Characterization of the in-plane mechanical behaviour of human skin

a mixed numerical-experimental approach employing a structural skin model

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Abstract

At the Personal Care Institute of the Philips Research Laboratories in Eindhoven the shaving process is studied. An important feature is the interaction between skin and apparatus. It is therefore needed to gain insight in the mechanical behaviour of human skin.

This report presents an in-plane mechanical characterization of human skin in vivo, employing a structural skin model, i.e. Lanir's Skin Model [Lanir, 1983]. For this purpose a mixed numerical-experimental technique was employed. This method, presented by Hendriks [Hendriks, 1991], is based on the confrontation of measured field data with calculated field data, eventually leading to a determination of the material parameters of an assumed constitutive model, here Lanir's Skin Model.

An experimental set-up was designed and some software for easy conversion of experimental data into finite element quantities was developed. An in vivo experiment was performed generating measured field data. Employing the numerical-experimental method, a limited set of four parameters, describing the in-plane mechanical behaviour, was estimated. This was done for two different configurations. It appeared to be possible to estimate these parameters but the estimation results should be considered with care; the results seem to be influenced by modelling errors.

A verification experiment was performed to examine the estimation results on consistency. One configuration demonstrated consistency but the results of the second configuration were not consistent, indicating the possible presence of modelling errors.

Different experiments were simulated numerically to find indications for possible improvements of the experimental set-up. From these experiments it can be concluded that, in case no modelling errors are present, the experiment performed (a one-dimensional loading situation) contains enough information to track back the limited set of four parameters. No direct evidence was found that a two-dimensional loading situation would improve the convergence speed of the parameter estimation process.
Notation

\( a \) scalar
\( \hat{a} \) column of scalars
\( \vec{a} \) vector
\( \tilde{a} \) column of vectors
\( A \) matrix
\( A^T \) transpose of matrix
\( A^{-1} \) inverse of matrix
\( I \) unit matrix
\( A \) second order tensor
\( A^c \) conjugate of tensor
\( A^{-1} \) inverse of tensor
\( I \) unit tensor
\( E(a) \) expected value of \( a \)
\( \hat{a} \) estimate of \( a \)
\( \bar{a} \) mean of \( a \)
Chapter 1

Introduction

1.1 Problem Definition

At the Personal Care Institute of the Philips Research Laboratories in Eindhoven the shaving process is studied. The aim is to improve existing shavers and develop new shaving systems in order to enhance the shaving performance. An important feature is the interaction between skin and apparatus. This is investigated by means of \textit{(in vivo)} experiments as well as numerical simulations in which one tries to get insight in the phenomena that occur during the shaving process. Essential for these numerical simulations is the availability of a good constitutive model of human skin. The constitutive model currently used in these simulations is a phenomenological model, \textit{i.e.} a Mooney-Rivlin (rubber) model. The parameters of this model are estimated from indentation tests. The disadvantage of this test is that the amount of information is rather limited. Only two quantities are measured: an indentation force and the indentation. This may result in a set of parameters that is not unique. Nevertheless, when simulating the indentation of a shaving head, the occurring phenomena are described satisfactory. However, the indentation of a shaving head is a similar loading case as the indentation test. When simulating more complex movements, for example moving the shaving head along the surface of the skin, the results are often non-consistent. This, among other shortcomings, is attributed to the limitations of the phenomenological model and the limited amount of information obtained from the indentation test.

1.2 Skin: a Complex Material

From an anatomical point of view, skin is often defined as a fibrous connective tissue. From an engineers point of view it can be seen as a very complex composite material. Figure 1.1 reflects a schematic representation of skin. From its stratified structure three main layers can be distinguished, which will be shortly outlined below.

The top layer is called epidermis. This is a rather stiff but very thin layer (0.07-1.4 mm [Cook, 1975]) and consists of four or five layers, distinguishing themselves by the amount of differentiation and keratinisation of their cells. The second layer, the dermis, is a dense fibrous connective tissue layer and is generally an order of magnitude thicker than the epidermis. It consists of cells, groundsubstance and fibres. The fibres form a dense network and are surrounded by the groundsubstance. The third layer, the hypodermis, is a fatty layer of loose connective tissue.
Chapter 1. Introduction

The mechanical behaviour of skin depends primarily on the response of the constituents of the dermis [Lanir, 1983]. Its main constituents are collagen and elastin fibres embedded in a matrix of amorphous ground substance. Collagen accounts for about 75% of the fat-free dry weight [Brown, 1972], [Manschot, 1985], [Savenije, 1982]. Morphological observations [Finlay, 1969], [Ridge et al., 1966] suggest that the collagen fibres are arranged in a three-dimensional but primarily planar wavy array [Finlay, 1969]. This waviness or undulation is considered as an important feature in skin mechanics. Upon stretch the collagen fibres become gradually straight, until they straighten completely. This process is accompanied by an increase of stiffness of the skin.

Collagen has been subjected to numerous mechanical investigations. These investigations have demonstrated the high strength, low extensibility and viscoelastic properties of collagen. There is a considerable variation in the magnitude of the obtained material parameters, both between different investigators and within any investigation. The elastic modulus of collagen is approximately $1.10^8 \text{N} / \text{m}^2$ [Savenije, 1982].

Elastin fibres are less abundant in skin than collagen and cover approximately 4% of the fat-free dry weight [Brown, 1972], [Lanir et al., 1990]. There is very little evidence on the structure of the elastin fibres. The most striking difference between elastin and collagen is that elastin exhibits long-range elasticity with little viscoelastic effects [Savenije, 1982]. The elastin fibres are frequently interwind around the thicker collagen fibres [Brown, 1972], [Finlay, 1969]. They are believed to act as energy storage and, after unloading, they pull the collagen fibres back to their original position [Manschot, 1985]. The elastic modulus of elastin is approximately $1.10^6 \text{N} / \text{m}^2$ [Savenije, 1982].

The ground substance is a complex matrix and can be characterized as a semi-fluid amorphous material. Its main constituents are glycoaminoglycans (GAG) and bind the present water in a molecular form. The ground substance is also thought to be partly responsible for the viscous part of the mechanical response of skin.

From the previous part it is clear that the mechanism of deformation of skin is a very complex one; it depends on the alterations in, and the interactions between its structural components. Fibres unfold, rotate and stretch and with that they squeeze the matrix that reacts through a build-up of pressure. Summarizing it can be stated that the overall response
1.3 Mechanical Characterization of Skin: Scope

of skin to macroscopic deformation contains the following effects:

- nonlinearity
- viscoelasticity
- anisotropy
- inhomogeneity

1.3 Mechanical Characterization of Skin: Scope

For the characterization of mechanical behaviour of connective tissue, two approaches have been adopted [Lanir, 1983]: the phenomenological approach, in which mathematical expressions are suggested, that fit the specific behaviour and the structural approach which seeks to develop constitutive relations based on the tissue's structure. The structural approach has several merits:

1. It is physical in the sense that all material functions are physical quantities.
2. The resulting models facilitate our understanding of the tissue's function and provide an insight into the tissue's response to a given deformation.
3. Structural models have a distinct advantage in tissue characterization since the structure is often known or can be investigated. Besides this the mechanical properties of some of the tissue components can be determined independently by isolating them from the tissue.

However, the structural approach also has some disadvantages. For example, structural models often contain a large number of parameters, making the model rather complex; this can result in an increase of computing effort.

A model based on structural grounds is Lanir's Skin Model [Lanir, 1983]. It is based on the assumption that the tissue's response is the sum of the responses of its constituents. Hence if the constituents' structure, their mechanics and interactions are known, then the overall tissue's response can be evaluated. Since it is believed that such an approach can result in a better constitutive model for human skin, much value is attached to this model here.

The objective of the present research is to characterize the mechanical behaviour of human skin \textit{in vivo}, employing Lanir's Skin Model. It will be restricted to the characterization of the in-plane mechanical behaviour. Lanir's Skin Model is an advanced model containing numerous material parameters. For the characterization of these parameters, it is essential to obtain more information from an experiment than the indentation test, as described in section 1.1, provides. In the current study extra information is gathered from field data.

1.4 Method and Overview

To reach the goal described in section 1.3, a mixed numerical-experimental technique is used. The method used here, presented by Hendriks [Hendriks, 1991], has proven to be
successful in numerous investigations concerning the characterization of complex materials [Oomens et al., 1993], [Rattingen, 1994] and opens up the way for in vivo measurements [Vossen, 1994]. The method is based on the confrontation of measured field data with calculated field data, eventually leading to a determination of the material parameters of an assumed constitutive model, here Lanir’s Skin Model.

Chapter 2 gives a description of the identification technique [Hendriks, 1991], where the estimation algorithm employed is different from the original work, where the algorithm is based on a sequential minimum-variance estimator. The estimation algorithm employed in this study is based on a constrained sequential maximum-likelihood approach. This method takes account for possible equality and inequality constraints as well as possible linear dependencies of the parameters. Further, a detailed description of Lanir’s Skin Model, used to characterize the behaviour of human skin in vivo, is given. In chapter 3 the experimental set-up for the in vivo measurements is presented. It is based on an optical measuring system used in previous research [Hendriks, 1991], [Rattingen, 1994], in order to obtain enough field data from an experiment. Chapter 4 presents the results of the actual in vivo experiment. Consequently, a limited set of parameters, describing the in-plane mechanical behaviour, is estimated using the numerical-experimental method as described in chapter 2. This chapter ends with the results of a verification experiment.

In order to obtain evidence for possible improvements of the experimental set-up, different experiments are simulated numerically. The results will be outlined in chapter 5. Finally, chapter 6 gives the conclusions of the present research and some recommendations for further work.
Chapter 2

Theory

To determine the in vivo behaviour of human skin, a numerical-experimental technique is used; it will be outlined in section 2.1. In section 2.2 the structural skin model of Lanir is elaborated.

2.1 Identification Technique

Hendriks [Hendriks, 1991] presented an identification technique for the characterization of the mechanical behaviour of complex materials. In this method it is assumed that a sufficiently accurate constitutive model is available and its purpose is to estimate the material parameters in these constitutive equations. In contrast with standard methods, the approach no longer demands a homogeneous stress and strain field in some part of the loaded specimen under consideration. Even more, it is preferable to obtain an inhomogeneous strain field [Hendriks, 1991], [Ratingen, 1994]. There are two arguments for this:

- it can be expected that an inhomogeneous strain field contains more information about the material properties than a homogeneous strain field does,
- when inhomogeneous strain fields are allowed, freedom in the design of the experimental set-up is gained.

The fact that no homogeneous stress and strain fields are demanded makes this technique extremely suitable for characterizing anisotropic and inhomogeneous materials like biological tissue. Furthermore, the freedom in the design of the experimental set-up is especially convenient for in vivo measurements. In vivo measurements have the advantage that one does not disrupt the internal structure, occurring when samples have to be excised for in vitro measurements.

