Modeling of a wetplate-clutch in a driveline

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<th>symbol</th>
<th>name</th>
<th>dimension</th>
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<tr>
<td>$T_{\text{engine}}$</td>
<td>torque delivered by the engine</td>
<td>[Nm]</td>
</tr>
<tr>
<td>$T_{\text{clutch}}$</td>
<td>torque transmitted by the clutch</td>
<td>[Nm]</td>
</tr>
<tr>
<td>$T_{\text{max}}$</td>
<td>maximum transmittable torque</td>
<td>[Nm]</td>
</tr>
<tr>
<td>$T_l$</td>
<td>torque to the left of the CVT</td>
<td>[Nm]</td>
</tr>
<tr>
<td>$T_r$</td>
<td>torque to the right of the CVT</td>
<td>[Nm]</td>
</tr>
<tr>
<td>$T_s$</td>
<td>secondary torque</td>
<td>[Nm]</td>
</tr>
<tr>
<td>$T_{\text{ext}}$</td>
<td>external torque</td>
<td>[Nm]</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>primary rotational speed</td>
<td>[rad/s]</td>
</tr>
<tr>
<td>$\omega_{\text{slip}}$</td>
<td>sliprate</td>
<td>[rad/s]</td>
</tr>
<tr>
<td>$\omega_l$</td>
<td>rotational speed to the left of the CVT</td>
<td>[rad/s]</td>
</tr>
<tr>
<td>$\omega_r$</td>
<td>rotational speed to the right of the CVT</td>
<td>[rad/s]</td>
</tr>
<tr>
<td>$\omega_s$</td>
<td>secondary rotational speed</td>
<td>[rad/s]</td>
</tr>
<tr>
<td>$P_{\text{feed}}$</td>
<td>feedpressure</td>
<td>[bar]</td>
</tr>
<tr>
<td>$P_{\text{clutch}}$</td>
<td>clutch pressure</td>
<td>[bar]</td>
</tr>
<tr>
<td>$I$</td>
<td>current</td>
<td>[A]</td>
</tr>
<tr>
<td>$v$</td>
<td>charge</td>
<td>[V]</td>
</tr>
<tr>
<td>$x_{\text{stem}}$</td>
<td>stemposition</td>
<td>[-]</td>
</tr>
<tr>
<td>$\phi$</td>
<td>flow</td>
<td>[l/s]</td>
</tr>
<tr>
<td>$\Phi_{\text{leak}}$</td>
<td>flow leakage</td>
<td>[l/s]</td>
</tr>
<tr>
<td>$b_1$</td>
<td>damping coefficient of the primary flywheel</td>
<td>[kgm²/rad²]</td>
</tr>
<tr>
<td>$b_2$</td>
<td>damping coefficient of the secondary flywheel</td>
<td>[kgm²/rad²]</td>
</tr>
<tr>
<td>$J_p$</td>
<td>inertia of the primary flywheel</td>
<td>[kgm²/rad]</td>
</tr>
<tr>
<td>$J_s$</td>
<td>inertia of the secondary flywheel</td>
<td>[kgm²/rad]</td>
</tr>
<tr>
<td>$\eta$</td>
<td>efficiency</td>
<td>[-]</td>
</tr>
<tr>
<td>$R$</td>
<td>resistance of the coil</td>
<td>[Ω]</td>
</tr>
<tr>
<td>$L$</td>
<td>coefficient of induction of the coil</td>
<td>[H]</td>
</tr>
<tr>
<td>$\mu$</td>
<td>coefficient of friction</td>
<td>[Nm/bar]</td>
</tr>
<tr>
<td>$A_{\text{piston}}$</td>
<td>cross-sectional area of the piston</td>
<td>[m²]</td>
</tr>
<tr>
<td>$k$</td>
<td>coefficient of stiffness of the clutch</td>
<td>[N/m]</td>
</tr>
<tr>
<td>$k_{\text{axle}}$</td>
<td>coefficient of stiffness of the flexible axle</td>
<td>[Nm/\text{rad}]</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature of the oil</td>
<td>[° Celsius]</td>
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Summary

The new trends in the world of automobiles are efficiency and ‘green’. The last trend implicates that the emission has to be clean and small. The first trend is related to the consumption of the fuel. These two trends are not totally separated, because less use of fuel consumption also means less emission.

Hybrid drivelines are developed to meet these trends. At the Faculty of Mechanical Engineering of the University of Technology in Eindhoven a flywheel-hybrid driveline has been developed. The main advantage of this driveline is the recycling of energy. During deceleration of the vehicle the energy is stored in a flywheel. When the vehicle has to be accelerated the energy stored in the flywheel can be used.

The main problem of this system is to keep the output torque at a desired level. During the clutch slips this can be done by controlling the pressure on the clutchplates. After this period it must be controlled by changing the transmission.

It is clear by now, that a good model of the system’s behavior is necessary for the design of a satisfying controller. So the main topic of this report is the development of a model for the mechanical behavior of the clutch.

The model is based on physical principles and has been verified and improved by use of experimental data. It is not tried to estimate each parameter by itself, but to estimate the overall response of the system and the dependency on several external circumstances. Some of the investigated aspects appeared to be significant, others seemed to be negligible. Some of the external circumstances could be enclosed in the model, while others have been used to formulate bounds on the behaviour.

