Time-frequency analysis of position-dependent dynamics in an Iteratively Controlled Waferstage

B.A. Hennen

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Traineeship report

Coach: Dr. I. Rotariu†

Supervisor: Prof. M. Steinbuch‡

†Australian Centre for Field Robotics
The University of Sydney, Australia

‡Eindhoven University of Technology
Department of Mechanical Engineering
Dynamics and Control Group

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Abstract

In this report signal processing, applied for the analysis of control signals, is exploited in the investigation of position-dependent dynamics in a motion system, i.e., a wafer stage apparatus with Iterative Learning Control (ILC). Based on the Matching Pursuit [1],[2] algorithm, which is used to decompose signals with respect to a multiple complex dictionary of atoms, a high-resolution signal energy distribution is derived in the time-frequency plane, which does not include cross-terms, like Wigner distribution. A thorough high-resolution time-frequency analysis of servo error signals of the controlled system will provide much insight in the appearance and propagation of the low energetic position-dependent dynamics encountered.

Keywords: atomic decomposition, cogging, time-frequency analysis, Iterative Learning Control, position-dependent dynamics, wafer stage
## Contents

1 Introduction ........................................... 1

2 Iterative Learning Control .......................... 4
   2.1 Standard Iterative Learning Control ......... 4
   2.2 Adaptive Iterative Learning Control ......... 6

3 Time-frequency atomic decomposition of servo errors ... 9

4 Analysis of the position-dependent dynamics in a wafer stage 12
   4.1 Position-dependent dynamics in a wafer stage with acceleration feed-forward 13
   4.2 Position-dependent dynamics in a wafer stage with adaptively learned feed-forward 16

5 Conclusions ........................................... 20

Bibliography ............................................ 22
Chapter 1

Introduction

In most control applications, linear time-invariant (LTI) models of dynamical systems are known to be used. Time-varying non-linear and stochastic behavior of such systems can be modelled and than studied analytically and numerically, but, for complex systems, these models often give poor representations of the real system, which implies that wide robustness margins have to be defined in order to ensure stabilisation of the system.

Instead of using such LTI, nonlinear or stochastic models for control purposes, one could also focus on the analysis of sensor signals obtained via system measurements. Whereas signal processing has been exploited in many disciplines, application of advanced signal processing methods in control engineering has not yet been investigated to its full extent [3]. Nevertheless, the physical features of a dynamical system can be extracted from measured signals, analysed and exploited for use in advanced intelligent identification and control schemes, which will maximize the throughput and performance of such a system by reducing the otherwise wide robustness margins.

In this report we will investigate the position-dependent dynamics of a mechanical motion system, e.g. a wafer stage [4], see Figure 1.1, with Iterative Learning Control (ILC) [5] using signal processing techniques, applied for the analysis of the control signals. In [5] it has been shown that ILC improves the tracking performance of a wafer stage. Despite this fact, standard ILC has liability to deal with setpoint trajectory changes and noise [6]. Furthermore, standard ILC has liability to deal with the position-dependent dynamics in a wafer stage [6]. The first two problems have been solved using so-called piecewise ILC [7],[8], but compensation of position-dependent
dynamics is not solved yet.

Figure 1.1: Schematic view of a wafer stage.

The position-dependent behavior of the wafer stage with ILC will be studied using the analysis of the learned servo error signals obtained when mass feed-forward, and standard and adaptive ILC [8] are applied at different points on the wafer stage. For this analysis, we are interested in high-resolution time-frequency representations of the servo error signals.

While the well-known Fourier analysis of sensor signals mainly gains information about the frequency content of a signal without retrieving intelligence on the actual time moment at which a certain frequency occurs, time-frequency analysis methods offer information about the frequency content of signals at any time instant [9]. We refer here to time-frequency techniques like the low-resolution short-time Fourier Transform (STFT), the higher resolution quadratic Wigner distribution, that suffers from cross-terms and Choi-Williams distribution, which is introduced to deal with this deficiency, but leads to unacceptable frequency resolution results [9].
In order to retrieve a higher resolution time-frequency representation of a signal without cross-terms, waveform based decomposition might first be applied [10]. Signal decomposition can be performed with respect to wavelet bases, but these are known to be not well adapted to represent functions whose Fourier transforms have narrow high frequency support [1]. It has been shown that signal decomposition with respect to time-frequency atoms, i.e. modulated versions of wavelets [1],[2], adds more flexibility to signal decomposition and identifies narrow-band high frequent effects, such as vibrations, present in the servo error signals. The shape of the atoms is adapted to represent the local properties of the signal as good as possible.

