TECHNICAL NOTE

DIRECT MECHANICS ASSESSMENT OF ELASTIC SYMMETRIES AND PROPERTIES OF TRABECULAR BONE ARCHITECTURE

B. Van Rietbergen,* A. Odgaard,† J. Kabel† and R. Huiskes*‡

*Biomechanics Section, Institute of Orthopaedics, University of Nijmegen, P.O. Box 9101, 6500 HB Nijmegen, The Netherlands; and †Biomechanics Laboratory, Department of Orthopaedic Surgery, Aarhus University Hospital, Denmark

Abstract—A method is presented to find orthotropic elastic symmetries and constants directly from the elastic coefficients in the overall stiffness matrix of trabecular bone test specimens. Contrary to earlier developed techniques, this method does not require pure orthotropic behavior or additional fabric measurements. The method uses high-resolution computer reconstructions of trabecular bone specimens as input for large-scale FE-analyses to determine all the 21 elastic coefficients in the overall stiffness matrix of the specimen, using a direct mechanics approach. An optimization procedure is then used to find the coordinate transformation that yields the best orthotropic representation of this matrix. The method is illustrated here relative to two trabecular bone specimens. The techniques developed here can be used to obtain a complete characterization of the mechanical properties of trabecular architecture. With the development of *in vivo* reconstruction techniques, even *in vivo* measurements will be possible. Copyright © 1996 Elsevier Science Ltd.

Keywords: Trabecular bone; Bone mechanics; Bone architecture; Bone reconstruction; Finite element analysis.

INTRODUCTION

The anisotropic mechanical behavior of trabecular bone is largely determined by its trabecular architecture. Precise determination of mechanical behavior is of particular importance to assess relationships between morphometric parameters and mechanical properties, which are used to characterize the mechanical fitness of bone in normal and pathological conditions. To evaluate these relationships in a particular specimen, the constitutive parameters governing its mechanical behavior must be determined experimentally.

For most purposes, bone can be considered as a linear elastic material, for which the mechanical behavior is characterized by a fourth-rank stiffness tensor $\mathbf{E}$ in the generalized Hooke's law that relates the stress to the strain tensor. The stiffness tensor is usually represented by a symmetric 6×6 matrix E [Fig. 1(a)]. In its most general form, this matrix involves 21 independent elastic coefficients that must be determined from experiments. If planes of elastic symmetry exist, some of these coefficients are interdependent or zero when measured in a coordinate system aligned with the normals to the symmetry planes (Bunge, 1982; Cowin and Mehrabadi, 1987, 1989; Nye, 1957). In the case of orthotropy, three orthogonal planes of symmetry exist, leaving nine independent elastic coefficients to be determined from experiments [Fig. 1(b)]. The number of independent elastic constants is reduced further to five for the case of transverse isotropy and to two for the case of isotropy.

Elastic symmetries are usually determined indirectly by considering symmetries in the texture or fabric of the material. Fabric can be evaluated, for example, by mean-intercept length (MIL) measurements (Harrigan and Mann, 1984; Whitehouse, 1974). For trabecular bone it was found that the MIL fits well to an ellipsoid, which has three planes of symmetry. Based on this

finding, it was proposed that trabecular bone architecture exhibits orthotropic material behavior (Ashman *et al.*, 1984; Cowin and Mehrabadi, 1989; Harrigan and Mann, 1984; Snyder *et al.*, 1989; Whitehouse, 1974). The nine orthotropic elastic coefficients in a coordinate system aligned with the fabric axes can be determined from nine elastic coefficients measured by compression tests or ultrasound experiments in the specimens' coordinate system, using a coordinate transformation (Cowin *et al.*, 1991). The accuracy of the material characterization thus determined depends on the accuracy of the mechanical test and the fabric measurement, and on the adequacy of the orthotropy assumption.

