Noise and vibration reduction for a mass-spring-damper system

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Abstract

In this work four control strategies are implemented in a mass-spring-damper system in order to suppress undesired vibrations. The first is a passive control strategy while the other three are active control strategies. Two of them are classical, namely the Linear Quadratic Regulator (LQR) and PI-control. Generalized PI is the fourth strategy and can be qualified as a modern control strategy. The performances of the controllers are compared both experimentally and in simulations.
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Chapter 1

Introduction

Vibration control is a valuable tool in attenuating undesired vibrations in mechanical systems. Vibration control can be categorized in three main concepts: passive, semi-active and active vibration control.

Passive control requires no control input. When the parameters and the dynamics of a system are known and when a perturbation exists of a known frequency then passive control can be a very useful tool. The main idea is to change the frequency response of the system by adding stiffness and damping to the system. When damping and stiffness are added depending on the operating conditions one can speak about semi-active control. Physically it is difficult to add stiffness to a system, so in practice springs with adjustable spring characteristics are used.

In the past many active control strategies are developed. Active control can be divided in two groups, namely classical and modern control. A Linear Quadratic Regulator is a classical active control strategy. The working principle of the Linear Quadratic Regulator is to change the pole locations of the system by using state feedback. The difference with ordinary state feedback control is that the gains of the controller are determined by minimizing a cost function. This cost function depends on the control effort and the performance of the controller. In many mechanical systems it is not possible to determine all the states of the system that are needed in the LQR control. PI control only needs an output and the same output that is integrated. The gains for the controller can not be determined, but an indication of the magnitude of the gains can be obtained by the method of Ziegler-Nichols.

An example of a modern strategy that has recently been developed is generalized PI control. When a control system is differential flat it is possible to reconstruct the states of the system by integrals of the output and the input and also guarantees controllability. This property simplifies the analysis and design of the Generalized PI controller that is able to achieve asymptotic output tracking.

A mass-spring-damper system that consists of mass carriages that are connected with springs is used as a carrier to compare the control strategies that are mentioned before. The comparison is done regarding robustness and performance both experimentally and in simulations.
CHAPTER 1. INTRODUCTION

This report is organised as follows. In the first chapter passive control is explained in theory and implemented in simulations. The vibration absorber that is used for the passive control is maintained for the other controllers which makes the system underactuated. In chapters 3, 4 and 5 are respectively the classical LQR and PI controller and the modern Generalised PI controller explained and simulated. In chapter 6 are some experiments made. The report is closed with a conclusion and some recommendations.
Chapter 2

Passive control

The vibration absorber that is discussed in this chapter is a useful tool to control vibrations with a single known frequency without any control effort. First the system without an absorber is discussed before the system with the vibration absorber is discussed.

2.1 The system without absorber

The dynamical behavior of the system without absorber is analyzed by a frequency response (FRF) plot and a stability analysis. In order to obtain the plot it is necessary to derive a mathematical model of the system. The system consists of a mass that is coupled to the world with a spring. The mass is a mass carriage with a suspension consisting of anti-friction ball bearings (see [1]). The friction can be modeled by a linear damper. The model can be found in figure 2.1.

Newton's law is used to obtain the model of:

\[ m_2 \ddot{x}_2(t) + c_2 \dot{x}_2(t) + k_2 x_2(t) = F(t) \]  

(2.1)

Where \( m_2 \) is the mass of the carriage, \( c_2 \) is the damping constant for the linear damper,

![Figure 2.1: The system without vibration absorber](image)
\( k_2 \) is the linear spring coefficient and \( x_2 \) the position of the mass with \( \dot{x}_2 \) and \( \ddot{x}_2 \) its velocity and acceleration. \( F(t) \) represents the perturbation. In order to obtain a FRF a Laplace transformation is performed on (2.1). This results in the following transfer function:

\[
G(s) = \frac{X_2(s)}{F(s)} = \frac{1}{(m_2s^2 + c_2s + k_2)}
\]  

(2.2)

The parameters are taken from the experimental setup as:

\[
m_2 = 2.839 \text{ kg}, \ k_2 = 340 \text{ N/m}, \ c_2 = 0.01 \text{ Ns/m}
\]  

(2.3)

Substituting the frequency \( s = j\omega \) and taking \( \|G(j\omega)\| \) an expression is found for the magnitude of the frequency response. A FRF plot can be obtained using Matlab, see figure 2.2.

The resonance frequency of the system is at \( \omega_r = \sqrt{\frac{k_2}{m_2}} = 10.944 \text{ rad/sec} \). A transformation of (2.1) to state space representation is carried out to obtain the eigenvalues of the matrix \( A \). The eigenvalues represent the location of the poles of the system, if all the poles are located in the left half plane the system is asymptotically stable. The state space representation of the system:

\[
\dot{z}(t) = \begin{bmatrix}
0 & 1 \\
-\frac{k_2}{m_2} & -\frac{c_2}{m_2}
\end{bmatrix} z(t) + \begin{bmatrix}
0 \\
\frac{1}{m_2}
\end{bmatrix} F(t), z(t) = \begin{bmatrix}
x_2(t) \\
\dot{x}_2(t)
\end{bmatrix}
\]  

(2.4)

Computing the eigenvalues of system matrix \( A = \begin{bmatrix}
0 & 1 \\
-\frac{k_2}{m_2} & -\frac{c_2}{m_2}
\end{bmatrix} \) yields the following...
2.2. PASSIVE CONTROL WITH VIBRATION ABSORBER

Figure 2.3: The system with absorber

pole locations:

\[ p_{1,2} = -1.7612 \times 10^{-3} \pm 10.943i \]

The real parts of the eigenvalues are small but negative which means that the system is asymptotically stable. The imaginary part can be recognized as the resonance frequency of the system.

2.2 Passive control with vibration absorber

The system with vibration absorber consists of a primary mass \( m_2 \) and an absorber \( m_1 \) (see figure 2.3). \( U(t) = 0 \) because the control is passive. The main idea of passive control is that the response on vibrations for the primary system is minimized when the excitation frequency coincides with the uncoupled natural undamped eigenfrequency of the absorber.

In order to obtain a FRF plot it is necessary to describe the system mathematically. Newton's law yields:

\[
\begin{align*}
  m_1 \ddot{x}_1(t) + c_1 \dot{x}_1(t) + k_1 x_1(t) - k_1 x_2(t) &= U(t) \\
  m_2 \ddot{x}_2(t) + c_2 \dot{x}_2(t) + k_2 x_2(t) + k_1 (x_2(t) - x_1(t)) &= F(t)
\end{align*}
\] (2.5)

By taking the Laplace transform of (2.5), with \( U = 0 \), the transfer function \( G(s) \), that gives a relationship between the amplitude of movement of the primary system and the excitation, is given by:

\[
G(s) = \frac{X_2(s)}{F(s)} = \frac{m_1 s^2 + c_1 s + k_1}{(m_1 s^2 + c_1 s + k_1)(m_2 s^2 + c_2 s + k_2) - k_1^2}
\] (2.6)

Substituting \( s = i \omega \) and taking \( \|G(i \omega)\| \) gives an expression for the magnitude of the FRF of the primary system. The parameters are taken from the experimental setup as:

\[
m_1 = 1.586 \text{ kg}, \ k_1 = 745 \text{ N/m}, \ c_1 = 0.01 \text{ Ns/m}
\] (2.7)
The other parameters are the same as in the system without absorber. The FRF of the passive control can be compared with the FRF of the system without absorber in figure 2.4.

![Figure 2.4: The FRF of system (2.7) (solid) and of system (2.1) (dashed)](image)

For the system with absorber two resonance frequencies can be recognized namely close to the natural eigenfrequencies of the system because the damping is very small (see [2]). From the figure it can be seen that the eigenfrequencies are near 9 and 28 Hertz and the anti-frequency is near 22 Hertz. The anti-resonance frequency should be near the uncoupled natural eigenfrequency of the absorber, \( \omega_{n1} = \sqrt{\frac{k_1}{m_1}} = 21.673 \) rad/sec. The eigenfrequencies can be determined by the pole locations. Equation (2.5) can be transformed to a set of first order differential equations by introducing the state vector \( z(t) = [x_1 \ x_1 \ x_2 \ x_2]^T \).

\[
\dot{z}(t) = \begin{bmatrix}
0 & 1 & 0 & 0 \\
\frac{k_1}{m_1} & -\frac{c_1}{m_1} & \frac{k_1}{m_1} & 0 \\
0 & 0 & 0 & 1 \\
\frac{k_3}{m_3} & 0 & -\frac{k_1 + k_3}{m_3} & -\frac{c_2}{m_2}
\end{bmatrix} z(t) + \begin{bmatrix}
\frac{1}{m_1} \\
0 \\
0 \\
\frac{1}{m_2}
\end{bmatrix} U(t) + \begin{bmatrix}
0 \\
0 \\
0 \\
\frac{1}{m_2}
\end{bmatrix} F(t)
\]

(2.8)

\[
\dot{z}(t) = Az(t) + BU(t) + EF(t)
\]

(2.9)

The eigenvalues of matrix A give the following pole locations:

\[
p_{1,2} = -2.543 \times 10^{-3} \pm 27.924i
\]

\[
p_{3,4} = -2.3708 \times 10^{-3} \pm 8.4938i
\]
The eigenfrequencies of the system are indeed near 9 and 28 Hertz according to the pole locations, namely 27.934 and 8.4938 Hertz. The location of the poles indicate that the system is asymptotically stable.

