Adaptive Control of a CFT
Master-Slave Robot System

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Traineeship report

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Summary

In a lot of manufacturing processes the tasks that have to be performed are so complex that a single robot cannot execute them. In such cases synchronization can be used to accomplish such a task. In this report Master-Slave synchronization of two identical pick and place robots developed by Philips Centre for Manufacturing Technology (so-called CFT-transposers) is discussed.

The objective of this report is to achieve Master-Slave synchronization, using an adaptive controller for the friction parameters of the Slave, which appear linear in the friction model. The non-linear friction parameters are estimated using a Kalman filter and are fixed. To accomplish this goal, a Model Reference Adaptive Controller (MRAC) is used to control the Slave. The derived adaptation law appears not robust for measurement noise. To make the adaptation law robust for measurement noise, the regressor has to be based on a noise-free velocity signal.

Before passing on to Master-Slave synchronization, adaptive control of single CFT-transposer is considered. To make the MRAC robust for measurement noise the regressor is based on the velocity of the reference trajectory. From simulations and experiments it can be concluded that the Slave can be controlled with the robust MRAC. It appeared that the position error of the Slave using a MRAC can be reduced compared to a model based controller with estimations for the friction parameters obtained in earlier research with a Kalman filter. However, because of the choice of the friction model, it takes a long time before the friction parameters have converged.

To achieve real Master-Slave synchronization, the controller of the Slave has to depend only on the information obtained from the Master. However, since the velocity of the Master is not a noise-free signal, also the velocity signal of the reference trajectory is used, to base the regressor on. It can be concluded using this reference signal, Master-Slave synchronization is achieved. The synchronization error using the adaptive controller, is smaller compared with a model based controller, using estimations of the friction parameters with a Kalman filter.
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Chapter 1

Introduction

1.1 Scope of the assignment

Flexibility and manoeuvrability of systems that have to perform a certain task in a production process, are very important topics in modern manufacturing. In some cases, the tasks which have to be performed are so complex, that they cannot be executed by a single system. In those cases, a multi-composed system can be used. A multi-composed system is a group of individual systems that work together to execute a certain task.

In this report Master-Slave synchronization of two identical CFT-transposers will be discussed. The CFT-transposer is developed by Philips Centre for Manufacturing Technology (CFT). The CFT-transposer is a cartesian pick and place robot of four degrees of freedom. The robot is actuated by four servomotors. The position of the four degrees of freedom can be measured with incremental encoders. At the end of the outer link a tool is mounted, which is constructed in such a way that it remains horizontal all the time.

Synchronization of robots with only position measurements was analytically investigated in a 4 years Ph.D. program carried out at the Eindhoven University of Technology, [4]. The pair of CFT transposers was used to experimentally validate the synchronization controllers. The synchronization controller that was used to achieve this goal was a model based controller. For this controller a model of the CFT transposer is derived, and the model parameters, such as the friction coefficients, are estimated using a Kalman filter and least squares techniques. Because the values of the friction parameters depend on external influences, e.g. temperature, it is likely that for the specific multi-robot system performance can be improved using an adaptive controller for the friction parameters.

The objective of this study is to achieve Master-Slave synchronization using an adaptive controller for the friction parameters of the Slave. To accomplish this goal, first the principle of Master-Slave synchronization has to be explained and the synchronization
error has to be defined. Further, an adaptive controller for the friction parameters of
the Slave has to be derived. Before passing on to Master-Slave synchronization, the
controller will be tested on a single CFT-transposer.

1.2 Outline

The topic of the first part of chapter 2 is synchronization. The principle of Master-Slave
synchronization is explained and an error index is given, which is a measure for the per-
formance of the Master-Slave system. In the second part of chapter 2 the experimental
setup is discussed. The keynote of Chapter 3 is adaptive control. In this chapter first
the principle of adaptive control is explained, after which the Model Reference Adap-
tive Controller (MRAC) is introduced and its components are derived. In chapter 4
the adaptive controller of chapter 3 is used control the Slave. Before experiments are
executed, a simulation model is made to test the adaptive controller. The results from
the simulation model are compared with the results obtained from the experiment. In
chapter 5 Master-Slave synchronization will be discussed, whereby the Slave is con-
trolled with the MRAC. Finally, some conclusions are drawn and recommendations are
given in chapter 6.
Chapter 2

CFT Master-Slave Robot system

2.1 Synchronization

Synchronization can be defined as the mutual time conformity of two or more processes. Depending on the type of interconnections in the system, several kinds of synchronization can be defined. The first type of synchronization is natural synchronization. In this case the systems that present synchronous behaviour are disconnected. The second type of synchronization is called self-synchronization. This type of synchronization occurs when synchronization is achieved by proper interconnections in the system. The last type of synchronization is controlled-synchronization.

Within the group of controlled-synchronized systems, a difference can be made between internal synchronization and external synchronization. In the first case, the synchronous motion occurs as a result of interaction of elements of the system. All synchronized objects are treated equal in the unified multi-composed system. In external synchronization, there is one object in the multi-composed system, which is more powerful than the others in the system, and its motion can be considered as independent of the motion of the others. The dominant system is also called the Master system, the follower is then called the Slave system. The synchronous motion is predetermined by the dominant Master system.

In this report, an adaptive controller is designed to achieve Master-Slave synchronization. This means that we are dealing with external synchronization of a controlled-synchronized system. The experimental setup consists of two CFT-transposers. One robot will be treated as the Master, the other CFT-transposers as the Slave.

In Master-Slave synchronization the goal is to design a control law for the Slave robot, such that the motion of the Slave synchronizes with the motion of the Master. In other words, the goal is to design a control law to minimize the synchronization error and its derivative.
The synchronization error and its derivative are defined by

\[ e_q = q_s - q_m \quad (2.1) \]
\[ \dot{e}_q = \dot{q}_s - \dot{q}_m \quad (2.2) \]

Subscripts \( s \) and \( m \) denote Slave and Master respectively.

In order to express the performance of the Master-Slave system, an error index is defined. Since only the position of the Master and the Slave can be measured, the error index is chosen as a function of the synchronization error (2.1).

\[
\text{error index} = \frac{1}{t_2 - t_1} \cdot \int_{t_2}^{t_1} e_q^2(r) \, dr \quad (2.3)
\]

In (2.3) \( t_1 \) and \( t_2 \) represent the time interval in which the error index is determined. The error index is inversely proportional to the performance of the Master-Slave system.

### 2.2 Experimental setup

As mentioned in the introduction, a multi-robot system is considered, in order to experimentally validate the adaptive synchronization controller. This system consists of two robots. The robots in the experimental setup are industrial transposer robots designed by the Centre for Manufacturing Technology (CFT) Philips laboratory. Both robots are identical in structure and design. However, the physical parameters are different for each robot. For implementation of the controllers and communication with the robots, the experimental setup is equipped with a DS1005 dSPACE system. Throughout the experiments the sampling frequency of the DS1005 dSPACE system was set to 2 kHz. The experimental setup is shown in Figure 2.1.
Chapter 2. CFT Master-Slave Robot system

The CFT robot is an industrial pick and place robot, which is used for assembling. The CFT transposer robot is a Cartesian basic elbow configuration robot. It consists of a two links arm that is placed on a rotating and translational base. The tool connected at the end of the outer link is a kinematically constrained planar support. The tool is passively actuated and designed to remain horizontal at all time.

The kinematics of the CFT transposer can be written in Cartesian space and in joint space. The CFT transposer has 4 degrees of freedom. In Cartesian space these degrees of freedom are denoted by $x_{c,i}(i = 1, \ldots, 4)$. The 4 Cartesian degrees of freedom are up and down, forward and backward of the arm, rotation and translation of the base, on which the two links arm is mounted. In joint space there are 7 degrees of freedom, denoted by $q_j(j = 1, \ldots, 7)$. Although the robot has 7 degrees of freedom in the joint space, 3 of them are kinematically constrained. The set of constrained joints is given by $(q_3, q_6, q_7)$. Therefore the robot can be represented in the joint space by 4 degrees of freedom $(q_1, q_2, q_4, q_5)$. The degrees of freedom are shown in Figure 2.2.

The CFT transposer is actuated by 4 DC brushless servomotors. Although the shaft of the motors and the corresponding links are connected by means of belts, the servomotor-link pair proved to be stiff enough to be considered as a rigid joint. The robot is equipped with encoders attached to the shaft of the motors with a resolution of 2000 PPR, which results in an accuracy of ±0.5 [mm] in all motion directions. The velocity of the robot will be obtained from numerical differentiation of the position.

2.2.1 CFT-transposer

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Chapter 3

Adaptive Control

An adaptive controller differs from an ordinary controller in that the controller parameters are time-varying and there is a mechanism for adjusting these parameters. The basic idea in adaptive control is to estimate uncertain plant parameters online, based on the measured system signals and use the estimated parameters in the controller computation. For adaptive control the unknown parameters have to be constant or must vary considerably slower than the parameter adaptation. The goal of an adaptive controller is to achieve proper tracking or set-point control behaviour.

There are two main approaches for constructing adaptive controllers. These types are the model-reference adaptive control method and the self-tuning method. In this report, only the model-reference adaptive control method will be treated.

Generally, the model reference control system is composed of four parts.

- The plant (system to be controlled)
- A reference model
- A controller
- An adaptation mechanism for parameters

3.1 Mathematical model of the CFT transposer

A mathematical model for the CFT transposer is needed for the adaptive control design. A full mathematical description of the robot includes the kinematic and dynamic model and a set of physical parameters of the robot. The model is derived in [3]. Since the derivation of the model is beyond the scope of this study, only a brief summary of the results is presented below. The model is also given in appendix A.
3.1.1 Kinematics

The kinematics of the CFT transposer are defined in Cartesian and in joint space. From [2] it follows that in Master-Slave synchronization the performance of the controller in joint space is better than in Cartesian space. Therefore, the model in joint space will be used to design the adaptive controller. To obtain the kinematics of the CFT transposer in joint space, Denavit-Hartenberg parameters are used.

