Description of a Flowing Cascade Arc Plasma

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The plasma in a cascaded arc in argon with flow is studied both experimentally and theoretically. The plasma pressure has been measured as a function of axial position in the arc channel with a Baratron pressure transducer. The electron density and the electron temperature have been determined as a function of axial position using Hg-Stark broadening and line-continuum emissivity ratio, respectively. Comparison of the gas pressure measurements with an equilibrium model suggests that the flow is laminar. A one-dimensional nonequilibrium model based on the electron- and heavy-particle number balances and the heavy-particle energy balance is presented. The measured axial profiles of the electron density agree well with the model predictions, especially in the most upstream part of the arc channel. The plasma is strongly ionizing. Temperature equilibration takes about 20 mm of arc length, depending on the arc flow.

KEY WORDS: Cascade arcs; flowing plasmas; nonequilibrium.

1. INTRODUCTION

The cascaded arc was introduced in 1956 by Meecker.1 Until now it has been applied almost exclusively for investigations of scientific interest. This type of wall-stabilized arc plasmas can be operated under a wide range of pressures (0.1–1000 bar) and currents (5–2000 A), and is characterized by large electron densities, high degrees of ionization, and moderate temperatures. Mostly noble gases or hydrogen are used to feed the plasma, but sometimes molecular gases like SF6 are added.2 The deviations from local thermal equilibrium (LTE) in nonflowing arcs have been studied by various

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authors.\(^{(3-5)}\) At high densities the nonideal plasma behavior has been explored.\(^{(6,7)}\)

The arc studied here forms an essential part of a reactor for plasma deposition.\(^{(8)}\) The arc is operated in argon. The flowing arc plasma emanates in a large vacuum system where it expands supersonically. If a gas like CH\(_4\) is injected in the arc, a carbon film will be formed on a substrate in the vacuum system. The expansion and deposition are discussed elsewhere.\(^{(9)}\)

Recently de Haas has investigated the physics of a strongly flowing, but fully equilibrated, cascaded arc plasma.\(^{(9)}\) His model has been used to analyze the pressure drop in the arc channel. However, the evolution of the electron density as a function of axial position in the arc channel indicates that the plasma is not in LTE, especially in the beginning of the channel. Therefore a simple one-dimensional model has been developed which includes ionizational and thermal nonequilibrium. It makes use of the measured axial profiles of the plasma pressure and of the electron temperature.

In this paper, we will outline that model and we will compare its predictions with results from measurements of the electron density and the electron temperature. Also the validity of de Haas’ models to explain the pressure drop in the channel will be demonstrated.

2. MODEL

A description according to partial local thermodynamic equilibrium (PLTE) is used. According to this concept the deviations from complete local thermodynamic equilibrium\(^{(10)}\) (LTE) may comprise temperature differences between several species present in the plasma and also an over- or underpopulation of the ground level.

The electrons and the heavy particles are all assumed to have Maxwel-\(\text{l}\)ian energy distributions, each at their own temperatures. The gas temperature and the electron density gradually approach their equilibrium values while the plasma flows through the arc channel.

As is usual in the description of duct flows, a one-dimensional approach will be followed. This implies that the transport equations are integrated over the cross section.

2.1. Basic Equations

The intrinsic continuum, momentum, and energy equations for each particle species present in the plasma read\(^{(11)}\)

\[
\frac{dn}{dt} = \frac{\partial n}{\partial t} + \nabla \cdot (nw) = S_c
\]  \hspace{1cm} (1)
\[
\frac{nm}{\partial t} \frac{\partial w}{\partial t} + nm(w \cdot \nabla)w + \nabla p + \nabla \cdot \pi - qn(E + w^* B) = S_m
\] (2)

and

\[
\frac{\partial}{\partial t} \left( \frac{3}{2} nkT \right) + \nabla \cdot \left( \frac{3}{2} nkTw \right) + nkT \nabla \cdot w + \pi \cdot \nabla w + \nabla \cdot q = S_e
\] (3)

respectively, where \( k \) is Boltzmann's constant and \( S_m, S_e \), and \( S_r \) represent the source terms for volume production of particles, momentum, and energy. The thermal heat flux \( q \) can be calculated by \( q = -\kappa \nabla T \), where \( \kappa \) is the thermal conductivity; \( \pi \) is the viscosity tensor.

