Valve Selection for Compressor Surge Control

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Report No. WFW 98.042

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Eindhoven, November 26, 1998

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Abstract

Surge is an aerodynamic flow instability which can lead to the catastrophic failure of the compressor system. One way to cope with this compressor flow instability is active control. For a laboratory-scale gas turbine installation, an active surge control system is proposed which consists of a plenum pressure sensor and a bleed/recycle valve. This work focuses on the selection of a control valve. More specifically, the required bandwidth and capacity of the valve are specified.

To study the influence of the control valve on system behavior, simulations are done with the Greitzer model which describes the behavior of the nonlinear compression system during surge. This model is extended with a control valve model which accounts for the control valve dynamics and the effect of valve saturation. A linear static output feedback controller is used to stabilize the compression system around its nominal operating point.

The main contributions of this study are the following. First, a systematic approach is presented to determine the required bandwidth and capacity of a control valve for active surge control. Second, an efficient control strategy is applied: one-sided control. In this control strategy, the control valve is nominally closed and only opens to stabilize the system in its nominal operating point. Third, a stability analysis of the constrained linearized compression system is presented.

Using the bounded static output feedback controller, surge limit cycles are stabilized in the nominal operating point. Stable operation can be sustained in this point with zero control valve mass flow if disturbances are absent. From simulations, the minimal bandwidth and capacity of the control valve needed for stabilization are determined. Surge point mass flow is seen to reduce by approximately 15% with the applied one-sided control strategy. Furthermore, it is concluded that nonlinear flow curves of the control valve are unfavorable for active surge control.
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Chapter 1

Introduction

Compressors are used for pressurization of fluids. Applications involve compression of air for use in aircraft engines and pressurization and transportation of gas in the process and chemical industries. Towards low mass flows, the stable operating region of axial and radial compressors is bounded due to the occurrence of aerodynamic flow instabilities: *rotating stall* and *surge*. These instabilities can lead to the failure of the compressor system because of large mechanical and thermal loads in the blading and casing, and limit the performance and efficiency (de Jager, 1995; Willems, 1997; Willems and de Jager, 1998b). Suppressing these instabilities will improve life span, maintainability, and performance of the machine (CCC, 1997; Epstein et al., 1989). In addition, enlarging the stable operating region may reduce the required number of compressors, thus, the investment costs, and increases the effectiveness of the compression system.

![Diagram](image)

Figure 1.1: Scheme of a compressor characteristic emphasizing the difference between surge avoidance and active control (Willems, 1997).

Stable operation can be guaranteed by operating the compressor at a safe distance from the unstable region using, e.g., a bleed or recycle valve (*avoidance control*). Surge control systems currently used in industry are based on this control strategy. The main drawback of this approach is the wasteful safety margin which constrains the performance and overall efficiency of the system, as shown in Fig. 1.1. To enlarge the stable region beyond its "natural" stability boundary, Epstein *et al.* (1989) proposed a strategy which aims at
suppressing the instabilities by so-called active control. The basic idea behind this strategy is to modify the dynamics of the compression system by feedback. Experimental studies (Ffowcs Williams and Huang, 1989; Pinsley et al., 1991) show that approximately 20% reduction in surge point mass flow can be realized using a complex-valued proportional feedback controller (Willems, 1997, Chapter 4).

This work focuses on active control of surge in a laboratory-scale gas turbine installation. Surge is a one-dimensional flow instability which affects the entire compression system. This unstable flow phenomenon is characterized by large plenum pressure rise fluctuations and unsteady annulus-averaged mass flow. In the compressor map, surge is seen as a limit cycle oscillation, see Fig. 1.2. The surge dynamics of the studied compression system are reasonably described by the Greitzer compression system model as reported in (Meuleman et al., 1998). For active surge control in the gas turbine installation, we propose a control valve in combination with a plenum pressure sensor. The objective of this work is to specify the required capacity and bandwidth of the control valve for active surge control.

![Figure 1.2: Compressor map with deep surge cycle (de Jager, 1995).](image)

This report is organized as follows. First, the choice of the applied sensor and actuator is briefly motivated in Chapter 2. Also, important criteria for valve selection are discussed. In Chapter 3, the compression system and control valve models are presented and an efficient control strategy is introduced: one-sided control. Furthermore, the stability of the linearized compression system is analyzed. Simulation results are shown and discussed in Chapter 4. Finally, conclusions are drawn and directions for future research are suggested.
Chapter 2

IO-Selection for Surge Control

This chapter deals with the selection of sensors and actuators (IO-selection) for surge control in a laboratory-scale gas turbine installation. First, the examined compression system is discussed. Second, the selected IO-set is motivated and several valves used in experimental surge control systems are compared with each other. Finally, the control valve specifications are determined.

2.1 Experimental set-up

Active surge control strategies will be evaluated on the gas turbine installation in the Energy Technology Laboratory of the Faculty of Mechanical Engineering. For surge experiments, the installation is used in the configuration shown in Fig. 2.1. The radial compressor is part of a turbocharger (BBC VTR 160L) and similar to (Fink et al., 1992) it is driven by the turbine. External supplied compressed air flows via the vessel into the combustion chamber where natural gas is added and burned. The hot exhaust gasses expand over the axial turbine and deliver the power to drive the compressor. Detailed information about the gas turbine installation can be found in (van Essen, 1995).

Figure 2.1: Scheme of the gas turbine installation.
2.2 Motivation of the selected IO-set

For the realization of a surge control system, an appropriate number, place, and type of sensors and actuators have to be selected. In the literature, various types of sensors and actuators are proposed for surge control, as shown in Fig. 2.2. An overview is presented in, e.g., (de Jager, 1995; Willems and de Jager, 1998b). In (van de Wal et al., 1997), the selection of sensors and actuators (IO-selection) for active surge control is discussed for four candidate sensors (plenum pressure, compressor mass flow, and static or total compressor face pressure) and three candidate actuators (close-coupled valve (CCV), plenum bleed valve, and movable wall). From this theoretical study, it is concluded that a mass flow sensor in combination with a CCV and a movable wall is the best IO-set. These results are in line with the results found by Simon et al. (1993); they conclude that a mass flow sensor and a CCV are most promising among their investigated IO-sets.

![Figure 2.2: Candidate sensors and actuators for surge control (de Jager, 1995).](image)

In the gas turbine installation, mass flow measurements are not available (yet), so plenum pressure measurements will be used for control. The compressor mass flow is controlled by a relatively large compressor blow-off valve (ECONOSTO STEVI-E 12.440/CS23: $K_{cv} = 160$ [m³/hr], stroke 30 [mm], and operating speed 25 [mm/min]). It is seen from simulations that this electrically powered globe valve is too slow for surge control in the studied compression system. Therefore, analogous to (DiLiberti et al., 1996) it is suggested to place a small control valve in parallel with the compressor blow-off valve, as shown in Fig. 2.3; the large blow-off valve represents the system's pressure requirements, e.g. downstream processes or losses due to resistance in the piping, whereas the small control valve has to stabilize the compression.

![Figure 2.3: Scheme of the modified gas turbine installation.](image)
2.3 Control valve selection

system around its nominal operating point. However, for the large plenum system studied in (van de Wal et al., 1997), the results are poor for a plenum bleed valve. But, the BBC compression system examined in this study has a relatively small plenum volume, see Table 2.1\(^1\), so the control valve is expected to behave more or less like a compressor bleed valve. In addition, promising results are reported for similar systems in (Badmus et al., 1995; Pinsley et al., 1991) in which plenum pressure measurements are used in combination with a fast throttle for surge control. However, in many cases the throttle represents the system’s load (Greitzer, 1981), so it is difficult to control. Therefore, the system shown in Fig. 2.3 is a better representation of a real process.

Table 2.1: System parameters of four studied compression systems.

<table>
<thead>
<tr>
<th></th>
<th>( V_p ) [m(^3)]</th>
<th>( \beta ) [-]</th>
<th>( f_H ) [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>van de Wal et al. (1997)</td>
<td>2.66</td>
<td>10.27 - 25.68</td>
<td>2.43</td>
</tr>
<tr>
<td>BBC compression system</td>
<td>0.0375</td>
<td>0.49 - 0.96</td>
<td>27.34</td>
</tr>
<tr>
<td>Badmus et al. (1995)</td>
<td>0.92</td>
<td>0.54 - 1.32</td>
<td>3.90</td>
</tr>
<tr>
<td>Pinsley et al. (1991)</td>
<td>0.027</td>
<td>1.26 - 1.93</td>
<td>12.5</td>
</tr>
</tbody>
</table>

The meaning of the Greitzer stability parameter \( \beta \) and Helmholtz frequency \( f_H \) listed in Table 2.1 is discussed in detail in Chapter 3.

2.3 Control valve selection

2.3.1 Control valves used for surge control

In the literature, six cases are found which report about control valves used for surge control in an experimental set-up. Table 2.2 gives an overview of the occurring surge frequency in the system, and the applied valve type and actuation. Figure 2.4 illustrates the basic working principle of three valve types. Detailed information can be found in, e.g., (Smith and Vivian, 1995; Whitehouse, 1993).

Table 2.2: Control valves applied in experimental set-ups for surge control.