The proposed model can be used to design a controller for the transmitted torque, with the objective to improve its behavior, for instance to improve the drivecomfort.
Chapter 1

General introduction

1.1 Goal

Within the scope of the research in flywheel-hybrid drivelines, a model has to be developed for the clutch in these drivelines. The main function of this wetplate-clutch is to transfer the power, in the driveline. On the basis of this model a controller can be designed to ensure optimal torque response during acceleration from standstill. Van Nistelrooij [1] already did some research on this problem. The main goal of this report can be defined as

*Model the wetplate-clutch in the TUE flywheel-hybrid vehicle and design a controller for the torque in the driveline during acceleration from standstill.*

To achieve this goal the following steps are proposed

- Develop an analytic model for both driveline and clutch
- Identify and validate the model
- Design a controller
- Perform simulations
- Investigate the controller performance.

The torque response has to satisfy the following requirements

- The steady state error of the torque response is not allowed to exceed a margin of 10 %
- The responsetime, defined as the time until 95 per cent of the static response value has been reached, should be less than 0.2 seconds
- Changes in the behavior of the system are not allowed to result in large changes in the response.
1.2 Test rig and its behaviour

The test rig of the system is located in building W-laag of the Faculty of Mechanical Engineering. It mainly consists of seven components in series: a DC-motor, a flywheel, a wetplate-clutch, a continuously variable transmission (CVT), a flexible axle, a flywheel to take the vehicle's inertia into account and an eddycurrent-brake (see Figure 1.1). Besides several measurement and control devices are used. A personal computer is connected to the test rig so the data can be manipulated. The interface between this PC and the measurement and control devices is a dSpace-autobox, which collects the measurement data and controls the control devices.

The DC-motor accelerates the primary flywheel up to a desired velocity. During the experiments the motor is decoupled. The flywheel now contains enough kinetic energy to accelerate the load. Initially the clutch is open. It can be closed by putting an electric voltage on the inputdevice. This inputdevice, a proportional valve, realizes a pressure behind the valve, responding to the input-setpoint. By this, the clutchplates are pressed against each other and due to the friction between the plates, a torque can be transmitted.

As soon as the system is synchronized, i.e. the rotational speeds on both sides of the clutch are the same, the secondary flywheel can no longer be accelerated by the primary flywheel. This means that no longer a torque is transmitted through the clutch. This is not a consequence of the limited friction, but of the amounts of kinetic energy on both sides of the clutch. Keeping the output torque at the desired level now means, that the CVT-ratio has to be controlled. This is behind the scope of this report.
The limited stiffness of a real vehicle driveline is represented by the flexible axle between the CVT and the secondary flywheel. The brake can be set to any value and so the brake torque can represent the external roadload.

The clutch contains sixteen plates, which are successively attached at the input axle and the output axle. Between the plates there is a paper layer. This layer is lubricated with oil.
Chapter 2

Analytic model

2.1 Subsystems

The system can be split into two parts, being the driveline without the clutch and the clutch itself. The clutch can be split into an electrical, a mechanical and a hydraulic part. In this chapter simplified\(^1\), analytical models will be derived for each part.

2.2 Driveline

A schematic model of the complete driveline is sketched below in Figure 2.1. To arrive at a mathematical model the relevant equations for the component are determined.

Figure 2.1

\(^1\) Non-modeled phenomena will be discussed in a subsequent chapter.
First, the primary flywheel is considered. The equation of motion for this element is given by

$$J_p \ddot{\omega}_p = -b_1 \omega_p^2 + T_{\text{engine}} - T_{\text{clutch}}$$  \hspace{1cm} [2.1]

where (see Figure 2.1) $J_p$ is the moment of inertia of the primary flywheel, $\omega_p$ is the angular velocity of the primary flywheel, $b_1 \omega_p^2$ is the dissipative torque due to airdrag (Spijker [2]), $T_{\text{engine}}$ is the torque exerted by an external motor and $T_{\text{clutch}}$ is the torque in the axle between the flywheel and the clutch.

Since the external motor is decoupled from the driveline during the experiments the external couple $T_{\text{engine}}$ is equal to zero.

In general the torques on both sides of the clutch will not be equal to each other. Making the assumption that the inertia of the clutch can be neglected, the following equation yields

$$T_{\text{clutch}} = T_1$$  \hspace{1cm} [2.2]

where $T_1$ is the couple in the axle between the clutch and the CVT. The neglect of the clutch’s inertia can be justified when seen in contrast to the large inertias of both flywheels. The torque $T_{\text{clutch}}$ will be discussed in a following section.

The angular slip velocity, which denotes the difference between angular velocity of the input axle of the clutch $\omega_p$ and the angular velocity of the output axle of the clutch $\omega_1$, can be described in the following equation

$$\omega_{\text{slip}} = \omega_p - \omega_1$$  \hspace{1cm} [2.3]

where the assumption is made that both axles are rigid.

Considering the power balance on both sides of the CVT by which the output power of the CVT $T_r \omega_r$ is related to the input power $T_1 \omega_1$

$$\eta T_1 \omega_1 = T_r \omega_r$$  \hspace{1cm} [2.4]

The parameter $\eta$ is the efficiency of the CVT and is assumed to be constant with a value equal to 95%. This equation only yields when the power flow through the CVT is from the primary flywheel to the secondary flywheel. Also the assumption is made that no slip does occur in the CVT.
With the controllable CVT-ratio \( i \) defined as

\[ i = \frac{\omega_r}{\omega_t} \]  

[2.5]

the relation between the torques can be written as

\[ T_r = \frac{\eta T_1}{i} \]  

[2.6]

The connection between the CVT and the secondary flywheel is flexible. It is modelled as a massless, linear elastic axle with torsional stiffness \( k_{\text{axle}} \), so

\[ \dot{T}_r = k_{\text{axle}} (\omega_r - \omega_s) \]  

[2.7]

where \( \omega_s \) is the angular velocity of the secondary flywheel. Since the axle is massless the torque \( T_s \) on this flywheel equals \( T_r \). Therefore, the equation of motion for the secondary flywheel is given by

\[ J_s \ddot{\omega}_s = -b_2 \omega_s^2 - T_{\text{ext}} + T_r \]  

[2.8]

where \( J_s \) is the inertia of the secondary flywheel, \( b_2 \omega_s^2 \) is the dissipative torque due to the airdrag (Spijker [2]) and \( T_{\text{ext}} \) is the torque, exerted on the flywheel by the controllable brake.

