The economic lot size and relevant costs

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Abstract

In many accounting textbooks it is strongly argued that decisions should always be evaluated on relevant costs; that is variable costs and opportunity costs. Surprisingly, when it comes to Economic Order Quantities or Lot Sizes, some textbooks appear to be less straightforward. The question whether described EOQ-models are in fact based upon variable costs and opportunity costs is generally not explicitly dealt with. This paper will investigate relevant costs if lot sizes in a components manufacturing plant are changed. The nature of opportunity costs is examined and several kinds of (semi-)variable costs regarding the use of (infrastructural) capacity and material flow are discussed.

In this manner, it is shown that in EOQ-models sunk costs are always included. It also occurs that variable costs must be analyzed carefully. This is also the case for more advanced models as described by for instance Wagner and Withn or Bertrand. Opportunity costs are in fact neglected, which nevertheless is not always that dramatic as some authors suggest.

1. Introduction

Few topics in management science have been discussed more intensively as the Economic Order Quantity or Lot Size. From an Operations Research point of view, one may discern several research issues [1]. The first phase begins in 1915 with the derivation of the EOQ-formula [2, 3]. The main assumptions of this basic model (see Appendix) are:

(1) Single stage production planning, not time phased.
(2) Known steady demand based on historical use.
(3) Costs vary linear with the amount of set ups per year or with the mean amount of inventory.

After World War II, a second phase in lot size research is distinguished, which may be labeled as "The EOQ-formula Revisited". Research in this phase can be characterized by

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optimal lot size in a JIT-system. Yet, we still need the economic grounds to justify the investments which are needed to change to JIT.

In textbooks, the different subjects of lot size research are also visible. In Management Accounting textbooks for instance, EOQ-models, MRP and Just-in-Time are treated as equally important (e.g. Ref. [9]), although some authors tend to give less attention to EOQ in favour of JIT [10]. In Financial Management textbooks, the problem is mainly related to working capital management; Gitman [11] for instance deals with the basic EOQ-model and MRP in this context. In Operations Management textbooks, we see EOQ-models, MRP as well as JIT (e.g. [12]).

When it comes to decision making, accounting textbooks strongly argue that decisions should be evaluated against relevant costs; that is variable costs and opportunity costs (e.g. Ref. [9, p. 341]).

Surprisingly, the question whether the EOQ-model is based on relevant costs (assumption (3)) is not that often dealt with explicitly. Some authors of the above mentioned references (like Vonderembse and White [12]) indeed point out that fixed (or sunk) costs have to be excluded, but they do not pay much attention to opportunity costs. Other authors do mention opportunity costs but we feel that they remain somewhat vague about the kind and size of opportunity costs that may be expected using the Economic Lot Size. An exception must be made for some recent contributions regarding the opportunity costs of capital and inventory policies (in general), see for instance Ref. [13]. But then again, these are not the only lot size opportunity costs that may occur.

Our purpose is to show relationships between lot sizes and relevant costs, not to present a new mathematical model for the determination of lot sizes. In this paper we will investigate what relevant costs may occur if lot sizes in a components plant are changed. Variable costs will be examined and gathered into a conceptual scheme in Section 2; opportunity costs will be dealt with in Section 3. Section 4 consists of concluding remarks.

2. The economic lot size and variable costs

2.1. A production situation

We will examine in this section what variable costs may occur if we change lot sizes in a component factory. We assume that this component factory is combined with an assembly line. It is furthermore assumed in our situation that the Customer Order Decoupling Point (CODP) is placed between the component factory and the assembly line. This implies that assembly activities only take place against real customer orders, whereas production in the components plant is based upon forecasts. This is in fact a strict “assemble-to-order” production method. See Ref. [14] or Ref. [15] for a more detailed description of CODP. The production situation is summarized in Fig. 1.

The required delivery time by customers is somewhat longer than the assembly throughput time. However, we assume that it is not much longer, so it is necessary to have finished components available (in the CODP) at the moment a customer order is received. Production of components must therefore be forecasted. We assume that these forecasts are finally summarized in an MPS after some rough capacity check. In principal, the components plant does not have to deliver more components per period than the MPS requires for that period. By doing so, the components plant would minimize inventory and maintain its required service level; almost corresponding to “Just-in-Time”.