<table>
<thead>
<tr>
<th>( f_{surge} ) [Hz]</th>
<th>Valve type</th>
<th>Valve actuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Badmus et al. (1995)</td>
<td>2-10</td>
<td>butterfly</td>
</tr>
<tr>
<td>DiLiberti et al. (1996)</td>
<td>31</td>
<td>gate</td>
</tr>
<tr>
<td>Jungowski et al. (1996)</td>
<td>10-12</td>
<td>sleeve</td>
</tr>
<tr>
<td>Pinsley et al. (1991)</td>
<td>13.5</td>
<td>gate</td>
</tr>
<tr>
<td>Nakagawa et al. (1994)</td>
<td>5</td>
<td>butterfly</td>
</tr>
<tr>
<td>Yeung and Murray (1997)</td>
<td>1.4</td>
<td>butterfly</td>
</tr>
</tbody>
</table>

In the LICCHUS multi-stage axial compressor test rig, low-inertia, high tolerance butterfly valves are applied (Badmus et al., 1995). These valves are reported to have a 3 [dB] bandwidth of 70 [Hz]. The surge frequency can be modified between 2 and 10 [Hz] by varying the plenum and duct geometry of the test rig. To suppress surge oscillations at 31 [Hz] in the turbocharger test facility in (DiLiberti et al., 1996), a fast response gate valve is used which is actuated by a loudspeaker. This loudspeaker-valve combination

\(^{1}\)Helmholtz frequency \( f_H \) in (Badmus et al., 1995) is based on a different definition compared to other three cases.
does not respond to signals above 60 [Hz] and observes an increasing phase shift from 0 to 180 [deg] for increasing frequencies. A complex-valued proportional feedback controller is implemented to compensate for this phase shift. Oscillations at frequencies between 10 and 35 [Hz] are reported to be damped out with this control system. Jungowski et al. (1996), on the other hand, apply a so-called low-inertia sleeve valve for surge control; this valve consists of two sleeves with opposed windows. Rotation of one sleeve relatively to the other determines a changing resistance to the air flow. In (Pinsley et al., 1991), a rotary gate valve with a flat frequency response characteristic up to 80 [Hz] is used for surge control. Nakagawa et al. (1994) combined a large valve with a by-pass valve at the suction-side of the compressor. Analogous to (DiLiberti et al., 1996), the performance of their control system is specified in terms of phase shifts, e.g. a phase lag of 7 [deg] for a frequency of 5 [Hz]. Finally, Yeung and Murray (1997) use a butterfly valve actuated by a servo motor. The bandwidth of this valve is specified in terms of angle variations: small signal bandwidth of 50 [Hz] (±5 [deg] modulation) and large signal bandwidth of 15 [Hz] (±90 [deg] modulation). In summary, it is concluded that a minimal control valve bandwidth of approximately 3 to 4 times the surge frequency is required for active surge control.

### 2.3.2 Control valve specifications

Several criteria have to be considered to make an appropriate valve selection, see e.g. (Smith and Vivian, 1995; Whitehouse, 1993). Details about valve and actuator sizing can be found, e.g. in (Fitzgerald, 1995, Chapter 8). Analogous to (Whitehouse, 1993), the following selection criteria are discussed.

**Valve function**  In this study, the control valve is used for surge control. The valve will be operated when the compressor outlet pressure (i.e., plenum pressure) differs from a set-point value.

**Valve actuation**  According to (Whitehouse, 1993), valves can be operated by hydraulic, pneumatic, and electric actuators. Hydraulic actuators can realize very high operating forces and fail-safe operation can be arranged due to the constant hydraulic pressure. However, the costs of the hydraulic power supply system can be high. Most important drawback of pneumatic actuation is the compressibility of air which limits the ability to hold a valve position and to realize fast displacements. Furthermore, the relatively low operating pressure of the pneumatic supply constrains the power delivered by the pneumatic actuator.

Most control valves listed in Table 2.2 are operated by an electric actuator. The main advantages of this type of actuation are the linear motor characteristics and the inherent ability to keep the valve in a specified position. However, backlash, friction, and flexibility can be a problem in case an electro motor is used in combination with a gear box. *Direct-drive motors* can overcome the problems associated with the gear box.
2.3 Control valve selection

Bandwidth  Because high-frequent pressure fluctuations can occur during surge, a high control valve bandwidth is required. As seen in the previous section, a minimal bandwidth of 3 to 4 times the surge frequency is demanded. The limited bandwidth of the control valve with actuator is the result of inertia effects and flexibility in the mechanical parts.

In the examined BBC compression system, the pressure fluctuations associated with surge have a frequency between 18 and 24 [Hz] (Meuleman et al., 1998). As a result, the minimal required control valve bandwidth will be on the order of 60-80 [Hz]. This is in line with the bandwidth of the valves discussed in (Badmus et al., 1995; Pinsley et al., 1991). Note that the occurring surge frequencies in our system are relatively large compared to most values found in other systems, see Table 2.2.

Process medium  In the compression system, air is processed so relations for compressible flows have to be applied to determine the flow characteristics of the control valve. Compared to liquids, significantly smaller mass flow rates can be realized for gases (for the same pressure drop and flow through areas) due to density differences.

Process conditions  Preferably, the control valve has to be used for the compression system with the BBC as well as the small GARRETT turbocharger. Details about the GARRETT turbocharger can be found in (Lahoye, 1996). The GARRETT compression system exhibits surge at 15 [Hz] for N= 70,000 [rpm], so the BBC system demands the largest control valve bandwidth. In the configuration shown in Fig. 2.1, the BBC turbocharger can be runned with a rotational speed between N= 18,000 and 25,000 [rpm] due to the limited power supplied by the turbine. Then, the radial compressor delivers air at a maximal pressure of $p_{co} \approx 1.5$ [bar] and a temperature of $T_{co} \approx 345$ [K]. For the GARRETT turbocharger, measurements during surge are done up to N= 86,000 [rpm]. This corresponds with operating conditions of $p_{co} \approx 2.5$ [bar] and $T_{co} \approx 420$ [K].

Table 2.3: Surge control valve specifications.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Spec/Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valve function</td>
<td>control</td>
</tr>
<tr>
<td>Valve actuator</td>
<td>electric</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>60-80 [Hz]</td>
</tr>
<tr>
<td>Process medium</td>
<td>air</td>
</tr>
<tr>
<td>Maximal process temperature</td>
<td>420 [K]</td>
</tr>
<tr>
<td>Maximal pressure drop</td>
<td>1.5 [bar]</td>
</tr>
<tr>
<td>Process flow rate</td>
<td>-</td>
</tr>
</tbody>
</table>

Important data for selecting a control valve are the required capacity, which is expressed by the so-called $K_v$-value, and the maximal pressure drop across the valve. According to (Smith and Vivian, 1995), the $K_v$-value represents the amount of water (in [m³/hr]) which can pass the valve for a pressure drop of $\Delta p = 1$ [bar] across the valve. This results in the following equation for subsonic flow of a compressible fluid across a valve (van Essen, 1995):

$$K_v = 7.0 \sqrt{\frac{T_{bu}}{\rho_n p_{av}} \frac{\dot{m}}{\sqrt{\Delta p}}}$$

where $\dot{m}$ is the mass flow through the valve (in [kg/s]), $T_{bu}$ is the temperature before the valve (in [K]), $\rho_n$ is the normalized density at 1.013 [bar] and 273 [K] (in [kg/Nm³]), and $p_{av}$ is the pressure after the valve (in [bar]). In this relation, the term $\sqrt{\frac{T_{bu}}{\rho_n p_{av}}}$ can be interpreted as a "density-correction". Due to physical limitations, the maximal pressure drop across the valve is constrained. As the control valve discharges into
the laboratory \( p_{av} = 1 \text{ [bar]} \), the maximal pressure drop across the valve is approximately 1.5 [bar]. The compressors of the BBC and GARRETT turbochargers can realize a maximal mass flow rate of approximately 0.4 and 0.5 [kg/s], respectively. Because of the relatively small compressor delivery pressure \( p_{co} \), the BBC compression system will demand the largest control valve capacity. However, the actual required control valve capacity for surge control is not known. A summary of important specifications is given in Table 2.3.

Note that accuracy and resolution will also be of significant relevance. Furthermore, the flow characteristic of the chosen valve is important, see Fig. 2.5. For surge control, a linear characteristic is preferred since

Figure 2.5: Scheme of flow characteristics for constant pressure drop: A - quick opening globe valve; \( A_1 \) - linear globe valve; B - butterfly valve (Fitzgerald, 1995; Smith and Vivian, 1995).

this makes control easier; the valve shows the same change in mass flow in the entire operating range for a specific displacement variation. Nevertheless, in case the nonlinear flow characteristics are known the controller can correct for these nonlinearities of the valve. In Section 4.2, the influence of flow characteristics on compression system performance is studied in detail.

In summary, it is concluded that active surge control in the BBC compression system demands the largest control valve bandwidth and capacity. As seen from Table 2.3, the actual required process flow rate and actuator bandwidth are not (exactly) known. Therefore, simulations will be done to get an impression of the required values.
Chapter 3

Compression System Model

In this chapter, the Greitzer lumped parameter model is introduced to describe the dynamics of the examined compression system. This model is extended with a control valve model to study the influence of valve dynamics on system performance. Based on linearizations of this extended model, the stability of the system is studied in case a static output feedback controller is used for stabilization.