Now the relevant equations for the physical model of the driveline are summarized

\[ J_p \dot{\omega}_p = -b_1 \omega_p^2 - T_{\text{clutch}} \]  

[2.1]

\[ \omega_{\text{slip}} = \omega_p - \omega_t \]  

[2.3]

\[ i = \frac{\omega_r}{\omega_t} \]  

[2.5]

\[ T_r = \frac{\eta T_1}{i} \]  

[2.6]

\[ \dot{T}_r = k' (\omega_r - \omega_s) \]  

[2.7]
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\[ J_s \dot{\omega}_s = -b_2 \omega_s^2 - T_{ext} + \frac{\eta_i}{I} T_{clutch} \quad [2.8] \]

2.3 Clutch

Figure 2.2 gives some details of the clutch with the hydraulic system.

![Diagram of clutch](image)

Figure 2.2

2.3.1 Mechanical part

Initially the clutch is open, which means that the piston is pulled back to the piston stops. This is done by the resetspring. This spring warrants that the piston is at the utmost left position at that moment, so the clutchplates can be assumed to be not in contact with each other. At this point no torque will be transmitted. The initial volume \( V_0 \), which is the volume to the left of the stops is filled with oil. At this point there is no pressure within the clutch \( (p_{clutch} = 0 \text{ bar}) \). Now when the valve opens oil flows in the clutch chamber.

Due to the friction between the piston and the house and the force on the piston by the resetspring the piston will remain in the same position \( (x_{piston} = 0 \text{ m}) \) despite the increase of the
amount of oil in the chamber. The flow of oil into the clutch will increase the pressure and the
piston starts moving when the pressure becomes larger than the sum of the friction and spring
forces. The friction is assumed to be negligible in contrast to the force caused by the
resetspring. At this point the pressure can be denoted as

\[ P_{\text{clutch}} = P_{\text{prestress}} \frac{k_{\text{reset}} x_{\text{stop}} + F_{\text{friction}}}{A_{\text{piston}}} \approx \frac{k_{\text{reset}} x_{\text{stop}}}{A_{\text{piston}}} \]  \[2.9\]

where \( k_{\text{reset}} \) is the stiffness of the resetspring, \( x_{\text{stop}} \) the position of the piston stops and \( A_{\text{piston}} \) the
cross-sectional area of the piston.

For \( P_{\text{clutch}} > P_{\text{prestress}} \) the piston moves to the right and reduces the distance between the
clutchplates and the piston. Assume that inertia effects can be neglected; the relation between
the piston displacement \( x_{\text{piston}} \) and the pressure \( P_{\text{clutch}} \) is given by

\[ (P_{\text{clutch}} - P_{\text{prestress}})A_{\text{piston}} = k x_{\text{piston}} \]  \[2.10\]

In this relation the parameter \( k \) denotes the stiffness of the subsystem. At a certain point the
contact between the clutchplates is recovered. Now the pressure is denoted by \( P_{\text{contact}} \), so

\[ P_{\text{contact}} = P_{\text{prestress}} + \frac{k x_{\text{contact}}}{A_{\text{piston}}} \]  \[2.11\]

Due to the large stiffness of these plates the piston position is fixed again.

\[ x_{\text{piston}} = x_{\text{contact}} \quad \text{for} \quad P_{\text{clutch}} \geq P_{\text{contact}} \]  \[2.12\]

Now the plates are pressed together with a compressionforce \( F_{\text{compress}} \) as formulated in Equation
2.13 and a torque can be transmitted.

\[ F_{\text{compress}} = (P_{\text{clutch}} - P_{\text{contact}})A_{\text{piston}} \]  \[2.13\]
Summarizing,

\[0 \leq p_{\text{clutch}} < p_{\text{prestress}} \quad x_{\text{piston}} = 0, \quad F_{\text{compress}} = 0\]

\[p_{\text{prestress}} \leq p_{\text{clutch}} < p_{\text{contact}} \quad x_{\text{piston}} = \frac{A_{\text{plates}}}{k} (p_{\text{clutch}} - p_{\text{prestress}}), \quad F_{\text{compress}} = 0\]  \[2.14 \text{ a,b,c}\]

\[p_{\text{clutch}} \geq p_{\text{contact}} \quad x_{\text{piston}} = x_{\text{contact}}, \quad F_{\text{compress}} = (p_{\text{clutch}} - p_{\text{contact}})A_{\text{piston}}\]

Now the torque at the input axle is transmitted to the output axle by means of friction between the clutchplates. The friction is modelled as a Coulomb-friction. Let \(\mu\) be the coefficient of friction. The maximum torque \(T_{\text{max}}\) is now given by

\[T_{\text{max}} = \mu F_{\text{compress}}\]  \[2.15\]

The proportionality factor \(\mu\) in this relation depends on the surface of the plates \(A_{\text{plates}}\), on the number of plates \(z\), the oil temperature \(T\), the compression force \(F_{\text{compress}}\) and probably also on the slip velocity \(\omega_{\text{slip}}\), so

\[\mu = \mu(A_{\text{plates}}, z, T, F_{\text{compress}}, \omega_{\text{slip}})\]  \[2.16\]

Further information on this dependence has to be gained from experiments.