For materials that exhibit pure orthotropic or higher symmetries, Cowin and Mehrabadi (1987, 1989) and Cowin (1989) have developed a method to determine the principal orthotropic axes directly from the 21 components of the stiffness tensor. Since no mechanical tests have been developed yet from which all 21 elastic coefficients can be measured, the application of this method has been limited to theoretically derived stiffness tensors. With this method, inaccuracies in the fabric measurements are eliminated. However, errors due to inaccuracies in mechanical tests or due to the inadequacy of the orthotropy assumption remain. If the material is not purely orthotropic, or if inaccuracies in mechanical tests defy the symmetries, no unique symmetry basis is found.

In this article, a method is presented to estimate elastic symmetries of bone specimens, without the need for compression test experiments, without the need to assume pure orthotropic behavior and without the need for fabric measurements. For this purpose, high-resolution computer reconstructions of trabecular bone specimens are used as input for large-scale FE-analyses to determine all the 21 elastic coefficients in the overall stiffness matrix of the specimen, using a direct mechanics approach. An optimization procedure is then used to find the coordinate transformation that yields the best orthotropic representation of this matrix. The method is illustrated here relative to two trabecular bone specimens.

$$\begin{array}{ll}
 \text{a)} & E = \begin{bmatrix} e_{11} & e_{12} & e_{13} & e_{14} & e_{15} & e_{16} \\ e_{12} & e_{22} & e_{23} & e_{24} & e_{25} & e_{26} \\ e_{13} & e_{23} & e_{33} & e_{34} & e_{35} & e_{36} \\ e_{14} & e_{24} & e_{34} & e_{44} & e_{45} & e_{46} \\ e_{15} & e_{25} & e_{35} & e_{45} & e_{55} & e_{56} \\ e_{16} & e_{26} & e_{36} & e_{46} & e_{56} & e_{66} \end{bmatrix} & \text{Anisotropic} \\
 \\
 \text{b)} & E^{\text{ORT}} = \begin{bmatrix} e_{11} & e_{12} & e_{13} & 0 & 0 & 0 \\ e_{12} & e_{22} & e_{23} & 0 & 0 & 0 \\ e_{13} & e_{23} & e_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & e_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & e_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & e_{66} \end{bmatrix} & \text{Orthotropic}
 \end{array}$$

Fig. 1. In its most general form the stiffness matrix holds 21 different elastic coefficients (a). For materials with orthotropic elastic symmetries this number is reduced to 9 (b).

METHODS

Material

Two cubic specimens were harvested from a whale vertebral body. The first specimen was cut such that its sides were aligned with the body longitudinal axis, the second specimen such that its sides were rotated 45° relative to this axis. Both specimens measured approximately 10 mm in size at each side.

Three-dimensional reconstruction

The morphology of each specimen was digitized in a serial sectioning procedure (Odgaard *et al.*, 1994). This procedure uses an automated microtome and a digital camera to obtain a set of digitized images of sequential cross-sections. The magnification of the camera was set such that the pixels were 20 μm in size. The slice thickness was also chosen as 20 μm. After segmentation of the images, the morphology of each cube was numerically reconstructed in a three-dimensional voxel data set. To exclude boundary artifacts due to the cutting, sections near the boundaries with an area fraction less than 75% of the volume fraction were removed from the data set. In this way smooth surfaces were ensured such that all cut trabeculae were load-carrying.

FE-modeling

The voxel data sets were used as the geometries for the microstructural FE-models. For this purpose, the original data sets were remeshed into a coarser grid by grouping 4 × 4 × 4 voxels into a new one. The new voxel was considered to represent bone tissue if more than half of its original voxels represented bone tissue, and was deleted otherwise. FE-models were generated by directly converting the new bone voxels to equally sized eight-node brick elements (Fig. 2). The elements in the FE-model thus measured 80 μm on each side. In an earlier mesh-convergence study (Van Rietbergen *et al.*, 1995a), it was found that with this element size the model was sufficiently converged, whereas it allowed the modeling of a reasonably large volume of bone (Van Rietbergen *et al.*, 1994, 1996). The number of elements in the FE-models was 566,967 for the first specimen and 653,533 for the second. A special purpose FE-code was used to solve these large FE-problems. This solving

algorithm makes use of an iterative solver in combination with a 'row-by-row' matrix-vector multiplication algorithm (Van Rietbergen *et al.*, 1994, 1996). In this way the memory required for solving could be limited to 222 Mbyte for the largest problem. The cpu-time in this case was 17 min for a single analysis when using a Cray C90 supercomputer for the calculations.