This numerical-experimental technique is based on the use of three elements (figure 2.1):

- measurements of field data on a specimen with arbitrary geometry and boundary conditions,
- finite element modelling of the experiment,
- an iterative scheme to obtain an estimate for the material parameters in the constitutive model employed.
Chapter 2. Theory

Figure 2.1: Scheme for the identification technique.

In the following sections the three elements of this numerical-experimental technique are described.

2.1.1 Measurement of Field Data

A common experimental set-up in order to measure (inhomogeneous) strain fields is to place a large number of markers on the surface of the specimen. The positions of the markers are determined with an optical system. The advantages of this method are that it is a non-contacting method and that a large amount of information can be obtained. However, a disadvantage of this method is that it is only possible to measure displacements on the outer surface of the specimen. This means that strain field measurement is only relevant when the surface strain field contains enough information for a characterization of the whole.

2.1.2 Finite Element Modelling

The next part in the mixed numerical-experimental method is a finite element model of the experiment. For this purpose the finite element code MARC [MARC, 1994] is used. Because of the in vivo measurements, the so-called local approach is used [Hendriks, 1991], solely using kinematic boundary conditions. Here, only a part of the skin is modelled and a selected set of markers are used to define the edges of the part under consideration (figure 2.2). The displacements of these markers are then used as kinematic boundary conditions for the finite element model. As a consequence it may be clear that because forces are not part of the boundary conditions and the measured data, the stiffness parameters cannot be determined absolutely. Nevertheless it is still possible to estimate the ratios between different stiffness parameters. The finite element calculations can only be carried out for a given set of parameters, thus an initial set of parameters must be available.

2.1.3 Parameter Identification

The essential part of this numerical-experimental technique is the iterative procedure that is used to adjust the parameters of the constitutive model employed. The simulation of the experiment results in a set of calculated data which can be compared with a set of data measured in the experiment. The difference (residual) between the two sets contains
2.1. Identification Technique

Figure 2.2: A part of the sample is modelled by a finite element model.

information about the modelling error, which can be used in order to adjust the parameters. The algorithm employed in this study is essentially different compared to the one presented in the original work [Hendriks, 1991], based on a sequential minimum-variance estimator. The present algorithm is based on a constrained sequential maximum-likelihood approach. The solution procedure and sequential estimator are treated more separately, resulting in a sequential maximum-likelihood estimator, combined with a minimization procedure suited for constrained problems. This minimization procedure takes account for possible equality and inequality constraints as well as possible linear dependencies of the parameters. The algorithm employed will be outlined shortly in the remainder part of this section.

Assume that a priori information on the previous \((k^{th})\) estimate for the parameters, \(\hat{x}_{k}^{*}\), is available and can be modelled by:

\[
\hat{x}_{k} = x_{t} + w_{k},
\]

with \(x_{t}\) a column with the true parameter values and \(w_{k}\) the estimation error with zero mean and covariance matrix \(P_{k}\):

\[
E(w_{k}) = 0, \quad E(w_{k}w_{k}^{T}) = P_{k},
\]

with \(E\) the expected value operator. The nonlinear model for the measured or observational data is represented by:

\[
y_{k+1}^{*} = h_{k+1}(x_{t}) + v_{k+1},
\]

The nonlinear function \(h_{k+1}(x_{t})\) describes the dependence of observation \(k + 1\) on column \(x_{t}\), if there are no observation errors. Function \(h_{k+1}(x_{t})\) symbolizes the finite element calculation with use of material parameters in column \(x_{t}\). Column \(y_{k+1}^{*}\) contains the observational data from the experiment. Column \(v_{k+1}^{*}\) contains observational errors with supposed zero mean and covariance matrix \(R_{k+1}\):

\[
E(v_{k+1}) = 0, \quad E(v_{k+1}v_{k+1}^{T}) = R_{k+1}.
\]

Further it is assumed that \(w_{k}\) and \(v_{k+1}\) are uncorrelated, i.e.

\[
E(w_{k}v_{k+1}^{T}) = 0.
\]
Besides equations (2.1) and (2.3), the parameters have to obey a set of equality and inequality constraints,

\[ b(\hat{x}_e) = 0, \quad c(\hat{x}_e) \geq 0. \]  

(2.6)

The estimation problem is formulated as an optimization problem and can be written as the following weighted minimization formulation [Starmans, 1994]:

\[
\min_{\hat{x}_{k+1}} \{ (y_{k+1} - h_{\sim_k} (\hat{x}_{k+1}))^T R_{k+1}^{-1} (y_{k+1} - h_{\sim_k} (\hat{x}_{k+1})) + (\hat{x}_{k} - \hat{x}_{k+1})^T (P_k + Q)_{k+1}^{-1} (\hat{x}_{k} - \hat{x}_{k+1}) \mid b(\hat{x}_{k+1}) = 0 \land c(\hat{x}_{k+1}) \geq 0 \}. 
\]  

(2.7)

It can be derived that matrix \( P_k \) is updated according to [Starmans, 1994]:

\[
P_{k+1} = Z^T [Z[H_{k+1}^T (\hat{x}_{k+1}) R_{k+1}^{-1} H_{k+1} (\hat{x}_{k+1}) + (P_k + Q_k)^{-1}] Z^T]^{-1} Z. 
\]  

(2.8)

The matrix \( H_{k+1} \) expresses the sensitivity of the model output for the parameter variations and is defined as:

\[
H_{k+1} = \left( \frac{\partial h_{\sim_k} (\hat{x})}{\partial \hat{x}} \right)_{\hat{x} = \hat{x}_k} 
\]  

(2.9)

The matrix \( Z \) accounts for the active constraints, i.e. the constraints which hold as equality. For further details on this it is referred to [Starmans, 1994]. Matrix \( Q_k \) represents an artificial modelling error covariance which is added in order to assure convergence.

The described estimator is implemented in the program PARFIT [Starmans, 1994], employing the NAG library [NAG, 1993]. The implemented algorithm is designed to minimize an arbitrary smooth function subject to constraints. Per estimation step, first the minimization according to equation (2.7) is performed. Secondly the estimate covariance matrix is updated according to equation (2.8). In this way a sequence of guesses for the solution, i.e. estimates, is generated, converging to the solution of the minimization problem.

2.2 Lanir’s Skin Model

Lanir’s Skin Model [Lanir, 1983] is a constitutive skin model employing a structural approach. This 3-D anisotropic nonlinear skin model is based on the assumption that the tissue’s response is the sum of the responses of its constituents. Hence, if the constituents’ structure, their mechanics and interactions are known, then the overall tissue response can be evaluated. In the following paragraph some definitions and assumptions of Lanir’s Skin Model are stated.

The mechanical behaviour of skin is assumed to be dominated by the dermis; the influence of the epidermis and hypodermis is neglected. In this model structure is defined in terms of the fibres’ orientation. To each type of fibre, \( k \), a (discrete) density distribution function, \( R_k(\hat{r}_0) \), is associated, where \( \hat{r}_0 \) is a unit vector tangent to the fibre in reference configuration. This leads to a mathematical approach based on the following assumptions:

- Each fibre is thin and perfectly flexible. It has no compressive stiffness and, if contracted, will buckle under zero load. It does not bear any load if it is undulated.
• If a fibre is stretched, it is subjected to a uniaxial strain which is the tensorial transformation of the overall strain in the fibre's direction (affine deformation).

• The fibres are elastic.

• Skin is incompressible.

• The effect of the matrix due to deformation is that of a hydrostatic pressure.

• Upon stretching the fraction of fibres that are straightened and stretched rises, providing nonlinear behaviour.

The stress-strain relation is described with:

$$\sigma = \tau - pI,$$

(2.10)

with $\sigma$ the Cauchy stress tensor, $\tau$ the so-called extra stress tensor, $p$ a hydrostatic pressure and $I$ the second order unit tensor. The mechanical behaviour of the fibres is accounted for in $\tau$, the contribution of the groundsubstance is accounted for in $p$. The stress-strain relation is then defined by [Feron, 1993]:

$$\sigma = \frac{1}{J} \sum_k \sum_{\vec{r}_0} S_k R_k(\vec{r}_0) \frac{1}{\lambda(\vec{r}_0)} f_k^x(\lambda(\vec{r}_0)) F \cdot \vec{r}_0 \cdot F^c - pI$$

(2.11)

with:

• $F$ the deformation tensor and $J = \det(F)$, a measure for the volume strain,

• $\vec{r}_0$ a unit vector tangent to the fibre in reference configuration,

• $\lambda(\vec{r}_0)$ the elongation ratio of the fibres oriented in the direction $\vec{r}_0$ in the reference state,

• $S_k$ the volumetric fraction (out of the total volume) of fibres of type $k$ in the unstrained state,

• $R_k(\vec{r}_0)$ the fraction of all fibres of type $k$ oriented in direction $\vec{r}_0$ in the reference state; it should hold that $\sum_{\vec{r}_0} R_k(\vec{r}_0) = 1$,

• $f_k^x(\lambda(\vec{r}_0))$, the load per unit undeformed cross-sectional area, which also accounts for the undulation of the fibres, according to a normal distribution $N(\mu, \sigma)$ (see Appendix A for details).

This model is implemented [Feron, 1993] in the finite element code MARC [MARC, 1994] and uses 8-node, isoparametric trilinear brick elements. The use of the hydrostatic pressure as unknown results in a mixed finite element formulation, where the incompressibility constraint is forced using the penalty function method. A Newton-Raphson iteration scheme is applied to solve the system of nonlinear equations.
Chapter 3

Experimental Set-up

A common way to measure inhomogeneous field data, is to place a large number of markers on the surface of the specimen under investigation and register their position during deformation with an optical system. In this way an entire displacement field can be measured. For this purpose an experimental set-up is developed using a standard CCD video camera and an image processing and analysis system to obtain the field data, and a small surface tensile device to create different states of deformation. The next section deals with this tensile device and section 3.2 explains the procedure to obtain the field data.

3.1 Tensile Device

To impose a certain state of deformation, a small surface tensile device is used. This device was developed originally in order to carry out in vivo pre-tension measurements [Jong, 1995]. To meet the needs of our purpose the device had to be adjusted in such a way that the camera has a clear view on a, still to be applied, marker pattern. A schematic representation of the device as used for our purpose is shown in figure 3.1. The pads of the device can be glued to the surface under investigation using cyanoacrylate resin. A spindle is used to drive the translating arms, and the pads attached to them, in opposite directions. The displacement

Figure 3.1: Schematic representation of the tensile device.
of the pads can be prescribed with a constant velocity by means of an electric motor. The surface of a pad glued to the skin is $10 \times 5 \ mm^2$. The displacement of the pads is registered with a linear variable differential transformer (LVDT) and the force on each pad is measured using strain gauged leaf springs.

