Friction Compensation
on the RRR-robot

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Chapter 1

Introduction

For experimental evaluation of control schemes for so-called underactuated systems, where the number of actuators is less than the number of degrees-of-freedom, it is desirable to employ the second joint of the RRR-robot as an underactuated joint. It is not sufficient in this case to not drive the motor, because internal forces, like friction, will act as a significant disturbance and will prevent realistic evaluation of control of underactuated systems. The task is therefore to come up with a control scheme that eliminates all internal joint torques, so the actuated joint acts as a free joint.

The dynamical behaviour of the second joint is nonlinear, so it is difficult to get insight in friction models and identification techniques. To focus on the friction characteristics, this report first describes friction compensation of the linear first joint.

One of the interesting properties of friction is the force in presliding displacement, spring-like behaviour near zero velocity. Not all friction models provide a good estimation in this region. We will describe the friction using a LuGre model. This dynamical model has the benefits of describing the presliding behaviour as well as the behaviour in low and high velocity regimes.

The LuGre model parameters are estimated using two different approaches, a time domain, proposed by [1], and a frequency domain approach, proposed by [2]. Validation of the model is done by tracking experiments.

The friction shows to be dependent of the position of the first joint. A position dependent LuGre model is proposed to obtain better tracking. An experimental problem is that the RRR-robot does not have a reference position, so that implementing this model is not possible.

In chapter 5 the same techniques used for the first joint, are used to identify and compensate the friction forces which act on the second joint.
Chapter 2

Friction compensation

There are many ways to compensate for friction. The method used depends on the goal of the particular problem. For normal control purposes, like tracking problems, the friction needs not to be eliminated completely, but the effect on the dynamics must be reduced. For these problems a very simple way to eliminate some effects of friction is to use a dither signal, that is a high frequency signal added to the control system. The effect of this signal is that it introduces extra forces that make the system move before the static friction level is reached.

A more effective way of eliminating all the effects of friction, is to use a model based friction compensation. A good model will estimate the friction at every position for every velocity very accurate. For tracking problems the desired velocity is known in advance, and the friction can be compensated by feedforward only. This has the advantage of eliminating the lag and noise effects of the velocity measurement.

In our control problem the goal is that the first joint of the RRR-robot will act as a free joint. To achieve this, all the friction forces need to be eliminated. A simple dither signal will not do for this problem, and we will use a friction model based compensation. A disadvantage in this problem is that the desired velocity is not available in advance, so the real velocity has to be measured or predicted and fed back to a friction observer. At the RRR-robot the velocity is predicted from filtering the position measurements. Because of the high resolution and frequency of these measurements, 1e-5 [rad] and 1 [kHz] respectively, we do not expect to have problems with the lag and noise introduced. However, we must take care not to over-compensate the friction, because this may lead to instabilities due to the feedback. The friction compensation problem is represented by a block diagram in figure 2.1.

In the next section a few friction models will be described. These models will provide a

![Figure 2.1: Block diagram of the friction compensation scheme.](image-url)
good estimation if the friction is not position dependent, that is, if the friction has the same
properties at every position of the joint. An extension of these models will be discussed in
chapter 3.4, so position dependency can be taken into account.

2.1 Friction models

Friction can be modeled as a static relation between velocity and force, but also as a dynamic
relation. The classical static friction model is the Coulomb and Viscous friction model

\[ F = F_C + F_v v \]  \hspace{1cm} (2.1)

where \( F_C \) is the Coulomb friction, \( F_v \) is the viscous friction constant and \( v \) is the velocity.

Besides these two effects, friction has another property, the Stribeck effect. This means that
the friction force may decrease continuously from the static friction level as shown in figure
2.2. The classical model takes no account of this effect. A better description of friction is

\[ F = F_C + (F_S - F_C)e^{-(v/v_S)^2} + F_v v \]  \hspace{1cm} (2.2)

where \( v_S \) is the Stribeck velocity, \( F_S \) is the static friction force and \( F_C \) and \( F_v \) as in 2.1.

Models like 2.1 and 2.2 have been used for a long time for control purposes and are easy to
implement. They provide a good estimate of the friction force when the relative velocity is not
zero, the sliding region. A disadvantage is that it doesn’t model the presliding behaviour, the
behaviour near zero velocity. The friction force shows in this region a nonlinear behaviour,
called stiction. There are some statical models which take this effect into account, but they
generally consist of two separate models, one for sticking and one for sliding. A better way
to model this behaviour is with a dynamic model, like the LuGre model.