In the following, these transport equations will be used for a variety of particles. The source term of each equation will be evaluated. Table I lists the symbols that are used to represent the several densities and temperatures.

Furthermore we will use the equation of state:

\[
p = \sum x n_x kT_x
\] (4)

The Saha equation describes the equilibrium relation between neutral and ionized species of the same kind\(^{(12)}\):

\[
n_x^2 = \frac{g_x W_x}{W_s} \left( \frac{2 \pi m_k T_x}{h} \right)^{3/2} \exp \left( -\frac{E_{ix} - \Delta E_{ix}}{kT_x} \right)
\] (5)

where \( W_s \) represents the partition function of the neutral system and \( E_{ix} \) is the ionization energy of species \( x \); \( \Delta E_{ix} \) is the ionization potential lowering.\(^{(13)}\)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_x )</td>
<td>density of argon atoms</td>
</tr>
<tr>
<td>( n_{ix} )</td>
<td>density of argon ions</td>
</tr>
<tr>
<td>( n_e )</td>
<td>electron density</td>
</tr>
<tr>
<td>( n_i )</td>
<td>density of species ( y ) according to the Saha equation</td>
</tr>
<tr>
<td>( w_y )</td>
<td>velocity of species ( y )</td>
</tr>
<tr>
<td>( T_e )</td>
<td>electron temperature</td>
</tr>
<tr>
<td>( T_x )</td>
<td>temperature of the argon atoms and ions</td>
</tr>
<tr>
<td>( u )</td>
<td>velocity of the plasma: ( \sum n_i m_i w_i / \sum n_i m_i )</td>
</tr>
<tr>
<td>( p_x )</td>
<td>partial pressure of particle ( x )</td>
</tr>
<tr>
<td>( p )</td>
<td>total system pressure</td>
</tr>
<tr>
<td>( \mathbf{e}_x )</td>
<td>axial unity vector</td>
</tr>
</tbody>
</table>
2.2. Heating and Ionization

To analyze the evolution of the initially relatively cold, flowing arc plasma in the arc channel, a set of differential equations will be constructed that describes the development of the electron density and the gas temperature. The plasma consists of argon atoms, argon ions, and electrons. The physical phenomena which will be taken into account are direct or indirect ionization by electron impact, three-particle recombination, radiative recombination, and energy exchange between different species by means of elastic collisions. It has been shown [7] that for sufficiently large electron densities almost every excitation of an atom in an arc plasma eventually leads to ionization. Therefore excitation will be treated as ionization.

The neutral argon atoms and the argon ions will be treated together as far as their transport equations are concerned. The heavy particle density \( n_h \) is defined by

\[
n_h = n_{at} + n_{ar}^+
\]  

(6)

Since ionization and recombination do not change the total heavy-particle density \( n_h \), the combined mass balance is

\[
\frac{dn_h}{dt} = \frac{\partial n_h}{\partial t} + \nabla \cdot (n_h \mathbf{w}_h) = 0
\]  

(7)

In the case of a stationary plasma

\[
\nabla \cdot (n_h \mathbf{u}) = 0
\]  

(8)

This can be transformed to

\[
\nabla \cdot \mathbf{u} = \mathbf{u} \cdot \left( \frac{\nabla T_h}{T_h} - \frac{p}{p_h} \nabla p + \frac{p_e}{p_h} \nabla p_e \right)
\]  

(9)

In this case we can neglect the last term, because the gradient lengths for electron pressure and total system pressure are comparable. Furthermore, the degree of ionization is at maximum 3% and consequently \( p_e \ll p, \) \( p = p_h \).

