Modelling the tibia-knee prosthesis interface
a finite element analysis

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Summary

Replacement of joints by artificial joints is often done on patients suffering from osteoarthritis (arthrosis), rheumatoid arthritis, congenital deformations or traumatic disorders. It consequences mostly in a pain-free motion of the joint during normal daily activity.

Loosening of the artificial joint is one of the major causes of failure [Ryd, 1986]. Loosening is dependent on the age of the patient, the type of implant and the load-situation. It is assumed that, under normal load-situations, the cells interfacing the hydroxy-apatite coated prosthesis are bone-cells. A biological fixation between the tibia and the prosthesis is then expected. Overloading of the prosthesis initiates damaging of the healthy bone-cells, and the development of soft-tissue surrounding the prosthesis. This process is called the interface bone-resorption process.

Initially, four models describing the constitutive behaviour of the soft-tissue interface, have been formulated. The models were implemented in a 2-dimensional model of the tibia, and have been judged by using five criteria. Two models have been judged to be adequate to describe the soft-tissue interface. These models are a description of the interface by means of non-linear foundations and a description by means of non-linear hypo-elastic elements in combination with contact. Eventually, the first model has been selected, because it was easier to expand to a 3-dimensional model.

The constitutive models of the soft-tissue interface and the bone-interface have been implemented in a 3-dimensional model of the tibia. The soft-tissue interface was presented by non-linear foundations, and the bone-interface by constraints in x-, y-, and z-direction.

It was concluded that the model of the interface is a good phenomenological model, which can be used to predict the load-transmission through the tibia, dependent on the presence of a soft-tissue interface or a bone-interface. Simulations showed that a soft-tissue interface results in a dramatically change of load-transmission, compared to a bone-interface. The magnitude of the stresses and the strain-energy density appeared to be much higher when considering the soft-tissue interface. It is therefore not unlikely that the presence of soft-tissue causes loosening of the prosthesis.
## List of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<td>$E$</td>
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<td>$[N/mm^2]$</td>
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<td>$E_0$</td>
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<td>$\sigma$</td>
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<td>$P$</td>
<td>2\textsuperscript{nd} Piola-Kirchhoff stress-tensor</td>
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<td>$g$</td>
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<tr>
<td>$u$</td>
<td>displacement</td>
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</tr>
<tr>
<td>$t$</td>
<td>thickness interface</td>
<td>$[mm]$</td>
</tr>
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</table>
Nomenclature

\( a \) scalar a
\( A \) tensor A
\( \Delta \) matrix A
\( A_x \) A in the x-direction
\( A_y \) A in the y-direction
\( A_z \) A in the z-direction
\( \vec{a} \) column a
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Chapter 1

General introduction.

At the Eindhoven University of Technology (E.U.T.), research is conducted on the process of loosening of an artificial joint. To gain more insight in this process, the proximal part of a hydroxyapatite coated prosthesis of the tibia, implanted in bone, is implemented in the finite element code MARC. This report reviews the constitutive description of the interface between prosthesis and bone.

Replacement of joints by artificial joints is often done on patients suffering from osteoarthritis (arthrosis), rheumatoid arthritis, congenital deformations or traumatic disorders. It mostly consequences in a pain-free motion of the joint during normal daily activities.

Loosening of such an artificial joint is one of the major causes of failure. Studies [Ryd, 1986] indicated that 10 - 12 percent of the implanted knee prostheses failed by loosening. Loosening is dependent on the age of the patient, the type of implant and the load-situation [Weinans, 1991]. In 1960, Willmer discovered how the histology of bone cells depend on the load-situation. Beside bone-cells, he distinguished two kinds of cells, which were originally bone-cells, but obtained a different macro-structure because of overloading. These cells were called soft-tissue and fibro-cartilage. Soft-tissue is formed when the bone-cell has been damaged. Recovery changes the soft-tissue into successively fibro-cartilage and bone-cells [Willmer, 1960]. It is assumed is that the process described by Willmer also applies on the cells surrounding the prosthesis. Under normal conditions, the prosthesis is surrounded with bone-cells. Overloading of the bone initiates damage to bone-cells and the development of soft-tissue. This process is called the interface bone-resorption process [Weinans, 1991]. This means that an overloaded prosthesis can be (partially) surrounded by soft-tissue. Loosening of the prosthesis can then be expected, because soft-tissue is not able to transmit shear forces or tensile forces [Hori et al., 1982].

Judged by the fixation technique, four types of implants are distinguished. Fixation of the first type is accomplished by tight fit, which means that the outer geometry of the prosthesis must fit perfectly in the inner geometry of the bone. The second type of implant is fixated by cement (PMMA), which is inserted between the bone and the prosthesis. The third type of prosthesis is the porous coated prosthesis. The rough surface, obtained by the porous coating, and the bone surrounding the prosthesis are responsible for the geometric confinement of the prosthesis. The fourth type of implant, which has recently been developed, is covered with a coating which realizes biological fixation between prosthesis
1. General introduction.

and bone. The coating used to accomplish the biological fixation is called hydroxy-apatite coating, and is composed of the same main substances as bone. Still, phenomena of particularly the fourth type of prosthesis are unknown, particularly with respect to the influence of the hydroxy-apatite coating on the interface between bone and implant. Research has been done to determine the extent of ingrowth of bone into the hydroxy-apatite coating [Soballe et al., 1992], [Stephenson et al., 1991], [Geesink et al., 1987], and to ascertain the influence of this coating on the constitutive behaviour of the interface between the bone and prosthesis [Soballe et al., 1992], [Geesink et al., 1987].

1.1 Objective.

The objective of this study is to gain more insight in the influence of the interface bone resorption process on the mechanisms of implant loosening. The type of prosthesis which is considered, when studying this objective, is the hydroxy-apatite coated knee-prosthesis. Based on this objective, an assignment has been formulated,

*Develop a model which describes the behaviour of the interface located between bone and prosthesis, and implement this in the finite element code MARC.*

The influence of the interface model on the stress- and strain-distribution in bone will be analysed by implementing the interface model in a finite element model of the tibial part of a hydroxy-apatite coated knee-prosthesis. This part of the human body is considered because of the ongoing research on the process of loosening of knee prostheses, conducted at the Eindhoven University of Technology.
Chapter 2

Bone and its behaviour on prosthesis implementation.

The constitutive behaviour of bone is non-homogeneous, anisotropic and visco-elastic, and therefore very complex [Weinans, 1991]. The function of bone seems trivial: transfer of load from one object to another within the body. Joints, in combination with muscles, tendons and ligaments provide the force transmission between two bones. To understand the force transmission through bone, it is necessary to "learn" about the structure of bone.

When a joint itself is not able to transmit forces properly, a part of the bone is often replaced by a prosthesis. It is assumed that this replacement consequences in a change of structure of the bone in two ways. First, bone cells interfacing the implant and bone will be resorbed when high contact stresses between implant and bone occur. This process results in a fibrous tissue layer interposed between bone and prosthesis and is called interface bone resorption [Weinans, 1991]. Second, because of a changing force transmission through the bone, the structure and geometry of bone are adjusted by resorption (activated by cells called osteoclasts), or apposition (activated by osteoblasts). This process is called remodelling [Bourne, 1972] [Weinans, 1991]. Both bone remodelling and interface bone resorption will be described in this chapter.

Judging by their method of fixation, four different types of prostheses exist, as mentioned in the general introduction. During this thesis, only one type of prosthesis has been considered, namely the prosthesis which is covered with a hydroxyapatite coating. Direct bonds between the hydroxyapatite coating and bone can be achieved because of the similar chemical structure. Namely, the calcium phosphates, of which hydroxyapatite consists, are also the main substance of bone. This is called biological fixation. The coating is not only compatible to bone, but is also non-toxic, and bioresorbable.

2.1 Anatomy and histology of bone.

In this section, only information about the anatomy of bone is provided which is considered useful to gain full understanding about the force transmission through bone. This means that the description of the anatomy is not complete, but satisfactory to the purpose it is meant for.
At molecular level, bone is a true composite [Weinans, 1991]. It consists of proteins, which are mostly collagen fibres, water, vessels and a matrix of minerals which are mainly calcium-phosphates (chondrocytes). Based on the difference of architecture and arrangement of the collagen fibres and calcium phosphates, two different sorts of bone can be distinguished, namely woven bone and lamellar bone. Woven bone is a pre-mature, temporary phase of bone, and can rapidly grow or decrease by respectively apposition and resorption. It appears, amongst others, in healing processes. The collagen fibres in woven bone are relatively loose-packed, and randomly ordered. This means that it can be considered isotropic. Lamellar bone, however, is more regularly arranged in line with stress-patterns. The fibres of collagen and the associated calcium phosphates are orientated in sheets. This results in anisotropic properties of the lamellar bone fibres.

Regarding the histology at macroscopic level, bone can be subdivided into cortical bone and cancellous bone which have been depicted in figure 2.1.

![Figure 2.1: The location of the different types of bone [Bourne, 1972].](image)

Cortical bone is almost solid, except for some spaces for blood vessels and osteocytes. The volume fraction of fibres in cortical bone is about 80 to 95 percent [Little, 1973]. The mechanical behaviour of cortical bone is highly anisotropic and its stiffness is high compared to other bone ($E=1.7 \times 10^4 N/mm^2$)[Weinans, 1991]. Cancellous bone has the lowest density. The structure of cancellous bone looks like randomly arranged "struts" or in one direction orientated "plates". The "struts" and "plates" are called trabeculae, which is the reason that cancellous bone is also called trabecular bone [Weinans, 1991]. It is not as anisotropic as cortical bone and its stiffness is much smaller ($E=4 \times 10^2 N/mm^2$)[Bourne, 1972][Huson, 1992].

The histology of the tibia is considered to obtain knowledge about the location of the cortical bone, spongious bone and cancellous bone. In figure 2.1 a tibia has been depicted. The shaft of the tibia exists of the cortex, which is cortical bone, and the medullary cavity consisting of bone marrow. The expansions at the extremities of the tibia are composed of spongious bone and cancellous bone. Cancellous bone is located between the medullary cavity and the end of the tibia. It's density becomes smaller near the medullary cavity. The ends of the tibia are covered by cartilage, which forms the gliding surface of the joint. It provides the possibility of motion between the particular long bone and the adjacent bone in the skeleton.
2. Bone and its behaviour on prosthesis implementation.

2.2 The interface bone-resorption process.

The possibility of the interface bone-resorption process near an artificial joint, and its influence on the loosening-process of a prosthesis have been generally accepted [Geesink, 1987][Weinans, 1991][Soballe, 1992]. However, the process itself and the mechanisms inducing this process are less understood, so assumptions have to be done concerning these two aspects.

