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TECHNICAL NOTE

A STEREOPHOTOGRAMMETRIC METHOD FOR MEASUREMENTS OF LIGAMENT STRUCTURE

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Abstract—A stereophotogrammetric method is presented to reconstruct the course of a curve in the three-dimensional space. This method is exclusively suitable as a non-destructive tool to determine the surface fiber-structure of ligaments, tendons and other organised collagenous structures. In addition, it is a convenient tool to measure the geometry of articular surfaces and other complicated surface shapes.

NOMENCLATURE

F, G	camera stations
K	curve to be reconstructed
A, B, C	points on K
R	reconstruction of point C
E	perpendicular projection of C on the plane FAB
P	arbitrary point in space
K′, A′, B′ etc. and K″, A″, B″ etc.	denote the projections of K, A, B etc. on the fiducial plane by the camera stations F and G respectively.
a, **b**, **c** etc.	denote the position vectors of A, B, C etc.
x	arbitrary position vector
h	normal vector of the fiducial plane
a	constant in the fiducial plane description
q	direction vector of an arbitrary line l
q′	direction vector of the projection of this line l by camera station F
M	matrix in the central projection formula
ε	reconstruction error
u	distance E–C
n	unit normal vector on the plane FAB
ω	magnification factor
ϕ	angle of incidence of camera station F
u'	distance E′–C′
n′	projection of the normal vector n of FAB through E, which is the direction of E′C′
t	direction of A′B′
u^*	component of E′C′ perpendicular to A′B′
n*	estimate for **n**′
ω^*	estimate for ω
ζ	enclosed angle between A′B′ and E′C′
h	distance A′–B′
ε^*	estimate for ε
J	configuration parameter in the error estimation
u^*_{max}	maximal deviation of K′ from the straight line segment A′B′
ρ	radius of curvature of K′ over A′B′
ε^*_{max}	estimate for the maximal reconstruction error over A′B′

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INTRODUCTION

Essential for the understanding of the mechanical behavior of complicated collagenous structures, such as ligaments and tendons, is a quantitative description of their geometry and collagen structure. The present paper introduces a non-destructive and remote method to determine the geometry and the superficial fiber orientation of these structures.

The method is called SCR (Stereophotogrammetric Curve Reconstruction) and uses principles of traditional photogrammetry. Huiskes *et al.* (1985) emphasized the advantages of close range photogrammetry for surface shape reconstruction, in the sense that all the necessary information can be recorded after preparation, whereafter the specimen can be discarded. The surfaces are not touched during the measurements, so no local damage or deformation can occur.

The development of SCR for ligaments and tendons is based on the observation, that ligaments consist of individual fibers, with an almost parallel‡ course between the insertions. The method itself and its application to ligaments are discussed here. In addition, the application to complicated surfaces is illustrated and an error analysis for the method is presented.

THEORETICAL BASIS

The principles and practice of stereophotogrammetry have been described previously (Selvik, 1974; Ghosh, 1979; Ghosh, 1983; Huiskes *et al.*, 1985). The SCR-method is similar to traditional stereophotogrammetry in the sense that it uses photographic exposures of the object and a calibration cage, taken from two angles. Whereas object points are reconstructed in traditional stereophotogrammetry, SCR reconstructs an object curve, such as a ligament fiber.

An outline of a possible experimental setting, used to elucidate the method presented here, is shown in Fig. 1. The calibration cage has two sets of easily identifiable points, of which the relative three-dimensional positions are *a priori* known with respect to the cage-fixed laboratory coordinate system.

The set contains the fiducial markers, all of them occupying positions in one fiducial plane. These markers are used

‡Parallel in this sense means, that fibers, if they are neighbors anywhere in the ligament, they are neighbors over their total length.

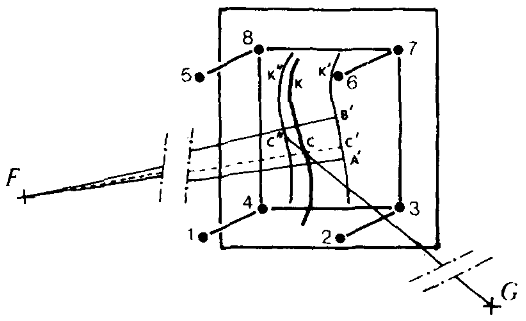

Fig. 1. Schematic drawing, elucidating the method. The points 3, 4, 7 and 8 of the calibration cage represent the fiducial plane.

to calculate the projective transformation constants for each photograph with respect to the fiducial plane, in order to calculate for each digitized point of a photograph its fiducial 'ghost' image (Huiskes *et al.*, 1985).

