CO₂ COLLECTIVE LASER SCATTERING ON MOVING DENSITY PERTURBATIONS IN A PLASMA

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Abstract—CO₂ laser scattering from plasma density perturbations which are not wave-like in nature moving through the beam is calculated for various geometries and scale-lengths. Exact expressions are found for the heterodyne detector currents as a function of time, and for the corresponding frequency spectra. In particular, the cases of small-scale rotating density perturbations in cylindrical plasmas and off-axis rotation of the entire plasma column are analyzed in detail. By way of illustration, the theoretical results are compared with some measured scattering spectra from a magnetized arc.

1. INTRODUCTION

IN THE PAST few years several papers have been published on the measurement of density fluctuations in Tokamaks and other plasmas making use of forward scattering of CO₂ laser beams (HOLZHAUER and MASSIG, 1978; SLUSHER and SURKO, 1980; SLUSHER *et al.*, 1982; SURKO and SLUSHER, 1980; EVANS *et al.*, 1982; POTS *et al.*, 1981a; MEYER and MAHN, 1981), and also far-infrared laser beams (LEE *et al.*, 1981). The interpretation of scattering measurements in these experiments is usually done in terms of plasma waves with a definite frequency and wave number spectrum, characterized by the Fourier amplitudes $n_e(\mathbf{k}, \omega)$ or the spectral density distribution $S(\mathbf{k}, \omega)$ associated with plasma density fluctuations. To this end a relation is derived between the scattered power and corresponding detector current and the spectral characteristics of density fluctuations. The frequency spectrum is measured by heterodyning and filtering the detector current, while the wave number spectrum is found by varying the orientation and scattering angle of the scattered beam with respect to the incident beam. From the thus observed ω and $\mathbf{k}$ dependence of the scattered signal one arrives at conclusions regarding the propagation direction and dispersion of the plasma waves responsible for the scattering.

This procedure is most useful in situations where the fluctuation amplitude is only a weak function of the position in the plasma. The scale length of the amplitude variation must be large compared to the wavelength, so that a relatively well defined spectrum characterizes the fluctuations. Thus, the description of the phenomena in terms of waves is most useful in situations where the scattering is done on high frequency turbulence, for which the wavelengths are usually small. A pertinent example is scattering on lower hybrid waves injected into the plasma for heating purposes (SLUSHER *et al.*, 1982).

The situation is more complex when scattering is used to study low frequency phenomena characterized by relatively large scale-lengths. Apart from serious practical complications arising from the necessity of scattering at very small angles, the interpretation of the results in terms of the spectral parameters ω and k becomes difficult. For example, azimuthally propagating low-frequency drift waves, which fall into this class, generally have a radial extent which is not much larger than the azi-

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muthal wavelength. In addition, many plasma perturbations are not wave-like in character and are more meaningfully described by a space-time variation of the density than by a spectral density function. This class of phenomena would include the off-axis rotation of an entire column, or rotating flutes whose scale-length is small compared to the plasma radius. In such cases it is worthwhile to re-examine the situation by calculating scattering spectra from density fluctuations specified in time and space, which do not necessarily have a wave-like character and hence are not meaningfully described by a spectral density function. Thus, one would calculate the relation between the observed heterodyne detector current and a given density perturbation $n_e(\mathbf{r}, t)$.

Since in earlier scattering work on magnetized arcs (POTS, 1979) and Tokamaks (SURKO and SLUSHER, 1980; DOYLE *et al.*, 1982) it has been observed that most appreciable scattering arises from fluctuations propagating in a direction perpendicular to the magnetic field, we shall limit ourselves to this case. It is thus the purpose of this paper to investigate the scattering of a CO₂ laser beam resulting from rotating density perturbations, including off-axis rotation of the entire plasma column. As will be shown, plasma columns which exhibit this kind of gross plasma motion of a periodic nature give rise to significant scattering and unless detailed measurements are made of the spectral characteristics of the scattered beam as a function of scattering geometry, it may be difficult to distinguish experimentally between wave motion and moving density perturbations.

The calculation of laser scattering from moving plasma structures is of interest because in addition to wave-like instabilities, many magneto-plasmas exhibit flute-like rotation of density structures and/or rotation and wobbling of the entire plasma column. In a laser scattering experiment recently published (POTS *et al.*, 1981a), independent measurements of the plasma dynamics point to the presence of such moving plasma structures. As will be shown in a later section, some of the features of the scattering measurements carried out in that experiment may be explained qualitatively by the theoretical description of scattering from moving density perturbations.