The Matching Pursuit algorithm [1],[2] is used here to perform atomic decomposition of the learned servo error signals with respect to a multiple complex dictionary of atoms [11]. High-resolution time-frequency analysis of servo errors based on this kind of decomposition is suited to give considerable information on the system dynamics. Throughout this report, it will be shown, that this analysis is particularly useful to obtain insights in the low energetic position-dependent dynamics of a wafer stage with ILC, which are not easy to detect using wavelet-based decomposition of the signal or low-resolution time-frequency analysis with crossterms.
Chapter 2

Iterative Learning Control

This chapter will give a brief introduction on the theoretical backgrounds of both standard and adaptive ILC strategies applied for control of the wafer stage.

2.1 Standard Iterative Learning Control

Consider a causal, LTI closed loop system with plant $P$ and stabilizing feedback controller $C$. A setpoint trajectory $r$, representing the desired response of the control-loop, is defined on the interval $(t_0, t_0 + T)$, where $T < \infty$. With setpoint $r$ acting as the only disturbance on the loop (i.e.
where $S$ is the sensitivity function, defined by $S = \frac{1}{1 + PC}$.

The goal of ILC [5] is to create a feed-forward signal $u^*$ in an iterative way such that, after several iterations, the servo error $e^r$ is greatly reduced and $r = Pu^*$. Standard ILC design comprises the definition of a learning filter $L$ and a robustness filter $Q$, see Figure 2.1. A sequence of inputs $u_k$ is sought, with the condition:

$$u^* = \lim_{k \to \infty} u_k.$$  \hfill (2.2)

Index $k$ denotes the trial number, where a trial is one complete movement along the prescribed trajectory. The error $e_k$ is used to update the feed-forward $u_{k+1}$ in the next iteration according to the prototype update law:

$$e_k = e^r - S_p u_k - S n_k,$$  \hfill (2.3)

$$u_{k+1} = Q(u_k + L e_k),$$  \hfill (2.4)

where $S_p = \frac{P}{1 + PC}$ is the process-sensitivity function, and $n_k$ an output disturbance. Note that the product $L(s)Q(s)$ compresses the noise in the error signal at iteration $k$.

Using (2.3) and (2.4) and the uniform fixed point theorem, a sufficient but not necessary condition for the convergence of standard ILC can be derived:

$$|Q(s)(1 - L(s) S_p(s))| < 1 \quad \forall \quad s \in j\mathbb{R}$$  \hfill (2.5)

From this convergence criterion, it follows that the learning filter $L$ should approximate a stable inverse of the process-sensitivity function $S_p$, such that $L(s) S_p(s) \approx 1$. Furthermore, it follows that the robustness filter $Q$ ensures that the convergence criterion is also satisfied for frequencies where $|1 - L(s)S_p(s)| \not< 1$. In standard ILC, $Q$ is defined as a low-pass filter with constant cut-off frequency, in order to increase the robustness against high-frequent noise amplification and plant/model mismatch.
2.2 Adaptive Iterative Learning Control

In adaptive ILC [8] the fixed, steady-state robustness filter $Q$ of standard ILC, which remains the same for each iteration, is replaced by a time-varying filter $Q_k(s,\bar{t},\Omega_k(\bar{t}))$ [12],[13], namely a zero-phase Butterworth filter of order $n$ with cut-off frequency $\Omega_k(\bar{t})$, where $\bar{t} \in [t_0(k),t_0(k)+T]$, $t_0(k)$ is the initial time of the $k$-th trial, $T$ is the time required to perform the setpoint trajectory. The cut-off frequency $\Omega_k = \Omega_k(\bar{t})$ may vary throughout the length of each trial. Therefore, at each time instant $\bar{t}$, the $Q$-filter might change its cut-off frequency. Note that in adaptive ILC, the learning filter $L$ remains the same as for standard ILC.

