TECHNICAL NOTE

A NUMERICAL-EXPERIMENTAL METHOD FOR A MECHANICAL CHARACTERIZATION OF BIOLOGICAL MATERIALS

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Abstract—For the determination of material parameters, it is a common practice to extract specimens with well-defined geometries. The design of the samples and the choice of the applied load are meant to lead to a homogeneous stress and strain distribution in a part of the sample. When applied to biological materials, this raises a number of problems: homogeneous strains cannot be obtained because the materials have inhomogeneous properties, and the manufacturing of samples is hard or sometimes impossible. In this technical note a different approach is presented, based on the use of a digital image technique for the measurement of nonhomogeneous strain distributions, finite element modeling and the use of a minimum-variance estimator. The method is tested by means of experiments on an orthotropic elastic membrane of a woven and calendered textile. Five parameters are identified using the experimental data of one single experiment.

INTRODUCTION

Traditional ways for a quantitative determination of the material parameters have some features in common that lead to insoluble difficulties when applied to biological materials. A closer look at the familiar uniaxial strain test will make this clear. Specimens with a well-determined shape are manufactured under the assumption that they are representative for the mechanical properties of the material. The design of the samples and the choice of the applied load are meant to lead to a homogeneous strain distribution in the central region of the sample. Due to the homogeneous strain distribution, a fairly large area can be used to measure the displacements and, indirectly, the strain in the central region. Another key element in such experiments is the hypothesis of a homogeneous stress distribution, which enables the determination of the stress in the central region by equilibrium considerations. The development of constitutive theories for biological materials requires a re-examination of this kind of testing. Peters (1987) demonstrated that special care must be taken to see that the desired information is obtained. Figure 1 shows the positive principal strains in a collagenous connective tissue specimen measured in a uniaxial strain test. Clearly, the strains are far from homogeneous. Peters also showed that averaging the strains to obtain averaged properties is not worthwhile. Because of the large difference in stiffness between fibers and matrix, it appeared that inhomogeneous boundary conditions due to clamping affect the strains in the whole tissue. St Venant's principle is not valid for these types of materials. Moreover, fibers are not unidirectional. By disrupting the structure, due to cutting fibers in the manufacturing of the samples, only part of the fibers are loaded in a uniaxial strain test.

The problems with tensile testing for complex materials also apply to other common mechanical tests, such as circular rods in torsion, beams in bending and some biaxial tests. These will be referred to as 'traditional tests'.

A different method will be proposed that is not based on homogeneous stress and strain fields, but uses inhomogeneous strain distributions in test specimens. Such an

Fig. 1. Measured strain distribution on a specimen obtained from the overlying fascia of the extensor digitorum muscle (Peters, 1987).

approach creates more freedom for experiments and offers a way out of the above-mentioned problems with biological tissues.

Early attempts for a comparable approach have been published by Kavanagh and Kavanagh and Clough (1971) (1972). Their method was based on combining a numerical analyzing method with experiments on statically undeterminate structures. Yettram and Vinson (1979) tried to use this technique for the determination of orthotropic, elastic moduli for the left ventricle, but were hampered by large model errors and large observation errors. Liu and Ray (1978) and Lin *et al.* (1978) minimized a criterion based on the sum of squares of the differences between the calculated and the measured displacements to obtain material properties of an intervertebral joint. The method had limited success, because the data did not contain enough information to identify all parameters.

Common interest of the papers mentioned above is to accomplish a less tight framework than is used in traditional testing. Consequently, more observational data are required and it is necessary to perform a numerical analysis. The basic ideas are appealing, but some of the authors immediately applied their method to rather complex problems. Their efforts were hampered by model errors, observation errors and the amount of computer time and memory they needed. Furthermore, it is remarkable that, in contrast to the typical numerical-experimental nature of the subject, a majority of the authors present numerical experiments only. This may explain why a number of investigators stopped further developments.

The numerical-experimental approach described in the next section can, in principle, be used for nonlinear, visco-elastic materials. Although the method has the potential for a broad application area, in this paper only experiments on a linear elastic material will be described. These experiments are used as tests of the method in practical situations and are considered necessary before more complex materials can be approached. For this test an orthotropic membrane of a woven textile, with a low stiffness ratio in the principal directions, was used.