3.1 Greitzer compression system model

In (Meuleman et al., 1998), the Greitzer model is applied to describe the behavior of the compression system shown in Fig. 2.1. This lumped parameter model is modified to account for the influence of the control valve, see Fig. 3.1.

![Compression system model diagram]

Figure 3.1: Compression system model.

The flow in the ducts is assumed to be incompressible and one-dimensional. Compressibility effects are associated with the isentropic compression of the gas in the plenum while inertia effects are lumped on the
acceleration of the gas in the compressor duct. Moreover, temperature effects and the influence of rotor speed variations on system behavior are neglected. The original model is described concisely in (Greitzer, 1976).

Assuming quasi-steady compressor behavior leads to the following set of dimensionless equations:

\[
\frac{d(\delta \Phi_c)}{dt} = \beta [\Psi_c(\Phi_c + \delta \Phi_c) - \psi_0 - \delta \psi] \tag{3.1}
\]

\[
\frac{d(\delta \psi)}{dt} = \frac{1}{\beta} [\Phi_c + \delta \Phi_c - \Phi_t - \Phi_b] \tag{3.2}
\]

where \( \Phi_c \) is the dimensionless compressor mass flow and \( \psi \) is the dimensionless plenum pressure rise defined as:

\[ \Phi_c = \frac{\dot{m}_c}{\rho_a A_c U_t} \quad \text{and} \quad \psi = \frac{\Delta p_p}{\frac{1}{2} \rho_a U_t^2}, \]

with compressor mass flow \( \dot{m}_c \) in [kg/s], plenum pressure rise \( \Delta p_p = p_p - p_a \) in [Pa], and the dimensionless time \( t = \frac{t}{\omega_H} \) is obtained using the Helmholtz frequency:

\[ \omega_H = a \sqrt{\frac{A_c}{V_p L_c}}. \]

The so-called Greitzer stability parameter \( \beta \) is given by:

\[ \beta = \frac{U_t}{2\omega_H L_c}, \]

and the subscript 0 in (3.1) and (3.2) indicates quantities in the nominal operating point whereas \( \delta \) expresses deviations from the nominal operating point. The meaning and values of the parameters used in the Greitzer model can be found in Table 3.1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning/Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho ) [kg/m(^3)]</td>
<td>1.2</td>
</tr>
<tr>
<td>( V_p ) [m(^3)]</td>
<td>( 3.75 \times 10^{-2} )</td>
</tr>
<tr>
<td>( A_c ) [m(^2)]</td>
<td>( 9.56 \times 10^{-3} )</td>
</tr>
<tr>
<td>( L_c ) [m]</td>
<td>1.00</td>
</tr>
<tr>
<td>( a ) [m/s]</td>
<td>340</td>
</tr>
<tr>
<td>( f_H ) [Hz]</td>
<td>27.34</td>
</tr>
<tr>
<td>( c_t ) [-]</td>
<td>0.2994</td>
</tr>
<tr>
<td>( r_t ) [m]</td>
<td>0.09</td>
</tr>
<tr>
<td>( \omega ) [rpm]</td>
<td>18 - 35</td>
</tr>
<tr>
<td>( U_t ) [( m/s )]</td>
<td>1.70 - 3.30</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.49 - 0.96</td>
</tr>
</tbody>
</table>

Steady-state characteristics are applied to describe the behavior of the compressor, throttle, and control valve. Similar to (Moore and Greitzer, 1986), the dimensionless compressor characteristic \( \Psi_c(\Phi_c) \) is
approximated by a cubic polynomial in $\Phi_c$:

$$
\Psi_c(\Phi_c) = \Psi_c(0) + H \left[ 1 + \frac{3}{2} \left( \frac{\Phi_c}{F} - 1 \right) - \frac{1}{2} \left( \frac{\Phi_c}{F} - 1 \right)^3 \right]
$$

The parameters $\Psi_c(0), H,$ and $F$ are determined from steady-state measurements of the compressor characteristics where the peak of $\Psi_c(\Phi_c)$ corresponds to $\Phi_c = 2F$ and the valley point is laid at the plenum pressure rise axis: $\Psi_c(\Phi_c = 0) = \Psi_c(0)$. Note that steady-state measurements are not available for low mass flows: $\Phi_c < \Phi_{surge}$. Using the data of the parameters for different rotational speeds, they are interpolated by polynomials in $N$. For the operating region investigated in the sequel, the results are shown in Fig. 3.2. The compressor characteristic $N_4$ is obtained from extrapolation.

For subsonic flow conditions, the dimensionless throttle and control valve characteristics are given, respectively, by:

$$
\Phi_t = c_t u_{t0} \sqrt{\psi_0 + \delta \psi}
$$

$$
\Phi_b = c_b (u_{b0} + \delta u_b) \sqrt{\psi_0 + \delta \psi}
$$

where $u_{t0}$ and $u_{b0}$ are the nominal dimensionless throttle and control valve positions, respectively. Both valves are supposed to have linear flow characteristics, i.e., the $K_c$-value is linear proportional to the dimensionless valve position $u_t$ and $u_b$. The throttle and control valve parameter $c_t$ and $c_b$ represent the capacity of the fully opened valve, whereas $c_t$ is estimated from available steady-state measurements.

Note that this model can describe a system with a bleed valve as well as a recycle valve since the (ambient) pressure at the inlet of the compressor duct and after the control valve are identical.
3.2 Control valve model

To determine the influence of the valve dynamics on the compression system behavior, analogous to (Botros et al., 1991) the control valve is modeled by a second order model:

\[
\frac{d^2(\delta u_{bf})}{dt^2} + 2\zeta \tilde{\omega}_{co} \frac{d(\delta u_{bf})}{dt} + \tilde{\omega}_{co}^2 \delta u_{bf} = \tilde{\omega}_{co}^2 \delta u_b
\]  

(3.3)

where:

\[
\tilde{\omega}_{co} = \frac{2\pi f_{co}}{\omega_H}
\]

This control valve model can be interpreted as a mass-spring-damper model of the core mass of a solenoid valve, see Fig 3.3; \(\delta u_b\) is the dimensionless input signal from the controller and \(\delta u_{bf}\) is the unconstrained dimensionless control valve position. The core mass is excited by the Lorentz force which is generated by the coil. Valve actuator dynamics are neglected since the electronics are assumed to be relatively “fast” compared to the valve dynamics. Consequently, the following relation can be derived for the bandwidth of the control valve:

\[
f_{co} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}
\]

with the core mass \(m\) and the stiffness \(k\) of the retract spring.

![Figure 3.3: Scheme of a solenoid valve.](image)

The actual dimensionless valve position \(\delta u\) can be varied between 0 (closed) and 1 (fully open). In (Willems and de Jager, 1998a), this is modeled by a constraint on \(\delta u_{bf}\) after integration, see Fig. 3.4. As a result, the closed control valve opens when the unconstrained valve position \(\delta u_{bf}\) becomes positive. Analogously, the fully opened valve starts to close if \(\delta u_{bf} < 1\). This is not in accordance with practice; the closed control valve \((\frac{d\delta u_{bf}}{dt} = 0)\) starts to open when the net force becomes positive. Therefore, the control valve model is
3.3 Linear stability analysis of the unconstrained system

In order to compare the compression system behavior for various control valves, analogous to (Simon et al., 1993) the simplest control law, a linear static output feedback controller, is used to stabilize the system, see Fig. 3.4: This control strategy is applied since it is supposed to be most limiting; other strategies are assumed to (i) stabilize the linearized system in the entire compressor’s operating region (e.g., full state feedback or $H_\infty$ control (van de Wal et al., 1997, Chapter 3)), (ii) realize a larger domain of attraction, and (iii) require smaller control actions. This seems plausible because, e.g., nonlinear control strategies deal directly with the nonlinearities (Slotine and Li, 1991) and can employ useful nonlinearities. Consequently, the valve sizing will be conservative so the valve is not chosen too small. Moreover, this controller facilitates an analytical stability analysis of the unconstrained linearized compression system, i.e., the system with unconstrained dimensionless valve positions $\delta u_{bf}$. This analysis is done to get an impression of the values of control gain $K$ which stabilize the linearized system. As a result, the local behavior of the nonlinear compression system in the vicinity of the nominal operating point can only be predicted; essentially nonlinear phenomena, e.g., limit cycles, can not be described by linear systems (Khalil, 1996).