The advantage of the passive control is clear: it is possible to tune the absorber in a way that the gain at the excitation frequency is very low. This is achieved with no control effort. Figure 2.4 shows two resonance peaks. This implies that in the presence of some noise in the excitation the primary system will excite with higher amplitudes.

### 2.3 Simulations for passive control

In order to show the conclusions of the previous section some simulations are carried out with Matlab's Simulink. In passive control are initial conditions very important to tune the absorber. The simulations are carried out with the initial conditions that are calculated below.

To obtain good results in simulation the initial conditions are very important. However in experiments this is not the case because friction is not taken into account in the model. For the system of (2.5) the next solution (and its derivatives) is proposed:

\[
\begin{align*}
    x_1(t) &= A \cos(\omega t), \\
    \dot{x}_1(t) &= -A \omega \sin(\omega t), \\
    \ddot{x}_1(t) &= -A \omega^2 \cos(\omega t) \\
    x_2(t) &= B \cos(\omega t), \\
    \dot{x}_2(t) &= -B \omega \sin(\omega t), \\
    \ddot{x}_2(t) &= -B \omega^2 \cos(\omega t)
\end{align*}
\]

Substituting this and \( \omega = \sqrt{\frac{k_1}{m_1}} \) (because this is the excitation frequency that the absorber is tuned to) in (2.5) at \( t = 0 \) (where \( \sin(\omega t) = 0, \cos(\omega t) = 1 \) and \( F(t) = F_0 \cos(\omega t) = F_0 \)) yields:

\[
\begin{align*}
    -k_1 A + k_1 A - k_1 B &= 0 \\
    -m_2 B k_1 \frac{k_1}{m_1} - k_1 A + k_1 B + k_2 B &= F_0
\end{align*}
\]

The first equation gives \( B = 0 \), which is desirable because the amplitude of the movement of the primary system should be 0. Substituting \( B = 0 \) in the second equation yields:

\[
A = -\frac{F_0}{k_1}
\]

The initial conditions used in the simulations are \( x_1(0) = A, \dot{x}_1(0) = x_2(0) = \ddot{x}_2(0) = 0 \). The Simulink schemes can be found in appendix B. The excitation is \( F = 3 \cos(21.673 t) \).

Simulations without the addition of noise are carried out on both the system without (2.1) and with (2.3) absorber. From these simulations it can be concluded that the absorber is a very useful tool since the amplitude of the vibration for the system without absorber is \( 5 \cdot 10^{-3} \) m and for the system with absorber \( 3 \cdot 10^{-6} \) m. The amplitude of the vibration for the system with absorber when white gaussian noise with mean zero and variance 0.02 is added is \( 1 \cdot 10^{-3} \) m. This indicates that the passive control is not robust to noise but still shows better results than the system without absorber. In figure 2.5 the result of a simulation for the passive controller without the addition of noise can be found.
Figure 2.5: Response of primary system, see figure (2.3), without the addition of noise.
Chapter 3

Linear Quadratic Regulator

The first control strategy used to control the primary system by actuating the absorber is the Linear Quadratic Regulator (LQR). LQR is an optimized state feedback control strategy. First state feedback control is explained using pole placement. Then LQR is used to find suitable pole locations and the corresponding gains.

3.1 Pole assignment

In order to place the poles of the system in arbitrary positions the system should be observable and controllable. The system to be controlled is represented by figure 2.3 and the mathematical model of (2.8). It has already been proven that the system is stable. To prove observability the observability matrix \([ C \ C A \ C A^2 \ C A^3 ]^T\) should be of full rank (4). To prove controllability the controllability matrix: \([ B \ AB \ A^2B \ A^3B ]\) should be of full rank. Here the system is given by (2.8) with the output:

\[
y(t) = Cz(t) + DU(t)
\]

\[
y(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 262.42 & 0 & -382.18 & -3.5224 \times 10^{-3} \\ -0.9243 & 262.42 & 1.3462 & -382.18 \end{bmatrix} z(t)
\]

Resulting in the observability matrix and its rank:

\[
\text{obs} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 262.42 & 0 & -382.18 & -3.5224 \times 10^{-3} \\ -0.9243 & 262.42 & 1.3462 & -382.18 \end{bmatrix}, \text{rank} = 4
\] (3.2)

And the controllability matrix and its rank:

\[
\text{cont} = \begin{bmatrix} 0 & 0.63052 & -3.9755 \times 10^{-3} & -296.18 \\ 0.63052 & -3.9755 \times 10^{-3} & -296.18 & 3.7349 \\ 0 & 0 & 0 & 165.46 \\ 0 & 0 & 165.46 & -1.626 \end{bmatrix}, \text{rank} = 4
\] (3.3)
Thus it can be concluded that the system is stable, observable and controllable. This implies that the poles can be placed at arbitrary positions using state feedback control. The control law used for state feedback control is:

\[ U(t) = -Kz(t) \]  

(3.4)

The result of substituting \( K = \begin{bmatrix} K_1 & K_2 & K_3 & K_4 \end{bmatrix} \) in (2.8) yields:

\[
\begin{bmatrix}
0 & 1 & 0 & 0 \\
-k_1K_1/m_1 & -k_2K_2/m_1 & k_3K_3/m_1 & K_4/m_1 \\
0 & 0 & 0 & 1 \\
-k_1K_1/m_2 & 0 & -k_2K_2/m_2 & -k_3K_3/m_2 \\
\end{bmatrix} z(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/m_2 \end{bmatrix} \]

(3.5)

\[ y(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} z(t) \]

The characteristic polynomial can be derived from system matrix A:

\[
p^4 + 9.8276 \times 10^{-3}p^3 + 851.92p^2 + 4.0643p - 0.63052K_2p^3 \\
-2.2209 \times 10^{-3}K_2p^2 - 240.97K_2p + 56256 - 0.63052K_4p^2 \\
-2.2209 \times 10^{-3}K_1p - 240.97K_1 + 165.46K_3 + 165.46K_4 \]

The gains of the system can be found by substituting the desired pole locations in the characteristic polynomial.

### 3.2 Linear Quadratic Regulator

The LQR determines the gains by minimizing a cost function. First the minimization procedure will be highlighted. Then the LQR is applied to the system, a FRF plot is made and discussed and simulations are carried out.

The cost function that is to be minimized is a quadratic time domain performance index:

\[ J = \int_0^\infty (z^TQz + U^TRU) dt \]  

(3.6)

It is proven (see [6]) that (3.6) is minimized by choosing the feedback gain \( K \) to be:

\[ K = R^{-1}B^TS \]  

(3.7)

Where \( S = S^T > 0 \) is the solution of the Riccati equation:

\[ 0 = A^TS + SA - SBR^{-1}B^TS + Q \]  

(3.8)

\( A \) and \( B \) are the system matrices as determined in (3.5) and \( Q \) and \( R \) are the user defined weighting matrices of the cost function. The performance of the controller will depend on the values for the weighting matrices. The \( Q \) matrix puts more emphasis on
3.2. LINEAR QUADRATIC REGULATOR

Figure 3.1: Bode plots for system with absorber (dash-dot), without absorber (solid) and LQR (dashed)

the performance of the controller and the $R$ matrix puts more emphasis on the control effort. The objective is to control the position of the primary system without a big control effort. It appeared that the following weighting matrices suffice this demand:

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, R = 0.001$$

To determine the gain the Matlab command lqr.m is used. This results in the feedback gain:

$$K = \begin{bmatrix} 89.5126 & 16.8403 & -6.0366 & 15.4141 \end{bmatrix}$$

The resulting pole locations $p = \begin{bmatrix} -1.2896 \pm 28.0851i \\ -4.0243 \pm 9.0000i \end{bmatrix}$. This implies that the eigenfrequencies will be $\omega_{n1} = 9 \text{ rad/sec.}$ and $\omega_{n2} = 28.0851 \text{ rad/sec.}$ A frequency response plot or Bode plot can easily be obtained by using the bode.m command in Matlab, see figure 3.1. From figure 3.1 can be concluded that the control is more robust to noise because the resonance peaks are lower but the feature of the vibration absorber in the passive control in terms of the anti resonance is not preserved. Simulations are made to justify this conclusion.
3.3 Simulations for LQR

It is necessary that the primary system can be controlled to a reference position. For this case an additional control effort has to be made. This additional control effort can be determined by considering the system in state space notation:

$$\ddot{z}(t) = Az(t) + BU(t)$$

The control force will be defined as:

$$U(t) = -Kz(t) + Gy_r$$

Where $y_r$ is the reference point and $G$ the additional gain to bring the system in the desired equilibrium position. Plugging the control force into the system yields:

$$\ddot{z}(t) = (A - K) z(t) + BGy_r$$

In the equilibrium $\ddot{z}(t)$ is zero. This yields the equilibrium:

$$\ddot{z} = -(A - BK)^{-1}BGy_r$$

The output at the equilibrium is defined as:

$$\bar{y} = C\ddot{z} = -C(A - BK)^{-1}BGy_r$$

So $\bar{y} = y_r$ yields for $G$:

$$G = -\frac{1}{C(A - BK)^{-1}B}$$  \hspace{1cm} (3.10)$$

It should be noted that this addition in the control effort will make the performance index (3.6) to go to infinity.