METHOD

It is assumed that a sufficiently accurate constitutive model is available. The problem is to determine quantitatively the material parameters in these constitutive equations. Of course, results from experiments may lead to adjustments of the constitutive model, but this will not be discussed here. There is one important difference between this method and the traditional methods. It is no longer necessary that the strain field is homogeneous in some part of the loaded specimen under investigation. On the contrary, it is preferable that the strain field is inhomogeneous. Arguments for this are:

- Inhomogeneous strain fields contain more information about the material properties of some specimen than a homogeneous strain field does. This opens the way to a much more effective determination of properties than is possible with traditional tests.

- When inhomogeneous strains are allowed, extra freedom arises for the design of experiments with optimized performance. In the long run it may even be possible to consider *in vivo* tests.

Although the use of inhomogeneous strain fields opens up new perspectives, it raises three new problems:

- The inhomogeneous strain field has to be measured and it is necessary to apply loads in a more general way than in traditional testing. In the experiment described in this paper small markers are attached to the surface of the specimen. The displacements of these markers are measured optically by a video tracking system (Video interface 84.330, Hentschel GmbH, Hannover, Germany).

- The analysis of the experimental setup becomes more complex and can be done only in a numerical way with a computer. This, however, can only be performed for a given set of the (unknown) parameters. Therefore, initial values have to be available, which can be improved, using an iterative procedure. Thus, it is necessary to use some finite element code as a subroutine. For this DIANA (Borst *et al.*, 1985) was used.

- A method has to be developed to confront the experiments with the (numerical) analysis and to determine the unknown material parameters from the data. Because most readers will be unfamiliar with these methods, we will extend a bit on this subject below.

Assume that the experimental data consist of a finite sets of columns $\mathbf{y}_k$, with $k=1, \dots, N$. The index k indicates a load case of the experiment under investigation. Each column $\mathbf{y}_k$ can represent a displacement field at some load increment, but also (for example, in case of visco-elastic materials) a displacement field at some point in time. Column $\mathbf{y}_k$ can also contain other measurable properties, like forces, velocities and pressures. The quantitative behavior of the material is represented by a finite set of unknown quantities x_i , with $i=1, \dots, n$. These parameters define a column $\mathbf{x}$ of unknown material parameters and are determined by the type of constitutive model that is chosen. Column $\mathbf{x}$ will be called the 'parameter column' and contains, for example, Young's moduli, time constants, Poisson's ratios or a nonlinear function of these material properties. It is assumed that some algorithm is available to calculate $\mathbf{y}_k$ when $\mathbf{x}$ is known. This algorithm, based on the finite element method, is symbolized by a (highly nonlinear) function $\mathbf{h}_k(\mathbf{x})$. Function $\mathbf{h}_k(\mathbf{x})$ describes the dependence of the k th observation on $\mathbf{x}$ if there were no observation errors. These errors will be presented by a column $\mathbf{v}_k$. The model equation can be written as

$$\mathbf{y}_k = \mathbf{h}_k(\mathbf{x}) + \mathbf{v}_k. \quad (1)$$

The problem is to find $\mathbf{x}$ for given $\mathbf{y}_k$ and for given statistics of $\mathbf{v}_k$. Many papers, usually applied to dynamical systems, are devoted to this estimation problem. An excellent review can be found in Norton (1986). Since the objective of the present paper is not to develop new estimation algorithms but to apply an existing algorithm in a different application field with its own problems, we only give the line of approach.

Because of the enormous amount of data available from the strain distribution measurements (in case of time-dependent problems), a sequential algorithm is chosen. This means that, instead of using all available experimental data at once, at each new iteration step, new experimental data are used to update the current estimation. Thus, at some step in the process, an estimate $\hat{\mathbf{x}}_k$ for the parameters is available, based on the experimental data $\{\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_k\}$. A new estimate $\hat{\mathbf{x}}_{k+1}$ can be found by means of the following equation (Norton, 1986; Hendriks, 1991):

$$\hat{\mathbf{x}}_{k+1} = \hat{\mathbf{x}}_k + \mathbf{K}_{k+1} \{\mathbf{y}_{k+1} - \mathbf{h}_{k+1}(\hat{\mathbf{x}}_k)\}. \quad (2)$$

