Watt-1 Linkages with Shunted Chebyshev-Dyads, Producing Symmetrical 6-Bar Curves

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Received for publication 10 September 1980

Abstract
The five "symmetry-conditions" to be met by a non-symmetrical Watt-1 linkage, producing a symmetrical curve, may be derived through a cognate-transfiguration of a symmetrically driven Peaucellier-Lipkin cell (see Figs. 6–8). A further derivation shows that such a curve may also be produced by a Watt-1 linkage containing a "kite-cell" that exists of a 4-bar kite with two isosceles similar triangles attached to adjacent sides of the kite. Such a kite-cell transforms a circle as well as a symmetrical 4-bar coupler curve into a symmetrical 6-bar Watt-1 curve of order 8 (see Figs. 9, 10).

1. General Introduction
Algebraic curves, produced by points of linkage mechanisms, may be classified through the type of chain or network involved in the mechanism, in addition to the location of the coupler-plane in the chain to which the tracing point is attached. From Grüber's formula we derive that planar and constrained linkage mechanisms, having no pin-slot connections or sliding pairs, comprise only an even number of links \(n\); thus \(n = 2, 4, 6, \ldots\). The order or degree of the curve, however, appears to increase progressively with the number of links. For instance, if \(n = 4\), the 4-bar coupler curve is of order 6, whereas for a 6-bar mechanism, depending on the type that is used, the order could be even 14, 16 or 18.

Notwithstanding this though, it is possible to diminish this order by considering only symmetric curves. Technical simplifications for the design of the mechanism in conjunction with an easier access to practical applications, usually go along with such a restriction.

It is not necessary, however, to restrict oneself to symmetric mechanisms: the mechanisms themselves, namely, do not have to be symmetric in order to produce symmetric curves. In order to obtain the capability to drive the mechanism, one even prefers non-symmetric mechanisms: otherwise, they may not have a turnable shaft to drive the mechanism with.

It is the purpose of this paper, therefore, to investigate the conditions for the dimensions of non-symmetric mechanisms in this case of type Watt-1, necessary to produce symmetric Watt-1 curves.

If we further restrict ourselves to the Watt-1 curves of lowest order, we may prove that they are derivable from a particular 8-bar mechanism, containing a Peaucellier cell. For abbreviation's sake, however, we follow the road in the opposite direction and start with Peaucellier's cell to come out, finally, with the 6-bar mechanism in which two isosceles triads (also named Chebyshev-dyads) are shunted parallel.

It appears that chain-networks of this type are able to produce symmetric curves of order eight; thus, only two degrees more than the order of a common coupler curve of a 4-bar linkage. The additional degree of freedom in design, caused by this increment in degrees of the curve, could possibly lead to applications in areas where a singular 4-bar linkage has failed thus far.

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2. Historical Introduction

It is known\[2\] that Chebyshev-dyads may be used to transform symmetrical curves into other ones of mostly higher order. A common example is one for which the circle as the input-curve transforms itself into a symmetrical (4-bar) coupler curve through a Chebyshev-dyad (see Fig. 1). Such a dyad consists of a bar that is hinged at the apex of an isosceles triangular link for which the uprising sides have the length of the bar (see Fig. 2). A necessary condition for the output-curve to be symmetrical is that the loose end of the bar, taken as a fixed center, should lie on the symmetry-axis of the input-curve. This condition is also clearly demonstrated in another, second example, for which the symmetrical 4-bar coupler curve in its turn is taken as the input-curve of another Chebyshev-dyad. In practice, this is realized by a series-connection of the dyads from the first and this second example (see Fig. 3). Then, build together, a 6-bar linkage of type Stephenson-3 is created, for which the output-curve identifies a symmetrical 6-bar curve\[3–5\].