### 3.2 Measurement of Field Data

This section deals with the procedure followed to measure field data. Section 3.2.1 explains the marker pattern employed. Section 3.2.2 is about the determination of the marker positions. Further, section 3.2.3 explains in what way markers at different times are matched.

#### 3.2.1 Marker Pattern

The size of the marker pattern is restricted to the dimensions of the tensile device. However, this restriction is not the only point of attention. In order to identify each marker relatively easily in successive states of deformation, the distance between the markers has to be a few times the diameter of the marker; this will be explained in section 3.2.3. Furthermore, to obtain enough field information a sufficient number of markers has to be applied. On the other hand, the variance on the determination of the position of a marker is inversely proportional to the diameter of the marker [Peters, 1987]. These conflicting demands resulted in a marker pattern as shown in figure 3.2. The pattern consists of 72 markers, equidistantly placed, with a diameter of approximately 0.4 mm. The pattern can be applied on the skin site under investigation using white oil-paint (Royal Talens, titanium white), in order to obtain enough contrast with respect to the skin. The marker pattern is applied by employing a template. The pads can be glued to the surface on either side of the marker pattern as shown in figure 3.3, schematically representing the proposed camera view.

![Figure 3.2: Design of the marker pattern as used in the experiments.](image)

#### 3.2.2 Determination of Marker Positions

The marker pattern is observed with a CCD video camera, connected to a super-VHS video recorder and an on-line video screen. The camera is positioned perpendicularly above the specimen at a working distance of approximate 580 mm, using a 105 mm lens. The individual positions of the markers are determined using the image processing and analysis system Quantimet 500+. The procedure followed will be outlined below shortly.
3.2. Measurement of Field Data

Figure 3.3: Schematic representation of the camera view: pads and marker pattern.

Figure 3.4: Digitized images of a marker pattern before and after image processing.  

a: original image.  b: binary image using a threshold operation.

From the video recording an image is taken by a frame-grabber resulting in a digitized image. The image size is expressed in pixels (picture elements). The size of the whole image is $752 \times 512$ pixels. The image obtained contains (a maximum of) 256 different grey levels (figure 3.4.a). To identify the white spots as markers the image is made binary using a threshold operation. This operation sets every grey level above the threshold to grey level 255, i.e. white, and every grey level below this threshold to grey level 0, i.e. black. After minor additional image processing it results in a black and white image as shown in figure 3.4.b. The image is colour inverted for the sake of clarity. The positions of the markers are determined using a routine provided by Quantimet 500+ which determines the centres of gravity of a set of (selected) objects of the binary image.

The accuracy of reconstructed marker positions proved to be 0.004 mm for the x-coordinate and 0.003 mm for the y-coordinate (see Appendix B for details).

3.2.3 Matching of Markers

In order to generate the deformation state from the marker positions at different times, each marker has to be followed in time. Therefore, different sets of data for different states of deformation have to be matched. A general way is to use images of successive states of
Chapter 3. Experimental Set-up

definition such that for the change in deformation holds [Peters, 1987]:

\[ \|\vec{u}_i\| < \frac{1}{2} \min_{i,j} \{\|\Delta \vec{x}_{ij}\|\} \quad \forall i; \quad i, j \in \{1, 2, ..., n\} \quad , \quad \text{(3.1)} \]

with:

\( \vec{u}_i \) : displacement vector of the centroid of the i-th marker between two consecutive states,

\( \Delta \vec{x}_{ij} \) : the vector pointing from the centroid of the i-th marker to the centroid of the j-th marker in the first of two consecutive states.

The two consecutive states of the skin site under investigation are denoted as \( C_0 \) and \( C_1 \), and the position vectors of marker \( i \) corresponding to these configurations as \( \vec{x}_0i \) and \( \vec{x}_{1i} \), respectively. For the case defined by equation (3.1), a combination of vectors \( \vec{x}_0i \) and \( \vec{x}_{1j} \) can be found for which \( \Delta_{ij} \) is minimal, where \( \Delta_{ij} \) is defined as

\[ \Delta_{ij} = \|\vec{x}_{1j} - \vec{x}_0i\|. \quad \text{(3.2)} \]

This combination belongs to the same marker in the two different configurations \( C_0 \) and \( C_1 \). With a minimal marker distance of 2 mm, a displacement of the pads of 2 mm may be (theoretically) allowed between two images, neglecting contraction effects of the material.
Chapter 4

Characterization of Human Skin

In this chapter the identification method, as explained in chapter 2, is used to estimate a set of parameters of Lanir's Skin Model. For this purpose an in vivo experiment was carried out. The procedure applied in this experiment will be outlined in the first section. Results of the experiment will be presented and suitability of the experiment will be discussed by means of strain distribution measurements. Section 4.2 describes the numerical model as it is used in the estimation process. In section 4.3 the estimation results for two different assumptions with respect to the in-plane fibre orientation are outlined. Further, section 4.4 describes a verification experiment and discusses the results with respect to the results described in section 4.3. This chapter ends with discussion and concluding remarks.

4.1 In Vivo Experiment

The marker pattern of 72 markers is applied to the medial site of a human forearm of a test person. It has to be noted that the resulting marker pattern is not perfectly rectangular, due to deformation of the skin during appliance. The arm is positioned on a cushion of natural rubber, filled with small polystyrene balls. This cushion can be sucked vacuum, providing a fixation of the arm. Movement of the arm will be reduced to a minimum. The pads are then glued to the surface of the skin, along the two long sides of the marker pattern, as shown in figure 3.3. In this way the load can be applied in longitudinal direction of the arm. The initial distance between the two pads is measured to be approximately 12.5 mm. The prescribed displacement of the pads and the reaction force on the pads are shown in figure 4.1. The pads are moved apart with a constant velocity of 0.24 mm/s for about 5 s. Subsequently, the skin is given time to relax for a period of ca. 100 s. This cycle is repeated twice, with the notice that after the last step the time to relax covers a period of ca. 175 s. It can be seen that the skin is not fully relaxed and the end of each relaxation period. From the video recording four pictures are taken. One from the starting-point of the experiment and three at the end of each relaxation period. From now on, these four situations will be referred to as state 0, 1, 2, and 3.

4.1.1 Strain Distribution Measurements

From marker coordinates, two-dimensional principal Green-Lagrange strain fields can be estimated [Peters, 1987], as is outlined in Appendix C. The principal strain domain, i.e. prin-
Chapter 4. Characterization of Human Skin

Figure 4.1: Prescribed displacement of the pads and reaction force on the pads.

Principal Green-Lagrange strain $E_1$ versus $E_2$ of the current state with respect to the reference state (state 0), for all measured material points, reflects the degree of inhomogeneity of the strain field. A homogeneous strain distribution would be characterized by a single point in the principal strain domain. Figure 4.2 represents the principal strain domain of the first load case, i.e. state 1. The principal strain domain clearly reflects an inhomogeneous strain field. The area covered by the cloud of dots can give an indication about the amount of information the strain field contains [Ratingen, 1994]. A different way to show these results can be seen in figure 4.3 where at every marker position except the outer ones principal strains are represented as line segments, where solid lines represent positive principal strains and dotted lines represent negative strains. Length and direction of these line segments correspond with the (absolute) value and direction of the principal strains.

Considering the principal strain domain obtained, it is assumed that the experiment contains enough information to estimate a limited set of parameters. The fact that the experiment does contain enough information, will be shown in chapter 5 (see also Appendix D).

4.2 Numerical Model

Lanir’s Skin Model is used to describe the material behaviour of the skin under consideration. The experiment is assumed to be two-dimensional. The nodes of the finite element mesh are defined by the measured marker coordinates. This is a pragmatic choice, but effective in case of
1.2. Numerical Model

Principal strain domain

Figure 4.2: Principal strain domain of state 1, based on the measured marker coordinates.

Figure 4.3: Principal strain distribution of state 1. Solid lines represent positive principal strains, dotted lines represent negative principal strains.

simple geometries. Figure 4.4 shows the measured marker field in the reference configuration, i.e. state 0. Distinction is made between observation markers (o) and boundary markers (+), of which the latter will be used to prescribe the kinematic boundary conditions in the finite element model. Figure 4.5 shows the finite element mesh in the reference configuration. Because Lanir's Skin Model is implemented in a 3-D brick element, the one element thick mesh is given an arbitrary thickness of 1 mm. The same in-plane kinematic boundary conditions will be prescribed to the nodes of top and bottom plane. In thickness direction free contraction is allowed, providing a plane-stress situation. Furthermore, the properties of skin within the modelled area are assumed to be homogeneous.

Lanir’s Skin Model contains a lot of different parameters. Because computing effort increases with the number of parameters that has to be estimated, only a limited set of parameters will be used. Because of the local approach (section 2.1.2), it is only possible to estimate stiffness ratios.
Chapter 4. Characterization of Human Skin

4.2.1 Parameters

The implementation of Lanir's Skin Model allows the user to define a discrete fibre density function \( R_k, \rho_0 \). In our case the number of fibre directions is eight. A possible configuration of these eight directions is shown in figure 4.6, and is defined by:

\[
\tilde{r}_0 = \begin{pmatrix}
\tilde{r}_{01} \\
\tilde{r}_{02} \\
\tilde{r}_{03} \\
\tilde{r}_{04} \\
\tilde{r}_{05} \\
\tilde{r}_{06} \\
\tilde{r}_{07} \\
\tilde{r}_{08}
\end{pmatrix} = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\
-\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\
\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\
-\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}}
\end{pmatrix} \begin{pmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z
\end{pmatrix}.
\]

(4.1)

From literature it is known [Finlay, 1969] that the collagen fibres are oriented in a three dimensional but primarily planar wavy array. A reasonable assumption therefore is to model the greater part of the collagen fibres in the xy-plane. For numerical reasons it is needed to model some stiffness in all directions. That is why the four mean diagonal directions are modeled. It is assumed that a small part of the collagen fibres is distributed among these four directions. The elastin fibres are assumed to be distributed isotropically in the skin, i.e. equally divided among the four diagonal directions.

One of the advantages of the use of a structural model is that parameters are physical quantities and some of them can be derived from literature. For collagen and elastin fibres the physical properties are found to be [Finlay, 1969], [Savenije, 1982], [Manschot, 1985], [Lanir et al., 1990]:

- Volume fraction \( S_{\text{collagen}} = 0.35 \) [-]
- Volume fraction \( S_{\text{elastin}} = 0.02 \) [-]
- Stiffness modulus \( K_{\text{collagen}} = 100 \) [N/mm²]
- Stiffness modulus \( K_{\text{elastin}} = 1 \) [N/mm²]
Furthermore it is assumed that the elastin fibres are already straightened in reference configuration [Lanir 1979].

