Model Predictive Compressor
Surge Control

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DCT 2007.111

Master’s thesis

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Eindhoven, August, 2007
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Abstract

Below a certain minimum massflow in a compression system, a stable operating point cannot be maintained and rotating stall or surge may occur, which are flow instabilities that may lead to severe damage of the machine due to the large and thermal loads. The focus in this report is on surge control using active control. The stable operating region, robustness and noise rejection properties of a compression system are investigated using a lumped two-state Greitzer model to model a centrifugal compressor with a Positive Feedback Stabilization (PFS) control system. The goal is to investigate if Model Predictive Control (MPC) can improve the stable operating region, robustness and noise rejection properties compared to PFS. Therefore a linear and a form of hybrid MPC are both used and all control types are compared in the end. Simulation results show that the stable operating region cannot be increased compared to PFS by implementing MPC. The compression system with PFS and hybrid MPC controller used simultaneously seems to be significantly more robust to system parameter mismatches than when solely PFS is used. Implementation of noise on the last system with hybrid and PFS control combined does not change the results qualitatively.
Chapter 1

Introduction

Below a certain minimum massflow in a compression system, a stable operating point cannot be maintained and rotating stall or surge may occur, which are flow instabilities that may lead to severe damage of the machine due to the large and thermal loads. The focus in this report is on surge control in centrifugal compressors using Model Predictive Control.

1.1 Centrifugal compressor

In a centrifugal compressor, see Figure 1.1, the entering fluid is accelerated by the impeller, increasing the total pressure of the fluid. Then the kinetic energy is converted into potential energy by decelerating the fluid in diverging channels, which results in a static pressure rise of the fluid. In a centrifugal compressor, the pressurized fluid leaves the compressor in a direction perpendicular to the rotational axis.

In a compression system steady-state operating points with constant rotational speed are indicated by speed lines or compressor characteristics and the rotational speed increases in the direction of the arrow, see Figure 1.2. The load or throttle line represents the pressure requirements of the system. The steady-state operating point of a compression system is the intersection point of the compressor characteristic and this load line.

The operating range for a compressor is bounded for high mass-flows by the Stonewall line, this is due to chocked flow. For low mass flows the operating range is limited by the occurrence of rotating stall and surge. The transition from this stable to unstable region is marked by the so-called Surge line. The unstable region is located at the top of the compressor characteristic or near the top at a point with a specific positive slope of the speed line. While rotating stall is a local instability, surge affects the compressor system as a whole, where large amplitude pressure rise and annulus averaged mass-flow fluctuations occur (de Jager [1995]).
1.1 Centrifugal compressor

Figure 1.1: Centrifugal compressor scheme.

Figure 1.2: Compressor map (Willems [2000]).
1.2 Compressor performance

Rotating stall and surge restrict the performance (pressure rise) of a compressor (de Jager [1995]), since the compressor needs to be operated at a safe distance from the surge line and therefore the maximally achievable pressure is smaller than the peak pressure. This margin is necessary because off-design conditions may lead to flow instabilities (Willems [2000, Section 1.2, p. 7]). Load variations such as shutting off downstream processes or temporal changes in the rate of production can make the compressor’s operating point move towards the surge line.

Linear stability analysis predicts, that the system will become unstable when the slope of the compressor characteristic for constant speed exceeds a certain positive value determined by the characteristics of the compressor and the slope of the load line (Fink et al. [1992]).

1.3 Literature

A short survey on rotating stall and surge control in compressors is performed in this section and the possibilities for applying Model Predictive Control for this goal. The focus of the chosen control type is on its practical value, meaning its applicability in laboratory and industry.

1.3.1 Rotating Stall and Surge Control

Control systems in industry use a method based on surge avoidance mainly (Botros and Henderson [1994]) see Figure 1.3. If the desired operating point is A, which has the largest pressure rise, in case of surge avoidance this point is shifted to B, which guarantees stability and a safety margin from the surge line. For downstream processes, the compression system appears to operate in point C with a smaller pressure and mass-flow than desired (the compressor speed can be increased to correct this). Surge avoidance limits the performance of the compressor, since the maximal pressure is obtained close to the surge line.

Using active control a control system feeds back perturbations into the flow field (Epstein et al. [1989]) which causes that the surge line is shifted to the left and the result is that the stable operating region is enlarged, see Figure 1.3. The operating point here remains in A.

The Greitzer compression system model Greitzer [1976] is a non-linear model which describes surge in axial compression systems and has been widely used for surge control design. Hansen et al. [1981] showed that it is also applicable to centrifugal compressors. The model of Moore and Greitzer [1986] dominates the recent study on rotating stall and surge control, since it is a low order non-linear model which can describe the development of both rotating stall and surge and the coupling between these instabilities.

Gu et al. [1999] gives a survey of the research literature and major developments in the field of modeling and control of rotating stall and surge for axial
flow compressors. According to this survey rotating stall and surge control is effective in low speed compressor machines. However rotating stall and surge control in high-speed compressors is being researched with reasonable success. The most interesting result of all theoretically seems to be the non-linear back-stepping method (Krstic et al. [1995]). This method gives a non-linear feedback system which is globally stable at any setpoint in the presence of large uncertainties in the compressor model. This only works for cubic compressor maps of the type commonly described in the literature, else no stability can be guaranteed. The controller described in (Krstic et al. [1995]) also requires a certain equilibrium structure of the open-loop plant, which may not in general exist. In this case a new concept for a control law needs to be designed. For these reasons this method does not seem easy to apply in practice however. In the survey on rotating stall and surge de Jager [1995], it is also concluded that control of rotating stall for high speed axial machines is ineffective and not used in research laboratories and has no practical value. The active control of surge, also for high speed machines, is a proven effective technology and seems to be an approach that can be applied profitably in industrial practice (de Jager [1995]).

In Willems [2000] active surge control of a centrifugal compressor is simulated and implemented on a gas turbine installation. Using a bounded feedback controller, surge limit cycles are stabilized in the desired set-point. In the simulations the Greitzer compression system model (Greitzer [1976]) is used to describe the development of deep surge in the compression system. The form of surge which is stabilized in the simulations is deep surge, characterized by reverse flow over part of the cycle and a large amplitude limit cycle oscillation, see Figure 1.4. In (1) the flow becomes unstable and goes very fast to the negative flow characteristic at (2). It descends until the flow is approximately zero (3) (in this step the plenum is emptied). Then it proceeds very fast to the normal characteristic at (4), where it starts to climb to point (1) (in this step the plenum is filled) and the cycle repeats.

These results of Willems seem very promising, resulting in a control strategy which is successfully implemented on the examined compression system and therefore the compressor model with the bounded feedback controller will be used in this work as a basis to design a Model Predictive Control system. There seems hardly any literature available on MPC in the field of stall and surge control. A short overview of Model Predictive Control is given in the next section.
Figure 1.3: Difference between surge avoidance and active control (Willems [2000]).

Figure 1.4: Deep surge (de Jager [1995]).
1.4 Goals of research

In this work a centrifugal compression system is modeled with a two state Greitzer lumped parameter model, this system and model used are taken from the work of Willems [2000]. Active control is used to stabilize surge limit cycles in a desired set-point. The first goal here is to reach a mass-flow which is as small as possible. The purpose of this is to obtain a large as possible stable operating region and thus giving a large as possible safety margin when the compressor operates at maximum performance (maximum pressure point in the compressor map). Next to this attention is paid to disturbance rejection and robustness of the closed-loop.

First a one-sided controller is designed, as described in Willems [2000]. Output and state-feedback are both investigated using this controller. Both a linear and hybrid Model Predictive Controller are designed on the one-sided control system to investigate if improvements in the stable operating region, disturbance rejection and robustness of the closed-loop system can be achieved.

The work has the following structure:
In Chapter 2 the two state Greitzer lumped parameter model is presented, together with a one-sided controller and closed-loop simulations are performed, these are numerically compared to the ones in Willems [2000]. Both output and state-feedback are used here. Also two scenarios are introduced in this Chapter, in which robustness and disturbance rejection of the closed-loop will be investigated. These same scenarios are also adopted to investigate the linear and hybrid Model Predictive Controllers.

The design of a linear Model Predictive Controller is discussed in Chapter 3 and simulations are performed.

Chapter 4 introduces a hybrid Model Predictive Control strategy. The design of a hybrid Model Predictive Controller is described here and simulated. All the different types of control in this work are compared and conclusions are drawn in Chapter 5.
Chapter 2

Active Surge Control

In this chapter the used compressor model with the one-sided controller described in (Willems [2000, Section 3.3.3, p. 38]) is presented and explained. Simulations are performed of deep surge control for the output and feedback controlled cases. Also robustness and disturbance rejection of the closed-loop systems are investigated. The results are compared with Willems.

2.1 Greitzer lumped compressor model

The model of the centrifugal compression system which is used is given in Figure 2.1. It is represented by a duct in which the compressor works that discharges in a large volume (plenum). The compressed fluid flows via the plenum through the throttle and control valve into the atmosphere.

![Compression system](image)

Figure 2.1: Compression system (Willems [2000]).
To describe the dynamic behavior of the examined compression system, the Greitzer lumped parameter model (Greitzer [1976]) is used, this model is originally designed for axial compressors and was proven to be usable for centrifugal compressors as well (Hansen et al. [1981]).

The following assumptions are made in Greitzer [1976]: 1) the flow in the ducts is one-dimensional and incompressible 2) in the plenum the pressure is uniformly distributed and the gas velocity is neglected 3) the temperature ratio of the plenum and ambient is assumed to be near unity: therefore an energy balance is not required 4) the influence of the rotor speed variations on the system behavior is neglected.

The dimensionless mass-flow \( \varphi \), dimensionless pressure difference \( \Psi \) and dimensionless time \( \tilde{t} \) are defined as

\[
\varphi = \frac{\dot{m}}{\rho_a A_c U_t} \quad \Psi = \frac{\Delta P}{\frac{1}{2} \rho_a U_t^2} \quad \tilde{t} = t \omega_H
\]  

with the Helmholtz frequency

\[
\omega_H = a \sqrt{\frac{A_c}{V_P L_c}}
\]

here \( \dot{m} \) is the mass-flow, \( \Delta P \) the pressure difference between the pressure in the system and the ambient pressure, \( \rho_a \) the air density at ambient conditions, \( A_c \) the compressor duct area, \( a \) the speed of sound, \( U_t \) the rotor tip speed, \( V_p \) the plenum volume and \( L_c \) the equivalent compressor duct length.

The following set of dimensionless equations that describe the non-linear compression system are

\[
\begin{align*}
\frac{d \varphi_c}{dt} &= B[\Psi_c - \psi] \\
\frac{d \varphi_t}{dt} &= \frac{B}{C}[\psi - \Psi_c] \\
\frac{d \psi}{dt} &= \frac{1}{B}[\varphi_c - \varphi_t] \\
\frac{d \Psi_c}{dt} &= \frac{1}{\tau}[\Psi_{c,ss} - \Psi_c]
\end{align*}
\]  

(2.3)

The equations for the behavior of the dimensionless mass-flow \( \varphi_c \) in the compressor duct and \( \varphi_t \) in the throttle duct are essentially the momentum equations for each duct. \( \Psi_c \) is the dimensionless pressure rise across the compressor and \( \Psi_t \) gives the dimensionless pressure drop across the throttle. The equation for the pressure rise in the plenum \( \psi \) gives the mass conservation in the plenum. The expression for the dimensionless pressure rise across the compressor \( \Psi_c \) is a first order transient response model with time constant \( \tau \) and \( \Psi_{c,ss} \) the steady-state dimensionless compressor pressure rise given in the compressor map. In
2.2 Closed-loop two-state compressor surge model

These equations the Greitzer stability parameter is defined as

\[ B = \frac{U_t}{2\omega H L_c} \]  \hspace{1cm} (2.4)

and the dimensionless parameter

\[ G = \frac{L_t A_c}{L_c A_t} \]  \hspace{1cm} (2.5)

with the throttle duct length \( L_t \) and area \( A_t \).