A second condition for maintaining symmetry is, that the tracing point of the input-curve, the dyad-joint and the fixed center of the binary link of the (Chebyshev-)dyad, may never be aligned. If, namely, this condition is not met, symmetry of the curve will be lost, as demonstrated in the example of a non-Grashof 4-bar, for which all points of the coupler trace a complete coupler curve which is symmetrical with respect to the frame-line (see Fig. 4). By taking such a curve as the input-curve of a Chebyshev-dyad, the output-curve, indeed, will be non-symmetrical. (If a hinge at \(K\) is thought to be necessary the 4-bar should be replaced by one of the two curve cognates according to Roberts' Law.)

\[\text{Figure 1. Transformation of a circle into a symmetric 4-bar coupler curve.}\]

\[\text{Figure 2. Chebyshev-dyad.}\]
Figure 3. Transformation of a symmetric 4-bar coupler curve into a symmetric 6-bar curve, series connection of two Chebyshev-dyads, Stephenson-3 mechanism.

Figure 4. A Chebyshev-dyad connecting symmetric and non-symmetric 4-bar coupler curves.
Thus, by excluding stretched positions and subsequent running on of the Chebyshev-dyad(s), precisely five symmetry-conditions for the dimensions of a Stephenson-3 mechanism are necessary and sufficient to generate a symmetrical output-curve.

In case, like investigated here, two Chebyshev-dyads are shunted parallel, it will be shown, that then also five conditions for the 6-bar mechanism are necessary and sufficient to produce a symmetrical output curve (see Fig. 5). The creation of such a mechanism, that proves to be a Watt-I mechanism, is carried out by starting from a so-called, Peaucellier-cell[6, 7] which is normally used as a sub-chain for the composition of inversion mechanisms.

Symmetrical curves, produced by 6-bar mechanisms, may be generated, generally speaking, by different types, such as the Watt-1, the Watt-2 mechanism[8] and also the Stephenson-1, -2 and -3 mechanism[9]. In this paper, special attention will be given to the Watt-I mechanism, for which the coupler-plane contains the tracing point as well as the one point that traces a circle.

3. Peaucellier-Lipkin’s cell, symmetrically driven

Peaucellier-Lipkin’s cell, comprised of a rhombus and a kite, assembled as shown in Fig. 6, may be driven symmetrically using a Chebyshev-dyad (see again Fig. 6). Clearly, for this cell, the turning-joints $B_0$, $B^*$ and $E$ remain aligned, whereas also $EB^*$. $EB_0 = BE^2 - BB_0^2 = \text{constant}$.

Further, the turning-joint $B_0$, forever located on the symmetry-axis of the cell, should be taken as a fixed center on the frame, so that the cell may be driven symmetrically by moving joint $B^*$ along a symmetric curve. The latter may be realized, for instance, if we move a point $A$ of plane $BB^*$ along a circle. For then, point $B^*$ is forced to move along a 4-bar coupler curve, which indeed may be made symmetrical by taking $BA = BB_0 = BB^*$. By doing this, the axis of symmetry of the curve, will join the connecting line $B_0B^*$ as soon as the crank-pin $A$ joins the frame-line $A_0B_0$.

The configuration as a whole, represents an 8-bar mechanism, for which point $B^*$ and, hence, also point $E$, will describe symmetrical curves. For convenience sake, we will name the one produced by $E$, the kite-curve, as apparently always a kite is involved to produce this curve. We further state that the kite-curve is an algebraic curve of order 8. To prove this, we determine her number of intersections with the axis of symmetry of the curve.

Proof. If the 4-bar $(A_0ABB_0)$ has a rotatable crank $(A_0A)$, then the 4-bar coupler curve, traced by $B^*$, consists of two branches, each of them symmetrical with respect to the axis of

![Figure 5. Two Chebyshev-dyads shunted parallel; Kite-cell.](image-url)
symmetry. If further, \( A_0B_0 - AB^* < A_0A \), one of these branches will have the figure 8 with a double-point at \( B_0 \). (The other branch of the curve, then is the isogonal transformed one of the figure 8 with respect to the triangle of the double points of the curve). We also know, that the 4-bar coupler curve is of order 6. Thus, apart from the double-point at \( B_0 \), each branch has to intersect the symmetry-axis at two points, each of them giving rise to two points \( E \) of the kite-curve on the axis of symmetry. (The two points \( E \), each time being each others' reflected image with respect to the diagonal \( BB^* \) of the rhombus for that position.)