Define $\Gamma_k(t,\bar{t},\Omega_k(\bar{t}))$ to be the inverse Fourier transform of the Butterworth filter $Q_k(s,\bar{t},\Omega_k(\bar{t}))$ as a function in the variable $s$:

$$Q_k(s,\bar{t},\Omega_k(\bar{t})) \xrightarrow{F^{-1}} \Gamma_k(t,\bar{t},\Omega_k(\bar{t})).$$  \hfill (2.6)

The converging prototype feed-forward update law of standard ILC (2.4) may now be changed to the following convolution [8]:

$$u_{k+1}(t) = \Gamma_k(t,\bar{t},\Omega_k(\bar{t})) * (u_k(t) + Le_k(t))|_{t=\bar{t}},$$  \hfill (2.7)

while (2.3) remains the same. The sufficient but not necessary condition for the convergence of standard ILC (2.5) can as well be rewritten to the following convergence criterion for adaptive ILC:

$$\sup_{k,\bar{t}} |Q_k(s,\bar{t},\Omega_k(\bar{t}))(1 - L(s)Sp(s))| < 1, \quad \forall \ s \in j\mathbb{R}.$$  \hfill (2.8)

Again the adaptive $Q$-filter makes sure that the convergence criterion is satisfied for all frequencies.

A bandwidth profile for the previously introduced time-varying robustness filter can be found through time-frequency analysis of a measured servo error $e_0$, obtained after implementation of a learned feed-forward signal $u_0$. Define $H_{e_k}$ as the high-resolution time-frequency representation obtained after atomic decomposition of the learned servo error signal $e_k$ with respect to a multiple complex dictionary of time-frequency atoms, i.e. modulated versions of wavelets [11]. Chapter 3 will discuss this time-frequency analysis method, which is used throughout this report, in more detail.
An adaptive bandwidth update algorithm [8] is used to derive a bandwidth profile from the time-frequency energy representation \( H_{ek} \) of the learned servo error signal. This algorithm proceeds in an iterative way. The time-varying bandwidth \( \Omega_k(t) \) of the robustness filter \( \Omega_k(s, \tilde{t}, \Omega_k(\tilde{t})) \) for any iteration \( k \) contains the frequency envelope \( F_{max,k}(\tilde{t}) \) as gain, which encompasses the frequencies of all signal components at each time-instant whose energy exceeds a level \( C_e \):

\[
F_{max,k}(\tilde{t}) = \max(\omega_k(\tilde{t})), \text{ for } H_{ek}(\tilde{t}, \omega_k(\tilde{t})) \geq C_e,
\]

where \( \omega_k(\tilde{t}) \) is the cross-section of \( H_{ek} \). The pre-defined energy level \( C_e \) discriminates between noise and deterministic signal components, provided that the energy of the noise components is significantly smaller than the energy of deterministic disturbances. Note that the frequency envelope \( F_{max}(\tilde{t}) \) remains the same during further iterations of the bandwidth update algorithm.

After a bandwidth profile \( \Omega(\tilde{t}) \) has been derived, the algorithm proceeds by implementing the profile to obtain a new measured error. The newly obtained error is compared to the error of a previous iteration using the \( l_2 \) norm of both errors according to:

\[
\Delta N_k(t) = N_k(t) - N_{k-1}(t),
\]

where

\[
N_k(t) = \frac{i+T_w/2}{j=i-T_w/2} \sum e_k^2(t_j).
\]

