Yaw rate feedback by active rear wheel steering

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Master's thesis

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Preface

This master thesis took place in the period from October 2005 till August 2007 at the Eindhoven University of Technology, section Dynamics and Control Technology. The primary reason for this long graduation period can be dedicated to RSI, which I developed throughout my study. I would like to thank my supervisor and coaches for their understanding in giving me the freedom and time to deal with this handicap.
For the structural support during the master thesis period, I would like to thank Prof. Dr. H. Nijmeijer, Dr. Ir. F.E. Veldpaus, Dr. Ir. I. Besselink and Dr. Ir. A.J.C. Schmeitz. Besides on the usual theoretical graduation work, much time has been spent on the practical part, i.e. the implementation of the control strategy on the test vehicle. However, this does not show off in this report.
I would like to thank ing. W. J. Loor, ing. R. v.d. Bogaert and E. Meinders for the practical support in the electronics, the data acquisition equipment and the transport of the test vehicle.
Abstract

In this thesis a four wheel steering (4WS) control strategy is introduced, which is meant to improve the vehicle handling quality and ultimately vehicle safety. The control strategy incorporates a yaw rate reference model, which calculates a desirable yaw rate response depending on the driver’s steering angle and the vehicle speed. The reference yaw rate is chosen to be the product of a first order transfer function and the driver’s steering angle.

The linear controller, whose task it is to minimise the error between the reference yaw rate and the actual yaw rate, is designed using a technique called ‘loopshaping’. For this purpose the programme called DIET is used in Matlab to shape the open-loop of the feedback system, which consists of the controller to be designed and the linear vehicle model the controller will be based upon. The controller’s performance is validated using a multi-body vehicle model whose tyre characteristics are described by the Magic Formula.

Finally, experiments have been carried out with the 4WS test vehicle at the military airport ‘De Peel’. The dynamic steering response of the test vehicle, a Citroën BX, has been investigated by performing a double lane change test. The results of these experiments are unsatisfactory in the sense that the handling quality gets worse instead of improves. The bandwidth of the yaw rate feedback is too low for the rear wheel steering system to be adequate. This is attributed to:

1. Noise on the yaw rate sensor, which makes it necessary to use different controller settings in the experiments than the initial controller settings determined in the loop-shaping process. As a result the open-loop gain decreases and so does the bandwidth.

2. The dynamics of the rear wheel steering system, which introduces additional phase lag. This directly limits the control potential.
Chapter 1

Introduction

1.1 Motivation and background

For about hundred years since the introduction of the first automobiles in the late 19th century, front wheel steering has been used to control the direction of vehicles. This way of steering, which performs quite well, has been assumed to be the way automobiles have to be steered. In the late 1970s people began to realize that in order to change the vehicle’s direction not only the front wheels, but also the rear wheels can be steered. Up to this point in time the rear tyres could only participate in generating tyre forces by having a certain slip angle, which resulted from the vehicle’s motion (yaw motion and sideslip). The advantage of directly controlling both steering angles of the front and rear tyres is that the lateral movement can be changed more quickly. Directly steering the front and rear tyres can also help to reduce the vehicle’s yaw motion during transient manoeuvres, which in turn improves the driving workload. It can be said that four wheel steering (4WS) has great potential upon conventional two wheel steering (2WS) and therefore it was given a lot of researchers’ attention. This resulted in a few passenger cars equipped with 4WS.

In the early nineties TNO Road-Vehicles was involved in a project about four wheel steering. For this purpose a vehicle was modified to incorporate an active rear wheel steering system. After the project had ended, the vehicle ended up at the University to become an experimental vehicle.

1.2 Aim and scope

The purpose of this thesis is to develop a 4WS control strategy, which will try to improve the vehicle stability by making small adjustments in the steering angle of the rear wheels. Improving the vehicle stability will make the vehicle easier to handle. It will pay off in a reduced driver effort and so in an increased vehicle handling quality. Improving the vehicle stability will also reduce the chance for a driver to reach critical lateral driving conditions, such as extreme oversteer or understeer. This will be accomplished by making adjustments in the steering angle of the rear wheels before reaching these critical conditions. So basically, the development of the 4WS control strategy is in the scope of safe driving.
1.3 Contents of this thesis

Chapter 2 contains an overview of frequently used vehicle models. One of the linear vehicle models will be used later to design a controller, which will steer the rear wheels. The most complex vehicle model will be used for validating the controller’s performance. In Chapter 3 a literature review on four wheel steering will be presented. The purpose is to find out what has already been investigated in the past and which control objectives have been used. This chapter is strategically positioned after the previous chapter, containing a description of the vehicle models, as much of the control techniques in literature are based upon such vehicle models. In Chapter 4 the control structure to be used, consisting of a yaw rate reference model and a controller, is described. Linear controllers will be designed, based upon a linear vehicle model with and one without the rear wheel steering actuator dynamics. The performance of these controllers will be validated briefly. Chapter 4.7 contains more elaborate simulations. In one of these simulations a driver model is adopted to steer the vehicle through a predefined course. The simulations show some interesting features of the rear wheel steering system and point out what can be expected in the experiments. Subsequently, the test vehicle with the rear wheel steering actuator and additional instrumentation will be described in Chapter 5. It also contains the experiments conducted with this test vehicle at military airport ‘de Peel’. Finally, in Chapter 6 conclusions about the rear wheel steering system will be drawn and recommendations for improvement will be given.
Chapter 2

Vehicle modelling and validation

This chapter will introduce a few common vehicle models. One of the linear vehicle models will be used in Chapter 4 to base the controller upon. The more complex nonlinear two-track model will be used for validating the controller’s performance. At last the vehicle models to be introduced, will be fitted to approximate the vehicle dynamics of the test vehicle.

2.1 Vehicle modelling

2.1.1 The bicycle model

The bicycle or single track model is a relatively simple vehicle model. However it is used quite often in studies on 4WS to assess the potential of steering strategies (see Table 3.1 in Chapter 3). The bicycle model is a mathematical model of a two-wheel in-plane vehicle with two degrees of freedom, i.e. yaw motion and lateral displacement. The following assumptions apply on the bicycle model:

- The left and right tyre characteristics have been lumped into an equivalent tyre characteristic, which describes the axle’s lateral tyre force as a function of the slip angle.
- The forward velocity is considered to be constant.
- Body roll and pitch are not taken into account. The normal forces exerted from the ground onto the wheels is constant.
- The only external forces on the vehicle are lateral tyre forces, which, under the assumption that the slip angles are small, are proportional to the slip angles of the tyres. The proportionality constant is called the cornering stiffness $C$.
- All slip angles and steering angles are assumed to be small and so the model will become linear. This assumption implies that the model will only describe the vehicle behaviour sufficiently well up to lateral accelerations of about 4 m/s$^2$.

The following set of equations defines the model:

$$m(\dot{\nu} + ur) = F_{yf} + F_{yr} \quad (2.1)$$

$$I\dot{\gamma} = aF_{yf} - bF_{yr} \quad (2.2)$$
\[ F_{yf} = C_f \alpha_f \]  
\[ F_{yr} = C_r \alpha_r \]  
\[ \alpha_f = \delta_f - \frac{v + ar}{u} \]  
\[ \alpha_r = \delta_r - \frac{v - br}{u} \]

where \( m \) is the mass of the vehicle, \( I \) is the moment of inertia about a vertical axis through the center of gravity (cog), \( a \) is the distance from the cog to the front axis, \( b \) is the distance from the cog to the rear axis, \( u \) is the longitudinal velocity and \( v \) is the lateral velocity of the cog, \( r \) is the yaw rate, \( \delta_f \) and \( \delta_r \) are the steering angles of the front, respectively the rear wheels, \( \alpha_f \) and \( \alpha_r \) the slip angles at these wheels and \( F_{yf} \) and \( F_{yr} \) are the lateral forces at these wheels. These equations can be combined into two coupled first order differential equations.

\[ m \dot{v} + \frac{1}{u}(C_f + C_r)v + \left\{ mu + \frac{1}{u}(aC_f - bC_r) \right\} r = C_f \delta_f + C_r \delta_r \]  
\[ I \ddot{r} + \frac{1}{u}(a^2C_f + b^2C_r)r + \frac{1}{u}(aC_f - bC_r)v = aC_f \delta_f - bC_r \delta_r \]

The output quantities of interest are the lateral acceleration \( a_y = \dot{v} + ur \), the yaw rate \( r \) and the sideslip angle \( \beta \). Written in state space form with state \( x \), input \( u \) and output \( y \) the relevant equations are given by:

\[ \dot{x} = Ax + Bu \]  
\[ y = Cx + Du \]
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\[
x = \begin{bmatrix} v \\ r \end{bmatrix}, \quad u = \begin{bmatrix} \delta_f \\ \delta_r \end{bmatrix}, \quad y = \begin{bmatrix} a_y \\ \beta \end{bmatrix}
\]

\[
A = -\frac{1}{u} \begin{bmatrix} (C_f + C_r)/m & u^2 + (aC_f - bC_r)/m \\ (aC_f - bC_r)/I & (a^2C_f + b^2C_r)/I \end{bmatrix}, \quad B = \begin{bmatrix} C_f/m & C_r/m \\ aC_f/I & -bC_r/I \end{bmatrix}
\]

\[
C = -\begin{bmatrix} (C_f + C_r)/(mu) & (aC_f - bC_r)/(mu) \\ 0 & 1/u \end{bmatrix}, \quad D = \begin{bmatrix} C_f/m \\ 0 \\ 0 \end{bmatrix}
\]

The steady-state yaw rate gain \( H_{r0} \) of a regular front wheel steered vehicle \((\delta_r = 0)\) can be derived from (2.9) by making the derivatives \( \dot{r} \) and \( \dot{v} \) equal to zero:

\[
H_{r0} = \frac{\dot{r}}{\delta_f} = \frac{V}{l} \frac{1}{1 + \eta u^2 V^2}
\]

(2.10)

In this equation the longitudinal velocity \( u \) has been approximated by the total vehicle velocity \( V = \sqrt{u^2 + v^2} \) whereas \( \eta \), the so-called understeer coefficient, is defined by:

\[
\eta = \frac{mg}{l} \cdot \left( \frac{b}{C_f} - \frac{a}{C_r} \right)
\]

(2.11)

From the step response in Figure 2.2 an equivalent time constant \( \tau_r \) for the yaw rate can be defined by the ratio between the steady-state yaw rate \( r_{ss} \) and the derivative of the yaw rate at \( t = 0, \dot{r}(0) \). This derivative of the yaw rate can be derived from (2.9) and so the equivalent time constant is described by:

\[
\tau_r = \frac{r_{ss}}{\dot{r}(0)} = \frac{IV}{aC_f l (1 + \eta u^2 V^2)} = \frac{I}{aC_f} \cdot H_{r0}
\]

(2.12)

The characteristic equation \( \text{det}(sI - A) = 0 \) of the uncontrolled system is given by

\[
s^2 + \left( \frac{a^2C_f + b^2C_r}{l} + \frac{C_f C_r}{m} \right) s + \frac{l^2C_f C_r}{mu^2} (1 + \eta u^2 g^2) = 0
\]

(2.13)

From this characteristic equation it is seen that the uncontrolled system is unstable if \( \eta u^2 < -gl \), i.e. if \( \eta < 0 \) and \( u > \sqrt{-gl/\eta} \).

In stationary situations, i.e. for steady-state cornering, it follows after some calculation that

\[
(1 + \eta u^2 g^2) \cdot \begin{bmatrix} v \\ r \end{bmatrix} = \frac{u}{l} \cdot \begin{bmatrix} b - \frac{m u^2}{C_r} \\ l(1 + \eta u^2 g^2) \end{bmatrix} \cdot \begin{bmatrix} \delta_f - \delta_r \\ \delta_r \end{bmatrix}
\]

(2.14)

For realistic values of the vehicle parameters \( a, b, m \) and \( \eta \), of the longitudinal velocity \( u \) and of the steering angles \( \delta_f \) and \( \delta_r \), the lateral velocity \( v \) in absolute value is much smaller than the longitudinal velocity \( u \). This means that the total vehicle velocity \( V = \sqrt{u^2 + v^2} \) of the center
CHAPTER 2. VEHICLE MODELLING AND VALIDATION

Figure 2.2: Step response of the yaw rate to the front wheel steering angle

of gravity may be approximated by \( u \) and that the sideslip angle \( \beta = -v/u \) is very small. Since \( V = Rr \), where \( R \) is the radius of the driven circle, it follows from (2.14) that

\[
\delta_f - \delta_r = \frac{rl}{u} \cdot (1 + \frac{u^2}{gl}) = \frac{Vl}{Ru} \cdot (1 + \eta \frac{u^2}{gR}) \approx \frac{l}{R} + \eta \frac{u^2}{gR} \tag{2.15}
\]

or, using the relation \( a_y = V^2/R \approx u^2/R \) for the lateral acceleration, that

\[
\delta_f - \delta_r = \frac{l}{R} + \eta \frac{a_y}{g} \tag{2.16}
\]

This relation for the bicycle model with front and rear wheel steering reduces to the well-known relation \( \delta_f = l/R + \eta a_y/g \) for the model with front wheel steering only if \( \delta_r = 0 \) is substituted. The understeer coefficient \( \eta \) is basically a quantity derived for front wheel steered vehicles. In that case the sign of the understeer coefficient determines whether the vehicle is understeered (+) or oversteered (-). This means that during steady-state cornering the driver has to respectively increase or decrease the steering wheel angle when the lateral acceleration increases. It is noted that statements like “the vehicle is oversteered if \( \eta \) is negative” lose much of their significance for a four wheel steered vehicle, as the rear wheel steering angle \( \delta_r \) is present in the left part of (2.16).

