of lobes that appear for those cycles}}. \]

Thus, if the curve needs $n_1$ cycles to repeat itself, the maximum number of lobes that would fit for $n_1$ cycles equals $n_1 \cdot n$. Therefore, the actual number* of lobes $s$ that appear in the curve equals
\[ s = \frac{n_1 \cdot n}{m+1}. \]

For the protracted hypocycloid we so find that**
\[ \frac{1}{k} = \left(\frac{n_1}{s}\right) = \frac{m+1}{n} (k > 2). \]

For the contracted hypocycloid we then have
\[ \frac{1}{k'} = 1 - \frac{m+1}{n} (1 < k' < 2). \]

Similarly, we find for the epi-cycloid the relationship
\[ \frac{1}{k} = -\frac{m+1}{n} (k < 0). \]

And for the peri-cycloid
\[ \frac{1}{k'} = 1 + \frac{m+1}{n} (0 < k < 1). \]

These equations agree with the fact that either
\[ 2\pi R = \pm (m + 1) R_0, \]

or
\[ 2\pi R' = \{2\pi - (m + 1)\} R_0'. \]

So, if we choose the values $m$ and $n$, in addition to the kind of curve we are going to apply, the gear-ratio is fixed. This can be done using the equation (20) that corresponds to our choice of curve or mechanism.

7. PRACTICAL INDICATIONS

Clearly, we may confine ourselves to the protracted hypocycloid and to the epi-cycloid driven Geneva wheel mechanisms. If necessary, we can always apply the cognate transformation and use the curve-cog-

* Numbers $n_1$ and $s$ are positive integers only
** For each revolution of $\pi m$ a knot is produced. Thus $s(2\pi R) = n_1 \cdot (2\pi R_0)$. 
nates instead of the one mentioned. The dimensions for the curve-cognate mechanisms are then easily derived from the source mechanisms through the cognate transition formulas already given in this paper. So, for briefness’ sake we shall only refer to equation (20) if it is written in the form

\[ k = \pm \frac{n}{m + 1}. \]  

If we substitute this value into eq. (18) we arrive at the relation

\[ \left( \pm 2 \frac{m + 1}{n} - 1 \right) \sin \pi v_0 - \sin \left\{ \pi v_0 \left( \pm 2 \frac{m + 1}{n} - 1 \right) + \frac{2\pi}{n} \right\} = 0. \]

(22)

For each integer \( n \) and rational number \( m \) it is then possible to calculate the ratio \( v_0 \). From graphs that are made that way, we choose the practical values \( v_0 \) and \( n \) and then determine the number \( m \) from which we calculate the gear-ratio, using eq. (20).

We then determine the values for \( \alpha \) and \( \gamma \), according to the equations (16) and (6) respectively.

The remaining dimensions, such as \( M_0C/R_0 \) and \( MC/R \) are finally calculated through relations

\[ \frac{M_0C}{R_0} = \cos \frac{1}{2} (\alpha + \gamma), \quad (k < 0 \text{ or } k > 2) \]  

(7)

and

\[ \frac{MC}{R} = \frac{k}{2} \cdot \frac{\sin (\alpha + \gamma)}{\sin \frac{1}{2} k \alpha}, \quad (k < 0 \text{ or } k > 2), \]  

(23)

the last one being derived through equations (2) and (11). The graphs we have been referring to just now are demonstrated herewith under the numbers 1 and 2. The dots in these graphs represent mechanisms having only positive integer values for \( m \), including the number zero. (Later, we will show that also rational numbers of \( m \) are allowed as soon as we allow lobes to be unused.)

If the lobes that appear in the curve are all used to drive the wheel \( v = v_0 = k\alpha/2\pi \) otherwise \( v \neq v_0 = k\alpha/2\pi \). But even if we use them all, the designer of this kind of intermittent motion mechanism is still left with a large number of values \( v \) that are equal or less than one.

Examples are given in figures 6 to 15.

8. THE MAXIMUM NUMBER OF SLOTS \( g_0 \) ON THE WHEEL THAT COULD BE USED

The number of slots or grooves \( g_0 \) that have to be made in the wheel, does not necessarily has to be identical to the number of stations \( n \) of the mechanism.
Fig. 6. — Epicycloidal-driven Geneva wheel.

Fig. 7. — Epicycloidal-driven Geneva wheel.
Fig. 8. — Epicycloidal-driven Geneva wheel.

Fig. 9. — Hypocycloidal-driven slot with instantaneous dwell in the output-motion.
Fig. 10. — Hypocycloidal-driven Geneva wheel.

Fig. 11. — Epicycloidal-driven Geneva wheel.
**Fig. 12.** Epicycloidal-driven Geneva wheel.