We will start our discussion with a theoretical description of the problem in Section 2. In Section 3 a summary of some relevant experimental data from the experiment of POTS *et al.* (1981) will be given. Section 4 will contain a discussion of the application of the theory to these measurements and our principal conclusions.

2. THEORY

A. Calculation of the heterodyne detector current

We consider real time density perturbations exhibiting cylindrical symmetry of the form

$$n(\mathbf{r}, t) = n_1 \exp - \left[\frac{(x \pm \Delta \sin \Omega t)^2 + (y - \Delta \cos \Omega t)^2}{\Lambda^2} \right]. \quad (1)$$

This expression describes a whole range of quite different plasma density perturbations. For $\Delta \ll \Lambda$, it corresponds to off-axis rotation of a Gaussian electron density profile, of radial extent Λ , in which case Δ is the rotation amplitude. On the other hand for $\Delta \gg \Lambda$, it could represent a local plasma perturbation of scale length Λ rotating at mean radius Δ and frequency Ω . For an illustration of these two cases, see Fig.

FIG. 1.—Laser scattering geometry.

(7 a, b). We assume a forward scattering geometry as shown in Fig. (1), with the incident and scattered beams propagating in the xy plane nearly parallel to the x -axis and incident on the plasma at the lateral position $y = y_0$. We assume (although this is not really necessary) that the dimensions in the x -direction of the plasma are sufficiently small so that the beams superimpose in the plasma, and thus we write for the incident and scattered beams respectively

$$\mathbf{E}_i(x, y, z) = \mathbf{z}_u E_i U_i(y, z) \equiv \mathbf{z}_u E_i \exp \left[-\frac{(y - y_0)^2 + z^2}{a_w^2} \right] \quad (2)$$

$$\mathbf{E}_s(x, y, z) = \mathbf{z}_u E_s U_s(y, z) \equiv \mathbf{z}_u E_s \exp \left[-\frac{(y - y_0)^2 + z^2}{a_w^2} \right] \quad (3)$$

where $\mathbf{E}$ is the electric field associated with the laser beam, $\mathbf{z}_u$ is the unit vector in the z -direction, i and s are indices denoting incident and scattered, and a_w is the beam waist radius.

If we arrange the optics in such a way that the scattered beam at $\theta_s = k/k_i$ is mixed with a local oscillator signal of strength E_{LO} , then the intermediate frequency signal on the detector is given by (HOLZHAUER and MASSIG, 1978)

$$i_{if}(t) = jC_{opt} \int d^3\mathbf{r} n_e(\mathbf{r}, t) [U(\mathbf{r}) \exp(-j\mathbf{k} \cdot \mathbf{r}) - U^*(\mathbf{r}) \exp(j\mathbf{k} \cdot \mathbf{r})] \quad (4)$$

where $U(y, z) = U_i(y, z)U_s(y, z)$; U^* is the complex conjugate of U , and $C_{opt} = 2r_0 S(P_i P_{LO})^{1/2}/R$, with r_0 = classical electron radius, R = Rayleigh length $\equiv \pi a_w^2/\lambda_i$, S the detector sensitivity, P_i the incident laser power, and P_{LO} the local oscillator power. Substituting the expressions for n_e and U gives

$$i_{if}(t) = jC_{opt} \int dx dy dz n_1 \exp \left[-\frac{(x \pm \Delta \sin \Omega t)^2 + (y - \Delta \cos \Omega t)^2}{\Lambda^2} \right] \exp \left[-\frac{2(y - y_0)^2 + 2z^2}{a_w^2} \right] \{ \exp[-jk(y - y_0)] - \exp[jk(y - y_0)] \}. \quad (5)$$

The integrals over x and z are straight forward, and we obtain, assuming that the beams nearly overlap in the plasma,

$$i_{if}(t) = j\pi a_0 \Lambda n_1 C_{opt} \int dy \exp \left[-\frac{(y - \Delta \cos \Omega t)^2}{\Lambda^2} \right] \exp \left[-\frac{(y - y_0)^2}{a_0^2} \right] \{ \exp[-jk(y - y_0)] - \exp[jk(y - y_0)] \} \quad (6)$$

where $a_0 \equiv a_w/\sqrt{2}$ is the effective beam width.

It is of interest to note that apart from the complex exponential factors this integral is in fact the beam-weighted line integral of the electron density perturbation, here derived for a Gaussian profile. One would obtain an expression of the same form for any other density profile having cylindrical symmetry in the rotating coordinates.