In the LuGre model, the friction force during stiction is modeled as the average force applied
by a set of elastic springs under a tangential microscopic displacement. Physically this phe-
nomenon can be interpreted as two moving surfaces that are in contact by a large number
of bristles with a certain stiffness. This is illustrated in figure 2.3. The model also includes
rate dependent phenomena such as varying break-away force and frictional lag.
The LuGre model has the form

\[
\dot{z} = v - \sigma_0 \frac{|v|}{g(v)} z \\
F = \sigma_0 z + \sigma_1 \dot{z} + \alpha_2 v \\
g(v) = \alpha_0 + \alpha_1 e^{-|v/v_0|^2}
\] (2.3)

where \( z \) denotes the average deflection of the bristles, \( v \) the relative velocity between the surfaces, \( g(v) \) the Stribeck effect, \( v_0 \) the Stribeck velocity, \( \sigma_1 \) the bristle damping, \( \sigma_0 \) the bristle stiffness and \( \alpha_2 \) the viscous damping. The model behaves like a spring for small displacements. For constant velocity the steady state friction is

\[
F_{ss} = g(v) \text{sgn}(v) + \alpha_2 v
\] (2.4)

which has the same characteristics as equation 2.2. Figure 2.4 shows the estimated friction force against velocity when the system undergoes a sine displacement.

The complete friction model is thus characterized by the four static parameters \( \sigma_0, \sigma_1, \alpha_2 \) and \( v_0 \) and the two dynamic parameters \( \sigma_0 \) and \( \sigma_1 \). The identification of this model is described in chapter 3.
Chapter 3
Identification of the LuGre model

3.1 Experimental setup

The RRR-robot (figure 3.1) considered here, consists of three rotating arms, all driven by a direct drive servo without transmission. Sliprings are applied to realize unconstrained rotation of all joints. The measurement system consists of a position encoder for each joint, all with $6.5536e5$ increments per revolution. Using WinCon and a MultiQ I/O board these measurements are downloaded to a PC at a frequency of 1 [kHz]. To obtain the velocity, the position measurement is differentiated and filtered by a 4th order Butterworth filter with a cut-off frequency of 80 [Hz]. The control-schemes can be built using Matlab Simulink. The control signal is send from the I/O board to a Dynaserv motor driver, where it is transferred to a three phase signal and fed to the servo. To perform on-line frequency domain measurements, the input signal and the angular displacement are processed by a SigLab system which operates on a 2nd PC. A schematic representation of the setup is given in figure 3.2.

The first joint of the RRR-robot can be modeled as

$$J\dot{\theta} = -F + k_t u$$

(3.1)

where $J$ is the inertia, $\theta$ is the angular displacement, $u$ the input voltage, $k_t$ the motor constant and $F$ the friction torque. Measurements were done to identify the parameter $J/k_t$. To eliminate the effect of the friction force, constant velocity experiments under PD control

![Figure 3.1: 3D view of the RRR-robot](image)
are performed. Random noise, with a bandwidth of 200 [Hz] and a RMS value of 0.300 [V], is added by the SigLab system to the control signal. The frequency response function (FRF) is estimated by averaging 50 time series of 4096 samples at a sample frequency of 1 [kHz] with a Hanning window and 50% overlap. Because the measurements were done over the complete revolution of the link, the position dependent effects are eliminated. Figure 3.3 shows the measured and the estimated FRF. From this we can see that the systems phase turns away from the expected $\pi$ [rad]. This is due the Dynaserv motor driver, which has a delay of approximately 2 [ms]. The FRF is fitted using frfit. From this estimation we get $J/k_t = 5.8e-2$ [V s rad$^{-1}$]. Because all the parameters are lumped with the motor constant $k_t$, we shall normalize this parameter at 1 [Nm V$^{-1}$], so the other parameters can be written in their normal units. Here we can say $J = 5.8e-2$ [Nm s rad$^{-1}$].
3.2 Static parameter estimation

To estimate the static parameters of the LuGre model, the friction force is measured at constant velocity. From this data a friction-velocity map is constructed. To obtain constant velocity motion of the first link of the RRR-robot, closed loop experiments were performed under PD-control. The system shows an eigenfrequency at 13 [Hz], so the closed loop bandwidth is kept at approximately 8 [Hz].