For a coupler-point \( B^* \) at \( B_0 \), the points \( B \) and \( B^* \) are each others' reflected image with respect to one of the two path-tangents at \( B_0 \), not with respect to the axis of symmetry. Therefore, the points \( E \) that correspond to a point \( B^* \) at \( B_0 \), do not join the axis of symmetry. We conclude,† that the kite-curve intersects the symmetry-axis at not more than \( 2 \times 2 \times 2 = 8 \) points, so that indeed the kite-curve is of order 8.

4. The Alternative Watt-1 Mechanism (coupler alternative mechanism for plane BE)

By application of *cognate* theory[10] it is possible to transform the 8-bar of Fig. 6 into a 6-bar mechanism of type Watt-1. Point \( E \) of the 6-bar then produces the same symmetric curve as produced by \( E \) of the initial 8-bar mechanism. One derives the 6-bar through successive stretch-rotations about the turning-joints \( B \) and \( B_0 \) (see Fig. 7). The instructions are successively:

(a) Choose a *random* joint \( B^* \) in the plane \( B_0B \),
(b) Stretch-rotate the 4-bar kite \( B_0BEB^* \) about \( B \) into \( B^*BD^*K^* \) (the stretch-rotation factor \( f_B = BB^*/B_0B \)),
(c) Form the rigid and similar triangles \( B_0BB^* \) and \( EBD^* \),
(d) Stretch-rotate \( A_0ABB_0 \) about \( B_0 \) into \( A^*A^*B^*B_0 \) (the stretch-rotation factor \( f_{B_0} = B_0B^*/B_0B \)),
(e) Form the rigid \( A^*B^*K^* \).

The resulting mechanism is a 6-bar of type Watt-1 that consists of a 4-bar \( A^*A^*B^*B_0 \) to which a dyad \( K^*D^*C^* \) with a coupler triangle \( EC^*D^* \) is connected. The derivation shows that the

†The conclusion is the same as the one obtained through an algebraic derivation, given by G. R. Veldkamp, emeritus-professor in kinematics and mathematics of the Eindhoven University of Technology.
Figure 7. Derivation of the alternative 6-bar mechanism.

alternative 6-bar mechanism, just obtained, produces the same eighth order and symmetric Watt-I curve as generated by the initial 8-bar mechanism.

The five symmetry-conditions, which are necessary for this Watt-1 mechanism, are obtained directly from this derivation. They are

1. $B^*A^* = B^*B$
2. $K^*B^* = C^*B^*$ with $C^* = B$
3. $D^*K^* = D^*C^*$
4 and 5. $\triangle EBD^* \sim \triangle B_0BB^*$.

In the design-position of the mechanism, point $A^*$ joins the frame-line $A^*_0B_0$, whereas the tracing-point $E$ joins the axis of symmetry. Instead of starting off from the Peaucellier-Lipkin cell, one may also determine the dimensions in a more direct way. The instructions to obtain the mechanism then are successively (see Fig. 8):

(a) Draw the 4-bar $A^*_0A^*B^*B_0$ with $A^*$ on $A^*_0B_0$ and $A^*B^* = B_0B^*$,
(b) Choose the turning-joints $K^*$ and $D^*$ randomly,
(c) Reflect $K^*$ with respect to $B^*D^*$ into the point $C^* = B$ and form the rigid triangles $A^*B^*K^*$ and $B_0BB^*$,
(d) Form the 4-bar kite $K^*B^*B_0D^*$ and make $\triangle EBD^* \sim \triangle B_0BB^*$.