Equation (6) corresponds to the difference of two Fourier integrals of the form

$$J_{\mp} = \int dy \exp \left[-\frac{(y - \Delta \cos \Omega t)^2}{\Lambda^2} \right] \exp \left[-\frac{(y - y_0)^2}{a_0^2} \right] \exp[\mp jk(y - y_0)]. \quad (7)$$

Letting $\eta = y - y_0$, we can rewrite these integrals as

$$J_{\mp} = \exp \left[-\frac{(y_0 - \Delta \cos \Omega t)^2}{\Lambda^2 + a_0^2} \right] \int d\eta \exp \left[-\frac{\left\{ \eta + \frac{(y_0 - \Delta \cos \Omega t)a_\Lambda^2}{\Lambda^2} \right\}^2}{a_\Lambda^2} \right] \exp(\mp jk\eta) \quad (8)$$

where

$$a_\Lambda^2 \equiv \frac{a_0^2 \Lambda^2}{\Lambda^2 + a_0^2}. \quad (9)$$

The integrals $J_{\mp}$ can be simply calculated, giving

$$J_{\mp} = a_\Lambda \sqrt{\pi} \exp \left[-\frac{(y_0 - \Delta \cos \Omega t)^2}{\Lambda^2 + a_0^2} \right] \exp \left[-\frac{a_\Lambda^2 k^2}{4} \right] \exp \left[\mp jk \frac{(y_0 - \Delta \cos \Omega t)a_0^2}{a_0^2 + \Lambda^2} \right]. \quad (10)$$

We thus obtain

$$i_{if}(t) = -\frac{2\pi^{3/2} a_0^2 \Lambda^2 n_1 C_{opt}}{\sqrt{a_0^2 + \Lambda^2}} \exp \left[-\frac{a_\Lambda^2 k^2}{4} \right] \exp \left[-\frac{(y_0 - \Delta \cos \Omega t)^2}{a_0^2 + \Lambda^2} \right] \sin \left[\frac{ka_0^2}{a_0^2 + \Lambda^2} (y_0 - \Delta \cos \Omega t) \right]. \quad (11)$$

We can find the spectrum corresponding to this detector current by noting that $i_{if}(t)$ is a periodic function with period $T = 2\pi/\Omega$.

Hence we may write formally:

$$i_{if}(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\Omega t}. \quad (12)$$

The calculation of the coefficients c_n gives the spectrum, since the spectral distribution of $i_{if}(t)$ is then given by

$$I_{if}(\omega) = \sum_{n=-\infty}^{\infty} c_n \delta(\omega - n\Omega). \quad (13)$$

We thus need to calculate the Fourier coefficients c_n which are given by

$$c_n = \frac{i_0}{T} \int_{-T/2}^{T/2} dt \exp \left[-\frac{(y_0 - \Delta \cos \Omega t)^2}{a_0^2 + \Lambda^2} \right] \sin [k_\Lambda (y_0 - \Delta \cos \Omega t)] \exp(-jn\Omega t) \quad (14)$$

where

$$i_0 \equiv -\frac{2\pi^{3/2} a_0^2 \Lambda^2 n_1 C_{\text{opt}}}{\sqrt{a_0^2 + \Lambda^2}} \exp \left[-\frac{a_\Lambda^2 k^2}{4} \right] \quad (15)$$

and

$$k_\Lambda = k \left[\frac{a_0^2}{a_0^2 + \Lambda^2} \right]. \quad (16)$$

The integral for c_n can be performed exactly, as is shown in Section 2D below. However, the exact solution is not very instructive, and more simple expressions can be obtained for several limiting cases of interest. We therefore study the situation first for two cases, namely $\Delta^2 \gg a_0^2 + \Lambda^2$ and $\Delta^2 \ll a_0^2 + \Lambda^2$. The first case corresponds to a small density perturbation rotating in the plasma at relatively large radius, while the second corresponds to an off-axis rotation of the entire plasma column. We also assume a value of k such that for the first case $k_\Lambda \Delta \gg 1$ and for the second $k_\Lambda \Delta \lesssim 1$.

B. Calculation of spectral distribution of detector current for the case $\Delta^2 \gg a_0^2 + \Lambda^2$ (small structures)

For this case, the exponential factor in the integrand is only appreciable in the neighbourhood of $\Omega t = \cos^{-1}(y_0/\Delta)$; the general form of $i_{if}(t)$ is given in Fig. (2) for various choices of y_0 and for a particular combination of beam and plasma parameters. Hence a good approximation to the function $i_{if}(t)$ in the range $-T/2 < t < T/2$ may be found by approximating the function $(y_0 - \Delta \cos \Omega t)$ using a Taylor expansion to first order in t in the neighbourhood of $t = \mp t_0$, where $\Omega t_0 \equiv |\cos^{-1}(y_0/\Delta)|$.