According to the Coulomb model the following relation holds for the transmitted torque \(T_{\text{clutch}}\)

\[|T_{\text{clutch}}| = \begin{cases} T_{\text{max}} & \text{if } \omega_{\text{slip}} \neq 0 \\ \leq T_{\text{max}} & \text{if } \omega_{\text{slip}} = 0 \end{cases}\]  \[2.17\]

### 2.3.2 Hydraulic part

The hydraulic part consists of a tube, a variable restriction and a hydraulic cilinder. Suppose that initially the valve is closed (i.e. \(x_{\text{stem}}=0\) and \(\phi=0\)) and the clutch is in a neutral position (i.e. \(p_{\text{clutch}}=0\) and \(x_{\text{piston}}=0\)). Furthermore suppose that at a certain point in time \(t=t_0\) the valve is opened. This will result in an oil flow \(\phi\neq 0\), which is described by

\[\phi = c x_{\text{stem}} \sqrt{\frac{P_0 - P_{\text{clutch}}}{\rho}}\]  \[2.18\]
in which $p_0$ denotes the constant pressure delivered by the external pump, $p_{\text{clutch}}$ the pressure within the clutch, $\rho$ is the oil mass per unit volume and $x_{\text{stem}}$ the valvestem position (see Figure 2.3). The variable $x_{\text{stem}}$ is a measure of the restriction in the pipeline. The parameter $c$ is a constant factor.

When the oil flow is not equal to zero the pressure in the clutch starts rising and at a certain moment a piston displacement occurs. We can analyse this hydraulic event by considering the law of conservation of mass

$$[V_0 + A_{\text{piston}} x_{\text{piston}}(t)] \rho(t) = V_0 \rho_0 + \int_{t=0}^{t} [\phi(t) - \phi_{\text{leak}}(t)] \rho(t) \, dt$$

$$V_0$$ represents the initial oil volume behind the valve if $x_{\text{piston}}=0$, $A_{\text{piston}}$ is the piston cross-sectional area and $\phi_{\text{leak}}$ is the leakage flow. Some variables are denoted as time variable, the other are assumed to be constant in time.

Because this law must be satisfied for all $t>0$ it is to be seen that after differentiation in time

$$[V_0 + A_{\text{piston}} x_{\text{piston}}(t)] \dot{\rho}(t) + A_{\text{piston}} x_{\text{piston}}(t) \rho(t) = [\phi(t) - \phi_{\text{leak}}(t)] \rho(t)$$

The oil mass per unit volume $\rho$ depends on both oil temperature $T$ and pressure $p_{\text{clutch}}$ (and thus on the time)

$$\rho = f(T, p_{\text{clutch}})$$

The time derivative of this variable $\rho$ can be described as

$$\dot{\rho} = \frac{\partial f}{\partial T} \dot{T} + \frac{\partial f}{\partial p_{\text{clutch}}} \dot{p_{\text{clutch}}}$$

Both temperature and clutch pressure will change in time but the pressure will change within milliseconds, while the temperature only changes significantly within minutes, so the time derivative of the latter can be neglected. Still the mass per unit volume $\rho$ depends on the oil temperature by the function $f$. Notice that also the flow $\phi$ in Equation 2.18 depends on the temperature $T$ by depending on $\rho$. 
Neglecting the time derivative of the temperature and dividing Relation 2.22 by \( \rho \), it can be written as

\[
\frac{\dot{p}}{p} = \frac{\dot{p}_{\text{clutch}}}{k_{\text{oil}}} \tag{2.23}
\]

in which \( \frac{1}{k_{\text{oil}}} \) denotes the isothermal compressibility of the oil. In general \( k_{\text{oil}} \) will depend on the pressure \( p_{\text{clutch}} \) and the temperature \( T \) in the normal operation phase. For the moment it is assumed that \( k_{\text{oil}} \) is constant.

By substitution of Equation 2.23 in Equation 2.20, it is clear now that the following relation yields

\[
\dot{p}_{\text{clutch}} = \frac{k_{\text{oil}}}{V_0 + A_{\text{piston}} x_{\text{piston}}} \left[ -A_{\text{piston}} \dot{x}_{\text{piston}} + \phi - \phi_{\text{leak}} \right] \tag{2.24}
\]

In the previous Section 2.3.1 it is shown that closing the clutch can be separated in three parts: the time between opening the valve and the piston starts moving, the time in which the piston moves towards the clutchplates but does not yet contact them, and the time after the clutchplates have recovered contact. These three parts will again be analysed separately for the hydraulic part.