It is assumed that the bone resorption process is initiated by a mechanical stimulus. The mechanical stimulus is induced by external forces which are transmitted through an adjacent bone. It is not clear what stimulus initiates the interface bone-resorption process. In this thesis, it is assumed, that the stimulus is a mechanical stimulus.

![Figure 2.2: A hypothetical description of the interface bone-resorption process.](image)

Full understanding of the bone-resorption process has not yet been accomplished. Publications of Willmer in 1960 about the dependence of bone-cells on the load-situation are used to understand this process. He discovered that overloading of bone damages the bone-cells, which induces the formation of a tissue called soft-tissue. When recovery occurs, this tissue changes into successively fibro-cartilage and bone-cells [Willmer, 1960]. It is assumed is that the process described by Willmer can also be used to describe the interface bone-resorption process. When the prosthesis is not overloaded, a natural fixation exists between the hydroxy-apatite coated prosthesis and the surrounding bone-cells. Overloading of the prosthesis initiates damaging of bone-cells and the development of soft-tissue. Recovery initiates the development of successively fibro-cartilage and bone. In figure 2.2, a scheme is depicted in which a graphical representation of the preceding hypotheses is given.

A histological description, and a description of the mechanical properties of respectively a bone interface, a fibro-cartilage interface and a soft-tissue interface will be given in the following subsections.
2. Bone and its behaviour on prosthesis implementation.

2.2.1 A bone-interface.

When the interface between the prosthesis and the tibia is not overloaded, the artificial joint is surrounded with bone-cells. The cells which surround the prosthesis are not damaged, so a biological fixation exists between these cells and the hydroxy-apatite coated prosthesis.

It is assumed is that when a bone-interface surrounds the prosthesis, the histology of this bone corresponds with the histology of cancellous bone. Therefore, the mechanical behaviour of the bone-interface is assumed to be the same as the mechanical behaviour of cancellous bone, which has already been discussed in section 2.1.

2.2.2 A fibro-cartilage interface.

Fibro-cartilage consists of an intercellular material, which is a matrix of minerals (chondrocytes), and a network of collagenous fibres and elastic fibres. Figure 2.3 shows a schematic illustration of the histology of fibro-cartilage.

![Figure 2.3: The histology of fibro-cartilage [Little, 1973].](image)

The histology of the fibro-cartilage can be used to explain the mechanical behaviour of the fibro-cartilage. The stiffness of fibro-cartilage is smaller than the stiffness of bone, because of the presence of elastic fibres in fibro-cartilage. The randomly ordered collagenous fibres induce an approximately isotropic behaviour [Jenkins, 1991][Livingstone, 1986].

2.2.3 A soft-tissue interface.

Like the fibro-cartilage interface, the soft-tissue interface is composed of a matrix of minerals, which are mainly calcium phosphates, collagenous fibres and elastic fibres. However, the volumetric ratio between the collagenous fibres and the elastic fibres is lower when a soft-tissue interface is regarded. This consequences in a more elastic behaviour of the soft-tissue. Another important difference between soft-tissue and fibro-cartilage is the orientation of the fibres. The soft-tissue fibres are orientated parallel to the interface surface,
contrary to the fibres of fibro-cartilage, which are randomly ordered. In this direction, parallel to the interface surface, the orientation of soft-tissue is defined. A schematic representation of the histology of soft-tissue is shown in figure 2.4.

![Schematic representation of the histology of soft-tissue](image)

**Figure 2.4:** *The structure of soft-tissue [Little, 1973].*

The amount of information available concerning the mechanical behaviour of soft-tissue is very small. Compression tests have been performed by Hori and Lewis [Hori et al., 1982] to obtain insight in the mechanical and constitutive behaviour of soft-tissue. The results of these tests are depicted in figure 2.5.

This figure shows a growing stiffness with increasing compression. The curve depicted in this figure should only be interpreted phenomenologically, because of discrepancies during the experiments which could not be controlled. This is representative of the current knowledge of the constitutive behaviour of soft-tissue: only the phenomenological behaviour is known while no proper information is available about the quantitative mechanical behaviour of soft-tissue.

Until now, only the mechanical behaviour of soft-tissue in compression has been investigated. The mechanical behaviour on tensile- and shear-load has been based on the histology of the soft-tissue. It is assumed that soft-tissue is not able to offer any resistance against shear- and tensile-forces [Weinans et al., 1991].

Various studies claim that the histology of soft-tissue, and thus the mechanical properties, can be influenced by the presence of hydroxy-apatite [Geesink et al., 1987] [Soballe et al., 1992]. These studies reported that soft-tissue contains more cartilage fibres when it interfaces a hydroxy-apatite coated implant. This would mean that soft-tissue is stiffer in combination with a hydroxy-apatite
coated prosthesis.

It is believed that, because of its mechanical properties, the presence of soft-tissue between bone and prosthesis causes loosening of the prosthesis [Albrektsson, 1987] [Soballe, 1992][Weinans, 1991].

2.3 Remodelling of bone.

First, an explanation will be given of the processes indicated in this report by remodelling and modelling of bone, because no unequivocal meaning of these processes is available in literature. Remodelling is defined as the process of formation of new bone or resorption of existing bone. This means that this process consequences a change of bone-mass. Two types of remodelling are distinguished, namely internal remodelling and external remodelling. Internal remodelling is the adaptive process relative to the internal morphology of the bone, while external remodelling is the formation of new bone at the cortical surface. Modelling is defined as the continuous process of bone resorption and formation with no net change of bone-mass. This means that the amount of bone resorbed by osteoclasts is the same as the amount of bone formed by osteoblasts. Unlike interface bone-resorption, remodelling and modelling of bone occurs throughout the tibia.

The processes of remodelling, as well external remodelling as internal remodelling, and modelling are held responsible for the change of histology of bone, when the load-situation on bone is changed. These processes are, just like the interface bone-resorption process, induced by a mechanical stimulus. Different mechanical stimuli are reported as being the stimulus of the remodelling of bone and modelling of bone, but none has been proved. Possible stimuli are strain [Arramon, 1993] and strain energy density [Weinans, 1991].

Remodelling of bone, initiated by an artificial joint, will be explained here. A tibia without a prosthesis transmits forces to the cortical bone via the line of the least resistance. A prosthesis may change this force transmission, which initiates remodelling of bone. The mechanical stimulus induces bone formation, where the load on bone has been increased, and resorption of bone, where the load on bone has been decreased. This leads to a changing histology of the tibia. Assumed is that the remodelling of bone plays an important role in the loosening of the prosthesis. However, this thesis deals with the interface bone-resorption process so the remodelling process of bone will not be considered closer.

2.4 Discussion

This chapter first considered the anatomy and histology of the tibia. This is important to understand force transmission through the tibia. When considering a prosthesis implanted in the tibia, two other processes may also become important, namely the interface bone-resorption process, and the process of remodelling and modelling of bone. Both processes are induced by a mechanical stimulus which is initiated by external forces. It is assumed that the interface bone-resorption process changes overloaded bone-cells into soft-tissue, which is believed to cause loosening of the prosthesis. The second process, remodelling of bone, is the formation of new bone, and resorption of existing bone. It may also cause loosening of the prosthesis when the force transmission through bone is
drastically changed after implantation of a prosthesis.
The first process, the interface bone-resorption process, has to be modelled in the finite element code MARC. To do this, constitutive models of the tibia, prosthesis and interface, particularly the soft-tissue interface, have to be developed.
Unfortunately, only qualitative data, and no quantitative data are available about the mechanical properties of the soft-tissue. The model will therefore be a phenomenological model.
Chapter 3

Model-descriptions of tibia, prosthesis and interface.

In this chapter, a finite element model will be presented which describe the constitutive behaviour of the tibia, prosthesis and interface. The in the preceding chapter presented description of the mechanical properties of the tibia and the three types of interface will be used to define the constitutive models. The next section will describe what assumptions and simplifications are made to model the three parts.

Because of the objective of the thesis, the most important part of the model is the definition of the interface bone-resorption process. This means that the constitutive properties of the soft-tissue, as assumed in section 2.2, are included in the constitutive model of the interface. As already determined in this section, the constitutive model describing the interface is a phenomenological model. Three fundamentally different models have been developed to describe the constitutive behaviour of soft-tissue. After creating the three constitutive models of the interface, they will be judged not only on their capability of describing the constitutive behaviour of the soft-tissue, but also on their numerical stability.

3.1 Assumptions and simplifications.

To model the proximal part of the tibia including prosthesis, three parts are distinguished: the prosthesis, the tibia and the interface, which have been depicted in figure 3.1. Assumptions and simplifications will be done concerning these parts of the model. In figure 3.1, number 1 indicates the tibia. As already mentioned in chapter 2, bone is an extremely complex material, because of its material properties. It is non-homogeneous, anisotropic, visco-elastic and able to model and remodel itself (section 2.3). The model would become very complex when all these properties would be considered. To avoid a complex model and since not the tibia, but the interface is the most important part of the model, the bone of which the tibia consists is assumed to be homogenous, isotropic and linear-elastic. The model of the tibia will only exist of two types of bone, namely cortical bone and cancellous bone, which are characterized and distinguished by Young's modulus $E$, and Poisson's ratio $\nu$. 

Figure 3.1: The three parts of which the model consists.

Number 2 represents the interface. The mechanical and constitutive behaviour of this part varies, dependent on its histology. In section 2.2 has been described how external forces determine the histology of the interface. Three main types of interfaces have been distinguished, namely a bone-, fibro-cartilage and soft-tissue interface. To simplify the model, only two types of interface will be modelled, which are the bone-interface and the soft-tissue interface. A constitutive model of the fibro-cartilage interface is considered redundant in this phase of model-definition of the interface. This is why the existence of the fibro-cartilage interface will be neglected.

First, assumptions and simplifications regarding the soft-tissue interface will be discussed. In section 2.2 has been described what assumptions have been done concerning the mechanical and constitutive behaviour of soft-tissue. Soft-tissue is assumed to be highly anisotropic, and will therefore be modelled as such. Its mechanical behaviour will be modelled as non-linear. Other assumptions are that soft-tissue behaves elastic and homogeneous.

As assumed in section 2.2, the material properties of the bone-interface resemble the material properties of cancellous bone. The assumptions regarding the bone-interface will therefore be the same as the assumptions concerning the cancellous bone of the tibia. This means that the mechanical and constitutive behaviour of the bone-interface is assumed to be homogenous, isotropic and elastic.

The prosthesis has been indicated by number 3 and is composed of cobalt-chrome. A hydroxyapatite coating has been applied on the stem and the under-surface of the plateau. The material properties are mainly determined by the cobalt-chrome part. Cobalt-chrome is a metal with a much higher stiffness than bone. This means that deformations of the prosthesis is small compared to the deformations of the tibia and interface. This, and the fact that there is no interest in the stress- and strain-distribution and deformation of the prosthesis, is the reason why the prosthesis is modelled as a rigid.
3.2 Mechanical and constitutive models in MARC.