Not much is known about the collagen fibre distribution in the xy-plane and their undulation. As a start, an attempt is made to estimate the four parameters that describe the collagen fibre distribution in the xy-plane. The other parameters are assumed to be known, and set to a fixed value. An overview of all parameters is given in table 4.1. The choice for the parameters that describe the undulation of the collagen fibres, i.e. $\mu$ and $\sigma$, is rather arbitrary. They are assumed to be equal in all directions.

The choice of the distribution of the four fibre directions in the xy-plane according to equation (4.1) is arbitrary as well. It is decided to evaluate a second configuration. The two
Chapter 4. Characterization of Human Skin

<table>
<thead>
<tr>
<th>parameter</th>
<th>collagen</th>
<th>elastin</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K )</td>
<td>100</td>
<td>1</td>
<td>N/mm²</td>
</tr>
<tr>
<td>( S )</td>
<td>0.35</td>
<td>0.02</td>
<td>-</td>
</tr>
<tr>
<td>( R_1 )</td>
<td>t.b.e.</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>( R_2 )</td>
<td>t.b.e.</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>( R_3 )</td>
<td>t.b.e.</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>( R_4 )</td>
<td>t.b.e.</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>( R_5 )</td>
<td>0.05</td>
<td>0.25</td>
<td>-</td>
</tr>
<tr>
<td>( R_6 )</td>
<td>0.05</td>
<td>0.25</td>
<td>-</td>
</tr>
<tr>
<td>( R_7 )</td>
<td>0.05</td>
<td>0.25</td>
<td>-</td>
</tr>
<tr>
<td>( R_8 )</td>
<td>0.05</td>
<td>0.25</td>
<td>-</td>
</tr>
<tr>
<td>( \mu )</td>
<td>1.2</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.2</td>
<td></td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4.1: Overview of all parameters that are present in Lanir’s Skin Model, where ‘t.b.e.’ stands for: to be estimated.

Figure 4.7: Two different configurations for the collagen fibre distribution in the xy-plane; \( R_1 \) to \( R_4 \) are the fractions of collagen fibres oriented in the directions indicated in the reference state.

configurations, denoted as Configuration I and Configuration II, are represented in figure 4.7. It has to be noticed that these configurations can not be resolved into each other.

4.3 Parameter Estimation

To initialize the estimation sequence an initial guess for the parameter values to be estimated, \( \hat{x}_0 \), and an initial guess for the parameter covariance matrix \( P_0 \) is needed. All parameters are considered initially to be mutually independent, i.e. \( P_0 \) is considered to be diagonal.

The three sets of observation marker coordinates of state 1, 2 and 3 are collected in one single observation column of dimension \( 240 \times 1 \) defined as:

\[
y^T = [(x_1)_1, (y_1)_1, (x_2)_1, \ldots, (x_{40})_1, (y_{40})_1, (x_1)_2, \ldots, (x_{40})_2, (y_{40})_2, \ldots, (x_1)_3, (y_3)_{40}].
\]  

This column of observations will be used in the estimation process. Using all experimental data of successive increments in each estimation step has proven to be effective [Starmans, 1994].
The accuracy of the measured positions of the observation markers can be taken into account in the observational covariance matrix $R$. All observational covariances between different observational quantities are assumed to be zero; $R$ is considered to be diagonal.

Confidence in the model can be expressed by the model covariance matrix $Q$. Again, all model covariances between different parameters are assumed to be zero; $Q$ is considered to be diagonal.

Summarizing, this means that only the standard deviations of the parameters, the observational standard deviations, and the model standard deviations have to be defined in the estimation input file.

The various standard deviations have a more or less physical meaning. However, it may also be advantageous to choose their values based on convergence considerations. Especially, when choosing the model standard deviations too small, this may slow down the convergence enormously [Hendriks, 1991], [Starmans, 1994] (Appendix D).

The column of parameters to be estimated is defined by:

$$\mathbf{x} = [R_1 \ R_2 \ R_3 \ R_4]^T.$$  \hspace{1cm} (4.3)

The initial guess is set to:

$$\hat{\mathbf{x}}_0 = [0.2 \ 0.2 \ 0.2 \ 0.2]^T.$$  \hspace{1cm} (4.4)

Further, the parameters have to obey one equality constraint,

$$\sum_i R_i = 0.8, \quad i = 1, 2, 3, 4$$  \hspace{1cm} (4.5)

and four additional inequality constraints based on physical grounds,

$$R_i \geq 0.001, \quad i = 1, 2, 3, 4.$$  \hspace{1cm} (4.6)

Due to historic reasons, the right-hand-side value of the inequality constraints is set to an arbitrary chosen 'small' number, instead of zero.

The initial standard deviations of the parameters to be estimated are set to $10^{-1}$. The choice of $Q$ can be arbitrary but the diagonal elements are often set to $P_{0ii}/100$ [Ratingen, 1994]; the model standard deviations are set to $10^{-2}$. Initially, the standard deviations of the observations are set to the values according to the experimentally determined standard deviations of the marker positions:

$$\sigma_x = 0.004 \text{ mm}, \quad \sigma_y = 0.003 \text{ mm}.$$  \hspace{1cm} (4.7)

However, this choice resulted in bad convergence. From numerical simulations it proved to be effective to set the standard deviation for all observations to $10^{-4} \text{ mm}$ (see Appendix D for details).

For the sake of clarity, the remainder of this section is divided in two main parts:

- **Evaluation I** gives the estimation results in case of the fibre directions in the xy-plane are defined according to Configuration I.

- **Evaluation II** gives the estimation results in case of the fibre directions in the xy-plane are defined according to Configuration II.
4.3.1 Evaluation

In this section the four fractions of the collagen fibres distributed in the xy-plane according to Configuration I are estimated. The estimation sequence of the four parameters is shown in figure 4.8, starting with the initial guess $\hat{x}_0$. It can be observed that all parameters converge to a stable value. Two fractions, i.e. $R_1$ and $R_2$, are forced on the boundary of their inequality constraint. For $R_1$ a possible explanation is that the direction of $R_1$ is defined approximately perpendicular to the loading direction; it does (almost) not contribute to the force transmission. Even more, because the skin contracts in that direction. No such explanation can be found for the fact that it is also hard to attach a value to $R_2$.

![Figure 4.8: Estimation results of Configuration I (Case I).](image)

In an attempt to exclude convergence to a local minimum, the procedure is repeated for two other sets of initial guesses. The various defined standard deviations stay the same. The results of all three attempts are summarized in table 4.2. It can be observed that in all cases the parameters converge to the same value. The only difference is the needed number of estimates, determined by a relative convergence criterium:

$$\chi^2_{\text{rel}} = \frac{\chi^2_k - \chi^2_{k-1}}{\chi^2_k} \leq 10^{-10},$$

where $\chi^2_k$ is defined by:

$$\chi^2_k = (y_k - h_k(\hat{x}_k))^T R_k^{-1} (y_k - h_k(\hat{x}_k)).$$

### Examination of the residuals

The difference between experimental and calculated marker positions, i.e. the residuals, can give an indication of the reliability of the parameters and the constitutive model employed. Defining the column of residuals as:

$$\xi = h(\hat{x}) - y$$

where $h(\hat{x})$ is the calculated position and $y$ is the experimental position.
4.3. Parameter Estimation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case I</th>
<th>Case II</th>
<th>Case III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Initial</td>
<td>Estimate</td>
<td>Initial</td>
</tr>
<tr>
<td></td>
<td>guess</td>
<td>$5^{th}$</td>
<td>guess</td>
</tr>
<tr>
<td>$R_1$</td>
<td>0.2</td>
<td>0.001</td>
<td>0.35</td>
</tr>
<tr>
<td>$R_2$</td>
<td>0.2</td>
<td>0.001</td>
<td>0.25</td>
</tr>
<tr>
<td>$R_3$</td>
<td>0.2</td>
<td>0.618</td>
<td>0.15</td>
</tr>
<tr>
<td>$R_4$</td>
<td>0.2</td>
<td>0.180</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 4.2: Estimation results for three different initial guesses (Configuration I).

results in an easy interpretation of the residuals. A positive residual implies a deviation of the calculated position compared to the measured position in positive x- or y-direction. For the sample mean of the residuals, a similar conclusion holds with respect to the 'mean' orientation of the residuals. In the ideal case of no modelling errors, the distribution of the residual field should be random, and the order of magnitude of the residuals should be equal to the observational error (see appendix D for details). Figure 4.9 shows the residual fields for each state of deformation, using the final estimates for the parameters of Case I presented in table 4.2. The measured positions of the markers are represented by circles ($\circ$). The plus-signs (+) represent the estimated positions of the markers. In order to clarify the picture, the residuals are multiplied by a factor 10.

Figure 4.9: Residuals for the three successive states of deformation 1, 2 and 3 (Configuration I, Case I). The measured positions of the markers are represented by circles ($\circ$). The plus-signs (+) represent the estimated positions of the markers. The residuals are multiplied by a factor 10.

To evaluate the size of the residuals and their structure, the sample mean and standard deviation of the residuals are determined. For this purpose, distinction is made between the x- and the y-component of the residuals as well as the state of deformation. The results for the three different cases are summarized in table 4.3. From these results it can be observed that the size of the (standard deviations of the) residuals increases with increasing load. The increase in x-direction is stronger than in y-direction, which is also reflected by figure 4.9. Note the relatively large standard deviation in x-direction of state 3. Furthermore, it can be
observed that the standard deviations in table 4.3 differ significantly from the experimentally determined standard deviations, i.e. $\sigma_x = 0.004 \text{ mm}$ and $\sigma_y = 0.003 \text{ mm}$. This, and the fact that the residual fields seem to exhibit some structure, is an indication for possible modelling errors.

<table>
<thead>
<tr>
<th>State x</th>
<th>1</th>
<th>y</th>
<th>State x</th>
<th>2</th>
<th>y</th>
<th>State x</th>
<th>3</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu \text{ mm}$</td>
<td>-0.010</td>
<td>0.030</td>
<td>-0.010</td>
<td>0.030</td>
<td>-0.010</td>
<td>0.030</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma \text{ mm}$</td>
<td>0.021</td>
<td>0.034</td>
<td>0.021</td>
<td>0.034</td>
<td>0.021</td>
<td>0.034</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu \text{ mm}$</td>
<td>-0.015</td>
<td>0.049</td>
<td>-0.015</td>
<td>0.049</td>
<td>-0.015</td>
<td>0.049</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma \text{ mm}$</td>
<td>0.021</td>
<td>0.046</td>
<td>0.021</td>
<td>0.046</td>
<td>0.021</td>
<td>0.046</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu \text{ mm}$</td>
<td>-0.026</td>
<td>0.107</td>
<td>-0.026</td>
<td>0.107</td>
<td>-0.026</td>
<td>0.107</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma \text{ mm}$</td>
<td>0.028</td>
<td>0.060</td>
<td>0.028</td>
<td>0.060</td>
<td>0.028</td>
<td>0.060</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.3: Sample mean and standard deviation of the residuals (Configuration I).