2.2 Closed-loop two-state compressor surge model

The model that is used to describe the system during surge in all the simulations performed in this report, is the two state Greitzer lumped parameter model. The assumptions made for the use of this model are (Willem [2000, Section 2.2, p. 23-24]):

1) \( A_t \approx A_c \) and \( L_t \) is significantly smaller than \( L_c \) therefore \( G \) in Equation 2.5 is small
2) the compressor behaves quasi-stationary and therefore in Equation 2.3 the fourth formula can be neglected
3) rotational speed variations are negligible
4) overall temperature ratio of the plenum and ambient temperature is near unity.

The system is now described by

\[ \frac{d\varphi_c}{dt} = B[\Psi_c - \psi] \] \hspace{1cm} (2.6)
\[ \frac{d\psi}{dt} = \frac{1}{B}[\varphi_c - \varphi_t] \] \hspace{1cm} (2.7)
\[ \frac{d\varphi_b}{dt} = c_t u_t \sqrt{\psi} \] \hspace{1cm} (2.9)

The cubic polynomials \( \Psi_c(\varphi_c) \) from Willem [2000, Section 2.3.1, p. 24] are used to approximate the steady-state compressor characteristic as determined from experiments. These are modified versions of the cubic polynomials in Moore and Greitzer [1986], only these modifications give deep surge behavior and improve prediction of the surge frequency.

The throttle behavior which will be used is given by

\[ \varphi_t(u_t, \psi) = c_t u_t \sqrt{\psi} \] \hspace{1cm} (2.9)

in which \( c_t \) is the dimensionless throttle parameter and \( u_t \) the dimensionless throttle position.

A surge control system is implemented, as can be seen in Figure 2.2, with the valve behavior similar as the throttle behavior

\[ \varphi_b(u_b, \psi) = c_b u_b \sqrt{\psi} \] \hspace{1cm} (2.10)
this gives the following set of equations:

\[
\begin{align*}
\frac{d\Phi_c}{dt} &= B[\Psi_c(\Phi_c) - \psi] \\
\frac{d\psi}{dt} &= \frac{1}{B}[\Phi_c - \Phi_t(u_t, \psi) - \Phi_b(u_b, \psi)]
\end{align*}
\tag{2.11}
\]

Linearization around the operating point \((\Phi_{c0}, \psi_0, u_{t0}, u_{b0})\) where the subscript 0 indicates the nominal value, with the following perturbed variables

\[
\begin{align*}
\tilde{\psi} &= \psi - \psi_0 \\
\tilde{\Phi}_c &= \Phi_c - \Phi_{c0} \\
\tilde{u}_b &= u_b - u_{b0}
\end{align*}
\tag{2.12-2.14}
\]

gives the state space model

\[
\dot{x} = \begin{pmatrix} \frac{\tilde{\Phi}_c}{\tilde{\psi}} \\ \psi \end{pmatrix} = \begin{pmatrix} BM_c & -B \\ \frac{1}{\beta} & -\frac{1}{\beta \beta \beta} \end{pmatrix} \begin{pmatrix} \tilde{\Phi}_c \\ \tilde{\psi} \end{pmatrix} + \begin{pmatrix} 0 \\ -\frac{V}{\beta} \end{pmatrix} \tilde{u}_b
\tag{2.15}
\]

The dimensionless slope of the compressor characteristic is

\[
M_c = \frac{\partial \psi_c}{\partial \Phi_c} \bigg|_{\Phi_{c0}}
\tag{2.16}
\]
2.3 Parameters used in simulations

The parameters which are used in all simulations according to Willems [2000, table 2.5 on p. 31], are given in Table 2.1. The compressor curve ($\Psi_c(\phi_c)$) is essentially described by the cubic polynomial in Willems [2000, Section 2.3.1, p. 24], however a shifted valley point for the deep surge case is introduced in Willems [2000, Section 2.3.2, p. 26] and therefore the compressor curve used here consists of two different polynomials. This data of the compressor curve for a compressor speed of 25000 rpm is plotted in Figure 2.3. A polynomial fit is made here to be able to easier implement the compressor curve as one function instead of two in the simulations. This fit is also shown in the same Figure. In Willems [2000] compressor speeds are used in a range from 18000-25000 rpm. A compressor speed of 25000 rpm is chosen in this work, since a high-speed system is in general more difficult to stabilize from deep surge and the restrictions found for high speeds are therefore also valid for lower speeds.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressor duct area $A_c$ [$m^2$]</td>
<td>7.9e-3</td>
</tr>
<tr>
<td>Compressor speed $N$ [rpm]</td>
<td>25000</td>
</tr>
<tr>
<td>Throttle parameter $c_t$ [-]</td>
<td>0.3320</td>
</tr>
<tr>
<td>Control valve capacity $c_b$ [-]</td>
<td>0.07$c_t$</td>
</tr>
<tr>
<td>Speed of sound $a$ [m/s]</td>
<td>340</td>
</tr>
<tr>
<td>Plenum volume $V_p$ [$m^3$]</td>
<td>2.03e-2</td>
</tr>
<tr>
<td>Equivalent compressor duct length $L_c$ [m]</td>
<td>1.8</td>
</tr>
<tr>
<td>Rotor tip radius $R_t$ [m]</td>
<td>0.09</td>
</tr>
<tr>
<td>Greitzer stability parameter $B$ [-]</td>
<td>0.41</td>
</tr>
</tbody>
</table>

the dimensionless slope of equivalent throttle parameter

$$M_{te} = \left[ \left( \frac{\partial(\varphi_t + \varphi_b)}{\partial \psi} \right)_{(\phi_c, u_t, u_b)} \right]^{-1}$$

(2.17)

and the dimensionless slope of bleed valve characteristic

$$V = \frac{\partial \varphi_b}{\partial u_b} \bigg|_{(\psi, u_{b0})}$$

(2.18)

The output feedback is implemented as can be seen in Figure 2.2 and is given by the following relationship

$$\tilde{u}_b = -K \cdot \tilde{\psi}$$

(2.19)
A point on the compressor curve has a specific massflow and corresponding pressure, in this report the points on this curve will be expressed using the massflow as a function of $F$. Here the distance $2F$ is defined as the value of the compressor massflow in the point on the peak of the compressor curve, see Figure 2.4.
The dimensionless sample time $T_{s,\text{dim}}$ used in the simulations, for the discrete linear models is: $T_{s,\text{dim}} = 0.1$, which corresponds to a sample time $T_s = 0.0040 \ \text{s}$. This relationship is given by:

$$T_s = \frac{T_{s,\text{dim}}}{\omega_H} \quad (2.20)$$

where Equation 2.2 is used and a value $\omega_H = 158.1 \ \text{rad/s}$ is found.
2.4 One-sided control

In this work one-sided control is used, in which the nominal control valve position \( u_{b0} = 0 \). The motivation for this choice is discussed in Section 2.5.

As the control input \( u_b \) of the compression system is bounded between 0 (closed) and 1 (fully opened) it is also desired to deal with input constraints in the stability analysis. Therefore the theory of positive feedback stabilization (Heemels and Stoorvogel [1998]) is used.

2.4.1 Positive Feedback Stabilization

The theory of positive feedback stabilization is restricted to linear systems and does not deal with an upper constraint on the control input.

If given the following linear system:

\[
\dot{x}(t) = A \cdot x(t) + Bu(t)
\]  

(2.21)

positive feedback can be constructed of the form:

\[
u(t) = \max(0, -Kx(t))
\]  

(2.22)

with the state \( x(t) \) of the system and the feedback gain \( K \).

If the linear system defined in Equation 2.21 has a scalar input and \( A \) has at most one pair of unstable, complex conjugated eigenvalues, the closed-loop system can be stabilized using positive feedback stabilization if \((A, B)\) is stabilizable and \(\sigma(A) \cap \mathbb{R}_+ = \emptyset\). Here \( \sigma(A) \) is the set of eigenvalues of \( A \).

If using the positive feedback defined in Equation 2.22, such that the eigenvalues of the closed-loop system \( \sigma \pm j\omega \) are contained in

\[
\left\{ \lambda = \sigma + j\omega \in \mathbb{C} \mid \sigma < 0 \text{ and } \left| \frac{\omega}{\sigma} \right| < \left| \frac{\omega_0}{\sigma_0} \right| \right\}
\]  

(2.23)

this guarantees stability for the closed-loop system. In here \( \sigma_0 \pm j\omega_0 \) are the poles of the open-loop system, see Figure 2.5. The mirror images of the open-loop poles determine the boundary of the stabilizing cone, in which the closed-loop poles should lie, to guarantee positive feedback stabilization.

Because the control input of the compression system is bounded between 0 and 1 and the positive feedback does not take an upper bound into account, this means that it has to be tried that the control signal \( u(t) \) generated by positive feedback controller does not exceed 1. This can be done by choosing the feedback gain \( K \) not too large and thus placing the closed-loop poles of the controlled linear model of Equation 2.21 not too far in the left-half plane within the stabilizing cone (Figure 2.5). Specifically a controller can be designed,
2.4 One-sided control

Figure 2.5: Stability region for closed-loop poles.

which places the closed-loop poles closely near the mirror images of the open-
loop poles. According to LQ-control in this case the energy of $\tilde{u}_b$ is minimized
and therefore $\tilde{u}_b$ is kept as small as possible (Willems [2000, p. 57]).

2.4.2 Problems with Positive Feedback Stabilization

There are a few problems when using positive feedback control. The first is that
the open-loop poles need to be complex. The second is the domain of attraction
of a stabilized equilibrium point. Thirdly the robustness of the system and the
fourth the disturbance and noise rejection properties. The focus in this work on
the second, third and fourth point and if MPC can give improvements there.

If the closed-loop poles are placed inside the cone, local stability of a stabi-
lized operating point is guaranteed. As long as the perturbed system stays in
the domain of attraction of this stabilized equilibrium point, stable compressor
operation can be guaranteed (Willems [2000, p. 35]). According to Pinsley et al.
[1991] stabilization is possible using proportional feedback if the surge limit cycle
is contained in the domain of attraction of a nominal operating point. There-
fore, the use of linear and hybrid MPC will be used in this work to investigate if
the domain of attraction of a nominal operating point can be enlarged compared
to positive feedback control. This is done by determining if nominal operating
points with a smaller mass-flow can be stabilized from deep surge using MPC
(hence meaning the surge limit cycle is contained in the domain of attraction
of those nominal operating points), than when solely using positive feedback
control.

When considering robustness of the system, if positive feedback control is
used there is a restriction in the mismatches of system parameters to still be able
to achieve stability from deep surge in the nominal operating point. Therefore
MPC will be used to investigate if the robustness of the system can be improved
2.4 One-sided control

compared to using solely positive feedback control.