The second set of calibration points contains the control points, occupying positions in a plane parallel to the fiducial plane. These are used to calculate the positions of the camera stations with respect to the laboratory coordinate system.

Considered is now a curve K, projected from two camera stations F and G on the fiducial plane (Fig. 1). For any point of the projected curve K″, there exists at least one point C′ on K′, so that their projection lines intersect. The intersection, related to C″ and C′, is the original C.

In traditional stereophotogrammetry, the points C″ and C′ must be identified and measured directly from the exposures. In the present case, the projected curves on both exposures are digitized individually, in an arbitrary sense. For every point C″ on K″, the computer program will select two points A′ and B′ on K′, which define the curve section on which C′ must be located. This selection is linearized, and the spatial plane FA′B′ is reconstructed. Intersection of projection line GC″ with this plane delivers an estimate of the position of the original C.

In SCR the two projections of a curve K, K_1 and K_2, are digitized individually, in an arbitrary sense. For every point C″ on K″, which successively is K_1 and K_2, the computer program will select two points A′ and B′ on K′, being respectively K_2 and K_1, which define the curve section on which C′ must be located, and calculate the reconstructions.

Evidently, the equation system determining the estimated position of C is ill-conditioned, if the angle between the projection line GC″ and the plane FA′B′ and the line through the camera stations is small. For that reason, it can be necessary to make a second filmpair for a certain setup with the pair of cameras a quarter turn rotated around an axis perpendicular to the fiducial plane.

Another possible reason to take more than one filmpair concerns the restricted visibility of an entire object. In such a case it is necessary to rotate the object within the laboratory coordinate system. If necessary, the position of the camera stations can be changed as well. The presence of sufficient markers on the object surface, which enables the coupling between the filmpairs, is then imporant.

EQUIPMENT AND LABORATORY SETTING

The method has been applied to three different subjects: (i) the measurement of the articular surface geometry of a human tibial plateau of the knee joint, (ii) to acquire the cross-sectional surface geometry of the stem of a hip prosthesis, and (iii) to record quantitatively the surface-fiber structure of cruciate ligaments.

The equipment used in these experiments is the following.

Camera. A SINAR P professional camera with a Rodenstock-Sironar N lens ($f = 240$ mm); Kodak Ektapan Chromatik film plates (black and white negatives), 4×5 in. for the tibia plateau and the hip prosthesis, Kodak Ektachrome Professional film plates (colour positives, tungsten), 4×5 in. for the ligaments. In case of the ligaments a halogen spot with a polarization filter was used in combination with a polarization filter in front of the camera lens in order to improve fiber visibility.

Calibration devices. (i) A stainless-steel calibration cage with plastic targets for eight fiducial and eight control markers was used for the hip prosthesis. The size of the fiducial plane was 200×200 mm, the depth between the fiducial and the control plane was 50 mm, and the relative position of the markers are known with a precision of 2 μm. The objects are located within the cage. For the tibial plateau (ii) and the ligaments (iii), an aluminium calibration block with engraved targets for six fiducial and three control markers was used. The size of the fiducial plane was 30×30 mm, the depth between the fiducial and the control plane was 20 mm, and the block was manufactured with a tolerance of 50 μm. The block is placed as closely as possible to the objects.

Projected grid. In the case of the tibial plateau (i), an ordinary slide projector was used, with an engraved grid slide with a sufficient number of lines, giving sufficiently sharp lines at the articular surface.

Coordinate digitizer. An Aristomat 104 M two-dimensional mechanical coordinate digitizer, with an accuracy of 20 μm, and a resolution of 5 μm.

The human tibia was placed in the calibration cage and a grid was projected on the articular plateau. One pair of photographs was sufficient to cover the whole articular surface. The angle of incidence with respect to the fiducial plane was approximately 60°.

The hip prostheses were marked by lines perpendicular to the longitudinal axis of the stem, and these lines, representing cross-sections, were reconstructed. Four filmpairs were necessary to cover the whole outline of the stem. For the coupling procedure 32 object points were used.

In a pilot experiment the fiber structure of the medial side of an anterior cruciate ligament of a right pig knee was measured. The capsule, the collateral ligaments, the posterior cruciate and the menisci were removed. By a sagittal cut through the femur the medial condyle was removed to obtain a clear view at the anterior cruciate. Synovial and loose tissue was removed. The femoral and tibial ends were fixed in a rig with six degrees of freedom, allowing a search for the position of the bones, whereby all the fibers of the ligament were tightened without the application of high forces.

