Identification of the inertias of the H-bridge

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Bachelor report

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Summary

To experimentally validate solutions for a central problem in control theory (namely the nonlinear output regulation problem) a machine that is present at the department of Mechanical Engineering at the Technical University of Eindhoven has recently been adapted in a master thesis study by B.R.A. Janssen. This machine, the H-bridge, has been equipped with a pendulum to form a TORA, a Translation Oscillator with a Rotational Actuator. This is a benchmark problem for this central problem in control theory. The solutions for this problem (the output regulation controllers) have been implemented and tested on the H-bridge.

Since these controllers are model-based, knowledge on the parameters of this model is required. The more accurate the system parameters are known, the higher the performance of the output regulation controllers is.

The aim of this study is to accurately identify a subset of the parameters that are used in the model used by the output regulation controllers. These parameters are:

- the mass of the x-cart of the H-bridge,
- the mass of y-cart of the H-bridge,
- and the moment of inertia of the pendulum around the driving axle.

This can be done by an identification method which is based on moving the machine part of which the mass or moment of inertia has to be determined with a constant velocity and applying noise on the input of this part. The noise input will cause the velocity to vary around the constant velocity. The required parameters can be estimated based on knowledge on the effect of the noise on the movement of the cart.

This identification method was first tested on a simulation level in Simulink to obtain skill in identifying parameters with this method. The following trade off became clear during the simulation. The noise may not disturb the constant velocity of the moving part too much since the velocity of the part may not come close to zero in order to ensure that the friction remains approximately linearly related to the velocity. On the other hand the effect of the noise may not be suppressed too much either; otherwise the required parameters can not be estimated anymore. A closed-loop control scheme was used to maintain the constant velocity. The controller in this scheme has to ensure the right balance in terms of the trade off mentioned above.

Next the actual experiments were conducted. The identification of the mass of the y-cart resulted in a more reliable measurement then the identification of the mass of the x-cart.
Chapter 1: Introduction

One of the machines that is being used for research and education within the department of Mechanical Engineering at the Technical University of Eindhoven (TU/e) is the so called H-bridge. This machine was formerly used by Philips as a pick-and-place robot. The H-bridge picked up parts, moved them with a high speed and put them down again on a specific place with high accuracy. Now, the H-bridge is used as an experimental benchmark system for control problems.

One of the central problems in control theory is the output regulation problem. Many researchers have tackled this problem which has lead to controllers that solve the output regulation problem. A benchmark problem for nonlinear control design is a cart with only one translational degree of freedom which is actuated by a eccentric rotational actuator mounted on the cart. This system is called a TORA-system: Translation Oscillator with a Rotational Actuator. Figure 1.1 shows a TORA.

![Figure 1.1: Schematic representation of a TORA system.](image)

To experimentally validate nonlinear output regulation controllers the H-bridge was recently modified into a TORA. Figure 1.2 is a photograph of the modified H-bridge. Figure 1.3 is a schematic representation of the modified H-bridge where the relevant parts and their names for this report can be seen. Since the translational oscillator of a TORA exist in the form of the H-bridge’s x-carriage (called x-cart in this report) a rotational actuator had to be added to form a TORA. This was done by attaching a pendulum to the cart. An electronic motor drives the pendulum (a description of the H-bridge is given in chapter 2).

As a master’s thesis study B.R.A. Janssen modified the H-bridge and implemented output regulation controllers experimentally [2].
1.1 Problem formulation

The nonlinear output regulation controllers are model-based. This means that several masses and moments of inertia (model parameters) are used in a model of the H-bridge that is needed in the output regulation controllers. The more accurate those parameters are known, the higher the performance of the nonlinear output regulation controllers will be.

The purpose of this research is to experimentally identify the following masses and moments of inertia (most of them are used in the model used by the output regulation controllers):
- the mass that moves when the x-cart is moved by actuating its motor. This includes the x-cart with the attached pendulum,
- the mass that moves when the Y-carts move. This mass includes the Y1- and Y2-cart, the beam between those carts and the x-cart with pendulum. This mass is called the y-mass or the mass of the y-cart,
- the moment of inertia of the pendulum around the center of its driving axle.

1.2 Outline of this report

First in chapter 2 a description of the H-bridge is given. In chapter 3, the identification method used to obtain the parameters mentioned above is explained. In chapter 4, a simulation of the experiments is described. This simulation was conducted to test the identification method. In chapter 5, the experiments to identify the required parameters are described and the related results are presented. Finally, conclusions and recommendations are presented in chapter 6.
Chapter 2: Description of the H-bridge

In this chapter, relevant information about the H-bridge concerning the identification of the required parameters is presented.

Figure 2.1 shows the H-bridge. The name ‘H-bridge’ is derived from the view from above of the machine, as can be seen in figure 2.1 and figure 2.2. The H-bridge consists of two parallel beams, positioned in the Y1- and Y2-direction (see figure 2.2). On each beam a cart can move in the y-direction. The coordinates that describe this movement are Y1 and Y2. The average of Y1 and Y2 is denoted as Y. For most movements of the H-bridge Y1 and Y2 are synchronised.