The number balance for the electrons now becomes

\[
\frac{dn_e}{dt} = \frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{w}_e) = K_{1+} n_e (n_{at} - n_{at}) - n_e n_{at}^+ (K_{1+}; \Lambda)
\]  

(10)

where \( K_{1+} \) represents the combined rate coefficient for excitation and ionization, and \( (K_{1+}; \Lambda) \) is the rate coefficient for radiative recombination corrected for local absorption. Three-particle recombination is taken into account in (10) by applying the method of detailed balancing [14]:

\[
n_e^2 n_{at} K_{1+} = n_e n_{at} K_{1+}
\]  

(11)
where $K_1^+_r$ is the rate coefficient for three-particle recombination. Since we consider a stationary situation, $\nabla \cdot j = 0$.

If $w_e = w_{ar} = w_{ar}^t = u$, then still

$$\nabla \cdot (n_e u) = \nabla \cdot (n_e u) - \nabla \cdot j/e = \nabla \cdot (n_e u)$$  \hspace{1cm} (12)

The left-hand side (lhs) of Eq. (10) can be written as

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e u) = \frac{\partial n_e}{\partial t} + u \cdot \nabla n_e + n_e (\nabla \cdot u)$$  \hspace{1cm} (13)

The one-dimensional, stationary continuity equation for $n_e$ now results from the combination of (9), (10), and (13):

$$\frac{\partial n_e}{\partial x} = \frac{1}{u} \left( K_{\text{ar}} n_e (n_{ar} - n_{ar}) - n_{ar}^+ (K_{\text{ar}} + \lambda) \right)$$

$$+ n_e \left( \frac{1}{p} \frac{\partial p}{\partial x} - \frac{1}{T_h} \frac{\partial T_h}{\partial x} \right) - \delta n_e$$  \hspace{1cm} (14)

where $\delta n_e$ is the negative production, i.e., loss of electrons by recombination at the arc wall per unit length. From here on the symbol $n_e$ has to be interpreted as a radially averaged value of the electron density. The energy equation of the heavy particles can be written as

$$\frac{\partial}{\partial t} \left( \frac{3}{2} n_h k T_h + \nabla \cdot \left( \frac{3}{2} n_h k T_h w_h \right) + n_h k T_h \nabla \cdot w_h + \pi_h : \nabla w_h + \nabla \cdot q_h \right)$$

$$= 3 \frac{m_e}{m_{ar}} n_e \left( \frac{1}{\tau_{el}} + \frac{1}{\tau_{eo}} \right) k (T_e - T_h)$$  \hspace{1cm} (15)

where the collision times are given by

$$\tau_{el} = \frac{3 \sqrt{m_e} (k T_e)^{3/2} (4 \pi \sigma)_{eo}^2}{4 \sqrt{2} \pi e^4 \ln \Lambda n_e}$$  \hspace{1cm} (16)

$$\tau_{eo} = \frac{1}{n_e \sigma_{eo}}$$  \hspace{1cm} (17)

Here $e$ is the elementary charge, $(\sigma v)_{eo}$ is the cross section for electron-neutral collisions weighted with the Maxwellian electron velocity distribution, and $\ln \Lambda$ represents the Coulomb logarithm, which can be approximated by

$$\ln \Lambda = \ln (12 \pi n_e \lambda_d^2)$$  \hspace{1cm} (18)

using the Debye length $\lambda_d$:

$$\lambda_d = \left( \frac{\varepsilon_0 k T_e}{n_e e^2} \right)^{1/2}$$  \hspace{1cm} (19)
We have implicitly used the fact that the energy exchange between ions and neutrals is more effective than the direct energy transfer from electrons to ions and to neutrals. This can be shown by calculating the ratio of the elastic energy exchange \( Q_{ab}/n_e \) and the temperature difference \( T_a - T_b \) for the cases of electron-ion \((e-i)\), electron-neutral \((e-o)\), and ion-neutral \((i-o)\) interaction. If we take
\[
Q_{ab} = 3 \frac{m_e}{m_o} n_e \frac{1}{\tau_{ab}} k(T_a - T_b)
\]
the result for a typical case \( n_e = 10^{22} \text{ m}^{-3}, n_{ar} = 3.10^{23} \text{ m}^{-3}, T_e = 12,000 \text{ K}, T_a = 8000 \text{ K} \) is