First, the stability of the unconstrained linearized system without valve dynamics is considered: $\delta u_b = \delta u$. Linearization of (3.1) and (3.2) around $(\Phi_0, \psi_0)$ results in the following set of equations:

$$\frac{d(\Phi)}{dt} = \left[ \begin{array}{c} \beta m_c \\ \frac{1}{\beta} \end{array} \right] + \left[ \begin{array}{c} -\beta \\ \frac{1}{\beta m_c} \end{array} \right] \frac{\delta \Phi_c}{\delta \psi} + \left[ \begin{array}{c} 0 \\ -\frac{V}{\beta} \end{array} \right] \delta u_b$$

The meaning of the used symbols can be found in Table 3.1. Substitution of (3.5) in this state-space
description finally leads to the following closed-loop transfer function:

$$H_c(s) = \frac{-\frac{V}{\beta} K (s - \beta m_c)}{s^2 + \left(\frac{1}{\beta m_{re}} - \beta m_c - \frac{V}{\beta} K\right) s + \left(1 - \frac{m_c}{m_{re}} + V K m_c\right)}$$  \hspace{1cm} (3.7)

Application of the Routh-Hurwitz stability criterion learns that the applied controller can only stabilize the linearized system around a nominal operating point ($\Phi_0$, $\psi_0$) if and only if the following condition holds (see also (Simon et al., 1993)):

$$m_c < \frac{1}{\beta}$$  \hspace{1cm} (3.8)

Consequently, the stability of a desired set-point depends on the shape of the compressor characteristic $\Psi_c(\Phi_c)$ in the nominal operating point, the system's geometry ($V_p$, $L_c$, and $A_c$), and the rotor tip speed $U_r$. For the linearized system under investigation, this implies that the stable operating region can be extended towards $\Phi_0 = 1.21 F$ and $1.64 F$ for N= 18,000 and 25,000 [rpm], as shown in Fig. 3.5. The straight lines in this figure indicate $\frac{1}{\beta}$. As seen in (Greitzer, 1981), the uncontrolled linearized system ($K = 0$) becomes unstable near the peak of the compressor characteristic, i.e., $\Phi_c \approx 2 F$, in a point on the positively sloped part of the characteristic. As a result, for N= 25,000 [rpm] an extension of approximately 20% of the stable operating region is feasible compared to the uncontrolled situation.

In order to determine values of $K$ for which the linearized system is stabilized around a nominal operating point, the root-locus technique is applied. Further details about this technique can be found, e.g., in (Bosgra and Kwakernaak, 1995, Chapter 2). A typical result is shown in Fig. 3.6 for $\Phi_0 = 1.7 F$, N= 25,000 [rpm], $u_{\phi 0} = 0$ and $c_b = 0.1 c_t$. It is seen from this figure that the system has a right-half plane zero $\beta m_c = 0.8644$. For decreasing $\Phi_0$ values, the open-loop poles ($K = 0$) move further away from the imaginary axis into the complex open right-half plane, as shown in Fig. 3.7. In addition, the negative real parts of the closed-loop poles move to smaller controller gains and the range of stabilizing controller gain values decreases.

For given values of $\Phi_0$, $c_b$, $c_t$ and $u_{\psi 0}$, the nominal throttle position is determined from:

$$u_{\phi 0} = \frac{1}{c_t} \left[\frac{\Phi_0}{\sqrt{\psi_0}} - c_b u_{\psi 0}\right]$$

with:

$$\psi_0 = \Psi_c(\Phi_0) \quad \text{and} \quad \Phi_0 = [c_t u_{\phi 0} + c_b u_{\psi 0}] \sqrt{\psi_0}$$

As a result, the root-locus does not depend on the desired nominal control valve position $u_{\psi 0}$ since the parameters $m_c$, $m_{re}$ and $V$ only depend on the desired nominal operating point ($\Phi_0$, $\psi_0$) and the control
3.3 Linear stability analysis of the unconstrained system

Figure 3.6: Typical example of a root-locus for the unconstrained compression system without valve dynamics (\(\times\): open-loop poles; \(\circ\): zero).

Figure 3.7: Real parts of closed-loop poles for system without valve dynamics (N = 25,000 [rpm], \(c_b = 0.1c_1\), and \(u_{b0} = 0\)).
valve capacity $c_b$:

$$m_c = \frac{\Phi_c}{\Phi_{c0}}$$

$$m_{te} = \frac{\Phi_{c0}}{\Phi_c}$$

$$V = c_b \sqrt{\Phi_0}$$

(3.9)

Hence, in case $VK$ in (3.7) is kept constant the same root-locus will be found for different $c_b$ values in one specific nominal operating point.

Similar results can be generated for the unconstrained linearized system with valve dynamics, see Figs. 3.8 and 3.9. Compared to the system without valve dynamics a double open-loop pole is added at $s = -\delta_{\phi_0} (\zeta$

Figure 3.8: Typical example of a root-locus for the unconstrained compression system with valve dynamics ($\times$: open-loop poles; $\circ$: zero).

is set to 1). A typical example of a root-locus for the system with valve dynamics is shown in Fig. 3.8. The influence of the valve dynamics on system stability is investigated for one specific case: $N= 25,000$ [rpm], $\Phi_{c0} = 1.7T$, $u_{b0} = 0$, and $c_b = 0.1c_t$, see Fig. 3.9. The real parts of the poles are seen to move towards larger values for decreasing $f_{co}$ values. For $f_{co} = 70$ [Hz], the range of stabilizing $K$ is equivalent with the values found for the system without valve dynamics. The range of stabilizing $K$ reduces for decreasing $f_{co}$ values and, finally, in case of $f_{co} = 50$ [Hz] the examined nominal operating point can not be stabilized by static output feedback based on plenum pressure measurements. But, this does not hold in general; e.g., a small set of stabilizing controller gains can be found for $N= 18,000$ [rpm], $\Phi_{c0} = 1.6T$, and $f_{co} = 50$ [Hz].

3.3.1 Influence of valve dynamics on stability

In this section, the influence of the control valve dynamics on the closed-loop stability is further examined. This stability analysis is based on frequency response analysis. For one specific nominal operating
3.3 Linear stability analysis of the unconstrained system

Figure 3.9: Influence of $f_c$ on closed-loop stability ($N = 25,000$ [rpm], $\Phi = 1.7 F$, $c_b = 0.1 c_f$, and $u_{b0} = 0$).

Point, $N = 25,000$ [rpm] and $\Phi = 1.7 F$, three cases are investigated: (i) system without valve dynamics ($f_{c0} = \text{unlimited}$), (ii) $f_{c0} = 70$ [Hz], and (iii) $f_{c0} = 50$ [Hz]. In all studied cases, $K = -19.9$, $u_{b0} = 0$ and $c_b = 0.1 c_f$ and the Bode plots of the loop gain transfer function $L(j\omega) = P C$ are shown in Fig. 3.10 whereas Fig. 3.11 shows the Nyquist diagrams of $\text{det}(I + PC)$. The different line types in Fig. 3.11 correspond to the same cases shown in Fig. 3.10. In the used notation, $C(j\omega)$ is the control gain $K$, and $P(j\omega) = P_{\text{compr}} \cdot P_{\text{value}}$ is the linearized compression system with valve dynamics:

$$P_{\text{compr}}(j\omega) = \frac{\frac{V}{\beta}(s - \beta m_c)}{s^2 + \left(\frac{1}{\beta m_c} - \beta m_c\right)s + \left(1 - \frac{m_c}{m_{te}}\right)}$$

$$P_{\text{value}}(j\omega) = \frac{\omega_{c0}}{(s + \omega_{c0})^2}$$

whereas for the system without valve dynamics $P_{\text{value}} = 1$. Note that due to the scaling in the Greitzer model, $\tilde{\omega}$ is defined as:

$$\tilde{\omega} = \frac{\omega}{\omega_f}$$

Figure 3.10 illustrates that the valve dynamics results in an undesirable decrease of magnitude and phase lead for $\tilde{\omega} > 0.1$ [rad/s] (i.e., $f > 2.7$ [Hz]) compared to the situation without valve dynamics. As seen in Fig. 3.8, the open-loop system has two unstable poles. Consequently, the generalized Nyquist criterion is applied to investigate closed-loop stability (Bosgra and Kwakernaak, 1995, Chapter 1):

**Generalized Nyquist criterion** Suppose that the loop gain transfer function $L$ is proper and has no poles on the imaginary axis. Assume also that the Nyquist plot of $\text{det}(I + L)$ does not pass through the origin, then:
Number of unstable closed-loop poles = 
Number of times the Nyquist plot of \( \det(I + L) \) encircles the origin 
(+: clockwise; -: anti-clockwise) 
+ 
Number of unstable open-loop poles 

It is illustrated in Fig. 3.11 that the origin is not encircled for case (iii); the closed-loop system is unstable for \( f_{co} = 50 \) [Hz]. This is the result of the phase lead reduction for \( \phi > 0.1 \). In the other two cases, the origin is encircled two times anti-clockwise, so the closed-loop system is stable.

Performance of the linearized system

To study the performance of the unconstrained linearized compression system, the sensitivity \( S \), complementary sensitivity \( T \), and input sensitivity \( M \) functions:

\[
S = \frac{1}{1 + L}, \quad T = \frac{L}{1 + L}, \quad M = \frac{C}{1 + L}
\]

are plotted in Fig. 3.12. This analysis is based on (Bosgra and Kwakernaak, 1995, Section 1.5).

Disturbance attenuation and bandwidth To determine the capability of the system to attenuate disturbances, the sensitivity function \( S \) has to be studied: the smaller \(|S(j\omega)|\) is, the more the disturbances are attenuated. \(|S(j\omega)|\) is small if the magnitude of \( L(j\omega) \) is large, see Fig. 3.10. However, it is known from the Freudenberg-Looze equality that the sensitivity function \( S \) can only be made small up to frequencies equal to the magnitude of the smallest right-half plane zero (Bosgra and Kwakernaak, 1995, Section 1.7);
3.3 Linear stability analysis of the unconstrained system

Figure 3.11: Nyquist diagrams of \( \det(I + PC) \) for \( -\infty < \omega < \infty \).