FIG. 2.—General form of the function $i_{if}(t)$ as given in equation (11) for the case $\Delta^2 \gg a_0^2 + \Lambda^2$ for various choices of y_0 , with $a_0 = 0.7 \times 10^{-3}$ m, $\Lambda = 1 \times 10^{-3}$ m, $\Delta = 2 \times 10^{-2}$ m and $k = 1 \times 10^4$ m $^{-1}$.

Performing this expansion gives

$$c_n \cong \frac{i_0}{T} \int_{-T/2}^{T/2} dt e^{-jn\Omega t} \left\{ \exp \left[-\frac{(\Delta^2 - y_0^2)\Omega^2(t - t_0)^2}{\Lambda^2 + a_0^2} \right] \sin[k_\Lambda \Omega \sqrt{\Delta^2 - y_0^2}(t - t_0)] \right. \\ \left. + \exp \left[-\frac{(\Delta^2 - y_0^2)\Omega^2(t + t_0)^2}{\Lambda^2 + a_0^2} \right] \sin[-k_\Lambda \Omega \sqrt{\Delta^2 - y_0^2}(t + t_0)] \right\}. \quad (17)$$

This linear approximation is equivalent to approximating the trajectory of a rotating density structure by a straight line as it passes through the laser beam. This approximation is adequate provided $(\Delta^2 - y_0^2)\Omega^2 t_0^2 \gg \Lambda^2 + a_0^2$. For $\Delta^2 \gg \Lambda^2 + a_0^2$ this implies that the approximation is good for all values of y_0 except those very close to Δ . The case $y_0 = \Delta$ will be discussed separately below.

Using standard Fourier analysis it is now possible to integrate (17). We note first that for $\Delta^2 \gg \Lambda^2 + a_0^2$ we may extend the integration to plus and minus infinity. If we define

$$\alpha^2 \equiv \frac{\Lambda^2 + a_0^2}{(\Delta^2 - y_0^2)\Omega^2} \quad \text{and} \quad \beta \equiv k_\Lambda \Omega \sqrt{\Delta^2 - y_0^2},$$

we obtain

$$c_n = \frac{\pi a_0^2 \Lambda^2 n_1 C_{\text{opt}}}{\sqrt{\Delta^2 - y_0^2}} \exp\left[-\frac{a_\Lambda^2 k^2}{4}\right] \sin(|n|\Omega t_0) \left\{ \exp\left[-\frac{(|n|\Omega + \beta)^2 \alpha^2}{4}\right] + \exp\left[-\frac{(|n|\Omega - \beta)^2 \alpha^2}{4}\right] \right\}. \quad (18)$$

Rewriting this expression, and substituting for α , β , and a_Λ , we obtain

$$I(\omega) = \sum_{n=-\infty}^{\infty} \sin(|n|\Omega t_0) S_n \delta(\omega - n\Omega) \quad (19)$$

where

$$S_n = \frac{\pi a_0^2 \Lambda^2 n_0 C_{\text{opt}}}{\sqrt{\Delta^2 - y_0^2}} \exp\left[-\frac{k^2 a_0^2 \Lambda^2}{4(a_0^2 + \Lambda^2)}\right] \left\{ \exp\left[-\frac{(n - n_{\text{max}})^2}{(\delta n)^2}\right] + \exp\left[-\frac{(n + n_{\text{max}})^2}{(\delta n)^2}\right] \right\} \quad (20)$$

and

$$n_{\text{max}} = k \frac{a_0^2}{a_0^2 + \Lambda^2} \sqrt{\Delta^2 - y_0^2} \quad (21)$$

and

$$\delta n = 2 \sqrt{\frac{\Delta^2 - y_0^2}{\Lambda^2 + a_0^2}}. \quad (22)$$

In (19) S_n is an amplitude envelope and $\sin(|n|\Omega t_0)$ is a rapidly varying phase factor which results from the correlation of signals obtained at $t = -t_0 + 2p\pi$ and $t = t_0 + 2p\pi$. As the detection of this phase modulation requires very sharp filtering and very stable rotation frequencies, we may ignore the phase modulation from an experimental point of view. In Fig. (3) we show S_n for several choices of y_0 , using a typical experimental situation of scattering from a laboratory plasma such as discussed in Section 3. The curve given for $y_0 = \Delta$ was obtained by calculating the integral in equation (14) numerically.