The coordinate frames are assigned as in Figure 3.1, the corresponding set of Denavit-Hartenberg parameters is listed in Table 3.1. The variables $L_i$ represent the length of link $i$, $d_i$ is the offset of each link along the $z_i$-axis. The values for these parameters are given in appendix A. The joint variables $q_1$ and $q_3$ are the translations along $z_1$ and $z_3$ respectively, the joints $q_i$ ($i=2,4,5,6,7$) are the rotations about the $z_i$-axis. As mentioned in the introduction, $q_3$, $q_6$ and $q_7$ are kinematically constrained and can be expressed as function of the other joints.

\[
\begin{align*}
q_3 &= L_4(\cos(-q_4 - q_5 + \frac{\pi}{2}) + \cos(-q_4 + \frac{\pi}{2})) + d_{2,g}\quad (3.1) \\
q_6 &= -q_5 \\
q_7 &= \pi - q_4
\end{align*}
\]
Chapter 3. Adaptive Control

Table 3.1: Denavit-Hartenberg parameters for the CFT-robot

<table>
<thead>
<tr>
<th>i</th>
<th>$a_i$</th>
<th>$\alpha_i$</th>
<th>$d_i$</th>
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</tr>
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<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>$-\frac{\pi}{2}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>$\frac{\pi}{2}$</td>
<td>$q_1$</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>$\frac{\pi}{2}$</td>
<td>$L_2$</td>
<td>$q_2$</td>
</tr>
<tr>
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<td>0</td>
<td>$\frac{\pi}{2}$</td>
<td>$g_3$</td>
<td>$-\frac{\pi}{2}$</td>
</tr>
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</tr>
<tr>
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<td>$L_8$</td>
<td>0</td>
<td>0</td>
<td>$\frac{\pi}{2}$</td>
</tr>
<tr>
<td>9</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

3.1.2 Dynamics

The dynamics of the CFT-transposer are derived from the Euler-Lagrange approach and the Denavit-Hartenberg parameters. The dynamics are given by:

$$M(q_i)\ddot{q}_i + C(q_i, \dot{q}_i)\dot{q}_i + G(q_i) + F(q_i) = \tau_i$$  \hspace{1cm} (3.2)

with $i = m, s$

In (3.2), $q_i \in \mathbb{R}^n$ is the vector of joint variables, with $n$ the number of joints. $M(q_i) \in \mathbb{R}^{n \times n}$ is the inertia matrix. The inertia matrix is symmetric and positive definite. $C(q_i, \dot{q}_i) \in \mathbb{R}^n$ represents a matrix, which accounts for the Coriolis and centrifugal torques and $G(q) \in \mathbb{R}^n$ denotes the conservative torques due to gravity. $\tau_i \in \mathbb{R}^n$ is a vector of input torques. The vector $F(q_i) \in \mathbb{R}^n$ represents the friction forces.

3.1.3 Friction Forces

The friction forces $F(q_i)$ generally consist of forces due to static friction and forces due to dynamic friction. In this report only static friction forces are considered, because the use of dynamic friction models is not justified for medium and high velocities. Furthermore, it is assumed that the friction forces are uncoupled among the joints, because friction is a local effect. To tackle the problem of the discontinuity that the Coulomb friction represents at zero velocity, the model used is an approximation based on exponential functions. The friction model is given by:

$$F(q_i) = B_v \dot{q}_i + B_{f1,i} \left(1 - \frac{2}{1 + e^{2w_1,\dot{q}_i}}\right) + B_{f2,i} \left(1 - \frac{2}{1 + e^{2w_2,\dot{q}_i}}\right)$$  \hspace{1cm} (3.3)

In (3.3) $B_v$ represents the diagonal viscous friction coefficient matrix and the other terms approximate the Coulomb and Stribeck friction effects.
Chapter 3. Adaptive Control

In this report, an adaptive controller is designed to adapt the friction parameters. However, since a linear parameterization of the friction forces is needed in the adaptation law, and the parameters $w_1$ and $w_2$ are found in the argument of the exponential function, these parameters will not be adapted. The values of the non-linear friction parameters $w_1$ and $w_2$ and the inertia parameters are obtained from [3], in which these parameters are estimated using Kalman filters and least squares techniques.

3.2 Reference model

In adaptive control a reference model is used to specify the desired response of the adaptive control system. In this report the objective is to achieve Master-Slave synchronization. Therefore, the synchronization error (2.1) and its derivative (2.2) have to be minimized. This means that the Master can be seen as reference model. The trajectory, described by the actual position of the Master robot, represents the desired trajectory for the Slave robot.

3.3 The controller

The controller should have perfect tracking capacity in order to allow the possibility of tracking convergence. This means that $e_q \to 0$ for $t \to \infty$ and $\dot{e}_q \to 0$ for $t \to \infty$. The position error is defined as the difference between the real trajectory and the desired trajectory. The position error and the time derivative of the position error are given as:

$$e_q = q - q_d$$

$$\dot{e}_q = \dot{q} - \dot{q}_d$$

with $q$, the position of the system and $q_d$ the desired position. Substitution of (3.4) and (3.5) in (3.2) results in the error dynamics.

$$M(q)\ddot{e}_q + M(q)\dot{q}_d + C(q, \dot{q})\dot{q} + G(q) + F(q, \Theta) = \tau$$

The control law has to be chosen in such a way that $e_q(t) \equiv 0$ is a globally asymptotically stable equilibrium point of the closed loop error equation. The choice of the control law can be based on the 2nd method of Lyapunov. A candidate Lyapunov function is:

$$V = \frac{1}{2} \dot{e}_q^T M \dot{e}_q + \frac{1}{2} e_q^T K_p e_q$$

In (3.7) $K_p$ is a constant, symmetric and positive definite matrix. The time derivative of the Lyapunov function (3.7) is given by:

$$V = \dot{e}_q^T \left( M \ddot{e}_q + \frac{1}{2} \dot{M} \dot{e}_q + K_p e_q \right)$$

(3.8)
Using the error equation (3.6) and the property that \((\dot{M} - 2C)\) is skew symmetric, \(\dot{V}\) becomes
\[
\dot{V} = \dot{e}_q^T \left( \tau - M(q)\ddot{q} - C(q, q)\dot{q} - G(q) - F(q, \Theta) + K_p e_q \right)
\] (3.9)

The expression for the control input \(\tau\) follows from the goal that \(\dot{V} \leq 0\) for all \(q, \dot{q}\).
\[
\tau = M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + F(q, \Theta) - K_p e_q + \nu
\] (3.10)

with:
\[
\nu = -K_v \dot{e}_q
\] (3.11)

With this choice of \(\nu\), a derivative action is added to the control law. This means that a computed torque control law is obtained. With this computed torque law the Lyapunov function and its time derivative become
\[
V = \frac{1}{2} \dot{e}_q^T M \dot{e}_q + \frac{1}{2} \dot{e}_q^T K_p e_q
\] (3.12)
\[
\dot{V} = -\dot{e}_q^T K_v \dot{e}_q
\] (3.13)

from which can be concluded that \(\dot{e}_q(t) \to 0\) for \(t \to \infty\). However, there is no guarantee yet that \(e_q \to 0\) for \(t \to \infty\). If the friction parameters are fully known, the closed loop error dynamics, with substitution of the control law, become:
\[
M(q)\ddot{e}_q + C(q, \dot{q})\dot{e}_q + K_v \dot{e}_q + K_p e_q = 0
\] (3.14)

From (3.14) it follows that \(e_q(t) \to 0\) for \(t \to \infty\), because \(\dot{e}_q(t) \to 0\) for \(t \to \infty\) and \(K_p\) is regular. The conclusion can be drawn that the choice of the computed torque control law and the assumption that all model parameters are known, leads to a globally asymptotically stable equilibrium point \(e_q(t) = 0\).

### 3.4 Adaptation law

When the model parameters are unknown, the computed torque control law cannot be used. Therefore, it is necessary to use a suitable adaptation to estimate the model parameters. In this report, only the linear friction parameters will be adapted. Substituting the control law based on adapted parameters, \(\hat{\Theta}\), the closed loop error dynamics become:
\[
M(q)\ddot{e}_q + C(q, \dot{q})\dot{e}_q + K_v \dot{e}_q + K_p e_q = \hat{F}(\dot{q}, \hat{\Theta}) - F(q, \Theta)
\] (3.15)

Because the unknown parameters appear linearly in (3.15), the right hand side can be written as \(W \theta e_\theta\) with:
\[
e_\theta = \hat{\Theta} - \Theta
\] (3.16)
Chapter 3. Adaptive Control

The goal is to adapt parameters $\hat{\Theta}$ so that $e_q(t) \rightarrow 0$ for $t \rightarrow \infty$. The adaptation mechanism can be based on the 2nd method of Lyapunov. A candidate Lyapunov function is given by:

$$V(t) = \frac{1}{2} e_q^T M e_q + \frac{1}{2} e_\theta^T K_p e_\theta + \frac{1}{2} e_\phi^T \Gamma e_\phi$$

with $\Gamma = \Gamma^T$ and $\Gamma > 0$.