<table>
<thead>
<tr>
<th>Interaction</th>
<th>e-i</th>
<th>e-o</th>
<th>i-o</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q/n\Delta T \ [\text{J K}^{-1} \text{s}^{-1}] )</td>
<td>4.8 ( \cdot 10^{-17} )</td>
<td>2.3 ( \cdot 10^{-17} )</td>
<td>3.18 ( \cdot 10^{-16} )</td>
</tr>
</tbody>
</table>

It is clear that the energy exchange between ions and neutrals is much more effective than that between electrons and heavy particles. The small remaining differences (of the order of a few percent) are uninteresting in our model and we will use the \( T_a \) defined as \( T_a = (n_i T_i + n_o T_o)/(n_i + n_o) \). Using (9), we can write (15) as
\[
\begin{align*}
\frac{\partial}{\partial t}(\frac{1}{2}n_k T_h) + \frac{1}{2}k T_h \nabla \cdot (n_k \mathbf{w}_h) + \frac{1}{2}k n_k \mathbf{w}_h \cdot \nabla T_h + n_k T_h \mathbf{u} \cdot \left( \frac{1}{T_h} \nabla T_h - \frac{1}{p} \nabla p \right) \\
+ n_h : \nabla \mathbf{w}_h + \nabla \cdot \mathbf{q}_h = 3 \frac{m_e}{m_{ar}} n_e \left( \frac{1}{\tau_{ei}} + \frac{1}{\tau_{co}} \right) k(T_e - T_h)
\end{align*}
\]
(21)

Neglecting the effects of viscosity for the heavy particles and using (8), we write the one-dimensional, stationary equation for \( T_h \) as
\[
\frac{\partial T_h}{\partial x} = \frac{2}{5n_h k u} \left( \frac{3 m_e}{m_{ar}} n_e \left( \frac{1}{\tau_{ei}} + \frac{1}{\tau_{co}} \right) k(T_e - T_h) + \frac{n_k T_h u}{p} \frac{\partial p}{\partial x} \right) + \delta Q
\]
(22)

where \( \delta Q \) is the amount of energy per unit arc length that the heavy particles gain or lose at the arc wall; it represents the heat loss by conduction to the wall. As with \( n_e \) from here on, the symbol \( T_h \) has to be interpreted as radially averaged value of the heavy-particle temperature.

Equations (14) and (22) describe the evolution of the electron density and the heavy-particle temperature as a function of the axial position in the arc. The argon atom density \( n_{ar} \) is calculated using the equation of state, the known values of the electron density, and the temperatures \( T_e \) and \( T_a \) and the measured gas pressure: \( n_{ar} = (p - n_e k T_e)/(k T_a) - n_e \). The plasma velocity \( u \) is obtained from the known gas flow \( \phi \), the heavy-particle density \( n_h \), and the cross-sectional area of the arc channel \( A \): \( u = \phi/(n_h A) \).
As a first approximation, the losses at the arc wall (\( \delta n_e \) and \( \delta Q \)) are assumed to be negligible. The validity of this assumption will be checked later on. Equations (14) and (22) can now be solved numerically by Runge-Kutta integration.

As mentioned above, de Haas\(^9\) recently developed a model to describe the flow of a fully equilibrated argon plasma in a cascaded arc. If the continuum, momentum, and energy equations are summed over all species present in the plasma, they read (one-dimensionally)

\[
\frac{\partial p}{\partial t} + \frac{\partial}{\partial x} (pu) = 0
\]  
(23)

\[
\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = -\frac{1}{2} \rho u^2 \frac{f}{D}
\]
(24)

and

\[
\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} + u \frac{\partial u}{\partial t} + u^2 \frac{\partial u}{\partial x} - \frac{1}{\rho} \frac{\partial p}{\partial t} = Q
\]
(25)

respectively, where \( \rho \) is the mass density, \( D \) is the diameter of the arc channel, \( f \) is the wall friction factor, \( h \) is the specific enthalpy, and \( Q \) is the source term for the total energy balance. Equations (23)-(25) can be transformed into a set of linear differential equations in \( p, \rho, \) and \( T \). If the boundary conditions are taken into account, this set can be solved numerically, again by Runge-Kutta integration.