The graphs show the general structure of the spectra, characterized by a broad Gaussian envelope. The most interesting features of these results are the following:

1. Small scale density structures rotating at large radii in the plasma can give rise to significant heterodyne scattering signals at frequencies many times the fundamental rotation frequency. Calculated signal strengths correspond to a scattered power of about 5×10^{-14} W over the band-width of the signal envelope. The spectra have a broad character, with $\Delta\omega \approx \omega$.
2. The frequency of maximum amplitude decreases as a function of the vertical intercept of the laser beam, y_0 , while the corresponding signal strength for a given band-width increases.
3. At $y_0 = \Delta$, the spectrum is relatively narrow, with a maximum at zero frequency.

C. Calculation of the spectral distribution of detector current for the case $\Delta^2 \ll \Lambda^2 + a_0^2$ (plasma wobble)

FIG. 3.—General form of the function $S_n(\omega)$ as given in equation (20) for the case $\Delta^2 \gg a_0^2 + \Lambda^2$, for various choices of y_0 , with $a_0 = 0.7 \times 10^{-3}$ m, $\Lambda = 1 \times 10^{-3}$ m, $\Delta = 2 \times 10^{-2}$ m and $k = 1 \times 10^4$ m $^{-1}$, $n_1 = 10^{20}$ m $^{-3}$.

This case, which corresponds to the off-axis rotation of the entire plasma column with relatively small eccentricity (plasma wobble), may be treated simply by assuming $k_\Lambda \Delta < 1$, and $y_0 \gg \Delta$. We then approximate $\sin k_\Lambda(y_0 - \Delta \cos \Omega t)$ by $\sin(k_\Lambda y_0) - k_\Lambda \Delta \cos(\Omega t) \cos(k_\Lambda y_0)$ in equation (14) and consider the integral

$$c_n = \frac{i_0}{T} \int_{-T/2}^{T/2} dt \exp \left[-\frac{(y_0 - \Delta \cos \Omega t)^2}{a_0^2 + \Lambda^2} \right] (\sin(k_\Lambda y_0) - k_\Lambda \Delta \cos \Omega t \cos(k_\Lambda y_0)) \exp(-jn\Omega t).$$

Using the expansion

$$e^{\chi \cos \Omega t} = \sum_{m=-\infty}^{\infty} I_m(\chi) \exp(jm\Omega t), \quad (23)$$

where I_m is the modified Bessel function of order m , we obtain, for $\Lambda^2 + a_0^2 \gg \Delta^2$, and $y_0 \gg \Delta$:

$$c_n = i_0 \exp \left[-\frac{y_0^2}{a_0^2 + \Lambda^2} \right] \sum_{m=-\infty}^{\infty} I_m \left[\frac{2y_0\Delta}{a_0^2 + \Lambda^2} \right] \left\{ \sin(k_\Lambda y_0) \delta(n-m) - \frac{k_\Lambda \Delta}{2} \cos k_\Lambda y_0 (\delta(n-m-1) + \delta(n-m+1)) \right\}. \quad (24)$$

Thus

$$c_n = i_0 \exp \left[-\frac{y_0^2}{a_0^2 + \Lambda^2} \right] \left\{ \sin(k_\Lambda y_0) I_n \left[\frac{2y_0\Delta}{a_0^2 + \Lambda^2} \right] - \frac{k_\Lambda \Delta}{2} \cos(k_\Lambda y_0) \left(I_{n+1} \left[\frac{2y_0\Delta}{a_0^2 + \Lambda^2} \right] + I_{n-1} \left[\frac{2y_0\Delta}{a_0^2 + \Lambda^2} \right] \right) \right\}. \quad (25)$$

Noting that $y_0\Delta/(a_0^2 + \Lambda^2) \ll 1$ (provided $y_0 \lesssim \Lambda$), we may write

$$I_n \left[\frac{2y_0\Delta}{a_0^2 + \Lambda^2} \right] \approx \frac{1}{n!} \left(\frac{y_0\Delta}{a_0^2 + \Lambda^2} \right)^n.$$

Then we find, substituting for k_Λ :

$$c_n = i_0 \exp \left[-\frac{y_0^2}{a_0^2 + \Lambda^2} \right] \frac{1}{n!} \left(\frac{y_0\Delta}{a_0^2 + \Lambda^2} \right)^n \left\{ \sin \left[\frac{ka_0^2 y_0}{a_0^2 + \Lambda^2} \right] - \frac{ka_0^2 \Lambda^2 n}{2y_0(a_0^2 + \Lambda^2)} \cos \left[\frac{ka_0^2 y_0}{a_0^2 + \Lambda^2} \right] \right\}. \quad (26)$$