The time derivative of $V$ is:

$$\dot{V} = -\dot{e}_q^T K_p \dot{e}_q + e_\phi (\dot{e}_\phi + W_d^T \dot{e}_q)$$

The first part of (3.19) is negative definite. The second part can be forced to 0 with parameter adaptation law:

$$\dot{e}_\phi = -\Gamma^{-1} W_d^T \dot{e}_q$$

This means that the time derivative of the Lyapunov function is a negative definite function. So, $\dot{e}_q(t) \rightarrow 0$ for $t \rightarrow \infty$. According to (3.20) this implies that $\dot{e}_\phi \rightarrow 0$. Because $\dot{e}_q(t)$ and $\dot{e}_\phi$ are uniform continuous functions on $[0, \infty]$ and their limits for $t \rightarrow \infty$ go to 0, this implies that

$$\lim_{t \rightarrow \infty} \int_0^t \dot{e}_q(\tau) d\tau$$

exist and are finite. In other words, $e_q(t)$ and $e_\phi(t)$ become constant if $t \rightarrow \infty$. From the error equation (3.15) it appears that in the limit $t \rightarrow \infty$ the tracking error and the parameter error are coupled.

$$K_p e_q \rightarrow W_d e_\phi, \text{ for } t \rightarrow \infty$$

Thus, for $t \rightarrow \infty$, both $W_d e_\phi$ and $e_\phi$ are constant. From this it follows that $W_d e_\phi \rightarrow 0$ and therefore, if the time derivative of $W_d$ is persistently exciting, $e_\phi \rightarrow 0$. 

$$W_d(\dot{q}) = \left( \begin{array}{cccccc} \dot{q}_1 & 0 & 0 & 0 & 0 \\ 0 & \dot{q}_2 & 0 & 0 & 0 \\ 0 & 0 & \dot{q}_4 & 0 & 0 \\ 0 & 0 & 0 & \dot{q}_5 & 0 \\ (1 - \frac{2}{1 + e^{2\pi^2 q_1^2}}) & 0 & 0 & 0 & 0 \\ 0 & (1 - \frac{2}{1 + e^{2\pi^2 q_2^2}}) & 0 & 0 & 0 \\ 0 & 0 & (1 - \frac{2}{1 + e^{2\pi^2 q_4^2}}) & 0 & 0 \\ 0 & 0 & 0 & (1 - \frac{2}{1 + e^{2\pi^2 q_5^2}}) & 0 \\ 0 & 0 & 0 & 0 & (1 - \frac{2}{1 + e^{2\pi^2 q_5^2}}) \end{array} \right) \tag{3.17}$$
3.5 Measurement noise

From Section 3.4 it appears that the adaptation of the first four friction parameters, consists of multiplication of the velocity with the derivative of the position error. This means that the velocity appears quadratic in the adaptation law. Since the velocity will be obtained from numerical differentiation of the position measurements that contain measurement noise, the velocity will also contain measurement noise. The squared measurement noise will disrupt the parameter adaptation if the relevant signals are not persistently exciting enough. So, it can be concluded that the derived controller has to be modified in such a way that it becomes robust for measurement noise.

A controller that avoids parameter drift is presented in [1]. To avoid the disruptive effect, the term \( \dot{q}_d^2 \), which appears in the adaptation law for the viscous friction parameters, has to be removed. This is possible by using the velocity of the reference signal, \( \dot{q}_d(t) \), in the regressor (3.17) instead of the velocity of the CFT-transposer. Besides that, the derivative of the position error in the adaptation law, has to be replaced by

\[
s = \dot{e}_q + \Lambda e_q \tag{3.22}
\]

The advantage of taking \( s \) (3.22) instead of \( \dot{e}_q \) (3.5), is that the adaptation is also based on the position error. This results in a more consistent estimation of the friction parameters.

With these measures the control law and the adaptation law become:

\[
\tau = M(q)\ddot{q}_d + C(q, \dot{q}_d)\dot{q}_d + G(q) + F(\dot{q}_d, \dot{\Theta}) - K_p e_q - K_v \dot{e}_q \tag{3.23}
\]

\[
\dot{\Theta} = -\Gamma^{-1}W_d^T(q_d)s \tag{3.24}
\]

Because the regressor is now based on the reference signal instead of the velocity of the robot, actually a new stability analysis is needed. However, because this is very difficult and assumptions with respect to measurement noise are needed, this is not discussed in this report. Another consequence of basing the regressor on the velocity of the reference trajectory instead of the velocity of the robot, is that the adaptation of the friction parameters is not properly related to the position error. If measurement noise is not taken into consideration, this results in a slower parameter convergence of the robust controller compared to the parameter convergence of the original controller.
Chapter 4

Adaptive control of the Slave

Before passing on to Master-Slave synchronization, the controller as derived in Chapter 3, will be used to control a single CFT-transposer. In this case, the Slave will be considered. Before the controller will be used to control the Slave, a simulation model in MATLAB/SIMULINK is made to test the controller. The resulting friction parameters obtained from adaptive control of the Slave are used to compare the simulation with the experiments.

4.1 Simulation

As stated in chapter 3, the model reference control system consists of a reference model, a plant, a control law and a adaptation law. The structure of these parts is implemented in a SIMULINK model as depicted in Figure 4.1. However, the parameters of the parts of the control system still have to be specified.

4.1.1 Settings

Reference model
For Master-Slave synchronization, the actual position of the Master is the reference position for the Slave. Since now the Slave is considered individually, a trajectory \( q_d(t) \) has to be prescribed for the Slave to follow. To overcome the problem of an unacceptable control input when \( q_d(t) \) is not differentiable, a new trajectory \( q_m(t) \) can be described using four second-order systems.

\[
m(\ddot{q} + 2\zeta \omega_n \dot{q} + \omega_n^2 q) = r(t)
\]

By choosing the signal \( r(t) = \omega_n^2 q_d(t) \), it can be accomplished that \( q_m(t) \rightarrow q_d(t) \). With this transformation the tracking error becomes \( e_q(t) = q(t) - q_m(t) \) instead of \( e_q(t) = q(t) - q_d(t) \). In (4.1), the mass \( m \) is set to 1 kg, the damping ratio \( \zeta \) is 0.7 Ns/m
Chapter 4. Adaptive control of the Slave

The desired trajectory, \( q_d(t) \), used in this report is a sine wave with a frequency of 0.085 rad/s for \( q_1 \) and 0.3 rad/s for \( q_2, q_4 \) and \( q_5 \), and amplitude of 0.2, 0.5, 0.12 and 0.25 for \( q_1, q_2, q_4 \) and \( q_5 \) respectively. These sine waves are added to the initial positions of each degree of freedom robot, which are \(-0.08, 0, -1.04 \) and \( 2.12 \). Because of the use of the reference model, the initial positions of the new trajectory, \( q_m(t) \), are equal to the initial positions of the desired trajectory, \( q_d(t) \), but the initial velocities of \( q_m(t) \) are zero such that an unacceptable control input is avoided.

Mathematical model

The difference between the simulation and the actual experiment is that a mathematical model of the CFT-transposer will replace the Slave. The structure of the mathematical model is given by:

\[
\ddot{q} = M(q)^{-1}\tau - M(q)^{-1}C(q, \dot{q})\dot{q} - M(q)^{-1}G(q) - M(q)^{-1}F(\dot{q}, \Theta) \tag{4.2}
\]

From (4.2) it appears that the values of the friction parameters in the mathematical model have to be specified. It was chosen to take the same values as obtained in [3] using a Kalman filter. These values are also given in appendix B.

Control law

For the control law, the proportional gain matrix \( K_p \) and the derivative gain matrix \( K_v \) have to be specified. \( K_p \) and \( K_v \) are diagonal matrices. The diagonal elements are also obtained from [3] and are given in Table 4.1.
Chapter 4. Adaptive control of the Slave

Table 4.1: Control parameters

<table>
<thead>
<tr>
<th>DOF</th>
<th>$K_p$</th>
<th>$K_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_1$</td>
<td>15000</td>
<td>20</td>
</tr>
<tr>
<td>$q_2$</td>
<td>2000</td>
<td>50</td>
</tr>
<tr>
<td>$q_4$</td>
<td>12000</td>
<td>50</td>
</tr>
<tr>
<td>$q_5$</td>
<td>12000</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 4.2: Adaptation gains simulation

<table>
<thead>
<tr>
<th>Element</th>
<th>Value</th>
<th>Element</th>
<th>Value</th>
<th>Element</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_{1,1}$</td>
<td>$2 \times 10^{-4}$</td>
<td>$\Gamma_{5,5}$</td>
<td>$2 \times 10^{-2}$</td>
<td>$\Gamma_{9,9}$</td>
<td>$2 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\Gamma_{2,2}$</td>
<td>$4 \times 10^{-2}$</td>
<td>$\Gamma_{6,6}$</td>
<td>$1 \times 10^{-2}$</td>
<td>$\Gamma_{10,10}$</td>
<td>$1 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\Gamma_{3,3}$</td>
<td>$1 \times 10^{-3}$</td>
<td>$\Gamma_{7,7}$</td>
<td>$2 \times 10^{-2}$</td>
<td>$\Gamma_{11,11}$</td>
<td>$2 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\Gamma_{4,4}$</td>
<td>$1 \times 10^{-3}$</td>
<td>$\Gamma_{8,8}$</td>
<td>$2 \times 10^{-2}$</td>
<td>$\Gamma_{12,12}$</td>
<td>$2 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

Adaptation law

Furthermore, the initial choices for the friction parameters in the adaptation mechanism and the $\Gamma$-matrix have to be specified. It was chosen to take the initial values of the friction parameters 2 times the value of the friction parameters of the mathematical model. The $\Gamma$-matrix is a diagonal matrix. The diagonal elements are given in Table 4.2. It must be remarked that the adaptation gains are obtained by trial and error, in order to get a fast convergence of the friction parameters. The parameter $\Lambda$ in (3.22) determines the weight of the position error and velocity error on which the adaptation is based. The value of this parameter is set to 2.