Measurements were done for 20 different constant velocities ranging from $\theta_1 = 0.0$ to $\theta_1 = 3.0$ [rad s$^{-1}$], for both positive and negative direction. To estimate the LuGre parameters, we will take the average value of the friction force over a complete revolution. The friction force shows a large dependence on the position, as can be seen from figure 3.8. However, the variation of the averaged force at constant velocity in different measurements is very small. We can say that $F = \bar{F} \pm 1e-3$ [Nm], where $\bar{F}$ is the mean friction force at a certain velocity. The variance $1e-3 < F_C \cdot 0.5\%$, is very small and can be neglected.

To obtain a good approximation for the Stribeck velocity a large number of points were collected at low velocity. For identification of $F_s = \alpha_0 + \alpha_1$ break-away experiments were performed at several positions, also for both negative and positive direction.

The non-linear optimization algorithm fminu in the Matlab optimization toolbox has been used to fit equation 2.4 to the data shown in figure 3.4. The function to be minimized is

$$\min_{\alpha_0, \alpha_1, \alpha_2, \alpha_0} \sum_{i=1}^{n} [F_{ss}(\theta_1) - \hat{F}_{ss}(\theta_1)]^2$$

where $F_{ss}$ is the measured steady state friction force and $\hat{F}_{ss}$ is the estimated steady state friction force. The parameters are estimated for positive and negative direction separately, and for both together. These nominal values are taken for compensation. The results are shown in table 3.1.

Figure 3.4: The friction velocity map

Identification of the LuGre model
Identification of the LuGre model

3.3. DYNAMIC PARAMETER ESTIMATION

<table>
<thead>
<tr>
<th>Friction parameter</th>
<th>$v &gt; 0$</th>
<th>$v &lt; 0$</th>
<th>Nominal value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$ [Nm]</td>
<td>2.27e-1</td>
<td>2.30e-1</td>
<td>2.29e-1</td>
</tr>
<tr>
<td>$\alpha_1$ [Nm]</td>
<td>4.94e-2</td>
<td>4.90e-2</td>
<td>4.93e-2</td>
</tr>
<tr>
<td>$\alpha_2$ [Nm s rad$^{-1}$]</td>
<td>9.44e-2</td>
<td>9.27e-2</td>
<td>9.36e-2</td>
</tr>
<tr>
<td>$v_0$ [rad s$^{-1}$]</td>
<td>3.25e-3</td>
<td>2.66e-3</td>
<td>3.18e-3</td>
</tr>
</tbody>
</table>

Table 3.1: Static friction parameters

<table>
<thead>
<tr>
<th>RMS level [V]</th>
<th>$\sigma_0$ [Nm rad$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.010</td>
<td>500</td>
</tr>
<tr>
<td>0.020</td>
<td>600</td>
</tr>
<tr>
<td>0.030</td>
<td>700</td>
</tr>
<tr>
<td>0.050</td>
<td>700</td>
</tr>
<tr>
<td>0.100</td>
<td>700</td>
</tr>
<tr>
<td>0.120</td>
<td>700</td>
</tr>
</tbody>
</table>

Table 3.2: Stiffness $\sigma_0$ at different noise levels

3.3 Dynamic parameter estimation

As mentioned before, in the presliding region the LuGre model behaves as a mass-spring system with damping. Linearization of equation 2.3 around zero velocity and zero state gives

$$\dot{\theta}_1 = -\sigma_0 \theta_1 - (\alpha_1 + \alpha_2) \dot{\theta}_1 + u$$

(3.3)

where $\sigma_0$ represents the stiffness of the bristles and $\alpha_1 + \alpha_2$ the damping [2].

With the linearization 3.3 and equation 3.1 this model can be written in frequency domain as

$$\frac{\theta(j\omega)}{U(j\omega)} = H(j\omega) = \frac{k_t}{-j\omega^2 + (\alpha_1 + \alpha_2)j\omega + \sigma_0}$$

(3.4)

To estimate this frequency response function, the system is excited with random noise of a bandwidth up to 200 [Hz] with a RMS level below the static friction level to prevent the system from leaving the presliding region. The measured $G(j\omega)$ is obtained by averaging 50 time series of 4096 samples at a sample frequency of 1 [kHz] with a Hanning window and 50% overlap.