- \( n = 12 \)
- \( m = 7 \)
- \( k = -\frac{1}{2} \)
- \( g = g_0 = 4 \)
- \( \alpha = 33^\circ 50' \)
- \( \psi = 0.141 \)
- \( M_C = 1.049 R_0 \)
- \( M_{G_C} = 0.849 R_0 \)

**Fig. 13.** Epicycloidal-driven Geneva wheel.

- \( n = 18 \)
- \( m = 5 \)
- \( k = -3 \)
- \( g = g_0 = 18 \)
- \( \alpha = 24^\circ 11' \)
- \( \psi = 0.21 \)
- \( M_C = 0.589 R_0 \)
- \( M_{G_C} = 0.927 R_0 \)
Fig. 14. — Epicycloidal-driven Geneva wheel.

Fig. 15. — Epicycloidal-driven Geneva wheel.
Clearly, the number of slots needed equals either \( n, \frac{n}{2}, \frac{n}{3}, \frac{n}{4}, \ldots \), or 1. Which number it actually is, is decided by the next reasoning: If the driving pin leaves a slot, it enters the next one just \((m + 2) \frac{2\pi}{n}\) radians further on the wheel. So, on the wheel, that is to say for \(2\pi\) radians, there are at least \(\frac{n}{m + 2}\) slots. If \(\frac{n}{m + 2}\) is a positive integer, \(g_0 = \frac{n}{m + 2}\).

If it is not, we have to multiply it with the smallest possible positive integer so as to make it one. Thus

\[
g_0 = \frac{n}{(\text{greatest common divisor of } n \text{ and } m + 2)} \tag{24}
\]

(For applications: see the given examples in figures 6 to 15).

**REDUCTION OF \(v\)**

In order to reduce the value of \(v\), we may diminish the number of slots. The lower values of \(v\), obtained in this way, are sometimes very practical, since they represent the circumstances in which only a small portion of time is needed for the actual motion of the wheel. Naturally, if there are fewer slots, the locking time of the wheel will be enlarged and more time is available for completion of products for instance that are moving around with the wheel.

How to find the number of slots in those cases will be explained through the next reasoning:

If the driving pin leaves a slot, it may find the next one \((m + 2) \frac{2\pi}{n}\) rad. further on the wheel. However, if no slot is available at that position, it may still find another one \((m + 1) \frac{2\pi}{n}\) rad. then further on the wheel. Again, if no slot is present at that position, we may find the next one \((m + 1) \frac{2\pi}{n}\) radians further on the wheel, and so on.

Therefore, the slots that are used subsequently are either \((m + 2) \frac{2\pi}{n}\) rad., \((2m + 3) \frac{2\pi}{n}\) rad., or \((3m + 4) \frac{2\pi}{n}\) rad., etc. . . set apart on the wheel. As before, we find that the number of slots \(g\), that are engraved in the wheel, has to meet the equation:

\[
g = n / g_{cd}(n, m + 2), \text{ or } g = n / g_{cd}(n, 2m + 3), \text{ or } g = n / g_{cd}(n, 3m + 4) \text{ etc.} \tag{25}
\]

where \(g_{cd}\) resembles the greatest common divisor of two positive integers, one of them being \(n\), which is the number of stations. Which integer the other one has to be depends on the number of lobes that are unused.
in the mechanism. For instance, if 3 lobes are unused \( g = n/g_0 \) \((n, 4m+5)\). As an example, we observe the case for which \( n = 8 \); \( m = 0 \) and \( k = 8 \). (See figure 16.) If all lobes are used then to drive the wheel intermittently, we find that \( g_0 = 8/2 = 4 \). Thus, the maximum number of slots \( g_0 \) equals 4, as is demonstrated in the figure. In addition \( \nu = \nu_0 - k\pi/2\pi = 0.65 \) \( (\nu_0 \) then resembles the maximum ratio if the maximum number of slots \( (g_0) \) is applied). If we want to diminish the number of slots, it is necessary to skip at least two lobes. Then, \( g = n/g_0 \) \((n, 3m + 4) = 2/4 = 2 \) slots. Such a possibility is demonstrated in figure 17. Then \( \nu = \frac{1}{3} \nu_0 = 0.22 \) since now the cyclic time is three times as large in comparison to the cyclic time needed if \( g_0 \) slots were in use.

If more lobes are skipped, the slot number decreases even further. For instance, if we skip 6 lobes, then \( g = n/g_0 \) \((n, 7m + 8) = 8/8 = 1 \) slot in such a case. This is demonstrated in figure 18. Here, \( \nu = \frac{1}{7} \nu_0 = 0.094 \).

**Remarks:** Contrary to what is demonstrated with the last figures, it is not in all cases possible to create redundant lobes. For instance, if we consider the case where \( n = 8 \), \( m = 1 \), as shown in figure 19, \( g = g_0 = 8 \). It is then not possible to reduce the number of slots, unless we allow the subsequent dwell periods to be unequal \( (3) \). Therefore, we are not looking for cases where this may or may not be done for a given curve, but we are looking for curves instead that may contain redundant lobes.