4.1.2 Simulation results

The simulation is carried out using the controller parameters of section 4.1.1. The results of this simulation are depicted in Figure 4.2 and Figure 4.3. Figure (4.2) shows the values of the estimated friction parameters as function of time. During the first 100 seconds, the adaptation is turned off. By this, the error and error index belonging to the initial value of the friction parameters can be determined. It appears that the friction parameters, have not converged after 2500 seconds. Although, if we look carefully at these parameters, it can be seen that they tend to go to parameters of the mathematical model, although it takes a very long time before they will reach their adjusted value.

In Figure 4.3 the position error is given as a function of time. In this figure it can be seen that the position error of all joints decreases as function of time. It can be expected that the position error would go to 0, when the friction parameters are converged. Since
Chapter 4. Adaptive control of the Slave

Figure 4.2: Friction parameters simulation

Figure 4.3: Position error simulation
χτηπερχαπεε χεπεη πεεαπεηε ηηε επεηεηεηε εηηεηε εηε ηεηεηε ηε απεηεηεηε χεηεηεηε εηεηεηε ηε ηεηεηεηε χεηεηεηε.

The reason for this slow converging behaviour is that the modelling of Coulomb and Stribeck friction is based on exponential functions

\[ B_{f1,i} \left(1 - \frac{2}{1 + e^{2w_{1,i}q_i}}\right) + B_{f2,i} \left(1 - \frac{2}{1 + e^{2w_{2,i}q_i}}\right) \]

Since the exponential functions appear in the regressor (3.17) and behave as an approximation for a sign function, the adaptation for \( B_{f1,i} \) and \( B_{f2,i} \) will be of almost the same magnitude for high velocities. The signs of the non-linear friction parameters \( w_{1,i} \) and \( w_{2,i} \), as shown in appendix B, determine the direction of adaptation.

For joints \( q_1 \) and \( q_2 \), \( w_1 \) and \( w_2 \) are of opposite sign. This means that during the first part, the adaptation of \( B_{f1} \) and \( B_{f2} \) is also of opposite sign. For joints \( q_4 \) and \( q_5 \), \( w_1 \) and \( w_2 \) have the same sign and so, the adaptation also has the same sign. Initially this results in an adaptation such that the sum of the Coulomb and Stribeck friction forces is equal to the sum of the Coulomb and Stribeck friction of the mathematical model.

This behaviour of the non-viscous friction parameters can also be seen in figure 4.2. After 200 seconds, it can be seen that the non-viscous friction parameters of joint \( q_2 \) remain almost constant. The adaptation of the non-viscous friction parameters of joint \( q_1 \) have the same sign, and the adaptation of \( q_4 \) and \( q_5 \) have opposite signs. This means that the sum of the non-viscous friction is converged to the sum of the mathematical model. This can also be concluded from Figure 4.3. It can be seen that within the first 100 seconds of adaptation, the error is reduced significantly, especially for joint \( q_1 \) and \( q_2 \). After 200 seconds can be seen that the error reduction is much smaller as before.

From this it can be concluded that the adaptation mechanism works, since it shows the expected adaptation behaviour. However, the time after which the parameters are converged is very large.

### 4.2 Experiment

From Section 4.1 it appeared that the controller works, although it takes a very long time before the friction parameters have converged. The following step is to control the Slave. This can be done by taking the same structure as given in Figure 4.1, whereby the only difference is that the simulation model is replaced by the real system.

The reference model and the reference signal, used for the simulation, will again be used to describe the desired trajectory. The parameters of the control law, \( K_p \) and \( K_v \), and the adaptation law parameter, \( \Lambda \), are the same as used for the simulation. The adaptation gains are obtained by trial and error. The adaptation gains are given in Table 4.3. The initial conditions for the linear friction parameters are chosen equal to the values obtained from [3].
Table 4.3: Adaptation gains experiment

<table>
<thead>
<tr>
<th>Element</th>
<th>Value</th>
<th>Element</th>
<th>Value</th>
<th>Element</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_{1,1}$</td>
<td>$1 \cdot 10^{-4}$</td>
<td>$\Gamma_{5,5}$</td>
<td>$1 \cdot 10^{-3}$</td>
<td>$\Gamma_{9,9}$</td>
<td>$1 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>$\Gamma_{2,2}$</td>
<td>$1 \cdot 10^{-3}$</td>
<td>$\Gamma_{6,6}$</td>
<td>$1 \cdot 10^{-2}$</td>
<td>$\Gamma_{10,10}$</td>
<td>$1 \cdot 10^{-2}$</td>
</tr>
<tr>
<td>$\Gamma_{3,3}$</td>
<td>$1 \cdot 10^{-4}$</td>
<td>$\Gamma_{7,7}$</td>
<td>$1 \cdot 10^{-3}$</td>
<td>$\Gamma_{11,11}$</td>
<td>$1 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>$\Gamma_{4,4}$</td>
<td>$1 \cdot 10^{-4}$</td>
<td>$\Gamma_{8,8}$</td>
<td>$1 \cdot 10^{-3}$</td>
<td>$\Gamma_{12,12}$</td>
<td>$1 \cdot 10^{-3}$</td>
</tr>
</tbody>
</table>

Figure 4.4: Friction parameters experiment
Chapter 4. Adaptive control of the Slave

Figure 4.5: Position error Slave first 200 sec.

Figure 4.6: Final position error Slave
Chapter 4. Adaptive control of the Slave

Table 4.4: Final value friction parameters Slave

<table>
<thead>
<tr>
<th>Parameter</th>
<th>value</th>
<th>Parameter</th>
<th>value</th>
<th>Parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{v1}$</td>
<td>40</td>
<td>$B_{f1,1}$</td>
<td>0</td>
<td>$B_{f2,1}$</td>
<td>-30</td>
</tr>
<tr>
<td>$B_{v2}$</td>
<td>5</td>
<td>$B_{f1,2}$</td>
<td>20</td>
<td>$B_{f2,2}$</td>
<td>10</td>
</tr>
<tr>
<td>$B_{v4}$</td>
<td>-18</td>
<td>$B_{f1,4}$</td>
<td>1</td>
<td>$B_{f2,4}$</td>
<td>-6</td>
</tr>
<tr>
<td>$B_{v5}$</td>
<td>-27</td>
<td>$B_{f1,5}$</td>
<td>27</td>
<td>$B_{f2,5}$</td>
<td>-10</td>
</tr>
</tbody>
</table>

The results of the experiments are given in Figure 4.4, Figure 4.5 and Figure 4.6.

In Figure 4.4 the linear friction parameters are given as function of time. It can be seen that after 2000 seconds the mean value of the friction parameters doesn't change much anymore. However, this does not automatically mean that the friction parameters have reached their final value. Looking at joint $q_1$ it can be seen that the adaptation of the non-viscous friction parameters is in the same direction, in spite of the opposite sign of the non-linear friction parameters. Thus it can be concluded that the friction parameters of joint $q_1$ almost have reached their final values. Because of the adaptation in opposite direction in combination with non-linear friction parameters of the same sign, the same conclusion can be drawn for joint $q_4$ and $q_5$. The remaining adaptation has a periodical behaviour, but the mean value of the parameters stays the same. For joint $q_2$ it cannot be concluded that the non-linear friction parameters are converged, because the adaptation as well as the sign of these parameters have the same sign. However, the effective friction force generated by the sum of $B_{f1}$ and $B_{f2}$ does not change anymore. So, it only can be concluded that the viscous friction parameter and the combination of the other friction parameters is converged. The values of the friction parameters after 2000 seconds are given in Table 4.4.

Also it can be concluded that the remaining adaptation still has a large amplitude. Moreover, the viscous friction parameters of the 4th and 5th joint have a negative value. An explanation for this is that the chosen friction model, doesn’t represent the friction of the Slave well. This can be due to the fixed values of the non-linear friction parameters or due to the chosen structure of the friction model.

In Figure 4.5 and Figure 4.6 the position error of the Slave is given as function of time. In Figure 4.5 the position error during the first 200 seconds is shown. During approximately the first 100 seconds there is no adaptation, but when the adaptation is turned on, the amplitude of the position error becomes smaller. In Figure 4.6 the position error at the end of the experiment is shown. In chapter 2.1 an error index (2.3) is defined to have a measure for the performance of the Master-Slave system. Replacing the position of the Master with the position of the reference model, a similar error index for the Slave can be defined.

$$\text{errorindex} = \frac{1}{t_2 - t_1} \cdot \int_{t_2}^{t_1} e_q(\tau)^2 d\tau$$  \hspace{1cm} (4.3)
Chapter 4. Adaptive control of the Slave

Table 4.5: Error indices Slave

<table>
<thead>
<tr>
<th>DOF ( q )</th>
<th>Error index start ( e_{q}) ( \cdot 10^{-5} )</th>
<th>Final error index ( e_{q}) ( \cdot 10^{-6} )</th>
<th>Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_1 )</td>
<td>( 5 \cdot 10^{-5} )</td>
<td>( 8 \cdot 10^{-6} )</td>
<td>84 %</td>
</tr>
<tr>
<td>( q_2 )</td>
<td>( 5 \cdot 10^{-6} )</td>
<td>( 4 \cdot 10^{-6} )</td>
<td>20 %</td>
</tr>
<tr>
<td>( q_4 )</td>
<td>( 1.2 \cdot 10^{-5} )</td>
<td>( 1.2 \cdot 10^{-6} )</td>
<td>0 %</td>
</tr>
<tr>
<td>( q_5 )</td>
<td>( 5.7 \cdot 10^{-6} )</td>
<td>( 4.4 \cdot 10^{-6} )</td>
<td>21 %</td>
</tr>
</tbody>
</table>

with:

\[ e_q = q_s - q_{ref} \]  \hspace{1cm} (4.4)

The error index for the initial values of the friction parameters and the error index for the final values of the friction parameters are given in Table 4.5. From Table 4.5 it appears that no reduction of the error index for joint \( q_4 \) can be achieved. Nevertheless, from Figure 4.5 it appears that the amplitude of the position error is decreased. A maximum reduction of 84\% is achieved for joint \( q_1 \).