Because the linearization is only valid locally, the nonlinear behaviour is investigated by varying the RMS value of the noise. Measurements are done at 4 different positions, each with 7 different input voltages. From figure 3.5 we can see that $H(j\omega)$ increases with increasing noise level, as expected. At $\omega = 0$ [Hz] we get an estimate for $\sigma_0$ as equation 3.4 gives $H(j0) = \frac{k_t}{\sigma_0}$. Table 3.2 shows estimates of $\sigma_0$ at different noise levels. To get the least effect of the nonlinear behaviour on $\sigma_0$, the noise level should be as low as possible. To estimate all the parameters, the FRF at $u = 0.010$ [V] RMS are fitted at each position using frfit. Here the measurements with the largest coherence value are taken. The results are shown in table 3.3.

Another way to determine the parameter $\sigma_0$ is in the time domain. If we again consider the presliding behaviour of the system as a mass-spring like behaviour, then the initial slope of a friction-position relation in break-away experiments indicates the stiffness of the bristles.

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3.3. DYNAMIC PARAMETER ESTIMATION

Figure 3.5: FRF of presliding behaviour at 1 position and 7 different $u_{RMS}$

Figure 3.6: FRF of presliding behaviour at 4 positions at $u_{RMS} = 0.030[V]$
Identification of the LuGre model

3.4. POSITION DEPENDENCE OF THE FRICTION

Here we shall only use this identification for validation of the parameter value obtained in the frequency domain experiments. A break-away experiment is shown in figure 3.7. If the average slope at 4 different positions of 3 experiments each is estimated, we get $\sigma_0 = 0.98e3$. We can conclude from this that the frequency domain identification provides a good estimation for this parameter.

### Table 3.3: Dynamic friction parameters

<table>
<thead>
<tr>
<th>Friction parameter position</th>
<th>position 1</th>
<th>position 2</th>
<th>position 3</th>
<th>position 4</th>
<th>Nominal value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_0$ [Nm rad$^{-1}$]</td>
<td>1.16e3</td>
<td>1.22e3</td>
<td>0.98e3</td>
<td>1.33e3</td>
<td>1.17e3</td>
</tr>
<tr>
<td>$\sigma_1$ [Nm s rad$^{-1}$]</td>
<td>4.40</td>
<td>2.59</td>
<td>4.48</td>
<td>3.23</td>
<td>3.67</td>
</tr>
</tbody>
</table>

Figure 3.7: Break-away experiment

Here we shall only use this identification for validation of the parameter value obtained in the frequency domain experiments. A break-away experiment is shown in figure 3.7. If the average slope at 4 different positions of 3 experiments each is estimated, we get $\sigma_0 = 0.98e3$. We can conclude from this that the frequency domain identification provides a good estimation for this parameter.

### 3.4 Position dependence of the friction

Figure 3.8 shows the friction-position relation during constant velocity experiments under PD-control for velocities ranging from $\theta_1 = 1.0$ to $\theta_1 = 3.0$ [rad s$^{-1}$]. As can be seen from this figure, the friction force depends significantly on the position of the link. This is probably due to the mounting of the bearing and the cogging of the direct drive motor. If nominal values are taken for parameters of the friction model, as described in sections 3.2 and 3.3, the friction force is not predicted accurate at all positions. This can result in an unstable system, because at some positions more energy is supplied to the system by the friction compensation then absorbed by the real friction. To overcome this problem, we can make the friction model also position dependent. As we can see from figure 3.8, the friction-position relation shows the same pattern at different velocities. It can also be shown that this relation is repetitive, as we can see from the 3 complete revolutions in figure 3.8. Since these two conditions are valid, it is possible to get a more accurate friction estimate for compensation by using a friction profile.
3.4. POSITION DEPENDENCE OF THE FRICTION

The new friction model becomes

$$\hat{F} = f_s(x)F_L$$

where $\hat{F}$ is the new predicted friction force and $F_L$ is the LuGre predicted friction force from equation 2.3. $f_s(x)$ can be determined by measuring the static friction at each position. It can also be determined by estimating a scaling factor for the friction force at each position from the friction-position relation (figure 3.8). The scaling factor at the nominal friction level is then normalized on 1.