Now simulations can be made with a reference point. The used Simulink models can be found in appendix B. Simulations are carried out with and without noise. From the simulations it could be concluded that the amplitude of the vibration is almost the same in the noisy case and in the simulations without noise. This amplitude is about 1000 times higher than in the simulations for passive control. This justifies that the anti-resonance as well as the resonances disappeared. Another disadvantage of the LQR control is that the complete state has to be known. Fast asymptotic tracking is achieved with a small control input (about 6 Newton) with this controller. In figure 3.2 can the result of the simulation with the addition of noise be found. The noise and perturbation are the same as in the simulations for passive control.

It is necessary to find a control strategy that conserves the anti resonance and provides robustness to noise. In the next chapter another classical control strategy will be applied to the system, namely PI control. Using this control strategy only the position of the primary system has to be known to control the system.
Figure 3.2: Response of primary system with LQR control in presence of noise
Chapter 4

PI control

The second classical control theory to be implemented is a PI controller. First the controller is proposed. The magnitude of the gains follow from a method using root-locus and Ziegler-Nichols (see [4]). A frequency response plot is made and finally simulations are made.

The standard form of the PI controller is:

\[ U(t) = K(e_p(t) + \frac{1}{T_i} \int_0^t e_p(t) dt) \] (4.1)

Where \( e_p(t) = y_r(t) - y(t) \) and \( K \) and \( T_i \) are the gains. To obtain an idea of the magnitude of the gains the Ziegler-Nichols method is used.

4.1 Determining the gains of the PI control

To use the Ziegler-Nichols method it is necessary to determine the critical times and the critical gain. The critical times can be derived from the resonance frequencies of the system:

\[ t_{c1} = \frac{2\pi}{\omega_{n1}} = \frac{2\pi}{8.4937} = 0.73975, \quad t_{c2} = \frac{2\pi}{\omega_{n2}} = \frac{2\pi}{27.924} = 0.22501 \] (4.2)

The critical gain of the system is that gain (in \( U(t) = Ky \)) that makes the system instable. To determine this gain the rlocus.m program in Matlab is used. As an input again the system matrices of (2.8) are used as well as a prescribed vector of gains. The critical gains can be seen as the lower and upper bounds of the gains for which the system is stable:

\[ K_{c1} = -340, \quad K_{c2} = 755 \] (4.3)

The Ziegler-Nichols frequency response method (see [4]) gives the next indication of the gains of the system:

\[ 0.4K_{c1} = -136 < K < 302 = 0.4K_{c2} \]

\[ T_{i1} = 0.8t_{c1} = 0.5918 \quad \text{or} \quad T_{i2} = 0.8t_{c2} = 0.18001 \]
The system is very close to instability due to low damping. That makes it more difficult to find gains that keep the system stable. To find gains PIfindgain.m (see appendix A) in Matlab is used. The algorithm computes the pole locations for the closed loop system for different gains. The state space representation for the closed loop system can be obtained by adding a state to the system: \( \tilde{z}(t) = \begin{bmatrix} x_1(t) & \dot{x}_1(t) & x_2(t) & \dot{x}_2(t) & \int_0^t x_2(t) \end{bmatrix}^T. \)

The open loop system in state space representation is given in equation (2.8). With the additional state and \( U(t) = K(-x_2(t) + \frac{1}{T_1} \int_0^t (-x_2(t)) \) the closed loop state space representation is:

\[
\begin{align*}
\dot{z}(t) &= \begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
-k_1/m_1 & -c_1/m_1 & k_1-K/m_1 & 0 & -K/T_1 m_1 \\
0 & 0 & 1 & 0 \\
k_2/m_2 & 0 & -k_2+k_2/m_2 & -c_2/m_2 & 0 \\
0 & 0 & 1 & 0 & 0
\end{bmatrix} \begin{bmatrix}
x_1(t) \\
\dot{x}_1(t) \\
x_2(t) \\
\dot{x}_2(t) \\
\int_0^t x_2(t)
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
1/m_2
\end{bmatrix} F(t) \quad (4.4)
\end{align*}
\]

\[
y(t) = \begin{bmatrix}
0 & 0 & 1 & 0 & 0
\end{bmatrix} \begin{bmatrix}
x_1(t) \\
\dot{x}_1(t) \\
x_2(t) \\
\dot{x}_2(t) \\
\int_0^t x_2(t)
\end{bmatrix}
\]

The pole locations are determined by computing the eigenvalues of the matrix \( A \). This process is repeated for different gains. One example of gains that result in a stable closed loop system is: \( K = 0.2 \) and \( T_1 = 0.18001 \). This results in the following pole locations:

\[
\begin{align*}
p_{1,2} &= -0.0027 \pm 27.9235i \\
p_{3,4} &= -0.0006 \pm 8.4965i \\
p_5 &= -0.0033
\end{align*}
\]

The imaginary parts of the pole locations indicate resonance behavior at the natural eigenfrequencies \( \omega_{n1} = 8.4965 \text{ rad/sec.} \) and \( \omega_{n2} = 27.9235 \text{ rad/sec.} \). This can also be read from the frequency response plot (figure 4.1) that can easily be obtained using the system matrices of equation (4.4) in the Matlab command bode.m. The PI controller maintains the anti resonance but it is not robust to noise because of the resonance peaks.

### 4.2 Simulations for PI control

The Simulink scheme for the simulations can be found in appendix D. The perturbation is again \( F(t) = 3 \cos(21.673t) \). White Gaussian noise with mean zero and a variance of 0.02 is added to the perturbation. From the simulations without the addition of noise it can be concluded that the amplitude of the response for PI control is very low (see Figure 4.2) and can be compared with the response of the passive control. This implies that the property of the anti resonance of the absorber is maintained. However simulations also show that the control is not robust to noise and its response is very slow. The amplitude of the movement of the primary mass is in the order of 0.01 m.
4.2. SIMULATIONS FOR PI CONTROL

Figure 4.1: Bode plot for closed loop system in PI control (solid), passive control (dashed) and system without absorber (dash dot)

Figure 4.2: Response of the primary system with reference point without noise for PI control
Chapter 5

Generalized PI control

After the classical control techniques LQR and PI control a modern control technique is to be implemented to the system to conserve the anti-resonance and to get a more robust controller. It is usual that not all states of a system are measurable. It is usual to estimate the states by either using an asymptotic observer or online calculations of time derivatives of the available output systems. The idea of the Generalized PI controller is to construct an integrated state feedback controller based on structural estimates of the states of the system. The errors that occur in that way can be compensated by integral actions. The flat output will first be derived because the controller is derived from this output. Also the reconstruction of the states is derived from the flat output. When all this is done the appropriate gains for the controller are derived using pole placement. The resulting bode plot will be derived and discussed. Finally simulations are carried out for the GPI controller.

5.1 Flat output

The system (2.8) has a flat output \( y = x_2 \). This means that the state variables and the input can be parameterized in terms of the output and a finite number of its time derivatives (see [12], [13] and [14]). From now on \( \alpha(t) \) will be written as \( \alpha \) to keep the expressions shorter. The parametrization is obtained from (2.8):

\[
\begin{align*}
x_1 &= -\frac{1}{k_1} (F - m_2 \ddot{y} - c_2 \dot{y} - k_2 y - k_1 y) \\
\dot{x}_1 &= -\frac{1}{k_1} \left( \dot{F} - m_2 \dddot{y} - c_2 \ddot{y} - k_2 \dot{y} - k_1 \dot{y} \right) \\
x_2 &= y \\
\dot{x}_2 &= \dot{y} \\
U &= \frac{1}{a_u} (-a_F F - a_{F1} \dot{F} - a_{F2} \ddot{F} + ay + a_1 \dot{y} + a_2 \ddot{y} + a_3 \dot{y}^{(3)} + y^{(4)})
\end{align*}
\]
Here the constants are given by:

\[
\begin{align*}
a_u &= \frac{k_1}{m_1m_2}, \quad a_F = \frac{k_1}{m_1m_2}, \quad a_{F1} = \frac{c_1}{m_1m_2}, \quad a_{F2} = \frac{1}{m_2}, \quad a = \frac{k_2k_1}{m_1m_2} \\
a_1 &= \frac{c_2k_1 + c_1k_2 + c_1k_1}{m_1m_2}, \quad a_2 = \frac{m_1k_2 + m_1k_1 + c_1c_2 + m_2k_1}{m_1m_2}, \quad a_3 = \frac{m_1c_2 + m_2c_1}{m_1m_2}
\end{align*}
\]  

(5.6)