If we look careful at Figure 4.6 the position error shows a periodical behaviour, with the same frequency as the frequency of the reference trajectory. Besides, except for joint \( q_2 \), the error is not symmetrical around \( e_q = 0 \). A possible reason for this is that it is due to the chosen friction model, with fixed non-linear friction parameters, which doesn’t represent the friction well.

### 4.3 Comparison simulation and experiments

In order to compare the experiments with the simulation, a new simulation will be run. The final values of the friction parameters of the Slave, as given in Table 4.4, are used in the mathematical model of the simulation plant. The initial estimates for the friction parameters and the parameters of the controller are the same as the initial values used for the experiment.

The results obtained with these settings are given in Figure 4.3 and Figure 4.8. Comparing the friction parameters of the simulation, as depicted in Figure 4.3, with the friction parameters of the experiment, Figure 4.4, it can be concluded that for joint \( q_1 \) and \( q_2 \) the progress of the friction parameters is similar. For joint \( q_1 \), it can be concluded that the non-viscous friction parameters are not converged yet. If we look again at Figure 4.4, it seems that the non-viscous friction parameters also haven’t reached their final value. So, it is possible for joint \( q_1 \) that the position error can be diminished. For joint \( q_4 \) and \( q_5 \) the progress of the non-viscous friction parameters in the simulation, differs from the experiment. A possible reason for the difference is that the structure and the non-linear friction parameters of the friction model, are the same for the controller as
Chapter 4. Adaptive control of the Slave

Figure 4.7: Friction parameters 2\textsuperscript{nd} simulation model

Figure 4.8: Position error 2\textsuperscript{nd} simulation model
well as for the plant during the simulation. For the experiment, the friction model of
the controller can be different from the friction of the Slave.

Looking at the position error of the Slave, Figure 4.5 and Figure 4.6, and the position
error of the simulation model, Figure 4.8, it can be seen that for joints $q_1$ and $q_2$ the error
reduction for the simulation model is much faster and larger than the error reduction of
the Slave. This can be due to measurement noise, which is not taken into consideration
in the simulation model. Looking at joint $q_3$ and $q_5$ it can be seen that the amplitude
of the error of the experiment decreases. For the simulation model, the position error
becomes larger instead of smaller. This can be due to the difference of the friction of
the Slave and the modelled friction. The modelling of the friction in controller and
mathematical model of the Slave is the same. The large adaptation gains lead to a
large amplitude of the adaptation, and induce an error.
Chapter 5

Master-Slave synchronization

From chapter 4 it appears that the MRAC can be used to control the individual CFT-transposers. The MRAC can be made robust for measurement noise by basing the regressor on the reference velocity that was obtained from the reference model. This signal was free of noise. When passing on to Master-Slave synchronization, it is not so easy to make the MRAC robust for measurement noise. For Master-Slave synchronization, the reference velocity for the Slave is the velocity of the Master. The velocity of the Master is obtained from numerical differentiation of the position of the Master. Since the position of the Master is obtained by using incremental encoders, the velocity of the Master is also disturbed with measurement noise. Basing the regressor on the velocity of the Master, will lead to disturbance of the adaptation, because the velocity of the Master appears quadratic in the adaptation law. To solve this problem, the velocity of the reference trajectory can be used instead of the velocity of the Master. This will lead to the following equations.

Control law:

\[ \tau = M(q_s)\ddot{q}_m + C(q_s, \dot{q}_s)\dot{q}_m + G(q_s) + F(\dot{q}_s, \dot{\Theta}) - K_pe_q - K_v\dot{e}_q \]  

Adaptation law:

\[ \dot{\Theta} = -\Gamma^{-1}W_d^T(\dot{q}_{ref})s \]  

with:

\[ s = \dot{e}_q + \Lambda e_q \]  
\[ e_q = q_s - q_m \]  
\[ \dot{e}_q = \dot{q}_s - \dot{q}_m \]

However, it has to be remarked that this is not real Master-Slave synchronization anymore, because the controller of the Slave is not only based on information of the Master, but also on information of the reference trajectory.
Chapter 5. Master-Slave synchronization

Table 5.1: Friction parameters for the Master

<table>
<thead>
<tr>
<th>Parameter</th>
<th>value</th>
<th>Parameter</th>
<th>value</th>
<th>Parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{e1}$</td>
<td>100</td>
<td>$B_{f1,1}$</td>
<td>4</td>
<td>$B_{f2,1}$</td>
<td>-21</td>
</tr>
<tr>
<td>$B_{e2}$</td>
<td>4</td>
<td>$B_{f1,2}$</td>
<td>22</td>
<td>$B_{f2,2}$</td>
<td>10</td>
</tr>
<tr>
<td>$B_{e4}$</td>
<td>-32</td>
<td>$B_{f1,4}$</td>
<td>-3</td>
<td>$B_{f2,4}$</td>
<td>-8</td>
</tr>
<tr>
<td>$B_{e5}$</td>
<td>-40</td>
<td>$B_{f1,5}$</td>
<td>30</td>
<td>$B_{f2,5}$</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5.2: Error indices master

<table>
<thead>
<tr>
<th>DOF</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_1$</td>
<td>$1.3 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>$q_2$</td>
<td>$1.3 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>$q_4$</td>
<td>$2.1 \cdot 10^{-6}$</td>
</tr>
<tr>
<td>$q_5$</td>
<td>$8.5 \cdot 10^{-6}$</td>
</tr>
</tbody>
</table>

5.1 Control of the Master

Since the adaptation law is also based on the velocity of the reference trajectory, it is desirable to have small difference between the velocity of the reference trajectory and the velocity of the Master. To control the Master a model based controller is chosen. To find the proper values of the friction parameters for the Master, first the Master is controlled with the same MRAC as used in chapter 3. The values obtained with this MRAC are shown in Table 5.1. The friction parameters and the position error of the Master are also depicted in appendix C. The error indices for the Master using the friction parameters of Table 5.1 are given in Table 5.2.

5.2 Results

The settings of the MRAC to achieve Master-Slave synchronization are the same as used to control the Slave robot individually. The results are shown in Figure 5.1, Figure 5.2 and Figure 5.3. In Figure 5.1 the linear friction parameters are given as function of time. Just like the experiment with only the Slave, the adaptation isn’t turned on at the start of the experiment but after about 130 seconds. It can be concluded that after 1500 seconds the parameters of joint $q_2$ and $q_5$ are almost converged. However, the friction parameters of joint $q_1$ and $q_4$ are still changing. From the decreasing slope of friction parameters of joint $q_1$, it is likely that these parameters also converge, but it will take some more time. From the convergence of the joints $q_1,q_2$ and $q_5$ it can be
Chapter 5. Master-Slave synchronization

Figure 5.1: Friction parameters Slave

Figure 5.2: Synchronization error first 200 sec.
Chapter 5. Master-Slave synchronization

expected that the friction parameters of joint $q_3$ also will converge. The value of the friction parameters after 1600 seconds is given in Table 5.3.

Comparing these values with the final values of the friction parameters obtained considering the Slave individually, it can be concluded that the friction parameters of joint $q_2$ are exactly the same for both experiments. For the other joints, the progress of the viscous friction parameters is the almost the same for both experiments, although the final values of these parameters are different. The values of the non-viscous friction parameters are also different. However, if the resulting friction force of these parameters is compared, it can be concluded that this force is almost the same for both experiments.

The reason for the difference between both experiments is that for Master-Slave synchronization the regressor is based on the reference trajectory, instead of the velocity of
Chapter 5. Master-Slave synchronization

Table 5.4: Error indices Slave

<table>
<thead>
<tr>
<th>DOF</th>
<th>Error index start</th>
<th>Final error index</th>
<th>Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_1$</td>
<td>6.0 $ \cdot 10^{-5}$</td>
<td>2.5 $ \cdot 10^{-5}$</td>
<td>58 %</td>
</tr>
<tr>
<td>$q_2$</td>
<td>1.3 $ \cdot 10^{-5}$</td>
<td>9.0 $ \cdot 10^{-6}$</td>
<td>31 %</td>
</tr>
<tr>
<td>$q_4$</td>
<td>1.8 $ \cdot 10^{-5}$</td>
<td>2.5 $ \cdot 10^{-6}$</td>
<td>86 %</td>
</tr>
<tr>
<td>$q_5$</td>
<td>3.4 $ \cdot 10^{-6}$</td>
<td>2.2 $ \cdot 10^{-5}$</td>
<td>35 %</td>
</tr>
</tbody>
</table>

the Master. Because of this error, the synchronization error and the error of the friction parameter will not be related correctly, and this will lead to a deviating adaptation.

In Figure 5.2 and Figure 5.3 the synchronization error is given as function of time. In Figure 5.2 the synchronization error during the first 200 seconds is shown. When the adaptation is turned on after 100 seconds, the amplitude of the synchronization error becomes smaller. In Figure 5.3 the position error at the end of the experiment is shown. The error index for the initial values of the friction parameters, and the value of the friction parameters after 1600 second is given in tabel 5.4.

From Table 5.4 it appears that a reduction of the synchronization error varying from 31 % for joint $q_2$ to 86 % for joint $q_4$ can be achieved. So, it can be concluded that Master-Slave synchronization can be achieved using the adaptive controller and that the performance of this controller is better than the model-based controller based on the friction parameters obtained with a Kalman filter.
Chapter 6

Conclusions and Recommendations

6.1 conclusions

- A Model Reference Adaptive Controller for the linear friction parameters of the Slave is derived, and is made robust for measurement noise. From simulations it can be concluded that, using this controller, adaptive control of the Slave is possible.

- Because of the chosen friction model, which consists of two exponential terms with fixed non-linear friction parameters, it takes a long time before the friction parameters are converged.