Figure 3.12: Sensitivity \( S \), complementary sensitivity \( T \), and input sensitivity \( M \) functions for \( N = 25,000 [\text{rpm}] \), \( \Phi_c = 1.7F \), \( c_b = 0.1c_l \), \( u_{b0} = 0 \) and \( K = -19.9 \).
in the studied case, $z = \beta m_c = 0.86$. As a result, the bandwidth of the closed-loop system is relatively small, so effective disturbance attenuation is possible in a limited frequency range. It is concluded from Fig. 3.12 that this bandwidth is approximately $\omega = 3$, i.e. $f = 82$ [Hz], and it increases for decreasing $f_{co}$ values.

**Measurement noise** The influence of the measurement noise on the system output, i.e., the plenum pressure rise $\psi$, can be examined from the complementary sensitivity $T$; for all frequencies, the influence of measurement noise on the output is small since $|T| < 1$. Decreasing $f_{co}$ results in an even smaller influence for large frequencies.

**Plant capacity** The input sensitivity function $M$ shows the sensitivity of the input $u = u_{id} + \delta u$ to disturbances and the reference signal. From Fig. 3.12 it is concluded that disturbances are amplified up to $\tilde{\omega} \approx 1$, so the inputs can become large. This is disadvantageous because the system has a limited capacity, i.e., can handle inputs of limited magnitude. For decreasing $f_{co}$ values, the bandwidth of $|M|$ slightly reduces.

In summary, it is concluded that the valve dynamics increase the bandwidth of the closed-loop system, but introduce a phase lag such that the closed-loop system can become unstable. This problem can be overcome by using a phase lead compensator (Bosgra and Kwakernaak, 1995, Chapter 2).

### 3.3.2 Lead-lag compensator

In order to get an impression of the possible performance improvement, simulations are done with a lead-lag compensator and the nonlinear compression system model. The applied lead-lag compensator is given in the frequency domain by:

$$C(j\tilde{\omega}) = -\frac{\omega^2_{co}}{\omega^2_{co}} \frac{(s + \tilde{\omega}_{co})^2}{(s + \tilde{\omega}_{c})^2}$$

where:

$$\tilde{\omega}_{c} = \frac{2\pi f_c}{\omega_H}, \quad \tilde{\omega}_{co} > \tilde{\omega}_{co}$$

This compensator consists of an inverse model of the valve dynamics and the desired valve dynamics. Figure 3.13 shows simulation results for a static output feedback controller and a lead-lag compensator with $f_c = 500$ [Hz]. This figure illustrates that the compression system can not be stabilized with the static output feedback controller in the studied nominal operating point for $f_{co} = 50$ [Hz]. This is in agreement with the results of the linear stability analysis. Stabilization can be realized by the lead-lag compensator. Similar results are found for $f_{co} = 30$ [Hz]. Note that both cases are ideal situations because disturbances are absent and the control valve dynamics are exactly cancelled. As a result, stabilization may not be obtained in practice.

### 3.4 Linear stability analysis of the constrained system

In the previous section, the stability analysis is restricted to the unconstrained linearized compression system. However, Heemels and Stoorvogel (1998) formulate easily verifiable necessary and sufficient conditions for positive stabilizability of a linear system given by:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
3.4 Linear stability analysis of the constrained system

with the input $u(t) \in \mathbb{R}^m$ and the state $x(t) \in \mathbb{R}^n$. The existence of a positive open-loop input $u(t)$ is examined that steers the state asymptotically to zero. In addition, $u(t)$ is assumed to belong to the Lebesgue space of square integrable, measurable functions, denoted by $L^2_t$. Note that the constraint $u(t) \geq 0$ is less restrictive than the actual constraint on the dimensionless control valve position: $0 \leq u \leq 1$.

In (Heemels and Stoorvogel, 1998), the following result is presented for positive stabilizability of a system with a scalar input ($m = 1$):

**Positive Stabilizability** $(A, B)$ is said to be positively stabilizable if and only if $(A, B)$ is stabilizable and $\sigma(A) \cap \mathbb{R}_+ = \emptyset$

The system $(A, B)$ is stabilizable if and only if:

$$\text{rank}(A - \lambda I, B) = n \quad \text{for all} \quad \lambda \in \mathbb{C}_+.$$  

For the studied compression system without valve dynamics (3.6), this results in:

$$\text{rank} \left( \begin{bmatrix} \beta m_c - \lambda & 0 \\ \frac{1}{\beta m_c} & -\beta \end{bmatrix} \right) = n.$$  

It can easily be verified that the compression system is stabilizable for $\Phi_{c0} < 2F$. Furthermore, $A$ has no eigenvalues on the nonnegative real axis, as shown in Fig. 3.14, so the linearized compression is positive stabilizable.

Besides the open-loop version of stabilizability, a linear state feedback variant is formulated in (Heemels and Stoorvogel, 1998) for systems with a scalar input $u = F x(t)$ and $n = 1, 2$. Consider the system:

$$\dot{x}(t) = Ax(t) + B \Pi (F x(t))$$  

where $A$ is unstable and has one pair of complex conjugated eigenvalues $\lambda = \sigma_0 \pm j \omega_0$. The projection operator $\Pi$ is defined for $v \in \mathbb{R}^n$ by:

$$(\Pi v)_k = \max(v_k, 0)$$
In case of a scalar input, the system (3.10) switches between two dynamics:

\[
\begin{cases}
\dot{x}(t) = Ax(t) & u(t) \leq 0 \\
\dot{x}(t) = (A + BF)x(t) & u(t) > 0
\end{cases}
\]

The following necessary and sufficient conditions are specified for positive feedback stabilizability (scalar input and maximal two states):

**Positive Feedback Stabilizability** The system (3.10) is positive feedback stabilizable, if \((A, B)\) is stabilizable and \(F\) can be designed such that the eigenvalues \(\lambda\) of \(A + BF\) are contained in:

\[
\lambda = \sigma + j\omega \in \mathbb{C} \mid \sigma < 0 \quad \text{and} \quad \left| \frac{\omega}{\sigma} \right| < \left| \frac{\omega_0}{\sigma_0} \right|
\]

(3.11)

Up to now, stability theory of hybrid systems is not capable of giving satisfactory answers for feedback stabilizability of higher order switching systems, i.e., \(n > 2\).

A schematic representation of (3.11) is shown in Fig. 3.15. According to (3.11), the closed-loop poles of \(A + BF\) have to be placed within the triangle in the open left-half complex plane for positive feedback stabilization. In order to meet the constraint on the dimensionless control valve position \((0 \leq u \leq 1)\), we assume that the control input has to be made as small as possible. Application of the Kalman-Jakubović-Popov equality learns that for LQ-control the least control effort is needed to stabilize the system if the closed-loop poles approach the mirror images of the "unstable" open-loop poles, as shown in (Bosgra and Kwakernaak, 1995, Chapter 3). Nevertheless, in that case the constraint on the valve position can still be violated because the control inputs are not bounded in the applied approach.

In the studied compression system in Section 3.3, plenum pressure measurements are only available for surge control. To avoid the application of filters, a static output feedback controller is used, which limits the
placement of the closed-loop poles, see Fig. 3.6 and 3.8. This can easily be verified from the denumerator of (3.7). For the linearized system without valve dynamics (N= 25,000 [rpm] and Φ,e0 = 1.7F), the root-locus is shown in Fig. 3.16. From this figure, it is concluded that the control gain K has to be chosen such that the closed-loop poles lay within the “triangle”. In addition, for the least control effort, the poles have to be placed as close as possible to the mirror images of the open-loop poles. Consequently, the complex conjugated closed-loop poles located near the intersection of the root-locus and the dash-dotted lines \(|\frac{\omega}{\omega_0}| = \frac{\omega}{\omega_0}\) will be the best solution. Note that for the case shown in Fig. 3.8, i.e., a compression system with \(f_{co} = 70 [Hz]\), this design is not applicable since the compression system with control valve dynamics is a fourth order system. Nonetheless, this system is positive stabilizable since:

\[
\text{rank} \left( \begin{bmatrix} \beta m_c - \lambda & -\beta & 0 & 0 & 0 \\ \frac{1}{\beta} & \frac{1}{\beta m_c} - \lambda & 0 & -\frac{\zeta}{\omega_{co}} & 0 \\ 0 & 0 & -2\zeta \omega_{co} - \lambda & -\omega_{co}^2 & 0 \\ 0 & 0 & 0 & -\lambda \\ 0 & 0 & 0 & 0 \end{bmatrix} \right) = n \quad \text{for all} \quad \lambda \in \mathbb{C}_+ \]

and \(A\) has no poles on the nonnegative real axis for \(0 \leq \Phi_{e0} \leq 3F\).
Figure 3.16: Validation of requirement (3.11) for positive feedback stabilizability: $K = [-30; 0]$, $c_b = 0.1c_i$ and $u_{i0} = 0$. 
Chapter 4

Simulation results

Simulations are done with the nonlinear compression system model and the advanced control valve model in MATLAB/SIMULINK to determine the required capacity and bandwidth of the applied control valve for surge control. Furthermore, the influence of other system parameters, e.g., nominal valve position and controller switch on time, and the shape of the control valve’s flow curves on the compression system behavior are examined.