\( T_w > 0 \) gives the width of the window where the signals are locally compared. The comparison of the errors of two subsequent iterations will be used to evaluate the benefit of changes in the bandwidth profile from one iteration to the other. Using both \( F_{max}(\tilde{t}) \) and \( \Delta N_k(\tilde{t}) \), the bandwidth update algorithm is now given by:

\[
\Omega_{k+1}(\tilde{t}) = \Omega_k(\tilde{t}) + \Delta \Omega_k(\tilde{t}), \\
\Delta \Omega_k(\tilde{t}) = F_{max,k}(\tilde{t}) \cdot \Delta N_k(\tilde{t}) \cdot K_k(\tilde{t}),
\]

where the term \( K_k(\tilde{t}) = -\text{sign}(\Delta \Omega_{k-1}(\tilde{t})) \) is introduced to add the following logic to the mechanism: if the bandwidth was previously increased (\( \Delta \Omega_{k-1}(t_i) > 0 \)), while the error decreased (\( \Delta N_k(t_i) < 0 \)), this change was beneficial and the bandwidth may be further increased. On the other hand,
if an increase in the bandwidth resulted in a larger error, this was obviously not meant to be so and the bandwidth should be lowered again. The combination $\Delta N_k(t) \cdot K_k(t)$ results in this kind of update behavior.

A high-resolution time-frequency energy distribution will result in an accurate bandwidth profile, where stochastic and deterministic characteristics in the control signals are separated and the adaptive ILC algorithm will perform an optimal tradeoff between tracking performance and noise suppression. Next a high-resolution time-frequency analysis method will be introduced, based on atomic decomposition of the learned servo error signal with respect to a multiple complex dictionary of atoms [11].
Chapter 3

Time-frequency atomic decomposition of servo errors

In this chapter high-resolution time-frequency atomic decomposition for analysis of servo error signals of a wafer stage with ILC is discussed, which makes use of a general atomic decomposition algorithm known as Matching Pursuit [1]. A time-frequency energy distribution is obtained using the Wigner distribution of all selected atoms for decomposition of the signal.

Define a family $D$ of time frequency atoms by scaling, translation and modulation of a single window function $g(t) \in L^2(\mathbb{R})$, where $g$ is real, continuously differentiable, non-zero $g(0) \neq 0$ and $g(t) \sim O\left(\frac{1}{1+t^2}\right)$. Furthermore we impose that $\|g\| = 1$ and $\int_{\mathbb{R}} g(t) dt \neq 0$. By definition a time frequency atom is given by:

$$g_{\gamma}(t) = \frac{1}{\sqrt{s}} g\left(\frac{t-u}{s}\right) e^{2\pi j \phi t}, \quad (3.1)$$

with frequency modulation $\phi$, $\gamma = (s, u, \phi) \in \mathbb{R}^+ \times \mathbb{R}^2$ and $\frac{1}{\sqrt{|\gamma|}}$ a factor which normalizes the $l_2$ norm of $g_{\gamma}$ to one for any scale $s > 0$.

The family $D = \{g_{\gamma}\}_{\gamma}$ is extremely redundant. By selecting an appropriate countable subset of atoms $\{g_{\gamma_n}\}_n$ and corresponding expansion coefficients $a_n$ any signal $h$ can be represented by:

$$h = \sum_{n=-\infty}^{\infty} a_n g_{\gamma_n}. \quad (3.2)$$
The elements of the dictionary $\mathcal{D} = \{g_\gamma\}_n$ need to be selected adaptively, depending on local properties of the signal $h$.

Next an outline of the Matching Pursuit algorithm [1] is given in the Hilbert space $L^2(\mathbb{R})$ where successive approximations in the sense of linear orthogonal projections of the signal $h \in L^2(\mathbb{R})$ over elements of $\mathcal{D}$ are accomplished in order to best match the inner structure of the signal, according to:

$$
    h = \langle h, g_\gamma \rangle g_\gamma + Rh,
$$

where $Rh \in L^2(\mathbb{R})$ represents the residual after approximating $h$ in the direction $g_\gamma$, orthogonal on $Rh$. Therefore it follows that:

$$
    \| h \|^2 = \langle h, g_\gamma \rangle^2 + \| Rh \|^2. \tag{3.4}
$$

In order to retrieve a good signal representation, the residual $\| Rh \|$ should be minimized by choosing $g_\gamma \in \mathcal{D}$ such that $| \langle h, g_\gamma \rangle |$ is maximum.