The two carts are connected with a third beam (the x-beam). When the y-carts are positioned halfway of the y-beams the machine looks like the letter ‘H’ (taking a top view). Moreover a cart is placed on the x-beam. This cart can move in the x-direction with corresponding coordinate X. The three carts combined can position the x-cart anywhere in the horizontal plane between the two y-beams. On the x-cart a pendulum is mounted. This pendulum can rotate around the axis normal to the horizontal plane.

The position and orientation of the machine can be described by four coordinates: X, Y1, Y2 and θ. Figure 2.2 schematically displays the H-bridge where the coordinates mentioned above are defined.

In this report a movement of the H-bridge relates either to a change in position of the x-cart, a change in position of the y-carts (with this experiment their positions are always the same) or a rotation of the pendulum.

To identify the mass of the y-part the H-bridge moves in such a way that Y1 and Y2 are identical. Therefore, the position of the y-cart is defined by:

\[
Y = \frac{Y_1 + Y_2}{2}. \tag{2.1}
\]

The positions of the y-carts and the x-cart are measured with linear encoders. Rulers are placed along the two y-beams and the x-beam (one on each beam). These rulers contain a large number of small stripes. When a cart moves the encoder ‘counts’ the stripes that it passes with the help of light. This counted number of passed stripes is converted to a distance: the distance that the cart has moved. The accuracy (the resolution) of this position sensor is 1.0 µm.
The position of the pendulum is also measured with an encoder: an angular encoder (the ruler of the pendulum is a circle) with a resolution of 0.18°.

The x- and y-carts are each driven by (a set of) \textit{Linear Motor Motion System(s)}, (LiMMS). The amplifiers and the LiMMS(s) convert an input of 1 Ampere into 74.4 Newton.

The pendulum is driven by a DC motor. The motor constant is $60.3 \times 10^{-3}$ N/A.

The H-bridge is connected to Simulink by using a Dspace system. DSpace, together with a computer that controls the H-bridge, is the connection between Simulink and the H-bridge. The combination of DSpace and Simulink enables the user to control the H-bridge and to measure the desired signals.

The moving parts of the H-bridge (carts and pendulum) endure friction when they move. For the x-cart the friction has been identified as a function of the velocity and the resulting model is inserted in a so-called 'look-up table' [2]. Now, the H-bridge can adjust the driving force of the x-cart in such a way that the friction is being compensated for by using the look-up table.

There is no friction compensation for the y-carts or for the pendulum.

The motors of the x-cart and y-cart endure another force: the cogging force. This is a force that results from the magnets in the linear motors. This cogging force depends on the position of the motor. This position-dependent cogging force was, for the x-cart, also identified and the resulting model is inserted in a look up table, see [2]. So, the H-bridge can compensate this force for the x-cart.

More (technical) information about the adapted H-bridge can be found in [2]. In the following chapter the method that was used to identify the inertial parameters is explained.
Chapter 3: Identification method

There are several methods to experimentally identify a mass or moment of inertia. The method that is used to identify the required parameters of the H-bridge is explained below. Advantages of this method are:
- the H-bridge did not have to be dismantled (for example: when one wants to put every part on a scale, the H-bridge has to be dismantled),
- with this method the inertia of a moving part can be estimated. The inertias that have to be determined (see chapter 1) are exactly the inertia’s of the moving parts. This method thus estimates exactly the right parameters.

In section 3.1, the equations of motion of the relevant parts of the H-bridge are presented. In section 3.2, the closed-loop identification method is explained using the identification of the mass of the x-cart as an example. The mass of the y-cart and the moment of inertia of the pendulum are identified in a similar way. In section 3.3, the method is briefly summarized to point out the essential steps in conducting a measurement.

3.1 Equations of motion

The x- and y-cart can be seen as a moving mass actuated by a driving force $F$ (imposed by the motor) and all other kinds of forces acting on it such as friction, cogging, air resistance, et cetera. The most essential forces that affect the motion of the cart are the driving force $F$, the cogging force and the friction force. The friction force and cogging force working on the x-cart can be compensated as mentioned in chapter 2. Only cogging compensation is being used since the friction can be neglected for certain reasons. This is explained in the next paragraph. For now, the friction is still taken in account because it is an essential force acting on the cart. Then the equation of motion for this system is:

$$F = m\ddot{x} + F_f ,$$

where $F_f$ is the friction force, $m$ the mass of the cart, $x$ the displacement and $\dot{x}$ the acceleration of the cart.

Let us model the friction force as being linearly dependent of the velocity of the cart:

$$F_f = b\dot{x} ,$$

with $b$ a constant, $b > 0$.

A plot of the friction force as a function of the velocity of a moving object can exhibit a non-linear part (especially when the velocity is around zero). Generally, when the velocity is high enough (this is called the viscous part of the graph), the friction force is linearly dependent on the velocity. Equation (3.2) describes this linear dependency.