Nevertheless, one is often obliged to enlarge the (minimal) MPS amounts to “economical” lots. For good reason, because production of minimal amounts could imply large setup times. An efficient utilization of capacity presupposes as much productive time as possible and minimal setup times. At this moment the lot size problem occurs. Increasing minimal lots to “acceptable” or “economic” lots is the application of a lot size rule.
We see that the lot size problem is twofold: on the one hand one would like to produce minimal (MPS-)amounts in order to obtain low inventory levels, on the other efficient utilization of capacity is at stake. The first aspect will be dealt with in Section 2.2, the capacity aspect is treated in 2.3.

2.2. Lot sizes and material flow

We define our components factory as a closed multi-item multi-machine jobshop. A prominent characteristic of such a shop are queues and delays. These queues are significant, in some cases waiting time is 90% of lead time. We will show that applying lot size rules will cause effects upon the following variables: 
- the average lead time (waiting plus manufacturing); 
- the lead time variance; 
- the level of Work-in-Process; 
- the safety stock level.

2.2.1. Lot sizes and lead time

The relationship between lot sizes and lead time has been investigated by Karmarkar et al. [16]. These authors make clear that the impact of lot sizes on lead time is much larger than application of priority rules (sequencing): “It is conventionally assumed that at the operational level, the dynamic performance of such shops with queues and delays is primarily controlled by sequencing and dispatching at machines. In fact, a major determinant is the lot size policy employed.” [16]. The relationship between lot size and lead time is given in Fig. 2.

Figure 2 makes clear that smaller lot sizes may lead to less lead time. Yet, the reverse relationship may occur as well; if the utilization rate of one or more work centers reaches

Fig. 1. Production situation.

Fig. 2. Relationship between lot size and lead time [16].
about 95%, smaller lot sizes will lead to quite extreme lead times.

2.2.2. Lot sizes and inventory

Lead time has an impact on several variables. At first, a relationship with Work-in-Process (WIP) inventory exists. The longer lead time, the more WIP. But this is not the only relationship. The larger lead time, the longer the horizon over which future component requirements have to be forecasted. The length of the forecast horizon influences forecast quality. The longer the horizon, the worse forecast accuracy. If we strive for keeping the components factory service level fixed, more safety stock is needed. The variance of lead time is also involved. The longer lead times, the more variance. This variance will cause for uncertainty about the exact moment in time components will be available for assembly. This uncertainty will also be warded off with safety stock.

Conclusion: Lot size have a large influence on lead time in a (job shop) components factory. This causes for Work-in-Process inventory and, indirectly, for safety stock. This is summarized in Fig. 3.

The EOQ-model only takes into account the relationship between lot size and lot size inventory in the above figure. Work-in-Process and safety stock are neglected. As for Work-In-Process inventory, Bertrand [17] has shown that this is rather a serious negligence. Safety stock is further dealt with in Section 3.

2.3. Lot sizes and capacity

Earlier we saw that the lot size decision in our production situation is actually a decision of increasing minimal (MPS-)amounts to "acceptable" lots. If one was to minimize material flow variable costs (2.1), one could produce minimal MPS-amounts. Mostly however, this is not possible because capacity constraints will occur. But then again, a lot size rule is not the only solution for abolishing capacity constraints. One could take measures for
temporary capacity enlargement such as work in overtime, work contracted out, additional shifts, etc. Logistics management has to decide explicitly whether to deal with constraints by a lot size rule or by temporary enlargement of capacity, because costs may differ significantly.

Some capacity constraints however, cannot be abolished by temporary measures. This type of capacity constraint has been given much attention to by Goldratt and Cox [18]. This type of constraint actually determines maximum output of the system. Such constraints are critical in lot size decision making, because every set up hour is loss of production (and sales) for the system as a whole. So, in case of a structural capacity constraints, opportunity costs will arise (see also Section 3).

Apart from this capacity aspect, some other effects may occur. Given the total number of lots one is to produce relationships exist with the following variables:
- required set up capacity;
- required internal transport and handling capacity;
- required level of (administrative) control.
If one is to produce in large lots, little set up capacity is needed. This is also the case for the required level of administrative control, as it is recently shown by Activity Based Costing authors. As for the required handling and internal transport capacity, the relationship also exists although it is often somewhat more fuzzy in practice. The relationships are summarized in Fig. 4.

Problem with the costs of these infrastructural capacities is that they are semi-variable with the lot size decision. Within certain boundaries, lot sizes can be modified without generating out-of-pockct expenses.

This semi-variable character of costs is not recognized in the Economic Lot Size model. Set up capacity costs are thought to be proportional; transport & handling costs and cost for administrative control are neglected.