Also the limitations of disturbance and noise rejection of the positive feedback controller will be investigated and MPC is used to determine if improvements can be made in these areas.

In the remainder of this Chapter the above limitations of positive feedback stabilization that were discussed here are investigated. First the applied compressor model is verified by comparing with Willems [2000].
2.5 Output feedback

In this section simulations are performed using output feedback to stabilize the non-linear compression system from deep surge. As was mentioned in Section 2.3 the control valve capacity used in this report is \( c_b = 0.07c_t \). However, to be able to compare results and verify the designed model for output feedback here with the one in Willems [2000], \( c_b = 0.1c_t \) is first also used in this section.

\[
\tilde{u}_b = \max(0, -K \tilde{\psi}) \tag{2.24}
\]

The second saturation block is used to model the control input of the compression system which is bounded between 0 and 1.

A root locus plot of the controlled linear compressor model in a specific operating point (Equation 2.15) is first made. In this way an estimation for a stabilizing controller gain for the non-linear compression system simulations can be made. The operating point used has a massflow \( \phi_{c,0} = 1.9F \) (see Figure 2.4 for the definition of the massflow), this point is chosen to compare the results directly with those of Willems [2000, Fig. 4.4 on p. 48] in the same operating point and therefore \( c_b = 0.1c_t \) is used here. The root locus plot is shown in Figure 2.11. The parameters chosen further are the ones in Table 2.1, with a compressor speed of 25000 rpm. In the upper figure the closed-loop poles \( \lambda \) of the linear compressor model with a linear feedback gain \( K \) are shown in the complex plane. To guarantee stability using positive feedback control as is
described in Section 2.4, the poles should be on the left side of the dotted cone. In the lower figure of the root locus plot the controller gains $K$ are shown as a function of the real parts of the closed-loop poles and it is concluded that the controller gains $K$ need to be in the range from approximately -20 up to -7, to place the closed-loop poles within the cone (upper figure) and assure positive feedback stability. Now this root-locus plot is compared with Willems [2000, Fig. 4.4 on p. 48], from which it can be observed that the range of control gains $K$ for the closed-loop poles to be placed inside the cone is approximately -12 up to -9.5. Hence it can be concluded that the range of control gains here is larger and therefore a larger operating region is expected than in Willems [2000]. This is investigated next.

![Root-locus plot](image)

**Figure 2.7**: Root-locus plot; $\phi_{c0} = 1.9F, c_b = 0.1c_t$, with the area to the left of the cone the stable area for the closed-loop poles required for positive feedback stability.

To determine the operating point with the smallest mass-flow that can be stabilized from deep surge the non-linear compression system shown in Figure 2.6 is simulated and again $c_b = 0.1c_t$ is used to compare the result with Willems [2000].

The simulation is setup as follows: the system is brought into deep surge with a pulse on the system input after 1 second, the pulse height is 1 [-] and the duration 0.1 s. The positive feedback controller is switched on after 2 seconds. To test the system’s response to a disturbance, the same pulse is repeated after 8 seconds.

To design the feedback controller the following procedure is used: 1) the
feedback gain $K$ of the linear model with linear feedback controller is chosen such that the closed-loop poles are within the stabilizing cone in the complex plane and 2) it is tried to keep the value of the control signal $u_b$ maximum 1 (Section 2.4), since higher values cannot be used and the positive feedback theory cannot be applied anymore because no upper bound is defined in this theory.

From simulations it appears that surge stabilization is not possible below $\phi_{c0} = 1.84F$. The gain that stabilizes this operating point is $K = -11$. In Figure 2.8 the position of closed-loop poles in the complex plane is shown. Figure 2.9 shows the dimensionless pressure and mass-flow as a function of time and Figure 2.10 also shows the controller actions of this simulation. In Willems [2000, p. 58] $\phi_{c0} = 1.87F$ is found to be the smallest value using output feedback and $c_b = 0.1c_t$. Hence the operating region found here is larger than in Willems [2000] as was expected. Here the surge point massflow is reduced with 8% versus 6.5% in Willems [2000]. Since all parameters used here are the same as in Willems [2000], it is expected that this is due to a mismatch in the compressor curves used here, likely due to the polynomial fit used. However, this difference is accepted and therefore the developed model here is used further in this work.

![Figure 2.8: Pole locations of open-loop (small) and closed-loop (big); $\phi_{c0} = 1.84F$, $K = -11$ (output feedback).](image-url)
Figure 2.9: Dimensionless pressure and massflow as a function of time; $\phi_{c0} = 1.84F$, $K = -11$ (output feedback).

Figure 2.10: Dimensionless mass-flow, dimensionless pressure and control signals; $\phi_{c0} = 1.84F$, $K = -11$ (output feedback).
Figure 2.11: Root-locus plot for $\phi_{c0} = 1.9F$, $c_b = 0.07c_t$; with the area to the left of the cone the stable area for the closed-loop poles required for positive feedback stability.

Now it is tried to stabilize the system from deep surge in the operating point $\phi_{c0} = 1.9F$ with the control valve capacity $c_b = 0.07c_t$, also using different controller gains $K$ and nominal control valve positions $u_{b0}$ to investigate the effect on surge stabilization. The simulation is setup as before: the system is brought into deep surge with a pulse on the system input after 1 second, the pulse height is 1 and the duration 0.1 s. The positive feedback controller is switched on after 2 seconds. To test the system’s response to a disturbance, the same pulse is repeated after 8 seconds.

The root locus plot of the linear model is depicted in Figure 2.11. Placing the closed-loop poles of the linear compressor model outside the cone with a gain $K = -6$, no matter if $u_{b0}$ is 0 or 0.5, the non-linear compressor with this gain cannot be stabilized. A gain $K = -17$ stabilizes the system, with both $u_{b0} = 0$ and 0.5. These results are consistent with what one would expect from the root locus plot in Figure 2.11 (although this is only accurate locally in $\phi_{c0} = 1.9F$). According to Willems [2000, Section 4.1, p. 43], closing the control valve below the nominal value has hardly any effect to stabilize the system from surge and for a non-zero $u_{b0}$ a non-zero stationary bleed valve mass-flow is required. It also appears that $u_{b0} = 0.5$ has disadvantages above $u_{b0} = 0$ because the switch-on time of the controller cannot be chosen arbitrarily to stabilize the system from surge, therefore one sided control in which $u_{b0} = 0$ is used in the rest of this work.

Using the same procedure as described earlier in this section when a mini-
2.5 Output feedback

The minimum mass-flow $\phi_{c0} = 1.84F$ was found using a control valve capacity $c_b = 0.1c_t$, it appears that surge stabilization is not possible below $\phi_{c0} = 1.87F$ with $c_b = 0.07c_t$. Therefore it can be concluded that the domain of attraction of an operating point can be enlarged here if $c_b$ is increased. The gain that stabilizes $\phi_{c0} = 1.87F$ is $K = -19$, in Figure 2.12 the position of closed-loop poles in the complex plane is shown and in Figure 2.13 the dimensionless pressure and mass-flow as a function of time and Figure 2.14 shows the controller actions of the simulation.

![Pole Locations](image)

Figure 2.12: Pole locations of open-loop (small) and closed-loop (big): $\phi_{c0} = 1.87F$, output feedback $K = -19$. 
2.5 Output feedback

Figure 2.13: Dimensionless pressure and massflow as a function of time; $\phi_{c0} = 1.87F$, $K = -19$ (output feedback).

Figure 2.14: Dimensionless mass-flow, dimensionless pressure and control signals; $\phi_{c0} = 1.87F$, $K = -19$ (output feedback).
2.6 Output-feedback and robustness

In this section the robustness of the compression system using output-feedback is investigated. This is done by introducing uncertainties in the Greitzer stability parameter $B$ and dimensionless slope of the compressor characteristic $M_c$ and determining their effect on the stability of the linearized closed-loop system. As was mentioned in Section 2.5 the control valve capacity used in this report (Section 2.3) is $c_b = 0.07c_t$, but to be able to compare results here concerning system robustness with (Willems [2000, Section 4.3.1 on p. 55-56]), $c_b = 0.1c_t$ is also used in this section. In the previous section it was concluded that there is mismatch in the used polynomial fit of the compressor curve used here, therefore it is expected that the results will not be same as in Willems [2000]

**LINEAR CASE**

![Diagram](image)

Figure 2.15: Robustness: feedback design such that both linear models are stabilized.

2.6.1 Robustness of linearized compressor model

The robustness of the linearized compressor model is determined in the following way: a linear compressor model in a specific operating point is designed, with $B_{old}$ and $M_{c,old}$ the original values for the Greitzer stability parameter and dimensionless slope of the compressor characteristic. Using this same linear model, the Greitzer stability parameter and dimensionless slope of the compressor characteristic are manipulated into: $B = xB_{old}$ and $M_c = xM_{c,old}$. Here the
gain \( x \) is defined as \( x = (1 + \frac{\%}{100}) \) or \( x = (1 - \frac{\%}{100}) \), where the percentage \( \% \) indicates the increase or decrease of the manipulated parameters. This is depicted in Figure 2.15. The maximum value of \( x \), for which both systems can still be stabilized using the same positive feedback stabilizing controller \( K \), determines the level of robustness. Thus a high achievable \( x \) means a good robustness.

First the robustness of the linearized compressor model with the following parameters is investigated: massflow \( \phi_{\text{c,0}} = 1.90F \) at a speed of 25000 rpm and \( c_b = 0.1c_t \), where \( B_{\text{old}} \) and \( M_{c,\text{old}} \) are the original values in the linear model. The maximum percentage that \( B_{\text{old}} \) and \( M_{c,\text{old}} \) can be manipulated, is determined and explained using the root-locus technique (which was explained in the previous Section 2.5). The closed-loop poles for varying control gains of the unmanipulated linear system are plotted in Figure 2.16 (which is the same as in Figure 2.7). In the same figure also the closed-loop poles of a second linear model are plotted, thus linearized in the same operating point, but the Greitzer stability parameter and dimensionless slope of the compressor characteristic are manipulated into: \( B = 1.42B_{\text{old}} \) and \( M_c = 1.42M_{c,\text{old}} \). From the root-locus plots of those two linear systems it can be determined that \( B = 1.42B_{\text{old}} \) and \( M_c = 1.42M_{c,\text{old}} \) are the maximum values that the second linear system can have to be able to stabilize the system by placing both closed-loop poles in the left-half plane using output feedback. Both parameters can thus be increased by 42\% (where in Willems [2000, Section 4.3.1 on p. 55-56] both parameters can be increased up to 29\%).

As an illustration the pole locations in Figure 2.18 show that the linear closed-loop system with a gain \( K = -19.38 \), \( B = 1.42B_{\text{old}} \) and \( M_c = 1.42M_{c,\text{old}} \) is stable and places the closed-loop poles near the origin inside its cone. While the linear model with \( B_{\text{old}} \) and \( M_{c,\text{old}} \) has the closed-loop poles much further in the left-half plane and inside the cone. It can thus be concluded that the closed-loop poles shift to the right in the complex plane when the manipulated parameters are increased and the area inside the systems corresponding cone becomes smaller, therefore only if these parameters are larger than expected the system tends to go to instability (as also seen in Section Willems [2000, Section 4.3.1 on p. 55-56]) and thus only this situation will be tested in this work.