Calculation of the overall stiffness matrix of a specimen

The FE-models were used to calculate the overall (or apparent) material properties of a specimen, as characterized by the stiffness tensor. For this purpose, a direct mechanics approach was used. With this approach the specimen's fourth-rank effective stiffness tensor $\bar{\mathbf{E}}$ in the generalized Hooke's law is obtained from the relationship (Hill, 1963; Hollister *et al.*, 1991; Hollister and Kikuchi, 1992)

$$\bar{\mathbf{E}} = \frac{1}{V} \int \mathbf{E} \mathbf{M} dV, \quad (1)$$

with V the volume of a representative volume of bone, $\mathbf{E}$ the tissue stiffness tensor and $\mathbf{M}$ the local structure tensor (Hollister *et al.*, 1991), also called the strain localization tensor (Suquet, 1985), which relates the local tissue strain tensor $\boldsymbol{\varepsilon}$ to the average strain tensor $\bar{\boldsymbol{\varepsilon}}$, from

$$\boldsymbol{\varepsilon} = \mathbf{M} \bar{\boldsymbol{\varepsilon}}. \quad (2)$$

The tissue stiffness tensor $\mathbf{E}$ is defined by the trabecular tissue properties. In the present study, linear elastic and isotropic tissue properties were assumed. In an earlier study, using one specimen from the present data set, a tissue Young's modulus of 5.33 GPa was found to give the best agreement between the apparent moduli calculated from the FE-model and those measured in experiments (Van Rietbergen *et al.*, 1995b). For this reason, all specimens in the present data set were assigned this tissue Young's modulus; the tissue Poisson's ratio was set to 0.3.

The local structure tensor $\mathbf{M}$ was determined from equation (2) by solving six FE-problems for six uniaxial strain cases. For each case, the tissue strains calculated from the FE-results then represent one of the six columns of the local structure matrix (Hollister and Kikuchi, 1992). The effective stiffness tensor can then be determined from equation (1).

Specimen 1

Volume fraction: 0.30
 # voxels: 124×121×121
 # elements: 540,217

Specimen 2

Volume fraction: 0.35
 # voxels: 125×125×120
 # elements: 653,533

Fig. 2. The FE-models generated from the 3-D reconstructions of the specimens.

Calculation of the principal orthotropy axes from the stiffness matrix

For materials with pure orthotropic symmetries a stiffness matrix of the form in Fig. 1(b) is found when the measurement axes are aligned with the normals of the symmetry planes. Hence, for such materials the determination of the principal orthotropy axes is equivalent to finding the coordinate transformation that transforms $\bar{E}$ in the form $\bar{E}^{\text{ORT}}$ of Fig. 1(b). This can be accomplished using, for example, the methods developed by Cowin and Mehrabadi (1987, 1989). For materials that do not have pure orthotropic symmetries, such a transformation does not exist. However, using a numerical optimization algorithm, it is possible to find the coordinate

transformation that yields the best orthotropic representation of the stiffness matrix

$$\bar{E}^{\delta} = \begin{bmatrix} e_{11} & e_{12} & e_{13} & \delta_{14} & \delta_{15} & \delta_{16} \\ e_{12} & e_{22} & e_{23} & \delta_{24} & \delta_{25} & \delta_{26} \\ e_{13} & e_{23} & e_{33} & \delta_{34} & \delta_{35} & \delta_{36} \\ \delta_{14} & \delta_{24} & \delta_{34} & e_{44} & \delta_{45} & \delta_{46} \\ \delta_{15} & \delta_{25} & \delta_{35} & \delta_{45} & e_{55} & \delta_{56} \\ \delta_{16} & \delta_{26} & \delta_{36} & \delta_{46} & \delta_{56} & e_{66} \end{bmatrix}, \quad (3)$$

where δ_{ij} is small.