2.1.2 The extended bicycle model

The extended bicycle model is the bicycle model extended to include the relaxation length of the tyres. The tyre model used within the bicycle model consists of a proportional relation between the lateral tyre force and the slip angle:

\[
F_y = C\alpha \tag{2.17}
\]

When taking the relaxation length into account, the tyre model changes to:

\[
\sigma \frac{\alpha'}{V} + \alpha' = \alpha \tag{2.18}
\]

\[
F_y = C\alpha' \]

The true slip angle \( \alpha' \) has become a first order function of the slip angle as defined in (2.5) and (2.6). This behaviour is caused by the finite lateral tyre stiffness. The constant \( \sigma \) is called relaxation length. The relaxation length depends on the type of tyre and is usually around 0.5 m. When the vehicle speed \( V \) increases, the time constant \( \sigma/V \) decreases and so does the response...
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Figure 2.3: The two-track vehicle model with four degrees of freedom: longitudinal, lateral, yaw and roll motion (source Pacejka [27])

The resulting vehicle model remains linear and describes the lateral vehicle behaviour quite accurately up to about 4 m/s².

2.1.3 The two-track model

A more complex vehicle model is the non-linear two-track model described by Pacejka [27]. Figure 2.3 depicts this model with four degrees of freedom: the longitudinal velocity \( u \), the lateral velocity \( v \) of point A, the yaw velocity \( r \) and the roll angle \( \varphi \). Point A is the projection on the ground plane of the center of gravity if the roll angle equals zero. The vehicle body can rotate around the roll axis, which is a virtual axis defined by the heights of the roll centers \( h_1 \) and \( h_2 \). Torsional springs and dampers in both roll centers represent the roll stiffness and damping, resulting from suspension springs, dampers and anti-roll bars.

A brief derivation of the equations of motion will be presented next. A more thorough derivation can be found in the report by Schouten [28].

Lagrange’s equations will be employed to derive the equations of motion. For a system with \( n \) degrees of freedom \( n \) coordinates \( q_n \) are selected to completely describe the system’s kinetic time. Besides this change no other changes have been made to the bicycle model. The equations of motion become:

\[
\begin{align*}
    m(\dot{v} + ru) &= C_f \alpha_f + C_r \alpha_r \\
    I_z \dot{r} &= aC_f \alpha_f - bC_r \alpha_r \\
    \sigma_f \dot{\alpha}_f &= -v - ar - u\alpha_f + u\delta_f \\
    \sigma_r \dot{\alpha}_r &= -v + br - u\alpha_r + u\delta_r
\end{align*}
\]

(2.19)

The equations of motion become:

\[
\begin{align*}
    m(\dot{v} + ru) &= C_f \alpha_f + C_r \alpha_r \\
    I_z \dot{r} &= aC_f \alpha_f - bC_r \alpha_r \\
    \sigma_f \dot{\alpha}_f &= -v - ar - u\alpha_f + u\delta_f \\
    \sigma_r \dot{\alpha}_r &= -v + br - u\alpha_r + u\delta_r
\end{align*}
\]

(2.19)
energy $T$ and potential energy $U$. External generalized forces $Q_i$ associated with generalized coordinate $q_i$ may act on the system. The Lagrange’s equation for coordinate $q_i$ reads:

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = Q_i,$$

(2.20)

The velocities $u, v$ and $r$ will be used as generalized motion variables in addition to the roll coordinate $\varphi$. The Lagrangean equations expressed in $u, v, r$ and $\varphi$ are given by [28]:

$$\frac{\partial}{\partial t} \frac{\partial T}{\partial u} - r \frac{\partial T}{\partial v} = Q_u$$

$$\frac{\partial}{\partial t} \frac{\partial T}{\partial v} + r \frac{\partial T}{\partial u} = Q_v$$

$$\frac{\partial}{\partial t} \frac{\partial T}{\partial r} - v \frac{\partial T}{\partial u} + u \frac{\partial T}{\partial v} = Q_r$$

$$\frac{\partial}{\partial t} \frac{\partial T}{\partial \varphi} - \frac{\partial T}{\partial u} \frac{\partial \varphi}{\partial u} + \frac{\partial T}{\partial v} \frac{\partial \varphi}{\partial v} + \frac{\partial U}{\partial \varphi} = Q_\varphi$$

(2.21)

The non-conservative generalized forces $Q_i$ follow from the virtual work as a result of the virtual displacement. The following non-conservative generalized forces $Q_i$ can be obtained from figure 2.4:

$$Q_u = \sum F_x = (F_{x1} + F_{x2}) \cos \delta_f - (F_{y1} + F_{y2}) \sin \delta_f$$

$$+ (F_{x3} + F_{x4}) \cos \delta_r - (F_{y3} + F_{y4}) \sin \delta_r$$

$$Q_v = \sum F_y = (F_{x1} + F_{x2}) \sin \delta_f + (F_{y1} + F_{y2}) \cos \delta_f$$

$$+ (F_{x3} + F_{x4}) \sin \delta_r + (F_{y3} + F_{y4}) \cos \delta_r$$

(2.22)

$$Q_r = \sum M_z = a(F_{x1} + F_{x2}) \sin \delta_f + a(F_{y1} + F_{y2}) \cos \delta_f$$

$$- b(F_{x3} + F_{x4}) \sin \delta_r - b(F_{y3} + F_{y4}) \cos \delta_r$$

Figure 2.4: View from above showing the non-conservative forces
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\[ + M_{x1} + M_{x2} + M_{x3} + M_{x4} \]
\[ + (F_{x1} \cos \delta_f - F_{y1} \sin \delta_f) s_1 - (F_{x2} \cos \delta_f - F_{y2} \sin \delta_f) s_1 \]
\[ + (F_{x3} \cos \delta_r - F_{y3} \sin \delta_r) s_2 - (F_{x4} \cos \delta_r - F_{y4} \sin \delta_r) s_2 \]
\[ Q_\varphi = \sum M_\varphi = -(k_{\varphi 1} + k_{\varphi 2}) \dot{\varphi}. \]

The generalized force \( Q_\varphi \) contains roll damping forces exerted at the front and rear roll center with damping coefficients \( k_{\varphi 1} \) and \( k_{\varphi 2} \).

The kinetic energy \( T \) of the vehicle becomes:
\[ T = \frac{1}{2} m \{(u - h'\varphi r)^2 + (v + h'\varphi)^2\} + \frac{1}{2} I_x \varphi^2 \]
\[ + \frac{1}{2} I_y (\varphi r)^2 + \frac{1}{2} I_z (r^2 - \varphi^2 r^2 + 2 \theta r \dot{\varphi}) - I_{xz} r \dot{\varphi}. \] (2.23)

in which \( h' \) is the distance from the center of gravity to the roll axis and \( \theta = (h_2 - h_1)/l \) the roll axis inclination angle.

The potential energy \( U \) consists of two parts: energy in the torsional springs and gravitational energy. The total potential energy is:
\[ U = \frac{1}{2} (c_{\varphi 1} + c_{\varphi 2}) \varphi^2 - \frac{1}{2} m g h' \varphi^2 \] (2.24)

in which \( c_{\varphi 1} \) and \( c_{\varphi 2} \) represent the torsional stiffness of the springs in the front and rear roll center. Using (2.22), (2.23),(2.24) the equations of motion become:
\[ m (\ddot{u} - rv - h'\varphi \dot{r} - 2h' r \dot{\varphi}) = (F_{x1} + F_{x2}) \cos \delta_f - (F_{y1} + F_{y2}) \sin \delta_f \]
\[ + (F_{x3} + F_{x4}) \cos \delta_r - (F_{y3} + F_{y4}) \sin \delta_r \] (2.25)
\[ m (\ddot{v} + ru + h' \ddot{\varphi} - h' r^2 \dot{\varphi}) = (F_{x1} + F_{x2}) \sin \delta_f + (F_{y1} + F_{y2}) \cos \delta_f \]
\[ + (F_{x3} + F_{x4}) \sin \delta_r + (F_{y3} + F_{y4}) \cos \delta_r \] (2.26)
\[ I_z \ddot{\varphi} + (I_z \theta - I_{xz}) \dot{\varphi} - m h' (\ddot{u} - rv) \varphi = a (F_{x1} + F_{x2}) \sin \delta_f \]
\[ + a (F_{y1} + F_{y2}) \cos \delta_f - b (F_{x3} + F_{x4}) \sin \delta_r \]
\[ - b (F_{y3} + F_{y4}) \cos \delta_r + M_{x1} + M_{x2} + M_{x3} + M_{x4} \]
\[ + (F_{x1} \cos \delta_f - F_{y1} \sin \delta_f) s_1 - (F_{x2} \cos \delta_f - F_{y2} \sin \delta_f) s_1 \]
\[ + (F_{x3} \cos \delta_r - F_{y3} \sin \delta_r) s_2 - (F_{x4} \cos \delta_r - F_{y4} \sin \delta_r) s_2 \] (2.27)
\[ (I_x + m h'^2) \ddot{\varphi} + m h' (\ddot{v} + ru) + (I_z \theta - I_{xz}) \dot{\varphi} - (m h'^2 + I_y - I_z) \varphi r^2 \]
\[ + (k_{\varphi 1} + k_{\varphi 2}) \dot{\varphi} + (c_{\varphi 1} + c_{\varphi 2} - m g h') \varphi = 0. \] (2.28)

Throughout this derivation it is assumed that the values of the roll axis inclination angle \( \theta \) and the roll angle \( \varphi \) are small.
CHAPTER 2. VEHICLE MODELLING AND VALIDATION

This vehicle model has been implemented in Matlab Simulink by Besselink [29]. The Magic Formula [27] is used to calculate longitudinal and lateral tyre forces and self-aligning moments of each tyre, depending on the longitudinal and lateral slip and the normal force on the tyre. As the non-linear Magic Formula accurately describes the tyre characteristics up to high levels of slip, this tyre model can be used for simulating manoeuvres at higher lateral acceleration levels. This in contrast to the bicycle model, in which lateral tyre forces are linear in the slip angle $\alpha_i$.

The same vehicle model including the roll axis has been built in Matlab SimMechanics (Multi-Body) by Besselink [29]. This has been done to eliminate some algebraic loops in the description, in which the differential equations stated above were programmed in Matlab Simulink. This multi-body version of the vehicle model containing a roll axis will be used in simulations and is referred to as two-track model.

2.1.4 The extended 3DOF model

The extended 3DOF model is basically equal to the extended bicycle model except that an extra degree of freedom has been introduced. Besides a lateral and yaw degree of freedom, vehicle roll is added as the third degree of freedom. The basic three equations of motion can be derived by eliminating all non-linear terms in the lefthand side of (2.26), (2.27) and (2.28). The differential equation for $u$ is omitted as $u$ is assumed to be constant. Furthermore, it is assumed that the longitudinal tyre forces $F_{xi}$ are small compared to the lateral tyre forces $F_{yi}$ and so they are neglected. The following linear equations of motion then described the extended 3DOF model:

$$m(\dot{v} + ru + h'\dot{\varphi}) = C_f\alpha_f + C_r\alpha_r$$

$$I_z\ddot{r} + (I_z \theta - I_{xz}) \ddot{\varphi} = aC_f\alpha_f - bC_r\alpha_r$$

$$(I_x + mh'^2) \ddot{\varphi} + mh' (\dot{v} + ru) + (I_z \theta - I_{xz}) \dot{r} + (k_{\varphi 1} + k_{\varphi 2}) \dot{\varphi} + (c_{\varphi 1} + c_{\varphi 2} - mgh') \varphi = 0$$

$$\sigma_f\dot{\alpha}_f = -v - ar - u\alpha_f + u\delta_f$$

$$\sigma_r\dot{\alpha}_r = -v + br - u\alpha_r + u\delta_r$$

(2.29)

2.2 Vehicle model validation

In the previous section four different vehicle models have been presented. In this section these vehicle models will be used to approximate the handling dynamics of the test vehicle, a Citroën BX, as good as possible. Essentially this means that the right parameters have to be determined. In the past TNO has put much effort in validating various vehicle models with different levels of complexity. The main goal has been to obtain a sufficiently accurate model with which steering strategies could be optimised. Of course a number of driving tests has to be carried out to obtain the necessary data for the validation process. The following two driving tests were carried out:

1. The random steering test: This is a standard ISO test in which a random steering input is generated by the test driver and the vehicle response is measured during 900 seconds. The lateral acceleration during the test remains below $4 \text{ m/s}^2$, the boundary below which the vehicle behaviour can be regarded as linear. The vehicle speed is kept constant at $80 \text{ km/h}$. This test provides an accurate vehicle system response in the linear range and the transfer functions for the yaw rate and the lateral acceleration are determined.
2.2. VEHICLE MODEL VALIDATION

2. The step steer input test: This test is similar to the standard ISO lateral transient response test. The step steer input is applied at the steering wheel with different steering angles. The propagation of the vehicle behaviour in the non-linear area is investigated with this test. Three different magnitudes of the steering input are applied at 80 km/h, reaching up to a lateral acceleration of 6 m/s².