5.2 Controller design

In the parametrization of the state variables and the input are derivatives of the output and inputs present. Also in the control law these derivatives are present. This is undesirable. A reconstruction of these derivatives is possible through integration of the last expression in (5.1). In order to keep the expressions shorter \( \int_0^t \beta dt \) will be written as \( \int_0^t \gamma dt \) as \( \int \int \gamma d\tau dt \) etc. This procedure yields a parametrization of the derivatives of the output in terms of the inputs, integrals of the inputs, the output and integrals of the output:

\[
\begin{align*}
y^{(3)} &= \Theta_3 + a_F \int F + a_{F1}F + a_{F2}F + y^{(3)}(0) \\
\dot{y} &= \Theta_2 + a_F \int \int F + a_{F1} \int F + a_{F2}F + y^{(3)}(0)t + \dot{y}(0) \\
\ddot{y} &= \Theta_1 + a_F \int \int \int F + a_{F1} \int \int F + a_{F2} \int F + 0.5y^{(3)}(0)t^2 + \ddot{y}(0)t + \dot{y}(0)
\end{align*}
\]  

(5.7)

With the structural estimates:

\[
\begin{align*}
\Theta_1 &= a_u \int \int \int U - a \int \int y - a_1 \int y - a_2 \int y - a_3y \\
\Theta_2 &= a_u \int \int U - a \int \int y - a_1 \int y - a_2y - a_3\Theta_1 \\
\Theta_3 &= a_u \int \int U - a \int y - a_1y - a_3\Theta_1 - a_3\Theta_2
\end{align*}
\]  

(5.8)

Here \( \Theta_1, \Theta_2 \) and \( \Theta_3 \) are structural estimates for \( \dot{y}, \ddot{y} \) and \( y^{(3)} \). The error made in these structural estimates are depending on the perturbation and the initial conditions. Now the estimates are known, a controller can be proposed that is independent of derivatives of the output.
5.2. CONTROLLER DESIGN

5.2.1 Proposed controller

The proposed controller is:

\[
U = \frac{1}{a_u} (a - K_4) y + \frac{1}{a_u} (a_1 - K_3) \Theta_1 + \frac{1}{a_u} (a_2 - K_2) \Theta_2 + \frac{1}{a_u} (a_3 - K_1) \Theta_3 \tag{5.9}
\]

The controller consists of three parts. The first part are the structural estimates \( (ay + a_1 \Theta_1 + a_2 \Theta_2 + a_3 \Theta_3) \) that are supposed to cancel the dynamical properties of part of the last expression of (5.1), that is \( (-ay - a_1 y - a_2 \dot{y} - a_3 \dot{y}^2) \). However in the structural estimates an error is made that is caused by the initial conditions and by the perturbation. The nature of the error caused by the initial conditions can be found in (5.7). These errors are of the type constant, linear \( (t) \) and parabolic \( (t^2) \). Applying a Laplace transformation to these errors gives for the constant: \( \frac{a_1}{s} \), for the ramp: \( \frac{a_2}{s^2} \) and for the parabolic function: \( \frac{a_3}{s^3} \). Here integrators can easily be recognized. In order to compensate these errors three integrators are added to (5.9). The third part of the control is an additional part of the dynamics that will cause the closed loop dynamics to be asymptotically stable, that is \( (-K_4 y - K_3 \Theta_1 - K_2 \Theta_2 - K_1 \Theta_3) \). When the integrators compensate the errors made in the structural estimates then the characteristic polynomial of the closed loop system is of the Hurwitz type which means that the poles are all located in the open left complex half plane.

5.2.2 Tuning of the controller

Choosing appropriate values for the gains \( K_1, K_2, K_3, K_4, K_5, K_6 \) and \( K_7 \), is done by pole placement. For this purpose the characteristic equation of the closed loop system has to be found. The characteristic equation is the denominator of the transfer function \( G(s) = \frac{y(s)}{F(s)} \). For this purpose a Laplace transformation is performed on (2.5) :

\[
(m_1 s^2 + c_1 s + k_1) X_1(s) - k_1 y(s) = U(s) \tag{5.10}
\]

\[
(m_2 s^2 + c_2 s + k_2 + k_1) y(s) - k_1 X_1(s) = F(s)
\]

To obtain the transfer function \( \frac{y(s)}{F(s)} \) (5.8) is substituted in (5.9) and this result is substituted in (5.10) after a Laplace transformation. The values for the masses, spring constants and damping constants are the same as in the previous chapters. The denom-
inat or, or characteristic equation, contains the following components:

\[
1.0s^7 + (4.6729 \times 10^{-8} + 1.0K_1)s^6 \\
(-4.5769 \times 10^{-7}K_1 + 1.4137 \times 10^{-2} + 1.0K_2)s^5 \\
(5.4573 \times 10^{-4} - 3.6305 \times 10^{-2}K_1 - 4.5769 \times 10^{-7}K_2 + 1.0K_3)s^4 \\
(5.1624 \times 10^{-2}K_2 + 1.6102 \times 10^{-7}K_3 + 2.457 + 1.0K_4 - 7.8532 \times 10^{-4}K_1)s^3 \\
(-35.781K_1 - 6.4656 \times 10^{-4}K_2 + 5.1659 \times 10^{-2}K_3 + 1.0K_5 + 0.72372)s^2 \\
(-2966.2 - 0.15102K_1 + 4.2753 \times 10^{-5}K_3 - 1.6262 \times 10^{-2}K_2 + 1.0K_6)s \\
-1943.3K_1 - 7.2514 \times 10^{-2}K_2 + 1.0K_7 + 63.297
\]

The desired dynamical behavior is:

\[
(s^2 + 2\xi\omega_n s + \omega_n^2)^3 (s + b)
\]

A common choice in engineering practice for the damping constant is \(\xi = 0.5\sqrt{2}\). The chosen value for both the natural eigenfrequency \(\omega_n\) and the additional pole \(b\) is 20. The resulting characteristic polynomial is:

\[
1.0s^7 + 104.85s^6 + 5297.1s^5 + 1.6251 \times 10^5s^4 + \\
3.2502 \times 10^6s^3 + 4.2376 \times 10^7s^2 + 3.3553 \times 10^8s + 1.28 \times 10^9
\]

The gains are then found as:

\[
K_1 = 104.85, \quad K_2 = 5297.1, \quad K_3 = 1.6251 \times 10^5, \quad K_4 = 3.2499 \times 10^6
\]

\[
K_5 = 4.2371 \times 10^7, \quad K_6 = 3.3553 \times 10^8, \quad K_7 = 1.2802 \times 10^9
\]

The nominator of the resulting transfer function \(G(s) = \frac{y(s)}{F(s)}\) is:

\[
0.35224s^5 + 36.932s^4 + 1731.1s^3 + 43122.s^2 + 7.3549 \times 10^5s + 1.2102 \times 10^7
\]

And the denominator:

\[
1.0s^7 + 104.85s^6 + 5296.9s^5 + 1.6251 \times 10^5s^4 + \\
3.2503 \times 10^6s^3 + 4.2378 \times 10^7s^2 + 3.3554 \times 10^8s + 1.2801 \times 10^9
\]

Due to numerical problems is the denominator of the resulting transfer function a little bit different from the desired characteristic polynomial. The biggest difference in the coefficients is of the order 0.02 percent. This has a big influence on the pole locations:

\[
s_1 = -20.002, \quad s_{2,3} = -15.006 \pm 13.631i
\]

\[
s_{4,5} = -14.113 \pm 15.153i, \quad s_{6,7} = -13.305 \pm 13.644i
\]

The desired pole locations are:

\[
s_1 = -20, \quad s_{2,3,4,5,6,7} = -14.142 \pm 14.142i
\]

The frequency response plot can be found in figure 5.1. The bode plot for the GPI control indicates that this control is able to meet the objectives of the project. It preserves the anti-resonance that is characteristic for the vibration absorber and is robust to noise. To justify these conclusions simulations are carried out for noisy and non-noisy cases with a reference trajectory.
5.3 Simulations for GPI control

Simulations are carried out with a perturbation of $F = 3 \cos(21.763t)$. In order to see whether the controller achieves fast asymptotic tracking is a reference signal created that the primary system should track. The structural estimates give estimates for the output and three time derivatives of the output. Therefore should the reference signal be smooth up until the fourth time derivative of the reference signal. Then the control effort is reduced. The reference is a zero signal up to 4.5 seconds. Then a smooth function is employed to bring the reference from zero to one cm. After this a reference of 1 cm is maintained.