The implementation of the constitutive models in FEM of tibia, prosthesis and interface, is discussed separately in this section.

3.2.1 Tibia.

Because of the assumptions and simplifications done in the preceding section, only cortical bone and cancellous bone, behaving isotropic, homogenous and linear-elastic, are modelled.

The cancellous bone is modelled with 8-node plain-strain elements in the 2-dimensional model. 20-node brick elements are used to describe cancellous bone in a 3-dimensional model. To prescribe the mechanical behaviour of these elements, Young's modulus $E$ and Poisson's ratio $\nu$ are used. Table 3.1 shows what values are attributed to $E$ and $\nu$ to characterize cancellous bone. Specifications about the 8-node plain-strain element and the 20-node brick element are included in respectively appendix A.1 and A.2.

The cortical bone is described by beam-elements in a 2-dimensional model and by shell-elements in a 3-dimensional model. The mechanical behaviour of these elements is also prescribed by Young's modulus $E$ and Poisson's ratio $\nu$, of which the values are shown in table 3.1. The beam element is a line element which has been further explained in appendix A.3. The shell-element is a 8-node element of which further specifications are given in appendix A.4.

<table>
<thead>
<tr>
<th>type of bone</th>
<th>cancellous (2D)</th>
<th>cancellous (3D)</th>
<th>cortical (2D)</th>
<th>cortical (3D)</th>
</tr>
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<tr>
<td>type of element</td>
<td>8-node plain-strain</td>
<td>20-node brick</td>
<td>beam</td>
<td>shell</td>
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<tr>
<td>$E$ [N/mm$^2$]</td>
<td>$4 \times 10^5$</td>
<td>$4 \times 10^4$</td>
<td>$1.7 \times 10^4$</td>
<td>$1.7 \times 10^4$</td>
</tr>
<tr>
<td>$\nu$ [-]</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 3.1: Material properties of cancellous bone and cortical bone.

3.2.2 Prosthesis.

In section 3.1 has already been discussed why the prosthesis is modelled as a rigid. Besides those arguments, more advantages can be mentioned, which favour a rigid modelled prosthesis. The first advantage is the fact that the number of elements is reduced. This consequences in a shorter processing time. The second advantage is that the ratio of the largest and smallest stiffness becomes smaller. This is beneficial to the stability of the finite element numerical process.

3.2.3 Interface.

Dependent on the magnitude of the external forces working on the prosthesis, the interface is modelled as a soft-tissue interface or a bone-interface. A bone-interface has the same mechanical and constitutive properties as cancellous bone. In the case of soft-tissue, the mechanical and constitutive behaviour are assumed to be non-linear, anisotropic,

elastic, and homogenous (see section 3.1). In MARC, three different descriptions have been tested to describe non-linear material behaviour: hypo-elastic elements, springs and foundations.

To test the fortran-subroutine and the prescribed non-linear "elements", test-problems are run in MARC. The test-problems consist of two parts. The first part is an isotropic, elastic element with a stiffness which is thousands times higher then the other part of the test-problem. It is only used to apply forces. The second part is the non-linear, elastic anisotropic hypo-elastic element, spring or foundation. The test-problems will be discussed more extensively in the following.

Non-linear hypo-elastic element.

A hypo-elastic element is an element which allows large strain, thanks to the use of the 2nd Piola-Kirchhoff stress. The subroutine HYPELA (see appendix B.1) can be used to prescribe a non-linear stiffness applied on a hypo-elastic element. To serve this purpose, the equation which describes the non-linear behaviour has to be included in the subroutine. In this subroutine, a non-linear stiffness-matrix $D$ is computed, which is updated during the Newton-Raphson iteration process (see appendix C). The incremental strain and the stiffness-matrix $D$ are used to calculate the incremental stress, according to equation 3.1.

$$
\delta \tilde{\sigma}_j^i = D_{j^k l}^i \delta \tilde{e}^{lk} + \tilde{g}_j^i
$$

(3.1)

$\tilde{g}$ is a coefficient with which temperature dependent processes can be described. The stress $\tilde{\sigma}$ is the 2nd Piola-Kirchhoff stress, which can be derived from the 2nd Piola-Kirchhoff stress tensor $P$, described by equation 3.2.

$$
P = \lambda \text{tr}(E) I + 2 \mu E
$$

(3.2)

The symbols $\lambda$ and $\mu$ represent the Lamé coefficients, while $E$ and $I$ stand for respectively the Green-Lagrange strain-tensor and the unit-tensor.

The strain $\epsilon$, used in equation 3.1, is the Green-Lagrange strain and is described by the following equations:

$$
\epsilon_{grl} = \frac{1}{2} (\gamma^2 - 1)
$$

(3.3)

with

$$
\gamma = \frac{l_r}{l_0}
$$

(3.4)

Equation 3.1 is valid for non-linear elastic material and suitable for large deformations.

Figure 3.2 shows the test-problems of the hypo-elastic element.

The shaded element is the element on which the forces are applied, and is only used for this purpose. The other element is the hypo-elastic element. A graphical presentation of the equation which describes the non-linear behaviour of this element has also been depicted in this figure. The graph has been based on the curve shown in figure 2.5, representing the behaviour of soft-tissue: an increasing stiffness when compression is applied, and a small stiffness in case of tension. The axes of the curve are labelled $E$ (Young’s modulus) and $\epsilon_x$ (strain in x-direction).

non-linear hypo-elastic element

Figure 3.2: The test-problem to test the non-linear hypo-elastic element, and the prescribe relation.

The results of the tests are shown in figure 3.3. This figure shows the stress-strain curve of node \( n \) (see figure 3.2), which is non-linear, as defined in the subroutine.

Concluded is that the non-linear stiffness has successfully been prescribed by the subroutine HYPELA.

However, two important problems were met while implementing the programmings of the subroutine. First, the hypo-elastic element is an isotropic element and not an anisotropic element, which is needed to model the soft-tissue. The second remark is about the prescribed strain, which should be dependent on the orientation of the hypo-elastic element. This makes it necessary that the strain is not determined with respect to the global coordinate system, but to the local (element) coordinate system.

Non-linear elastic spring.

In MARC, a spring is placed between two nodes, and its stiffness is given by the spring-stiffness \( k_1 \). The subroutine USPRNG (see appendix B.2) can be used to prescribe a non-linear spring-stiffness. To accomplish this, an equation describing the non-linear stiffness is included in the subroutine. The relative displacement of the two nodes, in the direction in which the spring is applied, is supplied to the subroutine. The subroutine calculates the force and the ratio of the spring stiffness supplied by the main-program to the spring-stiffness supplied by the subroutine. The subroutine calculates the spring-stiffness as a function of the displacement, according to equation 3.5.

\[
k_1(u) = \frac{dF_1(u)}{du}
\]

(3.5)

The Newton-Raphson iteration-process is used to iterate to the prescribed stiffness (appendix C). In figure 3.4 the test-problem of the non-linear elastic spring has been depicted. It consists of the shaded element on which the forces are applied, and two non-linear springs. The curve, which is also shown in this figure, represents the non-linear behaviour.

which is applied on the springs. The labels of the axes show the spring-stiffness $k_1$ and the displacement $u$. This graph has been conducted from the curve shown in figure 2.5.

In figure 3.5, the results of the tests are shown. The spring-force $F_1$ and the displacement $u$ of node $n$ have a non-linear relation, as prescribed in the subroutine USPRNG.

Two remarks have to be made concerning the non-linear elastic spring. First, springs need a local coordinate system when a randomly orientated soft-tissue has to be modelled. Namely, displacements of the nodes of one spring are now only detected in either the $x$-direction, or the $y$-direction, or the $z$-direction of the global coordinate system. (dependent in the orientation of the spring) These should be detected in the $x$-, $y$- or $z$-direction of the local coordinate system. Second, the parameters which describe the behaviour of the spring are the spring-force and the displacement. It is obvious that these parameters are not user-friendly because they are geometry-dependent (unlike for example stress and strain).

Non-linear elastic foundation.

In MARC, a foundation is applied to an edge or face of an element. It's stiffness is prescribed by $k_2$. To create a subroutine which results in a non-linear foundation, an equation which describes the non-linear behaviour has to be included. For this purpose the subroutine USPRNG is used (see appendix B.3). The displacement perpendicular to the edge or face of the element on which the foundation is mounted, is supplied to the subroutine, which then calculates the stress and the ratio of the foundation stiffness supplied by the main-programm to the foundation stiffness supplied by the subroutine. The subroutine determines the foundation stiffness as a function of the displacement, according to equation 3.6.

$$k_2(u) = \frac{dF_2(u)}{du}$$  (3.6)
The iteration-process used to obtain the prescribed stiffness, is the Newton-Raphson iteration process (appendix C).

Figure 3.6: The test-problem to test the non-linear foundation, and the prescribed relation.

In figure 3.6, the test-problem of the non-linear foundation has been depicted. The non-linear stiffness of the foundation is also depicted in this figure and represents the non-linear behaviour of soft-tissue as found by Hori and Lewis (see figure 2.5). The axes are labelled \( k_2 \), the stiffness per unit of volume, and \( u \), the displacement perpendicular to the face or edge.

Figure 3.7 shows the result of the test-problem. The relation between the stress and the displacement of node \( n \) appears to be non-linear, as prescribed by the subroutine.

Two remarks are made concerning the non-linear curve of node \( n \). The second remark also concerns the displacement. The displacement of node \( n \) depends on the thickness of the foundation, which is given a value in the subroutine. This means that the foundation stiffness, prescribed as a function of the displacement, is dependent on the geometry.

3.3 Discussion

This chapter described the implementation of the three parts of the proximal tibia with prosthesis in MARC. The prosthesis has been modelled as a rigid, and the bone of which the tibia consists has been modelled as being elastic, linear, isotropic and homogenous. The non-linear behaviour of the interface has been described with the aid of hypo-elastic elements, springs or foundations. Three fortran-subroutines are used to make these non-linear.
The three subroutines prescribed the non-linear behaviour very adequately. However, some important differences exist between the three models of the interface.

Table 3.2 shows the quantities and dimensions of the three subroutines. A very important disadvantage of the spring, is that it's stiffness is not calculated per unit of element area. It is therefore dependent on the geometry of the model.

<table>
<thead>
<tr>
<th>non-linear hypo-elastic element</th>
<th>non-linear spring</th>
<th>non-linear foundation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$ [N/m²]</td>
<td>$F_1$ [N]</td>
<td>$F_2$ [N/m²]</td>
</tr>
<tr>
<td>$E$ [N/m²]</td>
<td>$k_1$ [N/m]</td>
<td>$k_2$ [N/m³]</td>
</tr>
<tr>
<td>$\epsilon$ [-]</td>
<td>$u$ [m]</td>
<td>$u$ [m]</td>
</tr>
</tbody>
</table>

Table 3.2: The most important quantities and matching dimensions of the three subroutines.