The responses of the yaw rate and the lateral acceleration have been used for validating the vehicle models. Primary the yaw rate has been used since this quantity will be controlled by the active rear wheel steering system.

2.2.1 Random steering test

During the random steering test only the front wheels are steered. The following signals have been measured: the steering angle of the front wheels $\delta_f$, the lateral acceleration $a_y$ and the yaw rate $r$. The transfer functions for the yaw rate and the lateral acceleration have been determined and have been approximated with a 6th order transfer function. A number of key parameters has been calculated from these approximations. These are shown in Table 2.1 and some of them are explained below.

- $H_0$: Steady state response gain
- Bandwidth: The frequency with a gain of -3 dB ($=H_0/\sqrt{2}$)
- Peak/Dip Ratio: The ratio of the maximal/minimal frequency response and $H_0$
- Equivalent Frequency: The frequency where the response function has a phase of 45°

As mentioned earlier, TNO has validated various vehicle models to approximate the vehicle dynamics of the Citroen BX. Amongst those vehicle models are the bicycle model, the extended bicycle model and the two-track model. In fitting the vehicle models to the measured data the emphasis lies on matching the steady-state gain. As a result the relevant vehicle parameters are known. Hence, the relevant transfer functions can be calculated and compared to the measured transfer functions. The key parameters of the four vehicle models, discussed earlier, are listed in Table 2.1. The key parameters of the two-track model, which is a nonlinear model, have been determined after linearization while travelling in a straight line.

Figure 2.5 shows the transfer functions from the front wheel steering angle to the yaw rate for the mentioned models. It also shows a few data points of the measured transfer function as can be found in [7]. It can be seen that the transfer functions of the extended 3 DOF model and the two-track model approximate the measured data points quite well. The same conclusion can be drawn by looking at the key parameters in Table 2.1. The bicycle model and the extended bicycle model perform worse.

2.2.2 Step steer input test

The random steering test proves that the transfer function of the yaw rate for the extended 3DOF model and the two-track model show good similarity with the measured transfer function. However, as the lateral acceleration during this test is approximately 2 m/s², the vehicle response is within the linear range. Because the lateral acceleration in the step steer input test rises above the boundary of linear vehicle behaviour, i.e. 4 m/s², propagation of the vehicle behaviour in the non-linear area is investigated. Basically the step steer input test is used to verify the range of validity of the vehicle models.
Table 2.1: Key parameters of yaw rate and lateral acceleration transfer functions from the measured data and all four vehicle models.

Figure 2.6 shows the yaw rate and the lateral acceleration for three different step steer inputs. The measured data points taken from [7] represent the peak value, the dip value and the steady state value. The other responses result from simulations with all four vehicle models. The responses of the three linear models are quite good for the step steer input of 1 and 2 degrees. In that case the lateral acceleration remains below $4 \text{ m/s}^2$. The output of the linear models is too high when the steps steer input of 3 degrees is applied. The similarity between the measured data points and the two-track model is far better. This difference is caused by the different way the tyres are modelled in the linear models compared to the two-track model. Although the similarity between the measured data points and the response of the two-track model is quite good for all step steer input tests, there are still some differences which can partially be explained as follows:

- The exact steering wheel inputs as a function of time, applied on the steering wheel during the tests, are unknown. However the steady state values of these steering wheel inputs are known: 1, 2, and 3 degrees at the front wheels. Because a true step can not be realized, the step steer inputs, which are used in the simulations, have been limited to a steering rate of $10^\circ \text{/s}$ in order to approximate a realistic step steer input. The resulting difference between the transient steering inputs in the simulations and the truly applied transient steering inputs during the driving test may partly explain the difference in peak value between the measured data points and the step response of the two-track model.

- The differences between the measured data points and the two-track model may also result from a mismatch in the tyre property file used in the Magic Formula. As no tyre property file was available for the tyres mounted on the Citroën BX, a tyre property file of a tyre with nearly the same size is used in the simulations. As a result the tyre forces and moments can be slightly different and so will be the vehicle response.

In spite of the mentioned differences, it may be concluded that the two-track model can be used

<table>
<thead>
<tr>
<th>Key parameter</th>
<th>Units</th>
<th>Measured</th>
<th>Bicycle</th>
<th>Ext. bic.</th>
<th>Ext. 3DOF</th>
<th>Two-track</th>
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<tr>
<td>Yaw rate</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>$H_0$</td>
<td>1/s</td>
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<td>4.97</td>
<td>4.97</td>
<td>4.97</td>
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<tr>
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<tr>
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<td>1.08</td>
<td>1.18</td>
<td>1.17</td>
</tr>
<tr>
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<tr>
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<td>Hz</td>
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<td>3.16</td>
<td>2.96</td>
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</tr>
<tr>
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<tr>
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<tr>
<td>Equivalent Frequency</td>
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<td>-</td>
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<td>71.9</td>
<td>76.4</td>
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</table>
2.2. VEHICLE MODEL VALIDATION

Figure 2.5: The transfer function of the yaw rate to steering input of all four vehicle models and measured data
to predict the actual vehicle behaviour of the Citroen BX both in the linear area as well as in the non-linear area without causing too large deviations.
Figure 2.6: Yaw rate and lateral acceleration response to three step steer inputs
Chapter 3

Literature review

In this chapter a review will be given of some existing techniques for controlling the steering angle of the rear wheels. In the first section a review of somewhat earlier control techniques will be presented, based on the article by Furukawa et al. [1], which has been extended with some more recent control techniques. In the second section an overview of the more recent articles will be presented in tabular form.

3.1 Control objectives

In all studies on 4WS control techniques the following general objectives can be observed:

- reduction of phase lags in lateral acceleration and yaw rate responses
- reduction of the sideslip angle of the vehicle body
- stability augmentation
- better manoeuvrability at low speed
- achievement of the desired steering responses (model-following control)

A suitable chosen controller can achieve some of these objectives.

3.1.1 Reduction of phase lags in lateral acceleration and yaw rate responses

A motor vehicle is subjected to an increase in time delay in lateral acceleration and yaw rate responses to steering as its speed increases. To maintain its stability as a closed-loop system, the driver has to increase the phase lead in his steering control to compensate for increasing delays in vehicle responses. Since this compensation gives an additional workload to the driver, it is desirable to minimize the delay in vehicle steering responses.

From this viewpoint Sano et al.[2] proposed a feed-forward 4WS control to steer the rear wheels proportionally in the same direction as the front wheels in an attempt to reduce the delay in the vehicle’s lateral acceleration response. It appears that in case of 4WS the transient response characteristics of the yaw rate do not differ appreciably from those of the 2WS system. Only the transient response characteristics of the lateral acceleration vary significantly. Figure 3.1 shows the results of calculating the frequency responses of the lateral acceleration and yaw rate when the ratio $k$ between the steering angle $\delta_r$ of the rear wheels and $\delta_f$ of the front wheels is varied.
These calculations have been carried out with a vehicle model whose response characteristics are close to neutral steer. As can be seen in figure 3.1, steering the rear wheels proportionally in the same direction as the front wheels will reduce the phase lag of the lateral acceleration response. Besides, the reduction of the lateral acceleration’s gain with increasing frequency will be less severe in comparison with 2WS. It should be noted that by changing the ratio $k$ the steady state gains of the lateral acceleration and the yaw rate responses will be changed by a factor $(1 - k)$ compared to those for 2WS. However both characteristic equations of the lateral acceleration and the yaw rate in the feed-forward controlled 4WS are equal to those in the 2WS system. Therefore its open-loop stability with a fixed steering wheel angle does not differ from the 2WS system.

Vanderploeg et al.[3] have also studied a 4WS system designed to steer the rear wheels in proportion to the front wheels, using the linear bicycle model for a 4WS vehicle. According to their report, if the steering wheel operation needed to follow the desired path is found by a linear inverse model and, furthermore, the rear wheels are steered in the same direction as the front wheels, the steering wheel angle would contain less ‘high frequent’ content with an increasing front to rear steering ratio $k$. This supports the suggestion that a driver will find it more convenient to track closely a desired path with vehicles that have a positive $k$.

When a vehicle has a strong understeer character, steering the rear wheels in the same direction as the front wheels will slightly increase the phase lag in yaw rate response. As a 4WS control method for reducing phase lags in yaw rate as well as in lateral acceleration, Shibahata et al.[4] proposed a control technique that would delay rear wheel steering, compared with the front wheels. This method can not only reduce the phase lag in yaw rate but also more significantly decrease the phase lag in lateral acceleration in a low frequency range. In a high frequency range this system fails to reduce the lateral acceleration phase lag, because the delay in side force generation of the tires cannot be decreased effectively.
3.1. CONTROL OBJECTIVES

3.1.2 Reduction of sideslip angle of the vehicle body

The driver’s purpose of turning the steering wheel is to start cornering. Ideally then, the vehicle’s yaw rate and simultaneously the lateral acceleration will start to increase. In practice the transient lateral acceleration response lags the yaw rate response because the sideslip angle increases too. The lateral acceleration and the yaw rate are related by the sideslip angular velocity $\dot{\beta}$:

$$a_y = \dot{v} + rV \approx u(-\dot{\beta} + r) \quad (3.1)$$

So the lateral acceleration $a_y$ consists of two components, one the yaw rate $r$ and the other the sideslip angular velocity $\dot{\beta}$. As the vehicle speed $V$ increases, the lateral acceleration response delays more than the yaw rate response because the time constant for the sideslip angle decreases.

Quite some studies have been carried out on 4WS control techniques trying to achieve zero sideslip in steady state cornering and hereby aiming to minimize the delay in the lateral acceleration response with respect to the yaw rate response [4]. If the vehicle is described by the linear bicycle model, a feed-forward 4WS control technique can be derived which makes the steady state value of the sideslip angle zero. In that case the rear wheels are steered at a steering angle ratio $k$ to the front wheels:

$$k = \frac{\delta_r}{\delta_f} = -\frac{b - \frac{ma}{C_f}u^2}{a + \frac{mb}{C_f}u^2} \quad (3.2)$$

This technique is known as vehicle-speed-sensing 4WS. It should be noted that in a transient condition, the sideslip angle probably won’t be zero. Therefore Takeuchi et al. [5] extended (3.2) by calculating the transfer function between the sideslip angle and the steering wheel angle and choosing $k(s)$ such that the sideslip angle will be zero also in the transient state. The relation between the rear and front wheel steering angles will then become:

$$k(s) = \frac{\Delta_r(s)}{\Delta_f(s)} = -\frac{b - \frac{ma}{C_f}u^2 + \frac{L_s}{C_f}us}{a + \frac{mb}{C_f}u^2 + \frac{L_s}{C_f}us} \quad (3.3)$$

In this equation $s$ is the Laplace operator, $\Delta_r$ and $\Delta_f$ are respectively the Laplace transformed rear and front wheel steering angles $\delta_r$ and $\delta_f$.

Nalecz and Bindemann [6] analyzed different types of feed-forward 4WS control techniques. These techniques were simulated with a four wheel model, which covered the influence of kinematic effects of the suspension and lateral weight transfer. They concluded that the 4WS system could make the vehicle more responsive to the driver’s steering and reduce or even eliminate such undesirable motions of the vehicle body as sideslip and fishtailing.

3.1.3 Stability augmentation

The simplest 4WS feedback controller is one which steers the rear wheels proportionally to the yaw rate:

$$\delta_r = Pr \quad (3.4)$$

In a 4WS system, which controls the rear wheels by feeding back state variables like yaw rate, the characteristic equation of the system is changed. When the constant $P$ is negative, the roots of the characteristic equation will shift in the negative direction of the real axis, making the vehicle more stable.
The control strategy previously used by TNO [7] on the Citroën BX contains a term which feeds back the yaw rate with a speed dependent gain. If the gain is negative, the vehicle is stabilized by generating more understeer. It is said that this term is important for the system stability.

Sato et al. [8] have proposed a 4WS system which steers the rear wheels by feeding back yaw rate and feeding forward the front steering angle. At low speeds the rear wheels are steered in the opposite direction as the front wheels, but as the vehicle speed increases the system compensates for the sideslip angle by giving additional steering in the other direction to the rear wheels through yaw rate feedback. The vehicle’s response to an external disturbance from a side wind was simulated. The results indicate that, even with a fixed steering wheel, the 4WS system experienced less lateral displacement than the 2WS system.