The simulation starts at $t = -4.5$ seconds, which means that the reference trajectory starts at $t_0 = 0$ and ends at $t_f = 1$ second. The demands for the reference signal $r$ are:

\begin{align}
  r(t_0) &= r_0, \quad r(t_f) = r_f, \quad \dot{r}(t_0) = 0, \quad \dot{r}(t_f) = 0, \quad \ddot{r}(t_0) = 0 \\
  \dddot{r}(t_f) &= 0, \quad r^{(3)}(t_0) = 0, \quad r^{(3)}(t_f) = 0, \quad r^{(4)}(t_0) = 0, \quad r^{(4)}(t_f) = 0
\end{align}

(5.17)

Where $r_0 = 0$ and $r_f = 0.01$ m. With these demands for the function that describes the reference trajectory all reference states used by the controller are differentially continuous. Because there are ten demands a function with ten variables has to be proposed to solve the problem. The proposed function for the trajectory is:

$$ r = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 + b_6 t^6 + b_7 t^7 + b_8 t^8 + b_9 t^9 $$

(5.18)

Differentiating this signal four times yields a system of 10 equations because there are conditions at $t_0$ and $t_f$. Solving this system results in the following coefficients for the

![Bode plot for GPI control (solid), for no absorber (dash-dot) and for passive control (dashed)](image)
reference signal:

\[
\begin{align*}
 b_0 &= 0, \ b_1 = -9.1 \times 10^{-15}, \ b_2 = 0, \ b_3 = 2.37 \times 10^{-15}, \ b_4 = 0 \\
 b_5 &= 1.26, \ b_6 = -4.20, \ b_7 = 5.40, \ b_8 = -3.15, \ b_9 = 0.70
\end{align*}
\] (5.19)

Simulations are carried out for the noisy and the non-noisy case with different initial conditions. Gaussian white noise with zero mean and 0.02 variance is used. The reference signal is tracked by the primary system very well in both the noisy and non-noisy case. The amplitudes of the disturbance is in the noisy case in the order of 1 mm and in the case without noise $10^{-6}$ m. The results for the noisy case can be found in figure 5.2. In this figure represents the solid line the response and the dashed line the reference signal. The reconstruction of the states is evident for the GPI control. This reconstruction

![Figure 5.2: Response for the primary system for GPI control with noise](image)

has been carried out in a simulation with different initial conditions. Due to the initial conditions there was an offset in the reconstruction of the states.
Chapter 6

Experiments

Before the experiments for the described control strategies are carried out a short description of the experimental setup is given. After that experiments for the strategies are carried out.

6.1 The experimental platform

The experiments for the control strategies described before are carried out on the ECP\textsuperscript{TM} (Educational Control Products) rectilinear plant model 210a, see figure 6.1. This system is a three-degree-of-freedom mass-spring-damper system. The rectilinear plant is an educational tool that is designed to provide insight to control system principles through hands-on demonstration and experimentation. The system can be either used for introductory control courses or for advanced control.

The system consists of four main components: the electromechanical part, input output electronics, DSP based controller/data acquisition board and the system interface software, see figure 6.2.

The electromechanical part consists of three mass carriages interconnected by bidirectional springs. The mass carriage suspension is an anti-friction ball bearing type with approximately ±3 cm of available travel. The mass of the carriages can be changed by the user by adding or removing a number of brass weights (500 ± 5 grams each). The first carriage of the plant is connected to the brushless DC motor by a rigid driving bar. The position of the carriages is measured by incremental rotary shaft encoders. These optical sensors have a resolution of 4000 pulses per revolution. Because of the configuration of the sensor a 4\times resolution is generated what results in 16000 counts per revolution. That is equivalent to 2197.866 counts per cm. The configuration of the system can be changed by the user. This makes it possible to build a configuration for a two-degree-of-freedom plant that is similar to figure 2.3. The configuration for the experiments is the same as for the simulations. The parameters for the experiment differ from the parameters in the simulations, this implies that a good comparison with the simulations is not possible. In the experiment for active control the parameters for the
masses are:

\[ m_1 = 1.839 \text{ kg}, \ m_2 = 3.05 \text{ kg} \]

The friction is very small and can be assumed to be 0.01 N/s². The parameters for the spring constants are obtained through experiments. To determine the spring constant of the first spring, \( k_1 \), five times five experiments are carried out for forces ranging from 2 Newton until 10 Newton. In those experiments the elongation \( x \) of the springs is measured and the spring constant is obtained by the relation

\[ k = \frac{F}{x} \]

The average of the 25 experiments is taken and the result is \( k_1 = 745 \text{ N/m} \). The same experiment for forces ranging from 1 to 5 Newton is carried out for the second spring and the result is \( k_2 = 340 \text{ N/m} \).

### 6.2 Experiments for passive control

In order to do experiments for passive control the configuration is changed. The entire system is reversed because the brushless DC motor is used to apply the perturbation. First an experiment is carried out to find the anti-resonance frequency of the system. This frequency will be used in the experiment for the excitation frequency. In simulations noise is added however in experiments noise is present in the system. The characterization of the experimental noise is not done. This implies that the noise in the simulations is different from the noise that is experienced in the experiments.
6.2. EXPERIMENTS FOR PASSIVE CONTROL

6.2.1 Experiment to obtain the anti-resonance frequency

In the ECP\textsuperscript{TM} dyn executive a trajectory for the first mass can be specified that is called a sine sweep. A sine sweep excites the system in a prescribed frequency range. From the response of the sine sweep can be derived at which frequency anti-resonance occurs. For specifying the range of frequencies for the sine sweep it is good to have an indication where an anti-resonance frequency is located. This is at the uncoupled undamped natural eigenfrequency of the absorber, \( \omega_n = \sqrt{\frac{k_2}{m_2}} \). \( k_2 = 745 \text{ N/m} \) and \( m_2 = 1.586 \text{ kg} \) in this configuration. So \( \omega_n = 21.673 \text{ rad/sec} \). The sine sweep is to be specified in hertz: \( f_n = 21.673 = 3.4494 \text{ Hertz} \). The sine sweep is chosen to be linear ranging from the frequencies 2 Hz until 5 Hz. The response of the primary system on the sine sweep can be found in figure 6.3. The minimum is experienced at time 30 seconds. Since the sine sweep is linear and ranging from 2 hertz to 5 Hertz in 60 seconds, the excitation frequency at 30 seconds was \( \frac{\omega_n + \omega_2}{2} = 3.5 \text{ Hertz} \). This is very close to the calculated value of 3.4494 Hertz. Now the passive control experiments can be carried out.

6.2.2 Experiment for passive control

For the passive control experiments the perturbation is applied to the first mass. This implies that the configuration of the ECP\textsuperscript{TM} system is reversed see figure 6.4.

In the ECP\textsuperscript{TM} dyn system a sinusoidal force can be defined by the user. This force is applied to the primary system through the rigid coupling shaft between the motor and the primary system. The excitation will be defined as \( F = 3 \sin(3.5t) \). The experimental result can be found in figure 6.5. The amplitude of the steady state response is \( 3 \times 10^{-4} \text{ m} \). In simulations for the noisy case it was \( 6 \times 10^{-5} \) and for the simulations without
Experiments on the system without the absorber show an amplitude of $3 \times 10^{-3}$ m. So the passive control reduces the amplitude of the response with a factor 10.

### 6.3 Experiments with active control

For the active controllers the configuration of the plant is that of figure 2.3. In this configuration the control is applied by the brushless DC motor and the perturbation by the shaker that is mounted on the primary system see figure 6.6 and figure 6.8. The shaker consists of a DC motor that drives an eccentric mass. The shaker is controlled through a PID controller and a range of frequencies can be obtained for the shaker. Figure 6.7 represents the second mass carriage with the shaker mounted on top of it. A
disadvantage is that no display is available that indicates the frequency of the shaker. This means that the shaker has to be characterized. This is done by decoupling the second mass by removing the springs. By determining the acceleration of the mass carriage it is possible to determine the force that is acting on the carriage because the weight of the carriage is known. It is also possible to determine the frequency of the movement by taking the acceleration, but since the obtained acceleration is not a measured quantity, but a quantity that is obtained by a double numerical differentiation of the position it is better to determine the period time of the movement by taking the measurement of the position.

For each controller (LQR, PI and GPI control) two experiments are carried out, one at the anti-resonance frequency (about 3.4 Hertz see section 2.2) and one at a resonance frequency (4.5 Hertz). This is chosen because in the theoretically obtained frequency response plots for LQR and PI (see figure 3.1 and figure 4.1) resonant peaks are present in the region between 4 and 5 Hertz. The shaker (see figure 6.8) has to be tuned manually to those frequencies and checked by the procedure described before.

### 6.3.1 Experiments with LQR control

The resonance frequency for the experiments is 3.4 Hertz. It is difficult to find the right frequency for the shaker since no display is available that indicates the frequency of the shaker. The characterization of the shaker, as described before, showed that the frequency of the shaker is $\frac{12}{3.5} = 3.4$ Hertz.

The perturbation force is derived from the acceleration of mass carriage 2. The amplitude of the force is determined to be 0.46 Newton. First experiments are carried out at the anti resonance frequency for all three controllers. After that the shaker
Figure 6.6: Experimental setup for active control with shaker

Figure 6.7: Second mass with shaker
is tuned to a resonance frequency and again experiments are carried out for all three controllers.