Substitution for i_0 gives finally:

$$|I(\omega)| = 2\pi^{3/2} a_0^2 \Lambda n_0 C_{\text{opt}} \exp \left[-\frac{a_0^2 k^2 \Lambda^2}{4(a_0^2 + \Lambda^2)} \right] \exp \left[-\frac{y_0^2}{\Lambda^2} \right] \sum_{n=-\infty}^{\infty} \frac{1}{|n|!} \left(\frac{y_0\Delta}{a_0^2 + \Lambda^2} \right)^{|n|} \left\{ \sin \left[\frac{ka_0^2 y_0}{a_0^2 + \Lambda^2} \right] - \frac{ka_0^2 \Lambda^2}{2y_0(a_0^2 + \Lambda^2)} |n| \cos \left[\frac{ka_0^2 y_0}{a_0^2 + \Lambda^2} \right] \right\} \delta(\omega - n\Omega). \quad (27)$$

Figure 4 shows a plot of $I(\omega)$ using typical plasma and scattering parameters relevant to this particular case. It is to be noted that the scattering spectrum associated with this type of motion shows a drastic decrease in amplitude as a function of frequency; beyond the first few harmonics of Ω its amplitude becomes negligible. It can be shown that for $y_0 \lesssim \Delta$, the decrease in amplitude as a function of ω is even steeper. There thus appears to be little measurable signal for this case beyond $\omega \approx 5\Omega$. Since Ω is relatively small (usually of the order of 10–100 kHz), signals due to this type of plasma wobble should be strongest at frequencies less than a few hundred kHz, and decrease very rapidly as a function of ω .

D. Calculation of the spectral distribution of detector current for the general case

It is possible to integrate the expression given in equation (14) formally, making

FIG. 4.—General form of the function $I(\omega)$ as given in equation (27) for the case $\Delta^2 \ll a_0^2 + \Lambda^2$, for various choices of y_0 , with $a_0 = 0.7 \times 10^{-3}$ m, $\Lambda = 2 \times 10^{-2}$ m, $\Delta = 1 \times 10^{-3}$ m and $k = 1 \times 10^4$ m $^{-1}$, $n_1 = 10^{20}$ m $^{-3}$.

use of the developments given in equation (23), and the additional expansions

$$\cos(z \cos \theta) = J_0(z) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(z) \cos(2k\theta) \quad (28)$$

and

$$\sin(z \cos \theta) = 2 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(z) \cos[(2k+1)\theta], \quad (29)$$

where J_n is the Bessel function of order n .

We then obtain:

$$\begin{aligned} c_n = i_0 \exp \left[-\frac{y_0^2}{a_0^2 + \Lambda^2} \right] \exp \left[-\frac{\Lambda^2}{2(a_0^2 + \Lambda^2)} \right] \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} (-1)^l I_m \left[\frac{2y_0 \Delta}{a_0^2 + \Lambda^2} \right] \\ \times I_l \left[\frac{\Delta^2}{2(a_0^2 + \Lambda^2)} \right] \left\{ \sin(k_{\Lambda} y_0) \left[J_0(k_{\Lambda} \Delta) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(k_{\Lambda} \Delta) \right. \right. \\ \times \frac{1}{T} \int_{-T/2}^{T/2} dt \exp[-i\Omega t(n-m-2l)] \cos 2k\Omega t \left. \right] - \cos(k_{\Lambda} y_0) \\ \times \left[2 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(k_{\Lambda} \Delta) \frac{1}{T} \int_{-T/2}^{T/2} dt \exp[-i\Omega t(n-m-2l)] \cos(2k+1)\Omega t \right] \left. \right\}. \quad (30) \end{aligned}$$

Performing the integral gives

$$\begin{aligned}
 c_n = i_0 \exp \left[-\frac{y_0^2}{a_0^2 + \Lambda^2} \right] \exp \left[-\frac{\Lambda^2}{2(a_0^2 + \Lambda^2)} \right] \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} (-1)^l I_m \left[\frac{2y_0\Delta}{a_0^2 + \Lambda^2} \right] \\
 \times I_l \left[\frac{\Delta^2}{2(a_0^2 + \Lambda^2)} \right] \left\{ \sin(k_\Lambda y_0) J_0(k_\Lambda \Delta) + \sin(k_\Lambda y_0) \sum_{k=1}^{\infty} (-1)^k J_{2k}(k_\Lambda \Delta) \right. \\
 \times [\delta(n-m-2l-2k) + \delta(n-m-2l+2k)] - \cos(k_\Lambda y_0) \sum_{k=0}^{\infty} \\
 \left. \times \sum_{k=0}^{\infty} (-1)^k j_{2k+1}(k_\Lambda \Delta) [\delta(n-m-2l-2k-1) + \delta(n-m-2l+2k+1)] \right\}. \quad (31)
 \end{aligned}$$