3.1.4 Improvement of vehicle manoeuvrability at low speeds

Better vehicle manoeuvrability at low speeds can be achieved by steering the rear wheels in the opposite direction to the front wheels. As a result the radius of the smallest turning circle will decrease. At higher speeds this approach is not suitable since it produces a greater phase lag in the lateral acceleration response. A control technique should improve both vehicle manoeuvrability at low speeds and handling quality at high speeds. A control technique which obeys both criteria, is the vehicle-speed-sensing 4WS system [2]. This technique steers the rear wheels at a ratio $k$, which depends on the vehicle speed, to the front wheels. Figure 3.2 shows this speed dependent relation. It is clearly visible that at low speeds the rear wheels are steered opposed to the front wheels ($k < 0$), while at high speeds the rear wheels are steered in the same direction ($k > 0$).

Shibahata et al. [9] conclude that it is not very attractive to steer the rear wheels at a large angle opposed to the front wheels, since it makes the rear end of the vehicle stick out further towards the outside of the curve. Whitehead [10] in turn reports that in parallel parking the improved manoeuvrability is not desirable. Therefore improving the high speed handling quality is recognized as the main purpose of 4WS systems whereas the low speed manoeuvrability improvement is hardly relevant.
3.1. CONTROL OBJECTIVES

3.1.5 Achievement of desired steering response (Model matching/following control)

Over the last 10 years most studies on 4WS used a reference model, which somehow reflects the desired response characteristics of the vehicle. Depending on certain input quantities like vehicle speed and steering wheel angle, the reference model calculates the ideal vehicle response. Subsequently the controller tries to match the actual vehicle response to that of the reference model. Such a control technique is called model matching/following control.

A 4WS vehicle can be treated as a MIMO system. In the 2D plane the outputs are sideslip angle and yaw rate, whereas the inputs are the front and rear steering angles. Theoretically this system can be decoupled and the outputs can be controlled independently. However, in active rear wheel steering the only real input to be freely chosen is the rear steering angle, as the driver directly controls the front steering angle. The driver’s input can be considered as a kind of ‘disturbance’ which should make the vehicle’s output roughly approach the reference output. The purpose of the controller then is to control the rear wheels such that the vehicle’s output will better match the reference output. Since the controller can only influence one input, only one output can be chosen as the output which will be controlled to match the desired reference.

The controller itself can contain a feedback or a feed-forward part or a combination of both. To determine the feed-forward part, a model of the actual vehicle should be available. The 2 DOF bicycle model is often used. The feedback part can then be added to compensate for disturbances, unmodelled dynamics and parameter changes.

In 1997 Toyota [11] launched the Aristo, equipped with active rear wheel steering (ARS). The primary function of ARS in this case is to assist during normal driving conditions, which actually means within the linear region of the tyre characteristic. ARS complements the vehicle stability control (VSC) programme, which typically interferes during critical driving conditions. As ARS assists during normal driving conditions, it is believed to reduce the chance of reaching those critical driving conditions. In the same time it gives the driver a greater calmness and more allowance for driving, which in turn helps make him/her feel safer.

The controller used is a model matching controller, based on a 2 DOF linear vehicle steering model. The configuration is shown in figure 3.3. The command value for the rear wheels is
the sum of a feedforward and a feedback part. The feedforward part depends on the vehicle speed, the steering wheel angle, the vehicle steering model and the vehicle target value, which is determined by the driver’s steering action. The feedback part is determined by the difference between the target behaviour and the actual vehicle behaviour. In calculating the feedback term, the $H_{\infty} - \mu$ synthesis of modern control theory is applied in a state feedback. The vehicle sideslip angle is estimated by a linear observer, based on the linear 2 DOF vehicle steering model and the vehicle state variables that are measurable. The yaw rate is measured. As a result it is possible to ensure more optimum steering response and high-speed stability even with changes in driving environment like changes in the vehicle condition, in vehicle speed and in road surface friction. In addition good stability against external disturbances such as a crosswind is achieved.

Song et al. [12] have proposed a new 4WS system using a time delay control scheme which is suitable for the control of nonlinear systems. The controller consists of a combination of feedforward and feedback. The control scheme is based on a yaw reference model following control. The yaw reference is described by a first order system. Such a system displays suitable damping without resonance or overshoot. The steady state gain can be chosen to match a 2WS system or to cause zero sideslip. The vehicle itself is modeled by the bicycle model, extended with a vector representing disturbances, nonlinearities and unmodeled dynamics. This vector will be estimated by certain variables from the previous time sample. The actuator dynamics is also modeled as a first order system. A disadvantage of this control scheme is, that for certain values of the time constants of the actuator dynamics and reference model, the system can become unstable. Simulations are performed using a 16 DOF vehicle model. The results show that the proposed 4WS has a robust yaw damping to the steering input and a robust yaw rate gain against external disturbances.

A different method to control linear constant systems optimally is the Linear Quadratic Regulator (LQR) method. This method consists of a full state feedback, which is optimized by minimization of a cost function composed of the control effort and the control result. The influence of the control effort and the control result can be regulated by weighing matrices. Solving the Algebraic Riccati Equation (ARE) will result in a feedback matrix which minimizes the cost function. This control method has also been applied on 4WS systems [13][14][7]. The bicycle model is then used as the linear model of the vehicle. The controllable input to the vehicle model is the rear steering angle. The front steering angle is modeled as a known, uncontrollable input, as the driver directly controls its magnitude. The regulator problem can be extended to follow a reference signal. A yaw rate reference is used by [7] and [14]. By definition the problem then changes to a tracking problem instead of a regulator problem. The linear model for the control of the rear steering angle, including the yaw reference, can be extended to include the actuator dynamics, as is done in [7]. A disadvantage of the LQR method is that the system matrix of the bicycle model depends on the forward velocity and is therefore not constant over time. As a result the feedback matrix will be different at every velocity. Another drawback is that besides the velocity also the cornering stiffness, the position of the center of gravity and the mass of the vehicle may vary. The question which arises, is how robust the controller will be.

Another way to control the rear steering angle is presented by Chen et al. [15]. The control objective is to reduce the overshoot of yaw rate, sideslip angle and lateral acceleration in order to stabilize the transient gains of those responses and to improve the vehicle handling stability at high speeds. Again, the vehicle model is the bicycle model. The rear wheels are steered proportionally with the difference between the desired yaw rate and the measured yaw rate. The desired yaw rate is calculated by multiplying the 2WS yaw rate gain, which depends on the speed of the vehicle and on vehicle parameters, with the steering wheel angle at the front wheels. Therefore
3.2 OVERVIEW OF RECENT PAPERS ON 4WS

In this section an overview of more recent (from 1990) articles will be presented in a tabular form, see Table 3.1. The table shows per article: the name of the author, the year of publication, the used control technique(s), the used vehicle model(s) and whether the control technique has

<table>
<thead>
<tr>
<th>Name of author</th>
<th>Control technique</th>
<th>Vehicle model</th>
<th>Validation on real vehicle</th>
</tr>
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<td>6 DOF vehicle model</td>
<td>16 DOF vehicle model</td>
<td>x</td>
</tr>
</tbody>
</table>

Table 3.1: Overview of recent articles

the steady state value of the yaw rate in 4WS equals the one in 2WS. The proportionality constant is calculated through minimization of the $H_\infty$ norm of the three transfer functions of the responses mentioned above. This constant however has been calculated for one constant vehicle speed. From simulations using a stepsteer input it follows that the amount of overshoot of the three response signals decreases compared to 2WS, while the steady state values remain the same. Frequency response functions of those three signals show that the delay with increasing frequency is less than in the 2WS vehicle.
been validated on a real vehicle. Some of those articles have already been mentioned in the
previous section.

Almost all recent articles use some kind of reference model, which describes the desired vehicle
behaviour. The purpose is to make the response of the actual vehicle approach the one of the
reference model. These articles could be placed in the last group of the previous section, called
model matching/following control.

A few of the listed articles focused their attention on ARS as a particular type of 4WS. ARS ac-
tively controls the steering angle of the rear wheels, while the steering angle of the front wheels
is directly controlled by the driver. Of course those articles are of particular interest as this type of
4WS is present in the Citroën BX. However, when reviewing literature on 4WS, it is found that by
far the most papers deal with 4WS systems in which two inputs can be controlled independently
of each other in order to influence both the sideslip angle and the yaw rate of the 2 DOF vehicle
model. Those inputs can be for example the steering angles of the front and rear wheels or the
steering angle of the rear wheels and a yaw moment, exerted by braking or driving wheels indi-
vidually. In comparison with ARS those systems have greater potential in reaching the desired
steering response, as they control two inputs instead of one.

Generally speaking, one can say that the earliest papers on ARS focused on a feedforward control
aiming to minimize the vehicle’s sideslip angle. Later on a feedback loop for the vehicle’s yaw
rate was added to increase stability against external disturbances. The more recent papers use
a reference model to approach the desired steering response. Both feedforward and feedback
control is used to reach this goal. Now returning to Table 3.1, a few conclusions can be drawn.

It is clear that there is no general consensus about the control technique to be used. LQR con-
trol appears four times in the table. However this technique has the disadvantage that the state
feedback gain is different for every vehicle speed, as the linear differential equations of the bicy-
icle model depend on the vehicle speed. 'Tracking control’ is used three times. This is a way of
controlling the inputs such that the error, between the vehicle model and the reference model,
will exponentially approach zero. The state feedback gain can still be determined using LQR tech-
niques. $H_\infty - \mu$ synthesis control also appears three times in the table. In this control technique
the feedback gain is determined such that the system performance is robust for parameter per-
turbations. This property is called robustness of performance. Some more articles pay attention
to robust control, because in reality the parameters of the vehicle model may vary with respect
to the nominal values and because of unmodeled dynamics, as the vehicle model remains an
approximation of the real vehicle.

When having a look at the vehicle models, it is clear that the bicycle model is mostly used. Some
articles use the bicycle model to design the controller and test the controller’s performance on a
more complicated model, which should better match to reality. Two articles validate their control
technique on a real vehicle. Those articles present a control technique for ARS based on $H_\infty - \mu$
synthesis control.

Finally, it can be concluded that there is no general consensus about what control objective is to
be achieved by active rear wheel steering. Many approaches have been tried, from which a few
interesting aspects are discussed in the next chapter.
Chapter 4

Controlling the steering angle of the rear wheels

Based on the literature review in previous chapter a few conclusions can be drawn concerning the way the Citroën BX will be controlled:

1. The control scheme should contain a reference model, which describes the desired vehicle response depending on certain inputs, like vehicle speed and steering angle of the front wheels. Some freedom in changing the vehicle response should be built in this reference model, so that a desirable vehicle response can be obtained.

2. As only one input can be freely chosen, in this case the steering angle of the rear wheels, only one output can be directly controlled. The two quantities describing the vehicle’s state in a horizontal plane are yaw rate and sideslip angle. Together they determine the lateral acceleration. One of these three quantities should be the one to be controlled by the steering angle of the rear wheels.

3. As a result of a step steer by the driver a yaw moment is exerted by the front tyres. A yaw motion will be initiated and the sideslip angle of the vehicle will start to increase. Then the rear tyres will start to participate in generating lateral tire forces. Finally a stationary cornering situation will occur. The sideslip angle of the vehicle and the yaw rate are the two basic quantities controlled by steering, whereas the lateral acceleration is determined by those two. So either the sideslip angle or the yaw rate should be chosen as the quantity to be controlled. Early studies on 4WS focus on reducing the sideslip angle. A few more recent papers, like Song et al. [12], Nikzad [14] and Lv et al. [15], focus on the yaw motion. The main purpose of steering is changing the heading angle of the vehicle (i.e. integrated yaw rate) and therefore yaw rate will be the quantity to be controlled by the rear tyres.

4. According to the bicycle model, the yaw rate response to the steering angle of the front wheels is described by a second order dynamical system. The time response of a sub-critically damped second order system on a step input shows overshoot and damping. This leads to so-called ‘fishtailing’, a phenomena, occurring after a fast steering action at medium to high speeds, in which the back of the vehicle kind of behaves like the tail of a fish. Overshoot is not desirable, as the driver has to compensate it in order to quickly reach a constant value of the yaw rate. This results in an extra driver workload. In a few papers, like Song et al. [12], a yaw rate reference model, describing the reference yaw rate as a first
4.1 Control structure

As mentioned above model matching control will be applied in calculating the steering angle of the rear wheels. In this case the yaw rate of the vehicle will be controlled using a first order yaw rate reference. The control structure is displayed schematically in figure 4.1. The driver steers the front wheels and the measured steering angle $\delta_f$ is fed into the control scheme, together with the measured vehicle speed $V$ and the measured yaw rate $r$. Inside the control scheme the steering angle of the rear wheels $\delta_r$ is calculated, depending on the inputs of the control scheme, and is applied to the vehicle. The reference yaw rate $r_{ref}$ is calculated as a vehicle speed depending first order function of the steering angle of the front wheels. The actual yaw rate of the vehicle is fed back and subtracted from the reference yaw rate. The resulting error goes to the controller which calculates the steering angle of the rear wheels.