By moving the mechanism, point $E$ will trace a symmetric curve of order eight. The axis of symmetry of the curve then coincides with the line connecting the points $B_0$ and $E$ in the design-position of the mechanism. Further,

\[
\hat{\angle} A^*B^*K^* = \hat{\angle} B_0BB^* - \hat{\angle} ABB^* + \hat{\angle} B^*B_0B = 2\pi - \hat{\angle} B^*B_0B - \hat{\angle} ABB^*
\]

\[
= 2\hat{\angle} BAB^* - \hat{\angle} BB^*B_0.
\]
Figure 8. Alternative 6-bar mechanism of type Watt-1. Transformation of a circle into a curve of eighth order.

hence, with

$$\angle AB_0E = \pi/2 + \angle BAB^*$$

it follows that

$$\angle A^*B_0E = \angle B^*B_0B + \pi/2 + \frac{1}{2} \angle A^*B^*K^* + \frac{1}{2} \angle BB^*B_0.$$  

This value remains the same, even if $A^*_\delta$ does not join the line $A^*B_0$.

During the motion, therefore, the straight-lines $B_0A^*$ and $B_0E$ enclose a constant angle of the value given at the right-hand side of the equation, given above. In the symmetry-position, the enclosed angle identifies $\angle A^*_\delta B_0E$, which is the angle enclosed between the axis of symmetry and the frame-line $A^*_\delta B_0$. (Note, that the point $S^*$ of intersection between the axis of symmetry and the diagonal $B^*D^*$, joins the circle circumscribed about the triangle $\Delta B_0BB^*$.)

Clearly, a new cell has now been created, having the property to transform a circle (traced by point $A^*$) into a symmetric curve of order $\delta$ (traced by the coupler-point $E$). This cell consists of a 4-bar kite $BB^*K^*D^*$ and three rigid triangles attached to these sides, of which triangles there are two similar ones and a third which as two prescribed sides. According to the design of the mechanism as given under (a) to (d), ten free coordinates are left to the designer to choose from. Generally, fifteen degrees of freedom in design determines a Watt-1 mechanism. So, indeed, 5 coordinates remain that have been used to impose on the mechanism. They, in fact, are the five symmetry-conditions mentioned above.
5. Three Chebyshev-dyads Enclosed in a 6-bar Mechanism (the kite-cell driven symmetrically)

From section 4 we have seen, that the design of the alternative Watt-I mechanism did allow the free choice of turning-joint \( B^* \). It is possible, therefore, to choose this joint somewhere on the perpendicular bisector of the bar \( B_0B \) (see Fig. 9). Such a particular choice leads to a number of simplifications and a comprehensible network. For instance,

\[
B^*K^* = B^*B = B^*B_0 = B^*A^*
\]

and, since \( \triangle EBD^* \sim \triangle B_0BB^* \) we now find that

\[
D^*E = D^*B = D^*K^*.
\]

The resulting mechanism, therefore, is a particular 6-bar mechanism that includes at least three Chebyshev-dyads. Nevertheless, the mechanism still produces the same motion for the coupler-plane \( BE \).

As further,

\[
\angle BK^*B_0 = \frac{1}{2} \angle BB^*B_0 = \frac{\pi}{2} - \angle B_0BB^* = \pi/2 - \angle B^*BK^*,
\]

clearly, \( K^*B_0 \) runs normal to \( B^*B \). Therefore, \( K^*, B_0 \) and \( E \) do join the same straight-line. Since the derivation is independent of the position of the mechanism, the statement appears to be true for any position. Thus, for the design-position, which is a symmetric position of the mechanism, the coupler-point \( K^* \) does join the axis of symmetry. This axis resembles the axis of symmetry for the 4-bar coupler curve, traced by \( K^* \), as well as for the Watt-I curve, traced by joint \( E \) of the mechanism.

The mechanism as drawn in Fig. 9, consists of a "kite-cell", that is hinged at a turning-joint \( A^* \) to a crank \( A_0^*A^* \) and also pivots on a fixed center \( B_0 \) of the frame. The kite-cell in itself

\[\text{Figure 9. A kite-cell with a symmetric 4-bar coupler curve as input-curve.}\]
consists of a 4-bar kite with three attached and isosceles triangles, such as $\triangle EBD^*$, $\triangle B_0 BB^*$ and $\triangle K^*A^*B^*$. (The first two are similar to one another.) From the figure, it also appears that

$$\angle A^*B_0 K^* = \frac{1}{2} \angle A^*B^*K^*.$$

In order to obtain a symmetrical output curve through $E$, any point $A^*$, joining a circle about $B^*$ with radius $B^*B_0$, may serve for that purpose.