- The MRAC is used to control the Slave individually. After 2000 seconds, the friction parameters are almost converged. The error index of the Slave can be reduced with 80%, 20%, 0% and 21% for joint $q_1$, $q_2$, $q_4$ and $q_5$ respectively, using the MRAC instead of the model-based controller which was used in [3].

- The goal to achieve Master-Slave synchronization using an adaptation mechanism for the friction parameters is achieved basing the regressor in the adaptation law on the velocity of the reference trajectory.

- The error index, which is inversely proportional to the performance of the Master-Slave system, can be reduced with 58%, 31%, 86% and 35% for joint $q_1$, $q_2$, $q_4$ and $q_5$ respectively, using the MRAC instead of the model-based controller for which the friction parameters are obtained with a Kalman filter.
6.2 recommendations

- In this report an adaptation mechanism for the linear friction parameters is used. The non-linear friction parameters are obtained from [3] and are fixed. Since the non-linear friction parameters are also dependent on external circumstances, like temperature, the performance can possibly be improved if these parameters are again determined. This can be done by using a Kalman filter for the non-linear friction parameter, while the linear friction parameters are fixed to the values obtained with the adaptive controller.

- The velocities of the Master and the Slave are obtained by numerical differentiation of the position. Because the position is obtained with incremental encoders, the position is disturbed with measurement noise and so the velocity. A low pass filter can be used to reduce the measurement noise. This will lead to a better performance.

- In this report the control parameters as used in [3] are used. The parameters of the adaptation law are chosen by trial and error. The performance can likely be improved by tuning the parameters of the control law and the adaptation law.

- In this report Master-Slave synchronization is achieved using information of the reference trajectory to control the Slave. Since the difference of the velocity of the Master and the velocity of the reference trajectory results in disruptive adaptation behaviour, it can be considered to achieve Master-Slave synchronization using information of the Master only. However, because of measurement noise, quadratic application of the velocity signal also results in disruptive adaptation behavior. To avoid this disruptive adaptation behavior due to measurement noise, two sensors are needed to obtain the velocity of the CPT-transposer. If the measurement noise of both velocity signals has a zero mean, and the measurement noise of both velocity signals is uncorrelated, multiplication of these signal will not lead to disruptive adaptation behavior.

- Instead of Master-Slave synchronization, it can be considered to use the Model Reference Adaptive Controller to achieve Mutual Synchronization.
Appendix A

Dynamic model of the CFT robot

Entries of the inertia matrix $M(q)$

\[ M_{11} = \Theta_1 + \Theta_{11} + \Theta_{12} \quad (A.1) \]

\[ M_{12} = \left( -\Theta_{12}d_{20} - \Theta_{11}d_{20} - \Theta_3 \right) \sin(q_2) + \left( \Theta_2 + d\Theta_{11} \right) \cos(q_2) \]
\[ + \frac{1}{2} \left( (L_4 - L_5)(\Theta_{12} + \Theta_{11}) - \Theta_9 \right) \left( \cos(q_5 + q_2 + q_4) - \cos(q_5 - q_2 + q_4) \right) \]
\[ + \frac{1}{2} \left( \Theta_7 + \Theta_5 + \Theta_{12}L_6 \right) \left( \cos(-q_2 + q_4) - \cos(q_2 + q_4) \right) \]
\[ + \frac{1}{2} \left( \Theta_8 + \Theta_6 \right) \left( \sin(q_2 + q_4) - \sin(-q_2 + q_4) \right) \]
\[ + \frac{1}{2} \left( -\sin(q_5 - q_2 + q_4) + \sin(q_5 + q_2 + q_4) \right) \Theta_{10} \quad (A.2) \]

\[ M_{13} = \frac{1}{2} \left( -\Theta_5 - \Theta_7 - \Theta_{12}L_6 \right) \left( \cos(q_2 + q_4) + \cos(-q_2 + q_4) \right) \]
\[ + \frac{1}{2} \left( (L_4 - L_5)(\Theta_{12} + \Theta_{11}) - \Theta_9 \right) \left( \cos(q_5 - q_2 + q_4) + \cos(q_5 + q_2 + q_4) \right) \]
\[ + \frac{1}{2} \left( \Theta_8 + \Theta_6 \right) \left( \sin(q_2 + q_4) + \sin(-q_2 + q_4) \right) \]
\[ + \frac{1}{2} \left( \sin(q_5 + q_2 + q_4) + \sin(q_5 - q_2 + q_4) \right) \Theta_{10} \quad (A.3) \]

\[ M_{14} = \frac{1}{2} \left( (L_4 - L_5)(\Theta_{11} + \Theta_{12}) - \Theta_9 \right) \left( \cos(q_5 + q_2 + q_4) + \cos(q_5 - q_2 + q_4) \right) \]
\[ + \frac{1}{2} \left( \sin(q_5 + q_2 + q_4) + \sin(q_5 - q_2 + q_4) \right) \Theta_{10} \quad (A.4) \]
Appendix A. Dynamic model of the CFT robot

\[ M_{22} = ((L_5 - L_4)(\sin(q_5) + \sin(q_5 + 2q_4)) - 2\cos(q_4)d_{20})\Theta_8 + \Theta_4 \\
+(-2d_{20}\cos(q_4 + q_5) - L_4\sin(2q_5 + 2q_4))\Theta_{10} + \Theta_{12}d_{20} \\
+\left(\frac{1}{2} - \frac{1}{2}\cos(2q_5 + 2q_4)\right)(L_5^2 + L_4^2) + 2L_4d_{20}\sin(q_4 + q_5) + d_6^2 \\
+d_{20}^2 + ((\cos(2q_5 + 2q_4) - 1)L_4 - 2d_{20}\sin(q_4 + q_5))L_5)\Theta_{11} \\
+((\cos(2q_4) - \cos(q_5) - 1 + \cos(q_5 + 2q_4))L_4 \\
-2d_{20}\sin(q_4))\Theta_5 - 2(\sin(q_4 + q_5)L_5 + \sin(q_4)L_6)d_{20}\Theta_{12} \\
+((\cos(q_5 + 2q_4) - \cos(q_4))(L_4 - L_5) - 2d_{20}\sin(q_4))\Theta_7 \\
-\frac{1}{2}\Theta_{12}(\cos(2q_4) - 1)L_6^2 - \frac{1}{2}(\cos(2q_5 + 2q_4) - 1)(L_3^2 + L_4^2)\Theta_{12} \\
+(-L_4(\sin(2q_4) + \sin(q_5 + 2q_4) + \sin(q_5)) - 2\cos(q_4)d_{20})\Theta_6 \\
+((\cos(2q_5 + 2q_4) - 1)L_4 + (\cos(q_5) - \cos(q_5 + 2q_4))L_6)\Theta_{12} \\
+((\cos(2q_5 + 2q_4) - 1)L_4 - 2d_{20}\sin(q_4 + q_5))\Theta_9 \\
+(2\sin(q_4 + q_5)d_{20} - (\cos(q_5) + \cos(q_5 + 2q_4))L_6)\Theta_{12} \quad (A.5) \]

\[ M_{23} = -\Theta_7d_6\cos(q_4) + \Theta_8d_6\sin(q_4) + \Theta_{11}d_6(L_4 - L_5)\cos(q_4 + q_5) \quad (A.6) \]

\[ M_{24} = \Theta_{11}d_6(L_4 - L_5)\cos(q_4 + q_5) \quad (A.7) \]

\[ M_{33} = (((L_5 - L_6)L_4 + 2L_5L_6)\Theta_{12} + L_5L_4\Theta_{11} + (\Theta_9 - \Theta_5 - \Theta_7)L_4 \\
+2\Theta_7L_5\cos(q_5) + L_4(\Theta_6 + \Theta_8)\sin(2q_4)) \\
-L_4\left(\frac{1}{2}L_4 + L_6\right)\Theta_{12} + \frac{1}{2}L_4\Theta_{11} + \Theta_5 + \Theta_7\right)\cos(2q_4) \\
-L_4((L_6 + L_5)\Theta_{12} + L_5\Theta_{11} + \Theta_9 + \Theta_7 + \Theta_5)\cos(q_5 + 2q_4) \\
+(\Theta_8 + \Theta_{10} + \Theta_6)L_4\sin(q_5 + 2q_4) + ((2L_5 - L_4)\Theta_8 \\
-(\Theta_6 + \Theta_{10})L_4)\sin(q_5) + (L_4^2 + (L_6 - L_5)L_4 + L_3^2 + L_5^2)\Theta_{12} \\
+\left(\frac{1}{2}L_4 - L_5\right)\Theta_{12} + \Theta_{11} - \Theta_9\right)L_4\cos(2q_5 + 2q_4) \\
+(L_5^2 - L_6L_4 + L_4^2)\Theta_{11} + (\Theta_7 - \Theta_9 + \sin(2q_5 + 2q_4)\Theta_{10} - \Theta_3)L_4 \quad (A.8) \]

\[ M_{34} = \frac{1}{2}(\sin(q_5 + 2q_4) - \sin(q_5))L_4\Theta_6 + \frac{1}{2}\Theta_{12}(\cos(2q_5 + 2q_4) + 1)L_4^2 \]
Appendix A. Dynamic model of the CFT robot

Entries of the Coriolis matrix:

\[ C(q, \dot{q}) \]