Besides increasing the stability of the yaw motion of the vehicle, the control structure, as proposed here, can also exhibit ESP like features. ESP or electronic stability programme compares the measured yaw rate to the yaw rate, which should occur under normal driving conditions and similar driver inputs. If a certain critical threshold in the difference between those two is exceeded, the brake of an individual wheel will be applied in order to reduce the difference by exerting a yaw moment in the right direction. As the actual yaw rate of the vehicle is also compared to a reference value in the control structure presented here, the similarity between both systems is apparent. However the actions from both systems are quite different: Active rear steering (ARS) works in a continuous smooth way, while ESP interferes in a discrete, rather brute, way. Depending on the bandwidth of the overall control structure, ARS could complement ESP, making it harder for a driver to reach situations in which ESP interferes.

4.2 Reference model

The reference yaw rate $r_{ref}$ is the solution of a first order differential equation with the steering angle of the front wheels $\delta_f$ as the input, i.e.

$$\dot{r}_{ref} = -\frac{1}{\tau} r_{ref} + \frac{H_0}{\tau} \delta_f$$  \hspace{1cm} (4.1)
The problem is to choose reasonable values for the time constant \( \tau \) and the steady state gain \( H_0 \). Both a time constant and a steady state gain can be obtained from the differential equations of the bicycle model. Both quantities have already been derived in Chapter 2 and are given by:

\[
H_r(V) = \frac{V}{l + \frac{n}{g}V^2} \quad (4.2)
\]

\[
\tau_r(V) = \frac{IV}{aC_f(l + \frac{n}{g}V^2)} = \frac{I}{aC_f} \cdot H_r \quad (4.3)
\]

They both depend on the vehicle speed \( V \) and on the understeer coefficient \( n \) which is given by (2.11). Of course, other values for the time constant and the steady state gain can also be chosen. However, this choice seems very reasonable as it is based on the yaw rate response of the bicycle model.

If the desired understeer coefficient in (4.3) is chosen equal to the understeer coefficient \( n \) of the Citroën BX \( n_{\text{vehicle}} \), then the steady state yaw rate of the reference model will approximate the steady state yaw rate of the Citroën BX. The steering angle of the rear wheels will therefore be very close to zero in steady state conditions, as there is hardly any difference between the reference yaw rate and the actual yaw rate. This statement only holds while driving within the linear region of the tire characteristics, that is up to approximately \( 4 \, \text{m/s}^2 \), as the yaw rate reference is based on the linear bicycle model.

The transient part of the yaw rate reference will deviate from the transient part of the yaw rate response of the front wheel steered vehicle as a result of the difference between a respectively first and second order response. Figure 4.2 shows this with three yaw rate step responses at 120 km/h, one of the front wheel steered bicycle model, one of the front wheel steered multi-body vehicle model and one of the yaw rate reference (\( n_{\text{vehicle}} \) equals 0.0361).
CHAPTER 4. CONTROLLING THE STEERING ANGLE OF THE REAR WHEELS

Figure 4.3: Nyquist diagram of the transfer function from the steering angle of the rear wheels $\delta_r$ to the yaw rate $\tau$ at 120 km/h

4.3 Choice of type of controller

Lateral vehicle dynamics can be approximated quite accurately by a linear vehicle model in normal driving conditions, that is during low up to medium lateral acceleration levels. Therefore, and for reasons of simplicity, it makes sense to use a linear controller with transfer function $K$ in combination with a linear vehicle model with transfer function $H$. This controller $K$ can then be tuned using a technique called ‘loopshaping’, in which the open-loop transfer function $KH$ is shaped in order to achieve maximal performance. The best choice out of the three earlier described linear vehicle models for controller design, is the vehicle model with three degrees of freedom (yaw, lateral and roll motion) and including relaxation of the tyres. This model approximates the yaw rate response of the multi-body vehicle model better than the other two linear vehicle models within the linear lateral vehicle dynamics range. This is demonstrated by figure 4.3, which shows the Nyquist diagram of the transfer function from the steering angle of the rear wheels to the yaw rate at 120 km/h for three linear vehicle models and the multi-body vehicle model.

Basically it is not possible to display a transfer function of a non-linear model, like the multi-body vehicle model, in a Nyquist diagram. A transfer function of a nonlinear system always represents a linearization of the system in a working point. The transfer function of the multi-body vehicle model in figure 4.3 is obtained by applying small perturbations in the steering angle of the rear wheels during a straight line travel. It is noted that the Nyquist diagram of the multi-body vehicle model does not end in the origin, which seems to be very strange. This will be explained in the next section.

The term ‘extended’ in relation with the bicycle model means that relaxation of the tyres has been included. Relaxation causes the transfer function to enter the right half plane of the Nyquist diagram, which does have some implications on stability issues during controller design. The
4.4 Explanation for difference in vehicle models after 20 Hz

Figure 4.4 shows the Bode diagram of the transfer function from the steering angle of the rear wheels $\delta_r$ to the yaw rate $r$ at 120 km/h. There are no big differences up to about 15 Hz between the multi-body model and the two linear models. From about 20 Hz the difference between the multi-body model and the two linear models starts to increase rapidly. This might lead to problems, because the controller will be based upon the transfer function of the linear extended 3DOF bicycle model and will be validated with the multi-body model, whose transfer function apparently deviates from the former. As the frequency increases, so does the gain of the multi-body model whereas the phase goes to -90 degrees. As with almost any physical process, one should expect the gain to decrease at high frequencies.

The reason for this surprising behaviour lies in the way the vehicle is modelled in the multi-body model. Both the vehicle body and the wheels are modelled as bodies with a mass and moments of inertia. The transfer function of the multi-body model is obtained by prescribing a harmonic steering angle, determining the yaw rate and calculating the ratio between both using spectral densities. Prescribing a steering wheel angle $\delta$, which is in fact a relative angle between the vehicle body and the wheel, requires a moment $M$ on the wheel and, by the law of action and reaction, a moment $M$ on the vehicle body. This moment will influence the heading angle of the vehicle body, as can be seen in figure 4.5. The equations of motion for both the vehicle body and the wheel are given by

$$J_{z,\text{wheel}}(\ddot{\phi} + \ddot{\delta}) = M \tag{4.4}$$
CHAPTER 4. CONTROLLING THE STEERING ANGLE OF THE REAR WHEELS

Figure 4.5: Influence of inertia forces from the wheel upon the yaw rate of the vehicle body

\[ J_{z,\text{vehiclebody}} \ddot{\phi} = -M \] (4.5)

Adding those equations leads to:

\[ (J_{z,\text{wheel}} + J_{z,\text{vehiclebody}}) \ddot{\phi} = (J_{z,\text{wheel}} + J_{z,\text{vehiclebody}}) \dot{r} = -J_{z,\text{wheel}} \dot{\delta} \] (4.6)

Hence, the transfer function from the steering angle to the yaw rate is given by:

\[ H(s) = \frac{r(s)}{\delta(s)} = \frac{-J_{z,\text{wheel}} s}{J_{z,\text{wheel}} + J_{z,\text{vehiclebody}}} \] (4.7)

The Laplace operator in the numerator explains the behaviour, as it causes the yaw rate to increase with a slope of +1 and makes the phase angle reach -90 degrees. This explains the at first surprising behaviour of the transfer function of the yaw rate of the multi-body vehicle model. This behaviour does not occur with the extended bicycle model and the extended 3DOF bicycle model, as inertia of the wheels is not modelled at all. When an extra term, describing the inertia moment \( M = -2J_{z,\text{wheel}} \dot{\delta} \) from the wheels onto the vehicle body, is added to the differential equation of the yaw rate of the extended bicycle model, then the resulting transfer function will approximate the transfer function of the multi-body model including the behaviour, which can be seen in figure 4.4.

It is thought that this behaviour is purely a result of the relatively simple way the vehicle is modelled and that it will not occur in reality. The frequency range, in which the behaviour occurs, is not particularly interesting when looking at lateral vehicle dynamics, whose primary frequency range is up to about 10 Hz. The yaw rate of the vehicle is to be influenced by tyre forces and not by inertia forces through wheel movement. It should also be noted that the controller, needed for steering the rear wheels, will probably have very limited capabilities for frequencies higher than 20 Hz. This will be further discussed later. With these considerations taken into account it seems very reasonable to neglect the behaviour, explained above. This can be done by decreasing the moment of inertia of the wheel in the multi-body model to a very small value. The transfer function of the linearized multi-body model will then approximate the transfer function of the extended bicycle model and the extended 3DOF bicycle model up to about 200 Hz.

4.5 Controller design without actuator dynamics

As mentioned earlier a linear controller will be designed through loopshaping. The Matlab toolbox DIET (Do It Easy Toolbox) will be used for this purpose. The transfer function \( H \) of the
4.5. CONTROLLER DESIGN WITHOUT ACTUATOR DYNAMICS

Figure 4.6: Bode diagram of the yaw rate of both the extended 3DOF bicycle model and the multi-body model

extended 3DOF bicycle model can be imported and the open-loop transfer function \( KH \) can very easily be tuned using all kinds of controller combinations. A controller \( K \) will be designed with maximal performance and sufficient robustness. Performance will be measured by the magnitude of the bandwidth of \( KH \). The margin of robustness will be defined by the maximum \( S_{max} \) of the sensitivity function \( S = 1/(1 + KH) \). These are conflicting requirements: a high bandwidth will decrease robustness. The value \( 1/S_{max} \) equals the minimal distance from the plot of \( KH \) to the point \((-1,0)\) in the Nyquist diagram. The smaller \( 1/S_{max} \), the higher the risk that errors in the transfer function \( H \) will lead to instability. Therefore a certain minimal distance has to be required as a margin of robustness. A commonly used criterion for the margin of robustness is \( S_{max} = 6 \) dB, which means that the open-loop has to keep a minimal distance of 0.5 from the point \((-1,0)\). This criterion will be used as the margin of robustness.

A controller will be designed here at a vehicle speed of 120 km/h meeting the demands stated above. All steps are visualised in figure A.1 to figure A.12 in appendix A. Only the begin (figure 4.6) and the end situation (figure 4.7) will be displayed here. The controller will be based on the transfer function of the extended 3DOF bicycle model, however the transfer function of the linearized multi-body model, measured as mentioned earlier, will also be included in all figures.

First of all a gain of \(-1\), i.e. \( K = -1 \), is applied to mirror the Nyquist diagram in the origin. The resulting feedback loop is already stable. Through simulations, in which the steady state reference yaw rate did not equal the steady state yaw rate of the front wheel steered vehicle, it became clear that a large integrator had to be added to the controller to eliminate steady state errors. To achieve that, phase lead has to be created to allow phase lag from the integrator without violating the margin of robustness. Phase lead will be created using a lead/lag filter. It is assumed that a bandwidth of 10 Hz is high enough for the feedback loop to be adequate. To create about 45 degrees phase lead at 10 Hz, the zero of the lead/lag filter will be placed at 3 Hz and the pole at 30 Hz. The created phase lead of 45 degrees is cancelled by adding an integrator at 10 Hz. The gain will then be changed to \(-0.17\) in order to achieve a bandwidth close to 10 Hz. Finally a
second order lowpass filter will be added at a frequency of six times the bandwidth (60 Hz), so it will not interfere with the open-loop in the vicinity of 10 Hz. This filter is supposed to eliminate any highfrequent noise. The transfer function $K$ of the final controller is given by

$$K(s) = -0.17\left(\frac{1 + \frac{1}{30 \cdot 2 \cdot \pi} s}{1 + \frac{1}{10 \cdot 2 \cdot \pi} s}\right)\left(1 + \frac{1}{10 \cdot 2 \cdot \pi} s\right)\left(s^2 + 2 \cdot 0.5 \cdot 60 \cdot 2 \cdot \pi \cdot s + (60 \cdot 2 \cdot \pi)^2\right)$$  \hspace{1cm} (4.8)$$

The resulting Bode diagram of the open-loop $KH$ is displayed in figure 4.7. The frequency at which the magnitude equals 0 dB, i.e. the bandwidth, lies at 10.2 Hz.

In this case, where the actuator dynamics are not taking into account, it is not necessary to design a controller at the limit of the margin of robustness in order to achieve enough performance. A bandwidth of about 10 Hz can easily be achieved without approaching a maximal sensitivity of 6 dB. Later on, when taking into account actuator dynamics, controls will be designed at the minimal margin of robustness to achieve maximal performance.

### 4.5.1 Simulation results of the controller

The yaw rate feedback controller $K$ described by (4.8) is validated using the multi-body vehicle model with an ideal actuator. It has been designed for a vehicle speed of 120 km/h and it is tested at a vehicle speed of 120 km/h and 200 km/h. The input of this simulation is a step in the steering angle of the front wheels, which has been smoothed by a second order lowpass filter, as a true step in time is not physical, see figure 4.9. The understeer coefficient $\eta$ in the relation for the reference yaw rate is 0.0361, which is equal to that of the FWS vehicle. In that case the steady-state reference yaw rate will approximate the steady-state yaw rate of the FWS vehicle and so the steering angle of the rear wheels will be close to zero in steady-state conditions. Therefore the steady-state yaw rate response to steering remains almost unchanged and only the dynamic
4.6. CONTROLLER DESIGN WITH ACTUATOR DYNAMICS

Figure 4.8: Yaw rate response of both controllers to the steering angle of the front wheels, displayed in figure 4.9

The yaw rate response will be altered. The yaw rate responses of the closed-loop system is presented in figure 4.8 for both vehicle speeds. Similar figures for the lateral acceleration and the sideslip angle are given in appendix B. The steering angle of the rear wheels is depicted in figure 4.9. The similarity between the reference yaw rate and the yaw rate of the four wheel steered vehicle with yaw rate feedback is good at both vehicle speeds. So the feedback controller does what it is supposed to do, even at a vehicle speed of 200 km/h, at which it was not design for. The steady state yaw rate of the four wheel steered vehicle with yaw rate feedback approximates the steady state yaw rate of the front wheel steered vehicle, meaning that the same level of understeer is obtained. Only the dynamic yaw rate response is changed.