As long as they run along a circle, such points $A^*$ always lead to symmetrical 4-bar coupler curves through $K^*$, and to symmetrical curves of order 8 through the point $E$.

6. Two Chebyshev-dyads, Shunted Parallel

A further simplification may be obtained, if, additionally, the distance $K^*A^*$ equals zero. Thus, $K^* = A^*$. If the complete motion of the dyad $B_0BE$ has to be preserved, the coincidence of the two joints only occurs by special choice of the turning-joint $B^*$, which location is to be found using a series of vector-equations:

$$0 = K^*A^* = K^*B^* + B^*A^* = f_b \cdot B^*B_0 + f_{b_0} \cdot BA = (B^*B/B_0B)B'B + (B_0B^*/B_0B)BA,$$

hence,

$$B_0B^*/BB^* = -B'B/AB,$$

whence,

$$\triangle BB^*B_0 \sim \triangle A'B'B^* \text{ if } A'B = -AB.$$

Figure 10. The kite-cell with a circle as input-curve (alternative 6-bar mechanism containing two shunted Chebyshev-dyads).
From this, it follows that $B^*$ joins the perpendicular bisector of the bar $B_0B$, such that 

$\angle BB^*B_0 = (\angle A'B'B^* - 2 \angle BAB^*) = 2 \angle BAB^*$. 

Indeed, if this is true, then the double joint $K^* = A^*$ does meet the axis of symmetry $B_0E$.

Starting from the initial mechanism containing Peaucellier’s cell, one so finds this particular alternative mechanism that produces the same curve through the point $E$. The design of the mechanism is demonstrated in Fig. 10 and obtained from the initial one through the following instructions:

(a) Determine the turning-joint $B^*$ on the perpendicular bisector of $B_0B$ such that 

$\angle BB^*B_0 = 2 \angle BAB^*$.

(b) Form the rigid and isosceles triangle $\Delta B_0BB^*$.

(c) Stretch-rotate $\square A_0ABB_0$ about $B_0$ into $\square A*B*B_0$.

(d) Turn $A_0$ into a fixed pivot on the frame,

(e) Stretch-rotate $\Delta B_0BB''$ about $B$ into $\square B*BD*K^*$ (Note, that as a result $K^* = A^*$).

(f) Form the rigid $\Delta EBD^* \sim \Delta B_0BB^*$.

The five symmetry-conditions for this mechanism are:

- $B^*A^* = B^*B_0 = B^*B$
- $D^*A^* = D^*B = D^*E$
- $\angle EBD^* \sim \Delta B_0BB^*$

Factualy, the kite-cell of this mechanism consists of two Chebyshev-dyads, which are shunted parallel. For this cell, the points $A^*$, $B_0$ and $E$ always remains on a straight-line. Moreover, if point $B_0$ of the kite-cell is turned into a fixed pivot on the frame, and if also point $A^* = K^*$ moves along a circle. then point $E$ will trace a symmetric curve of order 8. In this case, the axis of symmetry of the curve coincides at the frame-line $A_0B_0$. In addition, one could say, that the 6-bar mechanism, here derived, having a double turning-joint at the crank-pin $A^*$, does indeed generate the same motion for the dyad-linkage $B_0BE$ as this dyad-linkage does if it is a sub-chain of the initial 8-bar mechanism. Provided this dyad-motion is preserved, one could say also, that the Peaucellier-Lipkin cell has been transfigurated into a singular kite-cell, similarly containing three marked points that remain on a straight-line.

Turning one of these points $B_0$ or $E$ into a fixed pivot on the frame, then a symmetric input-curve through the third point $A^*$ will be transformed into another symmetric output curve of higher order through one of the points $E$ or $B_0$.