Secondly the robustness of the linearized compressor model with the following parameters is investigated: massflow \( \phi_{\text{c,0}} = 1.90F \) at a speed of 25000 rpm, \( c_b = 0.07c_t \) with \( B_{\text{old}} \) and \( M_{c,\text{old}} \) the original values in the linear model. The same procedure as was described above for \( c_b = 0.1c_t \) is used. In Figure 2.17 the root locus plot of the controlled unmanipulated linear model with \( B_{\text{old}} \) and \( M_{c,\text{old}} \) is plotted (which is the same as in Figure 2.11) and also of the controlled linear model with manipulated parameters: \( B = 1.42B_{\text{old}} \) and \( M_c = 1.42M_{c,\text{old}} \). These are the same maximum values as were found for \( c_b = 0.1c_t \) that the manipulated parameters can have to be able to stabilize the linear model using output feedback. When comparing the root-locus plots with the two different control valve capacities described above, it can be seen that higher absolute feedback gains are needed to stabilize the system when using a smaller control valve capacity. From these results it can be concluded that the robustness of the
linear compressor model is not effected by changing the control valve capacity $c_b$, which can be explained by the fact that when the control valve capacity is reduced, a higher control gain can counteracts this effect (Equation 2.10).

Figure 2.16: Root-locus plots for $\phi_{c0} = 1.9F$ and $c_b = 0.1c_t$, $B_{old}$, $M_{e,old}$ (grey plots) and $B = 1.42B_{old}$, $M_e = 1.42M_{e,old}$ (black plots).
Figure 2.17: Root-locus plots for $\phi_{c0} = 1.9F$ and $c_b = 0.07c_l$; $B_{old}$, $M_{c,old}$ (grey plots) and $B = 1.42B_{old}$, $M_c = 1.42M_{c,old}$ (black plots).
Figure 2.18: Pole locations of open-loop (small) and closed-loop (big) for $\phi_{c0} = 1.90F$, $K = -19.38$ (output feedback); $B_{old}$, $M_{c,old}$ (grey) and $B = 1.42B_{old}$, $M_c = 1.42M_{c,old}$ (black).
2.6.2 Robustness of non-linear compression system

The goal here is to investigate the robustness of the non-linear compressor model, this is done as in the linear case by introducing uncertainties in the Greitzer stability parameter $B$ and dimensionless slope of the compressor characteristic $M_c$ and determining their effect on surge stabilization. The procedure is however different then in the linear case.

Uncertainties in the non-linear compressor model are more difficult to implement compared to the linear models of the previous Section 2.6.2, since a mismatch in the dimensionless slope of the compressor curve $M_c$ in the desired operating point cannot be easily implemented in the non-linear case, without changing a part or the entire compressor characteristic. Therefore the following procedure is used here to compare the robustness of the linear system of the previous section and the non-linear system in the same operating point $\phi_{c,0} = 1.90F$ and with the same relative parameter mismatch of $+42\%$ as in Section 2.6.1, see Figure 2.19:

Step 1. The linear model of the non-linear system is derived in the desired operating point.

Step 2. $B_{\text{old}}$ and $M_{c,\text{old}}$ of this linear model are manipulated and changed into $B = \frac{1}{1.42}B_{\text{old}}$ and dimensionless slope of the compressor characteristic $M_c = \frac{1}{1.42}M_{c,\text{old}}$.

Step 3. The positive stabilizing output-feedback is designed on the manipulated linearized model of the non-linear compression system with the parameters $B$ and $M_c$.

Step 4. The designed controller is now implemented on the non-linear model. The result is that the compression system on which the controller is implemented, has a Greitzer stability parameter $B$ and dimensionless slope of the compressor characteristic $M_c$ which are both $+42\%$ larger in the desired operating point, than the controller was designed for.

In Figure 2.20 the closed-loop poles of the linear system with $B = \frac{1}{1.42}B_{\text{old}}$ and $M_c = \frac{1}{1.42}M_{c,\text{old}}$ are shown. This is the system for which the controller was designed. In the same figure the closed-loop poles are shown of the linear model in the operating point of the non-linear system the controller is implemented on, where it can be seen that those poles shift more to the right and the cone for positive feedback stabilization becomes smaller.

The simulation results are shown in Figure 2.21, the system is brought into deep surge with a pulse on the system input after 1 second, the pulse height is 1 and the duration 0.1 s. The positive feedback controller is switched on after 2 seconds. To test the system’s response to a disturbance, the same pulse is repeated after 8 seconds. It can be seen that the controller stabilizes the system from deep surge in $\phi_{i,0} = 1.90F$ when switched on and also stabilizes the system after the disturbance is introduced.
Figure 2.19: Feedback design for parameter mismatch in non-linear compressor simulations.
2.6 Output-feedback and robustness

Figure 2.20: Pole locations of open-loop (small) and closed-loop (big) for $\phi_c = 1.90F$, $K = -13$ (output feedback); $B_{old}$, $M_{c,old}$ (grey) and $B = \frac{1}{1.12}B_{old}$, $M_c = \frac{1}{1.42}M_{c,old}$ (black).

Figure 2.21: Dimensionless mass-flow, dimensionless pressure and control signals, $\phi_c = 1.90F$, $K = -13$ (output feedback).
2.7 State feedback

It appears that in case of output feedback (see Section 2.5), stabilization from deep surge is not possible below $\phi_{c0} = 1.87F$. Therefore state-feedback is applied here to investigate if surge stabilization below this mass flow is possible. Here the state-feedback is given by the following relationship:

$$
\tilde{u}_b = \text{MAX} \left( 0, -K \cdot \left( \frac{\bar{\varphi}_c}{\bar{\psi}} \right) \right)
$$

(2.25)

![Diagram of state-feedback on compression system.](image)

Figure 2.22: Scheme of state-feedback on compression system.

First it is tried to stabilize the compression system from deep surge in $\phi_{c0} = 1.78F$, which is the lowest possible massflow that can be reached using state-feedback in Willems [2000, Section 4.3.2 on p. 56], however the valve dynamics used in the last are not infinitely fast, as is considered here. Therefore direct comparison cannot be made and also because the model here is not the same due to the mismatching compressor curve used. The feedback used here is $K = (120 \quad -50)$ and places the closed-loop poles on the real axis in the complex plane as is shown in Figure 2.23, which is near the closed-loop poles in the simulation Willems [2000, Figure 4.13 on p. 57], where they are -2 and -0.1 (although the compressor model used here is not the same surge stabilization is also expected here, since the closed-loop poles are not far in the left half plane). Simulating this controller as depicted in Figure 2.22, gives the massflow, pressure and control signals as a function of time, which are shown in Figure 2.24. In the simulations in this section the system is brought into surge with a pulse on the system input after 1 second, the pulse height is 1 $[-]$ and the duration 0.1 s. To test the system’s response to a disturbance, the same pulse is repeated after 8 seconds. It is clear from these results that switching on the feedback controller stabilizes the system from surge in $\phi_{c0} = 1.78F$ and also the
system is stabilized again in this operating point if the disturbance is introduced.

Figure 2.23: Pole locations of open-loop (small) and closed-loop (big) for $\phi_{c0} = 1.78F$, $K = [120, -50]$ (state-feedback).
Figure 2.24: Dimensionless mass-flow, dimensionless pressure and control signals for $\phi_{c0} = 1.78F$, $K = [120, -50]$ (state-feedback).
Secondly it is tried if a lower massflow can be stabilized than $\phi_{c0} = 1.78F$. After trial and error $1.75F$ seems the lowest massflow reachable with positive feedback. Three controller are described for this operating point, to give more insight into surge stabilization and the position of the closed-loop poles.

First a controller is designed, which places the closed-loop poles near the mirror images of the open-loop poles. As was discussed in Section 2.4.1, by placing the closed-loop poles at the mirror images of the open-loop poles the energy of $\tilde{u}_b$ is minimized and therefore $\tilde{u}_b$ is kept as small as possible. The closed-loop poles are shown in Figure 2.25. This does not stabilize the system from deep surge as can be seen in the simulation results shown in Figure 2.26 and the zoom of the control signals in 2.27. The second controller places the poles further in the left-half plane and both values are on the real axis, this is depicted in Figure 2.28. The simulation results in Figure 2.29 clearly show that the operating point can be stabilized and that the control signal $\tilde{u}_b$ achieves values up to 2.2. This means the upper control constraint of 1 is violated, however stability can still be achieved. The third controller shifts the poles even more into the left-half plane, shown in Figure 2.30. The system cannot be stabilized in this case, shown in Figure 2.31 and the zoom of the control signals in 2.32.

From these results it is seen that if the closed-loop poles are near the mirror images of the open-loop poles, the system cannot be stabilized from deep surge in $\phi_{c0} = 1.75F$. Placing the closed-loop poles further in the left-half plane and on the real axis, shows that the control actions become faster and the system can be stabilized. However if the closed-loop poles are real and too far in the left-half plane, the control signals becomes too large and the upper constraint becomes limiting and the system cannot be stabilized. This is according to Willems [2000, p. 57].
2.7 State feedback

Figure 2.25: Pole locations of open-loop (small) and closed-loop (big) for $\phi_{c0} = 1.75F, K = [153, -43]$ (state-feedback).

Figure 2.26: Dimensionless mass-flow, dimensionless pressure and control signals for $\phi_{c0} = 1.75F, K = [153, -43]$ (state-feedback).
2.7 State feedback

Figure 2.27: Dimensionless mass-flow, dimensionless pressure and zoomed plot of control signals for $\phi_{c0} = 1.75F$, $K = [153, -43]$ (state-feedback).

Figure 2.28: Pole locations of open-loop (small) and closed-loop (big) for $\phi_{c0} = 1.75F$, $K = [200, -60]$ (state-feedback).
Figure 2.29: Dimensionless mass-flow, dimensionless pressure and control signals for $\phi_{c0} = 1.75F$, $K = [200, -60]$ (state-feedback).

Figure 2.30: Pole locations of open-loop (small) and closed-loop (big) for $\phi_{c0} = 1.75F$, $K = [270, -70]$ (state-feedback).
2.7 State feedback

Figure 2.31: Dimensionless mass-flow, dimensionless pressure and control signals for $\phi_{c0} = 1.75F$, $K = [270, -70]$ (state-feedback).

Figure 2.32: Dimensionless mass-flow, dimensionless pressure and zoomed plot of control signals for $\phi_{c0} = 1.75F$, $K = [270, -70]$ (state-feedback).
2.8 State feedback and robustness

To investigate the robustness of the non-linear compressor system with feedback control, the same approach that was discussed for output feedback is used here. As in Section 2.6.2 the robustness for output-feedback on the non-linear compressor system was investigated by implementing uncertainties in the Greitzer stability parameter and dimensionless slope of the compressor characteristic and is repeated here.

2.8.1 Nominal operating point selection

The desired operating point chosen has a massflow of $\phi_{c,0} = 1.80F$, this operating point guarantees that the system is not on the edge of stability as $\phi_{c,0} = 1.75F$ was found to be the lowest massflow which could be stabilized in Section 2.7. A control signal with a maximum value of 1 is desired due to the upper constraint, guaranteeing that if the controller output is maximum 1, the stabilizing positive output feedback theory can be used. Therefore the closed-poles are placed near the mirror images of the open-loop poles (Figure 2.33). As was explained in Section 2.4.1, in this case the energy of $\tilde{u}_b$ is near its minimum and therefore $\tilde{u}_b$ is kept small. The results are shown in 2.34. As can be seen the system can be stabilized by placing the closed-loop poles near the mirror images of the open-loop poles in this case, because the system is not operating on the edge of stability as was the case with the first controller for $\phi_{c,0} = 1.75F$ in Section 2.7. Therefore a smaller $u_b$ is sufficient here. The system here is again brought into surge with a pulse on the system input after 1 second, the pulse height is 1 [-] and the duration 0.1 s. To test the system’s response to a disturbance the same pulse is repeated after 8 seconds.