4.6 Controller design with actuator dynamics

In the previous section a controller has been designed and tested at a vehicle speed of 120 km/h. During this process the influence of actuator dynamics has been neglected in order to investigate the potential of the feedback loop. In reality, the actuator dynamics has to be taken into account. Later on, in Chapter 5, the dynamic characteristics of the steering actuator will be investigated and modelled as a third order function in the Laplace domain. The dynamic model of the steering actuator will now be added to the extended 3DOF vehicle model and a new controller will be derived, which does take into account the influence of actuator dynamics. The Bode diagram of the extended 3DOF vehicle model at 120 km/h is shown in figure 4.10 with and without actuator dynamics. The gain shows no significant difference up to 10 Hz, while the phase lag of the model with actuator increases in this frequency range. After approximately 20 Hz the gain of the model with actuator decreases rapidly.

The goal now again is to design a controller with a bandwidth as high as possible, while maintaining a maximal sensitivity of 6 dB as a margin of robustness. The transfer function of the model
CHAPTER 4. CONTROLLING THE STEERING ANGLE OF THE REAR WHEELS

Figure 4.9: Steering angles of the front and rear wheels in the simulation with the multi-body vehicle model

Figure 4.10: Bode diagram of the extended 3DOF vehicle model at 120 km/h
with actuator dynamics shown in figure 4.10 will serve as the starting point. The controller designed previously for the vehicle model without actuator dynamics consisted of the following components:

1. a gain of -0.17
2. a lead/lag filter with a zero at 3 Hz and a pole at 30 Hz
3. a weak integrator with a zero at 10 Hz
4. a second order lowpass filter with an undamped eigenfrequency at 60 Hz

The new controller will have a similar structure. The second order lowpass filter will be omitted as the actuator basically takes over its role. Applying components 1 till 3 stated above to the transfer functions in figure 4.10 will lead to the Nyquist diagram in figure 4.11, which has been zoomed in around the origin. The point \((-1, 0)\), critical for stability, is encircled the wrong way around by the model with actuator, leading to an unstable feedback loop. As a result the gain, the lead/lag filter and integrator will require some tuning.

Because of the extra phase lag induced by the actuator, the integrator settings have to be changed. The zero of the integrator has to be decreased to allow for the phase lag from the actuator. As a consequence the bandwidth of the open-loop will decrease. Normally the main purpose of the lead/lag filter is to provide maximal phase lead at the bandwidth. As a rule of thumb the zero of the lead/lag filter is placed at \(\frac{\text{bandwidth}}{3}\) and the pole of the lead/lag filter at \(3 \times \text{bandwidth}\). If the bandwidth is decreased by lowering the zero of the integrator, a logical step would be lowering the zero and the pole of the lead/lag filter to provide maximal phase lead at the bandwidth. However, lowering the zero and the pole will have an increased low-frequent gain as negative side-effect. This will then have to be cancelled by decreasing the gain to ensure a maximal sensitivity of 6 dB.
CHAPTER 4. CONTROLLING THE STEERING ANGLE OF THE REAR WHEELS

Many different combinations of gain, integrator and lead/lag filter settings have been tried, but the ones in which only the gain and the zero of the integrator are lowered show the best potential. A maximal bandwidth of 4.8 Hz was achieved by a gain of -0.105, an integrator with the zero at 6 Hz and a lead/lag filter with the zero at 3 Hz and the pole at 30 Hz, while maintaining a maximal sensitivity of 6 dB (see table 4.1). The resulting Bode diagram of the open-loop is displayed in figure 4.12 and the Nyquist diagram in figure C.2 in appendix C. The phase lead, generated by the lead/lag filter, is almost of the same magnitude at the bandwidth of the new controller (i.e. 4.8 Hz) as at the bandwidth of the previously designed controller (i.e. 10.2 Hz). This approves an unaltered lead/lag filter setting.

This new controller has been designed at a specific vehicle speed of 120 km/h. The general idea behind the final controller is that a number of controllers are to be designed at a number of speeds in the range of normal vehicle speeds. The settings of these controllers should then be interpolated to cover the entire range of vehicle speeds. For this purpose 4 controllers have been derived at 40, 80, 120 and 160 km/h. The settings of these controllers are shown in table 4.1, together with the resulting maximal sensitivity and the bandwidth. As there are no large

<table>
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<th>velocity [km/h]</th>
<th>gain [-]</th>
<th>integrator [Hz]</th>
<th>lead/lag [Hz]</th>
<th>lead/lag [Hz]</th>
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</tbody>
</table>

Table 4.1: Controller settings at 40, 80, 120 and 160 km/h
4.6. CONTROLLER DESIGN WITH ACTUATOR DYNAMICS

Figure 4.13: Yaw rate response to the steering angle of the front wheels, displayed in figure 4.9


differences between the individual controllers, one controller could be used to cover the entire range of vehicle speed. The fact that there are no large differences between the controller settings, originates from the fact that there are no large differences in the transfer functions upon which those controllers are based. The transfer functions at 40, 80, 120 and 160 km/h are displayed in figure C.3 in the appendix. As can be seen, there are fairly small differences. Especially after 5 Hz, near the bandwidth, the differences are small. Therefore it is chosen that the controller derived at 80 km/h will be used at all speeds. The bandwidth and the maximal sensitivity of this controller, applied at all velocities, are also displayed in table 4.1 in the second row at each vehicle speed. The bandwidth remains almost constant at all velocities, while the maximal sensitivity increases at 120 and 160 km/h and so the margin of robustness is slightly decreased (see figure C.4 in appendix C).

4.6.1 Simulation results of the new controller

Like in section 4.5.1 the new controller is evaluated using the multi-body vehicle model. This time the actuator dynamics has been included in the model. The steering input will be the same as before, i.e. a smoothed step in the steering angle of the front wheels (see figure 4.9), and the vehicle speed equals 120 km/h. Both the controller derived for 120 km/h and the ‘general’ controller of 80 km/h are used. The yaw rate response of the four wheel steered vehicle to the steering input is displayed in figure 4.13 for both controllers, together with the reference response and the response of the front wheel steered vehicle. As can be seen, there is hardly any difference in the response obtained with these controllers. It should be said that the deviation between the response of the four wheel steered vehicle and the reference response is larger than the same deviation in case the actuator dynamics had not been taken into account (see figure 4.8). This is a direct consequence of the decreased bandwidth of the feedback loop. The controller simply lacks the responsiveness needed to achieve the reference
4.7 Some realistic driving situations

In the previous two sections controllers have been designed, using the extended 3DOF bicycle model with and without actuator dynamics. The performance of these controllers has been tested through relatively simple simulations, in which a smoothed step in the steering angle of the front wheels has been applied as steering input. In this section two more realistic simulations will be performed using the multi-body vehicle model including actuator dynamics. The controller used in these two simulations will be the controller derived in section 4.6. The purpose of these simulations is to demonstrate some interesting features of the active rear wheel steering system and to investigate what vehicle behaviour can be expected in the experiments.

4.7.1 Breaking in a corner

During this simulation the same steering input will be used as in the simulations of Chapter 4, i.e. a smoothed step in the steering angle of the front wheels. This steering input is prescribed as function of time and therefore it is called open-loop steering. If the steering angle of the front wheels would be calculated in time by some sort of driver model, then it is called closed-loop steering, since the driver is included in the final loop. This will be the case in the double lane change simulation.

The procedure during the simulation is as follows: initially the vehicle drives 120 km/h straight ahead, then the step steering input is applied and after steady-state cornering conditions have been reached, a longitudinal acceleration of $-5.5 \text{ m/s}^2$ is prescribed for 5 seconds. The yaw rate response of the front and the four wheel steered vehicle are shown in the left part of figure 4.14, together with the reference yaw rate. The steering angles of both the front and the rear wheels are shown in the right part. What normally happens to the front wheel steered vehicle after the brake has been applied, is that the vehicle will tend to oversteer (i.e. an increase in yaw rate) as a result of mass transfer from the rear to the front wheels. The nose of the vehicle will turn more towards

![Figure 4.14: The yaw rate response and the steering angles during the simulation](image-url)
the inside of the corner, thereby reducing the actual cornering radius. This is exactly what can be seen from the yaw rate response of the front wheel steered vehicle in figure 4.14.

In case of the four wheel steered vehicle, the rear wheels will try to counteract this motion. At the moment the brake is applied, i.e. at $t$ equals 3 seconds, a rapid increase of the yaw rate occurs. At this moment the rear wheels start participating in order to follow the reference yaw rate again. After about half a second the yaw rate is once again very close to the reference value and stays very close to it throughout the rest of the simulation. The reason why the reference yaw rate starts decreasing during the simulation lies in the fact that the vehicle speed also decreases as a result of the longitudinal deceleration. Finally, it can be concluded that the yaw rate feedback system shows a similar behaviour as an ESP system.

4.7.2 Double lane change

The double lane change test is standardised in [32] and simulates an obstacle avoidance maneuver, which will result in high lateral accelerations. The vehicle has to complete a path as shown in figure 4.15, in which the trackwidth depends on the width $w$ of the vehicle. For this purpose a relatively simple driver model has been developed, which will steer the front wheels such that the vehicle completes a trajectory within the bounds of figure 4.15. The driver model uses a look-ahead distance [30][31]: the look-ahead distance $l_{\text{look-ahead}}$ is determined by the product of the vehicle speed $V$ and a parameter called preview time $t_p$:

$$l_{\text{look-ahead}} = V t_p$$

A prescribed trajectory within the bounds of figure 4.15 has been calculated using the spline function in Matlab. In the beginning of the simulation the center of gravity of the vehicle lies exactly on the prescribed trajectory. The driver model now steers the front wheels proportional to the look-ahead angle $\varphi_{\text{look-ahead}}$, which is the angle between the vector in the longitudinal direction of the vehicle and the vector, originating in the center of gravity and with the length of the look-ahead distance, which crosses the prescribed trajectory, see figure 4.16. A first order lag function has been included to take into account neuromuscular delay. The transfer function from the look-ahead angle to the steering angle of the front wheels then becomes:

$$\delta_{\text{driver}} = \frac{c \cdot \varphi_{\text{look-ahead}}}{\tau_{\text{neuro}} s + 1}$$
The following three parameters can now be tuned in order to complete the course without exceeding the boundaries: the preview time $t_p$, the steering sensitivity $c$ and the neuromuscular time constant $\tau_{\text{neuro}}$. The choice for this type of driver model is motivated by the fact that it mimics the way a human driver steers. This makes tuning of the parameters very straightforward. Modelling the steering properties of the human driver cannot be treated in an exact manner. Large differences in steering angles can be encountered among various driver models. Different steering inputs in time can still lead to a successful completion of a given course. The main aspect when trying to investigate the contribution of active rear wheel steering is that the same driver model is used for both the front wheel steered vehicle and for the four wheel steered vehicle. The double lane change itself is performed at a vehicle speed of 80 km/h. The yaw rate response of the front wheel steered and the four wheel steered vehicle are shown in figure 4.17, together with the reference yaw rate. The yaw rate of the four wheel steered vehicle is very close to the reference value. The difference in yaw rate between the four wheel steered and the front wheel steered vehicle is quite small. However, at the end of the double lane change when the vehicle enters the exit, the front wheel steered vehicle shows an oscillation in the yaw rate, whereas the four wheel steered vehicle does not. The oscillation is the result of a similar oscillation in the steering angle of the front wheel steered vehicle, as can be seen in figure 4.18. This oscillation does not appear in the steering angle of the front wheels of the four wheel steered vehicle. Apparently, when the front wheel steered vehicle enters the exit part of the double lane change, a small oscillation in the yaw rate occurs on which the driver model reacts. The reaction of the driver model is stable, but subcritically damped. Since no oscillation occurs in case of the four wheel steered vehicle, the four wheel steered vehicle experiences a more damped yaw rate response when entering the exit part of the double lane change. This extra damping is provided by the rear wheels’ steering angle.

Finally, it is noted that the oscillation in the yaw rate of the front wheel steered vehicle does not have to occur when a human driver steers the vehicle. It can simply be introduced by the driver model. However, as the driver model steers based upon the vehicle motion, it can still be concluded that the yaw rate response of the four wheel steered vehicle is more damped than the yaw rate response of the front wheel steered vehicle.
4.7. SOME REALISTIC DRIVING SITUATIONS

Figure 4.17: Yaw rate response during the double lane change

Figure 4.18: Steering angles of the front and four wheel steered vehicle
Chapter 5

Vehicle layout and experiments

In this chapter the test vehicle, adapted to 4WS, will be described first together with the instrumentation. The experiments conducted with this test vehicle will be treated later. Normally, it would seem obvious to compare the simulation results of the double lane change from the previous chapter to the results of the double lane change during the actual experiments. However, this will not be done because of two reasons, which will become clear in this chapter.