Now the arc plasma is described by two sets of equations. Equations (23)-(25) represent an LTE description of a flowing plasma. They will be used to calculate the pressure drop over the arc channel. Equations (14) and (22) describe the evolution of the electron density and the gas temperature for a nonequilibrium plasma if the pressure and the electron temperature profiles are known.

3. EXPERIMENTAL SETUP

The arc consists of an anode, a stack of electrically isolated copper plates, and three cathodes (see Fig. 1). The cathodes are made of a sharpened pin of thoriated tungsten, which is pressed into a small copper screw. This screw is mounted in a water-cooled copper shaft, which is cemented into a quartz cylinder that isolates the cathode from the support. The cylinder is fixed in the support by a compressed, sliding O-ring of Viton elastomere, which also serves as a vacuum seal. The diameter of the tungsten tips is 1 mm for currents up to 30 A per cathode, and 2 mm for higher currents. The thorium present in the material segregates to the surface and increases
the electron emission. This allows operation at lower temperatures and prevents the material from melting.

The water-cooled cathode support also incorporates a window through which the arc channel can be observed end-on. The gas is fed through the shaft on which this window is mounted.

The cascade plates are made of copper and have a central bore of 4 mm, which is surrounded by a channel through which the cooling water is flowing. The plate thickness is 5 mm. Side-on observation of the plasma is possible through small quartz windows cemented in a narrow radial bore (Fig. 2). The diameter of this bore has to be chosen as small as possible as otherwise the flow pattern would be disturbed. To obtain radial information, lateral scanning would be required; this is inconceivable with the requirement that the laminar flow should not be obstructed. Therefore, we have chosen to measure integral properties along the line of sight. In fact this is also the information we look for: the development of the total flow of the plasma in the channel.

This bore is also connected to a pressure meter to allow for the measurement of the pressure in the arc channel as a function of the axial position. The arc channel is formed by the central bores of the 10 stacked plates that are kept at an interspacing of 1 mm by PVC spacing rings. Inside the spacers a Viton O-ring is mounted as a vacuum seal. The O-ring is prevented from melting by absorption of plasma radiation by a white ring made of Teflon or boron nitride.
The anode insert is made of copper, tungsten, or graphite and has a conical shape. It is pressed into the water-cooled support. The precision with which the conical hole is fabricated assures good electrical and thermal contact. This construction of the anode allows exchange of the nozzle insert in a few seconds.

The arc is fed continuously by supplying gas through the window support on the cathode mounting assembly and connecting the anode flange to a large vacuum system. The arc plasma that emanates in this vacuum system expands supersonically. The experimental and theoretical analysis of this expansion is discussed elsewhere.\textsuperscript{8}

4. DIAGNOSTICS

To determine the electron density, the Stark broadening of the well-known hydrogen-β line is used. Griem calculated the total width of the hydrogen lines as a function of electron density.\textsuperscript{15} The result is given by

\[ n_e = C \Delta \lambda^{3/2} \]  \hspace{1cm} (26)

where \( C \) is a constant that only slightly depends on the electron temperature, and \( \Delta \lambda \) is the full width at half maximum of the line. For the H\(_{β}\) line (\( \lambda = 486.1 \text{ nm} \)), for \( T_e = 1 \text{ eV} \), \( C \) is equal to \( 1.2 \times 10^{22} \text{ m}^{-3} \text{ nm}^{-3/2} \).