As an optimization criterion, an orthotropy objective function was defined as

$$\text{Obj} = \frac{\sum_{i,j} \delta_{ij}^2}{\sum_{i,j} e_{ij}^2}, \quad i, j = 1, \dots, 6, \quad (4)$$

where the summation applies only to the terms that exist in the matrix of equation (3). For purely orthotropic materials the objective of equation (4) will be zero. By finding the coordinate transformation that minimizes this objective function, the orthonormal basis for which the orthotropy assumption is best met is found (Appendix A). The best orthotropic material characterization E^{ORT} is then found by setting $\delta_{ij} = 0$.

An indication of the accuracy of the orthotropy assumption is possible by quantifying the errors in the stress-strain calculations based on this assumption. In the material characterized by matrix E , the stresses in the material are related to the strains by the generalized Hooke's law: $\sigma = E\varepsilon$. For a material which is characterized by the estimation E^{ORT} of E , the relationship will be $\sigma = E^{\text{ORT}}\varepsilon$. For a particular stress state σ the difference between the strain vector ε , corresponding to matrix E , and the strain vector ε^{ORT} , corresponding to E^{ORT} , can be written as:

$$\varepsilon - \varepsilon^{\text{ORT}} = (I - (E^{\text{ORT}})^{-1}E)\varepsilon, \quad (5)$$

with I the identity matrix. The relative error in the strain vector is then given by the matrix

$$D^{\text{ORT}} = I - (E^{\text{ORT}})^{-1}E. \quad (6)$$

The maximum error in the strain calculation due to the orthotropy assumption for any stress/strain state can be quantified as

$$\text{Error}^{\text{ORT}} = \max_{\varepsilon \neq 0} \frac{\|\varepsilon - \varepsilon^{\text{ORT}}\|}{\|\varepsilon\|} = \|D^{\text{ORT}}\|, \quad (7)$$

with $\|\cdot\|$ the Euclidean vector or matrix norm (Kreyszig, 1993).

RESULTS

The stiffness matrices for the first specimen, that was cut aligned with the anatomical main axes [Fig. 3(a)], resembled the orthotropic form shown in Fig. 1(b). The maximum error in the stress-strain calculation when the material was assumed as pure orthotropic with respect to the specimens' coordinate system was found to be 5.29%. This indicates that the material can be considered as orthotropic reasonably well, and that the orthotropy axes are almost aligned with the anatomical axes. For the

second specimen, that was cut at a 45° angle with the anatomic longitudinal axis, the matrix form was far from orthotropic. Hence, a calculation of the error in the stress-strain relation, as with the first specimen, could not even be considered.

After coordinate transformations to the bases found by the optimization procedure, the orthotropic matrix form of Fig. 1(b) was closely met for both specimens [Fig. 3(b)]. The orthotropy objective function with this transformation was 5.0E-6 and 1.15E-5 for the first and second specimen, respectively. The maximal errors in the stress-strain calculations when assuming pure orthotropy were reduced to 1.35% for the first specimen and 2.15% for the second.

DISCUSSION

Using three-dimensional microstructural FE-models generated from computer reconstructions of trabecular bone specimens, it is possible to calculate all elastic constants that determine the mechanical behavior of the trabecular architecture, as well as the planes of elastic symmetry.

A limitation of the methods presented here is that, with the assumption of homogenous and isotropic tissue material properties, the pure mechanical properties of the trabecular architecture are found, rather than the mechanical properties of trabecular bone. A value for the tissue modulus must still be based on experiments, in order to obtain realistic values for overall elastic moduli.