To investigate the robustness of the system, two scenarios are presented now. The controller that is in implemented in this section is solely state-feedback. To investigate the effects on the robustness using Model Predictive Control (MPC), two scenarios with the same parameter mismatch will be repeated including respectively a MPC (Chapter 3) and Multi Parametric Controller (MPT) (Chapter 4) further on in this work.

2.8.2 Scenario 1

The stabilizing state-feedback is designed on the manipulated linearized model of the non-linear compression system with a relative mismatch of $+25\%$ on the Greitzer stability parameter and dimensionless slope of the compressor characteristic: therefore $B = \frac{1}{1.25}B_{old}$ and $M_c = \frac{1}{1.25}M_{c,old}$. This chosen value of $+25\%$ is motivated in Section 4.4.4 when using hybrid MPC. The closed-loop poles are placed near the mirror images of the open-loop poles with the goal to minimize $\tilde{u}_b$, this is shown in (Figure 2.35). The simulation results show (Figure 2.36) that the system is not stabilized from deep surge. This can be explained
by the fact that the closed-loop poles of the linear model in \(\phi_{c,0} = 1.80F\) of the simulated model with \(B_{old}\) and \(M_{c,old}\) are in the right half plane.

### 2.8.3 Scenario 2

The second scenario represents an extreme situation, in which a relative mismatch of +100\% in the Greitzer stability parameter and dimensionless slope of the compressor characteristic is implemented. This gives: \(B = \frac{1}{2}B_{old}\) and \(M_c = \frac{1}{2}M_{c,old}\). The closed-loop poles are placed on the real axis in the left-half plane (Figure 2.37) by using larger controller gains than in scenario 1, with as goal to give a faster system response. The simulation result in Figure 2.38 show that the system cannot be stabilized from deep surge, which can be explained by the fact that the closed-loop poles of the linear model in \(\phi_{c,0} = 1.80F\) of the simulated model with \(B_{old}\) and \(M_{c,old}\) are in the right half plane.

### 2.9 Conclusions

It was determined in this Chapter that when using state-feedback and positive feedback stabilization, the smallest mass-flow in a nominal operating point which could be stabilized from deep surge is \(\phi_{c,0} = 1.75F\).

Investigating robustness it is clear that in the two scenarios presented the compressor model cannot be stabilized from deep surge with the off-design state-feedback used.

In the next chapter linear MPC is used to determine if improvements can be made in the above results.
Figure 2.33: Pole locations of open-loop (small) and closed-loop (big) for $\phi_{c0} = 1.80F$, $K = [94, -33]$ (state-feedback).

Figure 2.34: Dimensionless mass-flow, dimensionless pressure and control signals for $\phi_{c0} = 1.80F$, $K = [94, -33]$ (state-feedback).
2.9 Conclusions

Figure 2.35: Pole locations of open-loop (small) and closed-loop (big) for $\phi_{c0} = 1.80F$, $K = [25, -13]$ (state feedback); $B_{odd}$, $M_{c,odd}$ (grey) and $B = \frac{1}{1.25}B_{odd}$, $M_{e} = \frac{1}{1.25}M_{c,odd}$ (black).

Figure 2.36: Dimensionless mass-flow, dimensionless pressure and control signals for $\phi_{c0} = 1.80F$, $K = [25, -13]$ (state-feedback).
2.9 Conclusions

Figure 2.37: Pole locations of open-loop (small) and closed-loop (big) for $\phi_{c0} = 1.80F$, $K = [80, -25]$ (state feedback); $B_{old}$, $M_{c,old}$ (grey) and $B = \frac{1}{2}B_{old}$, $M_{c} = \frac{1}{2}M_{c,old}$ (black).

Figure 2.38: Dimensionless mass-flow, dimensionless pressure and control signals for $\phi_{c0} = 1.80F$, $K = [80, -25]$ (state-feedback).
Chapter 3

Linear Model Predictive Surge Control

3.1 Introduction

In this Chapter linear MPC is implemented on the state-feedback controlled compression of Chapter 2 to try and reduce the lowest achievable stable mass-flow \( \phi_{c0} = 1.75F \) found in Section 2.7.

In Section 2.8 two scenarios for robustness were presented and it was concluded that state-feedback was not able to stabilize the system in \( \phi_{c0} = 1.80F \) if there is a mismatch of respectively 25\% or 100\% in the Greitzer stability parameter \( B \) and dimensionless slope of the compressor characteristic \( M_c \). Here two new scenarios are simulated, using the same parameter mismatch and feedback-controllers as in Section 2.8, but now also a linear MPC controller is included. The goal is to see if prediction can improve the robustness of the system compared to the case with solely state-feedback.
3.1 Introduction

Compression system

Saturation

$-K$

$+ u_b$

$0 \leq u_b \leq 1$

Compression system

$\varphi_c, \psi$

$\tilde{\varphi}_c, \tilde{\psi}$

State feedback

$0 \leq \tilde{u}_{b, SF} < \infty$

$\tilde{u}_{b, SAT}$

$\tilde{u}_{b, SF}$

MPC controller

Figure 3.1: Schematic design of state-feedback and MPC on non-linear compression system, which is used for the surge control simulations.
3.2 Linear MPC

The MPC controller is designed with the MPC Matlab Toolbox (Mat [2004]) which solves a constrained quadratic predictive control problem, the resulting controller is an on-line one. The MPC controller uses a linear prediction model and since this controller does not guarantee stability for an unstable plant (Maciejowski [2002, Section 1.6, p. 22]), feedback control is used here to stabilize the linear compressor model system (which guarantees local stability in the desired operating point in this case). The linear model in the desired operating point with a linear state-feedback controller on it gives the closed-loop system, which is used in the MPC controller as prediction model (shown in Figure 3.2) and is derived here. Since only a linear model can be used in the MPC algorithm, the saturations cannot be implemented in this prediction model. Hard constraints are put on the MPC controller output \( \tilde{u}_{b,MPC} \), which is therefore restricted between 0 [\( \cdot \)] and 1 [\( \cdot \)] and this restricts \( \tilde{u}_b \) between the same values only if \( \tilde{u}_b \approx \tilde{u}_{b,MPC} \), which is the case if \( \tilde{u}_{b,SF} \ll \tilde{u}_{b,MPC} \).

![Figure 3.2: Linear model of the state-feedback controlled linear system, which is used in the MPC controller.](image)

The non-linear compression system is linearized in the desired operating point using Equation 2.15, where the control input is given by the following relationship (see Figure 3.1):

\[
\tilde{u}_b = \tilde{u}_{b,SF} + \tilde{u}_{b,MPC}
\]  

(3.1)

with the state-feedback:

\[
\tilde{u}_{b,SF} = -K \cdot \begin{pmatrix} \overline{\dot{\psi}_c} \\ \dot{\psi} \end{pmatrix}
\]  

(3.2)
using the controller gain $K = (K_{11} \ K_{12})$ and control signal $\bar{u}_{b,MPC}$ generated by the MPC controller, which is thus restricted between 0 [-] and 1 [-].

Combining the above, this results in the following linear closed-loop model that is used in the MPC controller

$$
\dot{x} = \left[ \begin{pmatrix} BM_c & -B \\ \frac{1}{\beta} & \end{pmatrix} - \frac{1}{BM_c} \begin{pmatrix} 0 \\ K_{11} \ K_{12} \end{pmatrix} \right] \cdot x + \begin{pmatrix} 0 \\ -\frac{\bar{u}_{b,MPC}}{B} \end{pmatrix} \bar{u}_{b,MPC}
$$

$$
y = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot x \tag{3.3}
$$

the state space matrices are defined as

$$
A_{MPC} = \begin{pmatrix} BM_c & -B \\ \frac{1}{\beta} & \end{pmatrix} - \frac{1}{BM_c} \begin{pmatrix} 0 \\ K_{11} \ K_{12} \end{pmatrix},
$$

$$
B_{MPC} = \begin{pmatrix} 0 \\ -\frac{\bar{u}_{b,MPC}}{B} \end{pmatrix}, \quad C_{MPC} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad D_{MPC} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{3.4}
$$

### 3.2.1 Prediction Horizon

The behavior of the linear system with no saturations (this is how the MPC controller sees the system) versus the non-linear system in $\phi_{c0} = 180^\circ F$ is investigated, to make an estimation of the prediction horizon in the MPC controller.

The linear model describes the dynamical behavior of the compressor model locally around the chosen operating point $\phi_{c0} = 180^\circ F$. The goal here is to estimate for how long the linear model approximates the compressor system when it is brought from stability into deep surge and to make an estimate of the prediction horizon $P$ from this.

The linear and non-linear model are brought into surge in $\phi_{c0} = 1.80F$, at $t = 0$ s, with a pulse on the system input of height 1 [-] with a duration of 0.1 s. The feedback controller $K = [94, -33]$ of Section 2.8 is implemented separately on both systems and activated on both systems after 0.8 s to prevent the linear model from reaching infinitely high values. The results of this test are depicted in Figure 3.3 and it is shown that after 0.4 s the linear model does not approximate the non-linear model anymore and therefore model prediction is expected to be useful up to this time. This time is considered to be approximately one period and first 0.1 s, which is a quarter of a ‘period’ is taken as the prediction time, since half a period appears to give large simulation times (in the order of hours). This means that a prediction horizon of 25 with the sample-time of 0.004s (see
Section 2.3) is first tried. It appears that making $P$ higher than 25 gives no improved results in the simulations performed in this chapter and a prediction horizon of 20 also gives the same results as 25, but lower values than 20 reduce the performance. This will be discussed in more detail in the simulation results.

Figure 3.3: Dimensionless massflow and dimensionless pressure of linear versus non-linear system.
3.3 State-feedback and linear MPC

In this section it is tried if a lower massflow than $\phi_{c0} = 1.75F$ can be stabilized from deep surge using MPC as in Figure 3.1. It is tried to stabilize a massflow of $\phi_{c0} = 1.73F$ from deep surge. Since no stabilizing state feedback controller can be found for $\phi_{c0} = 1.73F$, the controller chosen is the same which stabilizes $\phi_{c0} = 1.75F$ (Section 2.7), $K = [200, -60]$ and places the poles as shown in Figure 3.4.

The following simulation is setup:

$t = 1 \text{ s}$: a pulse with height 1 [-] and duration 0.1 s is given on the system input, to bring the system in deep surge.

$t = 2 \text{ s}$: the state-feedback controller is switched on, to see how this effects the system, no surge stabilization is expected.

$t = 4 \text{ s}$: the MPC controller is also switched on to investigate if both controllers can now stabilize the system from deep surge, since solely state-feedback is not able to.

$t = 6 \text{ s}$: MPC and state-feedback control are both switched off

$t = 8 \text{ s}$: the MPC controller is switched on to see if only this controller can stabilize the system.

$t = 10 \text{ s}$: the feedback is also switched on, to see if switching on the controllers the other way around makes any difference.