For the x-cart the friction force is identified as a function of the velocity and can indeed approximately be divided in a non-linear and linear part [2]. Although it is not verified with measurements, the friction force of the y-cart and the frictional torque acting on the pendulum are also assumed to be linearly dependent on the corresponding velocity for velocities sufficiently separated from zero. This is of course not valid when the velocity is close to zero as described above. When the assumption on the friction is correct, it is negligible at 'higher' frequencies. This is explained below. When this assumption is not correct, this will become clear in the results of the experiment.
A closed-loop system has to be used to control the H-bridge because the velocity has to be kept high enough such that the assumption that the friction force is linearly dependent on the velocity is correct. This closed-loop system is elucidated in section 3.2.

Using this assumption on the friction, equation (3.2) becomes:

\[ F = mx + bx. \]  

(3.3)

In the Laplace-domain equation (3.3) can be written as [1]:

\[ \frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs}. \]  

(3.4)

where \( s \in \mathbb{C} \) is the Laplace variable.

This equation expresses the transfer function from force to displacement for the cart. It describes the effect of a force \( F(s) \) on the displacement \( X(s) \) of the cart. The input is a force \( F \) and the output is the displacement \( x \). At frequencies smaller than \( b/m \) the friction is of much bigger influence than the mass \( m \). At frequencies higher than \( b/m \) the mass becomes dominant and the effect of the friction can be neglected. This can also be seen in a Bode magnitude plot of the transfer function. At the lower frequencies (as mentioned above) the plot has a slope of minus one. At higher frequencies the plot has a slope of minus two and the transfer equation can approximately be written as [1]:

\[ \frac{X(s)}{F(s)} = \frac{1}{ms^2}. \]  

(3.5)

Keeping this in mind, it is possible to determine the mass of the cart by measuring the frequency response function

\[ \frac{X(j\omega)}{F(j\omega)} = \frac{1}{-m\omega^2}, \]  

(3.6)

where \( \omega \) is the frequency, by moving the cart with a constant velocity, applying additional noise and measuring the resulting displacement.

The frequency response function describing the dynamics of the pendulum can be derived in a similar way. This results in the following equation:

\[ \frac{\theta(j\omega)}{T(j\omega)} = \frac{1}{-J\omega^2}, \]  

(3.7)

with \( \theta \) the angular displacement, \( T \) the applied torque, \( J \) the moment of inertia around the driving axle of the pendulum and \( \omega \) the frequency.

### 3.2 Closed-loop identification method

Feedback control is used within this report to control the H-bridge to ensure an approximately constant velocity of the carts (or pendulum). Using the strategy presented below it is possible to estimate the required parameters.

Figure 3.1 (see the next page) shows a closed-loop control scheme. The symbols of the relevant signals are also shown where \( C(s) \) is the controller, \( H(s) \) is the plant (also called the process), \( x_r \) is the reference signal, \( e \) is the error, \( w \) is the noise, \( u \) is the input of the plant and \( x \) is the output of the plant.
The sensitivity $S(s)$ is defined as [1]:

$$S(s) = \frac{e}{x_1}(s) = \frac{u}{w}(s) = \frac{1}{1 + C(s)H(s)},$$

(3.8)

and the process sensitivity is defined as:

$$PS(s) = \frac{e}{w}(s) = -\frac{H(s)}{1 + C(s)H(s)}.$$  

(3.9)

For the derivation of the sensitivity and process sensitivity functions, see appendix A.

Using these definitions the transfer function of the plant $H(s)$ can be determined by:

$$-\frac{PS(s)}{S(s)} = H(s).$$

(3.10)

Since the H-bridge was controlled by a similar scheme the plant $H(s)$ is of course the H-bridge dynamics described by equation (3.6). Equations (3.8), (3.9) and (3.10) are of course also valid in the frequency-domain. Therefore, by measuring $PS(j\omega)$ and $S(j\omega)$ the frequency response function $H(j\omega)$ of the H-bridge can be determined by:

$$-\frac{PS(j\omega)}{S(j\omega)} = H(j\omega).$$

(3.11)

Let us, for example, take a look at the x-cart since the identification of the other required parameters is similar to that of the mass of the x-cart. For the identification of each inertia an experiment has to be conducted.

The mass of the cart can be determined when the frequency response function of the plant $H(j\omega)$ is known by:

$$m = \frac{1}{\omega^2|H(j\omega)|},$$

(3.12)

since the cart's dynamics are described by equation (3.6).

$H(j\omega)$ can be estimated by using equation (3.11). $PS(j\omega)$ and $S(j\omega)$ can be estimated by measuring $e$, $u$ and $w$ as described by equation (3.8) and (3.9).

The strategy explained above is only valid when:

- the friction is approximately linear with the velocity (which is commonly true when the velocity is high enough),
- and for excitation frequencies high enough to guarantee that the friction can be neglected as described in section 3.2.

The variables $u$, $w$ and $e$ can be measured with DSpace and Simulink while executing the experiment.

The noise is the only signal that can directly be ‘chosen’. For example, one can apply a sinusoidal signal for $w$ and measure the output of the plant. Then the frequency response function $H(j\omega)$ is known for the frequency of the sinusoidal signal $w$. By choosing broad
banded noise as an input \( w \), the dynamics is excited excited at once in a broad frequency-range. So, by inserting noise and measuring \( u \), \( e \) and \( w \) the required parameters can be identified experimentally.