A considerable problem in applying this technique to ligaments is the quality of the photographic exposures. The trials to optimize the experimental conditions resulted in: (i) the usage of only one spotlight; for each camera position the optimal spot position was searched for, (ii) the use of polarization filters to suppress any reflexion; (iii) a calibration device of the same dimensions as the ligament, placed as closely as possible to the ligament, to obtain a large image of the ligament on the photograph; (iv) the use of colour positive film; and (v) the use of more than two photographs of one configuration, in order to have the opportunity to assemble the best photograph of a specific fiber to a photogrammetric pair. In this experiment six photographs were used.

RESULTS

Figure 2 shows the reconstruction of the tibial plateau. The advantage of SCR above traditional photogrammetry (Huiskes *et al.*, 1985) is two-fold: (i) the faster data-acquisition procedure, instead of object points only a small number of object curves has to be identified, and (ii) the variable sample density, useful if the curvature varies over the surface.

Fig. 4. Detail of photograph no. 4 of the six ACL photographs.

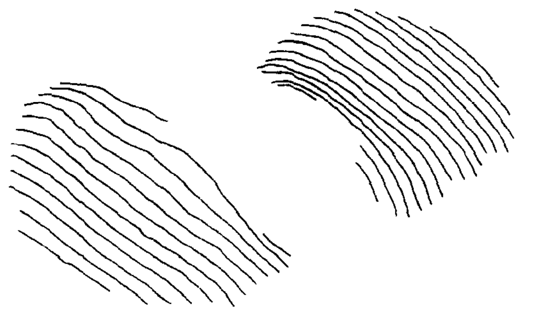

Fig. 2. SCR reconstruction of a tibial plateau in the knee joint.

Figure 3 shows the reconstructions of the cross-sections of the hip prosthesis stem.

Figure 4 shows a part of the fourth photograph of the anterior cruciate. Reconstructions of the digitized single photographs are shown in Fig. 5. These reconstructions include the contours of the femoral condyl, the tibial plateaus and the femoral cutting surface, which are not used for the actual three-dimensional reconstruction.

Eight filmpairs were assembled to reconstruct as many curves as possible. Figure 6 shows the reconstructed ligament, with the local fiber orientation indicated.

Error analysis

Apart from well known error sources in the photogrammetry (Huiskes *et al.*, 1985) due to lens distortions, unflatness of fiducial and photographic image planes, calibration and digitizing, the SCR method introduces a reconstruction error as well.

Any estimate R of a point C (Fig. 7) is not a point on the curve K, if the point R′ is not a point on the curve K′. Figure 7 shows the very position of R with respect to the curve K. The reconstruction error is

$$\varepsilon = \|\mathbf{c} - \mathbf{r}\|, \tag{1}$$

where **c** and **r** are the position vectors of C and R. In the following analysis it will be shown how this error can be estimated from the curvature of K′ between A′ and B′.

A point in the fiducial plane **x** is described by the dot product

$$\mathbf{h} \cdot \mathbf{x} = a, \tag{2}$$

where **h** is the normal vector of the fiducial plane, and a is a constant. The central projection formula for the fiducial image P′ of P by the camera station F gives

$$\mathbf{p}' = \frac{a\mathbf{f} - M\mathbf{p}}{\mathbf{h} \cdot (\mathbf{f} - \mathbf{p})}, \tag{3a}$$

with

$$M = \mathbf{f}\,{}^t\mathbf{h} + (a - \mathbf{h} \cdot \mathbf{f})I, \tag{3b}$$

and **f**, **p** and **p′** the position vectors of F, P and P′, respectively (${}^t\mathbf{h}$ = transposed vector **h**, I = unit matrix). Any line l through P, with direction vector **q**, is described by

$$\mathbf{x} = \mathbf{p} + \lambda \mathbf{q} \tag{4}$$

(λ real and arbitrary) and its fiducial projection by

$$\mathbf{x} = \mathbf{p}' + \mu \mathbf{q}' \tag{5a}$$

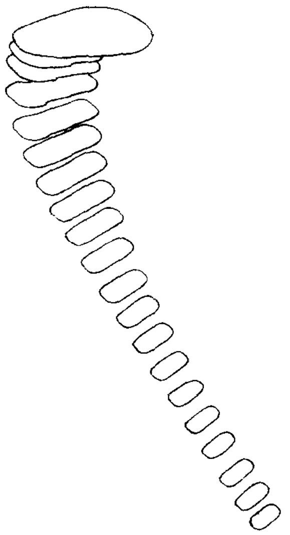

Fig. 3. SCR reconstruction of the cross-sections of a hip prosthesis.