3. SOME RELEVANT EXPERIMENTAL RESULTS

In this section we wish to summarize some experimental results already published elsewhere (POTS, 1979; POTS *et al.*, 1981a, 1981b; TIMMERMANS *et al.*, 1981; SCHRAM *et al.*, 1982) which will serve to illustrate that in some situations the assumption that scattering takes place from moving plasma density perturbations can explain several features of the observed scattering spectra. Measurements consisting of optical plasma dynamics studies and CO₂ laser scattering were performed on a magnetized current carrying plasma having densities in the 10²⁰ m⁻³ range and electron and ion temperatures of a few electronvolts. Both the density and electron temperature were measured with Thomson scattering.

The density profile is described by a Gaussian as shown in Fig. 5. As in all magnetized plasmas, the plasma rotates with an azimuthal velocity which at maximum is of the

FIG. 5.—Measured density profile of the magnetized arc used for the measurements described in Section 3; $B = 0.2$ T, $I_{\text{plasma}} = 50$ A, $p = 1$ mtorr.

order of one third of the ion thermal velocity (TIMMERMAN *et al.*, 1981). Ion drift velocities parallel to the magnetic field of this magnitude have also been measured (VOGELS *et al.*, 1982).

Superimposed on this equilibrium state a wide variety of plasma fluctuations were observed as measured by studying the temporal behaviour of optical emission from the plasma, and by CO₂ laser scattering. In the optical emission detection system, the line integrated plasma emission of the Argon II 4p–4s lines at lateral position y_0 is focussed on a detector with large quantum efficiency and low noise figure. It was shown (POTS, 1979) that the emission intensity fluctuations were relatively insensitive to the temperature fluctuations, and that the fluctuation amplitude of the detector signal $\tilde{I}$ is given by

$$\tilde{I}(y_0, t) = C \int dy \tilde{n}_e(y_0, t) \exp \left[-\frac{(y - y_0)^2}{a_{\text{opt}}^2} \right]$$

where C is a constant, $\tilde{n}_e$ is the electron density fluctuation amplitude, and the exponential weighting factor represents the (approximately) Gaussian profile of the optical detection system, a_{opt} being the effective aperture radius. We note that $\tilde{I}$ here is in fact proportional to the beam-weighted line integral of the density fluctuation already introduced in Section 2A.

By using appropriate filters, information on the frequency spectra was obtained for various values of y_0 . Measurements showed the existence of fluctuations in at least two frequency bands, as shown in Fig. 6. The lowest frequency component, referred to by POTS (1979) as a rotational instability, may be interpreted as an off-axis rotation of the plasma with eccentricity Δ . This interpretation is supported by the streak-camera picture (BOESCHOTEN *et al.*, 1976) shown in Fig. 7.

The second spectral feature corresponds to fluctuations in the neighborhood of the ion-cyclotron frequency. Correlation measurements on this disturbance using the

FIG. 6.—Typical spectrum of plasma fluctuations measured by optical emission from the plasma. $B_0 = 0.05$ Tesla, $y_0 \sim 10^{-2}$ m.

FIG. 7.—(a) Schematic diagram showing off-axis plasma rotation (plasma wobble) with $\Delta \ll \Lambda$. (b) Schematic diagram showing rotation of flute-like density perturbation, with $\Delta \gg \Lambda$. (c) Streak photograph showing fluctuations in light emission due to plasma motion as shown in (a) and (b).

optical detection system at different y_0 intercepts showed that this feature corresponds to a density perturbation which propagates azimuthally at the diamagnetic drift speed w_{de} which for this experiment is of the order of the ion-acoustic speed c_s (POTS *et al.*, 1981a). The original interpretation was that this feature corresponds to a $m = 1$ drift wave. However, an equally possible (and perhaps more plausible) explanation is that the measurements show the rotation of a flute-like density perturbation rotating at the diamagnetic drift speed as suggested by POTS (1979). Axial measurements show strong correlation at exactly the same azimuth, indicating the flute-like character of the perturbation. The small scale high frequency intensity fluctuations in the photograph (Fig. 7) support this interpretation.