2.4. Two more variables; an overview

As for material flow, two more variables have to be distinguished. At first, a (semi-variable) relationship exists between lot sizes and storage space capacity (warehouses and packaging). In EOQ-models, these costs are a part of the costs of inventory (see Appendix). This implies that storage costs are thought to be proportional with the lot size.

We have seen that lot size inventory, Work-in-Process inventory and safety stock are substantially determined by a lot size policy. This inventory carries a certain risk of getting obsolete, especially if lead times are long. This could cause for future costs. Mostly, this is also recognized in EOQ-models by some factor in the denominator. We will deal in Section 3 with the question whether this is a proper approximation of obsolescence costs.

All the mentioned effects can be gathered in a conceptual scheme (Fig. 5). As for costs, it may roughly be divided in the following four sections. At first, the maximum capacity of the components plant is at stake. Running a certain lot size policy could cause for temporary enlargement of capacity (i) or capacity constraints which may generate opportunity costs (ii), as we will see in Section 3.

The second section consists of infrastructural capacities which may induce semi-variable costs.

The third section includes inventory. Costs are here originated in the fact that capital is tied up in this inventory. We will deal with these so called opportunity costs of capital in the next part of this paper.
The fourth section consists of obsolescence risks, which could cause for future variable capacity and materials costs.

2.5. Conclusion

As for material flow, the basic EOQ-model only takes lot size inventory (finished components) into account. Relationships with Work-in-Process and safety stock are ignored. As for WIP, Bertrand [17] has shown that this is rather a serious negligence.

Although the application of a lot size rule is decisive on the utilization of capacity, and not on the amount of capacity (sunk costs!), it nevertheless may almost certainly generate (semi-)variable costs in a multi-item multi machine job shop. The capacity aspect of costs is twofold:

(i) semi-variable costs related to infrastructural capacities;
(ii) (semi-)variable costs for temporary enlargement of capacity (work in overtime, work contracted out, etc.).

As for the basic EOQ formula, these costs are in fact mostly neglected. Set up costs are usually based on (fixed) capacity costs which of course are sunk by nature. This implies negligence of extra costs for temporary enlargement of capacity and opportunity costs of set up time. As for infrastructural capacity costs, some are disregarded and others are thought to be variable instead of semi-variable.

3. The economic lot size and opportunity costs

Foster and Horngren [9, p. 314] define opportunity costs as "... the maximum contribution
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that is forgone by using limited resources for a particular purpose. Regarding the lot size decision two kinds of opportunity costs exist:
(i) opportunity costs concerning utilization of capacity;
(ii) opportunity costs of capital.

Our aim is to determine the order of magnitude of these opportunity costs. The first kind of opportunity costs arises if it appears that another lot size decision would have performed better. Profit loss is calculated by comparing the actual chosen lot size (policy) with the best possible alternative (which of course has not been chosen). This is dealt with in Section 3.1.

The second type of opportunity costs concerns capital that is tied up in inventory by lot size decisions (3.2). Some results are discussed in Section 3.3.

3.1. Opportunity costs regarding capacity

We will examine what opportunity costs may arise if customer demand is smaller than expected (3.1.2) and larger than expected (3.1.3). But first we introduce some important key variables (3.1.1).

3.1.1. Key variables

Actual customer demand. Main purpose of this section is to determine the order of magnitude of capacity opportunity costs. We will introduce important key variables here and we will make some confinements. In the previous section we saw that the components plant is obliged to deliver at least the MPS amounts for a certain period. At the moment a components lot is released, customer demand has been forecasted and is therefore uncertain. At the moment the finished components lot becomes available for assembly, we assume that actual customer demand is known and cannot be modified. If the actual customer demand is equal to the lot size in that period, no opportunity costs will occur if future service level will not jeopardized by this particular lot size policy. In this case, the best alternative has indeed been chosen (by chance).

Safety stock. Actual customer demand may be smaller or larger than the chosen lot size. This may generate opportunity costs.

If the actual customer demand is larger than the lot size, we must differentiate between a situation in which the amount of safety stock is sufficient for covering up extra demand, and situations in which it is not sufficient. One could observe here that lot size stock is partially safety stock as well, so the amount of free lot size stock during the period should be taken into account as well (see e.g. Ref. [19]). This is certainly important if demand within a period is fickle. Yet, our analysis is mainly aimed at determining the order of magnitude of opportunity costs. We will therefore confine ourselves to a situation in which it is determined whether safety stock was sufficient after a period has elapsed.