(μ real and arbitrary) whereby **p′** is the projection of **p** on the fiducial plane, and **q′** the direction of l' in the fiducial plane. From the observation that l' is the intersecting line of both fiducial plane and the plane through the camera station F and the line l itself, it follows, using equations (2), (4) and (5a),

$$\mathbf{q}' = \left(I - \frac{(\mathbf{f} - \mathbf{p})\,{}^t\mathbf{h}}{(\mathbf{f} - \mathbf{p}) \cdot \mathbf{h}} \right) \mathbf{q}. \tag{5b}$$

Referring to Fig. 8, E is the perpendicular projection of C on the plane through F, A′ and B′. The distance E–C is u. Let **n** be the unit normal vector on this plane FA′B′, then

$$\varepsilon = u |\cos(\mathbf{g} - \mathbf{r}, \mathbf{n})|^{-1} \tag{6}$$

where **g** is the position vector of camera station G.

The distance u is magnified to $\bar{u}$ (Fig. 8) by the factor ω, according to

$$\bar{u} = \omega u. \tag{7a}$$

From the geometrical configuration (Fig. 8) it follows

$$\omega = \frac{f}{f - e} = \frac{\mathbf{f} \cdot \mathbf{h} - a}{(\mathbf{f} - \mathbf{e}) \cdot \mathbf{h}}, \tag{7b}$$

where f and e are the distances of the points F and E to the fiducial plane.

The line segment $\bar{u}$ is projected to u' in the fiducial plane, according to (Fig. 8)

$$u' = \bar{u} \frac{\cos\phi}{\sin(\phi + \Delta\phi)} \tag{8a}$$

with ϕ the angle of incidence. If $|\Delta\phi| \ll |\tan\phi|$, we find from equation (8a)

$$u' \simeq \bar{u} \sin^{-1} \phi. \tag{8b}$$

Since the normal vector **n** is projected to **n′** in the fiducial plane, it follows according to equation (5b)

$$\mathbf{n}' = \left(I - \frac{(\mathbf{f} - \mathbf{e})\,{}^t\mathbf{h}}{(\mathbf{f} - \mathbf{e}) \cdot \mathbf{h}} \right) \mathbf{n}. \tag{9}$$

Sin ϕ is found from

$$\sin\phi = (\mathbf{n} \cdot \mathbf{n}')/\|\mathbf{n}'\|. \tag{10}$$

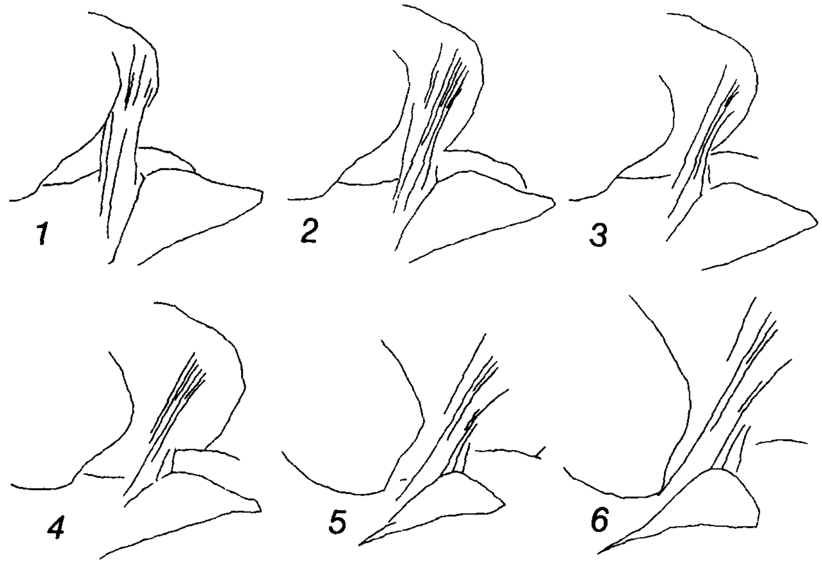

Fig. 5. Six plots showing the ACL photographs in digitized form.

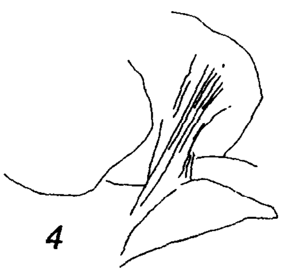

Fig. 6. The reconstruction of the ACL, plotted in photograph no. 4.