The same is valid for the displacement $u$ of the foundation and spring, which is not related to the original thickness of the interface. A great advantage of the foundation is the fact that no local (element) coordinate system has to be introduced. The displacement $u$ is always determined perpendicular to the element-surface of the element on which the foundation is mounted, unlike the hypo-elastic elements and the springs, which need a local coordinate system. The hypo-elastic element is an isotropic element. It should be anisotropic because of the properties of soft-tissue, which it represents. In the next chapter, this problem is solved by adapting the model of the interface described by non-linear hypo-elastic elements.

Decided is that the non-linear spring is not well suited to model the interface. The disadvantages mentioned (local coordinate system and the stiffness being dependent on the element-surface) make the use of the non-linear spring to complex. This means that in the next chapter, when modelling a 2-dimensional representation of the tibia, the non-linear hypo-elastic element and the non-linear foundation will be used.
Chapter 4

Interface descriptions included in 2-D FE-models of the tibia.

In this chapter, two-dimensional finite element models of the tibia with an interface will be presented. The main purpose of these models is to present the constitutive behaviour of the interface properly.

\begin{figure}[h]
  \centering
  \includegraphics[width=0.5\textwidth]{figure4_1.png}
  \caption{The three parts of which the models consist.}
\end{figure}

In chapter 3, the behaviour of the non-linear hypoelastic element and the non-linear foundation have been judged to be suitable to represent the behaviour of the non-linear interface.

Based on those two constitutive models, four 2-dimensional models of the tibia with prosthesis and interface have been developed. The model, which will be tested in this chapter, has schematically been depicted in figure 4.1. This 2-dimensional model is a simple representation of the tibia, but it is considered adequate to test the numerical behaviour of the interface model.

After introducing the models and performing calculations, the best description of the interface has to be selected. Therefore, the models will be judged on five different criteria, which are accuracy, solvability, possibility for expansion into a 3-dimensional model, processing time, and the ease of implementation.

4.1 Description of the models

As mentioned in the beginning of this chapter, four models will be presented in this section. The models are implemented in the finite element code MARC and consist of three parts: the prosthesis, the non-linear interface, and the tibia (see figure 4.1).
4. Interface descriptions included in 2-D FE-models of the tibia.

The models differ in the way the non-linear interface has been described. The prosthesis and the bone of the tibia however, has been modelled identically in each model. In figure 4.1, the location of the cortical bone and cancellous bone has been depicted. The thickness of the cortical bone has been taken homogeneous, and is one millimeter. Modelling the constitutive behaviour of cancellous bone, cortical bone and the prosthesis has been discussed in section 3.2.

Table 4.1 shows how the interface can be described in MARC by four different models. The initial stiffness of the interface, perpendicular to the orientation of the soft-tissue fibres of the interface (see chapter 2), has also been enumerated in this table. The initial stiffness of the interface is defined as the stiffness of the interface when no load has been applied. In figure 4.2 the four models have been depicted. This figure also shows the \( \sigma - \epsilon \) and \( \sigma - u \) plots of the interface in the direction perpendicular to the orientation of the interface. A description of the models is given below.

<table>
<thead>
<tr>
<th>Interface</th>
<th>model</th>
<th>implemented as</th>
<th>initial stiffness ( E_0 ) [N/mm(^2)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>model 1</td>
<td>foundation links</td>
<td>10(^2)</td>
<td></td>
</tr>
<tr>
<td>model 2</td>
<td>hypo-elastic element + contact</td>
<td>10(^2)</td>
<td></td>
</tr>
<tr>
<td>model 3</td>
<td>plain-strain elements + contact</td>
<td>10(^2)</td>
<td></td>
</tr>
<tr>
<td>model 4</td>
<td>gap + contact</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1: Four different descriptions of the constitutive behaviour of the interface.

In model 1 the interface is modelled with non-linear foundations. The foundations are mounted between the tibia and the ground, which consequences in a prosthesis represented by the ground. This means that the prosthesis is considered rigid. The non-linear behaviour of the interface has been imposed by including a non-linear equation in the fortran-subroutine USPRNG. The equation is the same equation as has been tested in section 3.2, and describes the non-linear behaviour of soft-tissue, as described by Hori and Lewis [Hori et al., 1982], phenomenologically. The first curve presented in figure 4.2 shows the mechanical behaviour of the soft-tissue interface as a result of the prescribed stiffness. Because of the properties of the foundations, the resistance of the interface against shear, is zero.

In model 2, the interface is described by hypo-elastic elements and contact. The elements have been attached to the tibia. The prosthesis is modelled as a rigid, which means that no deformation of the prosthesis is possible. To introduce anisotropy in the constitutive model of the interface, and because the rigid prosthesis allows no attachment to any element, the MARC contact-algorithm is applied between the prosthesis and the interface. An explanation of how the contact-algorithm works, has been given in appendix D. The curve describing the constitutive behaviour of the interface, in the direction perpendicular to the orientation of the interface, is also shown in figure 4.2. When contact exists
4. Interface descriptions included in 2-D FE-models of the tibia.

between the interface and the rigid prosthesis, the prescribed behaviour of the non-linear hypo-elastic elements will determine the behaviour of the interface. When tensile forces are applied, separation of the prosthesis and interface is possible, which results in a zero stiffness of the interface. The peak in this curve is caused by the force needed to separate two contact bodies. This separation force is positive, which means that a tensile force is needed to separate the two bodies. In this model, not only the resistance of the interface against tensile forces is zero after separation, but also the resistance against shear forces. This behaviour has also been introduced by the contact.

Model 3 has been based on the second model. The rigid prosthesis and contact between the prosthesis and interface have been maintained, but the non-linear hypo-elastic elements have been replaced by linear plain-strain elements. The curve which describes the behaviour of the interface is shown in figure 4.2. This curve shows that the non-linear behaviour is induced by the contact. When compression is applied, the behaviour of the interface is linear. Tensile load leads to separation of the rigid prosthesis and plain-strain elements of the interface, which induces the non-linear behaviour. When separation of the two bodies has occurred, the resistance against tensile-load is zero. Separation of the bodies is the cause of the peak in the curve. It is induced by the force needed to separate the two bodies. Just like model 2, this model has, induced by the presence of contact, no resistance against shear-forces. This is accomplished by rating the friction-coefficient of the contact zero.

In model 4, a totally different approach in modelling the interface has been employed. The interface is presented by a gap in combination with contact. A geometrical gap between the prosthesis and the tibia represents the thickness of the interface. When the prosthesis is loaded, the gap will close. The contact must prevent penetration of the prosthesis.
4. Interface descriptions included in 2-D FE-models of the tibia.

into the tibia. In this model, initially, the two bodies are separated, which means that, when performing a static analysis, the stiffness matrix will be singular. To prevent this, a dynamic analysis has to be performed, which adds a mass-matrix to the problem so it is not under-defined anymore. Figure 4.2 shows the non-linear behaviour caused by the geometrical gap and the contact. Until contact, no stresses will be detected. Again, the peak is caused by the force which is needed to separate the two bodies. Just like described in model 2 and 3, the zero value of the friction-coefficient induces no resistance against shear force.

4.2 Selection of the models

The four models presented in the preceding section will be tested in this section. Based on five selection criteria, the best model will be selected, so further analysis (3-dimensional) can be focussed on one model. The following criteria have been formulated to judge the models.

- accuracy of the model;
- solvability;
- possibility for expansion into a 3-dimensional model;
- processing time;
- ease of implementation.

The criteria have been placed in order of importance, and will be explained in the following.

The accuracy of a constitutive model can be tested by comparing its results with experimental data. In this case, the constitutive behaviour of soft-tissue must be verified. The only experimental data available concerning soft-tissue, is the data obtained from compression tests performed by Hori and Lewis [Hori et al., 1982]. In chapter 2 has already been explained why this data only can be used to learn about the phenomenological behaviour of soft-tissue. In subsection 4.2.1 will be described how data form this article is used to judge the behaviour of the models.

Solvability is expressed by two values. The singularity ratio is the first value which will be discussed. It is a measure of the conditioning of the system. It is related to the conditioning number, which is the ratio of the highest to the lowest eigenvalue of the system. The second value to express the solvability is the convergence ratio. It can also be expressed by a numerical value. The convergence ratio provides information concerning the accuracy of the solution. To get reliable generated data, this ratio should not be smaller then a given value. A singularity ratio larger then $10^{-4}$ and a convergence ratio larger then $10^{-4}$ are acceptable values.

The 2-dimensional model of the tibia has to be extended to a 3-dimensional model. The possibility of expansion of a 2-dimensional model into a 3-dimensional model is hard to predict. Namely, this criterium can only be judged, by the problems which were met when implementing the 2-dimensional description of the model. A 3-dimensional description
can entail new problems which can not be foreseen. This means that this criterium can not be qualified with a hundred percent certainty.

The processing time is the time needed to generate solutions. This is dependent on the number of elements used, the type of elements, the chosen calculation method, and the type of machine which is used to generate a solution.

To qualify the ease of implementation of a description of a model, the time needed for implementation and the complexity of the implementation will be considered. Implementation of a description of a model is possibly the most time-consuming part in the process of finite element modelling and is therefore useful to consider.

4.2.1 Judging the behaviour of the models.

The behaviour of the interface descriptions as modelled in the models of the tibia has to be judged. However, no applicable experimental data is available to verify the behaviour of the interface. The experimental data available are in-vivo experiments with a tibia and prosthesis [Ryd, 1986]. However, the data of the models can not be compared with the experimental data, because of the influence of tendons, ligaments and muscles, which have not been included in the model. In the description of the experiments performed by Ryd, also no information is supplied about the presence and location of soft-tissue.

This means that the behaviour of the interface descriptions as implemented in the model of the tibia, have to be verified alternatively. The method used has been based on the characteristic material behaviour of soft-tissue as found by Hori and Lewis [Hori Lewis, 1982]. The constitutive behaviour of the interface must resemble the constitutive behaviour of the soft-tissue. This means that it has a negligible stiffness in case of tensile- or shear-load, and increasing stiffness in case of pressure load. To control this behaviour of the interface in the models, the stress- and strain-transmission through the interface, to the tibia will be considered. The stress- and strain-distribution of the tibia illustrates the phenomenological behaviour of the interface.

To test the four constitutive model of the interface, two load situations will be regarded. The first load situation is designed to test the increasing stiffness of the soft-tissue when pressure is applied and the absence of shear-load transmission, while the second load situation is developed to test the soft-tissue's lack of tensile-load transmission.