4.1 Influence of system parameters

Before starting simulations, it seems worthwhile to examine the conditions which are most restrictive for valve sizing. More specifically, a worst case is defined which is supposed to require the largest control effort, so the maximal required control valve capacity can be determined. In this study, we assume that stabilizing from a surge limit cycle is more difficult than stabilizing from small deviations from initial conditions. As a result, in all simulations the uncontrolled system is initially disturbed from the desired operating point \( \Phi_{e,0}, \psi_0 \) which results in a limit cycle oscillation for \( \Phi_{e,0} < 2F \). Then, after \( t_s = 0.25 \) [s] the controller is switched on. A similar approach can be found in (Badmus et al., 1996; Pinsley et al., 1991).

4.1.1 Nominal control valve position \( u_{\text{bo}} \)

First, the influence of the nominal control valve position on system behavior is examined in a specific operating point: \( N = 25,000 \) [rpm] and \( \Phi_{e,0} = 1.7F \). As seen from Fig. 3.5, stabilization is relatively difficult to accomplish in this operating point compared to \( N = 18,000 \) [rpm]. Simulations are performed using a control valve with a capacity of 10% of the throttle (i.e., \( c_b = 0.1c_t \)) and relatively fast valve dynamics: \( f_{co} = 100 \) [Hz]. Recall from Section 3.4 that positive feedback stabilizability cannot be studied for this constrained system with valve dynamics. Therefore, the control gain \( K \) is obtained from the stability analysis of the unconstrained linearized compression system using the root-locus technique; for each nominal operating point, \( K \) is the control gain which shifts the closed-loop poles as far as possible to the left in the complex plane. For two \( u_{\text{bo}} \)-values, the results are shown in Fig. 4.1. The compressor and nominal equivalent throttle characteristic:

\[
\Phi_{e,0} = [c_t u_{\text{bo}} + c_b u_{\text{bo}}] \sqrt{\psi_0 + \delta \psi}
\]

\(^1\)The stability boundary is not exactly situated at 2F, as shown e.g. in Fig. 3.14, and can be determined from (3.8) (see also (van de Wal and Willems, 1996, Chapter3)).
Simulation results

\[ N = 25000 \text{ [rpm]}; \Phi_{c0} = 1.7F \]

\[ Q_{c0} = 1.7F; B = 0.1C; f_{c0} = 100 \text{ [Hz]}; K = -19.9 \]

Figure 4.1: Influence of \( u_{b0} \) on compression system behavior

are shown for reference in the compressor map. Because of the chosen nominal control valve position:

\[ u_{c0} = \frac{1}{c_1} \left( \Phi_{c0} \sqrt{\Psi_0} - c_b u_{b0} \right) \]

the nominal equivalent throttle characteristics \( \Phi_{te0} \) are identical for varying \( u_{b0} \), \( c_b \) and \( c_t \) values. In the lower right figure, time traces of the actual dimensionless valve position \( \bar{u} = u_{b0} + \delta u \) are shown. It is seen from additional simulations that for a large range of control gains \( K \) the compression system can not be stabilized if the control valve can be maximally opened and closed in both directions (\( u_{b0} = 0.5 \)). But, the amplitude of the mass flow and plenum pressure rise oscillations slightly decreases from the moment the controller is switched on, see Fig. 4.1. The nonlinearities obviously have a destabilizing effect since \( \Psi_{c}(\Phi_{c0} = 1.7F) \) is an asymptotically stable equilibrium point of the unconstrained linearized system, as seen in Section 3.3. In case of \( u_{b0} = 0 \), so-called one-sided control (Chen and Kuo, 1997), the compression system is stabilized within three surge cycle periods. One-sided refers to the sign of the control input; either positive or negative. Moreover, the actual control valve position \( u \) has also to be smaller than one. Note that in both studied cases the system first performs another limit cycle oscillation before the controller

Figure 4.2: Compressor and control valve mass flow for \( u_{b0} = 0 \) and 0.5 case (\( N = 25,000 \) [rpm], \( \Phi_{c0} = 1.7F, K = -19.9 \), and \( f_{c0} = 100 \) [Hz]).
becomes effective. For \( u_{B0} = 0 \), the desired operating point is finally reached with zero control valve mass flow in case disturbances are absent, see Fig. 4.2. This is beneficial for the total efficiency of the compression system. From Fig. 4.1, it is concluded that the domain of attraction of the nominal operating point includes a part of the surge limit cycle for \( u_{B0} = 0 \). Similar results are found for \( u_{B0} = 0.1 \) and 0.2 but these cases require a nominal nonzero control valve mass flow for stabilization. The one-sided control strategy is comparable with the strategy mentioned in (Behnken, 1997) where a bleed valve is operated in case \( \frac{d\Phi_e}{dt} < 0 \). In Fig. 4.2, the compressor mass flow \( \dot{m}_c \) and the required control valve mass flow \( \dot{m}_b \) to control the system are shown; stabilization requires a peak control valve mass flow of 24% of the nominal compressor mass flow \( \dot{m}_c \).

Additional simulations show that the compression system can not be stabilized for nominal valve positions \( u_{B0} \geq 0.4 \). The pressure and mass flow oscillations are seen to increase for increasing \( u_{B0} \) values. It is illustrated in Fig. 4.3 that in case of \( u_{B0} = 0.4 \), the oscillations become smaller although the control system is not capable of stabilizing the system in the nominal operating point. For \( u_{B0} = 0.3 \), on the other hand,

\[
N = 25000 \text{ [rpm]}; \Phi_{\phi_0} = 1.7F
\]

\[
c = 0.1; f_{\phi_0} = 100 \text{ [Hz]}; K = -19.9
\]

\[
\dot{m}_c, \dot{m}_b
\]

\[
\Phi_e
\]

\[
\delta u
\]

\[
time [s]\]

Figure 4.3: Influence of \( u_{B0} \) on compression system behavior

the domain of attraction is sufficiently large although the system’s response exhibits a strange loop in the compressor map. This might be the result of the asymmetric valve behavior, i.e., the valve can move further in positive \( \delta u \) direction than in negative direction.

### 4.1.2 Control Gain \( K \)

In addition to the influence of \( u_{B0} \), the system behavior is affected by the control gain \( K \). More specifically, limit cycles and different equilibrium points are found for the one-sided controlled nonlinear compression system by varying \( K \). For a small range of \( K \) and \( \Phi_{\phi_0} \), the nominal operating point is found to be stable, as seen in Section 3.3; the nonlinear compression system can be stabilized down to \( N = 25,000 \text{ [rpm]} \) and \( \Phi_{\phi_0} = 1.64F \) for \( K = -17.2 \). However, in case the nominal operating point is unstable, the following cases can occur:
For small $K$ values, a limit cycle persists as shown e.g. in Fig. 4.1 for $u_{b0} = 0.5$.

System is stabilized in a distinct equilibrium point for large $K$. An example of this type of system behavior is shown in Figs. 4.4 and 4.5. In the compressor map, the equivalent throttle characteristic is shown for reference. Linear stability analysis shows that the unconstrained controlled compression system can not be stabilized in $\Phi_{e0} = 1.6F$. But, for $K = -19.9$ and $-25$, equilibrium points are found corresponding to $\omega_{e} = 1.79F$ and $1.92F$, respectively. Figure 4.5 illustrates this behavior in more detail; the compressor and equivalent throttle characteristics are seen to intersect in two points if $K$ is sufficiently large. In the shown cases, these distinct equilibrium points are stable.

Oscillations with small amplitudes are found around a distinct equilibrium point for a specific range of large $K$, see Fig. 4.6. However, this type of behavior is not found for all nominal operating points. Obviously, the oscillations are the result of the high control gain in combination with the constraint $u \leq 1$; adding a differentiating action to the controller:

$$C(j\omega) = \frac{-K(1 + \frac{K_d}{K}\omega)}{1 + \frac{K_d}{\tau}}$$

with, e.g., $K = -150$, $K_d = -20$, and $\tau = 1 \cdot 10^5$, stabilizes the system in $\Phi_{e0} = 2.21F$.

### 4.1.3 Bandwidth $f_{co}$ of control valve

To select an appropriate valve for active surge control, the effect of the bandwidth on system behavior has to be examined. Therefore, the $f_{co}$ value applied in Fig. 4.1 is reduced to 60 and 50 [Hz] respectively.
4.1 Influence of system parameters

Figure 4.5: Compressor map in detail ($c_b = 0.1c_t, f_{co} = 100$ [Hz], and $u_{b0} = 0$)

Figure 4.6: Influence of $K$ on system behavior.
Comparison of Figs. 4.1 and 4.7 learns that the reduction of $f_{co}$ from 100 to 60 [Hz] increases the number of surge cycle periods before stabilization is accomplished. For $f_{co} = 50$ [Hz], the amplitude of the oscillations is only reduced.

\[
\begin{align*}
N &= 25000 \text{ [rpm]}; \Phi_{co} = 1.7c_f \\
c_b &= 0.1; u_b = 0; K = -19.9 \\
\end{align*}
\]

Figure 4.7: Influence of $f_{co}$ on system behavior.