### 4.3.2 Evaluation II

In this section the four fractions of the collagen fibres distributed in the xy-plane according to Configuration II are estimated. The same value for the relative convergence criterium, $\chi^2_{rel}$, is used.

The estimation sequence of the four parameters is shown in figure 4.10. It can be observed that all four parameters converge to a stable value. Unlike the previous section, none of the parameters is forced to its constraint. A possible explanation is that all fractions contribute to
4.3. Parameter Estimation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case I Initial Estimate</th>
<th>Case II Initial Estimate</th>
<th>Case III Initial Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>guess gth</td>
<td>Estimate 9th</td>
<td>Estimate 13th</td>
</tr>
<tr>
<td>$R_1$</td>
<td>0.2</td>
<td>0.1532</td>
<td>0.1467</td>
</tr>
<tr>
<td>$R_2$</td>
<td>0.2</td>
<td>0.3732</td>
<td>0.3753</td>
</tr>
<tr>
<td>$R_3$</td>
<td>0.2</td>
<td>0.2497</td>
<td>0.2496</td>
</tr>
<tr>
<td>$R_4$</td>
<td>0.2</td>
<td>0.0239</td>
<td>0.0284</td>
</tr>
</tbody>
</table>

Table 4.4: Estimation results for three different initial guesses (Configuration II).

the force transmission because none of the directions is defined perpendicular to the loading direction. Further, previous research found it to be effective to choose the material symmetry angle different from the loading angle [Ratingen, 1992].

The estimation results, and again the estimation results for different initial guesses, are summarized in table 4.4. It can be observed that in all cases the parameters converge approximately to the same values. Note the difference in the number of estimates needed to achieve these results.

**Examination of the residuals**

Figure 4.11 shows the residual fields for each state of deformation, using the final estimates for the parameters of Case I presented in table 4.4.

![State 1](image1.png)  ![State 2](image2.png)  ![State 3](image3.png)

Figure 4.11: Residuals for the three successive states of deformation 1, 2, and 3 (Configuration II, Case I). The measured positions of the markers are represented by circles (o). The plus-signs (+) represent the estimated positions of the markers. The residuals are multiplied by a factor 10.

The sample mean and the standard deviation are summarized in table 4.5. It can be observed that the size of the residuals increases with increasing load, although the increase is not as strong as the results of Configuration I. The size of the (standard deviations of the) residuals in y-direction of Configuration II are smaller than those of Configuration I. For the x-direction this can only be observed for state 3.
Chapter 4. Characterization of Human Skin

4.4 Verification Experiment

To verify the results obtained, a verification experiment is performed. Section 4.4.1 describes a first experiment, performed in longitudinal direction, and its estimation results. The estimation process is performed using both Configuration I and Configuration II of the collagen fibre distribution in the xy-plane as defined in section 4.2.1. Section 4.4.2 describes a second experiment that is performed in transverse direction, i.e. approximately perpendicular to the first experiment. This experiment is simulated numerically employing the final parameter estimation results of the longitudinal experiment. The difference between observed and predicted marker positions is used to verify the results of section 4.3 and the model employed. These two experiments are performed on the same day. It has to be noted that they are not performed on the same day as the experiment described in section 4.1. Ambient conditions and the physical condition of the test person could be different. Furthermore, the skin site under investigation is not likely to be exactly the same.

4.4.1 Longitudinal Experiment

The procedure followed is the same as described in section 4.1 with the only difference that the pads are moved apart with a constant velocity of 0.20 mm/s for about 5 s. This results in a displacement step of approximately 1 mm instead of 1.2 mm. The prescribed displacement of the pads and the reaction force on the pads are shown in figure 4.12. The displacement signal is slightly disturbed by some noise; this is ascribed to inappropriate filtering.

Parameter Estimation

The four fractions of the collagen fibre distribution in the xy-plane are estimated. The estimation process is performed for both Configuration I and Configuration II. The initial guess for both configurations is set to:

\[
\hat{x}_0 = [R_{10}, R_{20}, R_{30}, R_{40}]^T = [0.2, 0.2, 0.2, 0.2]^T. \tag{4.11}
\]

The same equality and inequality constraints have to be obeyed. The various standard deviations are set to the same values as used in Evaluation I, section 4.3.1. The same value for the relative convergence criterion, \(\chi^2_{rel}\), is used.
4.4. Verification Experiment

The estimation results are summarized in table 4.6. Employing different initial guesses for the parameters resulted in approximately the same estimation results. Comparing the results with the results of section 4.3, it can be stated that they are in reasonable agreement with our expectations, although the results of Configuration I seem to correspond better. This might be an indication of repeatability. Note that this experiment is not performed exactly on the same skin site and in the same direction as the experiment described in section 4.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Configuration I</th>
<th></th>
<th>Configuration II</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial guess</td>
<td>Estimate</td>
<td>Initial estimate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10th</td>
<td>19th</td>
<td>10th</td>
<td>19th</td>
<td></td>
</tr>
<tr>
<td>$R_1$</td>
<td>0.2</td>
<td>0.0010</td>
<td>0.2</td>
<td>0.0010</td>
</tr>
<tr>
<td>$R_2$</td>
<td>0.2</td>
<td>0.0540</td>
<td>0.2</td>
<td>0.3969</td>
</tr>
<tr>
<td>$R_3$</td>
<td>0.2</td>
<td>0.5250</td>
<td>0.2</td>
<td>0.2566</td>
</tr>
<tr>
<td>$R_4$</td>
<td>0.2</td>
<td>0.2200</td>
<td>0.2</td>
<td>0.1455</td>
</tr>
</tbody>
</table>

Table 4.6: Estimation results of the longitudinal experiment for both Configuration I and Configuration II.

For Configuration I it can be seen that $R_3$ is lowered by approximately 0.1. This is spread out approximately equally over $R_2$ and $R_4$. It can be seen that again $R_1$ is forced on the boundary of its inequality constraint. For Configuration II there seems to be an exchange
between $R_1$ and $R_4$, where $R_1$ is forced on the boundary of its inequality constraint.

The sample mean and the standard deviation of the residuals are summarized in table 4.7. It can be seen that the order of magnitude of the standard deviations of the residuals is the same as those presented in section 4.3.

<table>
<thead>
<tr>
<th></th>
<th>Configuration I</th>
<th></th>
<th>Configuration II</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu \text{ mm}$</td>
<td>$\sigma \text{ mm}$</td>
<td>$\mu \text{ mm}$</td>
<td>$\sigma \text{ mm}$</td>
</tr>
<tr>
<td>State x 1</td>
<td>0.043</td>
<td>0.041</td>
<td>0.039</td>
<td>0.035</td>
</tr>
<tr>
<td>2</td>
<td>0.019</td>
<td>0.027</td>
<td>0.015</td>
<td>0.018</td>
</tr>
<tr>
<td>State x 2</td>
<td>0.029</td>
<td>0.041</td>
<td>0.025</td>
<td>0.025</td>
</tr>
<tr>
<td>2</td>
<td>0.019</td>
<td>0.038</td>
<td>0.012</td>
<td>0.021</td>
</tr>
<tr>
<td>State x 3</td>
<td>0.017</td>
<td>0.099</td>
<td>0.016</td>
<td>0.062</td>
</tr>
<tr>
<td>2</td>
<td>0.006</td>
<td>0.059</td>
<td>0.000</td>
<td>0.031</td>
</tr>
</tbody>
</table>

Table 4.7: Sample mean and standard deviation of the residuals of the longitudinal experiment for both Configuration I and Configuration II.

### 4.4.2 Transverse Experiment

The procedure followed is the same as described in the previous section with the difference that it is performed in transverse direction, i.e. perpendicular to the longitudinal experiment. As accurate as possible, the transverse experiment is performed at the same position as the longitudinal experiment, i.e. the centre of gravity of the marker pattern is approximately the same for both experiments. The prescribed displacement of the pads and the reaction force on the pads are shown in figure 4.13.

**Simulation**

The transverse experiment is simulated numerically. The finite element mesh and the kinematic boundary conditions are deduced from the experiment carried out. The FE-analysis is performed employing the final estimates of the longitudinal experiment, taking into account the rotation of $90^\circ$ of the local coordinate system. The FE-analysis is performed for both Configuration I and Configuration II of the collagen fibre distribution in xy-plane. The four parameters that describe this distribution are set to:

\[
\tilde{\mathbf{z}} = [R_1 \ R_2 \ R_3 \ R_4]^T = [0.0540 \ 0.0010 \ 0.2200 \ 0.5250]^T. \quad (4.12)
\]

for Configuration I and

\[
\tilde{\mathbf{z}} = [R_1 \ R_2 \ R_3 \ R_4]^T = [0.2566 \ 0.1455 \ 0.0010 \ 0.3969]^T. \quad (4.13)
\]

for Configuration II respectively. The predicted marker positions can now be compared with the observed marker positions of the experiment. The sample mean and standard deviation of the residuals are summarized in table 4.8. It can be seen that the standard deviations for Configuration I are more or less the same as those presented in table 4.7. This could be an indication of consistency. The standard deviations for Configuration II show a significant increase compared to the standard deviations presented in table 4.7, except $\sigma_x$ of state 1.
This result does not support the previous stated concerning consistency; this could indicate the presence of modelling errors.

![Displacement of the pads against time](image)

![Force on the pads against time](image)

Figure 4.13: Prescribed displacement of the pads and reaction force on the pads of the transverse experiment.

<table>
<thead>
<tr>
<th></th>
<th>Configuration I</th>
<th>Configuration II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \mu ) mm</td>
<td>( \sigma ) mm</td>
</tr>
<tr>
<td>State x 1</td>
<td>-0.008</td>
<td>0.031</td>
</tr>
<tr>
<td>State x 2</td>
<td>-0.009</td>
<td>0.032</td>
</tr>
<tr>
<td>State x 3</td>
<td>-0.012</td>
<td>0.105</td>
</tr>
<tr>
<td>State y 1</td>
<td>-0.032</td>
<td>0.022</td>
</tr>
<tr>
<td>State y 2</td>
<td>-0.028</td>
<td>0.037</td>
</tr>
<tr>
<td>State y 3</td>
<td>-0.044</td>
<td>0.059</td>
</tr>
</tbody>
</table>

Table 4.8: Sample mean and standard deviation of the residuals of the transverse experiment for both Configuration I and Configuration II.

### 4.5 Discussion and Conclusions

In this chapter a mechanical characterization of skin was presented, employing a structural skin model, i.e. Lanir's Skin Model. An attempt was made to estimate a limited set of parameters, describing the collagen fibre distribution in the xy-plane. This was done for two
different configurations for this distribution. Further, a verification experiment was performed in order to examine the results on consistency.