The first load situation is a prescribed displacement applied in the axial direction as depicted in figure 4.3A. The expected transmission of load is also shown in this figure. Because of the axial displacement of the tibia, the soft-tissue orientated in the x-direction is compressed in the y-direction and the soft-tissue orientated in the y-direction is loaded with shear-stress. Because of the higher stiffness of the elements which represent the cortical bone, the largest compression of the hypo-elastic elements occurs where the cortical bone meets the interface. The increasing stiffness of the interface at this location is responsible for the stress-situation as depicted in figure 4.3A, in which the dashed lines represent iso-stress lines. The amount of stress transmitted through the elements representing the cortical bone will be larger then the amount of stress transmitted through the elements representing the cancellous bone. Further, no shear-force transmission should be observed along the stem, where the interface is loaded with shear-stresses.
4. Interface descriptions included in 2-D FE-models of the tibia.

The second load situation which is considered is shown in figure 4.3B. This load-situation results in a non-symmetric load of the interface: The soft-tissue located on the left of the symmetry-axis faces tensile load, while the soft-tissue located on the right of this axis faces pressure load. Because of the constitutive behaviour of the soft-tissue, which does not allow transmission of tensile load, the stress situation as indicated in figure 4.3B is the expected result of this load situation.

Appendix E.1 and appendix E.2 show the results of respectively the first and second load-situation (see figure 4.3A and B) of model 1 to 4, shown in figure 4.2.

4.2.2 Selection of an interface description.

The four models presented in section 4.1 are judged by the criteria introduced in the beginning of this section. Solvability and the processing time can be expressed by a numerical value, which is not possible when judging the accuracy, possibility for expansion into a 3-dimensional model, and the ease of implementation. These will be qualified by three possible values, which are + (= good), o (= neutral), and - (= bad). Table 4.2 shows for each criterium how the models have been qualified. Argumentation for these qualifications will be given in the following.

The accuracy of the models is tested by comparing the results, determined by means of finite element analysis (see appendix E), with the expected results as described in the preceding subsection.

First the axial load-situation will be discussed. As can be observed, the stress-distribution (appendix E.1) of the models differs significantly from the expected stress-distribution as depicted in figure 4.3A. Namely, the highest values of the stress $\sigma_y$ are not found along the cortical bone, but along the prosthesis-stem. When observing the x-displacements of the tibia (appendix E.2), it can be concluded that the deviating stress-distribution of the models is due to outward bending (away from the prosthesis-stem) of the parts of
4. Interface descriptions included in 2-D FE-models of the tibia.

<table>
<thead>
<tr>
<th>model</th>
<th>accuracy</th>
<th>solvability</th>
<th>possib. for expan. to 3-d</th>
<th>processing time</th>
<th>ease of implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>sing. rat.</td>
<td>conv. rat.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>model 1</td>
<td>+</td>
<td>$10^{-2}$</td>
<td>$10^{-2}$</td>
<td>$10^{-4}$</td>
<td>+</td>
</tr>
<tr>
<td>model 2</td>
<td>+</td>
<td>$10^{-2}$</td>
<td>$10^{-2}$</td>
<td>$10^{-2}$</td>
<td>0</td>
</tr>
<tr>
<td>model 3</td>
<td>+</td>
<td>$10^{-4}$</td>
<td>$10^{-13}$</td>
<td>$10^{-12}$</td>
<td>+</td>
</tr>
<tr>
<td>model 4</td>
<td>+</td>
<td>$10^{-7}$</td>
<td>$10^{-15}$</td>
<td>$10^{-12}$</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 4.2: Qualification of the four models judged on the five criteria

The tibia located left and right of the prosthesis-stem. This behaviour can be attributed to the constitutive behaviour of the soft-tissue: it is not able to transmit shear load and tensile load. Both loads are present at the prosthesis stem. Shear load is also located at the underside of the prosthesis-plateau.

Results of the second load-situation have been depicted in appendix E.3 and E.4, which show the stress-distribution and displacement in x- and y-direction. As expected, the large stresses $\sigma_y$ can be observed along the cortical bone, on which the pressure load has been applied. The same figure shows that the resistance against tensile-forces of most models is not zero. Model 4 is the only exception. This is the dynamic model, which has no initial constraints. The displacements show that the prosthesis forces the parts located left and right of the prosthesis-stem, to bend outward. This is also observed in the figures which show $\sigma_x$.

The displacements of model 3 are a factor 2 smaller, compared to the other models. This is explained by the fact that the stiffnesses of model 1 and 2 approach zero when tensile-load is applied, and the non-initial contact of model 4.

The ratios which represent the solvability, the singularity ratio and convergence ratio, are obtained from both load situations, discussed in subsection 4.2.1. Both load situations in this subsection show the same trends: the solvability of model 1 and model 2 are considered satisfying, while the solvability, especially expressed by the convergence ratios, of model 3 and 4 is very bad. This means that the output of these models can not be trusted.

Judged by the problems which were met when implementing the 2-dimensional model, expansion of the model into a 3-dimensional model will only be problematic for model 2. The stiffness of the hypo-elastic elements should be a function of the strain perpendicular to the orientation of the soft-tissue, which has been defined parallel to the interface surface. However, the stiffness is a function of the strain in x- or y-direction with respect to the global coordinate system. This means that the local coordinate system of every hypo-elastic element has to be determined. This is a very complex matter when implementing a 3-dimensional model.

The numerical values of the processing time need no further explanation.

The ease of implementation of all models is qualified equal, except for model 1. This is due to the fact that the thickness $S$ of the foundation is given a value in the subroutine, which is a global thickness. It is preferable that the thickness of the interface easily can be adjusted. The local thickness of the interface of the other models can easily be changed by
relocation of the nodes. However, the local change of thickness of the interface of model 1 is more complex. To do this, the subroutine USPRNG has to be extended so the thickness of the interface becomes dependent on an element parameter like the element-number, or element-stress.

Judged by the solvability, only two models qualify to represent the interface in further analysis. These models are model 1 and model 2.

In the next chapter, a 3-dimensional model of the tibia will be presented with a hundred percentage soft-tissue interface, of which the thickness is uniform. This is why the non-linear foundation (model 1) is chosen to model the interface.

4.3 Discussion

To test four constitutive models of a soft-tissue interface, tests have been done with 2-dimensional models of the tibia with soft-tissue interface and prosthesis. Two load-situations have been considered. The two load-situations were developed to test the constitutive behaviour of the soft-tissue. The first load-situation was constructed to analyse the behaviour of the soft-tissue on shear- and tensile-load, and the second load-situation to test the behaviour on a combination of tensile load and pressure load. The results of the tests have been judged by using five criteria.

Eventually, one constitutive description of the interface has been selected. The non-linear foundation appeared to be the best constitutive model to describe the behaviour of soft-tissue in the next chapter.

A logical next step in modelling the interface is an implementation of the non-linear foundation interface in a 3-dimensional model of the tibia. This way, the influence of the soft-tissue interface on the load-transmission through the tibia can be determined. In the 3-dimensional model, also the bone-interface will be modelled. The behaviour of both interfaces can then be compared.
Chapter 5

The constitutive interface model in a 3-D model of the tibia.

In the preceding chapter, the four 2-dimensional models have been tested to select the best constitutive description of the interface. Based on five criteria, the description of the interface by non-linear foundations has been selected.

To analyse the effect of the constitutive behaviour of the interface on the stress- and strain-distribution of the tibia, the non-linear foundation will be implemented in a 3-dimensional model of the tibia. A description of the 3-dimensional model, which is very similar to the 2-dimensional model, will be given in the next section.

5.1 Description of the model.

The constitutive models describing the tibia, prosthesis and interface, have already been presented in section 3.2. In this section, the constitutive behaviour of the three parts have to be implemented in a 3-dimensional model.

5.1.1 3-D model of the tibia.

The two types of bone of which the model of the tibia consists, are cancellous bone and cortical bone, and have been modelled by respectively 20-node brick elements and 8-node shell elements. Young’s modulus ($E$) and Poisson's ratio ($\nu$), corresponding to the cortical bone and the cancellous bone can be found in table 3.1. In the 2-dimensional model of the tibia, the thickness of the cortical bone has been considered uniform. The thickness of the cortical bone in the 3-dimensional model is presented more realistic. In distal direction, the thickness is assumed to increase logarithmically from a thickness of 2.5 mm. towards a thickness of 7 mm.. This has been conducted from a real tibia. The thickness of the cortical bone is dependent in the geometry of the tibia. The geometry of the model of the tibia is simplified, so the thickness of the cortical bone will
therefore not perfectly fit to the geometry of the model. However, the description of the thickness of the tibia as used in the model is considered satisfying.

The model of the tibia is rotational symmetric, as can be seen in figure 5.1. This means that the stem of the prosthesis is presented as a cylinder, and not with "wings" as shown in figure 3.1. This way, the prosthesis will not be able to resist torsion of the tibia about the y-axis.

5.1.2 3-D model of the interface.

As already explained in chapter 4, the prosthesis can only be presented as a rigid when foundations are used to describe the non-linear interface. Namely, the foundations are applied between the tibia and the ground, which means that the prosthesis is presented by the ground.

The interface is assumed to consist of bone or soft-tissue, dependent on the magnitude of the external forces, working on the prosthesis. When the interface consists of bone, a biological fixation is considered between the prosthesis and the tibia. Since the prosthesis is represented by the ground, the tibia will be fixated to the prosthesis by suppressing the displacement in x-, y-, and z-direction of the nodes "contacting" the prosthesis.

When the interface consists of soft-tissue, the non-linear foundations will represent the interface. The fortran-subroutine, which is used to prescribe the non-linear behaviour of the foundation, is the same as used in chapter 3, to test the non-linear foundation. The equation which describes the non-linear behaviour of the foundation has been inserted in the subroutine USPRNG, and represents the non-linear behaviour of soft-tissue as found by Hori and Lewis (see figure 2.5). The stiffness parallel to the face on which the foundation is applied, is zero. This represents the soft-tissue's absence of resistance against shear.

Parameters which describe the constitutive behaviour of the soft-tissue are the initial stiffness $k_{20}$ and the thickness $t$. The initial stiffness is assumed to be $10 \, N/mm^2$ [Weinans, 1982], and the thickness is assumed to be 2 millimeters. The initial stiffness is not zero, but very small compared to the stiffness of the cortical and the cancellous bone. A smaller initial stiffness is not possible because of the numericall problems which can then be expected.

5.2 The tested load-situations.

The tibia of the body undergoes different mechanical loads. In figure 5.2, six possible loads on the tibia have been depicted. These are forces in the x-, y-, and z-direction and moments about the x-, y-, and z-axes. Morrison did research on the mechanics of the knee joint in relation to normal walking [Morrison, 1969], and found that the following loads are important when considering normal walking: force in the y-direction, force in the z-direction, and a torque about the y-axis.