This is in agreement with the results of the linear stability analysis, see Fig. 3.9; for $f_{co} = 50$ [Hz], the nominal operating point can not be stabilized using static output feedback. However, distinct equilibrium points are found at $\Phi_{co} = 1.97c_f$ and $2.06c_f$ for $K = -35$ and $-50$ respectively, as shown in Fig. 4.8. Similar to the examined cases in the previous section, small amplitude oscillations (Case III) appear for $K = -100$.

### 4.1.4 Control valve capacity $c_b$

In the preceding sections, an arbitrary value of the control valve capacity $c_b$ is used. Figure 4.9 shows the influence of $c_b$ on the compression system behavior. Stabilization in the nominal operating point is realized for $c_b > 0.05c_f$ and $f_{co} = 60$ [Hz]. As expected, for decreasing control valve capacity more control effort, i.e., a larger control gain $K$, is required to stabilize the compression system, but the differences between $c_b = 0.1c_f$ and $0.08c_f$ are small. Further decrease of $c_b$ down to $0.06c_f$ results in more frequent saturation of the control valve. In case of $c_b = 0.05c_f$, the system can only be stabilized in an equilibrium point which corresponds with a fully opened control valve. Recall from Section 3.3 that the stability of the unconstrained linearized compression system with ideal control valve dynamics is independent of the chosen $c_f$-value; for an appropriate choice of $K$, the linearized system can be stabilized as long as $m_e < \frac{1}{\beta}$. However, increase of the control valve bandwidth $f_{co}$ up to 250 [Hz] does not change these results. Similar results are found for $N = 18,000$ [rpm], $\Phi_{co} = 1.6c_f$, and $f_{co} = 50$ [Hz].
4.1 Influence of system parameters

Figure 4.8: System behavior for different control gains $K$ ($f_{co} = 50 \text{ [Hz]}$).

Figure 4.9: Influence of control valve capacity $c_b$. 

$N = 25000 \text{ [rpm]}$; $\Phi_{co} = 1.7F$

$c_b = 0.1c_c$; $f_{co} = 50 \text{ [Hz]}; u_{bo}=0$

$c_b = 0.08c_c$; $K=-25$

$c_b = 0.06c_c$; $K=-32$

$c_b = 0.05c_c$; $K=-30$

$\omega = 0.1$; $f = 50 \text{ [Hz]}$; $u_0=0$

$\omega = 0.05$; $f = 60 \text{ [Hz]}$; $u_0=0$

$\omega = 0.01$; $f = 70 \text{ [Hz]}$; $u_0=0$
Determination of $K_v$-value

To determine the required $K_v$-value for active surge control experiments, the "instantaneous $K_v$-value" is computed during simulations:

$$K_v = 7.0 \frac{T_p}{\rho_n \rho_a} \frac{\dot{m}_b}{\sqrt{\Delta p}}$$

where the temperature $T_p$ before the valve is assumed to be $T_p = 397$ [K], $\Delta p$ is the pressure drop across the valve (in [bar]), and the flow characteristics of the valve are supposed to be linear, so $K_v$ can be written as:

$$K_v = \frac{U_c}{U_{c,max}} K_{v,max}$$

with the actual drive signal $U_c$ (in [V]), the capacity of the fully opened valve $K_{v,max}$ (in [m$^3$/hr]). Note that $\frac{U_c}{U_{c,max}} \in [0,1]$ is equivalent with the input $u$ of the compression system model with ideal valve dynamics. The instantaneous required capacity $K_v$ can be determined from simulations as long as no saturation occurs: $U_c < U_{c,max}$. If $U_c = U_{c,max}$, the results are affected by the chosen $c_b$-value, i.e., the chosen $K_{v,max}$ value. Therefore, the constraint on $u$ is dropped in the simulations to determine the required $K_v$-value, as shown in

![Figure 4.10: System behavior with unconstrained $u$.](image)

Fig. 4.10. The left part of this figure shows the phase plane of the control valve. The effect of the constraint on $u$ is demonstrated in Fig 4.11; if the upper constraint is reached, the control valve mass flow and pressure drop follow the control valve characteristic corresponding to $K_{v,max}$; in that case the computed maximal $K_v$ value is equal to the chosen $K_{v,max}$.

From Fig. 4.10 and 4.11, it is concluded that surge can be controlled in the studied compression system by using a control valve with a capacity of $c_b = 0.06c_t$, i.e $K_{v,max} = 10.7$ [m$^3$/hr]. The $\dot{m}_b$ and $\Delta p$ values corresponding to this maximal $K_v$-value are important for valve manufacturers to supply the desired valve since various definitions of the $K_v$-value are used (Smith and Vivian, 1995, Chapter 9). The chosen $K_{v,max}$-value also gives good results for $\Phi_{c0} = 1.6F$, $N = 25000$ [rpm], $K = -28$ and $f_{c0} = 50$ [Hz]. If the $K_v$-value of the valve is too small to realize the required $K_v$-value, the number of valves $n$ needed for control is given by:

$$n = \text{integer} \left\{ \frac{K_{v,required}}{K_{v,value}} \right\}$$
4.1 Influence of system parameters

4.1.5 Controller switch on time $t_s$

In all simulations, the controller is switched on after 0.25 [s]. This time $t_s$ will influence the performance of the controller. However, for all $t_s$ where $\delta \psi(t_s) < 0$ the system shows the same behavior as for $t_s = t_{opt}$ corresponding with the operating point $\delta \psi = 0$ and $\frac{d(\delta \psi)}{dt} > 0$. In the latter case, the controller is immediately effective and has the largest time span to stabilize the system. For three $t_s$-values, the results are shown in Fig. 4.12. If $t_s = 0.1955$ [s] the controller is activated in the point $\Phi_e = \Phi_{e0} + \delta \Phi_{e,max}$. For $t_{opt} = 0.2345$ [s], the controller stabilizes the system without performing an entire limit cycle oscillation. Both other cases need an extra surge cycle period before the oscillations are suppressed as illustrated in the upper right figure. From Fig. 4.12, it is concluded that in these cases the trajectory starts converging to the nominal operating point in the same point in the compressor map and follows the same trajectory as for $t_{opt}$. As a result, these responses can hardly be distinguished. Clearly, the amplitude of the mass flow and pressure oscillations during the extra cycle depends on $t_s$, but the system’s response is not fundamentally changed.

Figure 4.11: System behavior with constrained $u$.

Figure 4.12: Influence of $t_s$ for $u_{bo} = 0$. 
### 4.2 Influence of flow curves

Besides the nonlinearities of the compression system described by the Greitzer model, the nonlinear flow curves of the control valve can play an important role. So far, the flow curves are assumed to be linear, see (4.2). In this section, the effect of these nonlinear curves on compression system behavior is studied. Moreover, the flow curves can often be adjusted to the needs of the user. Therefore, it is useful to get insight into the compression system behavior for various flow curve settings, so an optimal setting can be chosen. Based on data supplied by valve manufacturers (ASCO-Joucomatic, 1997; TEKNOCRAFT, 1998), the flow curves are approximated by an arctan-function.

Four flow curves are applied in simulations, see Table 4.1 and Fig. 4.13. The modified simulation model is illustrated in Fig. 4.14; contrary to the model with linear flow curves shown in Fig. 3.4, the output signal of the control valve block first goes to the flow curve block before entering the compression system model.

Flow curve I is a simple approximation of the linear flow curve. However, for this flow curve a small amplitude oscillation remains after the controller is switched on, as shown in Fig. 4.15, whereas the system is stabilized by using the linear flow curve, see Fig. 4.7 for $f_{co} = 60$ [Hz]. Decreasing $K$ down to $-23$ results in a distinct equilibrium $\Phi_c = 1.72F$ close to the nominal operating point; asymptotic stability is not realized with flow curve I because $\frac{K_c}{K_{c,\text{max}}}$ is not exactly zero for $U_c = 0$. For flow curve II, the

<table>
<thead>
<tr>
<th>Number</th>
<th>Flow curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$\frac{1}{8} \arctan[0.3(U_c - 4)] + 0.455$</td>
</tr>
<tr>
<td>II</td>
<td>$\frac{1}{14} \arctan(U_c - 4) + 0.485$</td>
</tr>
<tr>
<td>III</td>
<td>$\frac{1}{53} \arctan(4(U_c - 6)) + 0.505$</td>
</tr>
<tr>
<td>IV</td>
<td>$\frac{1}{38} \arctan(U_c - 1) + 0.35$</td>
</tr>
</tbody>
</table>

![Figure 4.13: Flow curves used in simulations.](image-url)
4.2 Influence of flow curves

Figure 4.14: Block scheme of the modified simulation model.

Figure 4.15: Simulation results for different flow curves.
Simulation results

Oscillations are even larger after the controller is switched on. Modification of $K$ does not change this result significantly. Obviously, the reduced effective range, i.e., the range where a change in $U_c$ results in a significant change of $\frac{K_{e_{\text{max}}}}{K_{e_{\text{max}}}}$, degrades the performance of the control valve; the valve opens more slowly and $U_c$-values above 8 [V] have a minor effect on the control valve mass flow. Increasing $f_{co}$ up to 100 [Hz] does not fundamentally change the compression system behavior in both cases.