**Part 1:** \( 0 \leq p_{\text{clutch}} < p_{\text{prestress}} \)

\[
x_{\text{piston}} = \dot{x}_{\text{piston}} = 0 \\
\dot{p}_{\text{clutch}} = \frac{k_{\text{oil}}}{V_0} (\phi - \phi_{\text{leak}}) \tag{2.25}
\]

**Part 2:** \( p_{\text{prestress}} \leq p_{\text{clutch}} < p_{\text{contact}} \)

\[
x_{\text{piston}} = \frac{A_{\text{piston}}}{k} (p_{\text{clutch}} - p_{\text{prestress}}), \quad \dot{x}_{\text{piston}} = \frac{A_{\text{piston}}}{k} \dot{p}_{\text{clutch}} \] (see Equation 2.14b)
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With these relations for the piston displacement and the piston translational speed Equation 2.24 becomes

\[
\dot{p}_{\text{clutch}} \left[ \frac{V_0 + A^2}{k} \left( p_{\text{clutch}} - p_{\text{prestress}} \right) + \frac{A^2}{k} \right] = \phi - \phi_{\text{leak}} \quad [2.26]
\]

In practice always yields

\[
\left| p_{\text{clutch}} - p_{\text{prestress}} \right| << k_{\text{oil}} \quad [2.27]
\]

so \( \dot{p}_{\text{clutch}} \) can be approximated by

\[
\dot{p}_{\text{clutch}} = \left[ \frac{V_0}{k_{\text{oil}}} + \frac{A^2}{k} \right]^{-1} (\phi - \phi_{\text{leak}}) \quad [2.28]
\]

Part 3: \( p_{\text{clutch}} \geq p_{\text{contact}} \)

\[
x_{\text{piston}} = x_{\text{contact}}, \quad \dot{x}_{\text{piston}} = 0
\]

\[
\dot{p}_{\text{clutch}} = \frac{k_{\text{oil}}}{V_0 + A_{\text{piston}} x_{\text{contact}}} (\phi - \phi_{\text{leak}}) \quad [2.29]
\]

The flow \( \phi_{\text{leak}} \) out of the clutch will depend on the pressure \( p_{\text{clutch}} \) and oil temperature \( T \). In this report it is not tried to derive a model for this flow. For the moment it is assumed that \( \phi_{\text{leak}} \) can be neglected. A further investigation on the relation between \( \phi_{\text{leak}} \) on one hand and \( p_{\text{clutch}} \), \( T \) and possibly \( x_{\text{piston}} \) on the other hand will be performed only if the results for the experiments call for that.

Further it is assumed that the outlet flow is much larger then the leakage flow, so the pressure in the space behind the clutchplates is equal to zero.
2.3.3 Valve

A proportional valve is the input device of this system. It converts an electrical voltage into a pressure related to the setpoint of the charge. The charge induces a current in a coil (1) (see Figure 2.3). In the middle of the coil a magnet is positioned. When a voltage is put on the coil, a current will be produced. As a consequence of that a magnetic field is created, which will force the magnet in a different position. Attached to this magnet is a valvestem (3), which opens or closes the restriction in the oilpipe (4).

When the clutch is in an open position, i.e. the pressure within the clutch is atmospheric, and no input is set on the valve, the valve is closed, which means that a resetspring attached to the valvestem is in balance with the pressure of the oil before the valve on the stem. When a voltage has been put on the input, the valve will entirely open, letting a great oil flow through. As a consequence of this oil flow the pressure behind the valve starts rising as described in the section 2.3.2 Hydraulic part. Now the valve stem will be forced back by the rising oil pressure behind the valve and the valve starts closing again. Finally the pressure behind the valve becomes the value as set by the input.

Voltage V

Figure 2.3
The induction of the current within the coil can be modeled as

\[ L \frac{dI}{dt} + RI = v \]  \hspace{1cm} [2.30]

in which \( L \) is the coefficient of induction of the coil, \( R \) is the resistance of the coil, \( I \) is the current and \( v \) is the input voltage. During the current changes a magnetic field is produced within the coil, which forces the magnet into a new position. A relation for the magnetic force on the magnet will be strongly non-linear and complicated. Probably it will depend on the current \( I \) and its time derivative, the position of the magnet and dimensional parameters.

Besides a reset force reacts on the magnet by the resetspring which balanced the original, closed state. The stiffness of the resetspring is denoted as \( k_{\text{reset}} \).

\[ F_{\text{magnet}} = g(I, \frac{dI}{dt}, x_{\text{magnet}}, k_{\text{magnet}}) \]  \hspace{1cm} [2.31]

No simple model is yet known for this function \( g \).

The dynamics of the magnet are of a second order kind, which enclose inertia, damping and stiffness.

\[ m' \ddot{x}_{\text{stem}} + d' \dot{x}_{\text{stem}} + c' x_{\text{stem}} = F_{\text{magnet}} \]  \hspace{1cm} [2.32]

The mass represents the mass of the magnet, while the coefficient of damping and the coefficient of stiffness represent the damping within the oil respectively the friction within the valve.

So when an input is put on the valve, a current is induced in the coil. The change of the current creates a magnetic field, which will force the magnet into a new position. The valvestem connected to the magnet opens an opening through which the oil can flow into the part behind the valve. As a consequence of this oil flow the pressure behind the valve starts rising and the valvestem is forced back.
Chapter 3

Experiments

3.1 Experimental set-up

The main purpose of this report is the identification of the clutch. The necessary experiments can be split into several parts, similar to the parts of the clutch model in Chapter 2. The main difference is that the experiments concerning the hydraulic system can not be done directly by measuring the flow and the piston position but must be done on the entire clutch, thus incorporating the valve and the clutch itself.

Three kinds of input signals are used

- Step signal; coming from a static value at a lower level, the response shows a rise in time and a new, higher static value is reached.
- 'Relaxation' signal; coming from a higher level, the response descends to a new lower level. There is a great difference with the stepresponse, due to the behaviour of the valve.
- Staircase signals; in this case a new value is approached using little step changes at the input in a staircase format. Before each step is taken the response of the preceding step has to become stationary.

Comparison of a staircase in positive direction and in negative direction delivers some insight in the hysteresis of the system. The step and relaxation signals are used to obtain static values and timeconstants.