If point $B_0$ has been chosen as the fixed pivot on the frame, then the transformation of the input curve at $A^*$ into the output curve at $E$, may be decomposed by three successive transformations. In succession they are:

$$B_0A = (B_0B/B_0B^*) \cdot B_0A^*$$  \hspace{1cm} \text{(for the transformation of $A^*$ into $A$)}

$$B^*A^2 = B_0A^2 + B_0B^2 - 2 \cdot B_0A \cdot B_0B^* \sin \angle BAB^*$$  \hspace{1cm} \text{(for the transformation of point $A^*$ into point $B^*$)}

and, finally,

$$B_0E \cdot (B_0E - B_0B^*) = BE^2 - B_0B^2$$  \hspace{1cm} \text{(for the transformation of point $B^*$ into point $E$)}.$$

Thus, if point $A^*$ traces a circle, the exit-point $E$, finally, will trace an eighth order symmetric 6-bar curve.

**7. Approximate Straight-line Mechanism, Using a Kite-cell**

If, in the design-position of the mechanism, point $E$ coincides with an inflection point, then point $E$ has to be an undulation point or Ball’s point which point has a fourpoint-contact with its path-tangent. This immediately follows from the fact, that the curve produced by $E$, is a symmetric curve. Then, an odd number of coincident points between path and path-tangent on the axis of symmetry, is not allowed. The actual design of the mechanism, having $E = U$ for undulation-point, is given as follows (see Fig. 11).
Figure 11. Approximate straight-line 6-bar mechanism with kite-cell.

(a) Draw the 4-bar kite $ABCD$.

(b) Choose the fixed pivot on the frame $B_0$ somewhere in the plane attached to the bar $BC$.

(Note, that the transmission-angles $ADC$ and $ABB_0$ may not be smaller than $30^\circ$ or larger than $150^\circ$.)

(c) Intersect the straight-line $AB_0$ and the circle above $D$ having $DC$ for radius, at the undulation-point $E = E_u = U$.

(d) Draw the inflection circle $ic_{40}$ of plane $CDE$, joining $E_u$ and touching the pole-tangent $p_{40} = B_0C$ at $B_0$.

(e) Determine the point of intersection $P_{42} = F = (BC \times AD)$.

(f) Intersect the straight-line $FB_0$ and the inflection circle $ic_{40}$ at the point $F_u \neq B_0$.

(g) Draw the inflection circle $ic_{20}$ of plane $AB$; this circle has to join point $F_u$ and to touch the pole-tangent $p_{20} = B_0B$ at the point $B_0$.

(h) Intersect the inflection circle $ic_{30}$ and the straight-line $AB_0$ at the point $A_u \neq B_0$.

(i) Determine the location of the fixed pivot on the frame $A_0$ with the Euler-Savary-formula $P_{30}A_0^2 = AA_u \cdot AA_0$ in which $B_0 = P_{30} = P_{30} = P_{40} = P_{30}$.

(j) Even in the 2nd symmetry-position, the afore mentioned transmission-angles $ADC$ and $ABB_0$ have to lie between the values $30^\circ$ and $150^\circ$. This may be attained by a change of dimensions as chosen initially under the assignments (a) and (b). A repeated calculation of the location of point $A_0$ eventually leads to the right dimensions of the mechanism, we are looking for.

Explanation. For the relative motion of $CD$ with respect to $AB$, both, the Coriolis-acceleration as well as the relative acceleration vanish at the relative pole $F = P_{42}$. This is an immediate consequence of the fact that the relative velocity at the pole $F$ as well as the relative angular velocity, are zero in the design position. Hence, the vehicular—as well as the absolute acceleration are equal at $F$; which means that the points $F$ of the planes $AB$ and $CD$ have the same acceleration and so also have the same radii of curvature. Then, because of Euler-
Savary's formula, pole-ray $FB_0$ will intersect the two inflection-circles $ic_2$ and $ic_4$ at one and the same point $F_w$.

As a result, a crank-length $A_6A$ has been found such that the 4-bar $A_6ABB_0$ appears to be a double-crank mechanism (See Fig. 11). The symmetric path, traced by $E$, is a circle alike, although it is flattened out in the neighbouring of the symmetric position, which is the design position of the mechanism.