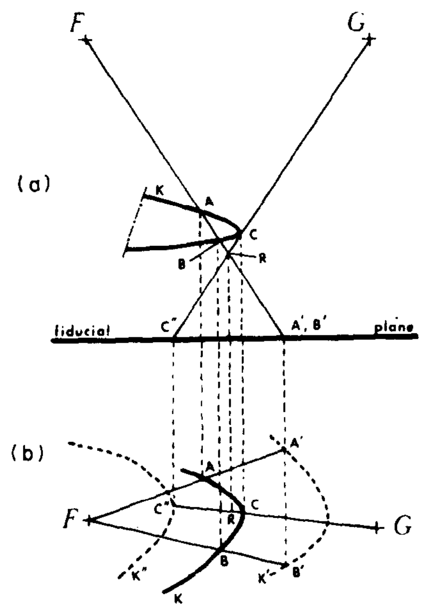

Fig. 7. The position of the inter-section R of (i) the plane through the camera station F and the points A′ and B′ of the projected curve K′, and (ii) the line through the camera station G and the point C″ of the projection K″; (a) side view, (b) top view.

Summarizing (6), (7a), (8b) and (10) gives for the reconstruction error

$$\varepsilon = u'|\mathbf{n}\cdot\mathbf{n}'|/|\omega\|\mathbf{n}'\|\cos(\mathbf{g}-\mathbf{r},\mathbf{n})|. \tag{11}$$

In (7b) and (9) the position of point E appears, but since the position of point C is unknown, point E is unknown as well, hence the precise value of E cannot be found from (11) in an experimental situation. An approximation can be found, however, when using the reconstruction R, instead of the original C. Because $\mathbf{f}-\mathbf{r}\simeq\mathbf{f}-\mathbf{e}$ (Figs 7 and 8), an estimate for $\mathbf{n}'$ can be found from the equation (9)

$$\mathbf{n}^* = \left(I - \frac{(\mathbf{f}-\mathbf{r})^t\mathbf{h}}{(\mathbf{f}-\mathbf{r})\cdot\mathbf{h}}\right)\mathbf{n}, \tag{12}$$

and an estimate for ω from equation (7b)

$$\omega^* = \frac{\mathbf{f}\cdot\mathbf{h}-a}{(\mathbf{f}-\mathbf{r})\cdot\mathbf{h}}. \tag{13}$$

In practice it is only possible to estimate the component u^* of u', perpendicular to A′–B′ (Fig. 9). So

$$u^* = u'\sin\zeta, \tag{14}$$

with ζ as the enclosed angle between the projected normal $\mathbf{n}'$ and the line segment A′–B′. The vector

$$\mathbf{t} = \mathbf{h}\times\mathbf{n} \tag{15}$$

gives the direction of A′B′, hence

$$\sin\zeta = \|\mathbf{t}\times\mathbf{n}'\|/(\|\mathbf{t}\|\ \|\mathbf{n}'\|). \tag{16}$$

The estimated reconstruction error then follows from equations (11), (14) and (16) as

$$\varepsilon^* = Ju^*, \tag{17a}$$

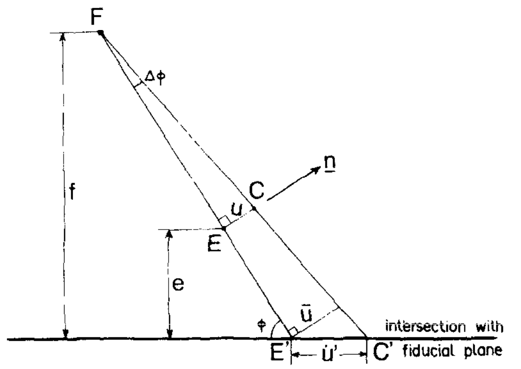

Fig. 8. Plane of drawing is the plane through the projection centre F, the point C on the curve and the normal on the plane through F, A' and B'. E is the perpendicular projection of C onto the plane through F, A' and B'. The distance E–C is u, but the point considered here is the distance E' – C', being u'.