After the first step of decomposition (3.3), the algorithm proceeds iteratively to sub-decompose $Rh$ by projecting it on a vector of $\mathcal{D}$ that matches $Rh$ the best, as has been done for the original signal $h$, according to:

$$
    R^n h = \langle R^n h, g_\gamma \rangle g_\gamma + R^{n+1} h. \tag{3.5}
$$

We can now inductively carry the decomposition of $h$ over $\mathcal{D}$ up to the order $m$:

$$
    h = \sum_{n=0}^{m-1} \langle R^n h, g_\gamma \rangle g_\gamma + R^m h, \tag{3.6}
$$

where $R^m h$ is the residual obtained at the $m^{th}$ order decomposition of $h$. Using (3.6), one can easily obtain the following important result:

**Theorem.** [1] If $\mathcal{D}$ is complete $(\text{span}(\mathcal{D})) = L^2(\mathbb{R})$ then

$$
    h = \sum_{n=0}^{\infty} \langle R^n h, g_\gamma \rangle g_\gamma \tag{3.7}
$$

and

$$
    \| h \|^2 = \sum_{n=0}^{\infty} | \langle R^n h, g_\gamma \rangle |^2. \tag{3.8}
$$
In this report a high-resolution time-frequency energy distribution of servo errors of the wafer stage with ILC is obtained by applying the Matching Pursuit algorithm [1], using decomposition with respect to a multiple complex atomic dictionary \( \mathcal{D} \) built with the asymmetric Daubechies’ wavelets and by their more symmetric and larger supported closely related cousins, i.e. Symmlets and Coiflets [11].

Based on decomposition of any signal \( h \in L^2(\mathbb{R}) \) over such a complex atomic dictionary, the Wigner distribution [14] of the obtained decomposition can be defined by:

\[
W_h(t, f) = \sum_{n=0}^{\infty} |< R^n h, g_{\gamma n} >|^2 W_{g_{\gamma n}}(t, f) + \sum_{n=0}^{\infty} \sum_{m=0, m\neq n} |< R^n h, g_{\gamma n} >| |< R^m h, g_{\gamma m} >| W_{n,m},
\]

where

\[
W_{n,m} = \frac{1}{2\pi} \int_{\mathbb{R}} g_{\gamma n}(t + \frac{\tau}{2}) g_{\gamma m}(t - \frac{\tau}{2}) e^{-2\pi j f \tau} d\tau
\]

(3.10)
denotes the cross Wigner distribution of the atoms \( g_{\gamma n} \) and \( g_{\gamma m} \). The double sum corresponds to the cross-terms of the Wigner distribution which should be removed in order to obtain a clear time-frequency distribution of the signal \( h \). We only keep the first sum and define the energy distribution of the signal \( h \) over the time-frequency plane as

\[
E_h(t, f) = \sum_{n=0}^{\infty} |< R^n h, g_{\gamma n} >|^2 W_{g_{\gamma n}}(t, f)
\]

(3.11)

By taking the absolute value of this energy distribution, one obtains the high-resolution time-frequency energy distribution of a signal. Next the high resolution time-frequency energy distribution will be used in a number of experiments to analyse the position-dependent dynamics of a wafer stage.
Chapter 4

Analysis of the position-dependent dynamics in a wafer stage

As mentioned earlier in this report an iteratively controlled wafer stage, i.e. an electromechanical servo system with scan speeds and accelerations of respectively 0.5 \( m/s \) and 10 \( m/s^2 \), is considered.

This motion system consists of a voice-coil actuated, [nm]-precision short stroke stage, supported by a [\( \mu \text{m} \)]-precision long stroke stage, which is actuated by linear motors. Performance of these permanent magnet motors can be reduced by cogging forces, which arise from the interaction of the permanent magnets in the stator with the ferromagnetic armature core and cause the armature to align in a minimal magnetic energy configuration when unexcited \([15],[16]\). Due to the slotted coil-spacing of exactly 20 [mm], the cogging forces in the linear motors are periodic and position-dependent. During movement of the long stroke stage, coil-transitions caused by cogging forces result in small disturbances on the short stroke. A number of experiments will now be used to detect and analyse periodic and position-dependent behavior in the control signals of a wafer stage to verify if these dynamics can indeed be due to cogging forces \([15],[16]\).
4.1 Position-dependent dynamics in a wafer stage with acceleration feed-forward