1. The multi-body vehicle model used in the previous chapter did not approximate the measured vehicle response close enough.

2. The controller derived in section 4.6 and used in the previous chapter has not been used in the experiments.

Finally, after the vehicle model has been adopted to coincide with the measured vehicle response, a comparison in control performance is made between the controller used in the experiments and the controller derived in section 4.6. Only a qualitative match is present between both, certainly no quantitative match is derived. The comparison shows what control performance could have been achieved if the controller derived in section 4.6 had been used in experiments with a far more accurate yaw rate sensor.

5.1 The test vehicle

The test vehicle, a Citroen BX 1.9 GTI, has been adapted to four wheel steering by TNO in 1990. The rear wheel steering rack basically consists of a default front wheel steering rack which has been rotated 180 degrees. The maximal steering angle is limited to 5 degrees. The steering angle of the rear wheels is actuated by a hydraulic servo system, whose layout is displayed in figure 5.1. The pump delivers pressurized oil, which is reduced to 100 bar and filtered. A 3-way servo valve drives an asymmetric motor, which in turn steers the rear wheels. Safety valves have been installed to block the piston’s motion and so freeze the rear wheels’ steering angle in case of an emergency. The safety system, which actuates these safety valves, is overruled as a manually operated safety switch is installed, which, if applied, returns the steering angle to the neutral position instead of freezing it in the current position.

The hydraulic servo system is controlled by a local position feedback loop. The steering angle of the rear wheels is fed back to the input current of the servo valve. As the steering system will be part of the global feedback loop, it is important that the response time is as small as possible.
An important factor in the response time is the natural frequency of the steering actuator. This frequency depends on the length of the oil column on both sides of the piston. As the asymmetric motor in the rear wheel steering system is normally used for the power steering of the vehicle, much of the available actuator stroke is unused, because the steering angle of the rear wheels is limited to 5 degrees. So the actuator had been modified to increase the effective oil column stiffness, thereby increasing the natural frequency and thus reduce the response time. Besides this modification, optimal controller settings have been determined, which further reduce the response time.

5.1.1 Modelling the rear wheel steering system

Here, the focus will be on modelling the dynamic steering characteristics of the hydraulic servo system. These characteristics play an important role during controller design, as the actuator will be embodied in a feedback loop. They are investigated by applying a sine sweep from 0 to 25 Hz to the input of the actuator. The output, i.e. the steering angle of the rear wheels, is measured and subsequently the frequency response function between the input and output is calculated. The results are shown in figure 5.2.
5.1. THE TEST VEHICLE

The behaviour below 3 Hz is the result of a low coherence in that region. Measurements with a steady-state sine as input indicated that the behaviour can be neglected. As can be seen, the gain of the response remains approximately 1 up to about 20 Hz, while the phase lag increases almost linearly in this frequency range. It can be concluded that the phase of the response of the hydraulic servo is much more important than the gain. The behaviour up to about 20 Hz can be very well modelled by a death time of 0.03 s.

In the past TNO conducted similar measurements on the hydraulic servo system. A fit in the Laplace domain was made by approximating the transfer function by a third order system. This approximation is also included in figure 5.2. There are some differences between the fit from the past by TNO and the recently measured data, however, in general the similarity is quite good. As the fit is in the Laplace domain, it can be easily added to a linear vehicle model in order to model the hydraulic servo for controller design purposes.

5.1.2 Instrumentation

A few sensors, needed for the final control law, have been installed in the test vehicle:

1. In the past TNO installed 2 linear variable differential transformers (LVDTs), which measure a one-dimensional displacement with high precision. The sensors are attached to the steering piston in the front and rear steering rack and so they measure both the front and rear wheel steering angles. The signals from these sensors are filtered in the hardware by a sixth order Butterworth filter at 60 Hz to eliminate any high frequency content.

2. The ABS-sensor mounted in the wheel hub of one of the undriven wheels is used to mea-
sure the vehicle speed. The sensor produces a fixed number of pulses per wheel revolution and counting the number of pulses in the short time interval will produce the vehicle speed. TNO used a printed circuit board to perform this task. However, after it broke down while testing, it has been replaced by a script in Matlab Simulink, which communicates with the event counter input of a TUEDACS.

3. The test vehicle contained a yaw rate sensor and a lateral acceleration sensor, which were installed long after the project at TNO had ended. These two sensors were quite old (25 years), were no longer used by TNO and so ended up in the vehicle. After some testing it appeared that the lateral acceleration sensor was broken and that the yaw rate sensor produced, besides some signal, lots of electrical noise around 20 to 40 Hz. The yaw rate sensor had last been calibrated in 1988 and only the sensor’s sensitivity was known. It was decided to use a newer type of yaw rate sensor. A Bosch yaw sensor was chosen, as it measures both the yaw rate and the lateral acceleration. Besides, all the sensor’s technical data was known and it was relatively cheap. Normally this type of sensor is part of an ESP system in passenger cars and so it was found at a car graveyard in a wrecked Mercedes A-class at a price of 20 euro (new price 450 euro). When comparing the output of the new yaw rate sensor to the output of the old one, it was discovered that the sensitivity of the old yaw rate sensor had dropped about 25 percent with respect to the sensitivity, calibrated in 1988. Besides, the electrical noise of the new sensor was reduced with respect to the old sensor. Finally, it should be said that yaw rate sensors with a better signal to noise ratio than the Bosch sensor do exist, however, they are quite expensive.

3 TUEDACS Microgiants are used to acquire all sensor data. They are connected to a laptop to calculate the steering angle of the rear wheels depending on the acquired data and the programmed control law.

5.2 Double lane change

During the testing phase it becomes clear very soon that the behaviour of the vehicle does not feel good while traveling in a straight line. The back of the vehicle vibrates and it gets worse when the vehicle speed increased. This vibration is caused by electrical noise from the yaw rate sensor. The sensor used for measuring both the yaw rate and the lateral acceleration is the earlier mentioned Bosch ESP sensor. The sensor’s yaw rate output contains quite some noise in a frequency band between 3 to 6 Hz, even when the vehicle stands still. Together with the sensor’s sensitivity, this means that the yaw rate feedback loop ‘senses’ a vibration in the yaw rate, which does not occur in reality. As a result the rear wheels’ steering angle reacts in order to cancel this fake vibration, thereby just inducing a real vibration in the yaw rate. The vibration in the steering angle of the rear wheels in a frequency band between 3 to 6 Hz is not felt as severe at a low vehicle speed as the same vibration at a high vehicle speed, because the time constant $\sigma/V$ in the first order transfer function from the tire slip angle $\alpha$ to the lateral tyre force $F_y$ is higher at a low than at a high vehicle speed. Tyre relaxation acts as a vehicle speed dependent lowpass filter and the filter function is higher at a low vehicle speed than at a high vehicle speed.

A first order lowpass filter with a pole at 4 Hz has been added to the controller in order to reduce the intensity of the vibration. Additional filtering does have implications for the controller settings, as the filter creates extra phase lag. Therefore the controller settings have been altered to maintain a maximal sensitivity of the feedback loop near 6 dB as a margin of robustness. These
5.2. DOUBLE LANE CHANGE

<table>
<thead>
<tr>
<th>setting</th>
<th>gain [-]</th>
<th>integrator [Hz]</th>
<th>lead/lag [Hz]/[Hz]</th>
<th>1st order [Hz]</th>
<th>$S_{max}$ [dB]</th>
<th>$f_b$ [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-0.13</td>
<td>4</td>
<td>3/30</td>
<td>-</td>
<td>6.0</td>
<td>4.8</td>
</tr>
<tr>
<td>B</td>
<td>-0.10</td>
<td>4</td>
<td>3/30</td>
<td>4</td>
<td>7.2</td>
<td>2.8</td>
</tr>
<tr>
<td>C</td>
<td>-0.10</td>
<td>2</td>
<td>-</td>
<td>4</td>
<td>5.1</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Table 5.1: Changes made to the controller settings (A: initial setting, B: first change, C: second change)

![Bode diagram of the controller with the 3 controller settings listed in table 5.1](image)

Figure 5.3: Bode diagram of the controller with the 3 controller settings listed in table 5.1

changes however lead to a reduced bandwidth. So basically the control performance is reduced as a result of sensor noise. The first change to be made is a reduction in the gain from -0.13 to -0.10. This only has a small effect in reducing the vibration. The second and final change is more drastic. The lead/lag filter is removed and the zero of the integrator is reduced from 4 Hz to 2 Hz. The vibration is now reduced to a sufficiently low level. However, there is a price to pay: the bandwidth of the feedback loop is reduced from initially 4.8 Hz to finally 1.6 Hz at a vehicle speed of 80 km/h, as can be seen in Table 5.1. The Bode diagram of the 3 controller settings is displayed in figure 5.3.

The double lane change is performed at a vehicle speed of 80 km/h. First a double lane change is driven with the front wheel steered vehicle. The steering angle of the front wheels, measured during this test, is used as the steering input in a simulation with the front wheel steered multi-body vehicle model at a vehicle speed of 80 km/h. The measured steering angle of the front wheels is displayed in figure 5.4 together with the lateral acceleration response. The yaw rate response is displayed in figure 5.5. As can be seen in this figure, there is a large difference between the measured yaw rate (i.e. FWS test) and the yaw rate from the simulation with the same steering input (i.e. FWS understeered). The used understeer coefficient $\eta$ of the vehicle model is 0.0361 rad. It has been calculated using vehicle parameters determined by TNO in the past after an optimisation process on measured vehicle data. It is clearly visible that both the
 CHAPTER 5. VEHICLE LAYOUT AND EXPERIMENTS

Figure 5.4: Steering angle of the front wheels on the left side, comparison between lateral acceleration responses of the test and the simulations on the right side

yaw rate response and the lateral acceleration response are not large enough to match with the measured data from the actual test. Apparently, the vehicle model with these parameters does not resemble the actual vehicle dynamics. As the vehicle’s reaction to steering is more severe than expected, it becomes clear that the vehicle is not as understeered as once thought. At first it is believed that the high friction asphalt of the runway of military airport ‘De Peel’, at which the test is performed, caused this mismatch. However, increasing both the front and rear cornering stiffness to compensate for the high friction between asphalt and tyre does not have an effect big enough to obtain a decent match between the simulation and the test. The cornering stiffness of the front wheels only is increased next in order to make the vehicle less understeered. This is done in a number of consecutive steps. The best match to the measured data from the double lane change is also plotted in figure 5.4 and figure 5.5. Amazingly, the final understeer coefficient of the vehicle model is very close to neutral steering ($\eta = 0.004$ rad) and for convenience ‘FWS neutral steered’ is used to address this setting in both figures. From now on this parameter setting will be used in the simulations.

A double lane change with the four wheel steered vehicle is performed next at a vehicle speed of 80 km/h. The yaw rate reference is calculated using a understeer coefficient of 0.0361 rad, which at that time was believed to be the understeer coefficient of the vehicle. So the reference yaw rate belongs to a much more understeered vehicle than the real vehicle. More understeer will mean a lower value of the reference yaw rate than one would expect in case of the front wheel steered vehicle. As a result it is thought that the rear wheels will steer in the same direction as the front wheels to reduce the yaw motion of the vehicle. The controller itself used control setting C, as listed in table 5.1.

The steering angles of both the front and rear wheels are shown in figure 5.6, together with the lateral acceleration response (i.e. 4WS test). The yaw rate response (i.e. 4WS test) and the yaw rate reference are shown in figure 5.7. Once again the measured steering angles of both the front and rear wheels have been used as steering input in a simulation which the four wheel steered vehicle in order to verify the 4WS vehicle model. The resulting yaw rate and lateral acceleration are also plotted in both figures as ‘4WS neutral steered’ and are very close to the yaw rate and lateral acceleration measured in the test. Another simulation has been performed using the front
5.2. DOUBLE LANE CHANGE

Wheel steered vehicle model with the steering angle of the front wheels only as steering input. These results are also included in both figures and addressed by 'FWS neutral steered'. As can be seen in figure 5.7, there is quite a difference between the yaw rate reference and the yaw rate of the simulation using the FWS vehicle model. The main reason is the difference in understeer coefficient between the reference model ($\eta = 0.0361$ rad) and the FWS vehicle model ($\eta = 0.004$ rad). Ideally, the rear wheels' steering angle should eliminate the difference in yaw rate. However, when comparing the yaw rate of the test (i.e. 4WS test) with the reference yaw rate, it becomes clear that the 4WS vehicle is not capable of tracking the reference value. It should be noted that the yaw rate of the 4WS vehicle in the test is much closer to the reference value than the yaw rate of the FWS vehicle model in the simulation. This is the result of steering the rear wheels roughly in the same direction as the front wheels, which is visible in the left part of figure 5.6. This behaviour is as expected. However the level of similarity between the 4WS vehicle in the test and the reference is poor. The feedback loop, which is responsible for steering the rear wheels, seems to be not fast enough.