Capacity constraints. The presence and position of capacity constraints is also important. We must differentiate here between capacity constraints that can be eliminated by means of temporary measures and "absolute" capacity constraints, which can only be eliminated by structural investments. The latter go beyond the lot size decision as lot sizing is in principal decisive on capacity utilization and not on the amount of capacity. "Evaporating" capacity constraints by structural investments can therefore not be taken into account.

Contribution margin. The last key variable is the contribution margin. If all costs are fixed, contribution margin is of course higher than if all costs are variable.

3.1.2. Opportunity costs in case customer demand is less than expected

In this case opportunity costs arise because more components have been delivered in a period than necessary to fulfill actual demand (in that period). In other words: one could have done with a smaller lot. Speaking
in opportunity terms, this has caused for out-of-pocket expenses for raw materials and variable capacity costs. Fixed capacity costs do not matter [15, 20].

The order of magnitude of opportunity costs is calculated by multiplying the above amount of cash by some percentage that expresses opportunity costs of capital for one period. (In Fig. 6, these costs are represented by “opportunity costs II”). Question however is: what will happen with this inventory in the next period, or, in other words: Will it still be marketable?

If no obsolescence risks exist, the above opportunity costs calculation must simply embrace some more periods. If however, obsolescence risks do exist, opportunity costs grow much larger: if items become obsolete they have not only caused for above opportunity costs of capital; but they will also cause for a loss of materials and variable capacity costs as well. It may therefore be stated that depending on the size of the raw materials and variable capacity costs, obsolescence risks transform opportunity costs to a larger order of magnitude. (Fig. 6: Opportunity costs I)

In this case safety stock is not used and so it need not be replenished in the next period. No safety stock opportunity costs occur. One could observe that opportunity costs do arise because one could have done without safety stock! This is however not the case. Keeping safety stock is in principal another decision with its own relevant costs. Nevertheless, both decisions are connected, as we will see in Section 3.3.

Conclusion: In the case customer demand is less than expected, opportunity costs are relatively small as long as no obsolescence risks exist. Furthermore, safety stock and capacity constraints do not cause for opportunity costs.

3.1.3. Opportunity costs in case customer demand is more than expected

In this case less components become available for assembly than necessary to fulfil actual customer demand. This implies that safety stock must be used. We must differentiate between the case in which safety stock is sufficient and the case in which it is not.

If safety stock is sufficient, it needs to be replenished the next period. If this is possible without any extra costs compared with normal lots, opportunity costs are nil. It could occur that this extra stock will cause for extra capacity costs such as work in overtime. (Fig. 6: Opportunity costs III)

Suppose the components factory is a structural capacity constraint. In that case safety stock cannot be replenished without market losses. In fact, keeping safety stock will here cause for opportunity costs in the order of magnitude of finished products’ contribution margin. In practice, this will lead to an adapted production method (logistics concept3) in which safety is no longer kept.

Opportunity costs as large as contribution margin losses are of course high if costs are mostly fixed. Moreover, they may cause for strategic marketing effects (which are not taken into account here).

Contribution margin losses will also occur if customer demand is higher than expected and safety stock is not sufficient. In this case it does not matter whether the assembly is a structural bottle-neck capacity or not. As back ordering and other demand management measures are not allowed, loss of sales will occur (Fig. 6: Opportunity costs IV).

One could observe here that production of unsold finished products inventory may avoid contribution margin losses, especially if customer demand is seasonal. In that case namely, the capacity constraint is not permanently structural. In a strict assemble-to-order production situation however, this is not allowed. One would in fact have to introduce another Customer Order Decoupling Point (position: finished goods inventory). Such measures are in fact also adaptations of the production method (logistics concept).

3 One could consider, if delivery time allows (which is not our case), an upstream shift of the Customer Order Decoupling Point. This is called an adaptation of the logistics concept. See Ref. [15] or Ref. [14].
Fig. 6. Opportunity costs regarding lot size decisions. Opportunity costs I: see section 3.1.2; opportunity costs II: see section 3.1.2; opportunity costs III: see section 3.1.3; opportunity costs IV: see section 3.1.3.
Conclusion: In the case customer demand is higher than expected, opportunity costs will be zero if safety stock is sufficient to cover up extra demand. In this case opportunity costs will only arise if safety stock replenishment causes for additional (capacity) costs compared with normal lots production. We consider these opportunity costs are rather low.

High opportunity costs will occur if safety stock is not sufficient. In this case contribution margin losses arise.