The transfer functions described by equation (3.8) and (3.9) are determined using the measurements on \( e \), \( u \) and \( w \) by using the Matlab-command tfe. This Matlab tool estimates the frequency response function between two signals (for example an input \( x \) and an output \( y \)) using the Welch’s averaged periodogram method [6]. The frequency response function is estimated by the quotient of the cross spectral density between \( x \) and \( y \) and the power spectral density of \( x \):

\[
T_{xy}(j\omega) = \frac{P_{xy}(j\omega)}{P_{xx}(j\omega)},
\]

(3.13)

with \( T_{xy} \) the frequency response function, \( P_{xy} \) the cross spectral density between \( x \) and \( y \) and \( P_{xx} \) the power spectral density of \( x \). More information about the Matlab-command tfe can be found in [4].

The sensitivity and process sensitivity describe linear systems in the frequency domain, or, in other words: the relation between \( e \) and \( w \) and between \( u \) and \( w \) should both be linear. By calculating the coherences between \( e \) and \( w \) and between \( u \) and \( w \) (that are measured during an experiment) one knows whether or not they are linearly related. So calculating those coherences is a check for the entire experiment.

As explained earlier the friction must be linear with the velocity. Otherwise the method described above to obtain the mass of the x-cart is not correct anymore since in that case we are trying to model a nonlinear system as being linear. Therefore the nominal (or desired) velocity of the x-cart must be well above 0 m/s. As a rule of thumb it can be said that the cart may not move backwards.

To keep the velocity of the cart constant a suited reference trajectory and a closed-loop control scheme are used: the controller makes the cart follow this trajectory and thus the velocity is almost kept constant.

The applied noise however can make the cart move backward. The applied noise is, as mentioned earlier, necessary for the experiment. So the reference trajectory may not result in a low velocity that, when noise is applied, makes the cart move backward. On the other hand the noise must be noticeable to identify the frequency response function of the plant. This trade off is also influenced by the controller (how well is the controller able to let the cart follow the reference trajectory despite the noise) which is discussed below.

The only part of the control scheme that is not yet discussed is the controller. This part of the control scheme has to stabilize the plant and make the cart follow the reference trajectory. These requirements can easily be accomplished with different types of controllers. But to make this experiment a success the controller has to meet more demands.

The controller affects both the sensitivity function \( S(s) \) and the process sensitivity function \( PS(s) \). As mentioned before, the sensitivity describes the relation between the noise \( w \) and the system input \( u \). Equation (3.8) expresses the fact that the controller influences this sensitivity. The controller tries to keep the effect of the noise as small as possible (how well it can do this depends on its bandwidth and its gain) by adjusting its output.

The process sensitivity describes the relation between the error \( e \) and the noise \( w \). Equation (3.9) expresses the fact that the controller influences the process sensitivity.
Figure 3.2 shows a (general form of a) sensitivity function of a closed-loop control system and figure 3.3 shows a (general form of a) process sensitivity of a closed-loop control system. These figures are not representative for the experiments described in this report but are only introduced to explain the trade off.

![Figure 3.2: Sensitivity.](image1) ![Figure 3.3: Process sensitivity.](image2)

The sensitivity and process sensitivity can oppose each other. For example, figure 3.2 and 3.3 show that for a frequency of 1000 rad/s the absolute value of the sensitivity (the relation between $u$ and $w$) is near 1 and this relation can readily be measured. The absolute value of the process sensitivity (the relation between $e$ and $w$) is much lower. Therefore the process sensitivity can not be measured very accurately. Consequently, the frequency response function of the plant ($H(j\omega)$) can not be measured in an accurate way and the required inertia can not be estimated accurately. By changing the controller in a control scheme the sensitivity and process sensitivity transfer functions change which can result in a good measurement. This trade off became clear during the simulation tests. By tuning the controller in a way that it takes care of this trade off the simulation resulted in a working identification method.

### 3.3 Summary

To experimentally identify the required inertia, the machine part in question is moved with a constant velocity by means of feedback control. Noise is applied on top of the control signal and the noise $w$, the system input $u$ and the error $e$ are measured. Using these signals the sensitivity and process sensitivity are determined. The sensitivity and process sensitivity can be used to determine the frequency response function of the plant, $H(j\omega)$. The only unknown that remains is the required inertia and which can be calculated.

The moving object (the carts and the pendulum) should not move with very low velocities because then the friction will not be linear with the velocity anymore. The mass is calculated at frequencies that are high enough to ensure that the influence of the (linear damping) friction on the transfer function is negligible.

In chapter 4, the simulation (that was conducted to test the identification method and to gain skill in using this method) is described. This simulation-study illustrates the method that is presented in this chapter.
Chapter 4: Simulation results

To test the identification method simulations are performed in Simulink. To simulate a measurement it has to be possible in the Simulink file to:

- let a plant (the x-cart) follow a reference trajectory that results in a constant velocity using a control scheme,
- be able to apply noise,
- gather the required signals ($e$, $u$ and $w$).

This can be done by a control scheme similar to that presented in figure 3.1. When the required signals are measured, the mass of the cart can be calculated with the use of Matlab as described in chapter 3. Appendix C shows the essential parts of the used Matlab-script.