Compared to using experimental tests alone, however, the methods described here have three advantages. First, the best fitting orthotropy axes are determined directly from the stiffness matrix, hence no additional fabric data are needed. Second, since all the 21 elastic constants are found, the method allows not only for the determination of the best orthotropic representation and elastic constants, but also for a quantification of the error in the stress-strain relationship due to this assumption. Third, with the direct mechanics approach for the determination of the stiffness matrix, experimental artifacts such as the effects of differences in specimen size and the effect of boundary artifacts are largely reduced. In this respect, the method developed here is the more complete tool for the determination of the pure mechanical properties of trabecular bone architecture, but a rigorous comparison of experimental and simulation data will be needed to determine if an 'effective' isotropic modulus can give accurate results for the calculation of the overall mechanical properties of bone.

Fig. 3. Stiffness matrices E^S for the two specimens in their coordinate system as sectioned (a), and in the optimized orthotropic coordinate system E^O (b). The transformation that interrelates both coordinate systems is given by matrices R .

With the methods developed here and with recent developments of *in vivo* three-dimensional reconstruction techniques (Kinney *et al.*, 1994, Müller *et al.*, 1994), it will ultimately be possible to estimate the mechanical properties of the trabecular architecture *in vivo*, which clearly is impossible with traditional mechanical tests. A more immediate purpose is the application to comparative evaluations of *in vitro* trabecular properties.

Acknowledgements—This project was sponsored by The Netherlands Foundation for Research (NWO/Medical Sciences), by the National Computing Facilities Foundation (NCF), by the Danish Rheumatism Association (233-588) and by the Danish Research Councils (5210563-1 and 9313411).