The simulation results are shown in Figure 3.5. As can be seen using feedback and MPC control at the same time, no matter if the feedback is activated before or after MPC is switched on, the system cannot be stabilized in $\phi_{c0} = 1.73F$. Also can be seen that only using MPC control cannot stabilize the system from deep surge in $\phi_{c0} = 1.73F$. It must be noted that the linear model used in the MPC controller includes the state-feedback dynamics, if these dynamics are not included in the linear prediction model, MPC cannot even stabilize in $\phi_{c0} = 1.75F$ from deep surge. Therefore it is concluded that using MPC in the way it is implemented here, the system cannot be stabilized from deep surge further than $\phi_{c0} = 1.75F$, which is here $\phi_{c0} = 1.73F$. This means that the domain of attraction of the nominal operating point cannot be increased by using linear MPC such that the domain contains the surge limit cycle.
3.3 State-feedback and linear MPC

Figure 3.4: Pole locations of open-loop (small) and closed-loop (big) for $\phi_{c0} = 1.73F, K = [200, -60]$ (state-feedback).

Figure 3.5: Dimensionless mass-flow, dimensionless pressure and control signals for $\phi_{c0} = 1.73F, K = [200, -60]$ (state-feedback).
3.4 Robustness using State-feedback and linear MPC

In this section it will be investigated if the robustness of the system can be increased compared to positive feedback by using linear MPC. Here two scenarios with the same mismatches in the Greitzer stability parameter and dimensionless slope of the compressor characteristic as in Section 2.8 are simulated, also using the same state-feedback controllers, but now also an MPC controller is implemented as well. The last uses the linear model of Equation 3.3 as its prediction model.

3.4.1 Scenario 1

In scenario 1 stabilizing state-feedback is designed in the same way as in Section 2.8 on the manipulated linearized compressor model, with a mismatch of 25\% in both the Greitzer stability parameter and dimensionless slope of the compressor characteristic: $B = \frac{1}{1.25} B_{old}$ and $M_c = \frac{1}{1.25} M_{c,old}$. The MPC controller uses these same values in its linear prediction model.

3.4.2 Scenario 2

In scenario 2 there is a mismatch of 100\% in the Greitzer stability parameter and dimensionless slope of the compressor characteristic. The state-feedback is also designed as in Section Section 2.8, with the MPC controller using $B = \frac{1}{2} B_{old}$ and $M_c = \frac{1}{2} M_{c,old}$ in its prediction model.

3.4.3 Disturbances and controller actions

The following simulation is performed for both scenarios:

$t = 1$ s: a pulse with height 1 [-] and duration 0.1 s is given on the system input, to bring the system in deep surge.

$t = 2$ s: the state-feedback controller is switched on, to see as in Section 2.8 how this effects the system.

$t = 4$ s: the MPC controller is also switched on to investigate if both controllers can now stabilize the system, since solely state-feedback was not able to.

$t = 8$ s: a second pulse similar to the first is given, to observe the system’s response to a disturbance, with both controllers acting on the non-linear system.

$t = 12$ s: the feedback is switched off, while the MPC controller is still switched on, this to investigate if in case the system is stable now, solely the MPC controller can keep it stable.
t = 14 s: a third pulse is given, similar to the first two, to determine what the system’s response to a disturbance is with solely MPC controlling the system.

t = 16 s: the feedback is switched on again to determine in case the system is not stabilized, if this can be improved if the feedback is on again.
3.4 Robustness using State-feedback and linear MPC

Table 3.1: Used parameters for robust check of non-linear compressor system with state-feedback and linear MPC controller: $\phi_{c0} = 1.80F$, $B = \frac{1}{1.25} B_{\text{old}}$ and $M_c = \frac{1}{1.25} M_{c,\text{old}}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressor speed [rpm]</td>
<td>25000</td>
</tr>
<tr>
<td>Desired mass-flow $\phi_{cs}$</td>
<td>$\phi_{c0}$ = 1.8F</td>
</tr>
<tr>
<td>Greitzer stab. par. $B$</td>
<td>$0.80^* B_{\text{old}}$</td>
</tr>
<tr>
<td>Compressor curve slope $M_c$</td>
<td>$0.80^* M_{c,\text{old}}$</td>
</tr>
<tr>
<td>State feedback controller gain $K$</td>
<td>[25, -13]</td>
</tr>
<tr>
<td>MPC weight on dim.less mass-flow $\varphi_c$</td>
<td>100</td>
</tr>
<tr>
<td>MPC lower limit and upper limit dim.less mass-flow $\varphi_c$</td>
<td>-5, 5</td>
</tr>
<tr>
<td>MPC weight on dim.less pressure $\psi$</td>
<td>0</td>
</tr>
<tr>
<td>MPC lower limit and upper limit dim.less pressure $\psi$</td>
<td>-5, 5</td>
</tr>
<tr>
<td>MPC prediction horizon P [-]</td>
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<tr>
<td>MPC control horizon M [-]</td>
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<tr>
<td>MPC weight on control signal $\tilde{u}_{b,MPC}$</td>
<td>0</td>
</tr>
<tr>
<td>MPC lower limit, upper limit and rate of change of $\tilde{u}_{b,MPC}$</td>
<td>0, 1, 100</td>
</tr>
</tbody>
</table>

3.4.4 Simulation results Scenario 1

The simulation results of the first scenario are presented here, the parameters used are given in Table 3.1. The closed-loop poles are shown in Figure 3.6, which is the same as in Scenario 1 in Section 2.8. The simulation results are given in 3.7. From these results it can be seen that using both MPC and state-feedback surge stabilization is not possible. Using solely the MPC controller, is not effective either. It has also been tried to use only the plant without state-feedback as the prediction-model, this gives no improved results however.

The prediction horizon $P$ is chosen 20 (Section 3.2.1), since larger values seem to give no improvements in the results and this value keeps the calculation time in the order of minutes. It is determined from simulations that also putting a weight on the dimensionless pressure, with the weight having the same value or higher than on the massflow, takes the system to the stable solution with the same pressure value on the right side of the top of the curve.
3.4 Robustness using State-feedback and linear MPC

Figure 3.6: Pole locations of open-loop (small) and closed-loop (big) for $\phi_{c,0} = 1.80F$, $K = [25, -13]$ (state feedback); $B_{old}$, $M_{c,old}$ (grey) and $B = \frac{1}{1.25} B_{old}$, $M_c = \frac{1}{1.25} M_{c,old}$ (black).

Figure 3.7: Dimensionless mass-flow, dimensionless pressure and control signals for $\phi_{c,0} = 1.80F$, $K = [25, -13]$ (state-feedback).
Table 3.2: Used parameters for robust check of non-linear compressor system with state-feedback and linear MPC controller: \( \phi_c = 1.80F \), \( B = \frac{1}{2}B_{old} \) and \( M_c = \frac{1}{2}M_{c,old} \).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Compressor speed [rpm]</td>
<td>25000</td>
</tr>
<tr>
<td>Desired mass-flow ( \phi_{cs} )</td>
<td>( \phi_{c0} = 1.8F )</td>
</tr>
<tr>
<td>Greitzer stab. par. ( B )</td>
<td>( 0.50 \times B_{old} )</td>
</tr>
<tr>
<td>Compressor curve slope ( M_c )</td>
<td>( 0.50 \times M_{c,old} )</td>
</tr>
<tr>
<td>State feedback controller gain ( K )</td>
<td>([80, -25])</td>
</tr>
<tr>
<td>MPC weight on dim.less mass-flow ( \phi_c ) [-]</td>
<td>100</td>
</tr>
<tr>
<td>MPC lower limit and upper limit dim.less mass-flow ( \phi_c ) [-]</td>
<td>([-5, 5])</td>
</tr>
<tr>
<td>MPC weight on dim.less pressure ( \psi ) [-]</td>
<td>0</td>
</tr>
<tr>
<td>MPC lower limit and upper limit dim.less pressure ( \psi ) [-]</td>
<td>([-5, 5])</td>
</tr>
<tr>
<td>MPC prediction horizon ( P ) [-]</td>
<td>20</td>
</tr>
<tr>
<td>MPC control horizon ( M ) [-]</td>
<td>1</td>
</tr>
<tr>
<td>MPC weight on control signal ( \bar{u}_{b,MPC} ) [-]</td>
<td>0</td>
</tr>
<tr>
<td>MPC lower limit, upper limit and rate of change of ( \bar{u}_{b,MPC} ) [-]</td>
<td>([0, 1, 100])</td>
</tr>
</tbody>
</table>

3.4.5 Simulation results Scenario 2

Here the simulation results of the second scenario are presented, the used parameters are given in Table 3.2. The closed-loop poles are shown in Figure 3.8 (which is the same as in Scenario 2 in Section 2.8). The simulation results are given in Figure 3.9, with a zoom of the control signals in Figure 3.10. From these results it can be concluded that only when state-feedback and MPC are activated simultaneously the amplitude of the surge limit cycle decreases significantly. When both controllers are switched on and the system has reached the smallest limit cycle, state-feedback is switched off to determine if only MPC can keep the system in its limit cycle. However, the system goes back into fully developed surge. The linear model used in the MPC controller includes the state-feedback dynamics and therefore it has also been tried to use only the plant without state-feedback as the prediction-model, this gives no improved results however.

The weight of 100 on the dimensionless massflow seems to be the minimum value to achieve the reduced limit cycle, since a lower weight causes this cycle to increase and higher values show no difference in its size. Lower values for the prediction horizon \( P \) than 20 also cause an increase in the limit cycle when MPC is switched on, higher values than 20 do not reduce the size of this limit cycle anymore.

Although there is a larger parameter mismatch in the second scenario than in the first (100% versus 25%), in the second the closed-loop poles of the state-feedback controlled system are placed on the real axis using larger controller gains and result in a faster system response and therefore the MPC controller
can reduce the limit cycle amplitude in this scenario (it must also be noted that in scenario 1 the poles of the closed-loop linear system with parameters $B_{old}$ and $M_{e,old}$ are in the right half plane, where in scenario 2 they are in the left-half plane). Scenario 2 also shows that MPC has to operate together with the state-feedback controller to reduce the limit cycle. Solely MPC cannot keep the system in its small limit cycle, this may be due to the fact that the linear plant is not stable anymore without the feedback controller and the MPC controller does not guarantee stability in this case (Maciejowski [2002, Section 1.6, p. 22]).

3.5 Conclusions

MPC showed not to be able to achieve a lower mass-flow ($\phi_{c,0} = 1.73F$ was tried to stabilize from deep surge) than $\phi_{c,0} = 1.75F$ which was achieved with positive feedback stabilization.

In both scenarios concerning the robustness of the system, the non-linear compressor model cannot be stabilized from deep surge in the nominal point $\phi_{c,0} = 1.80F$. However, the surge limit cycle is reduced significantly in the second scenario, while there is no reduction in the first scenario.

Although the control signal of the MPC controller is restricted between 0 and 1, the MPC controller does not take the saturations in the compression system into account and cannot optimally predict therefore. In the next chapter the Multi-Parametric Toolbox will be used to implement saturations in the prediction model and it is investigated if the results that are achieved here can be improved.
3.5 Conclusions

Figure 3.8: Pole locations of open-loop (small) and closed-loop (big) for $\phi_{c0} = 1.80F$, $K = [80, -25]$ (state feedback); $B_{old}$, $M_{c,old}$ (grey) and $B = \frac{1}{2}B_{old}$, $M = \frac{1}{2}M_{c,old}$ (black).