When the force in the y-direction is transmitted by both condyles, this load is called bicondylar load. When the knee joint is highly loaded (in the y-direction), the greater portion of the load is transmitted by the medial condyle [Morrison, 1969]. Morrison found that the maximum force, transmitted in y-direction during normal walking is in the range
of two to four times the body weight, with an average value of three times body weight (see figure 5.5). The force in the z-direction is called the medio-lateral force. The forces working in this direction during normal walking was found to be small, namely 0.26 times the body weight. The magnitude of the maximum torque about the y-axis, transmitted through the tibia is 15 Nm.

To test the 3-dimensional model of the tibia, in particular the interface, two load-cases will be analysed. The load-cases are a combination of the loads distinguished by Morrison when considering normal walking. The only loads which will not be considered are the torque about the y-axis and the anterior-posterior load. Because of the geometry of the model, no restriction against rotation about the y-axis is provided, and anterior-posterior load has the same affects as medio-lateral load.

The first load-case which will be considered is a load which is not equally divided over the two condyles. This load-case has been chosen because of the experimental findings of Morrison, which pointed out that a highly loaded knee-joint does not transmit the forces equally divided through the condyles. The force applied, is 2.5 times the body weight on one condyle, and 1.5 times the body weight on the other. This way, a highly loaded tibia is obtained according to Morrison, because 4 times the body weight is applied. The medio-lateral forces have not been applied in this load-case, to keep the results of the finite element analysis of the model understandable. The forces are applied on two models. One model with a hundred percentage bone-interface, and one with a hundred percentage soft-tissue interface. In figure 5.3, this load-case has been depicted.

Figure 5.2: *External loads acting on the tibia.*

Figure 5.3: *The two load-cases which are used to test the influence of the interface on the stress-distribution of the tibia.*
The second load-case considers a medio-lateral load. This load-case has been chosen to test the model on extreme loading conditions. This load is one times the body weight, which is four times the load in medio-lateral direction when normal walking is considered. This load-case is used to consider the behaviour of the interface when a combination of tensile-stress and pressure-stress is applied. This load-case is tested on the same models as the first load-case: one model with a bone-interface, and one model with a soft-tissue interface. This load-case has been depicted in figure 5.3.

5.3 Analysing the results.

The results of load-case 1 and 2 have been depicted in respectively appendix F.1 and appendix F.2. Both loadcases are symmetric with respect to the sagittal plane through the tibia (figure 5.4). This is why the results will be studied in this plane. To analyse the behaviour of the model, the first, and second component of stress will be studied. It is not useful to consider the von Mises equivalent stress, because it makes no distinction between tensile stresses and pressure stresses, which should be considered when analysing the behaviour of the soft-tissue interface. Also the strain energy density (SED) will be considered, because it is assumed to influence the interface bone-resorption process and initiate the remodelling process of bone (see chapter 2).

As already mentioned, loadcase 1 has been applied on a model with hundred percent bone-interface, and a model with hundred percent soft-tissue interface. Stresses are mainly transmitted through the cortical bone, which is correct, because this also happens when a tibia without prosthesis is considered. However, stresses can also be observed being transmitted at the end of the prosthesis-stem towards the cortical bone. The presence of soft-tissue reduces the amount of stress transmitted through the cortical bone near the prosthesis. This is probably caused by the lack of resistance against shear of the soft-tissue located under the plateau of the prosthesis.

The second component of stress of the model with bone-interface shows a notable distribution. Namely, stress is transmitted to the tibia, at the end of the prosthesis-stem. Also can be observed, that the load is not transmitted directly towards the cortical bone. The stresses of the model with soft-tissue interface are concentrated on one side of the prosthesis under the plateau. The magnitude of stresses is also much higher. Both can be explained by the fact that the prosthesis-stem does not transmit shear-stress, which consequences in a more heavily loaded plateau.

What has been discussed concerning the first and second component of stress, can also be observed when analysing the SED. Considering the model with bone-interface, the SED is concentrated at the end of the prosthesis-stem and in the cortical bone. The SED should not be concentrated at the end of the prosthesis-stem, because it consequences in an unnatural loaded tibia. The load should be transmitted through the cortical bone. The model with the soft-tissue interface shows that the SED is concentrated direct under
the plateau of the prosthesis. The magnitude of the SED is a factor five higher compared to the SED of the model with bone-interface. A very high SED is assumed to stimulate the growth of soft-tissue.

The second load-case is used to test medio-lateral loads on the tibia. The models tested are a model with a hundred percent bone-interface and a model with hundred percent soft-tissue interface.

The figures depicting the first and second component of stress of the model with bone-interface, show the same phenomenon as observed in load-case 1. The end of the prosthesis-stem transmits a large part of the stresses towards the cortical bone.

The model with the soft-tissue interface shows some interesting phenomena. As well the first component of stress as the second component, show that tensile forces are transmitted by the soft-tissue interface. This can be explained by the fact that the initial stiffness is not zero. Large displacements are needed to induce the tensile stiffness to zero (exponential decrease). The initial stiffness can not be decreased, because it will initiate numerical instabilities.

Compared to the model with bone-interface, the first component of stress of the model with soft-tissue interface is large. However, the amount of stress in the x- and y-direction, transmitted through the cortical bone is negligible.

The SED of the model with bone-interface shows that the loads are transmitted via the end of the prosthesis-stem, towards the cortical bone. Load should be transmitted from the prosthesis direct to the cortical bone.

The SED of the model with soft-tissue interface is 10 times larger compared to the SED of the model with bone-interface. The SED is very large and could initiate the interface bone-resorption process.

5.4 Possible extension of the interface description.

The model of the tibia with prosthesis and interface as presented in this chapter is, despite of its predictive capacity, a simple representation of the reality. To model a more realistic model, several problems will be met, which will be discussed in this subsection.

Until now, the interface is represented as either a bone-interface or a soft-tissue interface. Of course, it is more realistic to model an interface in which the presence of soft-tissue in the interface increases or decreases dependent on a mechanical stimulus. This is the first dilemma which is met. Namely, it is not obvious what quantity induces the development of soft-tissue. It is hard to predict what mechanical quantity is the stimulus of the growing process of soft-tissue. Research has to be done in this field.

When a mechanical stimulus has been chosen, the location of soft-tissue, dependent on the mechanical stimulus, can be determined. The model still is a static model, and the location of the soft-tissue has been determined for one particular load-case. The assumption done to perform this analysis, is that the development of soft-tissue is dependent on a certain maximum external load of the prosthesis.
To get a more realistic model, the interface bone resorption process should be modelled as a time-dependent process. When this is done, a more realistic load, caused by walking, can be applied on the tibia. Walking is a cyclic process, which causes a cyclic load on the tibia (see figure 5.5).

![Figure 5.5: The cyclic load on the tibia in y-direction and in z-direction [Morrison, 1969].](image)

When modelling the interface bone-resorption process as a time-dependent process, it is plausible that not only bone is resorbed, but also formed. This means that also the interface bone-formation process should be modelled. The interface bone-formation process is also initiated by a mechanical stimulus. This is not necessary the same quantity as the stimulus which induces the growth of soft-tissue.

When modelling the interface bone resorption process as a time-dependent process, a load-case has to be applied which causes the development of soft-tissue. This load-case can be measured on subjects with a pathological tibia.

### 5.5 Discussion

In this chapter, the 3-dimensional model of the tibia has been tested with soft-tissue interface and bone-interface. The results of these tests will be enumerated and commented upon in this section.

The model with the bone-interface appeared to transmit the loads through the end of the prosthesis-stem towards the cortical bone. When the bone remodelling process responds to the SED, bone will be formed at the end of the prosthesis-stem, so the density-distribution of the tibia will change. This is the consequence of transmission of load, which differs from the transmission of load in a tibia without prosthesis. It is therefore important that a prosthesis does not change the force-transmission through the tibia. At the moment, research is performed to develop a prosthesis, which transmits load only through the cor-
tical bone.

The magnitude of the SED of the model with soft-tissue interface are much larger, compared to the SED of the model with bone-interface. Sometimes, the difference is a factor 10. This is caused by the constitutive behaviour of the soft-tissue. A limitation of the constitutive model of the interface is the fact that the tensile forces in the tibia are not zero when the prosthesis is surrounded with soft-tissue. This is caused by the tensile-stiffness of the interface, which is small, but not zero. A second limitation of the present analysis is the choice for uniform layer thickness. It may be obvious that also an interrupted soft-tissue interface also drastically changes the load in the interface between the tibia and prosthesis.

The significance of the findings, finally, can be formulated in general terms, because interfaces occur in many different types, of which one example was studied here. It is evident from the results that the soft-tissue interface dramatically changed the load-transfer mechanism, effecting in high stress concentrations locally near the interface. It is not unlikely that these high stresses are responsible for the loosening of the prosthesis.
Chapter 6

General discussion.

The objective of this study, posed in chapter 1, is to gain more insight in the influence of the interface bone resorption process on the mechanisms of implant loosening. The formulated assignment was to

*develop a model which describes the behaviour of the interface located between bone and prosthesis, and implement this in the finite element code MARC.*

In this chapter will first be discussed what work has been performed to come to a satisfying model-description of the interface. Second will be discussed what the significance is of the interface model as modelled in this thesis.

6.1 Short review of the models.

In chapter 4, four interface models have been presented, of which two interfaces were judged adequate to represent the soft-tissue interface. The solvability of the two other models was unsatisfying in such an extent, that the other criteria on which the interface-models have been judged not even have been considered. The two remaining constitutive descriptions of the interface are a description by means of foundation links, and a description by means of hypo-elastic elements in combination with contact. Both models will be shortly reviewed.

Foundation links are mounted on an edge or face of an element. This way, the element is constrained in the direction perpendicular to the face or edge. The stiffness of the foundation $[N/mm^3]$ is prescribed by the displacement $[mm]$ perpendicular to the orientation of the face or edge. The resistance of the foundation against shear is zero. The foundation link is automatically mounted to the ground, which means that the prosthesis is a rigid. The hypo-elastic element is an isotropic element which allows very large strains $[-]$. The strain can be used to prescribe the stiffness $[N/mm^3]$ of the element. To create anisotropic behaviour of the interface, contact is applied between hypo-elastic elements and the prosthesis. In this case, the prosthesis is not necessary modelled as a rigid. Local coordinate systems have to be applied to correctly prescribe the stiffness as a function of the strain in the direction perpendicular to the orientation of the interface.

In the 3-dimensional model of the tibia, the non-linear foundation links have been used to prescribe the constitutive behaviour of the interface. The hypo-elastic elements were
6. General discussion.

not used, because it would mean that local coordinate systems had to be applied on the hypo-elastic elements.

6.2 Significance of the interface model.

In the 3-dimensional model of the tibia, two interfaces have been modelled, namely the soft-tissue interface and the bone-interface. The models of the interface describe the phenomenological behaviour of the interface, and its effect on the surrounding tibia, very adequate. The 3-dimensional model showed clearly the effect of the presence of soft-tissue on the load-transmission, which possibly consequences in prosthesis-loosening.