Two special situations have been distinguished which call for potential adaptation of the production method (logistics concept). These situations occur in the case of absolute capacity constraints. The above analysis is summarized in Fig. 6.

3.2. Lot sizes and opportunity costs of capital

Every change in lot size policy will change inventory level. We have also seen that semi-variable costs and temporary capacity costs may ensue. This will change capital structure.

If we want to analyse a proposed lot size change by using for instance a Net Present Value calculation, then the opportunity aspect of capital is important. Problem is that opportunity costs of capital are related with the lot size policy. Changing lot sizes will change opportunity costs of capital which may lead in its turn to a modified annual rate of interest. Mostly, this (indeed mostly marginal) effect is neglected in practice. NPV is calculated with a fixed annual rate of interest which is usually prescribed by the company’s financial management.

In principal this is not correct (see e.g. Ref. [13]), although many authors do prefer to calculate with a fixed interest rate [21, 22].

For the purpose of this paper, it is not necessary to deal with this problem any further. It is enough that one bears in mind that the annual rate of interest is not that fixed as it may seem. This is particularly interesting in the case of critical NPV values.

3.3. Lot sizes, safety stock and the purpose of this analysis

It may be clear that coping with opportunity costs is rather tricky. One could consequently question the objective of this analysis. We see however some reasoning.

(i) We have noticed that the EOQ-formula does not take opportunity costs into account. We observed that this sometimes is not all that severe, but in the case of contribution margin losses this negligence is serious. It is Goldratt (1984 and later) who has recently brought this into broad attention, although in some other terminology.

(ii) The order of magnitude of opportunity costs is very useful in the case safety stock level is calculated. We have seen that these are separate decisions, yet both influence service level. If safety stock levels are calculated, the opportunity costs’ order of magnitude makes it possible to obtain a realistic estimation of the so called “penalty costs” We feel that these penalty costs are often determined by rule of thumb, or even “uneducated guess”.

(iii) Finally, our analysis contains some (somewhat concealed) arguments for a hierarchical approach in lot size decision making (see Ref. [25]). At first sight, a lot size decision seems nothing more than a, rather trivial, local (components plant) decision on utilization of capacity. Yet, the decision has more than local consequences, so analysis on a higher hierarchical level is necessary. For instance: opportunity costs may occur concerning transactions at the sales market which is beyond the local components plant view. Moreover, we have seen that capacity constraints make it sometimes impossible to produce in conformity with a prescribed production method (such as “assemble-to-order”).

4. Conclusions

It may be clear that applying the EOQ as a lot size rule is not correct. All kinds of (semi-) variable costs are neglected or thought to be fully variable with lot size decisions.
Opportunity costs are practically neglected as well. The EOQ is moreover partially based upon sunk costs. In other words: EOQ and relevant costs simply do not match. Some case research was done which also indicated that (large) economical improvement could be obtained [23, 24]. Question remains: What other lot size rule must be applied?

If one is to determine (another) lot size rule, opportunity costs can of course not be avoided totally. In fact, one will enter a process of finding an adequate balance between all kinds of expected (semi-)variable costs and opportunity costs that might occur. As reality can never be fully predicted, calculating with (expected) opportunity costs will always be speculative. However, it is the opportunity costs' order of magnitude that really counts. In a given production situation, this order of magnitude is far more stable than the exact amount of opportunity costs. Therefore, we believe that is possible to determine a lot size policy which balances the magnitude of opportunity costs.

We have seen that in many textbooks EOQ and JIT are treated separately, as if no inter-connection exists. This is rather unsatisfactory because in practice only one lot size policy can be best on economic grounds. We feel that this ambiguity may be resolved by our proposed relevant costs approach. It makes clear why EOQ is not optimal and why pure JIT manufacturing may not be optimal as well. It is this interconnection on economic grounds that is so often missed.

Appendix. The EOQ-formula

The Economic Order Quantity-formula is used to derive a lot size for an item at the minimal total annual costs. The annual costs are thought to be twofold: set up costs and inventory costs.

The EOQ uses the next total cost function:

\[ C = \frac{FD}{Q} + \frac{1}{2}QkV \]

The definition of the parameters is:

- \( C \) = total lot size costs per year,
- \( F \) = fixed set up costs per lot,
- \( D \) = demand of item per lot,
- \( Q \) = quantity per lot,
- \( k \) = costs of inventory,
- \( V \) = value of item.

The EOQ is

\[ Q = \sqrt{\frac{2DF}{kV}}. \]

References