Figure 3.9: Dimensionless mass-flow, dimensionless pressure and control signals for $\phi_{c0} = 1.80F$, $K = [80, -25]$ (state-feedback).
Figure 3.10: Zoomed plot of control signals for $\phi_\theta = 1.80F$, $K = [80, -25]$ (state-feedback).
Chapter 4

Hybrid Model Predictive Surge Control

4.1 Introduction

Because of the ease of implementation of linear MPC this was first tried in the previous Chapter. However, using linear MPC it was not possible to stabilize the compression system from deep surge in $\phi_{c0} = 1.80F$, if there is a mismatch of $+25\%$ or $+100\%$ in the Greitzer stability parameter $B$ and dimensionless compressor characteristic $M_c$. Therefore in this chapter it is investigated if implementing the saturations in the system in a hybrid model predictive control strategy, better results can be achieved. The Multi Parametric Toolbox (MPT) is used for the design of a hybrid model predictive controller (described in Kvasnica et al. [2006]), which will be called MPT controller in the remainder of this chapter.

4.2 Implemented system in MPT

Here the hybrid prediction model which is implemented in the MPT-toolbox is derived. The simulation setup is the same as with linear MPC, only the controller is replaced by the MPT controller, see Figure 4.1. The saturations in this figure cannot be implemented the same way in the non-linear prediction model, since no direct constraints in the MPT toolbox algorithm that is used here can be placed on $\tilde{u}_b$. The MPT toolbox gives the possibility to provide constraints on the states and input variables to define regions with different dynamics. Therefore the non-linear prediction model which is used for the design of the hybrid prediction model is modified and shown in Figure 4.2. Here the the control signal of the MPT controller $\tilde{u}_{b,MPT}$ is restricted between 0 and 1.
4.2 Implemented system in MPT

Figure 4.1: Schematic design of State feedback and MPT on compressor system.

Figure 4.2: Schematic design of non-linear prediction model used in MPT controller.
This non-linear system is now modeled by 3 linear models, each active in a different part of the state space. The resulting hybrid prediction model is implemented in the MPT toolbox to generate a MPT controller. The three regions with the 3 linear models are defined as (see Figure 4.1 for the used signals)

region 1: \( \bar{u}_{b, SF, sat} = 0 \)

\[ \Rightarrow \dot{x} = A_1 \cdot x + B_1 \cdot \bar{u}_{b, MPT} \quad (4.1) \]

region 2: \( 0 < \bar{u}_{b, SF, sat} < 1 \)

\[ \Rightarrow \dot{x} = [A_1 - B_1 \cdot K] \cdot x + B_1 \cdot \bar{u}_{b, MPT} \quad (4.2) \]

region 3: \( \bar{u}_{b, SF, sat} \geq 1 \)

\[ \Rightarrow \dot{x} = A_1 \cdot x + B_1 \cdot \bar{u}_{b, MPT} \]

\[ - B_1 \quad (4.3) \]

The matrices used here, are those given in the linearization around the operating point (where the subscript 0 indicates the nominal value) \((\varphi_{c0}, \psi_0, u_{t0}, u_{b0})\):

\[ \dot{x} = \left( \begin{array}{cc} BM_c & -B \\ \frac{1}{\rho} & -\frac{1}{BM_{te}} \end{array} \right) \cdot x + \left( \begin{array}{c} 0 \\ -\frac{\nu}{\rho} \end{array} \right) \cdot \bar{u}_b \quad (4.4) \]

\[ y = \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \cdot x \quad (4.5) \]

with the state \( x = \left( \begin{array}{c} \varphi_c \\ \psi \end{array} \right) \) and the state space matrices are defined as

\[ A_1 = \left( \begin{array}{cc} BM_c & -B \\ \frac{1}{\rho} & -\frac{1}{BM_{te}} \end{array} \right), \quad B_1 = \left( \begin{array}{c} 0 \\ -\frac{\nu}{\rho} \end{array} \right) \quad (4.6) \]

\[ C_1 = \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \] and \( D_1 = \left( \begin{array}{c} 0 \\ 0 \end{array} \right) \quad (4.7) \]
4.3 Explicit solution

It must be noted that for region 2 and 3 the compressor control signal $u_b$ can reach values up to 2.

![Figure 4.3: Explicit optimal control system.](image)

4.3 Explicit solution

The controller designed in the MPT toolbox differs from the linear MPC controller used in the Matlab MPC toolbox. Where the MPC controller is implemented as a slow on-line algorithm that has to be solved every time-step, the MPT controller takes the form of a fast state-feedback law. When using MPT the state-space is partitioned into regions and for each of those regions the optimal control law is given as a function of the state. In the on-line implementation of such MPT controllers, computation of the controller action reduces to a sim-
4.4 Robustness of State-feedback and MPT controller

Similar as for linear MPC in Section 3.3 it has been tried using MPT control to reduce the minimum mass-flow \( \phi_{c0} = 1.75F \) that was found using positive feedback stabilization. Since the results were no different than with linear MPC and no lower mass-flow seemed achievable, these results are not presented here.

In this section two scenarios with the same mismatches in \( B \) and \( M_c \) and also using the same state-feedback controllers as described for the linear MPC control in Section 3.4 are simulated, but now the non-linear MPT controller is implemented in stead of the linear MPC controller. The 3 linear prediction models of Section 4.2, will use the manipulated Greitzer stability parameter \( B \) and dimensionless slope of the compressor characteristic \( M_c \). This means that the non-linear compressor model in Simulink uses \( B_{old} \) and \( M_{c,old} \) in the desired operating point \( \phi_{c0} = 1.80F \).

4.4.1 Scenario 1

In the first scenario the MPT algorithm uses the hybrid prediction model with a mismatch of 25\% in both the Greitzer stability parameter and dimensionless slope of the compressor characteristic: \( B = \frac{1}{1.25}B_{old} \) and \( M_c = \frac{1}{1.25}M_{c,old} \), for the design of the MPT controller. The state-feedback used is \( K = [25, -13] \).

4.4.2 Scenario 2

In the second scenario the MPT algorithm uses the hybrid model with a mismatch of 100\% in both the Greitzer stability parameter and dimensionless slope of the compressor characteristic: \( B = \frac{1}{2}B_{old} \) and \( M_c = \frac{1}{2}M_{c,old} \). The state-feedback used is \( K = [80, -25] \).

4.4.3 Disturbances and controller actions

The following simulation is performed with the designed MPT controller for each scenario:

\( t = 1 \text{ s}: \) a pulse with height 1 [-] and duration 0.1 s is given on the system input, to bring the system in deep surge.

\( t = 2 \text{ s}: \) the state-feedback controller is switched on.

\( t = 4 \text{ s}: \) the MPT controller is also switched on to investigate if both controllers can now stabilize the system, since solely state-feedback was not able to.
t = 8 s: a second pulse similar to the first is given, to observe the system’s response to a disturbance, with both controllers acting on the non-linear system.

$t = 12\ s$: the feedback is switched off, while the MPT controller is still switched on, this to investigate if in case the system is stable now, solely the MPC controller can keep it stable.

$t = 14\ s$: a third pulse is given, similar to the first two, to determine what the system’s response to a disturbance is with solely MPT controlling the system.

$t = 16\ s$: the feedback is switched on again to determine in case the system is not stabilized, if this can be improved if the feedback is on again.
4.4 Robustness of State-feedback and MPT controller

Table 4.1: Used parameters for robust check of non-linear compressor system with state-feedback and MPT controller: $\phi_{c0} = 1.80F$, $B = \frac{1}{1.25} B_{old}$ and $M_c = \frac{1}{1.25} M_{c,old}$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Compressor speed [rpm]</td>
<td>25000</td>
</tr>
<tr>
<td>Desired mass-flow $\phi_{cs}$</td>
<td>$\phi_{c0} = 1.8F$</td>
</tr>
<tr>
<td>Greitzer stab. par. $B$ of linear control model</td>
<td>$0.80^*B_{old}$</td>
</tr>
<tr>
<td>Compressor curve slope $M_c$ of linear control model</td>
<td>$0.80^*M_{c,old}$</td>
</tr>
<tr>
<td>State feedback controller gain $K$</td>
<td>$[25, -13]$</td>
</tr>
<tr>
<td>MPT weight on dim.less mass-flow $\phi_c$ [-]</td>
<td>100</td>
</tr>
<tr>
<td>MPT lower limit and upper limit dim.less mass-flow $\phi_c$ [-]</td>
<td>$-5, 5$</td>
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<tr>
<td>MPT weight on dim.less pressure $\psi$ [-]</td>
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<tr>
<td>MPT lower limit and upper limit dim.less pressure $\psi$ [-]</td>
<td>$-5, 5$</td>
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<td>MPT prediction horizon $P$ [-]</td>
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<tr>
<td>MPT control horizon $M$ [-]</td>
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<tr>
<td>MPT lower limit, upper limit of $\tilde{u}_{b,MPT}$ [-]</td>
<td>0, 1</td>
</tr>
</tbody>
</table>

4.4.4 Simulation results Scenario 1

The simulation results of scenario 1 are presented in Figure: 4.4. From these results it can be seen that if state-feedback and MPT are switched on at the same time, the system is stabilized from deep surge in $\phi_{c0} = 1.80F$. Now the sample-time in Simulink is reduced by a factor 2 to 0.002 s and the results are in Figure 4.5. This shows that lowering the sample-time reduces the time it takes the system to reach $\phi_{c0} = 1.80F$. Note that here the sample-time can be lowered in the simulations, due to the explicit controller form used, which is much faster on-line because of the look-up procedure. As for the on-line linear MPC case a sample-time reduction to 0.002 s causes the simulation time to become in the order of hours instead of minutes as here. Independent which sample-time is used it can be observed that the disturbances introduced are only rejected if both state-feedback and MPT control are both switched on.

The prediction horizon $P$ is chosen 16, which was 20 in the MPC controller. This seems to be the highest reachable value here, since no controller solution can found any more for higher horizons. The weight of 100 on the dimensionless massflow seems to be the minimum value, since lower values give worse results an higher values make no difference. Again (as was also seen for linear MPC) it is determined from simulations that also putting a weight on the dimensionless pressure, with the weight having the same value or higher than on the massflow, takes the system to the stable solution with the same pressure value on the right side of the top of the curve.

In the first scenario described here and also for full-state feedback (Section 2.8 and MPC (Section 3.4), the state-feedback controller is designed by placing the closed-loop poles near the mirror images of the open-loop poles of the linear
model. The goal of this is to keep $\bar{u}_b$ as small as possible because of the restriction of 1 on the control input. If it is tried to increase the parameter mismatch above 25% and using this same state-feedback design method by placing the closed-loop poles near the mirror images of the open-loop poles, it seems not possible anymore to stabilize the system in $\phi_{c0} = 1.80F$ using a MPT controller. Therefore this maximum value of 25% as mismatch in the parameters is found here.

Figure 4.4: Dimensionless mass-flow, dimensionless pressure and control signals for $\phi_{c0} = 1.80F$, $K = [25, -13]$ (state-feedback).
4.4 Robustness of State-feedback and MPT controller

Figure 4.5: Dimensionless mass-flow, dimensionless pressure and control signals for $\phi_{c0} = 1.80F, K = [25, -13]$ (state-feedback), $T_s = 0.002$ s.
Table 4.2: Used parameters for robust check of non-linear compressor system with state-feedback and MPT controller: $\phi_0 = 1.80F, B = \frac{1}{2}B_{old}$ and $M_c = \frac{1}{2}M_{c,old}$.