Although the position dependent friction model has shown good results in practical application and is easy to identify, it is difficult to implement it at the RRR-robot. This is because there is no reference sensor present. It is however possible to get a reference position by initializing through software. This can be done by measuring the velocity when a constant voltage is sent to the servo. Because the friction is repetitive, the reference position can be determined and will be the same every initialization. However, if the friction characteristics change, for example due to extensive use, the initialization through software will not give the same reference position anymore. It is then advisable to implement a reference sensor.
Chapter 4

Validation of friction compensation

For validation of the nominal friction model, experiments can be performed under friction compensation. If the friction is exactly predicted, the system will behave as a double integrator without energy dissipation. Because at the RRR-robot the friction depends on the position significantly and the nominal model is just an average over a complete revolution, the friction cannot be predicted exactly at any position. Due to this effect energy is dissipated at the positions where the real friction force is higher than the predicted friction force, and energy is supplied at the positions where the real friction force is lower than the predicted force. The system will never act exactly as a double integrator. To still get a indication of the performance of the friction model, experiments are performed under PD-control and mass feedforward with and without friction compensation. The experiments are further described in section 4.2. We will first describe the implementation of the LuGre model.

4.1 Implementation of the LuGre model

A difficulty of the bristle theory is that at time $t = 0$ the average deflection of the bristles, $z(0)$, is not known. In [4] an observer based friction compensation scheme is proposed as in equation 4.1. It is shown that with the use of an innovation signal $ke$, the predicted state $\dot{z}$ converges to the real state $z$.

$$\frac{d\dot{z}}{dt} = \dot{\theta}_i - \sigma_0 \frac{|\dot{\theta}_i|}{g(\theta_i)} \dot{z} - ke, \quad k > 0$$

$$\hat{F} = \sigma_0 \dot{z} + \sigma_1 \frac{d\dot{z}}{dt} + \alpha_2 \dot{\theta}_i$$

(4.1)

(4.2)

where $e = \theta_i - \theta_{ir}$ is the position error, $\theta_{ir}$ is the desired position, $\dot{z}$ is the bristle state observation and $\hat{F}$ is the friction estimate. A sufficient condition for stability is to choose a controller with all the poles in the left half plane. Numerical problems occurred when trying to integrate this friction observer. It can be shown that for high velocities ($\dot{\theta}_i \gg v_0$), the discrete version of 4.1 becomes unstable. It is then advisable to stop the integration of $\dot{z}$ at a certain velocity $v_{\text{max}}$ and use its steady state value $\dot{z}_{ss} = \alpha_0 / \sigma_0 \text{sign}(\dot{\theta}_i)$ instead. In the experiments we stop the integration at $v_{\text{max}} = 1e-2$ [rad s$^{-1}$]. The Simulink scheme is presented in appendix A.
**4.2 Experimental results**

Tracking experiments are performed using mass feedforward and a PD-controller. The control law becomes

\[ u = J s^2 \dot{\theta}_{tr} - (P + Ds)e + \hat{F} \]  

(4.3)

where the parameters P and D are tuned as

\[ s^2 + \frac{1}{J}Ds + \frac{1}{J}P = s^2 + 2\xi\omega_0 s + \omega_0^2 \]  

(4.4)

with \( \omega_0 = 11.5 \) and \( \xi = 0.65 \). Figure 4.1 shows the result of an experiment with reference \( \theta_r = 0.2\sin(2\pi t) \) and error \( e = \dot{\theta} - \dot{\theta}_r \). The LuGre friction estimation is shown in figure 4.1c. After 4 seconds the friction compensation is switched off, and we can see an immediate increase in tracking error. The result shows that friction compensation reduces the tracking error by 700% and at low velocities even by more than 1300%. Under PD-control the position \( \theta \) lags behind the reference \( \theta_r \). However, if the friction compensation is active, \( \theta \) keeps ahead of \( \theta_r \). This is due to over-compensation of the friction. The tracking error also shows a periodic behaviour. If a position dependent friction model is used, these effects can be reduced. Figure 4.2(a) shows a constant velocity experiment under PD-control with friction compensation. In this experiment the linear controller is switched off after 2 seconds. If the friction is exactly predicted by the observer, the velocity should maintain. We can see clearly that in some positions the friction is overestimated (energy is supplied) and the velocity increases. At
Validation of friction compensation

4.2. EXPERIMENTAL RESULTS

Figure 4.2: Constant velocity experiment with and without PD-control

In other positions the friction is underestimated (energy is dissipated) and velocity decreases. When the parameters identified in chapter 3 are used, overall more energy is supplied by the observer than energy is dissipated by the real friction, and using friction compensation without PD-control will result in unstable behaviour. If we reduce the compensation by a gain of $f = 0.96$ the resulting behaviour is stable and the system will slightly damp out, as shown in figure 4.2(b). This shows also that the extension of the LuGre observer with a friction profile function has effect on the behaviour and could give good results.