First the simulation setup is explained, then the results are presented and finally the results are discussed.

4.1 Setup

Figure 3.1 shows a standard closed-loop control scheme. The figure shows the controller, the applied noise, the plant and the input (a reference trajectory). This scheme is also used in the simulation of the experiment. In the next sections, the parts of the control scheme are elucidated one by one.

Plant: $H(s)$

The equation of motion for the x-and y-cart is given by (3.5) in the Laplace domain and by (3.6) in the frequency domain. As mentioned in chapter 3 the effect of friction can be neglected for higher frequencies. Therefore the x-cart is modelled as a system described by (3.5) (in the Laplace-domain).

In this simulation the mass of the cart is taken 20.3271 kg, a value taken from [2]. Of course it is not known whether this mass corresponds with the actual mass of the x-cart.

The input of the plant is of course a force in Newton. Its output is the position of the cart in meters.

Input: $x_s$

The input of the system ($x_s$ in figure 3.1) has to be a trajectory that, when followed by the cart, results in a constant velocity for the most of the trajectory (see page 10). The x-cart can move backward and forward on the x-beam. Therefore a simple repeating sequence (that is available in Simulink) can be used. This signal however returns back to its starting point in zero seconds. This is of course not possible for any part of the H-bridge. Therefore a special reference trajectory generator (for more information: [5]) was used. This program generates a signal that, when followed, lets the cart move backward and forward on the beam with a constant velocity. For the simulation and experiments this constant velocity was 0.1 m/s, a velocity that the H-bridge can easily perform. At the ends of the beam, where the cart turns around, the cart gently slows down and starts accelerating in the other direction. Because the braking and accelerating is done gently (it is a third-order trajectory) the motors are not heavily loaded.

Figure 4.1 shows, for a period of ninety seconds (the duration of the simulation), the trajectory generated by the reference trajectory generator. This is the input. Except for the turn arounds, the position increases linearly with time, which corresponds with a constant velocity. The simulation lasts for ninety seconds to obtain enough measurement data. Figure 4.2 shows a close-up of one of the turn arounds. It can be seen that the cart slows down and...
speeds up gently. During the turn arounds the velocity is indeed not constant but this is for such a short time compared to the part where the velocity is constant that it does not affect the simulation or experiments significantly.

Controller: $C(s)$

To stabilize the plant and to ensure that it will trace the reference trajectory, a wide variety of controllers can be used. With this simulation a lead-controller is used. The general form of a lead (or lag) controller is given by the following transfer function:

$$C(s) = K \frac{1}{\frac{1}{2\pi f_1} s + 1} \frac{s + 1}{\frac{1}{2\pi f_2} s + 1},$$

with $f_1$ and $f_2$ a frequency both in Hertz. For $f_1 > f_2$ it is a lag controller which gives phase lag and for $f_1 < f_2$ it is a lead controller which gives fase lead.

For this simulation the bandwidth of the controller is chosen to be 30 Hz. Therefore $f_1$ is set at 10 Hz and $f_2$ is set at 90 Hz. The gain $K$ is taken to be 200000 [N/m] to make the crossover frequency 30 Hz.

This controller stabilizes the plant and it enables the plant to follow a prescribed trajectory where the error $e$ converges to zero (without applied noise). With applied noise, the velocity of the cart does not come close to zero (except at the turn arounds). So this controller meets the demands that are required for a successful simulation of the identification method.

Noise: $w$

For the noise generation, a random number generator with an average of 0 [N] and variance of $5^2$ [N$^2$] was used. Figure 4.3 shows the output of the random number generator during the simulation.

The variance of the noise is a parameter that easily can be adjusted. This simulation is of course very accurate: Matlab ‘detects’ very little disturbances in the trajectory of the cart that result from the noise when estimating $H(j\omega)$ because Matlab is very accurate with calculations. Therefore the variance is a lot smaller then in the real experiment where the influence of a little bit noise is not detected. This because the encoders of the H-bridge do not have such a ‘high resolution’ as Matlab.
Matlab and Simulink

Appendix C shows the relevant part of the Matlab file to give an idea how the required parameter is calculated. The Simulink model is similar to figure 3.1.

During a period of ninety seconds the controller made the plant follow the reference signal while the noise was applied and the required signals were obtained. The results of the simulation are presented below.

4.2 Results

Figure 4.4 is a Bode magnitude plot displaying the absolute value of $H(j\omega)$ that was
calculated with the signals that resulted from the simulation (the red line). A Bode magnitude plot of a plant $1/\text{ms}^2$ has also been drawn in the figure (blue line), with the mass $m$ the same as used in the simulation (blue line). These lines should therefore ideally coincide. Figure 4.4 displays the coherence between the noise $w$ and the plant input $u$ (the sensitivity) and figure 4.5 displays the coherence between the noise $w$ and the error $e$ (the process sensitivity).

The used Matlabfile calculates the measured mass of the cart using the frequencies where the coherences are (almost) one. Then it is certain that reliable data are used to calculate the mass of the cart. The estimated mass of the cart is 20.4685 kilogram (obtained by using the identification method, described in chapter 3, in this simulation). The mass was taken to be 20.3271 kilogram in this simulation (see paragraph 4.1 setup). So, the mass obtained from the identification is quite close to the actual mass.