First the experimental results for LQR control are shown. In the ECP\textsuperscript{TM} software it is possible to program a controller and to implement it. The program for the LQR control can be found in Appendix F. In the ECP\textsuperscript{TM} software it is possible to prescribe a certain reference trajectory. A step reference trajectory is chosen for LQR control. The experimental results can be found in figure 6.9. The amplitude of the disturbance is about $3 \cdot 10^{-4}$ millimeter. This is a little higher than what was to be expected. Figure 3.1 shows that the gain at 3.4 Hertz is about $2 \cdot 10^{-3}$. This would imply a disturbance with $2 \cdot 10^{-3} \times 0.46 = 9.2 \cdot 10^{-4}$. This is three times as high as the actual amplitude. Because of the difference in parameters in the theoretical approach (the shaker is not accounted for) it is possible that the Bode plots that are used to compare experimental results with the theoretical results are not accurate. In experiments it has been shown that an anti resonance frequency is experienced at 3.5 Hertz. It is also possible that 4.5 Hertz is not exactly a resonance frequency. The shaker that is used in the experiments became available on the last day of my stay in Mexico. This is the reason that no more useful experiments are carried out.

The experiments are also carried out near to a resonance frequency. The chosen frequency is 4.5 Hz that corresponds to a force of 0.61 Newton. This experiment results in figure 6.10. From figure 3.1 can be read that the gain at 4.5 Hertz is $7 \cdot 10^{-3}$. This would have to result in an amplitude of $0.61 \times 7 \cdot 10^{-3} = 4.27 \cdot 10^{-3}$ while the amplitude of the disturbance is about $5 \cdot 10^{-4}$. This is a difference of factor 10.

It can be concluded that LQR-control is robust to disturbances. The amplitude of
Figure 6.9: Experimental results for LQR control

Figure 6.10: Experiment for LQR at 4.5 Hz
the disturbance seems large to be a result of a anti resonance. It can be concluded that
the anti resonance of the passive control is not present in the closed loop of LQR control.

6.3.2 Experiments for PI control

In the simulations for PI control is shown that the system converges very slowly to a
reference. In the ECP\textsuperscript{TM} software it is not possible to perform a step reference for the
time that is necessary for the PI controller to converge. Now only a constant reference
signal of 1 cm is used for the primary system. The algorithm that is used can be found
in Appendix G. The ECP\textsuperscript{TM} system is not capable of doing measurements that take a
long time. During the experiment with an perturbation with a frequency of 3.4 Hertz
ten measurements are made. For every measurement the time started at zero again.
It is not illustrative to show the measurements. In all measurements the amplitude of
the disturbance is about 3 \( \cdot 10^{-4} \) m. In the 3.4 Hertz experiments the applied force
equals 0.46 Newton. From figure 4.1 it can be seen that the gain at this frequency is
2 \( \cdot 10^{-6} \). This implies an amplitude of 9.2 \( \cdot 10^{-6} \) m. If the shaker excites the system
with a frequency that is only 0.2 Hertz shifted the gain is of the order 10\(^{-4}\). In that
case the calculated amplitude would be higher than the amplitude in the experiment.
As stated before in Chapter 6.3.1 and Chapter 6.2.2 it is possible that 3.4 Hertz is not
the anti-resonance frequency and that 4.5 Hertz is not a resonance frequency.

The amplitude of the disturbance is in the experiment with a perturbation frequency
of 4.5 Hertz about 4 \( \cdot 10^{-4} \) m. This is only a little higher than the disturbance in
the experiment with 3.4 Hertz. According to figure 4.1 is the gain at this frequency
of the order 10\(^{-1}\). The applied force is 0.61 Newton. This should result in a response
of 10\(^{-1}\) \( \cdot 0.61 = 0.061 \) m. This is a factor 100 larger than the experiment indicates.
However again it has to be noted that the control of the shaker is not very accurate. If
the shaker has a frequency of \( \frac{30}{2\pi} = 4.7746 \) Hertz than the gain is 3 \( \cdot 10^{-3} \). The resulting
amplitude would be 3 \( \cdot 10^{-3}\) \( \cdot 0.61 = 0.00183 \) m. This is only a factor 5 higher. With
a larger difference in the perturbation frequency the experimental result would be in
correspondence with the theoretical results. Also the determination of the applied force
is sensitive of errors.

6.3.3 Experiments for Generalized PI control

A measurement of the experiment for the GPI control with a perturbation with a fre-
quency of 3.4 Hertz can be found in figure 6.11. The resulting amplitude of the dis-
turbance is 2 \( \cdot 10^{-4} \) m. The gain in figure 5.1 at 3.4 Hertz is 1 \( \cdot 10^{-5} \) and the force
is 0.46 Newton. This results in a amplitude of 4.6 \( \cdot 10^{-5} \) Newton. It is possible that
the anti-resonance frequency is located at another frequency because of the difference in
parameters. Then the result of the experiment is plausible.

The amplitude of the vibration becomes very big after a long time. This is due to
errors in compensating the initial conditions. When in the beginning of the experiment
the integrals do not fully compensate will this error grow quadratically because of the
The measurement for the experiments with the GPI controller with a perturbation of 4.5 Hertz show also an amplitude of the disturbance of $2 \cdot 10^{-4}$ m. According to figure 5.1 this is very well possible since the gain at this frequency is $4 \cdot 10^{-4}$. The applied force has an amplitude of 0.61 Newton. This results in a disturbance of $4 \cdot 10^{-4} \times 0.61 = 2.4 \times 10^{-4}$ m. This experimental result is satisfactory because the part of the Bode plot at frequencies higher than the anti-resonance frequency experiences gains that are more or less constant. So a little change in parameters will not affect the experimental results.

### 6.3.4 Comparison of the experimental results

It is clear that experiments are not carried out at the anti-resonance frequency nor at the resonance frequencies. It can be concluded however that all controllers show asymptotic tracking behavior though PI control is very slow. LQR control can be preferred because of its simplicity while GPI control can be preferred over LQR control because only the position and the control effort have to be known.

Another problem is that it is not possible to tune the shaker exactly. Also it is not known whether the shaker excites the system with a constant frequency.

The objectives of the project were to design a controller that maintains the property of the passive control and is robust to perturbations and noise. The performance of the controllers at the frequencies used in the experiments in terms of disturbance rejection lie in the same order (between $2 \cdot 10^{-4}$ m and $5 \cdot 10^{-4}$ m).
Chapter 7

Conclusions and recommendations

The objectives of the project were to design a controller that maintains the anti resonance property of the passive control and is robust to perturbations and noise. GPI is the only control strategy in comparison with PI and LQR that, in theory, is robust to noise and preserves the anti resonance of the passive absorber. Also in the simulations shows GPI the best performance. It should be noted that more control strategies are available that possibly are simpler to implement and possibly perform better than GPI control.

A disadvantage of the GPI control is that it is a complicated controller to build. Also experimentally do errors in the initial conditions blow up to make the system instable. LQR shows good results in simulation however it does not conserve the anti resonance property of the vibration absorber. PI control preserves the property of the anti resonance of the vibration absorber but the gain at the resonance frequencies is still very large. Also the PI controller is not suitable since the system is underactuated. This however is not a surprise since PI is not a good control strategy on the fourth order mass-spring-damper system. PI is a very successful strategy on lower order systems. The conclusions that can be drawn from theory and simulation can not be justified by the experiments. More experiments are necessary.

It is possible to obtain better results, therefore some recommendations are made. In order to be able to make a comparison between the experimental and the simulation results it is necessary to use the same parameters in simulations and experiments. Also it is useful to use a shaker that excites the system with a constant frequency and that is easily adjustable to another frequency without the need to decouple the system. The measurements that are needed for the controllers to work enter the ECPTM programs in counts. It is recommended that the inputs are first converted to meters and Newtons before the control is executed. This control should be computed in Newtons, while the control force that is send to the DC motor is modified with a factor called the hardware gain. With these recommendations it should be possible to obtain better results.
Appendix A

M-files: PIfindgain

PIfindgain.m is used to compute pole-locations for different combination of the gains $K$ and $T_i$. $T_i$ has to be changed manually. If the poles are located in a suitable place is it possible to retrieve the gains from the vectors $K_M$ and $T_i M$.

```matlab
k1=745;
k2=340;
c1=0.01;
c2=0.01;
m1=1.586;
m2=2.839;
Td=0;
EAM=[ ];
KM=[ ];
TIM=[ ];
for K=-136:0.1:302
    for Ti=0.18001
        Ai=[0 1 0 0;kl/ml -cl/ml kl/ml-K/ml -K*Td/ml 1/ml
            ;0 0 1 0;k1/m2 0 (-k2-k1)/m2 -c2/m2 0;0 0 -K/Ti 0 0];
        EA=eig(Ai);
        EAM=[EAM EA];
        KM=[KM K];
        TIM=[TIM Ti];
    end
end
```

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Appendix B

Simulink model: Passive control

The simulink scheme used to do simulations for passive control is given in this appendix see Figure B.1. The properties of the blocks are given by:

Sine Wave 1
- Amplitude: 3 Newtons
- Frequency: 21.673 radians/sec
- Phase: $0.5 \times \pi$

Random Number
- Mean: 0
- Variance: 0
- Mean: 0
- Variance (noise): 0.02

Model

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0; -k1/m1 & -c1/m1 & k1/m1 & 0 & 0 & 0 & 1 \\
; k1/m2 & 0 & (-k2-k1)/m2 & -c2/m2 \\
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0; 0; 0; 1/m2 \\
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
1 & 0 & 0; 0 & 1 & 0; 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
0; 0; 0; 0 \\
\end{bmatrix}
\]

Initial conditions

[-3/k1; 0; 0; 0]

Figure B.1: Simulink model for simulations with passive control
Appendix C

Simulink model: LQR control

The Simulink model used in the simulations is given in figure C.1.