The emissivities of line- and continuum radiation, \( \varepsilon_l \) and \( \varepsilon_c \), respectively, can be written as

\[ \varepsilon_l(T_e) = n_e A_{\text{pe}} \left( \frac{\hbar \nu}{4 \pi} \right) P_1 = n_e \rho^{n_0} \left( \frac{E_{\text{pe}}}{kT_e} \right) A_{\text{pe}} \left( \frac{\hbar \nu}{4 \pi} \right) P_1 \]  \hspace{1cm} (27)

\[ \varepsilon_c(T_e) = C_t \frac{n_e n_i}{\lambda^3} \left( G_1(\lambda, T_e) + (1 - \exp(-\hbar \nu/kT_e)) \xi_1 \right) \]  \hspace{1cm} (28)
where \( n_e \) is the density of the excited level, \( r^p \) is the collisional radiative coefficient of the upper level of the transition, \( E_{1p} \) is the excitation energy of level \( p, \) \( A_{eq} \) is the transition probability, \( P_1 \) is the normalized line profile, \( C_1 \) is a constant \((1.63 \cdot 10^{-45} \text{ W m}^4 \text{ K}^{1/2} \text{ sr}^{-2})\), \( G_1 \) is the Gaunt factor \((\approx 1.2 \exp(-h\nu/kT_e) \text{ at } \lambda = 420.0 \text{ nm})\), \( \xi_1 \) is the Birnbaum factor \((\approx 1.55 \text{ at } \lambda = 420.0 \text{ nm})\), and \( n_i = n_e \) ion density (only singly charged ions are taken into account).

Taking into account the convolution of both emissivities with the apparatus profile, we can calculate the line-to-continuum intensity ratio in the top of the line profile. The collisional radiative coefficient \( r^p \) is not known accurately for high electron densities and low electron temperatures. Calculations according to extensive numerical collisional radiative models tested at lower densities\(^{16}\) yield values around 0.3 for a temperature of 1 eV and an electron density of the order of \(10^{22} \text{ m}^{-3}\). Therefore the interpretation of the intensity ratios into temperatures has been carried out by taking \( r^p \) equal to 0.3. Figure 3 illustrates the line-to-continuum ratio for an argon ion line \((\lambda = 480.6 \text{ nm})\). So, if the electron density is known, the electron temperature can be evaluated.

The electrical conductivity of the plasma in the arc channel according to Frost is given by\(^{(17)}\)

\[
\sigma = \frac{4\pi n_e e^2}{3kT_e} \int_0^\infty \frac{v^4 f_m}{v^f} dv
\]

where

\[
v^f = v_{eo} + v_{eo}^' = v_{eo} + 0.476 \frac{0.582 8\pi}{\gamma_e} \frac{e^2}{v^2} \left[ \frac{1}{4\pi e_i m_e} \right] \langle z \rangle n_e \ln \Lambda \left[ \frac{m_e}{2kT_e} \right]^{1/2}
\]

and \( v_{eo} \) is the electron-neutral collision frequency, \( v_{eo}^' \) is the electron-ion collision frequency according to Frost, \( \gamma_e \) is the peaking factor to compensate for the anisotropy in the velocity distribution,\(^{(18)}\) \( v \) is the electron velocity, \( \ln \Lambda \) is the Coulomb logarithm, \( \langle z \rangle \) is the effective charge of the ions, and \( f_m \) is the reduced Maxwell distribution. Equation (30) has been evaluated, and the result is illustrated in Fig. 4.

The experimental value of the conductivity is drawn from an approximation assuming a homogeneous current distribution over the effective channel diameter:

\[
\sigma_{exp} = Id/(\pi R^2 \Delta V)
\]

where \( I \) denotes the arc current, \( d \) is the distance of two adjacent cascade plates, \( \Delta V \) is the potential difference between those two plates, and \( R \) is the effective channel radius (about 0.9 times the actual channel radius\(^{(7)}\)).
5. RESULTS AND DISCUSSION

The plasma is flowing through the arc channel at a relatively large velocity. This flow is associated with a pressure drop. Friction with the wall causes the pressure to drop faster than would be expected for an accelerating duct flow without wall interaction. Figure 5 pictures the axial profile of the measured plasma pressure for three different gas flows. In the beginning of the channel, the pressure drops linearly with the axial position. At the end, the decrease is more than linear because of acceleration of the plasma toward sonic conditions.