However, it is important to realise that the model-description is a first-order description of the reality. The precise constitutive behaviour of soft-tissue is uncertain, which consequences in a phenomenological model. The results presented in chapter 5 are merely indicative, and can therefore be used to gain insight in the failure mechanisms of prosthesis loosening.

This model of the interface can also be used to study the effects of parameter- or geometry-variations of the prosthesis. For example, the influence of "wings" (see figure 3.1) or the absence of a prosthesis-stem can be examined. However, variation of the prosthesis-stiffness is not possible, because of the use of the foundation-links, which automatically consequences in a rigid modelled prosthesis. The model which describes the interface with hypo-elastic elements in combination with contact, is a very good alternative to study this parameter. The only problem which will be met, is the application of local coordinate systems at the hypo-elastic elements.

6.3 More complex models.

The model of the tibia as described in chapter 5 is a simple reflection of reality. However, the model appears to be adequate in predicting trends initiated by the soft-tissue interface and the bone-interface. The question now rises whether this model should be modelled more complex in order to obtain more realistic results. When, instead of a mechanical analysis, a dynamic analysis is performed, a cyclic, more natural load can be applied on the prosthesis, and the growth of the interface can be modelled dependent on time. A dynamic model will only be usefull if the stimulus, which initiates the interface bone-resorption process, is a time dependent quantity. If the stimulus is time-dependent, the interface bone-resorption process can be compared with fatigue of metals: A certain load will initiate the bone-resorption process, only if it is applied for a certain amount of time. If the stimulus is not time-dependent, but only dependent on the magnitude of the stimulus, a static model is satisfying.

Unfortunately, no publications were found which claim to have found the stimulus of the interface bone-resorption process. Research and experiments have to be performed to learn about the mechanical stimulus.

6.4 Conclusions and Recommendations.

It is concluded that the model of the interface is a good phenomenological model which can be used to gain insight in the process of prosthesis loosening. It is able to predict
the load-distribution in the tibia, dependent on the presence of a soft-tissue interface or a bone-interface.
From simulations appeared that a soft-tissue interface results in a dramatically change of load-transmission, compared to a bone-interface. Also the magnitude of the stresses and the SED, which is assumed to be the stimulus of remodelling of bone, appeared to be much higher when considering the soft-tissue interface. It is therefore not unlikely that the presence of soft-tissue stimulates loosening of the prosthesis.

To optimize the geometry of the prosthesis, the influence of the geometry on the load-transmission should now be judged by testing different geometries. Also parameter variations should be performed to gain more insight.
Also experiments have to be performed, not only to gain more insight in the constitutive behaviour of the soft-tissue, but also to validate the model. The latter can be done by ascertaining the load on the prosthesis of a person with a loosening prosthesis. This load should also be applied on the finite element model, to predict the direction in which the soft-tissue will grow. This data should be compared with measurements on the person, which show the soft-tissue growth after a few months.
Bibliography

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Appendix A

Used elements.

The information of all elements has been obtained from the MARC manuals [MARC-manual, 1994].

A.1 8-node plane-strain element

This element is an isoparametric, quadratic plain strain element. Each edge forms a parabola, so that 4 nodes define the corners of the element. Further, 4 nodes define the 'mid-point' of each edge, shown in figure A.1.

![Figure A.1: Plane strain, 8-node distorted quadrilateral element](image)

The thickness of this element has to be stored in the model definition of the programme description. This element has two degrees of freedom per node which are the displacements $u$ and $v$ in respectively the x- and y-direction.
A.2 The 20-node brick element

This element is an isoparametric, quadratic, brick. Each edge forms a parabola, so that 8 nodes define the corners of the element and a further 12 nodes define the position of the 'mid-point' of each edge, which has been depicted in figure A.2. This element is a rapidly converging element for three-dimensional analysis.

![Three-dimensional quadratic 20-node brick](image)

Figure A.2: *Three-dimensional quadratic 20-node brick*

This element has three degrees of freedom per node, which are the displacements $u$, $v$, and $w$ in respectively the $x$-, $y$-, and $z$-direction.
A.3 Beam element.

The beam element is a straight, Euler-Bernoulli beam in space with only linear elastic material response. Large curvature changes are neglected in the large displacement formulation. Linear interpolation is used along the axis of the beam (constant axial force) with cubic displacement normal to the beam axis (constant beam curvature). This element may be used for non-linear elasticity, where the material behaviour is given in subroutine UBEAM.

The cross-sectional area, and the moment of inertia about the local z-axis and the local y-axis (see figure A.3) have to be defined in the model-definition of the program-description. The element has 6 degrees of freedom per node: three degrees of freedom per node with respect to the global coordinate system, namely the displacements $u$, $v$, and $w$ in respectively the $x$-, $y$-, and $z$-direction; and three degrees of freedom per node with respect to the local coordinate system, namely the rotation $\theta_x$, $\theta_y$ and $\theta_z$ around the local $x$-, $y$-, and $z$-axis.

Figure A.3: Elastic beam element
A.4 Shell element.

This is a 8-node thick shell element with global displacements and rotations as degrees of freedom. Second order interpolation is used for coordinates, displacements and rotations. The membrane strains are obtained from the displacement field, while the curvatures are obtained from the rotation field.

![Diagram of a quadratic thick shell element]

Figure A.4: Quadratic thick shell element.

The thickness of this element has to be defined in the model-definition of the program description. This element has six degrees of freedom per node, which are six degrees of freedom per node with respect to the global coordinate system, namely the displacements $u$, $v$, and $w$ in respectively the $x$-, $y$-, and $z$-direction and the rotation $\theta_x$, $\theta_y$ and $\theta_z$ around the global $x$-, $y$-, and $z$-axis.
Appendix B

Subroutines.

B.1 HYPELA (non-linear hypo-elastic element).

*******************************************************************************
C The subroutine HYPELA is used to prescribe
C the non-linear behaviour of the hypo-elastic
C element
*
SUBROUTINE HYPELA(D,G,E,DE,S,TEMP,
 +DETEMP,NGENS,N,NN,KC,MATS,NDI,NSHEAR)
IMPLICIT REAL *8 (A-H,O-Z)
DIMENSION D(NGENS,NGENS), G(NGENS), E(NGENS),
 +DE(NGENS), S(NGENS), TEMP(1), DETEMP(1), N(2)
*
*******************************************************************************
C user coding
*
integer i,j
integer alta
real emod,nu,c1
real sum,E0
*
*******************************************************************************
C The initial stiffness of the non-linear
C hypo-elastic element E0. Alfa determines the
C increase of the stiffness-curve. nu is the
C Poisson ratio.
*
E0=10
alfa=1
nu=0.3
*
C The equation which prescribes Young's modulus.
C The variable E(1) is the incremental strain in the
C x-direction, and is provided by MARC. Young's
C modulus is needed to compute the matrix D.
*
emod=(1/((1+E(1))*((alfa+1))*E0
*
B. Subroutines.

C initialize G and D
*
do 11 i=1,NGEWS
   G(i) = 0
   do 10 j=1,NGEWS
      D(i,j) = 0
 10 continue
11 continue
*
*----------------------------------------------------
*
C Compose the isotropic elasticity matrix D for
C plain stress
*
*  c1 = emod / (1 - nu*nu)
*
D(1,1) = c1
D(1,2) = c1 * nu
D(1,3) = 0
D(2,1) = c1 * nu
D(2,2) = c1
D(2,3) = 0
D(3,1) = 0
D(3,2) = 0
D(3,3) = c1 * (1 - nu)
*
*----------------------------------------------------
*
C Calculate the stresses. This is done by using
C the matrix D, and the by MARC supplied
C incremental strain DE. The total stress S is the
C output of this subroutine, and is supplied to
C MARC.
*
*
do 30 i=1,NGEWS
   do 40 j=1,NGEWS
      sum = sum + D(i,j)*DE(j)
 40 continue
S(i) = sum
30 continue
*
*------------------------------------------------------
*
RETURN
END
*
*
*******************************************************************************
B. Subroutines.

B.2 USPRNG (non-linear elastic spring)

-------------
C    The subroutine USPRNG is used to prescribe
C    the non-linear behaviour of the non-linear
C    elastic spring
*
SUBROUTINE USPRNG(RATK,F,DATAK,U,TIME,W,N,NSPRNG)
IMPLICIT REAL *8 (A-H, O-Z)
DIMENSION RATK(2), DATAK(2), U(2), TIME(2), N(2)
*
*---------------------------------------------------------------------
* user coding
* integer i,j,h
integer NSPRNG,alfa
real k,l,Fv,Fb,t,q,k10
*
*---------------------------------------------------------------------
* initialize ratk
* do 20 j=1,2
  ratk(j)=0
20 continue
*
*---------------------------------------------------------------------
* The initial stiffness of the non-linear spring is E0. Alfa determines the
* increase of the stiffness-curve. nu is the
* Poisson ratio. t represents the length of the spring
* and thus the thickness of the interface.
* k10=10
  alfa=1
  nu=0.3
  t=1
*
* The equation which prescribes the spring's stiffness.
* The variable u(1) is the relative displacement of the
* two nodes connected by the spring, in the direction in
* which the spring is orientated. It is provided by MARC.
* *
  k=(1/((1+u(1)/t)**(alfa+1)))**k10
*
*---------------------------------------------------------------------
* Determining of the ratk and F which is the output of this
* subroutine and have to be provided to MARC.
*
B. Subroutines.

C Initialization of Fv (spring-force) and Fb (damping-force)

* Fv=0
  Fb=0

* ratk(1) = spring-stiffness of subroutine / spring-stiffness of MARC data-file

* ratk(1)=k/datak(1)
  ratk(2)=0

* Spring-force is determined from the spring-stiffness according to see equation 3.5.

* Fv=((-1/((1+(u(1)/t))*alfa))+1)*k10/alfa
  F=Fv+Fb

* return
  end
B. Subroutines.

B.3 USPRNG (non-linear elastic foundation)

******************************************************************************
C The subroutine USPRNG is used to prescribe
C the non-linear behaviour of the non-linear
C elastic foundation
******************************************************************************
SUBROUTINE USPRNG(RATK,F,DATAK,U,TIME,N,NW,NSPRNG)
IMPLICIT REAL *8 (A-H, O-Z)
DIMENSION RATK(2), DATAK(2), U(2), TIME(2), N(2)
******************************************************************************
C user coding
INTEGER i,j,h
INTEGER NSPRNG,alfa
REAL k,1,Fv,Fr,t,q,k20
******************************************************************************
C initialize ratk
DO 20 J=1,2
RATK(J)=0
CONTINUE
******************************************************************************
The initial stiffness of the non-linear
foundation is E0. Alfa determines the
increase of the stiffness-curve. nu is the
Poisson ratio. t represents the length of the foundation
and thus the thickness of the interface.