Column $\mathbf{y}_{k+1}$ represents the newly obtained observations (for example, a measured strain distribution at another load). Column $\mathbf{h}_{k+1}(\hat{\mathbf{x}}_k)$ is the model prediction on the basis of the last estimate $\hat{\mathbf{x}}_k$ for the material parameters. The difference $\mathbf{y}_{k+1} - \mathbf{h}_{k+1}(\hat{\mathbf{x}}_k)$ represents the new information and is used in a weighted sense to correct the old estimate $\hat{\mathbf{x}}_k$. The weighting occurs by means of a gain matrix $\mathbf{K}_{k+1}$, given by

$$\mathbf{K}_{k+1} = (\mathbf{P}_k + \mathbf{Q}_k) \mathbf{H}_{k+1}^T (\mathbf{R}_{k+1} + \mathbf{H}_{k+1} (\mathbf{P}_k + \mathbf{Q}_k) \mathbf{H}_{k+1}^T)^{-1}. \quad (3)$$

$\mathbf{K}$ depends on the estimated reliability of the parameters, characterized by the variance matrix $\mathbf{P}_k$, on matrix $\mathbf{H}_k = \partial \mathbf{h}_k / \partial \mathbf{x}$ (= sensitivity of the model for changes in $\mathbf{x}$) and on $\mathbf{R}$, the covariance matrix of the observation error $\mathbf{v}$. The matrix $\mathbf{Q}$ is an 'arbitrary' matrix, defined by the user and enables him to influence the convergence speed. The matrix

P_k in this expression is updated by

$$P_{k+1} = (I - K_{k+1}H_{k+1})(P_k + Q_k)(I - K_{k+1}H_{k+1})^T + K_{k+1}R_{k+1}K_{k+1}^T \quad (4)$$

In the examples given in the sequel, the components of H_k are determined numerically. R_k represents the covariance matrix of the error in observation y_k . The noise in all successive observations is assumed to be white. The initial conditions $\hat{x}_0$ and P_0 and the weighting matrices Q_k and R_k must be specified.

EXPERIMENTS

As a first experimental test, a woven and calendered textile was used. The warp and weft yarns were interlaced in a regular sequence. A square specimen was clamped along one edge and loaded on the other edges as shown in Fig. 2. The membrane ($100 \times 100 \text{ mm}^2$) was loaded with two forces ($F_1 = 0.1 \text{ kN}$ and $F_2 = 0.05 \text{ kN}$). This led to strains of up to 3%. The material behaved orthotropic and had homogeneous properties. The specimen was extracted in such a way that one principal material axis had a positive rotation of 45° from the clamped edge (Fig. 3). The displacement field after loading was measured by using 79 markers attached to the specimen. The marker displacements were measured by means of the earlier mentioned video-tracking system. The mean standard deviation of the measured displacements was $5 \times 10^{-3} \text{ mm}$ (the displacements were of the order of 0.4 mm).

The experiment was modeled by means of the finite element method with 4-noded, isoparametric, plane stress elements. The geometry of the sample was measured by putting additional markers on the edges of the surface and on the strings inducing the forces, in order to measure the direction of the forces. An element mesh with 100, 400 and 1600 elements was used to study the effect of mesh refinement. The material was assumed to be orthotropic and linear elastic. The quantitative behavior can be described with five parameters: two Young's moduli (E_1 and E_2), one Poisson's ratio (ν_{12}), one shear modulus (G_{12}) and the tangent of the positive rotation of the material axis from the arbitrary model axis [$\tan(\alpha)$]. This leads to a column of material parameters x :

$$x^T = \{E_1, E_2, \nu_{12}, G_{12}, \tan(\alpha)\}. \quad (6)$$

By means of the finite element method, the displacements in the nodes of the model can be calculated. The positions of the markers are calculated from the nodal displacements by interpolation.

To start the recursive identification method, an initial guess $\hat{x}_0$ for the parameter values and an initial guess for the error covariance of $\hat{x}_0$, matrix P_0 , are needed. The following

Fig. 2. Schematic drawing of the identification experiment. The dots represent the positions of the markers.

Fig. 3. The material orientation in the experimental setup.

value was chosen:

$$\hat{x}_0^T = \{2.0, 4.0, 0.25, 0.5, 1\} \quad (7)$$

with dimension $[\text{kN mm}^{-2}, \text{kN mm}^{-2}, -, \text{kN mm}^{-2}, -]$.