CO₂ scattering measurements on this plasma were reported in detail in an earlier paper (POTS *et al.*, 1981a). As a typical example of the scattering results for a central chord measurement ($y_0 = 0$) see Fig. 8 (POTS, 1979). Of particular interest for our purposes here is the form of the scattering spectrum as a function of y_0 . In Fig. 9 we show spectra obtained at six values of y_0 for a given value of k . One notes that as y_0 is increased, the maxima of the spectra shift to lower values of ω .

Measurements of I as a function of k and ω at fixed y_0 show a linear dispersion with phase velocity of the order of the ion acoustic speed c_s which for this experiment is of the same order as the electron diamagnetic drift speed w_{de} (Fig. 8). Therefore, these high frequency waves were labelled as drift-dissipative waves in the original publication (POTS *et al.*, 1981a). As we shall now show, an alternative interpretation in terms of the theory presented is possible.

FIG. 8.—Plot of the square of the detector current as a function of k and ω for a typical set of plasma parameters: $I_p = 50$ A, $B = 0.2$ T, $p = 0.6$ mtorr, $T_e = 3$ eV, $T_i = 5.5$ eV, $T_0 = 0.12$ eV, $n_e = 0.9 \times 10^{20} \text{ m}^{-3}$. Detector filter bandwidth $\Delta\omega/2\pi = 10$ MHz. (Replotted from POTS, 1979).

4. DISCUSSION AND CONCLUSION

There are a number of features of the reported results which lead us to propose that the observed scattering signals support the hypothesis of scattering from moving density perturbations rather than scattering from well defined waves. The presence of these moving density perturbations in this plasma was already discussed in the previous section, and it is striking that the spectra exhibit features which fit at least qualitatively with the theoretical predictions of Section 2. A comparison of Fig. 9 with the results shown in Fig. 3 show a strong similarity; as the beam is moved outwards (y_0 is increased), the maximum of the spectrum moves to lower frequencies. Assuming for the moment that the scattering is due to processes as calculated in Section 2B, we can calculate some of the features of the expected spectra. The relevant plasma and beam parameters give us the following values: $k = 2 \times 10^4 \text{ m}^{-1}$, $a_0 = 0.7 \times 10^{-3} \text{ m}$, $\Omega/2\pi = 70 \text{ kHz}$, $\Delta \sim 1 \times 10^{-2} \text{ m}$. From the variation of I_{max} with k as shown in Fig. 8 we find $\Lambda \sim 0.3 \times 10^{-3} \text{ m}$. We would expect broad spectral features at $y_0 = 0$ around $\omega/2\pi = (k_\Lambda \Delta/2\pi)\Omega = 12 \text{ MHz}$ as is indeed the case (Fig. 9). An approximate calculation of the expected detector current at $y_0 = 0$ yields $I = 5 \times 10^{-8}$ Amperes. This fits our observed maxima provided we assume $n_1/n_0 \sim 0.1$ which is not unreasonable. It is of interest to note that the spectral features of fluctuation measurements performed on a small Tokamak using FIR scattering (LEE *et al.*, 1981; PEEBLES *et*

FIG. 9.—Plot of the square of the detector current as a function of y_0 and ω for a typical set of plasma parameters with $k = 2 \times 10^4 \text{ m}^{-1}$, $I_p = 50 \text{ A}$, $B = 0.2 \text{ T}$, $p = 1.5 \text{ mtorr}$. Detector filter bandwidth $\Delta\omega/2\pi = 10 \text{ MHz}$. (Replotted from POTS *et al.*, 1981a).

al., 1981) are similar to those reported above, which leads us to the conclusion that possibly in their plasma similar phenomena play an important role.

Perhaps most importantly, our model predicts significant scattering for measurements made away from the central chord, as is indeed observed experimentally. The explanation of these peripheral chord measurements in terms of azimuthally propagating drift waves is at best difficult; one has to invoke relatively large radial wave numbers which physically violate the condition that the waves should have a radial extent which is large compared to the wave length. If, however, one assumes the presence of rotating plasma perturbation, the above theory explains quite adequately the presence of significant scattering at peripheral chords.

Thus we conclude that rotating plasma perturbations can give rise to significant laser scattering at frequencies large compared to the rotating frequencies and that this phenomenon may be responsible for scattering observed in magnetized arcs and Tokamaks, especially for scattering measurements made along peripheral chords.

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