K20=10
ALFA=1
NU=0.3
T=1

The equation which prescribes the foundation's stiffness.
The variable u(1) is the displacement perpendicular to
the element surface of the element on which the foundation has
been mounted. It is provided by MARC.

K=(1/(1+((U(1)/T)**(ALFA+1)))*K20

******************************************************************************
Determining of the ratk and F which is the output of this
subroutine and have to be provided to MARC.

INITIALISATION OF FV (foundation-stress) AND FB (DAMPING-STRESS)
Appendix C

Newton-Raphson iteration method.

The Newton-Raphson method is a second order iteration-method, which is derived from the Taylor-polynomial.

\[ F(x) \approx F(x^0) + \delta x^T \Delta F(x^0) + \frac{1}{2} \delta x^T (H(x^0))\delta x \]  \hspace{1cm} (C.1)

in which \( H(x^0) \) is the Hessian, and \( \delta(x) \) is written as:

\[ \delta(x) = x - x^0 \]  \hspace{1cm} (C.2)

differentiation of \( F(x) \) to \( \delta x \) leads to:

\[ \Delta F(x) \approx \Delta F(x^0) + H(x^0)\delta x \]  \hspace{1cm} (C.3)

For the variable \( x \), the function \( \Delta F(x) \) has to be zero:

\[ \Delta F(x) = 0 \]  \hspace{1cm} (C.4)

so equation C.3 becomes:

\[ H(x^0)\delta x = -\Delta F(x^0) \]  \hspace{1cm} (C.5)

This leads to, in combination with equation C.2, a linear equation:

\[ z = z^0 - [H(x^0)]^{-1} \Delta F(x^0) \]  \hspace{1cm} (C.6)

This can be written as the Newton-Raphson iteration-method:

\[ x^{t+1} = x^t - \alpha[H(x^0)]^{-1} \Delta F(x^t) \]  \hspace{1cm} (C.7)
Figure C.1 shows how the Newton-Raphson process iterates. Every step \( q \) \((q = 1, n)\), the function \( \Delta F(x^q) \) is zero, and the value \( x^{(q+1)} \) is determined.

Figure C.1: The Newton-Raphson iteration process.
Appendix D

Contact algorithm.

Whenever two or more bodies are in contact or come in contact with each other, it is necessary to impose non-penetration constraints. These constraints have been automated in MARC using the CONTACT model definition option.

The basic concept in the contact features of MARC is the definition of bodies. The boundary surfaces of these bodies contain all the geometrical information necessary to impose non-penetration. In most circumstances, the bodies correspond to the physical model being analyzed with finite elements. In some cases, however, a body may be present for the sole purpose of constraining other bodies, with no analysis required. This leads to the concept of defining both deformable bodies and rigid bodies. Deformable bodies are a simple collection of finite elements. Rigid bodies are pure geometrical entities. A requirement is that at least one deformable-body must be present for any finite element analysis to be performed.

The non-penetration constraint, as shown in figure D.1 is expressed as:

\[
U_A \cdot n \leq D
\]  

(D.1)

Figure D.1: The non-penetration constraint in CONTACT

Where \( U_A \) is the displacement vector, \( n \) is the normal vector, and \( D \) is the distance between the body and the rigid surface. In the finite element framework this constraint can be imposed by solver constraints. Whenever contact between a deformable-body and
D. Contact algorithm.

A rigid-body is detected, imposed displacements are automatically created. Whenever contact between two deformable bodies is detected, multipoint constraints (called TIES) are automatically created. No contact between rigid bodies is ever considered.

As stated above, a deformable-body is defined by the user as a set of elements. MARC determines all of the nodes on the boundary, which become the set of candidate nodes for contact. That is, the contact algorithm will attempt to prevent penetration of any of these nodes into any defined body. Simultaneously, the body's boundary surface is stored as a set of geometrical entities. All boundary nodes will be prevented from crossing this surface. There is no master/slave relation; the default is that every body will be checked for contact versus every other body.

The implementation of the contact algorithms requires that the following steps are performed every increment of an analysis.

1. Find all the nodes that are in contact. This is determined by the distance between the nodes and surfaces. Since the distance is a calculated number, there are always roundoff errors involved. Therefore, a contact tolerance is provided such that if the distance calculated is below this tolerance, a node is considered in contact. This tolerance is provided either by the user, or calculated by MARC as \( \frac{1}{15} \) of the smallest element size for solid elements or \( \frac{1}{2} \) of the thickness for shell elements. In general, the contact tolerance should be a small number compared to the geometrical features of the configuration being analyzed. Another important use of the contact tolerance is described in point five below.

2. For all the nodes that are in contact, determine either the tying constraints or the imposed displacement increments along the normal to the contacted surface, as well as a local transformation that defines such a normal. This step is repeated for each cycle of a nonlinear problem in order to permit a node to find its equilibrium position along the contact surface.

3. Once a convergent solution is found, contact forces are analyzed. If a contacting node has a tensile contact force greater than the separation tolerance, the node is released and another solution is found. The program will automatically calculate the separation tolerance force based upon the residual forces or you can enter it directly. A large separation tolerance ensures that there will not be any separation. If you want to move two contacting bodies apart, a solution requiring substantial artificial stretching might have to be found before the decision to separate is made. It is recommended that you use the RELEASE option to totally separate two bodies or that the separation is done in very small increments.

4. At this point, you must decide whether a node that was free at the beginning of the increment would go through the body by the end of the increment. If this is the case, the increment of time is reduced in size so that the first node barely reaches contact. In this way, the beginning of the following increment detects new contact and the analysis proceeds normally.

5. If the controls of the problem (HISTORY DEFINITION) require a set of equally spaced increments, the previous possible shrinking in size of an increment leads to an increment split. MARC repeats the same increment number in order to execute
the remainder part. You can easily see that if several nodes reach contact almost simultaneously, there is a potential for a very large number of increment splits. However, in spite of the first split being made with respect to the first contacting node, step 1 (above) of the remainder increment will detect and bring into contact all subsequent nodes that are within the tolerance bond of the surfaces they will contact. There is an advantage in making the contact tolerance as large as possible, so that the largest possible value is still small compared with the geometrical features of the problem.
Appendix E

Results of the 2-dimensional models.
E. Results of the 2-dimensional models.

E.1 Load-situation 1, x-y stress.

<table>
<thead>
<tr>
<th>Model</th>
<th>Interface Description</th>
<th>Stress $\sigma_x$ [N/mm$^2$]</th>
<th>Stress $\sigma_y$ [N/mm$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>interface: foundation - links</td>
<td>$\sigma_x = 2.6$</td>
<td>$\sigma_y = -33$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sigma_x = 20$</td>
<td>$\sigma_y = -42$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sigma_x = 2.6$</td>
<td>$\sigma_y = -43$</td>
</tr>
<tr>
<td>Model 2</td>
<td>interface: hypo-elastic elements &amp; contact</td>
<td>$\sigma_x = 2$</td>
<td>$\sigma_y = -20$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sigma_x = 28$</td>
<td>$\sigma_y = -49$</td>
</tr>
<tr>
<td>Model 3</td>
<td>interface: plain-strain elements &amp; contact</td>
<td>$\sigma_x = 8$</td>
<td>$\sigma_y = -23$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sigma_x = 23$</td>
<td>$\sigma_y = -48$</td>
</tr>
<tr>
<td>Model 4</td>
<td>gap &amp; contact</td>
<td>$\sigma_x = 2.8$</td>
<td>$\sigma_y = -5$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sigma_x = 30$</td>
<td>$\sigma_y = -32$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sigma_x = 2.8$</td>
<td>$\sigma_y = -1.5$</td>
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E. Results of the 2-dimensional models.

E.2 Load-situation 1, x-y displacement.

<table>
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<tr>
<th>Model 1</th>
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<td>$u_x$</td>
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<td>$u_y$</td>
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<table>
<thead>
<tr>
<th>Model 2</th>
<th>interface: hypo-elastic elements &amp; contact</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_x$</td>
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</tr>
<tr>
<td>$u_y$</td>
<td>-0.06</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model 3</th>
<th>interface: plain-strain elements &amp; contact</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_x$</td>
<td>-0.5</td>
</tr>
<tr>
<td>$u_y$</td>
<td>-0.06</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model 4</th>
<th>interface: gap &amp; contact</th>
</tr>
</thead>
<tbody>
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<tr>
<td>$u_y$</td>
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E. Results of the 2-dimensional models.

E.3 Load-situation 2, x-y stress.

<table>
<thead>
<tr>
<th>Model</th>
<th>Stress $\sigma_x$ [N/mm$^2$]</th>
<th>Stress $\sigma_y$ [N/mm$^2$]</th>
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</thead>
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<tr>
<td>Model 1</td>
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<td></td>
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<td>-11</td>
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E. Results of the 2-dimensional models.

E.4 Load-situation 2, x-y displacement.

<table>
<thead>
<tr>
<th>Model</th>
<th>Displacement $u_x$ [mm]</th>
<th>Displacement $u_y$ [mm]</th>
<th>Interface</th>
</tr>
</thead>
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<td>interface: foundation-links</td>
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</tr>
<tr>
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<td>interface: hypo-elastic elements &amp; contact</td>
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<tr>
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<tr>
<td>Model 3</td>
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<td>interface: plain-strain elements &amp; contact</td>
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<tr>
<td>Model 4</td>
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<td></td>
<td>gap &amp; contact</td>
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<tr>
<td></td>
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<td>0.8</td>
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<tr>
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<td>7</td>
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</table>
Appendix F

Results of the 3-dimensional models.
F. Results of the 3-dimensional models.

F.1 Load-case 1.

---

**Load-case 1 → 100% Bone-Interface**

**1st Comp of Stress**

---

**Load-case 1 → 100% Soft-Tissue Interface**

**1st Comp of Stress**
F. Results of the 3-dimensional models.

1. For Case 1, the 100% bone-interface model shows a distribution of stress with a significant concentration at the interface. The stress values range from $1.500 \times 10^{-1}$ to $1.200 \times 10^{-1}$.

2. The 100% soft-tissue interface model exhibits a spread of stress values from $3.000 \times 10^{-1}$ to $7.400 \times 10^{-1}$, indicating a different stress pattern compared to the bone interface model.
F. Results of the 3-dimensional models.

LOAD-CASE 1, 100% BONE-INTERFACE
Total Strain Energy Density

LOAD-CASE 1, 100% SOFT-TISSUE INTERFACE
Total Strain Energy Density
F. Results of the 3-dimensional models.

F.2 Load-case 2.
F. Results of the 3-dimensional models.

Load Case 2 - Bone-Interface

Load Case 2 - Soft-Tissue Interface
F. Results of the 3-dimensional models.