P_0 is chosen to be diagonal with 10^{-2} for all elements, corresponding to the expectation of the squared errors in the initial guess. For the diagonal elements of Q it is best to choose a value near the desired accuracy of the parameters; in our study all diagonal elements were chosen to be 10^{-4} . The accuracy of the measured displacements can be expressed by setting the covariance matrix R . The errors are considered mutually independent, which means that R also is diagonal. Based on the standard deviations of the measured displacement, an averaged value of 10^{-4} mm^2 was chosen.

Figure 4 shows estimations of the five material parameters as a function of the iteration counter, starting with the initial guess $\hat{x}_0$. It can be observed that the estimations converge. The resulting parameter column is

$$\hat{x}_1^T = \{0.56, 0.57, 0.22, 0.08, 1.05\}. \quad (8)$$

The above results were obtained with a finite element mesh with 100 elements. Mesh refinement up to 1600 elements led to the results given in Table I. It turned out that refinement led to a higher E_1 and E_2 , which could be expected. Further refinement no longer changed the results. For this particular material, it is possible to use traditional uniaxial tests for the characterization. Table 2 shows the results from the uniaxial tests and the identification results with a 1600-element mesh. The only inconsistency is found in E_2 , which has a rather large deviation between the two methods. From the structure of the woven textile, two equal Young's moduli would be expected; so, the results from the identification approach seems more reliable. Possible causes for the deviations in the traditional test are inhomogeneous properties or, more likely, problems with the internal coherence, which is disrupted by the sample extraction.

DISCUSSION AND CONCLUSIONS

In the present paper a new method is presented for a mechanical characterization of materials, which can possibly be used to characterize biological materials. It is a numerical-experimental approach using inhomogeneous strain distributions. Because of this, more freedom for the design of experiments is obtained than in traditional testing. This can be used to optimize the performance of experiments for specific types of materials. With this method, nonlinear, highly anisotropic and inhomogeneous materials can be approached and it is possible to design a new range of possible *in vivo* tests for biological materials. The algorithm for the confrontation of strain fields with the finite element

Fig. 4. Parameters versus the number of iteration steps.

Table 1. The influence of the finite element model on the estimation results

Parameters		100 elements		400 elements		1600 elements	
x_i	Units	$(\hat{x}_i)_i$	$\sqrt{(P_1)_{ii}}$	$(\hat{x}_i)_i$	$\sqrt{(P_1)_{ii}}$	$(\hat{x}_i)_i$	$\sqrt{(P_1)_{ii}}$
E_1	(kN mm ⁻²)	0.56	0.009	0.59	0.009	0.60	0.009
E_2	(kN mm ⁻²)	0.57	0.004	0.61	0.005	0.62	0.005
ν_{12}	(-)	0.22	0.006	0.19	0.007	0.18	0.007
G_{12}	(kN mm ⁻²)	0.080	0.0004	0.081	0.0004	0.081	0.0004
$\tan(\alpha)$	(-)	1.05	0.007	1.08	0.006	1.08	0.006

Table 2. Results of the comparison between traditional tests and the identification result for different finite element meshes

Parameters		Traditional testing		1600 elements	
x_i	Units	$(\hat{x}_i)_i$	$\sqrt{(P_1)_{ii}}$	$(\hat{x}_i)_i$	$\sqrt{(P_1)_{ii}}$
E_1	(kN mm ⁻²)	0.62	0.05	0.60	0.009
E_2	(kN mm ⁻²)	0.52	0.06	0.62	0.005
ν_{12}	(-)	0.21	0.01	0.18	0.007
G_{12}	(kN mm ⁻²)	0.080	0.005	0.081	0.0004
$\tan(\alpha)$	(-)	1.00	0.1	1.08	0.006

solution has proved to be efficient. An experiment on an orthotropic membrane has shown that it is indeed possible to determine five material parameters from the inhomogeneous strain field measured in one experiment. Although not yet tested in experiments, Hendriks (1991) has shown by means of simulations that the same technique can also be used to determine the local properties of fiber-reinforced materials with varying fiber directions, like most biological tissues. Our future research is aimed at finding characteristics of strain fields to optimize the performance for materials, reinforced with large fiber and high stiffness ratios and varying fiber directions as a next step to the characterization of biological materials.

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