In the experiments different values for the innovation term $k$ were used. Although it has effect on the LuGre friction estimation, it does not improve performance nor endanger stability. Even with $k = 0 \ [s^{-1}]$, the feedback compensation did not result in unstable behaviour. Figure 4.3 and 4.4 show the observer state $\hat{z}$ and the resulting friction estimation $\hat{F}$ when $k = 0 \ [s^{-1}]$ and $k = 10 \ [s^{-1}]$ respectively. In both the experiments the same reference is used. The resulting tracking error is, as we can see, of the same magnitude.
Validation of friction compensation

4.2. EXPERIMENTAL RESULTS

Figure 4.3: $k = 0$

Figure 4.4: $k = 10$

Friction compensation of the RRR-Robot
Chapter 5

Friction compensation on link 2

As shown in the previous chapter, friction compensation using the LuGre friction model gives good results in tracking problems. In the case of the second link of the RRR-robot, which should act as a frictionless underactuated system, no reference position is present. One of the advantages is that the LuGre model can be used as a feedback friction compensation, so no reference position or velocity is needed. This means that it is possible to implement the observer (4.1) at the 2\textsuperscript{nd} link of the RRR-robot.

5.1 Experimental setup

If the dynamic behaviour of both the first and the second link of the RRR-robot are considered, and the second link is acting as a free joint, the system behaves like a furuta pendulum. This system can be modeled by

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) + F = \tau$$

(5.1)

where

$$q = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}, \quad D = \begin{bmatrix} J_1 + m_1 (l_1^2 + l_2^2 \sin^2(\theta_2)) & m_2 l_2 l_1 \cos(\theta_2) \\ m_2 l_2 l_1 \cos(\theta_2) & J_2 + m_2 l_2^2 \end{bmatrix},$$

$$C(q, \dot{q}) = \begin{bmatrix} \frac{1}{2} m_2 l_2^2 \sin(2\theta_2) \dot{\theta}_2 & -m_2 l_2 l_1 \sin(\theta_2) \dot{\theta}_2 + \frac{1}{2} m_2 l_2^2 \sin(2\theta_2) \dot{\theta}_1 \\ -\frac{1}{2} m_2 l_2^2 \sin(2\theta_2) \dot{\theta}_2 & 0 \end{bmatrix},$$

$$g(q) = \begin{bmatrix} 0 \\ -m_2 g l_2 \sin(\theta_2) \end{bmatrix}, \quad F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \quad \text{and} \quad \tau = \begin{bmatrix} k_t u \\ 0 \end{bmatrix}$$

where $J_i$ is the inertia of link $i$, $m_i$ the mass, $l_i$ the distance to the center of gravity of the corresponding link, $\theta_i$ the angular displacement, $F_i$ the friction force, $k_t$ the motor constant, $u$ the input voltage. When the pendulum is in its stable equilibrium point, the angular rotation $\theta_2 = 0$.

Identification of the kinematic parameters shows $m_2 = 9.51$ [kg], $l_1 = 1.85e-1$ [m], $l_2 = 1.59e-1$ [m] where the normalized $k_t$ is used ($k_t = 1$ [Nm V\textsuperscript{-1}]). To identify the parameter $J_2$ the same experiment is performed as described in section 3.1. Figure 5.1 shows the measured FRF. The result of this identification shows $J_2 = 5.50e-2$ [Nm s rad\textsuperscript{-1}].
5.2 Identification of the LuGre model

As described in chapter 3, the LuGre parameters can be identified using time and frequency domain identification. For the second link, the same experiments are performed as for the first link. To identify the static parameters, constant velocity experiments were performed under PD-control and gravity compensation. The used controller is tuned with $\omega_n = 20$ and $\xi = 0.7$.

The identification of the dynamical parameters, as described in section 3.3, is here only done at one position. It is not possible to perform presliding experiments when the angular rotation $\theta_2 \neq 0$. Due to gravitational forces, the system will always come in the sliding region. Figure 5.2 shows the presliding behaviour at 3 different input voltages. The $\sigma_0$ and $\sigma_1$ at $\theta_2 = 0$ will provide a good estimation for the entire range of rotation. The results of the measurements are presented in table 5.1.