Evaluation of the results leads to the following general conclusions:

- The estimation process is successful for both configurations; the parameters converge to a stable value and the same parameter values are found for different initial guesses.
- Judging the residuals, the results seem to be influenced by modelling errors; the residuals are larger than expected and the residual fields exhibit structure, rather than randomness. Further, the size of the residuals show an increase with increasing load.

Modelling errors can be attributed to a number of reasons:

- The assumption that the measured surface is perfectly flat; recognition of slight curvature would be more realistic.
- The assumption that the influence of pre-tension is negligible.
- The assumption that the properties within the modelled area are homogeneous.
- Lanir’s Skin Model is restricted to the description of the dermis; the influence of the epidermis and hypodermis is neglected.
- Modelling elastic behaviour; skin exhibits strong viscoelastic behaviour.
- The fact that the skin was not fully relaxed at the time the marker coordinates were determined; even more, is it allowed at all to neglect the history.

The following conclusions result from the verification experiment:

- The estimation results seem to be consistent concerning Configuration I; the size of the residuals of the simulation of the transverse experiment correspond with the size of the residuals obtained with the final estimates of the longitudinal experiment.
- The previous stated, is not supported by the results of Configuration II; the residuals of the simulation of the transverse experiment are significantly larger than those obtained with the final estimates of the longitudinal experiment; this can indicate the presence of modelling errors.

In the next chapter different experiments are simulated to find indications for possible improvements of the experimental set-up.
Chapter 5

Numerical Simulations

To obtain evidence for possible improvements of the experimental set-up, different experiments are simulated numerically. A possible improvement of the experiment is a two-dimensional loading situation instead of the current one-dimensional loading situation. Indications for improved estimation results in case of a two-dimensional loading situation were found in previous research [Ratingen, 1994].

5.1 Numerical Experiments

Two numerical experiments are evaluated: An experiment subjected to a one-dimensional loading situation and an experiment subjected to a two-dimensional loading situation. These two experiments will be denoted as experiment A and experiment B respectively. For the simulation of these experiments, a mesh as obtained from a former performed pilot experiment is used in order to maintain resemblance with the real experiment. The finite element meshes, in undeformed and deformed state, as well as the principal strain domain of state 1 are shown in figure 5.1. From the principal strain domains it can be seen that experiment B exhibits a stronger inhomogeneous strain field than experiment A; judged by the area covered by the cloud of dots, experiment A seems to contain more information than experiment B. The same could be observed in the principal strain domains of state 2 and 3.

The kinematic boundary conditions prescribed in the finite element model of experiment A are obtained from the real pilot experiment; they are prescribed on all boundary nodes. The kinematic boundary conditions of experiment B are prescribed in such a way to simulate a two-dimensional loading situation. The kinematic boundary conditions in y-direction are more or less deduced from the real experiment. They are prescribed on the nodes marked with a circle (o). On the nodes marked with a cross (x), kinematic boundary conditions are prescribed in x-direction. The choice of these boundary conditions is rather arbitrary but of the same order of magnitude as the kinematic boundary conditions in y-direction. On the remaining nodes no kinematic boundary conditions are prescribed.

An overview of the parameters is given in table 4.1. In the FE-analysis performed to generate ‘measured’ data, Configuration I (figure 4.7) for the collagen fibre distribution is used. The parameters that will be estimated are set to:

\[
\mathbf{x} = [R_1 \ R_2 \ R_3 \ R_4]^T = [0.10 \ 0.25 \ 0.15 \ 0.30]^T.
\]

Observational errors are simulated by means of an artificial disturbance of the ‘measured’ data. The calculated positions are disturbed by adding a normally distributed noise with
a standard deviation according to the experimentally determined standard deviations, i.e. \( \sigma_x = 0.004 \text{ mm} \) and \( \sigma_y = 0.003 \text{ mm} \) (see Appendix B for details).

The four parameters mentioned will be estimated for both experiments. Note that modeling errors are not present, since exactly the same model is used for the estimation process as for the generation of 'measured' data. In order to compare the two different experiments the various standard deviations as well as the relative convergence criterium are set to the same values. The initial standard deviations of the parameters are set to 0.05 and the model standard deviations are set to 0.005. The observational standard deviations are set to \( 10^{-4} \text{ mm} \). The relative convergence criterium, i.e. \( \chi^2_{rel} \), is set to \( 10^{-5} \). The estimation results are summarized in table 5.1.

Figure 5.1: Experiment A and B; left: finite element mesh in undeformed and deformed state (state 1). Nodes of which the kinematic boundary conditions are prescribed in x-direction or y-direction are marked with a cross (x) or a circle (o) respectively. right: accompanying principal strain domain of state 1.

In both experiments the estimation process is successful; all parameters converge towards their true parameter values. Further, it can be stated that the final residuals were reduced to the observation error applied. Although the principal strain domain of the two-dimensional experiment suggests that it would contain more information, this is not supported by the needed number of estimates. The question that arises now is to what extent it is allowed to compare these two configurations; the number of kinematic boundary conditions differs strongly. This is investigated by simulating two more experiments. These experiments will be denoted as C and D.
5.1. Numerical Experiments

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Experiment A</th>
<th></th>
<th>Experiment B</th>
<th></th>
<th>True value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Initial guess</td>
<td>Estimate</td>
<td>Initial guess</td>
<td>Estimate</td>
<td>value</td>
</tr>
<tr>
<td>$R_1$</td>
<td>0.2</td>
<td>0.1005</td>
<td>0.2</td>
<td>0.1002</td>
<td>0.10</td>
</tr>
<tr>
<td>$R_2$</td>
<td>0.2</td>
<td>0.2504</td>
<td>0.2</td>
<td>0.2495</td>
<td>0.25</td>
</tr>
<tr>
<td>$R_3$</td>
<td>0.2</td>
<td>0.1501</td>
<td>0.2</td>
<td>0.1496</td>
<td>0.15</td>
</tr>
<tr>
<td>$R_4$</td>
<td>0.2</td>
<td>0.2989</td>
<td>0.2</td>
<td>0.3007</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Table 5.1: Estimation results of experiment A and B.

Figure 5.2: Experiment C and D; left: finite element mesh in undeformed and deformed state (state 1). Nodes of which the kinematic boundary conditions are prescribed in y-direction are marked with a circle (o). right: accompanying principal strain domain of state 1.

Experiment C is a fictitious one-dimensional experiment, where the kinematic boundary conditions prescribed in y-direction are the same as in experiment B; there are no kinematic boundary conditions prescribed in x-direction. Experiment D is an extension of experiment C, where the amount of input information is the same as experiment B, i.e. the same number of kinematic boundary conditions is prescribed. The finite element meshes, in undeformed and deformed state, as well as the principal strain domain of state 1 are shown in figure 5.2.
Chapter 5. Numerical Simulations

The amount of information, judged by the area covered by the cloud of dots, seems to be approximately the same for both experiments. Compared to experiment B, these experiments seem to contain less information.

The results of the estimation process are presented in table 5.2. Again, the estimation process is successful for both experiments. The parameters converge towards their true value and the final residuals were reduced to the observation error applied. The parameter values of experiment C converge slower towards their true values than experiment B. This supports the findings of van Ratingen [Ratingen, 1994], that a two-dimensional loading situation leads to faster convergence. However, the needed number of estimates of experiment D is much smaller than of experiment C, i.e. 9 versus 15 estimates. This could be an indication that not only the amount of information the principal strain domain contains, is of importance. Also the amount of kinematic input information provided to realize a certain principal strain field seems to be of major influence on the convergence speed.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Experiment C</th>
<th>Experiment D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Initial guess</td>
<td>15th estimate</td>
</tr>
<tr>
<td>$R_1$</td>
<td>0.2</td>
<td>0.0968</td>
</tr>
<tr>
<td>$R_2$</td>
<td>0.2</td>
<td>0.2516</td>
</tr>
<tr>
<td>$R_3$</td>
<td>0.2</td>
<td>0.1502</td>
</tr>
<tr>
<td>$R_4$</td>
<td>0.2</td>
<td>0.3013</td>
</tr>
</tbody>
</table>

Table 5.2: Estimation results of experiment C and D.

5.2 Discussion and Conclusions

In this chapter different experiments were simulated to find indications for possible improvements of the experimental set-up. Observation errors were simulated by means of an artificial disturbance of the 'measured' data. The four parameters, that describe the collagen fibre distribution in the xy-plane, were estimated.

The results obtained lead to the following conclusions:

- There is no direct evidence that a two-dimensional loading situation would improve the convergence speed.
- It is difficult to assess the amount of information an experiment contains on the principal strain domain, judged by the area covered by the cloud of dots.
- Indications were found that not only the degree of inhomogeneity of the principal strain field is of importance. Strong evidence was found that the amount of kinematic input information provided to realize a certain principal strain field is of major influence on the convergence speed.

One has to bear in mind that this conclusion is based on the results of these specific cases. More research on this would be appropriate in order to investigate its general validity.
Chapter 6

Conclusions and Recommendations

In this report an in-plane mechanical characterization of human skin in vivo, employing Lanir's Skin Model, was presented.

An in vivo experiment was performed generating measured field data. Employing a numerical-experimental method, a limited set of four parameters was estimated, describing the collagen fibre distribution in the xy-plane. This was done for two different configurations of this distribution. The results were examined on consistency by means of a verification experiment. Further, different experiments were simulated numerically to find indications for possible improvements of the experimental set-up.

It has to be noted that the characterization was restricted to the medial site of a forearm of one test person. Besides dependence on direction it is known that mechanical properties of skin can depend on site, sex, race, age, and all kinds of ambient conditions. Nevertheless, the results obtained gain insight in the mechanical behaviour of human skin.

Conclusions:

- It is possible to estimate the four parameters, describing the collagen fibre distribution in the xy-plane. This was demonstrated for both Configuration I and Configuration II of this distribution. Their values gained confidence due to the same results obtained starting from different initial guesses.

- The results seem to be influenced by modelling errors; the residuals are larger than expected and the residual fields exhibit structure, rather than randomness. Further, the size of the residuals demonstrate an increase with increasing load.

- The verification experiment showed that the results concerning Configuration I demonstrate consistency; the size of the residuals of the simulation of the transverse experiment correspond with the size of the residuals obtained with the final estimate of the longitudinal experiment. This is not supported by the results concerning Configuration II, where the difference in the size of the residuals may be an indication of the presence of modelling errors.

- Numerical simulations showed that the experiment as performed (a one-dimensional loading situation) contains enough information to track back the four parameters, describing the collagen fibre distribution, in case no modelling errors are present. No direct evidence was found that a two-dimensional loading situation would improve the convergence speed.
It is difficult to assess the amount of information an experiment contains on the principal strain domain, judged by the area covered by the cloud of dots. Indications were found that not only the degree of inhomogeneity of the principal strain field is of importance. Strong evidence was found that the amount of kinematic input information provided to realize a certain principal strain field is of major influence on the convergence speed.