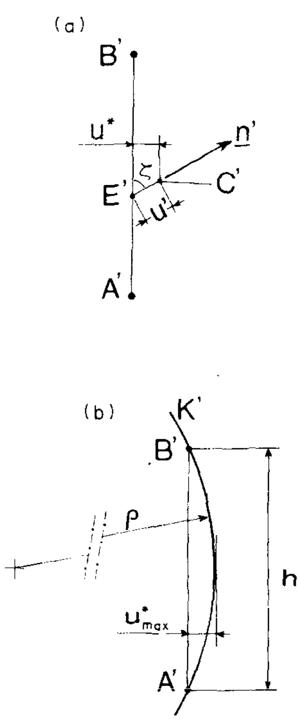

Fig. 9. (a) It is not possible to estimate the distance u' as the angle is not known, but it is to estimate the component u^* of u', perpendicular to A' – B'. (b) The upper bound of u^*_{max} as an estimate for u^* in the error analysis.

whereby

$$J = \left| \frac{(\mathbf{n} \cdot \mathbf{n}^*)\|\mathbf{t}\|}{\omega^* \|\mathbf{t} \times \mathbf{n}^*\| \cos(\mathbf{g} - \mathbf{r}, \mathbf{n})} \right|, \qquad (17b)$$

and $\mathbf{n}^*$ and ω^* can be found from equations (12) and (13) respectively.

If a projected curve K' has a constant curvature over A'B', radius of curvature ρ, the maximal deviation of K' from the straight line segment A'B' is (Fig. 9b)

$$u^*_{max} = \tfrac{1}{8} h^2 \rho^{-1} \qquad (18)$$

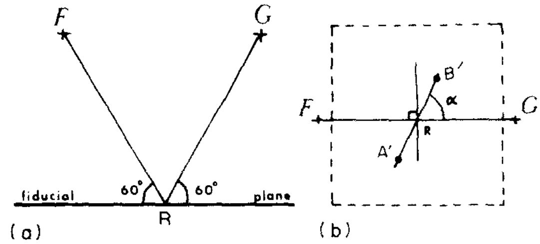

Fig. 10. A typical experimental setting, (a) side-view, (b) top-view.

with h as the distance A'B'. The maximal reconstruction error is then

$$\varepsilon^*_{max} = Ju^*_{max} = \tfrac{1}{8} Jh^2 \rho^{-1}. \qquad (19)$$

In the SCR program, the curvature ρ^{-1} of K' in the neighborhood of C' is estimated by curve fitting through six points, three on each side of C'. If the curvature varies only mildly from any segment to the next one, a reliable error estimation can be calculated with equations (17b) and (19).

In the reconstruction of the hip prosthesis, for instance, the maximal reconstruction error per curve had a mean value of 96 μm, and a maximal value of 0.20 mm. This relatively low and constant reconstruction error was due to the digitizing procedure: the stepwidth chosen was small and variable, depending on the curvature.

Figure 10 shows a schematic configuration of cameras, fiducial plane and object. Normally the object is close to the fiducial plane, here it is schematized as lying within the fiducial plane. The angle indicates the local direction of K'. In this case J remains small over a large range of α ($1.0 \leq J \leq 2.0$ for $\alpha \geq 30°$), but increases to infinity, if α tends to zero, indicating coplanarity of K', G and F. As mentioned above, in this case the equation system for the reconstruction is ill-conditioned. Here it is shown, that in this case it is necessary, not only because of the ill-conditioned equation system but particularly to avoid large reconstruction errors, to have a second filmpair available with the camera stations a quarter turn rotated around an axis perpendicular to the fiducial plane.

DISCUSSION

The SCR method extends the possibilities of the traditional close range photogrammetry. The advantages of close range photogrammetry, already mentioned above, are meaningful in soft tissue research particularly.

Current photogrammetric methods use object points, peculiar to the object itself or projected points (Selvik, 1984; Ghosh, 1979, 1983; Huiskes *et al.*, 1985), or lines, projected from a known focus (Frobin and Hierholzer, 1981), but none of these techniques can determine the course of specific object curves. Hence, the surface geometry of ligaments and other collagen structures could be measured with traditional methods, but the superficial fiber structure could not.

As indicated here, the application of the SCR method to ligaments requires a thorough experimental technique where optimization of the photographic quality is crucial. The pilot study, however, showed that SCR can be an effective non-destructive tool to measure ligament fiber orientation.

The SCR method is also suitable to measure and describe spatial surfaces without a natural fiber structure. In this case, lines must be drawn or projected on these surfaces. Because of the arbitrary scanning method, a variable sample density can be used, in order to reduce the reconstruction errors where the curvature is high. Advantages of SCR over traditional stereophotogrammetry in measurements of joint

surfaces (Huiskes *et al.*, 1985) are easy identification of object lines and a faster data-acquisition procedure.

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