The properties of the blocks are given by:

<table>
<thead>
<tr>
<th>Gain</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>[89.5126 16.8403 -6.0366 15.4141]</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>G</th>
<th>$y_r$</th>
<th>0.01 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>G (no step)</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>G (step)</td>
<td>464.3275</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sinewave</th>
<th>Amplitude</th>
<th>3 Newton</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>21.673 radians/sec</td>
<td></td>
</tr>
<tr>
<td>Phase</td>
<td>$0.5 \times \pi$</td>
<td></td>
</tr>
</tbody>
</table>

| Model | $A$ | $[0 \ 1 \ 0 \ 0; -k_1/m_1 \ -c_1/m_1 \ k_1/m_1 \ 0; 0 \ 0 \ 0 \ 1$ |
|       | B   | $; k_1/m_2 \ 0 \ (-k_1-k_2)/m_2 \ -c_2/m_2]$ |
|       | C   | $[0 \ 0; 1/m_1 \ 0; 0; 0 1/m_2]$ |
|       | D   | $[1 \ 0 \ 0 \ 0; 1 \ 0 \ 0 \ 0 1 \ 0; 0 \ 0 \ 0 \ 1]$ |
|       | Initial conditions | $[0 \ 0 \ 0 \ 0]$ |

| Random number | Mean (no noise) | 0 |
|               | Variance (no noise) | 0 |
|               | Mean               | 0 |
|               | Variance           | 0.02 |
Figure C.1: Simulink model for LQR control
**Appendix D**

**Simulink models: PI control**

The simulink model that is used for the PI control can be found in figure D.1. The properties of the blocks are given by:

<table>
<thead>
<tr>
<th>Block</th>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID</td>
<td>P</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>(\frac{0.2}{0.18001})</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>0</td>
</tr>
<tr>
<td>Sine wave</td>
<td>Amplitude</td>
<td>3 Newton</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>21.673 radians/sec</td>
</tr>
<tr>
<td></td>
<td>Phase</td>
<td>(0.5 \times \pi)</td>
</tr>
<tr>
<td>Random number</td>
<td>Mean (no noise)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Variance (no noise)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Variance</td>
<td>0.02</td>
</tr>
<tr>
<td>Constant</td>
<td>C (no step)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.01</td>
</tr>
</tbody>
</table>
| Model   | A                         | \[
  \begin{bmatrix} 0 & 1 & 0 & 0; & k_1/m_1 & -c_1/m_1 & k_1/m_1 & 0 & 0 & 0 & 1 \\
  :k_1/m_2 & 0 & (-k_1-k_2)/m_2 & -c_2/m_2 \\
  \end{bmatrix}
\] |
|         | B                         | \[
  \begin{bmatrix} 0 & 0; & 1/m_1 & 0 & 0 & 0 & -1/m_2 \\
  \end{bmatrix}
\] |
|         | C                         | \[
  \begin{bmatrix} 0 & 0 & 1 & 0 \\
  \end{bmatrix}
\] |
|         | D                         | \[
  \begin{bmatrix} 0 & 0 \end{bmatrix}
\] |
|         | Initial conditions        | \[
  \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}
\] |
Figure D.1: Simulation model for PI control
Appendix E

Simulink model: GPI control

The simulink model that is used in the simulations for Generalized PI control can be found in figure E.1.

The block yref + der’s loads the reference trajectory and its derivatives from the workplace. The properties of the blocks are:

- **Sinewave**
  - Amplitude: 3 Newton
  - Frequency: 21.673 Hertz
  - Phase: 0.5 * π

- **Random number**
  - Mean (no noise): 0
  - Variance (no noise): 0
  - Mean: 0
  - Variance: 0.02

- **Original**

  - **A**
    
    \[
    \begin{bmatrix}
    0 & 1 & 0 & 0; & -k_1/m_1 & -c_1/m_1 & k_1/m_1 & 0; & 0 & 0 & 1 \\
    k_1/m_2 & 0 & (-k_2-k_1)/m_2 & -c_2/m_2
    \end{bmatrix}
    \]

  - **B**
    
    \[
    \begin{bmatrix}
    0 & 0; & 1/m_1 & 0; & 0; & 0 & 1/m_2
    \end{bmatrix}
    \]

  - **C**
    
    \[
    \begin{bmatrix}
    1 & 0 & 0 & 0; & 0 & 1 & 0 & 0 & 0 & 1 & 0; & 0 & 0 & 0 & 1
    \end{bmatrix}
    \]

  - **D**
    
    \[
    \begin{bmatrix}
    0 & 0; & 0; & 0 & 0; & 0 & 1
    \end{bmatrix}
    \]

  - **Initial conditions**
    
    \[
    \begin{bmatrix}
    0.01 & 0.05 & -0.001 & 0.005
    \end{bmatrix}
    \]

  The subblocks reconstruction and errors theta can not be found in this appendix because the submodels are too big. The subblock reconstruction constructs the thetas that are used in the control law. The subblock errors theta determines the errors of the thetas and its corresponding reference signal. The subblock control law can be found in figure E.2. The gains of the subblock control law are given by:

  - \( K_1 = 104.85 \)
  - \( K_2 = 5297.1 \)
  - \( K_3 = 1.625 \times 10^5 \)
  - \( K_4 = 3.2499 \times 10^6 \)
  - \( K_5 = 4.2371 \times 10^7 \)
  - \( K_6 = 3.3553 \times 10^8 \)
APPENDIX E. SIMULINK MODEL: GPI CONTROL

Figure E.1: Simulink model for GPI control

Figure E.2: Subblock control law
Appendix F

ECP programs: LQR

This appendix gives the ECP<sup>TM</sup>-program that is used during the experiments for LQR control.

```
// user variables
#define K1 q1
#define K2 q2
#define K3 q3
#define K4 q4
#define G q5
#define posold1 q6
#define posold2 q7
#define vel1 q8
#define vel2 q9
#define deltat q10
#define khw q11

// initialize variables
posold1=0; // initial value for old position for mass carriage 1
posold2=0; // initial value for old position for mass carriage 2
vel1=0;   // initial velocity of mass carriage 1
vel2=0;   // initial velocity of mass carriage 2
deltat=0.000884; // remember to choose Ts=0.000884 in dialog box prior
to "Implement"

khw=12000; // hardware gain (see manual)
K1=89.5126; // gain 1
K2=16.8403; // gain 2
K3=-6.0366; // gain 3
K4=15.4141; // gain 4
G=430;     // value to get system to reference

// Real-Time User Code

begin
    vel1=(encl_pos-posold1)/deltat; // construction of the velocity of mass
```
vel2=(enc2_pos-posold2)/deltat ; carriage 1
  ; construction of the velocity of mass
  ; carriage 2
control_effort=(1/khw)*(-K1*(enc1_pos)-K2*vel1
  -K3*(enc2_pos)-K4*vel2+G*cmd_pos)
posold1=enc1_pos
posold2=enc2_pos
end
Appendix G

ECP programs: PI

This appendix gives the ECP™-program that is used during the experiments for PI control.

;********** user variables
#define K q1
#define Ti q2
#define yr q3
#define posold q4
#define deltat q5
#define khw q6
#define intpos q7
#define intold q8
#define intref q9
#define intrefold q10
#define refold q11

;******** Initialize variables
posold=0 ;initial old position for mass carriage 2
deltat=0.000884 ; remember to choose Ts=0.00442 in dialog box prior
to "Implement"
khw=8224 ;hardware gain
intpos=0 ;initial integrated position for mass carriage 2
intold=0 ;initial integrated old position for mass carriage 2
intref=0 ;initial integrated reference position
intrefold=0 ;initial integrated old reference position
K=0.2 ;Gain
Ti=0.18001 ;Integral time
refold=0 ;Old reference position
yr=2197.866*32 ;Reference position (equivalent to 1 cm)

;********** Real-Time User Code
begin
  intpos=(enc2_pos+posold)*deltat/2+intold ;constructing integrated

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APPENDIX G. ECP PROGRAMS: PI

intref=yr*deltat+intrefold

control_effort=(1/khw)*(K*(yr-enc2_pos)+(K/Ti)*(intref-intpos))

intold=intpos ; updating
intrefold=intref
refold=cmd_pos
posold=enc2_pos
end

************************************************************
Appendix H

ECP programs: Generalized PI

This appendix gives the ECP\textsuperscript{TM}-program that is used during the experiments for GPI control.

```plaintext
;************ user variables
#define m1 q1
#define m2 q2
#define c1 q3
#define c2 q4
#define k1 q5
#define k2 q6
#define P1 q7
#define a q8
#define a1 q9
#define a2 q14
#define a3 q15
#define au q16
#define th1 q17
#define th2 q18
#define th3 q19
#define deltat q20
#define khw q21
#define z1 q10
#define z2 q11
#define z3 q24
#define z4 q12
#define hup q26
#define pos q27
#define t q28
#define cr q29
#define kvtn q30
#define uy q13
```
APPENDIX H. ECP PROGRAMS: GENERALIZED PI