4.3 Discussion

The frequency response function $H(j\omega)$ estimated by the identification coincides with the Bode-magnitude plot of the $1/\text{ms}^2$ system for frequencies between 100 and 1000 rad/s. Therefore, we can conclude that the used identification method works in a simulation setting. The coherence of the sensitivity and process sensitivity confirm this, for frequencies higher than the bandwidth of the controller the coherences increase. The coherence of the process sensitivity becomes one for frequencies above 500 rad/s. The coherence of the sensitivity already becomes one for lower frequencies than 500 rad/sec.

The controller is able to correct for the noise of frequencies that are below its bandwidth. Then the reference trajectory is still being followed by the cart. Noise of frequencies higher than the bandwidth become a problem. The controller can not make the cart follow the reference trajectory anymore. The noise sufficiently perturbs the plant and by measuring the reaction of the plant to the noise, the frequency response function of the plant (and with that the mass of the cart) can be determined. This can also be seen in Bode magnitude plot (the lines start to coincide) and in both coherence-plots (the coherence becomes one) at frequencies of the noise at which the controller can not suppress the noise anymore.

This simulation has also been done with other controllers: lead-controllers with a lower bandwidth and a lower gain. Those simulations showed that a controller with a lower gain results in a more accurate result. This makes sense: a controller with a low gain (and thereby a lower bandwidth) can not suppress the noise input as well as a controller with a higher gain. The controller with a higher gain is able to correct noise of higher frequencies than the
controller with the lower gain. With a controller with a lower gain more noise sufficiently perturbs the closed-loop system; so more information about the plant becomes available. These simulation results confirm that the used identification method can be used to accurately identify masses and moments of inertia of the system.
Chapter 5: Experimental results

In this chapter the experiments to identify the mass of the x- and y-cart are described and the related results are presented and discussed. This is done in a similar way as was done for the simulation: first the experimental setup is described followed by a presentation and discussion of the experimental results.

5.1 Identification of the mass of the x-cart

5.1.1 Experimental setup

Almost the same control scheme as in figure 3.1 was used for the experiment, only minor adjustments have been made. The biggest difference is that the plant now of course is the actual H-bridge. This means that no longer the system input $u$ is a force in Newton but a signal consisting of three currents: one for the x-motor and one for each y-motor. The factor that converges a current into a force is the motor constant (see chapter 2). The noise $w$ has to be converted to a current also by using the motor constant.

The Simulink file is again similar to figure 3.1. Below, the different parts of the control scheme are explained.

**Plant: $H(s)$**

The plant is now, as mentioned above, the actual H-bridge. B.R.A. Janssen [2] designed a block in Simulink that could be used for controlling the H-bridge. The input is a current that makes the H-bridge move. The output is of course the movement of the H-bridge. It is possible to obtain information about the movement of the H-bridge (like the position, velocity, acceleration etcetera) from this block. To identify the mass of the x-cart the only information that is required, obtained out of this Simulink block, is the position of the x-cart. More information about this Simulink block can be found in [2].

The pendulum of the H-bridge was fixed; therefore it can not move (rotate). When it would rotate due to the movement of the x-cart this would influence the movement of the x-cart and with that the identification of the mass of the x-cart.

**Input: $x_s$**

Exactly the same input as the input of the simulation of the identification of the mass of the x-cart was used. This trajectory generator, when perfectly followed, makes the x-cart move back and forth from one side of the beam to another side. For a more detailed description and figures of the trajectory, see chapter 4.

The y-cart had to maintain its fixed position. Therefore the reference input of the y-cart was a constant position.

The cogging-compensation (for the x-cart) was activated during the experiment.

**Controller: $C(s)$**

The controller now consists of three controllers: one for the x-cart, one for the Y1-motor and one for the Y2-motor. Since the Y-motors only had to keep their position during the experiment, controllers designed by B.R.A. Janssen were used [2]. The x-controller however is more complicated.
A wide variety controllers for the x-cart were used to obtain good results. Some examples are lead-controllers (all with different bandwidths), PD-controllers, a combination of a lead-filter and a PID-controller (made by [2]), et cetera.

The controller determines, as explained in chapter 3, how well the reference trajectory can be followed and how much the noise affects the movement of the cart. The trade off, mentioned earlier in chapter 3, is the major problem when designing a controller. The best results were accomplished with a lead-controller with a bandwidth of 6 Hz. With this controller the trajectory was followed sufficiently close (the velocity of the cart did not come close to zero) and the effect of the noise was large enough to ‘notice’ a linear relation between the tracking error and the noise. The parameters of the controller are (see equation (4.1)) are: $K = 10000 \text{[N/m]}$, $f_1 = 2 \text{[Hz]}$ and $f_2 = 18 \text{[Hz]}$. The gain $K$ is of course converted into a current by using the motor constant.

This controller stabilizes the plant and results in a satisfying compromise between following the reference trajectory and ‘noticing’ the noise; so all the demands are met for a successful experiment.