No noise or sinusoidal inputs are offered to the system. In the near future this has to be done to get an idea of the dynamic response of the system.
3.2 Procedure

The global procedure of the experiments is already described in Chapter 1. Before an experiment is started the primary flywheel is accelerated until a rotational speed of 150 rad/s. The clutch is open and the vehicle equivalent flywheel does not move yet. In this way a large measuring range before synchronization is reached and acceleration of the vehicle equivalent flywheel from zero can be studied. Next the clutch is closed and the responses are measured.

The total driveline is used only for the experiments to estimate the coefficient of friction. The others do not need the flywheel to be loaded, because the only relevant variable is the pressure behind the valve. By doing this the pressure response can be measured more correctly.

An other point of interest in this case is the type of the input. Initial a DC-voltage is used for the experiments. In appendix B the use of pulswidth-modulation is discussed. This method should give a better response. Pulswidth-modulated inputs disable certain negative effects, but do not influence the systems behavior itself.

3.3 Measurement devices

The following variables can be measured during the experiments (see also Figure 1.1 and 2.1)
- Torque to the right of the CVT, $T_r$
- Rotational speed to the left of the CVT, $\omega_l$
- Rotational speed to the right of the CVT, $\omega_r$
- Rotational speed of the primary flywheel, $\omega_p$
- Pressure behind the valve, which is assumed to be equal to the pressure in the clutch, $p_{clutch}$.

Other variables as ratio, primary torque and angular slip velocity can be calculated from these measurements. Next to the measurement devices several devices like lowpass filters are used. It is assumed that these devices have no significant effects on the signals within the operational bandwidth and therefore their influence can be neglected.

3.4 Assumptions

In the preceding chapter several assumptions have been made to obtain a simple model. Yet the relevance of some phenomena has to be checked before it is justified to neglect them.

In this section some global assumptions are discussed. The dependency on variables will be discussed in Chapter 4.
Temperature
Temperature changes greatly influence the system’s behaviour, because the viscosity of the oil strongly depends on the oil temperature. For instance, suppose that the clutch starts in an opened position. At a high temperature a normal step response can be produced. However, if the system has just been started up, the oil temperature is low, about the environmental temperature. As a consequence of the high viscosity of the oil for low temperatures a torque is transmitted even if the clutch is fully open.
The temperature changes from 20°C at startup to about 80°C after some time. This means that this parameter should be measured frequently. In these experiments the temperature is known at each moment with a margin of three degrees and so it can be assumed to be constant during a single experiment.

Ratio
The transmission ratio of the CVT is assumed to be constant. This ratio can only be changed if the axles rotate. Essentially this should not be a problem. However, for low rotational speeds the speed measurements are very inaccurate, resulting in an inaccurate value for the transmission ratio. This can distort other calculated variables like the coefficient of friction. Several parts of the experiment data are not usable due to this effect.

Flowleakage
Imperfect sealings in the clutch result in a flow leakage. A positive effect of this phenomenon is that once the clutch-plates are brought in contact, the pressure does not rise less extremely than without leakage and the system’s behaviour is easier to control. However this flow is not modeled.

Inputmargins
The inputs are chosen to enclose the total range of the possible pressure values. There is an upper and a lower bound on this range. The upper bound has to do with the safety of the clutch. If the pressure exceeds 8 bar the clutch could be damaged and that is why the oil pressure before the clutch is limited. The lower bound is a consequence of the limited sensitivity of the measurement devices. Below 0.2 bar this effect will influence the measurement data too strongly. The bounds in the input are, as a consequence of the reasons given above respectively 5 Volts and 0.2 Volts.

CVT
Slip within the CVT is not modeled. Due to the stretching and pushing of the V-belt there will be a certain amount of slip between the belt and the pulleys, but this will be negligible compared to the slip within the clutch.
Secondly the efficiency of the CVT is taken constant and equal to 95 %.

Feedpressure
The pump delivers a nearly constant pressure of 8 bar.

Clutchpressure
The valve controls the pressure directly behind the valve. Between this point and the clutchplates there are some curves in the pipeline and a change of cross-sectional area at the point where the oil enters the clutch. This is not modeled to keep the model as simple as possible. The pressure drop due to the curves can be neglected in contrast to the pressure drop when the oil enters the clutch. The last one has to be modeled in a subsequent work if necessary.

Inertia
The inertia of the clutchplates is neglected because the mass and the accelerations of these plates are very small.

Coefficient of friction
It is assumed that the coefficient of friction depends amongst others on the pressure $p_{\text{clutch}}$, the temperature $T$ and the slipvelocity $\omega_{\text{slip}}$.

$$\mu = \mu(p_{\text{clutch}}, T, \omega_{\text{slip}})$$ [3.1]

A large pressure will lead to a large friction, but the relation between $p_{\text{clutch}}$ and the coefficient of friction $\mu$ is strongly non-linear. For low pressures, which in practice can be related to driving in a traffic-jam, the friction will be much less than in case of the linear expression and, due to stick-slip phenomena, the friction will be of a varying kind. The dependency on the sliprate will be small over the total range but is significant for small sliprate values. In the last mentioned area stick-slip is brought up again as main disturbance. The dependency on the temperature is caused by the strong dependency of the viscosity of the oil on the oil temperature.

Stiffness
The stiffness of the clutch depends on the position $x_{\text{piston}}$ of the piston and on the pressure $p_{\text{clutch}}$, i.e.

$$k = k(x_{\text{piston}}, p_{\text{clutch}}, T)$$ [3.2]
For small values of \( x_{\text{piston}} \) the stiffness is equal to the stiffness of the resetspring. Once the plates are in contact with each other the piston only moves on behalf of the stiffness of the plates. Earlier the oil was assumed to be incompressible, but now its stiffness becomes the most essential part of the parameter \( k \). Thus compressibility of the oil has to be taken into account. That explains the dependency on the pressure \( p_{\text{clutch}} \) and the temperature \( T \).