#define uyi q32
#define uyold q33
#define uyiold q34
#define uyi y q35
#define uyi yold q36
#define uyi yi q37
#define uyi yiold q38
#define uyi yi y q39
#define uyi yi yiold q40
#define uyi yi yi yi q41
#define uyi yi yi yiold q42
#define P2 q43
#define P3 q44
#define P4 q45
#define P5 q46
#define P6 q47
#define P7 q48
#define refdd q49
#define refddd q50
#define refi q51
#define refiold q52
#define refi i q53
#define refi iold q54
#define refi iii q55
#define refi iiiold q56
#define yr q57
#define b0 q58
#define b1 q59
#define b2 q60
#define b3 q61
#define b4 q62
#define b5 q63
#define b6 q64
#define b7 q65
#define b8 q66
#define b9 q67
#define t0 q68
#define cr q69
#define td q70
#define d1 q71
#define d2 q72
#define d3 q73
#define d4 q74
#define d5 q75
#define d6 q76
#define d7 q77
#define d8 q78
#define d9 q79
#define yrOId q80

; Initialize variables

; d0 = 0
; d1 = 0
; d2 = 0
; d3 = 0
; d4 = 0
; d5 = 0
; d6 = 0
; d7 = 0
; d8 = 0
; d9 = 0

; t0 = -3
; t = -3

b0 = 0; b’s are the coefficients for the reference
b1 = -0.00000001; trajectory as in the simulations
b2 = 0
b3 = -0.00000001
b4 = 0
b5 = 276931.1160000400
b6 = 923103.7200001000
b7 = 1186847.640000110
b8 = 692327.7900000500
b9 = 153850.6200000100

; td = 0

deltat = 0.001768; remember to choose Ts = 0.001768

; in dialog box prior to "Implement"

m1 = 1.839
m2 = 2.586
c1 = 0.01
c2 = 0.01
k1 = 745
k2 = 340
P1 = 104.85; P’s are gains as computed in theory
P2 = 5297.1
P3 = 162510
P4 = 3249900
P5 = 42371000
APPENDIX H. ECP PROGRAMS: GENERALIZED PI

\[ P6 = 335530000 \]
\[ P7 = 1280200000 \]
\[ au = k1/(m1*m2) \]
\[ a = (k2*k1)/(m2*m1) \]
\[ a1 = (c2*k1+c1*k1+c1*k2)/(m1*m2) \]
\[ a2 = (k1*m1+k2*m1+m2*k1+c1*c2)/(m1*m2) \]
\[ a3 = (c2*m1+c1*m2)/(m1*m2) \]
\[ th1 = 0 \]; \text{thetas} \]
\[ th2 = 0 \]
\[ th3 = 0 \]
\[ refd = 0 \]; \text{initialise derivatives of reference} \]
\[ refdd = 0 \]
\[ refddd = 0 \]
\[ refi = 0 \]; \text{initialise integrals of the reference} \]
\[ refio = 0 \]
\[ refii = 0 \]
\[ refiii = 0 \]
\[ refiiio = 0 \]
\[ yr = 0 \]
\[ pos = 0 \]
\[ cr = 32 \]; \text{factor to get meters from counts} \]
\[ hup = 0 \]
\[ au = k1/(m1*m2) \]
\[ a = (k2*k1)/(m2*m1) \]
\[ a1 = (c2*k1+c1*k1+c1*k2)/(m1*m2) \]
\[ a2 = (k1*m1+k2*m1+m2*k1+c1*c2)/(m1*m2) \]
\[ a3 = (c2*m1+c1*m2)/(m1*m2) \]
\[ th1 = 0 \]
\[ th2 = 0 \]
\[ th3 = 0 \]
\[ z1 = 0 \]
\[ z2 = 0 \]
\[ z3 = 0 \]
\[ z4 = 0 \]
\[ kwh = 3275 \]; \text{khw and kvtn are used to get voltage from Newton} \]
\[ kvtn = 6.5 \]
\[ uy = 0 \]; \text{integrals of the position of the second mass} \]
\[ uyi = 0 \]
\[ uyi = 0 \]
\[ uyi = 0 \]
\[ uyi = 0 \]
\[ uyi = 0 \]
uyold=0
uyiold=0
uyiyold=0
uyiyiold=0
yrold=0

;********** Real-Time User Code **********
begin
t0=t0+deltat
if (t0>2 and t0<3)
td=td+deltat
endif
d9=td*td*td*td*td*td*td*td*td

d8=td*td*td*td*td*td*td*td*td

d7=td*td*td*td*td*td*td*td*td

d6=td*td*td*td*td*td*td*td*td

d5=td*td*td*td*td*td*td*td*td

d4=td*td*td*td*td*td*td*td*td

d3=td*td*td*td*td*td*td*td*td

d2=td*td*td*td*td*td*td*td*td

d1=td
d
pos=(enc2_pos/cr)/219786.6; converts encoder position to position in meters
yr=(b0+b1*d1+b2*d2+b3*d3+b4*d4+b5*d5+b6*d6+b7*d7
+b8*d8+b9*d9)/219786.6
refd=(b1+2*b2*d1+3*b3*d2+4*b4*d3+5*b5*d4
+6*b6*d5+7*b7*d6+8*b8*d7+9*b9*d8)/219786.6
refdd=(2*b2+6*b3*d1+12*b4*d2+20*b5*d3
+30*b6*d4+42*b7*d5+56*b8*d6+72*b9*d7)/219786.6
refddd=(6*b3+24*b4*d1+60*b5*d2+120*b6*d3
+210*b7*d4+336*b8*d5+504*b9*d6)/219786.6
refi=(deltat*(yr+yrold))/2+refiold
refii=(deltat*(refi+refiold))/2+refiiold
refiii=(deltat*(refii+refiold))/2+refiiiold
hup=(1/au)*((a3-P1)*(th3-refddd)+(a2-P2)*(th2-refdd)+(a1-P3)*(th1-refd)
+(a-P4)*(pos-yr)-P5*(posi-refi)-P6*(posii-refii)-P7*(posiii-refiii))
yu=k1/(m1*m2)*(hup-k2*pos)
uyi=uyiold+(deltat*(uyold+uy))/2
uyyi=uyi-a1*pos
uyiy=uyiyold+(deltat*(uyiyold+uyiy))/2
uyiyi=uyiyi-a2*pos
uyiyiy=uyiyiyold+(deltat*(uyiyiyold+uyiyiy))/2
th1=(uyiyi-a3*pos)
\[ th2 = (uyiy - a2 \cdot pos - a3 \cdot th1) \]
\[ th3 = (uyi - a1 \cdot pos - a2 \cdot th1 - a3 \cdot th2) \]
\[ z1 = l/k1 \cdot (m2 \cdot th2 + c2 \cdot th1 + (k1+k2) \cdot pos) \]
\[ z2 = l/k1 \cdot (m2 \cdot th3 + c2 \cdot th2 + (k1+k2) \cdot th1) \]
\[ z3 = pos \]
\[ z4 = th1 \]
\[ \text{control\_effort} = hup * khw/kvt \]
\[ uyold = uy \]
\[ uyiold = uyi \]
\[ uyiyold = uyiy \]
\[ uyiyiold = uyiyi \]
\[ uyiyiyold = uyiyiy \]
\[ uyiyiyiold = uyiyiyi \]
\[ refiold = refi \]
\[ refiold = refii \]
\[ refiiodld = refiii \]
\[ yrold = yr \]
\[ end \]

************************************************************************************************************
Appendix I

Information on CINVESTAV-IPN

The traineeship is done in the period May 2002-August 2002 at CINVESTAV-IPN in Mexico-city. CINVESTAV-IPN is an abbreviation for Centro de Investigacion y de Estudios Avanzados del Instituto Politecnico National. The english translation is Centre for research and advanced studies of the National Polytechnical Institute. Cinvestav has been founded in 1961 as result of an initiative of the ministry of education and of IPN. The objective of the institute is to educate investigators to a postgraduate level and realise basic investigacion. Cinvestav consists of 8 units, three of them in Mexico city and five in other cities. These cities are Irapuato, Queretaro, Saltillo, Merida and Guadalajara. There are 571 investigators working full time of whom 95 percent are Ph.D. and 80 percent has a membership of the Sistema Nacional of Investigators, the national union of investigators. The 2000 students that are in CINVESTAV try to get their M. Sc. or their Ph. D.

The Mexican government is responsible for financing CINVESTAV for 95 percent through the Ministry of public education and the National council of science and technology. The remaining part is financed by companies whithout any direct benefit. The institute is a decentralized organization that therefore is independant of other educational institutes.

The activities of CINVESTAV are divided in four different areas which consist of different departments. The four areas are Social and Human science that consists of four departments, Exact and Natural science that also consists of four departments, Biological and Health science that consists of eleven departments and Technology and science for engineers that consists of nine departments. The traineeship is done in the latter area in the department of electrical engineering. This department is divided in the sections of bioelectronics, computation, communication, automatic control, solid state electronics and mecatronics. The project of this report is carried out in the mecatronics section in the unit Zacatenco in Mexico city.
Bibliography