First an acceleration feed-forward signal is implemented for the standard length illumination interval of the given system as shown in Figure 4.1, which results in a typical servo error as depicted in Figure 4.2. The high-resolution time-frequency energy distribution of this servo error is computed, using the multiple complex atomic decomposition as described in the previous chapter, and plotted in Figure 4.2. Note that this results in four expected high energy peaks where the jerk of the acceleration setpoint is nonzero. Furthermore, from this high-resolution time-frequency energy distribution a time-varying bandwidth profile $\Omega$ is derived using the method described in section 2.2. The bandwidth profile $\Omega$ is plotted in Figure 4.2 including a smoothed approximation of $\Omega$. 
Figure 4.2: Servo errors, time-frequency energy distributions and time-varying bandwidth profiles $\Omega$ for a standard illumination length acceleration feed-forward.

Figure 4.3: Servo errors, time-frequency energy distributions and time-varying bandwidth profiles $\Omega$ for an enlarged illumination length acceleration feed-forward.
Together with the standard length acceleration feed-forward signal another acceleration feed-forward signal is implemented with an enlarged length of the illumination interval. Servo error, high-resolution time-frequency energy distribution and time-varying bandwidth profile for this feed-forward are visualized in Figure 4.3. In both situations the starting position on the wafer stage is identical. Furthermore, in the derivation of the bandwidth profile $\Omega$ the same pre-defined energy level $C_e$ has been used to discriminate between noise and deterministic signal components.

Comparing the results for both cases, one can draw several conclusions. First note that the servo errors, energy distributions and bandwidth profiles of the system are identical for both situations during the acceleration part of the trajectory till the beginning of the illumination interval. Secondly one can see that this similarity ends when the illumination interval starts. During the illumination and deceleration part one can observe differences in the shape of the servo errors, the time-frequency energy distribution, the maximum energy level present in the servo errors and the bandwidth profile obtained for both cases.

Since the initialization and acceleration parts are identical for both situations, the differences encountered during the illumination and deceleration part can only be due to system dynamics or disturbances which are related to a certain position on the wafer. However, in this experiment only acceleration feed-forward has been applied, which does not compensate completely for all dynamics and periodic disturbances in the control system. Hence, for further analysis we will make use of servo errors obtained after implementation of learned feed-forward signals, by means of which the system dynamics and periodic disturbances are compensated.
4.2 Position-dependent dynamics in a wafer stage with adaptively learned feed-forward

Figure 4.4: Servo error signals obtained by implementing an adaptively learned feed-forward for one setpoint at different starting positions in y-direction on the wafer stage, but with the same step-length

A second experiment has been carried out, where an adaptively learned feed-forward signal, obtained for one setpoint, is implemented without variation in the illumination length at different starting positions in y-direction on the wafer stage. In other words, the same learned feed-forward signal is used to perform mutually shifted setpoints of 100 [mm] length. This leads to servo errors, that show peaks of another magnitude than those observed in the servo error of the learned setpoint, as shown in Figure 4.4.

If the adaptively learned feed-forward is applied on the learned trajectory, all system dynamics and periodic disturbances are taken into account in the feed-forward signal and are therefore compensated adequately. However, from the observed peaks one can conclude that this feed-forward (learned for one trajectory) is not suited to compensate all dynamics and disturbances on a different position on the wafer stage, which can only be due to the fact that the system exhibits position-dependent behavior.
Figure 4.5 shows the root mean square (RMS) values of the servo errors from Figure 4.4. The values on the horizontal axis represent the subsequent starting positions of the setpoint in $y$-direction. The adaptively learned feed-forward used here is obtained for the trajectory starting at $y_0 = -50$ [mm]. The RMS-values show a clearly periodic behavior, i.e. comparable values are obtained within 20 [mm] intervals at the learning position. Note that the coil-spacing of the linear motors that actuate the [$\mu$m]-precision long stroke of the wafer stage is exactly 20 [mm] and corresponds with the periodic behavior encountered in the RMS-values of the servo errors. Hence, this position-dependent and periodic behavior can indeed be due to cogging forces [15],[16] arising from the interaction of the permanent magnets in the stator with the ferromagnetic armature core in the long stroke linear motors of the wafer stage.