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<td>Greitzer stab. par. $B$ of linear control model</td>
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</tr>
<tr>
<td>Compressor curve slope $M_c$ of linear control model</td>
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</tr>
<tr>
<td>State feedback controller gain $K$</td>
<td>[80, -25]</td>
</tr>
<tr>
<td>MPT weight on dim.less mass-flow $\dot{\phi}_c$ [-]</td>
<td>100</td>
</tr>
<tr>
<td>MPT lower limit and upper limit dim.less mass-flow $\dot{\phi}_c$ [-]</td>
<td>-5, 5</td>
</tr>
<tr>
<td>MPT weight on dim.less pressure $\dot{\psi}$ [-]</td>
<td>0</td>
</tr>
<tr>
<td>MPT lower limit and upper limit dim.less pressure $\dot{\psi}$ [-]</td>
<td>-5, 5</td>
</tr>
<tr>
<td>MPT prediction horizon $P$ [-]</td>
<td>16</td>
</tr>
<tr>
<td>MPT control horizon $M$ [-]</td>
<td>1</td>
</tr>
<tr>
<td>MPT weight on control signal $\ddot{u}_{b,MPC}$ [-]</td>
<td>0.01</td>
</tr>
<tr>
<td>MPT lower limit, upper limit of $\ddot{u}_{b,MPT}$ [-]</td>
<td>0, 1</td>
</tr>
</tbody>
</table>

4.4.5 Simulation results Scenario 2

The parameters used in the second scenario are given in Table 4.2. The simulation results of scenario 2 are presented in Figure 4.6 and a zoom of the last in Figure 4.7. From these results it can be seen that the conclusion which can be drawn is the same as in scenario 1. If state-feedback and MPT are switched on at the same time, the system is stabilized from deep surge in $\phi_0 = 1.80F$. Also the sample-time in the simulation is reduced by a factor 2 to 0.002 s and the results are in Figure 4.8 and a zoom of the last 4.9. This again shows that lowering the sample time reduces the time it takes the system to reach $\phi_0 = 1.80F$ from deep surge. Once the system is stabilized and state-feedback is switched off it is observed that solely the MPT controller cannot keep the system stable, as was possible in Scenario 1. The same conclusions drawn for the used weights on the states and prediction horizon $P$ as in the previous scenario can also be drawn here and the same values are used.

It can also be observed that the maximum value of the compressor input is 2 in both scenarios as was described in Section 4.2, however stability can still be achieved (as was also found using only state-feedback in Section 2.7). From this the conclusion may be drawn the saturation mismatch in the implemented non-linear system in the MPT controller may cause for not so much performance deterioration as could be expected.
Figure 4.6: Dimensionless mass-flow, dimensionless pressure and control signals for $\phi_o = 1.80F$, $K = [80, -25]$ (state-feedback).

Figure 4.7: Zoomed plot of control signals for $\phi_o = 1.80F$, $K = [80, -25]$ (state-feedback).
Figure 4.8: Dimensionless mass-flow, dimensionless pressure and control signals for $\phi_{c0} = 1.80F$, $K = [80, -25]$ (state-feedback), $T_s = 0.002$ s.
Figure 4.9: Zoomed plot of control signals for $\phi = 1.80F$, $K = [80, -25]$ (state-feedback), $T_s = 0.002$ s.
4.5 Sensor Noise

Finally the effect of measurement noise is investigated. Therefore colored noise is implemented on both the pressure $\psi$ and massflow $\phi_c$ by using white noise and filtering this with a Butterworth 4th order filter using a cutoff frequency at 100 Hz. It is chosen to filter the noise before it is put into the system, this to have no effects of phase changes of the measured signals due to filtering. The cutoff frequency is chosen such that the bandwidth of the controller is not limited.

First of all a noise level of approximately 10% (after filtering) is implemented on the full state-feedback controlled system described in Section 2.8.1, with $\phi_{c0} = 1.80F$ and $K = [94, -33]$ extended with the use of MPT. The following simulation is performed:

$t = 1$ s: a pulse with height 1 and duration 0.1 s is given on the system input, to bring the system in deep surge.

$t = 2$ s: the state-feedback controller is switched on.

$t = 8$ s: a second pulse similar to the first is given, to observe the system’s response to a disturbance, with solely state-feedback.

$t = 10$ s: the MPT controller is also switched on to investigate if both controllers can now stabilize the system, since solely state-feedback was not able to.

$t = 14$ s: a third pulse is given, similar to the first two, to determine what the system’s response to a disturbance is with both state-feedback and MPT controlling the system.

The simulation results are depicted in Figure 4.10. In here both the noise level (in grey) and the actual mass-flow and pressure (in black) are plotted. It can be seen that state-feedback alone can stabilize the system, also after a disturbance, however without introducing a disturbance the system can still go into deep surge ($t = 4-6$ s) and therefore stability seems not to be guaranteed always. After switching on the MPT controller, such that both controllers act on the system, the systems seem to stabilize from deep surge. Therefore this might be an indication that the use of MPT increases the systems ability for noise rejection.

Now Scenario 1 described in Section 4.4.5 is repeated using as sample time of 0.002 and again a noise level of approximately 10%. The results are shown in Figure 4.11. Here it can be seen that the noise level is too high to always guarantee stability: at $t = 6$ s the system becomes unstable without introducing a disturbance. Therefore the simulation is repeated with a lower noise level of approximately 5 %, this seems to give a better surge stabilization guarantee. It must be noted that in all simulations performed here using measurement noise, the operating point $\phi_{c0} = 1.80F$ is shifted to a higher average mass-flow of...
4.6 Conclusions

Taking the saturations into account (although the prediction model used for the MPT controller does not exactly have the saturations implemented as in the non-linear compressor model) it was shown in this Chapter that MPT combined with positive feedback stabilization can significantly improve the robustness of the compression system, compared to linear MPC and positive feedback stabilization.

It also was shown that the disturbance and noise rejection properties of the system are increased when MPT and positive feedback stabilization are combined.

Figure 4.10: Measurement noise 10\%, \( \phi_{c0} = 1.80F \), \( K = [94, -33] \) (state-feedback).
Figure 4.11: Measurement noise 10%, $\phi_{x0} = 1.80$F, $K = [25, -13]$ (state-feedback), $T_s = 0.002$ s.

Figure 4.12: Measurement noise 5%, $\phi_{x0} = 1.80$F, $K = [25, -13]$ (state-feedback), $T_s = 0.002$ s.
Chapter 5

Conclusions and Recommendations

In this work a two-state Greitzer lumped parameter model is used to model deep surge in a centrifugal compression system, the model used is described in Willems [2000]. Active control is used to stabilize deep surge limit cycles in a desired setpoint. The control input is bounded between 0 (closed) and 1 (fully open), called one-sided control since it can only become positive. Positive feedback stabilization as described in Willems [2000], is first used to try and stabilize surge limit cycles. There are a few restrictions when using positive feedback stabilization for surge stabilization, on which the focus is in this work: 1) the limited domain of attraction of a stabilized equilibrium point 2) the robustness of the system 3) the disturbance and noise rejection properties. Both linear and hybrid Model Predictive Control are used to investigate if improvements can be made in these areas.

First of all the stable operating region of the compressor is determined using positive feedback stabilization by investigating the smallest mass-flow that can be stabilized from deep surge. Using output feedback of the pressure, the mass-flow can be reduced by 6.5% compared to the mass-flow at the maximum pressure point. Using full state-feedback of both mass-flow and pressure, this reduction is 12.5%. When linear MPC is implemented on the positive full state-feedback controlled system, it appears that the use of linear MPC cannot reduce the minimum mass-flow found compared to when using solely positive full state-feedback. Using hybrid MPC also no significant results were observed either.

Two scenarios for robustness were presented for a massflow \( \phi_{c0} = 1.80F \), where a mismatch of respectively +25% or +100% in both the Greitzer stability parameter \( B \) and dimensionless slope of the compressor characteristic \( M_c \) is introduced. First, using full state-feedback it was concluded that the system could not be stabilized for both scenarios or no significant reduction of the deep surge limit cycle could be achieved. Secondly, if linear MPC is combined with the
existing state-feedback to investigate if improvement in robustness of the system can be made, in the first scenario with +25% mismatch in $B$ and $m_c$ no surge stabilization could be achieved. For the second scenario with a mismatch of 100% in both parameters, a significant reduction in the limit cycle was observed. This can be explained by the fact that a faster state-feedback controller with higher control gains was used for the scenario with 100% mismatch, resulting in a faster system response using both full state-feedback and MPC. Thirdly, hybrid MPC combined with the original full state-feedback system was investigated. Here the hybrid MPC also models the saturations used in the positive feedback controller and the control valve, which the linear MPC controller was not able to do. The result is that for a mismatch of +25% respectively +100% in both $B$ and $m_c$ the system can be stabilized in the desired mass-flow $\phi_{c0} = 1.80F$. Therefore it can be concluded that the use of hybrid MPC can stabilize the compression system in $\phi_{c0} = 1.80F$ in case of significant parameter mismatches and therefore increases the systems robustness significantly compared to linear MPC and positive full state-feedback control.

Another advantage is the explicit control form of the hybrid MPC controller which reduces the average simulation time compared to the online MPC algorithm significantly (order of seconds instead of minutes). Therefore the sample time can also be reduced in the hybrid case resulting in faster stabilization from deep surge.

Disturbance rejection was also investigated by introducing a disturbance pulse on the compression system input. It was concluded that in the state-feedback case without parameter mismatches, this disturbance was rejected effectively, meaning that the system does not go into deep surge and stabilizes back into the desired operating point $\phi_{c0} = 1.80F$ after the disturbance is introduced. In case of hybrid MPC and parameter mismatches it appeared that once the system is stable and the disturbance is introduced both the state-feedback and hybrid MPC had to be switched on the be able to reject the disturbance effectively.

Finally the effect of sensor noise is investigated, by implementing colored noise on the system's states. Simulations shows that hybrid MPC and state-feedback can stabilize the system with noise values where solely state-feedback is not able to do this. However, it was observed that in all simulations the desired operating point is shifted to a stable point with a higher mass-flow due to the noise.

Concluding, the compression system with full state-feedback and hybrid MPT controller used simultaneously seems to be significantly more robust to system parameter mismatches, have significantly higher disturbance and noise rejection properties than when using solely positive feedback or combined with linear MPC. From these results at first sight implementation on experimental scale is expected to improve the results compared to one-sided positive feedback stabilization. The implementation of the hybrid MPT algorithm is expected to cause no computational problems contrary to the linear on-line MPC algorithm also used, since the explicit control law can be implemented in the form of a look-up table and reduces calculation time significantly. However, it must be
clear that comparison of the controllers in this work is purely based on a select set of simulation results and no theoretical foundation is given.

It must also be noted that in future experiments full state-feedback cannot be used without complications. This since only reliable pressure measurements are often available and the use of an observer is required. The effect of an observer was not included in this study.

For further research on this topic it is recommended to extend the use of the hybrid MPC algorithm used here. This can be done by not only using one operating point with different linear models, but also including more operating points with each different linear models. However it must be considered that the simulation time can become significantly larger because of the increase of complexity of the system. Calculating the explicit MPT controller in this work using a prediction horizon of 16 already took more than 3 hours.
Bibliography