**Noise: $w$**

Because the effect of noise of a very low level is not being noticed anymore, the noise had to be of a higher level than with the simulation. A signal with an average of 0 [N] and a variance of 500 [N$^2$] was used.

**Matlab and Simulink**

The Matlab file and Simulink model used for this experiment are quite similar to those of the simulation. The Matlab file is again an implementation of the identification method presented in chapter 3. The basis of the Simulink file is again similar to figure 3.1; adapted to conduct the experiment. The basis of the Matlab file is presented in appendix C.

During three minutes the controller made the plant follow the reference signal while the noise was applied. The required signals were measured and they were used by the Matlabscript to identify the mass of the x-cart. The results of the experiment are presented below.

**5.1.2 Results**

Figure 5.1 displays a Bode magnitude plot of the absolute value of the measured frequency response function $H(j\omega)$ (the red line). This is the frequency response function related to the x-cart of the H-bridge. The blue line is a bode magnitude plot of 1/ms$^2$ with $m$ the mass of the x-cart that was experimentally identified.
Figure 5.1: Bode magnitude plot of the x-cart.

Figure 5.2 and 5.3 display the relevant coherences, the coherence of the process sensitivity and the coherence of the sensitivity.

The mass is calculated using the frequencies with the highest coherences (from 80 to 600 rad/s). This results in a experimentally identified mass of 18.13 kg.

5.1.3 Discussion

It can be seen that the coherence of the process sensitivity is not as high as in the simulation results. The linearity of the relation between the error $e$ and the noise $w$ is not as obvious as the linearity of the relation between the plant input $u$ and the noise $w$. 
The mass of the x-cart was determined by B.R.A. Janssen by weighing the parts on a scale and by an estimate of Unigraphics (see [2]). Both identified the mass as being more than 20 kg. The experimentally identified mass therefore, is certainly not correct. The coherence of the process sensitivity emphasizes the fact that the reliability of the obtained estimate is rather limited.
Moreover, around a frequency of 105 rad/s, a strange phenomena occurs. This is no resonance since a resonance is a linear phenomenon and the coherence of the process sensitivity drops to low value around this frequency [3].
When the frequency increases, it is also remarkable that around 105 rad/s the line in the Bode-plot does continue on a smooth way. At 105 rad/s it jumps from 118 dB to 115 dB and continues with the same slope as the slope of the plot for frequencies smaller than 105 rad/s.
Resuming it can be concluded that the experiment to identify the mass of the x-cart is no reliable measurement since the coherence of the process sensitivity is too low to be reliable.

The quality of the experimental identification of the mass of the x-cart would improve when the coherence of the process sensitivity increases. Therefore, the relation between the error and the noise has to be guaranteed. This can be done by using a more suited controller: a controller that cannot correct the noise very well but that makes sure that the cart does not move backward. During this experiment we did not succeed to design such a controller, despite a lot of effort.
Because we adopted several assumptions about the friction which are possibly not met, compensating for the friction will also improve the experiment. This can be done activating friction compensation (chapter 2).

5.2 Identification of the mass of the y-cart

5.2.1 Experimental setup
An almost similar control scheme is used as discussed in the previous section. Again the parts of the control scheme are discussed below.

Plant: $H(s)$
With this experiment the y-cart is moved instead of the x-cart. Therefore, a current is applied to the motors Y1 and Y2. The average of the system input of motor Y1 and Y2 as well as the average of the error of Y1 and Y2 are measured. Therefore, the moving y-mass (which is driven by two motors) can be seen as one moving inertia.

Input: $x_s$
The input for the x-cart was of course the position in the middle of the x-beam to ensure that both y-motors were loaded (in theory) equally.
The input of the y-motors was almost the same as the input of the experiment to identify the mass of the x-cart. The only difference are the begin- and endpoints of this trajectory. The Y-carts moved back and forth on their beam from one side of the beam to another. The velocity was again 0.1 m/s, a velocity that the H-bridge can easily realize.

Controller: $C(s)$
For the x-cart the same controller as in the simulation was used. The y-motors used the same controller. This controller was designed by B.R.A. Janssen and is a combination of two lead-controllers in series. In appendix C, the controller that was used in
this experiment is discussed. The original gain is lowered to enhance the influence of the noise on the plant.

**Noise: w**
On both the Y-motors the same noise was applied as the noise that was applied on the x-motor during the experiment to identify the mass of the x-cart.

**Matlab and Simulink**
The Matlab file and Simulink model are almost similar to the files used for the identification of the mass of the x-cart but slightly modified for the identification of the y-mass, see figure 3.1 and appendix B.

Again, during three minutes the controllers made the plant follow the reference signal while the noise was applied. The required signals were measured and were used in the Matlabscript to estimate the mass of the y-cart. The results of the experiment are presented below.

### 5.2.2 Results
Figure 5.4 shows the Bode magnitude plot of the absolute value of the measured frequency response function $H(j\omega)$, (the red line). This is the frequency response function related to the y-cart of the H-bridge.

![Bode magnitude diagram of the y-cart](image_url)

*Figure 5.4: Bode magnitude plot of the y-cart.*

The blue line is a bode magnitude plot of $1/\text{ms}^2$ with $m$ experimentally identified.