As a result of this slow responsiveness, the yaw rate of the 4WS vehicle will overshoot the reference yaw rate time after time. This will induce even steeper fluctuations in the yaw rate and lateral acceleration than the ones which appear in case of the FWS vehicle, see figures 5.7 and 5.6. Generally, the driver will steer the front wheels based upon the actual lateral acceleration and yaw rate. If steep fluctuations in those two would trigger a driver steering response, then the entire system of driver, rear wheel steering and vehicle could become unstable. The system of driver, rear wheel steering and vehicle is shown schematically in figure 5.8. As can be seen there are two feedback loops: the inner loop, which is the rear wheel steering loop, and the outer loop, which is closed by the driver. The events leading to an unstable system can be explained as follows: An initial severe steering action by the driver is applied to the front wheels. This steering input and the vehicle speed are the two inputs to the reference model, which generates a reference yaw rate. The controller, which steers the rear wheels, should ideally eliminate the difference between the actual yaw rate and the reference yaw rate. However, this is not the case as the controller (i.e. the
Figure 5.6: steering angles of the front and rear wheels on the left side, comparison between lateral acceleration responses of the test and the simulations on the right side.

Figure 5.7: comparison between yaw rate responses of the test and the simulations.
5.2. **DOUBLE LANE CHANGE**

inner loop) lacks responsiveness, thereby inducing fluctuations in the yaw rate around the reference value. If the magnitude of these fluctuations becomes large enough, then the driver (i.e. the outer loop) will respond to reduce those fluctuations. This reaction will once again change the reference yaw rate and so it will induce new fluctuations. Now depending on the magnitude of the driver reaction, those fluctuations can either be smaller or larger and so the entire system of driver, rear wheel steering and vehicle can become unstable. This did happen once in another double lane change test. For an adequate rear wheel steering system, the inner feedback loop has to be at least faster than the outer feedback loop, which is closed by the driver.

Besides the slow responsiveness, the rear wheel steering did also show a positive feature. As the reference model's setting was chosen such that it described a much more understeered vehicle than the actual FWS vehicle, the yaw motion of the 4WS vehicle was reduced. This made the vehicle's reaction to steering less severe compared to the FWS vehicle and it also reduced body roll significantly.

In the previous simulation the steering angles of both the front and rear wheels, measured during the double lane change, were used as steering input. In the next simulation only the steering angle of front wheels will be used as input and the rear wheels' steering angle will be calculated depending on the chosen controller settings. The test has been conducted with setting C, listed in table 5.1, and a reference model using an understeer coefficient of 0.0361 rad. These settings are also used in this simulation. The yaw rate response is shown in figure 5.9 and the steering angles in figure 5.10. When comparing the steering angle of the rear wheels, calculated in the simulation and addressed by 'rear wheels 4WS controller setting C', with the steering angles of the rear wheels, measured in the test, it becomes clear that the difference between both is small. The difference in yaw rate between the 4WS vehicle in this test and the 4WS vehicle in this simulation is also small. They both show fluctuations around the reference yaw rate. From this it can be concluded that the yaw rate response to steering and the hydraulic servo system are modelled quite accurately.

Now a similar simulation will be performed with the same measured steering angle of the front wheels as steering input. However, in this case the initial controller setting A, listed in table 5.1 will be used instead of controller setting C. The yaw rate response and the steering angle of the rear wheels of this simulation have also been included in respectively figure 5.9 and figure 5.10. As can be seen, the yaw rate response as addressed by '4WS neutral steered controller setting A' is very close to the reference yaw rate. This response does no longer show fluctuations around the

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**Figure 5.8:** The entire system of driver (i.e. outer loop), rear wheel steering (i.e. inner loop) and vehicle.
CHAPTER 5. VEHICLE LAYOUT AND EXPERIMENTS

Figure 5.9: Yaw rate response in the test and in simulations using 2 controller settings

Figure 5.10: Steering angles of front and rear wheels in the test and of the rear wheels in simulations using 2 controller settings
reference yaw rate as was the case using controller setting C. When looking at the steering angle of the rear wheels, it follows that the rear wheels react more quickly using setting A than with setting C. The steering angle of the rear wheels is almost in phase with the steering angle of the front wheels, whereas with controller setting C the steering angle of the rear wheels displays a certain delay with respect to the steering angle of the front wheels. This explains why the yaw rate response of controller setting A is very close to the reference yaw rate. The reason why the rear wheels react more quickly using controller setting A, lies in the fact that the bandwidth of this setting is 3 times larger than the bandwidth of controller setting C (4.8 Hz versus 1.6 Hz, see Table 5.1). Initially it was decided to use setting A while testing, however, low-frequent sensor noise made the change to setting C necessary. The example demonstrated above shows that this change directly limits the control performance of the rear wheel steering feedback system. Finally, it can be concluded that the active rear wheel steering system, as implemented in the experiments, is inadequate. Because of the low bandwidth of the feedback loop, the reference yaw rate can not be closely followed. As a result fluctuations in the yaw rate around the reference value are induced, which are experienced negatively by a driver.
Chapter 6

Conclusions and recommendations

6.1 Conclusions

The 4WS control strategy, introduced in this thesis, contains a yaw rate reference model, which calculates a desirable yaw rate depending on the driver’s steering angle and the vehicle speed. The reference yaw rate is chosen to be the solution of a first order differential equation with the steering angle of the front wheels as the input.

A linear controller, whose task it is to minimise the difference between the reference yaw rate and the actual yaw rate, is designed through loopshaping. For this purpose the extended 3DOF vehicle model including actuator dynamics has been used. Simulations have been carried out with the nonlinear multi-body vehicle model to investigate the control performance of the controller. It is concluded that the active rear wheel steering system introduces additional yaw damping and that it shows features of an ESP system.

Finally, experiments have been carried out at the military airport 'De Peel'. The main conclusion that can be drawn on these experiments, is that the active rear wheel steering system, as implemented in the test vehicle, is not fast enough. As a result the yaw rate of the 4WS vehicle does not track the reference yaw rate close enough. Fluctuations in the yaw rate occur around the reference value and this behaviour does certainly not improve the handling quality of the 4WS vehicle compared to the FWS vehicle.

The rear wheel steering system consists of the inner feedback loop and the driver closes the entire system by the outer feedback loop. Generally, it can be said that, in cases where an inner and an outer feedback loop are present, the influence of the inner loop on the output of the process is determined by its bandwidth relative to the bandwidth of the outer loop. The bandwidth of the yaw rate feedback, which determines the level of responsiveness of the active rear wheel steering system, is in this case simply too low relative to the bandwidth of the human operator. The reason for the relatively low bandwidth of the yaw rate feedback loop is twofold:

1. Noise on the yaw rate sensor, which makes it necessary to use different controller settings in the experiments than the initial controller settings determined in the loopshaping process. As a result the open-loop gain decreases and so does the bandwidth.

2. The dynamics of the rear wheel steering actuator introduces additional phase lag which directly limits the control potential.
6.2 Recommendations

A new yaw sensor has been used to measure the yaw rate. This sensor was chosen because it measures both yaw rate and lateral acceleration. Normally it is used in ESP applications and ESP typically interferes in a discrete way when the output of the sensor is large. In that case the impact of some low frequent sensor noise is relatively low, as the noise to signal ratio is low. However, for this application, in which the yaw rate output is fed back continuously, it is not suitable. During straight line travel the noise to signal ratio is infinite and noise is directly fed back, which results in vibrating rear wheels. The fact that the sensor noise is very low frequent (3 to 6 Hz) makes the impact even worse as it falls in the frequency range from 0 to about 10 Hz, which is particularly interesting in vehicle dynamic terms. For this application a more accurate yaw rate sensor is recommended with a low noise to signal ratio. Any noise present should be high frequent, i.e. far away from the interesting frequency range for vehicle dynamics.

The rear wheel steering actuator consists of a hydraulic servo system. Hydraulic systems typically have a low bandwidth. TNO had optimised the hydraulic system in order to decrease the response time and so increase the bandwidth. The dynamic characteristics, however, still limit the control potential. After the dynamics of the actuator had been identified by means of a sine sweep input test, they were approximated by a third order function in the Laplace domain. As this transfer function is known, it is theoretically possible to drive the hydraulic system through the transfer function of the inverse dynamics. As a result the transfer function from input to output theoretically becomes 1 and so an ideal steering actuator would arise. So far this option has not been implemented and, if implemented, it should be very well tested in discrete time.

Concerning the test vehicle itself the following can be said: Although the vehicle is over 15 year old, it can still be used to investigate various 4WS control strategies quite well. During testing the electronic circuit board for measuring the vehicle speed broke down. This problem has been fixed in a provisional way. However, all other electronics still performed as they were designed to do. It is recommended to acquire a new set of accurate sensors including signal conditioning electronics for measuring vehicle dynamic quantities, which in turn can be used for control purposes. Finally, the importance of the availability of a test vehicle is highlighted. Within computational programmes the most complex controllers can be built, however, practical implementation usually yields lots of difficulties.
Bibliography


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[17] Gao, X., Mcvey, B., Tokar, L., "Robust controller design of four wheel steering systems using $\mu$ synthesis techniques", Proceedings of the 34th Conference on Decision and Control, p875-882


Appendix A

Controller design without actuator dynamics

\[ K_1 = -1 \]
\[ K_2 = \frac{1 + \frac{1}{30} \pi s}{1 + \frac{1}{2 \pi} s} \]
\[ K_3 = (1 + \frac{1}{10 \cdot 2 \pi s}) \]
\[ K_4 = \frac{(60 \cdot 2 \pi)^2}{s^2 + 2 \cdot 0.5 \cdot 60 \cdot 2 \pi s + (60 \cdot 2 \pi)^2} \]
\[ K_5 = -0.17 \]
Figure A.1: Bode diagram of the open-loop $KH$ for $K = K_1$

Figure A.2: Nyquist diagram of the open-loop $KH$ for $K = K_1$
Figure A.3: Bode diagram of the open-loop $KH$ for $K = K_1K_2$

Figure A.4: Nyquist diagram of the open-loop $KH$ for $K = K_1K_2$
APPENDIX A. CONTROLLER DESIGN WITHOUT ACTUATOR DYNAMICS

Figure A.5: Bode diagram of the open-loop $KH$ for $K = K_1K_2K_3$

Figure A.6: Nyquist diagram of the open-loop $KH$ for $K = K_1K_2K_3$
Figure A.7: Zoomed in Nyquist diagram of the open-loop $KH$ for $K = K_1K_2K_3$

Figure A.8: Bode diagram of the open-loop $KH$ for $K = K_2K_3K_5$
Figure A.9: Zoomed in Nyquist diagram of the open-loop $KH$ for $K = K_2K_3K_5$

Figure A.10: Bode diagram of the open-loop $KH$ for $K = K_2K_3K_4K_5$
Figure A.11: Zoomed in Nyquist diagram of the open-loop $KH$ for $K = K_2K_3K_4K_5$

Figure A.12: Bode diagram of the complementary sensitivity $T$ for $K = K_2K_3K_4K_5$
Appendix B

Simulation results without actuator dynamics
Figure B.1: Lateral acceleration response of both controllers to the steering angle of the front wheels, displayed in figure 4.9

Figure B.2: Sideslip response of both controllers to the steering angle of the front wheels, displayed in figure 4.9
Appendix C

Controller design with actuator dynamics

\[ K_1 = -1 \]
\[ K_2 = \frac{1+\frac{1}{\pi^2}}{1+\frac{1}{30^2}} \]
\[ K_3 = (1 + \frac{1}{10^2\pi^2}) \]
\[ K_4 = \frac{(60 \cdot 2\pi)^2}{s^2 + 2 \cdot 0.5 \cdot 60 \cdot 2\pi \cdot s + (60 \cdot 2\pi)^2} \]
\[ K_5 = -0.17 \]
\[ K_6 = -0.105 \]
\[ K_7 = (1 + \frac{1}{8^2\pi^2}) \]
\[ K_8 = -0.13 \]
\[ K_9 = (1 + \frac{1}{3^2\pi^2}) \]
APPENDIX C. CONTROLLER DESIGN WITH ACTUATOR DYNAMICS

Figure C.1: Zoomed in Nyquist diagram of the open-loop \( KH \) for \( K = K_2K_3K_5 \)

Figure C.2: Zoomed in Nyquist diagram of the open-loop \( KH \) for \( K = K_2K_6K_7 \)
Figure C.3: Bode diagram of the extended 3 DOF model with actuator dynamics at 4 speeds

Figure C.4: Zoomed in Nyquist diagram of the open-loop $KH$ with $K = K_2 K_8 K_9$, i.e. the ‘general’ controller, at 4 speeds
Appendix D

Simulation results with actuator dynamics
APPENDIX D. SIMULATION RESULTS WITH ACTUATOR DYNAMICS

Figure D.1: Steering angles of the front and rear wheels

Figure D.2: Lateral acceleration response to the steering angle of the front wheels, displayed in figure D.1
Figure D.3: Sideslip response to the steering angle of the front wheels, displayed in figure D.1