### 3.5 Measuring the separated parts

*Mechanical part*

Estimating the mechanical part means estimating the parameter of friction and stiffness. The Equations 2.16 and 3.2 deliver a suitable model for this part. In Section 3.2.1 the results of the measurements of the coefficient of friction are discussed. The main problem in the estimation of the stiffness \( k \) is that the piston position can not be measured. Van Nistelrooij [1] estimated the stiffness of the clutch itself.

*Hydraulic part*

As indicated above the piston position, like the flow, is not measured, meaning that little can be said about the hydraulic part of the system. The model has to be validated by looking at the response of the total system.

The influence of the temperature on the system's behaviour is investigated in a distinct experiment. The oil is heated to 80°C and during this process several stepresponses are measured. Thus the relation between the oil temperature and the system's behaviour can be found.

The point in time at which the plates contact can be found by comparing the pressure response and the torque response, because at this point the torque starts rising from zero. This can be used to estimate the dead time and thus the required time to pressurize the clutch.
Valve
As described in section 2.3.3 the valve is modeled as a third order system, consisting of a first order and a second order system in series. The first order system represents the induction and the second order system estimates the magnet dynamics. To eliminate the system behind the valve, the clutch is replaced by a restriction with a constant flow of 32 ml/min (see Figure 3.1).

Figure 3.1
Chapter 4

Results

4.1 Driveline

First of all the driveline itself is considered. This driveline can be modelled as a system with two masses and a spring in between. Initially the flywheel rotates, the clutch is open and the vehicle equivalent flywheel is at rest. Closing the clutch results in a damped response for the output torque in the axle between the CVT and the vehicle equivalent flywheel. After synchronization no torque is transmitted by the clutch (see Section 1.1). By closing the clutch a torque was introduced in the driveline, which wound up the system. After the point of synchronization there is a free response to the original state as a consequence of that. The resonance frequency corresponding to this effect is as shown in the Figure 4.1 about 2.1 Hertz.

Figure 4.1
4.2 Clutch

Next several aspects of the clutch will be discussed

- Estimating the coefficient of friction and its dependency on temperature $T$, slip rate $\omega_{\text{slip}}$ and pressure $p_{\text{clutch}}$.
- Determining the valve’s behaviour, especially the pressure response as a function of the input.
- Determining the behaviour of the entire clutch.
- Estimating the influence of the oil temperature.

4.2.1 Coefficient of friction

Dependency on the temperature

It turns out that there is hardly any relation between the coefficient of friction and the temperature. Both maximum and mean values of this coefficient are measured. The maximum values are distorted by measurement noise, while the mean values depend strongly on the range of datapoints. The mean values of the coefficient of friction varied between 15.1 and 12.3. These boundaries also depend on other circumstances. A better description of the relation between friction and temperature requires more experiments, especially at low temperatures. However, until now measurements at low temperatures give no reliable results.

Dependency on the pressure/electric voltage

The relation between these two variables also seems to be very weak over a large range. Only for small voltages a significant decrease in friction appears (see Figure 4.2). This figure gives the measured mean values. For voltages larger than 1 Volt the coefficient varies between 0.034 and 0.052. Hardly anything can be said about the coefficient of friction for voltages below 1 Volt. When such little inputs are used, the pressure stops rising when the plates revolve contact. So the compression force $F_{\text{compress}}$ is about zero.

The designer of the controller should take the proposed bounds into account.
Figure 4.2

Dependency on the sliprate
There is a strong relation between the sliprate and the coefficient of friction (see Figure 4.3). This value increases for low values of the sliprate, but decreases for large values of the sliprate. The relation can be estimated with a second order polynomial.

Figure 4.3
The dependency of the coefficient of friction on the sliprate can be fitted by a second order polynomial

\[ P = -1.7E - 6\omega_{\text{slip}}^2 + 2.7E - 4\omega_{\text{slip}} + 2.6E - 2 \]  

[4.1]

### 4.2.2 Proportional valve

The experimental procedure for the proportional valve is already described in Chapter 3. Several aspects of the valve's behaviour are considered now in more detail.

**Linearity**

The relation between the steady state value of the pressure behind the valve and the magnitude of a step input is approximately linear. However, for very small and for very large steps non-linear behaviour of the electric components occurs and some non-linear effects like magnetic whirls can no longer be neglected (see Figure 4.4). A straight line has been fitted, using a least squares method, to the data of the plot. This line represents the static gain between the input voltage \( v \) and the pressure in the clutch \( p_{\text{clutch}} \) and is given by

\[ p_{\text{stat}} = 24pdc - 1.38 \]  

[4.2]

*Figure 4.4*
The term 'power duty cycle' is explained in appendix B.

**Hysteresis**

There is some energy dissipation in the valve due to the friction between the magnet and its surroundings. The hysteresis is plotted in Figure 4.5

![Graph of hysteresis](image)

**Figure 4.5**

**Total response**

The behaviour of the valve does not correspond perfectly to the model proposed in Chapter 2. This is amongst others caused by the electric circuit (see also Appendix A).

Improvement of the model requires time-consuming research. Hong [4] did some research on this problem but could not bring up an analytic description of the effects.