The system available now is a useful tool to increase the knowledge on the in-plane mechanical behaviour of human skin, that can contribute to the improvement of existing shavers and the development of new shaving systems. Continuation of the work presented will be useful and offers numerous challenging possibilities for future research.

Recommendations:

- The measuring system used here is relatively simple, using a standard CCD camera and super-VHS video recordings. The measuring error can be reduced using a digital camera; this also will facilitate the overall use of the measuring system. However, it should be taken into consideration that this option is expensive. Further, an extension to 3-D marker coordinates can be achieved using two cameras instead of one. It should be realized that this adjustment implies a much more complex approach and thus makes it questionable if this option is feasible.

- The characterization presented in this report was restricted to the in-plane mechanical behaviour of human skin. It is questionable if it is allowed to restrict oneself to the description of the dermis. A pilot measurement, where the skin was hydrated, showed that the influence of the epidermis concerning the in-plane mechanical behaviour of skin cannot be neglected. It is suggested to investigate the influence of hydration of skin and with that the influence of the epidermis. The influence of hydration of skin can be a very interesting subject of research for all kinds of personal care applications.

- In the model employed the orientation distribution of the fibres is prescribed by a discrete density distribution function. It is suggested to implement a continuous density distribution function. Besides that it is more elegant, it has the important advantage that the number of parameters will stay acceptable. However, sufficient accurate integration over all directions will result in an increase of computing effort. An example of such a distribution function, using only two parameters, is given by Lanir [Lanir et al., 1996].

- It is desired to obtain more information about the description and the degree of the undulation. For this purpose information about the absolute stiffness will eventually be indispensable. The force signal of the pads can yield useful information. For example the measured force can be added to the column of observations; in the estimation process this can be compared with a force resulting from calculated reaction forces on nodes ‘near’ a pad. However, the exact course of action is yet uncertain.

- The experiments performed demonstrated strong viscoelastic effects. Within the Personal Care Institute a start has been made to investigate these effects; this should be continued. For future research it is suggested to add viscoelasticity to Lanir’s Skin Model, supplying the collagen fibres with viscoelasticity.
To incorporate the behaviour of the third dimension and with that the possible influence of hypodermis and underlying tissue, it is suggested to simulate indentation tests numerically employing Lanir's Skin Model; experimental data of these tests is already available, which is convenient.

Until now the displacement of the pads is prescribed more or less by hand. It is strongly suggested to provide the displacement of the pads with computer control, for example using LabVIEW. It will then be possible to prescribe displacement cycles repeatable and more accurate. Besides, in this way the force can be controlled, building in safety; this is important performing *in vivo* experiments. This adjustment will not only be useful for the sequel of this research. The surface tensile device can then be used in a much broader manner. The viscoelastic behaviour of skin can be investigated by means of relaxation curves. Further, it will be possible to investigate the effect of prescribed history on skin.

Finally, it is suggested to reconsider the anatomy of skin of the face and the neck. It was revealed that the anatomy of the skin of the face and the neck is essentially different from the rest of the body, in particular the musculature (making grimaces) and the connection to the underlying tissue [Huson, 1995]. There can be important differences moving a shaving head along the surface of the face or moving it along the surface of an arm.
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Appendix A

Stress-Elongation Law

In this appendix the relation between stress and elongation for the fibres present is given. First, this relation is given for straightened fibres (section A.1). Section A.2 explains in more detail the relation for wavy fibres.

A.1 Straightened Fibres

For straightened fibres linear elastic behaviour is assumed. Further, because the fibres are assumed to be thin and perfectly flexible, they have no compressive stiffness. For straightened fibres the stress-elongation relation $f_k(\lambda)$ is then given by:

$$f_k(\lambda) = \begin{cases} 0 & \text{for } \lambda \leq 1 \\ K_k (\lambda - 1) & \text{for } \lambda > 1, \end{cases} \quad (A.1)$$

with $K_k$ the stiffness constant of fibre type $k$.

A.2 Wavy Fibres

The undulation behaviour of fibre type $k$ is accounted for in the stress-elongation relation $f_k^*(\lambda)$. Figure A.1 illustrates how the effect of the undulation is taken into account for a set of fibres. In this figure the stress-elongation curves for four fibres of type $k$ with orientation vector $\vec{r}_0$ are shown. Fibre $i$ remains unstraightened until a stretch ratio of $X_i = l_i/l_0$. Here, $l_0$ is the fibre length along $\vec{r}_0$ in the unstrained undulated state and $l_i$ is the fibre length in the unstrained straightened state (figure A.2). From this point on the fibre is strained. To describe the behaviour of an undulated fibre a new variable, the effective stretch ratio $\lambda^*_i$, is introduced:

$$\lambda^*_i = \frac{l_i}{l_i} = \frac{l_i}{l_0} \frac{l_0}{l_i} = \frac{\lambda}{X_i} \quad (A.2)$$

The portion of fibres, that straightens between a stretch of $X$ and $X+dX$, equals $D_{k,\vec{r}_0}(X)dX$, where $D_{k,\vec{r}_0}(X)$ is a distribution function. So the contribution of the portion of fibres that
Appendix A. Stress-Elongation Law

![Stress-Elongation Curve Diagram]

Figure A.1: The stress-elongation curve for an undulated fibre.

<table>
<thead>
<tr>
<th>State</th>
<th>Stretch ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>undulated and unstrained</td>
<td>1</td>
</tr>
<tr>
<td>straightened and unstrained</td>
<td>( l_1/l_0 = X_i )</td>
</tr>
<tr>
<td>strained</td>
<td>( l_\lambda/l_0 = \lambda )</td>
</tr>
</tbody>
</table>

Figure A.2: Fibre shapes at different stretch states.

straighten between a stretch of \( X \) and \( X + dX \) to the total load per unit unstrained cross-sectional area (in fibres of type \( k \) that are oriented in direction \( \vec{r}_0 \)) with stretch ratio \( \lambda \) is equal to

\[
f_k(\frac{\lambda}{X}) \int D_k,\vec{r}_0(X)\,dX.
\]  

(A.3)

Performing a summation over all fibres with different straightening strain, the total load per unit unstrained cross-sectional area of fibres of type \( k \) that are oriented in direction \( \vec{r}_0 \) can be written as:

\[
f_k^*(\lambda) = \int_{X = 1.0}^{\lambda} D_k,\vec{r}_0(X) \cdot f_k(\frac{\lambda}{X})\,dX.
\]  

(A.4)
Given a portion \( a \) of fibres of type \( k \), oriented in direction \( \vec{r}_0 \) and already straightened at reference configuration, equation (A.4) can be written as:

\[
f_{k,\vec{r}_0}^*(\lambda) = a_{k,\vec{r}_0} f_{k,\vec{r}_0}(\lambda) + \int_{\lambda = 1.0}^{\lambda} D_{k,\vec{r}_0}(X) f_{k,\vec{r}_0}(\frac{\lambda}{X}) dX. \tag{A.5}
\]

The undulation is assumed to be normally distributed, \( N(\mu, \sigma) \). Then, the distribution function \( D_{k,\vec{r}_0}(X) \) is defined as:

\[
D_{k,\vec{r}_0}(X) = \frac{1}{\sigma_{k,\vec{r}_0} \sqrt{2\pi}} e^{-\frac{(X - \mu_{k,\vec{r}_0})^2}{2\sigma_{k,\vec{r}_0}}} \tag{A.6}
\]

Figure A.3: The distribution function \( D_{k,\vec{r}_0}(X) \).

Accounting for the portion of fibres that is already straightened in reference configuration, the distribution function can be represented as shown in figure A.3. In case of a normal distribution the portion of fibres that is already straightened in reference configuration equals

\[
a_{k,\vec{r}_0} = \int_{-\infty}^{X = 1.0} D_{k,\vec{r}_0}(X) dX. \tag{A.7}
\]
Appendix B

Accuracy of Marker Position Measurement

This appendix describes the experiment that was carried out in order to determine the accuracy of the reconstructed marker positions.

B.1 Measuring System

A schematic representation of the experimental set-up is shown in figure B.1. The entire measuring system is rather complex because the camera signal undergoes two more conversions before the marker positions are reconstructed (figure B.2). Because the complexity of the system, it will be considered as a black box.

B.2 Experimental Set-up

A piece of rubber, with the marker pattern on it, is positioned on an optical table. The CCD camera is positioned perpendicularly above the test specimen at a working distance...
of approximate 580 mm. The stationary marker pattern is recorded on video for about 50 seconds.

From the stationary situation ten images are taken with Quantimet 500+, the image processing and analysis system. For each image the positions of the 72 markers are determined. For each marker the standard deviation for the x-centroid as well as the y-centroid is determined. With these, the standard deviation $s$ on the mean position of a marker can be derived. The standard deviation measured in x-direction is $s_x = 0.004 \text{ mm}$. In y-direction a standard deviation $s_y = 0.003 \text{ mm}$ is measured. Peters [Peters, 1987] showed that the variance due to the discretization is given by:

$$
\sigma^2 = \frac{1}{22p^3d} \text{ mm}^2,
$$

where $p$, the scan resolution, is the number of pixels per millimeter and $d$ is the diameter of the marker. Given a scan resolution $p = 25.2 \text{ pixels/mm}$ and diameter $d = 0.4 \text{ mm}$, the theoretical standard deviation on the x- or y-centroid is $s = 0.0027 \text{ mm}$.

The difference between measured standard deviation in x- and y-direction can be explained by the mechanical instability of the video tape, i.e. 'jitter', that is known to be stronger in the x-direction than in the y-direction.

Figure B.2: Measuring system presented as a block scheme.
Appendix C

Strain Estimation from Measured Marker Coordinates

The present characterization method uses marker displacements to determine the material properties. From the marker displacements local strains can be estimated [Peters, 1987].

In this appendix a short outline is given of a general three dimensional model for the estimation of the deformation tensor $F$ and the Green-Lagrange strain tensor $E$. For this, a group of markers is considered. The strain at the position of the central marker of this group is based on the displacement of each of the other markers.