If we assume redundant lobes, generally, the \( \nu \)-value that corresponds to the mechanism may be obtained from the equation:

\[
\nu = \frac{\beta}{2\pi(1 + u_k)} \tag{26}
\]

in which \( u_k \) resembles the number of unused or redundant lobes that lie between two successively used ones. Therefore,

\[
\nu = \frac{\nu_0}{(1 + u_k)} \tag{27}
\]

in which \( \nu_0 = \beta/2\pi \) resembles the maximum ratio of times moving/cycling. This number is actually used as the \( y \)-coordinate in the graphs 1 and 2. So far, we have only displayed examples in which \( m \) represents an integer. However, as soon as we allow lobes to be unused this is no longer necessary.

For instance, if each time 1 lobe is unused as it is shown in the examples demonstrated in the figures 20 and 21, only \( 2m \) has to be an integer. This follows from the fact that for two successively used lobes \( (2m + 1) \) lobes could be thought to lie in-between. Therefore, if each time 1 lobe is unused, the quantity \( m \) may obtain the values \( m = 0, \frac{1}{2}, \frac{3}{2}, 2, 2\frac{1}{2}, \ldots \), etc.
Fig. 16. — Hypocycloidal-driven Geneva wheel.

Fig. 17. — Hypocycloidal-driven Geneva wheel.
Fig. 18. — Hypocycloidal-driven Geneva wheel.

Fig. 19. — Hypocycloidal-driven Geneva wheel.
Figures 20 and 21 show examples in which $m = 3/2$ and $m = 1/2$ respectively. Since in both cases, the number of slots $g$ answers the equation $g = n/g_{cd} (n, 2m + 3)$, one finds that

$g = 1$ if $n = 6, m = 3/2$, and similarly,

$g = 2$ if $n = 8, m = 2/1/2$

So the number of slots is very much reduced for these examples.

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Graph 1. — Hypocycloidal-driven Geneva wheel mechanisms.

Graph 2. — Epicycloidal-driven Geneva wheel mechanisms.
Fig. 20. — Epicycloidal-driven slot.

Each time 1 lobe is skipped.

Fig. 21. — Hypocycloidal-driven Geneva wheel.

Each time 1 lobe is skipped.
In order to find out if a mechanism could be composed at all for given values of \( n \) and \( m \), we first of all observe the ratio \( n/(m + 1) \). Only if the ratio is larger than 2 we may assemble a hypocycloid mechanism as well as an epicycloid mechanism. In case the ratio is less than 2, only an epicycloidal mechanism (or his cognate) could be assembled.

To decide if a hypocycloid mechanism could be used in case \( n/(m + 1) > 2 \), we further have to look at graph no. 1, and find out if for the given data of \( n \) and \( m \), a value for \( v_0 \) or \( \alpha \) could be found (In this connection see also the remarks made just after formula (13)).

In case, for instance, \( n = 6 \), \( m = 3/2 \), eq. (13) gives rise only to values \( \alpha \) for which \( \alpha > \pi - \gamma \). So there are no permissible values for \( v_0 \) or \( \alpha \) in this case as could also be seen from graph no. 1. Thus in this case only an epicycloidal mechanism may be applied as is carried out in figure 20.

In case \( n = 8 \), \( m = 1/2 \) graph No. 1 indeed provides us with a permissible value for \( v_0 \) or \( \alpha \). Hence, a hypocyclloid as well as an epicycloid may be used in this case. Figure 21 demonstrates how it works out for the hypocycloid. Once the values for \( k \) and \( \alpha \) are derived, the remaining dimensions may then be calculated using the equations (2), (7), (11) and (27).

If each time two lobes are unused, as is shown with the examples demonstrated in figures 22, 23, 24 and 25, \( g = n/g_{cd} \) \((n, 3m + 4)\), and furthermore, the quantity \( m \) may obtain the values \( m = 0, \frac{1}{3}, \frac{2}{3}, 1, \frac{1}{5}, \frac{2}{5}, 2, \ldots \) etc.

The examples displayed in figures 22 to 25 show cases with only a few number of slots. Technically easy to manufacture are those mechanisms with only 1 slot. Figures 22 and 23 show examples of them.

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Each time 2 lobes are skipped

Fig. 22. — Epicycloidal-driven slot.

n = 7
m = 1
k = \frac{7}{2}
g = 1 + 0.5
\alpha = 85.4°
\theta = 0.28
MC = 0.673 R_o
M_o C = 0.368 R_o

Fig. 23. — Hypocycloidal-driven slot.

Each time 2 lobes are skipped
Fig. 24. — Hypocycloidal-driven Geneva wheel.

Each time 2 lobes are skipped.

Fig. 25. — Hypocycloidal-driven Geneva wheel.

Each time 2 lobes are skipped.