In Figure 4.6 the valve pressure response to a step and relaxation input is plotted.
4.2.3 Entire clutch behaviour

The behaviour of the entire clutch is mainly determined by the response of the valve. Several aspects of the behaviour of the entire clutch are discussed below.

Dead time
The subsystem shows a dead time between the response of the pressure and the response of the output torque. This is caused by the fact that it takes some time to fill the clutch with oil. During this period the plates are not in contact with each other and thus no torque can be transmitted. The pressure $p_{\text{clutch}}$, output torque $T$, and slip velocity $\omega_{\text{slip}}$ as a response on a step in the input at time $t = 0$ s is plotted in Figure 4.7.
The dead time varies between 0.1 and 0.5 seconds. The variation is caused, amongst others, by the change in the oil viscosity and the value of the input. If a large input value is chosen, the valve will be opened more and then the clutch will be filled rapidly so the dead time will be small.

This dependency on the input shows up for small input values only: above the pdc-value of 0.1 the electric circuit is always distorted, which means that the valve will be opened entirely during the first moments, irrespective of the value of the input.

The dead time also depends on the oil temperature: for higher temperatures the viscosity of the oil is lower and the clutch will be filled faster.

The dependency of the dead time on the above proposed variables is described in Table 4.1

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Pdc = 0.5</th>
<th>Pdc ≥ 0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>T = 41°Celsius</td>
<td>0.50 s</td>
<td>0.2 s</td>
</tr>
<tr>
<td>T = 85°Celsius</td>
<td>0.35 s</td>
<td>0.1 s</td>
</tr>
</tbody>
</table>

*Table 4.1*
After this dead time the pressure $p_{\text{contact}}$ is reached, which through the measurements appears to be 0.7 bar.

**Hysteresis**

The hysteresis of the entire clutch is plotted in Figure 4.7 below.

![Response on a staircase-input: (+) = up (o) = down](image)

**Figure 4.7**

**Steady state gain**

One might expect that the steady state gain of the total clutch resembles the steady state gain of the valve. However Figure 4.8 shows a gain function, which is not similar to the one for the valve. Now a quadratic function instead of a linear function, has been fitted to the experimental data:

$$p_{\text{stat}} = 0.41v^2 + 0.29v + 0.20$$  \[4.3\]

One should notice the use of Volts on the horizontal axis in contrast to the use of pdc in Plot 4.4.
4.2.4 Temperature

There is a strong relation between the behaviour of the clutch and the temperature of the oil (see Figure 4.9 and 4.10).
All experiments are all done with an input of 0.2 pdc. But the steady state response dependency on the temperature can be modeled by the formula

\[ p_{\text{stat}} = 4.8E - 4T^2 + 4.1E - 4T + 4.2 \]  

[4.4]

*Figure 4.10*
Chapter 5

Concluding remarks and recommendations

5.1 Conclusions

- The clutch and the driveline can be modelled as a non-linear fourth order system with dead time, respectively as a second order system with non-linear damping.

- The coefficient of friction strongly depends on the slip rate. For low values of the slip rate, pressure or temperature the value of the coefficient is not well known. It is very difficult to perform a good experiment in such circumstances. Bounds on the value of the coefficient can be determined. The controller has to operate well within these bounds.

- The response of the entire clutch mainly depends on the behavior of the proportional valve. The steady state gain has been modeled.

- The clutch has to be filled and pressurized before a torque can be transmitted. Therefore a dead time appears. This dead time depends on both the electric voltage and oil temperature.

- The changes in pressure due to the changes in temperature can be described by a second order polynomial.
5.2 Recommendations

- Not only step and staircase inputs should be used to determine the model, but also banded noise and sinusoidal inputs to get the dynamic response in the frequency domain and more parameters can be estimated.

- More experiments have to be done for low values of, for instance, slip rate and temperature. Some strange phenomena appear for such low values. However, these experiments are very difficult.

- Design a controller, based on the available models for the clutch and the driveline.
Appendix A

Proportional valve

The input device is a proportional valve. The main advantage of this device is the proportionality between the electric voltage and the movement of the stem. A sketch of the electric circuit is given below.

\[ \tau_1 = \frac{L_1}{R} \]  

\[ [A.1] \]
This exponential rise lasts until $V_r$ cancels out the signal difference within the amplifier. After that the current remains constant.

When the input is decreased the transistor is blocked and the current decreases exponentially. This time the timeconstant is denoted by

$$\tau_2 = \frac{L_2}{R}$$  \[A.2\]

Equation 2.30 now becomes

$$L(v, t; t_0) \frac{dI}{dt} + RI = v$$  \[A.3\]

which implies that the induction depends on the history and the actual value of the voltage. The magnetic force is the output of a higher order system as can be seen in Figure A.2.

---

Figure A.2
Appendix B

Pulswidth-modulation

Pulswidth modulation (PWM) means that the information of the original signal is no longer in the absolute value of that signal, but is in the pulswidth of a basic signal, which is usually a block signal (see Figure B.1).

![Diagram of PWM signal]

*Figure B.1*

The first part of the PWM signal represents a low value: of the original signal the pulses are small. If the value of the original signal increases then the pulswidth increases.

The power duty cycle is a measure for this modulation; it indicates the size of the part of a fixed period $T_{PWM}$ during which the signal is ‘high’. This can be formulated as

$$Pdc = \frac{t_{\text{high}}}{T_{PWM}}, \text{ with } 0 \leq pdc \leq 1$$

[B.1]
The main advantage of this method for the considered valve is that stick-slip effects within the valve are avoided. The frequency \( f = \frac{1}{T_{PWM}} \) of the basic signal must be high enough to prevent the pulseffects in the output.
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