\( C_{11} = C_{21} = C_{31} = C_{41} = 0 \) \hspace{1cm} (A.11)

\[
C_{12} = \frac{1}{2}(\Theta_7 + \Theta_9 + L_5 \Theta_{12})(\dot{q}_2 - \dot{q}_4)\sin(q_5 - q_2) + (\dot{q}_2 + \dot{q}_4) \\
\sin(q_2 + q_4) - ((\Theta_{12} + \Theta_{11})d_2 + \Theta_3)\dot{q}_2 \cos(q_2) \\
+ \frac{1}{2}(\Theta_9 + (L_5 - L_4)(\Theta_{12} + \Theta_{11}))((\dot{q}_4 + \dot{q}_2 + \dot{q}_5) \\
\sin(q_5 + q_2 + q_4) + (\dot{q}_2 - \dot{q}_4 - \dot{q}_5)\sin(q_5 - q_2 + q_4)) \\
-(\Theta_2 + d_6 \Theta_{11})\dot{q}_2 \sin(q_2) + \frac{1}{2}(\Theta_8 + \Theta_6)((\dot{q}_4 - \dot{q}_2)\cos(q_4 - q_2) \\
+ (\dot{q}_2 + \dot{q}_4)\cos(q_2 + q_4)) + \frac{1}{2}((\dot{q}_2 - \dot{q}_4 - \dot{q}_5) \\
\cos(q_5 - q_2 + q_4) + (\dot{q}_4 + \dot{q}_2 + \dot{q}_5)\cos(q_5 + q_2 + q_4))\Theta_{10} \hspace{1cm} (A.12) \]

\[
C_{13} = \frac{1}{2}(\Theta_8 + \Theta_6)((\dot{q}_4 - \dot{q}_2)\cos(q_4 - q_2) + (\dot{q}_2 + \dot{q}_4)\cos(q_2 + q_4)) \\
+ \frac{1}{2}(\Theta_9 + (L_5 - L_4)(\Theta_{12} + \Theta_{11}))((\dot{q}_4 + \dot{q}_2 + \dot{q}_5) \\
\cos(q_5 + q_2 + q_4)) \]
Appendix A. Dynamic model of the CFT robot

\[
\sin(q_5 + q_2 + q_4) + (\dot{q}_4 + \dot{q}_2 + \dot{q}_5)\sin(q_5 - q_2 + q_4)) \\
+ \frac{1}{2}(\Theta_5 + \Theta_7 + L_6\Theta_{12})((\dot{q}_4 - \dot{q}_2)\sin(q_5 - q_2) + (\dot{q}_2 + \dot{q}_4) \\
\sin(q_2 + q_4)) + \frac{1}{2}((\dot{q}_4 + \dot{q}_2 + \dot{q}_5)\cos(q_5 + q_2 + q_4) \\
+(\dot{q}_4 + \dot{q}_5 - \dot{q}_2)\cos(q_5 - q_2 + q_4))\Theta_{10}
\]  
(A.13)

\[
C_{14} = \frac{1}{2}((\dot{q}_4 + \dot{q}_2 + \dot{q}_5)\cos(q_5 + q_2 + q_4) + (\dot{q}_4 - \dot{q}_2 + \dot{q}_5) \\
\cos(q_5 - q_2 + q_4))\Theta_{10} + \frac{1}{2}(\Theta_9 + (L_5 - L_4)(\Theta_{12} + \Theta_{11})) \\
((\dot{q}_4 + \dot{q}_2 + \dot{q}_5)\sin(q_5 + q_2 + q_4) \\
+(\dot{q}_4 + \dot{q}_5 - \dot{q}_2)\sin(q_5 - q_2 + q_4))
\]  
(A.14)

\[
C_{22} = -\frac{1}{2}(L_4\Theta_6 - L_5\Theta_8 + \Theta_8L_4)((2\dot{q}_4 + \dot{q}_5)\cos(2q_4 + q_5) \\
+\dot{q}_3\cos(q_5)) - \dot{q}_4d_20(\Theta_5 + \Theta_7 + L_6\Theta_{12})\cos(q_4) \\
-(\dot{q}_4 + \dot{q}_5)(\Theta_9 + (L_5 - L_4)(\Theta_{12} + \Theta_{11}))d_{20}\cos(q_4 + q_5) \\
-\frac{1}{2}(\dot{q}_4 + \dot{q}_5)(2L_4\Theta_9 - (L_5 - L_4)^2(\Theta_{12} + \Theta_{11}))\sin(2q_5 + 2q_4) \\
-\frac{1}{2}\dot{q}_4(2L_4\Theta_5 - L_6^2\Theta_{12})\sin(2q_4) - \dot{q}_4L_4\Theta_6\cos(2q_4) \\
+\frac{1}{2}(L_4(\Theta_5 + \Theta_7) + L_6\Theta_{12}(L_4 - L_5) - L_5\Theta_7)(\dot{q}_5\sin(q_5) \\
-(2\dot{q}_4 + \dot{q}_5)\sin(2q_4 + q_5)) + (\dot{q}_4 + \dot{q}_5)\sin(q_4 + q_5)d_{20}\Theta_{10} \\
-(\dot{q}_4 + \dot{q}_5)\cos(2q_5 + 2q_4)L_4\Theta_{10} + d_{20}\dot{q}_4(\Theta_8 + \Theta_6)\sin(q_4)
\]  
(A.15)

\[
C_{23} = \frac{1}{2}\dot{q}_2(L_5^2\Theta_{12} - 2L_4\Theta_5)\sin(2q_4) - \dot{q}_2\cos(2q_5 + 2q_4)L_4\Theta_{10} \\
+\frac{1}{2}\dot{q}_2((L_5 - L_4)^2(\Theta_{12} + \Theta_{11}) - 2L_4\Theta_9)\sin(2q_5 + 2q_4) \\
+(\dot{q}_2d_{20}\Theta_{10} + d_6\Theta_{11}(\dot{q}_4 + \dot{q}_5)(L_5 - L_4))\sin(q_4 + q_5) \\
-((L_5 - L_4)(\Theta_{12} + \Theta_{11}) + \Theta_9)d_{20}\dot{q}_2\cos(q_4 + q_5) \\
+\dot{q}_2((L_5 - L_4)(L_6\Theta_{12} + \Theta_7) - L_4\Theta_5)\sin(2q_4 + q_5) \\
+(\dot{q}_4d_6\Theta_8 - q_2(L_5\Theta_{12} + \Theta_7 + \Theta_5)d_{20})\cos(q_4) \\
+(\dot{q}_2(\Theta_8 + \Theta_6)d_{20} + \dot{q}_4d_6\Theta_7)\sin(q_4) \\
+\dot{q}_2((L_5 - L_4)\Theta_8 - L_4\Theta_5)\cos(2q_4 + q_5) - \dot{q}_2L_4\Theta_6\cos(2q_4)
\]  
(A.16)
Appendix A. Dynamic model of the CFT robot

\[ C_{24} = ((\dot{q}_4 + \dot{q}_5)(L_5 - L_4)d_6 \Theta_{11} + \dot{q}_2 d_2 \Theta_{10}) \sin(q_4 + q_5) \]
\[ + \frac{1}{2} \dot{q}_2((L_5 - L_4)^2(\Theta_{12} + \Theta_{11}) - 2L_4 \Theta_9) \sin(2q_5 + 2q_4) \]
\[ - ((L_5 - L_4)(\Theta_{12} + \Theta_{11}) + \Theta_9) d_2 \dot{q}_2 \cos(q_4 + q_5) \]
\[ - \dot{q}_2 L_4 \Theta_{10} \cos(2q_5 + 2q_4) \]
\[ - \frac{1}{2} \dot{q}_2 (L_4 \Theta_6 + \Theta_8(L_4 - L_5)) (\cos(q_5) + \cos(2q_4 + q_5)) \]
\[ + \frac{1}{2} \dot{q}_2 ((L_5 - L_4)(L_6 \Theta_{12} + \Theta_7) - L_4 \Theta_5) (\sin(2q_4 + q_5) - \sin(q_5)) \]  \hspace{1cm} (A.17)

\[ C_{32} = \dot{q}_3 L_4 \Theta_{10} \cos(2q_5 + 2q_4) + \dot{q}_2 L_4 \Theta_6 \cos(2q_4) \]
\[ + \dot{q}_2 d_2 (\Theta_9 + (L_5 - L_4)(\Theta_{12} + \Theta_{11})) \cos(q_4 + q_5) \]
\[ - \dot{q}_2 d_2 \sin(q_4 + q_5) \Theta_{10} + \frac{1}{2} \dot{q}_2 (2L_4 \Theta_5 - L_6^2 \Theta_{12}) \sin(2q_4) \]
\[ + \dot{q}_2 (L_4 \Theta_5 - (L_5 - L_4)(L_6 \Theta_{12} + \Theta_7)) \sin(2q_4 + q_5) \]
\[ + \frac{1}{2} \dot{q}_2 (- (L_5 - L_4)^2(\Theta_{12} + \Theta_{11}) + 2L_4 \Theta_9) \sin(2q_5 + 2q_4) \]
\[ + \dot{q}_2 d_2 (\Theta_9 + \Theta_7 + L_6 \Theta_{12}) \cos(q_4) - \dot{q}_2 d_2 (\Theta_8 + \Theta_6) \sin(q_4) \]
\[ + \dot{q}_2 (L_4 \Theta_6 + \Theta_8(L_4 - L_5)) \cos(2q_4 + q_5) \]  \hspace{1cm} (A.18)