<table>
<thead>
<tr>
<th>parameter</th>
<th>nominal value</th>
</tr>
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<tbody>
<tr>
<td>$\alpha_0$ [Nm]</td>
<td>3.50e-1</td>
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<tr>
<td>$\alpha_1$ [Nm]</td>
<td>7.00e-2</td>
</tr>
<tr>
<td>$\alpha_2$ [Nm s rad$^{-1}$]</td>
<td>7.00e-2</td>
</tr>
<tr>
<td>$\sigma_0$ [Nm rad$^{-1}$]</td>
<td>1.96e2</td>
</tr>
<tr>
<td>$\sigma_1$ [Nm s rad$^{-1}$]</td>
<td>6.32</td>
</tr>
<tr>
<td>$\nu_0$ [rad s$^{-1}$]</td>
<td>3.00e-3</td>
</tr>
</tbody>
</table>

Table 5.1: LuGre model parameters for link 2
5.3 Validation of friction compensation

In chapter 4.1 a friction observer is described which uses an innovation signal $ke$ to let $\dot{z}$ converge to $z$. In the case of the underactuated system no reference position or velocity is present, so the error $e$ can not be used in the implementation. Experiments have shown that taking $k = 0$ did not result in unstable behaviour of link 1, so we do not expect any problems here. A few experiments are done to validate the friction compensation.

First a tracking experiment of link 1 is performed under PD-control. In this experiment link 2 is left as a free joint with friction compensation only. From the measured position of link 1 it is possible to get an estimation for the position of link 2 using equation 5.1 and the identified parameters. The measured and estimated behaviour of link 2 are shown in figure 5.3. We can see that the measured motion is as expected. The LuGre friction estimation of link 1 and 2 is shown in figure 5.4. To complete the results, figure 5.5 shows the tracking reference and error of link 1 during this experiment. Despite of the large coriolis torques due to link 2, the error is very small.

A second experiment is performed to check whether the friction is under- or overestimated. Therefore link 2 is brought to a certain position under PD-control. When the PD-controller is switched off and the friction is exactly predicted, link 2 has to act as a double integrator where no energy is dissipated or supplied. Figure 5.6 shows such experiment. We can see that the friction is slightly underestimated, because the oscillation damps out. The resulting motion is a sine wave with frequency $f = 0.96$ [Hz]. This indicates a mathematical pendulum with length $L_2 = 2.67e-1$ [m] ($l = (1/f\pi)^2g$). This is almost twice of what we expect it to be, but different experiments show the same result. This could be due to the delay in position measurement and velocity reconstruction. If the real velocity of the link changes sign, the friction observer could still introduce torques of opposite sign. In this way the link is delayed and the period time is larger.

Figure 5.2: FRF of presliding behaviour at 1 position and 3 different $u_{RMS}$
5.3. VALIDATION OF FRICTION COMPENSATION

Figure 5.3: Measured and estimated position of link 2

Figure 5.4: LuGre friction estimation
3. VALIDATION OF FRICTION COMPENSATION

Figure 5.5: Tracking error $e$ of link 1

Figure 5.6: $\theta_2$ during free movement
Friction compensation on link 2  \hspace{1cm} 5.3. VALIDATION OF FRICTION COMPENSATION

An interesting thing to see is that the energy supplied by the friction observer and the energy absorbed by the real friction come in equilibrium when a certain oscillation with amplitude $a = 0.38 \, \text{[rad]}$ and frequency $f = 0.96 \, \text{[Hz]}$ is reached. Figure 5.7(a) shows an experiment where link 2 damps out to $a = 0.38 \, \text{[rad]}$ and figure 5.7(b) shows an experiment where the friction observer supplies energy to the system. This is due to the position dependent characteristic of the friction. The friction is overestimated at $\theta_2 \approx 0 \, \text{[rad]}$ and underestimated in the region around it.

To really see the difference in behaviour of link 2 with and without friction compensation, figure 5.8 shows an experiment when the friction compensation is switched off after 5 seconds. We can see that the system immediately damps out due to energy absorption. The system comes at rest when $\theta_2 = 0.08 \, \text{[rad]}$. This is only possible under large friction.
Figure 5.8: Free oscillation with and without friction compensation
Appendix A

Simulink scheme of LuGre Friction Observer

In this appendix the implementation of the LuGre friction observer in Simulink is shown.
Simulink scheme of LuGre Friction Observer

Figure A.1: LuGre friction observer
Bibliography