Figure 5.5 and 5.6 show the relevant coherences:
Again the mass was determined using information in a frequency range with a high coherence of the process sensitivity and sensitivity: from 100 rad/s to 500 rad/s. This results in an experimentally identified mass of 71.74 kilogram.

5.2.3 Discussion

These results are, in contradiction with the results of the experiment to identify the mass of the x-cart, reliable. This is supported by the fact that the coherences are close to unity in the frequency range used in the identification. B.R.A. Janssen also estimated this mass around 70 kg what corresponds with this experiment.
Chapter 6: Conclusions and recommendations

6.1 Conclusions

The goal of these experiments is to experimentally identify the inertias of the H-bridge. These inertial parameters are used in the output regulation controllers, see [2]. The identification of each parameter is discussed below.

The identification of the mass of the y-cart was performed successfully. The results are reliable as indicated by the coherences of the frequency response functions used in the identification process. Moreover, the results correspond with a estimation of B.R.A. Janssen [2].

The obtained result of the identification of the mass of the x-cart is not reliable, indicated by the low coherence between the error $e$ and the noise $w$ (the process sensitivity). B.R.A. Janssen identified the mass by weighing parts of the x-cart [2]. The mass should be between the 20.5 and 21 kg according to B.R.A. Janssen [2]. The result of the experiment that is described in this report was that the x-cart weighs 18.13 kg which is not correct when compared to the result of B.R.A. Janssen [2].

Phenomena that, until now no explanation has been found for, are present in the results of the experiment to identify the mass of the x-cart. A non-linearity is most probably present and the Bode-magnitude shows a strange phenomena around 105 rad/s (see section 5.1.3). A wide variety of controllers have been tried to obtain more reliable results but without success: the coherence of the process sensitivity does not increase and the non-linearity as well as the shifting of the line both remain present in the results.

6.2 Recommendations

To improve the output regulation controllers the moment of inertia of the pendulum has to be identified. When the moment of inertia of the pendulum is known more accurately a higher performance of the output regulation controllers may be attained.

A better controller for the x-cart must be designed to improve the quality of the identification of the mass of the x-cart. This controller should not suppress the noise to ensure that the x-cart does not move backward. Then, the relation between the noise and the error becomes more clearly linear and the coherence between those signals will become higher.

Cogging- and friction compensation for the x- and y-carts will improve both experiments. Models for the friction and the cogging are available for the x-cart. For the y-cart these should be developed.
Appendix A: Derivation of the sensitivity and process sensitivity

Figure A.1 shows a standard closed loop control scheme with $x_s$ the input, $e$ the error, $C(s)$ the controller, $w$ the noise, $u$ the system input, $H(s)$ the plant and $x$ the output. The sensitivity given by equation (3.6) is derived as:

$$u = w + Ce = w + C(x_s - x) = w + C(x_s - Hu)$$
$$u + CHu = w + Cx_s$$
$$u \frac{1}{1 + CH} = w + Cx_s$$
$$u = \frac{1}{1 + CH} w + \frac{1}{1 + CH} x_s$$

where the left term on the right side of the equal sign of the last equation in (A.1) represents the sensitivity.

The process sensitivity is derived below:
$$e = x_s - x = x_s - Hu = x_s - H(w + Ce)$$
$$e + HCe = x_s - Hw$$
$$e(1 + HC) = x_s - Hw$$
$$e = \frac{1}{1 + HC} x_s - \frac{H}{1 + HC} w$$

where the term most right on the left of the equal sign of the last equation in (A.1) represents the sensitivity.
Appendix B: Matlabscript

Below the (relevant part) of the used Matlabfile is presented to give an idea of how the simulation and experiments were conducted. First the data is loaded, then parameters for the calculation of the coherences and transfer function are defined, then the coherences and the transfer function are calculated and last but not least the mass of the cart is calculated.

```matlab
close all;
load u.mat
load w.mat
load y.mat
load e.mat;
load r.mat
u=u(2,:);
w=w(2,:);
y=y(2,:);
e=e(2,:);
r=r(2,:);

fs=1000;
tijd=[0:0.001:90];

% parameters
Np=14;
Npoints = 2^Np;
Noverlap = 2^(Np-5);

% calculate coherence
[CS,F] = cohere(w,u,Npoints,fs,[],Noverlap);
[CPS,F] = cohere(w,e,Npoints,fs,[],Noverlap);

% calculate measured frf
[S,F]  = tfe(w,u,Npoints,fs,[],Noverlap); % sensitivity: 1/(1+PC)
[PS,F] = tfe(w,e,Npoints,fs,[],Noverlap); % -process sensitivity: -P/(1+PC)

PS = -PS;

% calculate system frf
H = PS./S;

% Massa berekenen
mberekend=1./(Habs(2:50).*(2.*pi.*F(2:50)).^2);mean(mberekend)
mberekend=1./(Habs(410:650).*(2.*pi.*F(410:650)).^2);mean(mberekend)
```
Appendix C: Y-controller of the y-mass identification

Figure D.1: Y-controller of the measurement of the mass of the y-cart
Bibliography