The results for flow curves III and IV are shown in Fig. 4.16. Small amplitude oscillations persist in case of flow curve III. For flow curve IV, on the other hand, a distinct equilibrium point is found: $\Phi_c = 2.12F$. In this case, $U_c$ variations between 0 and 6 [V] are effective whereas variations for $U_c > 6$ [V] are supposed to have a minor effect on the control valve mass flow. Recall from Section 3.3 that the unconstrained linearized system can be stabilized by static output feedback for $N = 25,000$ [rpm], $\Phi_{co} = 1.7F$, $c_b = 0.1c_t$, and $f_{co} = 60$ [Hz]. Furthermore, for a linear flow characteristic the compression system is stabilized in the nominal operating point, as shown in Fig. 4.7. Hence, the quick opening characteristic is unfavorable for suppressing surge oscillations. Similar results are found for various $K$-values and $f_{co} = 100$ [Hz].

![Simulation results](image)

Figure 4.16: Simulation results for different flow curves.

4.3 Influence of measurement noise

In case measurement noise is present in the system, a control valve with the capacity determined in Section 4.1.4 ($c_b = 0.06c_t$) and linear flow characteristics gives reasonable results, as illustrated in Fig. 4.17. The noise power level in this simulation is equal to 1\% of the nominal plenum pressure rise $\psi_0$. This noise level is larger than the RMS-value mentioned in (van de Wal and Willems, 1996, Chapter 3): 220 [Pa]. As expected, the control input $u$ does not converge to zero because of the measurement noise on $\psi$. This problem can be circumvented by filtering the measurements before they enter the controller. However, the filter introduces a phase lag which can be disadvantageous for the compression system's stability.
4.4 Discussion

The performance of the surge control system proposed in Section 3.2 is examined for the control of surge limit cycle oscillations. This case is relevant since it is assumed that the surge control system will not be operated permanently in practice. As the limit cycle is seen to be stable, all initial perturbations and disturbances which drive the operating point outside the domain of attraction of a stable equilibrium will result in the same limit cycle oscillation in the uncontrolled situation. Consequently, the system can also be stabilized in those cases.

Simulations show that for \( N = 25,000 \) [rpm] surge can be stabilized by the bounded static output feedback controller down to \( \Phi_{co} = 1.7F \); this corresponds with a decrease in surge point mass flow of approximately 15% compared to the uncontrolled situation. In that case, a control valve with a capacity and bandwidth of \( K_p = 10.7 \) [m³/hr] (i.e., \( c_b \approx 0.06c_t \)), and \( f_{co} = 60 \) [Hz], respectively, is required for the examined compression system. Furthermore, from the results in Section 4.2, it is concluded that only a linear flow curve gives acceptable results for active surge control. This can be realized by adding an inverse model of the nonlinear flow curve to the controller.

To account for the effect of valve saturation, an advanced control valve model is applied in the simulations, see Appendix A. In (Willems and de Jager, 1998a), a simplified valve model is used which constrains the control valve position after integration and, in case of saturation, the valve stem velocity is not set to zero. Remarkably, the qualitative results of both model are approximately similar; for \( N = 25,000 \) [rpm], \( \Phi_{co} = 1.7F \), and \( c_b = 0.1c_t \), both systems are unstable for \( f_{co} = 50 \) [Hz] and \( K = -19.9 \) and are stabilized in the same operating point for \( K = -35 \). Additional simulations show that the simplified model is unstable for \( f_{co} = 60 \) [Hz] and various \( K \)-values whereas the advanced model is stabilized. Therefore, the nonzero valve velocity during saturation in the simplified model is expected to be harmful for system stability.
Simulation results

The extent of the stable operating region of the controlled compression system depends on the slope $m_c$ of the compressor characteristic and the Greitzer stability parameter $\beta$, see Section 3.3. Consequently, the stable region is expected to be enlarged, e.g., by decreasing the plenum volume $V_p$, as reported in (Fink et al., 1992). In the studied compression system, the compressor characteristic is not accurately known for $\Phi_c < 2F$ (Meuleman et al., 1998). Therefore, the results of this study have to be validated on the experimental set-up; as long as the actual compressor characteristic is not steeper than the approximated characteristic, the compression system is expected to be stabilized in the equilibrium point $(\Phi_{c0}, \psi_0)$. 
Chapter 5

Conclusions and Future work

5.1 Conclusions

5.1.1 Control valve sizing

In this study, the sizing of a control valve is examined which is proposed for surge control in an experimental set-up. More specifically, the required capacity and bandwidth of the control valve have to be specified. To accomplish this, a simulation model of the examined compression system is presented which accounts for the control valve dynamics and the effects of valve saturation. To stabilize surge, a linear static output feedback controller is applied. From simulations, it is concluded that the surge point mass flow can be reduced by 15% in the examined compression system if the control valve meets the following specifications:

- **Capacity**: \( K_v = 10.7 \text{ [m}^3\text{/hr]} \) (for \( \Delta p = 0.54 \text{ [bar]} \) and \( \dot{m}_b = 6.2 \cdot 10^{-2} \text{ [kg/s]} \))
- **Bandwidth**: 60 [Hz].

Additional valve specifications are listed in Table 2.3. It is seen from simulations that nonlinear flow curves of the control valve are disadvantageous for surge control.

5.1.2 One-sided control

Surge is stabilized using a bounded linear output feedback controller. Because of the applied one-sided control strategy, the nominal operating point can finally be reached with zero control valve mass flow. As a result, the overall efficiency of the compression system will be improved compared to studies which use a close-coupled valve or recycle valve, see e.g., (Botros *et al.*, 1991; Gravdahl and Egeland, 1997; Simon and Valavani, 1991).

The stability analysis of the one-sided controlled nonlinear compression system is based on the behavior of the *unconstrained* linearized system with control valve dynamics. In addition, the stability of the *constrained* linearized system is analyzed. However, theory is restricted to second order systems, so control

\[1\] It is noted that most control valve suppliers work with volume flow rates instead of mass flow rates.
valve dynamics can not be incorporated in the linearized model. Neglecting the control valve dynamics and dropping the requirement of static output feedback, i.e., allowing state-feedback or (observer-based) dynamic output feedback, facilitates a theoretical analysis of the stability properties of the nonlinear system using one-sided control, see, e.g., (Lefeber and Nijmeijer, 1997; Chen and Kuo, 1997). Moreover, more advanced control strategies such as sliding mode control or $H_\infty$ control are expected to increase the domain of attraction and to improve the performance of the controlled compression system.

5.2 Future work

- **Model validation** To determine if the proposed surge control system stabilizes surge in the experimental set-up, a suitable control valve has to be chosen and the control system has to be implemented. Then, the simulation model and the one-sided control strategy can be validated and compressor characteristics can be determined for compressor mass flows down to $\Phi_c = 1.7F$. Furthermore, the influence of hysteresis in the control valve on system behavior can be studied. So far, this effect is supposed to be negligible.

An important assumption in the applied control strategy is that the desired nominal operating point is known. However, in general this is not the case; if e.g. the pressure requirements of downstream processes are unknown, the nominal operating point (for zero control valve flow) can not be determined since the throttle characteristic is not available. Consequently, techniques have to developed to overcome this problem.

- **IO-selection** An experimental validation of different actuator and sensor sets is possible because, in addition to the proposed plenum pressure sensor, a compressor inlet pressure sensor and a hot wire positioned in the inlet can be used in combination with the control valve for surge control. Moreover, some of the assumptions and results in (van de Wal and Willems, 1996; van de Wal et al., 1997) can be experimentally verified.


Bibliography


The control valve model, see Fig. A.1, applied in the simulations is taken from the example Electrohydraulic Servomechanism of the MATLAB/SIMULINK Demos.

To realize the constraints on the dimensionless valve position:

\[
\begin{align*}
\delta u(i) = 0 & \quad \delta u(i - 1) = 0 \land \frac{d^2(\delta u(i-1))}{dt^2} \leq 0 \\
0 < \delta u(i) \leq 1 & \quad \delta u(i - 1) = 0 \land \frac{d^2(\delta u(i-1))}{dt^2} > 0 \\
0 \leq \delta u(i) \leq 1 & \quad \text{if} \quad 0 < \delta u(i - 1) < 1 \land \forall \frac{d^2(\delta u(i-1))}{dt^2} \\
0 \leq \delta u(i) < 1 & \quad \delta u(i - 1) = 1 \land \frac{d^2(\delta u(i-1))}{dt^2} < 0 \\
\delta u(i) = 1 & \quad \delta u(i - 1) = 1 \land \frac{d^2(\delta u(i-1))}{dt^2} \geq 0
\end{align*}
\]  

Equation (A.1)

a special double integrator block is used, as shown in Fig. A.2. In this block, the valve stem velocity is reset to zero by the "feedback" around the integrators if $\delta u$ is constrained. Furthermore, in case $\delta u$ is limited and the net force works in a specific direction, i.e. the first or last case in (A.1), the switch is flipped to zero, so the valve stem acceleration is set to zero.
Advanced valve model
Figure A.2: Double integrator block.