\[ C_{33} = \frac{1}{2} L_4 (\dot{q}_4 + \dot{q}_5)((2L_5 - L_4)(\Theta_{12} + \Theta_{11}) + 2\Theta_9) \sin(q_5 + 2q_4) \]
\[ - \frac{1}{2} \dot{q}_5 ((2L_5 L_6 - L_6 L_4 + L_5 L_4) \Theta_{12} + (2L_5 - L_4) \Theta_7 + \Theta_{11} L_5 L_4 \]
\[ + L_4 (\Theta_9 - \Theta_5)) \sin(q_3) + L_4 (\dot{q}_4 + \dot{q}_5) \Theta_{10} \cos(2q_5 + 2q_4) \]
\[ + \frac{1}{2} L_4 (2\dot{q}_4 + \dot{q}_3)((L_5 + L_6) \Theta_{12} + \Theta_7 + L_5 \Theta_{11} + \Theta_5 + \Theta_9) \]
\[ \sin(2q_4 + q_5) + L_4 \dot{q}_4 (\Theta_8 + \Theta_6) \cos(2q_4) \]
\[ + \frac{1}{2} L_4 \dot{q}_4 ((L_4 + 2L_6) \Theta_{12} + 2\Theta_7 + L_4 \Theta_{11} + 2\Theta_5) \sin(2q_4) \]
\[ + \frac{1}{2} L_4 (2\dot{q}_4 + \dot{q}_3)(\Theta_8 + \Theta_6 + \Theta_{10}) \cos(2q_4 + q_5) \]
\[ - \frac{1}{2} \dot{q}_5 (L_4 (\Theta_6 + \Theta_{10}) + \Theta_8 (L_4 - 2L_5)) \cos(q_5) \]  \hspace{1cm} (A.19)

\[ C_{34} = \frac{1}{2} (\Theta_8 + \Theta_{10} + \Theta_6) \cos(2q_4 + q_5) + \Theta_{10} \cos(2q_5 + 2q_4)) L_4 \]
Appendix A. Dynamic model of the CFT robot

\[(\dot{q}_4 + \dot{q}_5) - \frac{1}{2}(\dot{q}_4 + \dot{q}_5)(L_5L_4\Theta_{12} + (2L_5 - L_4)(\Theta_7 + L_6\Theta_{12})
+ \Theta_{11}L_5L_4 + L_4(\Theta_9 - \Theta_5))(\sin(q_5) + \frac{1}{2}L_4(\dot{q}_4 + \dot{q}_5)((L_5 + L_6)\Theta_{12}
+ \Theta_7 + L_6\Theta_{11} + \Theta_5 + \Theta_9)(\sin(2q_4 + q_5)
+ \frac{1}{2}L_4(\dot{q}_4 + \dot{q}_5)((2L_5 - L_4)(\Theta_{12} + \Theta_{11}) + 2\Theta_9)(\sin(2q_5 + 2q_4)
+ \frac{1}{2}(\dot{q}_4 + \dot{q}_5)((2L_5 - L_4)\Theta_8 - L_4(\Theta_{10} + \Theta_6))\cos(q_5) \tag{A.20}\]

\[C_{42} = \dot{q}_2d_{20}((L_5 - L_4)(\Theta_{12} + \Theta_{11}) + \Theta_9)\cos(q_4 + q_5)
+ \frac{1}{2}\dot{q}_2((L_5 - L_4)(L_6\Theta_{12} + \Theta_7) - L_4\Theta_9)(\sin(q_5) - \sin(2q_4 + q_5))
+ \frac{1}{2}\dot{q}_2(2L_4\Theta_9 - (\Theta_{12} + \Theta_{11})(L_5 - L_4)^2\sin(2q_5 + 2q_4)
+ \dot{q}_2L_4\Theta_{10} \cos(2q_5 + 2q_4) - \dot{q}_2d_{20}\Theta_{10}\sin(q_4 + q_5)
+ \frac{1}{2}\dot{q}_2(L_4\Theta_6 - (L_5 - L_4)\Theta_8)(\cos(q_5) + \cos(2q_4 + q_5)) \tag{A.21}\]

\[C_{43} = \frac{1}{2}\dot{q}_4(L_4(\Theta_9 - \Theta_5 + \Theta_{11}L_5) + (2L_5L_6 - L_6L_4 + L_5L_4)\Theta_{12}
+ (2L_5 - L_4)\Theta_7)\sin(q_5) + L_4(\dot{q}_4 + \dot{q}_5)\Theta_{10} \cos(2q_5 + 2q_4)
+ \frac{1}{2}\dot{q}_4L_4((L_3 + L_6)\Theta_{12} + \Theta_7 + L_5\Theta_{11} + \Theta_5 + \Theta_9)\sin(2q_4 + q_5)
+ \frac{1}{2}\dot{q}_4L_4(\Theta_8 + \Theta_6 + \Theta_{10})\cos(2q_4 + q_5)
+ \frac{1}{2}\dot{L}_4(\dot{q}_4 + \dot{q}_5)((2L_5 - L_4)(\Theta_{12} + \Theta_{11}) + 2\Theta_9)\sin(2q_5 + 2q_4)
+ \frac{1}{2}\dot{q}_4(L_4(\Theta_6 + \Theta_{10}) + \Theta_8(L_4 - 2L_5))\cos(q_5) \tag{A.22}\]

\[C_{44} = \frac{1}{2}L_4(\dot{q}_4 + \dot{q}_5)((2L_5 - L_4)(\Theta_{12} + \Theta_{11}) + 2\Theta_9)\sin(2q_5 + 2q_4)
+ L_4(\dot{q}_4 + \dot{q}_3)\Theta_{10} \cos(2q_5 + 2q_4) \tag{A.23}\]

Entries of the gravity vector: \(G(q)\)

\[g_1 = 0 \tag{A.24}\]
Appendix A. Dynamic model of the CFT robot

\begin{align}
  g_2 & = 0 \\
  g_3 & = -g(\Theta_9 + \Theta_{12}L_5 + L_5\Theta_{11})\sin(q_4 + q_5) - g(\Theta_6 + \Theta_3)\cos(q_4) \\
  & \quad -g(\Theta_5 + \Theta_{12}(L_6 + L_4) + L_4\Theta_{11} + \Theta_7)\sin(q_4) - g\Theta_{10}\cos(q_4 + q_5) \\
  g_4 & = -g(\Theta_9 + \Theta_{12}L_5 + L_5\Theta_{11})\sin(q_4 + q_5) - g\Theta_{10}\cos(q_4 + q_5)
\end{align}

Entries of the vector of friction forces: \( F(\dot{q}, \Theta) \)

\begin{align}
  f_1 & = \Theta_{13}\dot{q}_1 + \Theta_{17}(1 - \frac{2}{(1 + e^{(2\Theta_{25}\dot{q}_1)})}) + \Theta_{21}(1 - \frac{2}{(1 + e^{(2\Theta_{29}\dot{q}_1)})}) \\
  f_2 & = \Theta_{14}\dot{q}_2 + \Theta_{18}(1 - \frac{2}{(1 + e^{(2\Theta_{26}\dot{q}_2)})}) + \Theta_{22}(1 - \frac{2}{(1 + e^{(2\Theta_{30}\dot{q}_2)})}) \\
  f_3 & = \Theta_{15}\dot{q}_4 + \Theta_{19}(1 - \frac{2}{(1 + e^{(2\Theta_{27}\dot{q}_4)})}) + \Theta_{23}(1 - \frac{2}{(1 + e^{(2\Theta_{31}\dot{q}_4)})}) \\
  f_4 & = \Theta_{16}\dot{q}_5 + \Theta_{20}(1 - \frac{2}{(1 + e^{(2\Theta_{28}\dot{q}_5)})}) + \Theta_{24}(1 - \frac{2}{(1 + e^{(2\Theta_{32}\dot{q}_5)})})
\end{align}
Appendix B

Estimated parameters

The physical parameters of the CFT-transposer robot with plate number 677528, the Slave, in the Dynamics and Control Laboratory have been estimated by using an extended Kalman filter and least squares methods in [4]. The physical parameters are listed in Table B.1.
### Appendix B. Estimated parameters

**Table B.1: Estimated parameters for the Slave**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$\Theta_1$</td>
<td>$m_1 + m_2$</td>
<td>121.3049</td>
</tr>
<tr>
<td>$\Theta_2$</td>
<td>$m_2 l_{xc2}$</td>
<td>0.3107</td>
</tr>
<tr>
<td>$\Theta_3$</td>
<td>$m_2 l_{yc2}$</td>
<td>4.1955</td>
</tr>
<tr>
<td>$\Theta_4$</td>
<td>$m_2 (l_{zc2}^2 + l_{yc2}^2) + m_3 (l_{yc3}^2 + l_{zc3}^2) + m_4 l_{zc4}^2 + m_5 l_2^2 - zc5 + m_6 l_{zc6}^2 + m_7 (l_{yc7}^2 + l_{zc7}^2) + m_8 (l_{zc8}^2 + l_{yc8}^2) + I_{xx2} + I_{yy2} + I_{yy3} + I_{xx3} + I_{zz4} + I_{zz5} + I_{zz6} + I_{yy7} + I_{xx7} + I_{xx8} + I_{zz8}$</td>
<td>1.7453</td>
</tr>
<tr>
<td>$\Theta_5$</td>
<td>$m_4 l_{xc4}$</td>
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</tr>
<tr>
<td>$\Theta_6$</td>
<td>$m_4 l_{yc4}$</td>
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</tr>
<tr>
<td>$\Theta_7$</td>
<td>$m_6 l_{xc6}$</td>
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<tr>
<td>$\Theta_8$</td>
<td>$m_6 l_{yc6}$</td>
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<tr>
<td>$\Theta_9$</td>
<td>$m_8 l_{xc8}$</td>
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<tr>
<td>$\Theta_{10}$</td>
<td>$m_8 l_{yc8}$</td>
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<td>$\Theta_{11}$</td>
<td>$m_6$</td>
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<tr>
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</tbody>
</table>
Appendix C

Adaptive control of the Master

In Figure C.1, the friction parameters of the Master are shown as function of the time. It can be concluded that the friction parameters are converged after 2000 seconds. The values of the friction parameters will be used for the control of the Master to achieve proper tracking behaviour of the Master.

The position error is depicted in Figure C.2. It is important to have a small position error, since the velocity of the reference signal is used for the control of the Slave instead of the velocity of the Master.
Appendix C. Adaptive control of the Master
Bibliography