De Haas has numerically analyzed Eqs. (23)–(25) to evaluate the evolution of the gas pressure, the plasma velocity, and the temperature in the case of a plasma in complete LTE. The wall friction is taken into account by the friction factor $f$. This numerical program has been used to evaluate $f$ from the pressure measurements presented in Fig. 5. It appears that the friction factor is not constant throughout the plasma channel. In the first
part of the channel $f$ has values around 0.11, independent of the gas flow. In the last part of the channel, close to the sonic plane, $f$ has decreased to about 0.05. For large gas flows, $f$ appears to be slightly smaller than for small gas flows. In the case of perfect laminar flow, the friction factor $f$ can be expressed by\(^{(19)}\)

$$f = \frac{64}{R_d}$$  \(32\)

using the familiar Reynolds number $R_d$, based on the channel diameter $D$, defined by

$$R_d = \frac{\rho u D}{\eta}$$  \(33\)

where $\eta$ represents the viscosity coefficient. The viscosity coefficient $\eta$ is taken from a table given by Vargaftik\(^{(20)}\). Figure 6 shows the viscosity as a function of temperature for a number of plasma pressures. Figure 7 compares
the friction factor as a function of the Reynolds number according to (32) with accumulated values measured for the arc channel with and without the plasma being present for a variety of gas flows. As only minor differences occur between the ideal curve according to (32) and the experimental points, the corrections of the friction coefficient proposed by Prandtl and von Karman (described in Benedict’s book) for the transition toward turbulent flow are not needed. The flow can be regarded as laminar. The fact that the points representing a flow with plasma are below the ideal curve might indicate that the viscosity coefficient shown in Fig. 6 should be corrected for nonequilibrium effects. At ionizational nonequilibrium the electron density is smaller. This will result in a higher value for the viscosity and the friction factor $f$.

In Fig. 8, the plasma potential is illustrated as a function of the axial position. Again an almost linear profile has been obtained. From the known current density and the theoretical values of the electrical conductivity, the electron temperature has been estimated using this curve. The electron temperature has also been determined by the line-continuum ratio method.
The differences between the results of the two methods are negligible. Figure 9 shows the axial profile of the experimentally determined electron temperature (points), as well as the profiles of the gas temperature as they are calculated using (22) and (14) in combination with the measured profiles for pressure and electron temperature. Especially for large gas flows, the gas temperature is initially lower than the electron temperature. The plasma needs about 20 mm of axial length to relax toward temperature equilibrium. For small gas flows, this equilibration is established almost instantaneously. At the end of the channel the gas temperature decreases because of acceleration of the flow.

The calculations also yield axial profiles of the electron density. In Fig. 10 the calculated profiles are compared with the results of measurements using the Stark broadening of the H$_\beta$ line (486.1 nm), again for three values of the gas flow. The agreement between theory and experiment is good, the only exception occurring for a small gas flow at the end of the channel. Here the measured values are about 40% lower than the calculated values. This might be an indication that for these conditions the assumption that the axial gradients are much stronger than the radial gradients is no longer valid. Radial diffusion and recombination at the arc wall will lead to lower
calculated densities, which would be in better agreement with the experimental findings.

Regarding the optical measurements, one should remember that Abel inversion was impossible due to the chosen experimental configuration. The measured values of the electron density and the electron temperature might therefore deviate from the desired radially averaged value. An estimate based on radial profiles of stationary cascade arcs\(^{(7)}\) shows that the deviation is limited to about 10%.

To check whether the neglect of radial heat transport was justified, the heat flux to the arc wall is measured calorimetrically. At the beginning of the arc channel, about 7% of the dissipated electrical power is transferred to the channel wall. There the mentioned assumption is justified. At the end of the channel, the percentage increases to about 50%, and there the assumption becomes questionable.

Thus, there are two indications that the simplification of the equations to a one-dimensional system is justified only in the first (most upstream)
part of the arc channel: the discrepancy between measured and calculated values of the electron density and the measured heat flux to the arc wall.

