Case Study

Deriving route lengths from radial distances: Empirical evidence

C.F.M. Stokx
Krekel van der Woerd Wouterse B.V., Management Consultants, P.O. Box 20706, 3001 JA Rotterdam, The Netherlands

C.B. Tilanus
Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands

Abstract: Estimating expected route lengths for multi-drop trips (serving several destinations) from radial distances (as the crow flies) is relevant as a short cut to avoid actual vehicle routing, e.g., in transportation studies at a strategic level. Empirical evidence, using a model of Christofides and Eilon, is given for Belgium and The Netherlands.

Keywords: Transportation, routing, radial distance

1. Introduction

The infrastructure of ground transportation is roads; the infrastructure of transportation planning is road network databases. The latter infrastructure is catching up with the former, and one may wonder if the topic of this article, estimating vehicle route lengths along the road on the basis of known distances as-the-crow-flies, is still relevant today. It is.

Take for example the strategic transportation study performed by the first author for his Master’s thesis (Stokx, 1985). A petrol company wanted to optimise its distribution structure and therefore analyse the number and location of its petrol distribution depots in Belgium, given the location of some 200 petrol outlets throughout the country, and given a mixed fleet of third party tanktrucks and tanktrucks owned and operated by the oil company.

In order to find a solution to this classic problem, a simulation model was developed. In each of the simulation runs the total distribution cost was calculated using a different combination of the available depots. The total distribution cost was determined first by summarising the replenishment cost of each of the 200 retail outlets, and next by adding the fixed costs of the distribution system, e.g., depot costs. To determine the replenishment cost of each of the outlets, criteria had to be established to allocate the petrol outlets to either its own tanktrucks or third party tanktrucks.

Tariff rates of third party transportation were given. To compare the costs of its own transportation with the costs of transportation by third parties, unit costs, and hence distances between depots and destinations had to be assessed.

In this business, tankers sometimes deliver their full load at one address—then the problem reduces to finding the shortest route between depot and destination. But they may also deliver their load in portions at two, three or more addresses. This is
called the multi-drop problem and it is a true vehicle routing problem, though the number of addresses is small.

The petrol company had statistics available on the percentages of one-drop trips, two-drop trips, etc.

It would, of course, have been possible, for all strategic scenarios considered to generate a number of delivery schemes and simulate vehicle routing schedules and thus establish average distances and costs. But this was considered too clumsy. Rather, it was proposed to estimate route lengths from the available data on realized trip lengths and from readily measured radial distances between the depots and the petrol outlets.

2. Problem

The problem becomes the following:
- given the location of the depot and the destinations, and the radial distances between the depot and the destinations (these can be readily measured using a map and a ruler),
- given a sufficient number of realized trips, planned by hand, of one drop, two drops, three drops, etc.,
  *establish the expected route lengths, for trips of one drop, two drops, three drops, etc.

The problem is not to improve on the realized, hand planned trips. Let them be assumed optimal. The problem, thus, has nothing to do with the traveling salesman or vehicle routing problems, that have drawn so much interest (Lawler et al., 1985; Bodin et al., 1983; Florian, 1984); nor with the vehicle scheduling computer packages that become available at an increasing rate (Bocxe and Tilanus, 1985); nor with finding expected travel times by regression on realized travel distances (Kolesar et al., 1975). Instead, it is that realized trips are used as a regression basis for expected trip lengths.

A mathematical formula for the expected trip length in terms of radial distances was found in Christofides and Eilon (1969), referring back to Beardwood et al. (1959). Later elaborations are given by Eilon et al. (1971), Love and Morris (1972), Fernandez et al. (1974), Ginsburgh and Hansen (1974), Love and Morris (1979), Cooper (1983), Daganzo (1984a,b), Berens and Körling (1985). Empirical evidence reported remains scarce, however. Some results are given from the USA (Love and Morris, 1979), the UK (Cooper, 1983) and West-Germany (Berens and Körling, 1985).

In the experiments carried out by Christofides and Eilon (1969), randomly generated data were used. In this case, however, the original Christofides and Eilon model was estimated first by a regression of data based on 200 realized trips in Belgium.

The model was

\[ T = a \sum_{i=1}^{d} \frac{r_i}{d} + b \left( \sum_{i=1}^{d} r_i \right)^{1/2}, \]

where

- \( T \) is the expected trip length,
- \( d \) is the number of drops per trip (the number of destinations served in one trip),
- \( r_i \) is the radial distance (as-the-crow-flies) between the depot and the \( i \)-th destination,
- \( a, b \): regression constants.

The results for Belgium were:

\[ a = 1.48, \]
\[ 95\% \text{ confidence interval: } 1.09-1.87; \]
\[ b = 9.69, \]
\[ 95\% \text{ confidence interval: } 7.40-11.98; \]
\[ \text{correlation coefficient } R = 0.98. \]

The scatter diagrams and regression lines for \( d = 1, d = 2, \) and \( d = 3 \) are given in Figures 1, 2 and 3 (\( d > 3 \) is a negligible occurrence in this type of distribution).

The results were used in the construction of cost functions representing the annual replenishment costs of each of the petrol stations when using tanktrucks owned by the oil company.

The oil company was also operating in The Netherlands, but for this country, regression results were expected to be worse because of the many water barriers formed by the rivers Rhine and Meuse, the former Zuyderzee and the South-West Delta.

Nevertheless, another regression was made for The Netherlands on 200 observations and the results were:

\[ a = 1.81, \]
\[ 95\% \text{ confidence interval: } 1.54-2.08; \]
\[ b = 6.93, \]
Figure 1. Expected total trip length as a function of radial distance: one drop per trip. Note that if there is one drop per trip then $T \geq 2r_i$. In some observations, however, $T$ is smaller than $2r_i$ due to measurement faults, map inaccuracies, and the impossibility of pinpointing the exact outlet locations on a 1:250000 scale map.

95% confidence interval: 5.76–8.10; correlation coefficient $R = 0.99$.

The set of six regression lines is given in Figure 4. The results seem to imply that the presence of barriers does not present severe difficulties in estimation. However, since Dutch trips also appear to be shorter than Belgian, another explanation is possible. Examination of the depot and outlet locations reveals that Dutch petrol stations are more or less clustered around a relatively large number of depot locations, whereas Belgian outlet locations and depots are more equally distributed across the country. Apparently, the difficulties

Figure 2. Expected total trip length as a function of radial distance: two drops per trip.
presented by barriers have been overcome by the construction of additional depots.

Of course any residual effects of barriers on estimation from radial distances might be mitigated, by breaking the radial distances into segments, one from the depot to the point where the barrier is pierced and the other from there to the respective drop points.

Incidentally, the greatest eye opener of the strategic petrol distribution study (Stokx, 1985) was the role of the temperature of the petrol. The volume of petrol increases about 1 percent per degree Celsius. Since it is sold by the volume, the warmer the petrol is, the more revenues. The temperature is highest in the refinery and falls slowly in the depots and outlets, hence stocks in the latter two should always be kept as low as possible. A further lowering of stocks could only be obtained by reducing the number of depots and by reducing the dropsizes of the tanktrucks, thus by increasing distribution costs. Therefore the optimum is found by minimizing the sum of distribution costs and costs due to temperature conversion of the petrol.

We hope that deriving route lengths from radial distances is accepted as a relevant problem in strategic distribution studies like the one sketched. We also hope that some readers may benefit from our results.

References


Daganzo, C.F. (1984b), “The distance traveled to visit \( N \) points with a maximum of \( C \) stops per vehicle: An analytic model and an application”, *Transportation Science* 18, 331–350.