REFERENCES

- Ashman, R. B., Cowin, S. C., Van Buskirk, W. C. and Rice, J. C. (1984) A continuous wave technique for the measurement of the elastic properties of cortical bone. *J. Biomechanics* **17**, 349–361.
- Bunge, H. J. (1982) *Texture Analysis in Material Science*, Butterworths, London.
- Cowin, S. C. (1989) Properties of the anisotropic elastic tensor. *Q. Jl. Mech. Appl. Math.* **42**, 249–266.
- Cowin, S. C. and Mehrabadi, M. (1987) On the identification of material symmetry for anisotropic elastic materials. *Q. Jl. Mech. appl. Math.* **40**, 451–476.
- Cowin, S. C. and Mehrabadi, M. (1989) Identification of the elastic symmetry of bone and other materials. *J. Biomechanics* **22**, 503–515.
- Cowin, S. C., Sadeh, A. M., and Luo, G. M. (1991) Correction formulae for the misalignment of axes in the measurement of the orthotropic elastic constants. *J. Biomechanics* **24**, 637–641.
- Harrigan, T. and Mann, R. W. (1984) Characterization of microstructural anisotropy in orthotropic materials using a second rank tensor. *J. Mat. Sci.* **19**, 761–767.
- Hill, R. (1963) Elastic properties of reinforced solids: some theoretical principles. *J. Mech. Phys. Solids* **11**, 357–372.
- Hollister, S. J., Fyhrie, D. P., Jepsen, K. J. and Goldstein, S. A. (1991) Application of homogenization theory to the study of trabecular bone mechanics. *J. Biomechanics* **24**, 825–839.
- Hollister, S. J. and Kikuchi, N. (1992) A comparison of homogenization and standard mechanics analyses for periodic porous composites. *Comp. Mech.* **10**, 73–95.
- Kinney, J. H., Nichols, M. C. and Lane, N. E. (1994) In vivo, three-dimensional histomorphometry: new perspectives of sequential changes in bone structures. *Trans. Orthop. Res. Soc.* **19**, 56.
- Kreyszig, E. (1993) *Advanced Engineering Mathematics* (7th Edn). Wiley, New York.
- Nye, J. F. (1957) *Physical properties of crystals: their representation by tensors and matrices*. Clarendon Press, Oxford.
- Müller, R., Hildebrand, T. and Rüegsegger, P. (1994) Non-invasive bone biopsy: a new method to analyse and display the three-dimensional structure of trabecular bone. *Phys. Med. Biol.* **39**, 145–164.
- Odgaard, A., Andersen, K., Ullerup, R., Frich, L. H. and Melsen, F. (1994) Three-dimensional reconstruction of entire vertebral bodies. *Bone* **15**, 335–342.
- Press, W. H., Teukolsky, S. A., Vetterling, W. T. and Flannery, B. P. (1992) *Numerical Recipes in Fortran* (2nd Edn). Cambridge Univ. Press, Cambridge.
- Snyder, B. D., Cheal, E. J., Hipp, J. A. and Hayes, W. C. (1989) Anisotropic structure–property relations for trabecular bone. *Trans. Orthop. Res. Soc.* **14**, 265.
- Suquet, P. M. (1985) Elements of homogenization theory for inelastic solid mechanics. In *Homogenization Techniques for Composite Media* (Edited by Sanchez-Palencia, E. and Zaoui, A.). Springer, Berlin.
- Van Rietbergen, B., Weinans, H., Huiskes, R. and Odgaard, A. (1995a) A new method to determine trabecular bone elastic properties and loading using micromechanical finite-element models. *J. Biomechanics* **28**, 69–81.
- Van Rietbergen, B., Weinans, H., Huiskes, R. and Polman, J. W. (1994) A fast solving method for large-scale FE-models generated from computer images, based on a row-by-row matrix–vector multiplication scheme. *ASME/CED* **6**, 47–52.
- Van Rietbergen, B., Huiskes, R., Weinans, H., Odgaard, A. and Kabel, J. (1995b) The role of trabecular architecture in the anisotropic mechanical properties of bone. In *Bone Structure and Remodeling* (Edited by Odgaard, A. and Weinans, H.). World Scientific, Singapore.
- Van Rietbergen, B., Weinans, H., Huiskes, R. and Polman, J. W. (1996) Computational strategies for iterative solution of large FEM applications employing voxel data. *Int. J. Numer. Methods Eng.* (in press).
- Whitehouse, W. J. (1974) The quantitative morphology of anisotropic trabecular bone. *J. Microsc.* **101**, 153–168.

APPENDIX A

The components of the stiffness tensor in two different orthonormal coordinate systems E_{ijkl} in the e_i coordinate system and $E_{\alpha\beta\gamma\delta}$ in the e_α coordinate system are related by

$$E_{ijkl} = R_{i\alpha} R_{j\beta} R_{k\gamma} R_{l\delta} E_{\alpha\beta\gamma\delta}, \quad (A1)$$

with $R_{i\alpha}$ the components of an orthogonal transformation that interrelates both coordinate systems. Relative to the e_i coordinate system, R can be represented by a matrix:

$$R = \begin{bmatrix} \cos \alpha \cos \chi + \sin \alpha \sin \beta \sin \chi & \sin \alpha \cos \beta & -\cos \alpha \sin \chi + \sin \alpha \sin \beta \cos \chi \\ -\sin \alpha \cos \chi + \cos \alpha \sin \beta \sin \chi & \cos \alpha \cos \beta & \sin \alpha \sin \chi + \cos \alpha \sin \beta \cos \chi \\ \cos \beta \sin \chi & -\sin \beta & \cos \beta \cos \chi \end{bmatrix} \quad (A2)$$

with α , β and χ the angle of rotation about the z -, x - and y -axes, respectively.

An optimization procedure based on a Powells algorithm (Press *et al.*, 1992) is used to find the rotation angles that minimize the orthotropy objective function of equation (4). The columns of R then represent the basis for which the orthotropy objective is best met.