The choice of the starting value of the electron density for the Runge-Kutta integration is not trivial. A low value (e.g., 0) is not realistic because the cathode fall region does not at all behave like the bulk of the plasma. Therefore, the calculations have been limited to the plasma bulk: the starting point is the first cascade plate. The calculated profiles have been obtained by taking the measured value of the electron density in the first cascade plate as a starting value, while adjusting the starting value of the gas temperature to give a best fit to the measurements. Once the starting conditions (in this case the electron density and gas temperature at the beginning of the arc channel) are set, the complete axial profile is determined by Runge-Kutta integration of Eqs. (14) and (20).

A remarkable feature is that, for constant current, the measured electron density does not differ very much for the three values of the gas flow, in spite of the fact that the pressure does differ strongly (see Fig. 5). This can be explained by analyzing the electron energy balance terms in the first part of the arc channel. If ohmic input is included, and heat conduction, viscosity, radiative losses, and expansion terms are neglected, the net electron energy
Fig. 9. The various temperatures as a function of axial position. Parameter is the argon flow (50, 100, and 200 scc/s). The points represent the values of the electron temperature as measured with line-continuum ratio, whereas the curves correspond to the calculations for the gas temperature.

The balance reads (7)

\[ j \cdot E = n_e n_{ars} K_{i} + (b_1 - 1) E^{1+} + \frac{m_e}{m_{ars}} n_e \left( \frac{1}{\tau_{ei}} + \frac{1}{\tau_{co}} \right) k(T_e - T_h) \]  

(34)

where the overpopulation \( b_1 \) of the argon neutral ground level is defined by

\[ b_1 = n_{ars}/n_{ars} \]

(35)

The Saha density \( n_{ars} \) is defined by the Saha equation (5). The term on the left-hand side represents the ohmic heating, and the terms on the right-hand side represent the electron energy loss because of ionization of argon atoms and heat exchange between electrons and heavy particles because of elastic collisions, respectively.

In other words, the ohmic input balances the heating of the heavy particles by electrons and the net ionization. With an ohmic heating power of (in this case) \( 4 \times 10^6 \) W m\(^{-3}\) and a temperature difference between electrons and heavy particles of 3000 K (see Fig. 9), which leads to an energy exchange
term of $3 \cdot 10^9$ W m$^{-3}$, the value of the overpopulation factor $b_1$ can be estimated to be around 60. In the beginning of the arc channel, temperature equilibration takes about 60% of the ohmic input and ionization of argon atoms absorbs the other 40%.

Figure 11 shows the axial profiles of the overpopulation factor as it is obtained from the measured electron densities (Fig. 10) and gas pressures (Fig. 5) in combination with the calculated gas temperatures (Fig. 9). At
the beginning of the arc channel the value of this experimentally determined overpopulation factor is around 40, which agrees fairly well with the value estimated from the electron energy balance. At the end of the channel, \( b_i \) is about 10. The overpopulation factor \( b_i \) essentially determines the net electron production by ionization of argon atoms by electron impact. Therefore, we can conclude that at the beginning of the arc channel a large fraction (about 60%) of the energy supplied to the plasma is used to heat the originally cold gas until its temperature equals the electron temperature. The remaining energy (around 40%) is used to ionize the argon atoms. This gives rise to a constant production of electrons and thus of ions. Once the gas is heated and ionized, the (now smaller) net electron and ion production is compensated by expansion because of the pressure drop. While the total ion flux still increases slightly, the ion density does not.

6. SUMMARY AND CONCLUSIONS

Measurements of the gas pressure, the electron density, and the electron temperature as a function of axial position in a strongly flowing cascaded arc are reported for several values of the gas flow. The models of De Haas
adequately explain the pressure drop in the arc channel. The flow is laminar. The nonequilibrium models outlined in this paper satisfactorily describe the evolution of the electron density. The assumption that the flow can be treated one-dimensionally is, however, only justified in the most upstream section of the arc channel. The plasma is strongly ionizing: the overpopulation factor of the ground level is always larger than 10. The model calculations suggest that for large gas flows the gas temperature approaches the electron temperature within an axial distance of 20 mm.

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