**Remark.** For an eventual 6-point contact at Ball's point, point $C$ has to be the center of a circle, joining the points $B_0$ and $U$; which circle is a branch of the circling-point curve for that position. Then, however, $CB = CE$, so that $CB = CD$ and $ABCD$ identifies a rhombus. In that case, plane $CDE$ is nothing else than a coupler-plane of common 4-bar linkage, for which the adjoining of a linkage dyad is no longer necessary.

**Conclusion**

The five "symmetry conditions" to be met by 6-bar linkages producing symmetrical curves, may be derived through a transfiguration of symmetrical source mechanisms. For a Watt-I 6-bar, the source mechanism will be a symmetrically driven Peaucellier-Lipkin cell. The 6-bar derived from the source mechanism then produces the same symmetric curve like the one produced by the kite-apex of the mentioned cell. As a consequence of the symmetry, the order of such Watt-I 6-bar curves will be reduced from 14 to 8.

A further derivation shows that such 8th order curves may also be produced by 6-bar linkages with two shunted Chebyshev-dyads. The "kite-cell" therein contained, exists of a 4-bar kite with two isosceles similar triangles attached to adjacent sides of the kite. The property of the kite-cell being that the apices of the attached triangles remain aligned with the fourth joint of the kite which is not a vertex of the two triangles. The "kite-cell", as it is proved, transforms a circle as well as a symmetrical 4-bar coupler curve into a symmetrical 6-bar Watt-I curve of order 8.

One may turn the mechanism into an approximate straight-line mechanism if the tracing-point coincides with Ball's point. This is carried out by adjusting the length of the input-crank of the 6-bar.

**References**


**Symmetrische, sechsgliedrige Kurven erzeugt von Watt-1 Gelenkgetrieben mit zwei parallelgeschalteten Tschebyschev-Zweischlüssen**

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**Kurzfassung** Für ein unsymmetrisches sechsgliedriges Gelenkgetriebe, das eine symmetrische (höhere) Koppelkurve produzieren kann, sind
Für sogenannte "Symmetriebedingungen" notwendig. Solche Bedingungen sind abgeleitet worden durch Umwandlung eines symmetrischen Ausgangsgetriebes, wofür offenbar eine symmetrisch angetriebene Peaucellier-Lipin-Zelle geeignet ist. Beide Getriebe, d.h. das Ausgangsgetriebe und das sechsgliedrige Gelenkgetriebe, erzeugen dieselbe symmetrische Kurve wie ursprünglich vom Scheitelpunkt eines Drachenvierecks der genannten Zelle produziert worden ist. (Figuren 6, 7 und 8)

(In Folge der Symmetrie wird auch der Grad dieser sechsgliedrigen Watt-1 Kurven reduziert, und zwar vom 14. bis auf 8. Grad.)

Eine weitere Vereinfachung zeigt, dass solche Kurven achten Grades ebenfalls von sechsgliedrigen Gelenkgetrieben mit zwei parallelgeschalteten Tchebyschev-Zweischlägen produziert werden können. (Figur 10) Die dabei in einer solchen Gelenkgetriebe erhaltene "Drachenzelle", besteht aus ein Drachengelenkviereck mit zwei gleichschenkligen und ähnlichen Dreiecken, erweitert von zwei aufeinander folgenden und ungleichen Seiten des Drachenvierecks. (Offenbar hat die Drachenzelle die Eigenschaft, dass die zwei Scheitelpunkte dieser Dreiecken auf einer Gerade mit dem, nicht unmittelbar mit Dreiecken verbundenen vierten, Dreipunkt des Drachenvierecks bleiben.)

Wie auch bewiesen ist, transformiert eine solche "Drachenzelle" nicht nur einen Kreis, sondern auch eine symmetrische Viergelenk-Koppelkurve in einer ebenfalls symmetrischen sechsgliedrigen Watt-1 Kurve achten Grades. (Figur 9)

Eine Anwendung des neuen Getriebes wurde durch Wahl des Ballischen Punktes im geführten Punkt dieses Getriebes gefunden, was durch Anpassung der Länge des Antriebskurbels des sechsgliedrigen Gelenkgetriebes erreicht wird.