If the position-dependent dynamics encountered indeed originate from cogging forces, one should be able to detect a major periodic frequency component in the servo errors of 25 [Hz], since the scan speed of the stage during the experiments was approximately 0.5 [m/s] and the coil-spacing of
the linear motors actuating the stage is exactly 20 [mm]. The scan speed is maximum during illumination and therefore the frequency component of 25 [Hz] should appear in close vicinity of the illumination interval.

Figure 4.5 has shown that position-dependent dynamics are compensated appropriately when starting at a certain position while using other starting points leads to inadequate compensation. Hence, one will find that in some of the servo errors, obtained by implementing the same learned feed-forward signal for mutually shifted setpoints, a 25 [Hz] component is detectable while this frequency does not occur in other errors.

A final experiment has been performed where the high-resolution time-frequency energy distributions are calculated for the servo errors obtained by implementation of one learned feed-forward for several mutually shifted trajectories. An average high-resolution time-frequency energy distribution for all of these distributions is computed to avoid servo errors where the 25 [Hz] component is undetectable. This computation is justified because the length of each mutually shifted trajectory is the same and the subsequent acceleration, illumination and deceleration parts occur at exactly the same time instances during the trajectory.

The resulting average high-resolution time-frequency energy distribution is shown in Figure 4.6. Only essential atoms are taken into account. After observation of the figure a few more remarks can be made. The energy distribution shows frequency content around 25 [Hz], which is at a lower energy level than the system dynamics encountered so far and is already present during the acceleration part of the system. During the illumination interval a 25 [Hz] frequency component is observed at a slightly lower energy level. At the end of the illumination interval and during the beginning of deceleration, frequency content is present around 25 [Hz] at a relatively higher energy level. The difference in dissipated energy between the acceleration and deceleration part on the one hand and the illumination interval on the other can be explained by considering that excitation of the dynamics is more likely to occur when the acceleration and deceleration levels of the motion system are high.
Finally note that next to the 25 [Hz] component a frequency of 32 [Hz] is clearly visible in the image. The appearance of this component can be explained through the decomposition algorithm, which works with a computational framework in powers of two ($2^N$) because of computational efficiency reasons. It has been show that part of this framework can become visible through leakage of low-energy signal components from their actual time-frequency location to a frequency which is a power of 2.
Chapter 5

Conclusions

It has been shown that high-resolution time-frequency analysis based on atomic decomposition over multiple complex dictionaries of atoms can be of particular use in the investigation of certain dynamics encountered in control systems. Energy distributions based on complex atomic dictionaries are well-localized in time and frequency, their resolution is higher than other time-frequency analysis methods, they do not include crossterms and their high-energy deterministic components can be much better separated from low-energy stochastic components.

The approach can lead to a clearer identification of time-varying non-linear and stochastic servo error components, which is of particular use in the design of the bandwidth of a time-varying robustness filter in adaptive ILC. Furthermore, the method is able to detect non-stationary and deterministic chaotic vibrations present in servo error signals, which have narrow-band frequency characteristics.

Since the Matching Pursuit algorithm first selects atoms containing higher energy than the next ones, the order in which the selection of atoms propagates is of particular interest to investigate which parts of the servo error signals are dominant with respect to other parts, which can provide insight in the dynamical features of a control system.
The example of a wafer stage discussed throughout this report has shown that high-resolution time-frequency analysis can be used to obtain insights in the low energetic position-dependent dynamics encountered in this motion system. Several experiments have revealed that the position-dependent behavior can be related to so-called cogging forces, which arise in the linear motors actuating the wafer stage.

Future research may focus on detection of deterministic and stochastic components in control signals, but also on detection of their constituents, i.e. periodic, subharmonic, chaotic and transient for deterministic nonlinear time-varying components and non-stationary and stationary for stochastic components. Furthermore application of high-resolution time-frequency analysis in advanced intelligent identification and control schemes can provide much more challenging control engineering problems.
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