C.1 Introduction

Consider a body $M$ in the reference configuration at $t = t_0$ and in arbitrary configuration at $t = t_1$ (figure C.1). P and Q represent two marker centroids, whereas the vectors connecting P and Q in both configurations are denoted $\Delta \vec{x}_0$ and $\Delta \vec{x}_1$ respectively. Theoretically, when $\Delta \vec{x}_0$ and $\Delta \vec{x}_1$ are infinitely small (i.e. $\Delta \vec{x}_0 \to d\vec{x}_0$ and $\Delta \vec{x}_1 \to d\vec{x}_1$), the vector at time $t_1$ is related to the vector at time $t_0$ by

$$d\vec{x}_1 = F \cdot d\vec{x}_0,$$

(C.1)
where \( F \) is the deformation tensor. Considering finite length of vectors \( \Delta \bar{x}_0 \) and \( \Delta \bar{x}_1 \), a modelling error vector \( \bar{f} \) in \( P \) is written as
\[
\bar{f} = \Delta \bar{x}_1 - F \cdot \Delta \bar{x}_0.
\] (C.2)

This expression is the starting point for the estimation of strain quantities from measured marker coordinates. Due to the measurement errors and the modelling errors, an estimate \( \hat{F} \) of the deformation tensor \( F \) in a marker point can be derived. This is done by considering a group of markers close together. With \( \hat{F} \) an estimate \( \hat{E} \) of the Green-Lagrange strain tensor \( E \) can be determined.

### C.2 Estimation of Deformation Quantities

Consider a group of \( n \) markers close together of which the coordinates are measured in two configurations (figure C.2). Such a group of markers is denoted as a 'strain group'. The vectors \( \Delta \bar{x}_{0i} \) and \( \Delta \bar{x}_{1i} \), which connect the central marker \( P \) to the rest of the markers in the group, can be calculated. These vectors contain measurement errors. When the positions of the markers in a strain group are chosen arbitrarily, the corresponding \( \bar{f}_i \) at each point is assumed to consist of a constant part \( \bar{f} \) and an variable part \( \delta \bar{f}_i \). According to equation (C.2), we can now write:
\[
\Delta \bar{x}_{1i} - F \cdot \Delta \bar{x}_{0i} - \bar{f} = \bar{u}_i, \quad i = 1, \ldots, n,
\] (C.3)
where \( \bar{u}_i \) is defined as the sum of the measurement error \( \bar{w}_i \) and the variable part \( \delta \bar{f}_i \) of the modelling error,
\[
\bar{u}_i = \bar{w}_i + \delta \bar{f}_i.
\] (C.4)

It is assumed that the deviations \( \bar{u}_i \) are only related to \( \Delta \bar{x}_{1i} \), uncorrelated and normally distributed. The estimates for \( \bar{f} \) and \( F \) can be determined by the maximum likelihood method, which results in minimizing the scalar function \( J \):
\[
J = \frac{1}{n} \sum_{i=1}^{n} (\Delta \bar{x}_{1i} - F \cdot \Delta \bar{x}_{0i} - \bar{f}) \cdot (\Delta \bar{x}_{1i} - F \cdot \Delta \bar{x}_{0i} - \bar{f}).
\] (C.5)
This minimum is found by variation of \( J \) with respect to \( \tilde{f} \) and \( F \), which yields
\[
\tilde{f} = \overline{\Delta \tilde{x}_{1}} - \tilde{F} \cdot \overline{\Delta \tilde{x}_{0}},
\]
\[
\tilde{F} = D_{01}^{c} \cdot D_{00}^{-1},
\]
where
\[
D_{00} = \frac{1}{n} \sum_{i=1}^{n} \Delta \tilde{x}_{0i} \Delta \tilde{x}_{0i} - \overline{\Delta \tilde{x}_{0}} \cdot \overline{\Delta \tilde{x}_{0}},
\]
\[
D_{01} = \frac{1}{n} \sum_{i=1}^{n} \Delta \tilde{x}_{0i} \Delta \tilde{x}_{1i} - \overline{\Delta \tilde{x}_{0}} \cdot \overline{\Delta \tilde{x}_{1}},
\]
are tensors describing the distribution of the markers and
\[
\overline{\Delta \tilde{x}_{0}} = \frac{1}{n} \sum_{i=1}^{n} \Delta \tilde{x}_{0i},
\]
\[
\overline{\Delta \tilde{x}_{1}} = \frac{1}{n} \sum_{i=1}^{n} \Delta \tilde{x}_{1i},
\]
are the mean vectors of the markers in the reference configuration and in the current configuration respectively. It can be proven that \( \tilde{F} \) is an unbiased estimate of \( F \), thus
\[
E(\tilde{F}) = F,
\]
and \( \tilde{F} \) is normally distributed. Based on \( \tilde{F} \), different strain measures can be calculated. Restricting ourselves to the Green-Lagrange strain tensor, which is defined as
\[
E = \frac{1}{2} (F^c \cdot F - I),
\]
the estimate \( \hat{E} \) is then given by
\[
\hat{E} = \frac{1}{2} (\hat{F}^c \cdot \hat{F} - I).
\]
As \( \hat{E} \) is a non-linear function of \( \hat{F} \) it is a biased estimate. One can derive that holds
\[
E(\hat{E}) = E + \frac{\sigma^2}{n} D_{00}^{-1}.
\]
From equation (C.15) it follows that the accuracy of the estimate \( \hat{E} \) depends, among other things, on the number of markers \( n \) and their distribution \( D_{00} \).
Appendix D

The Influence of Observational Standard Deviation

This appendix will evaluate the effect of the choice of the observational standard deviations on the estimation result. For this purpose, a numerically simulated experiment will be used. Observation errors are simulated by means of an artificial disturbance of the 'measured' data. A limited set of parameters will be estimated. Two cases are evaluated: the first case describes the estimation with the observational standard deviations set to the standard deviations of the applied noise; in the second case the observational standard deviations are set to a much smaller value.

For the simulation of the experiment, the mesh as obtained from a former experiment is used. The kinematic boundary conditions obtained from the experiment are prescribed. An overview of the parameter values is given in table 4.1. In the FE-analysis performed to generate 'measured' data, Configuration I (figure 4.7) for the collagen fibre distribution is used. The parameters that will be estimated are set to:

$$\mathbf{x} = [R_1 \ R_2 \ R_3 \ R_4]^T = [0.10 \ 0.25 \ 0.15 \ 0.30]^T.$$  \hspace{5cm} (D.1)

The calculated positions are disturbed by adding a normally distributed noise with a standard deviation according to the experimentally determined standard deviations, i.e. $\sigma_x = 0.004 \ mm$ and $\sigma_y = 0.003 \ mm$ (see Appendix B), in order to simulate observational errors.

The four parameters mentioned, will be estimated. In the first case the observational standard deviations are set to the values according to the applied noise, i.e. $\sigma_x = 0.004 \ mm$ and $\sigma_y = 0.003 \ mm$. The second case, the observational standard deviations are set to $10^{-4} \ mm$ for both the x- and the y-data. The initial values of the parameters for both cases are:

$$\hat{\mathbf{x}}_0 = [R_{10} \ R_{20} \ R_{30} \ R_{40}]^T = [0.15 \ 0.20 \ 0.20 \ 0.25]^T.$$  \hspace{5cm} (D.2)

The initial standard deviations of the parameters are set to 0.05 and the model standard deviations are set to 0.005. The relative convergence criterium, $\chi^2_{rel}$, is set to $10^{-10}$. Note that modelling errors are not present, since exactly the same model is used for the estimation process as for the generation of 'measured' data.

The estimation sequence for both cases are shown in figure D.1 and figure D.2 respectively. It can be seen that with the first choice of observational standard deviations the parameter
Appendix D. The Influence of Observational Standard Deviation

Values do not converge. However, making the observational standard deviations relatively small, it results in convergence towards the true parameter values (table D.1).

![Figure D.1: Estimation results; observational standard deviations $\sigma_x = 0.004 \text{ mm}$ and $\sigma_y = 0.003 \text{ mm}$.](image1)

Figure D.1: Estimation results; observational standard deviations $\sigma_x = 0.004 \text{ mm}$ and $\sigma_y = 0.003 \text{ mm}$.

![Figure D.2: Estimation results; observational standard deviations $\sigma_x = \sigma_y = 10^{-4} \text{ mm}$.](image2)

Figure D.2: Estimation results; observational standard deviations $\sigma_x = \sigma_y = 10^{-4} \text{ mm}$.

It can be concluded that, although losing its physical meaning, significantly lowering the observational standard deviations can result in improved convergence. It has to be noted that the same effect can be expected when increasing the model standard deviations. This can be stated, having a closer look at equation (2.7). Lowering the observational standard
deviations, increases the first term, \textit{i.e.} the weighted residuals. The same effect is reached by enlarging the model standard deviations. This reduces the influence of the second term and thus enlarging relatively the first term.

An indication of the reliability of the parameters and the constitutive model employed, is a comparison between experimental and calculated positions, \textit{i.e.} the residuals. In the ideal case of no modelling errors, the distribution of the residual field should be random and the order of magnitude of the residuals should be equal to the observational error. Figure D.3 shows the residual fields for each state of deformation, using the final estimates for the parameters presented in table D.1. The measured positions of the markers are represented by circles (\textcircled{o}). The plus-signs (+) represent the estimated positions of the markers. In order to clarify the picture, the residuals are multiplied by a factor 100. No structure can be observed. This is confirmed by the sample mean and the standard deviation of the residuals, that are summarized in table D.2. It can be observed that the size of the (standard deviations of the) residuals corresponds with the standard deviations of the applied noise.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial \text{guess}</th>
<th>Estimate \text{11th}</th>
<th>True value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>0.15</td>
<td>0.1005</td>
<td>0.10</td>
</tr>
<tr>
<td>$R_2$</td>
<td>0.20</td>
<td>0.2504</td>
<td>0.25</td>
</tr>
<tr>
<td>$R_3$</td>
<td>0.20</td>
<td>0.1501</td>
<td>0.15</td>
</tr>
<tr>
<td>$R_4$</td>
<td>0.25</td>
<td>0.2989</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Table D.1: Estimation results; $\sigma_x = \sigma_y = 10^{-4} \ mm$.

Figure D.3: Residuals for the three successive states of deformation 1, 2 and 3 ($\sigma_x = \sigma_y = 10^{-4} \ mm$). The measured positions of the markers are represented by circles (\textcircled{o}). The plus-signs (+) represent the estimated positions of the markers. The residuals are multiplied by a factor 100.
### Table D.2: Sample mean and standard deviation of the residuals.

<table>
<thead>
<tr>
<th>State</th>
<th>( \mu , mm )</th>
<th>( \sigma , mm )</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>-0.0010</td>
<td>0.0039</td>
</tr>
<tr>
<td>1 y</td>
<td>0.0001</td>
<td>0.0035</td>
</tr>
<tr>
<td>x</td>
<td>0.0003</td>
<td>0.0033</td>
</tr>
<tr>
<td>2 y</td>
<td>-0.0005</td>
<td>0.0037</td>
</tr>
<tr>
<td>x</td>
<td>-0.0001</td>
<td>0.0043</td>
</tr>
<tr>
<td>3 y</td>
<td>-0.0001</td>
<td>0.0034</td>
</tr>
</tbody>
</table>