Aliasing in PNAH

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1 Introduction

1.1 What is nearfield acoustical holography?

Holography is a technique which can be applied in three dimensions. It is an imaging technique, using any waveform in the electro-magnetic spectrum. In history, the first thing ever done in this area, was lightwave holography, developed by Dennis Gabor [1]. He was trying to create a more detailed model of an electron microscope. His technique records light waves, which were emitted and reflected by the object of interest. By re-emitting laser-light on the developed film, it re-creates all the points of light that originally came from the object. The result, a hologram, has all the dimensions of the original object.

Hologram is a contamination of the Greek words “holos” (whole) and “gramma” (message).

If it could be done by light waves, it also could be done by sound waves, because both waves are part of the electro-magnetic spectrum. Acoustical holography used light wave holography as a basis and has first be mentioned by Hildebrand and Brenden [2] in 1974. It is an approximation to the inverse problem of the reconstruction of sound fields. By measuring the pressure at a certain distance of the object, called the hologram plane, it is possible to determine the pressure field, in both space and time, between the object (the source) and the plane. Using this technique it is not possible to detect source details smaller than the acoustic wavelength. A solution to this problem was brought up in 1980 by Williams and Maynard [3]. They introduced NAH, Nearfield Acoustic Holography. The resolution of this method was infinite in theory, due to the fact that NAH could solve the inverse problem exactly. The prefix “nearfield” has been derived from the fact, that the hologram is measured in the nearfield of a sound source making it possible to observe evanescent waves. It is also possible, to determine the intensity and particle velocities in the three dimensional space at the source plane.

Some applications of this method is for example the detection of vibration modes in large plates, or looking for the part, that causes high noise in computers or even mobile phones.

Figure 1-1: example of NAH: looking for which condensator causes high noise in mobile phones

Figure 1-2: Using NAH to visualize a plate mode. The red dots are maximum in amplitude (left). Plate mode visualized (right).
2 Theory of sound waves

2.1 Some quantities of sound

Sound waves can travel through a number of media, for example water and air. In these substances, the velocity of a wave through this medium is depending on what kind of material it is. In this project, the focus will be on sound waves propagating through air. As sound travels through air, it disturbs its surroundings, characterized by the deviations from the average pressure $p$ and average particle velocity.

Waves travel through air at a speed of $c_0$ at T=20.

The particle velocity is the speed of the particles at the time the disturbance passes.

The acoustic wave equation for an infinitesimal change in acoustic pressure from its equilibrium value is given by [4]:

$$\nabla^2 p - \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} = 0$$  \hspace{1cm} (2.1)

$\nabla^2$ is known as the Laplacian operator. This wave equation shows two dependences. Firstly, this equation depends on time: when we stand at a fixed point in the three-dimensional space we can check the domain behavior. Secondly we can freeze time and walk along a certain path in three-dimensional space.

One great advantage of sound waves is the property of linearity, which means that the effects of different sources of disturbance on one position can be added. Using this property makes it possible to separate a pressure signal in a number of sine signals and process them separately. One well-known method for describing the total sound pressure signal in separate sine waves, is the Fourier transform:

$$\tilde{p}(\omega) = \int_{-\infty}^{\infty} p(t) e^{-i\omega t} dt$$  \hspace{1cm} (2.2)

where $p(t)$ is a measured sine wave with angular frequency $\omega = 2\pi f$. The outcome of the Fourier transform $\tilde{p}(\omega)$, is a complex value of the pressure of the measured sine wave. When we want to study the spatial behavior of this $\tilde{p}(\omega)$, we need to get our hands on the second derivative of the pressure with respect to time.
Firstly we derive the inverse Fourier transform of $\tilde{p}(\omega)$:

$$p(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{p}(\omega)e^{i\omega t} d\omega$$  \hspace{1cm} (2.3)

when this equation above is differentiated with respect to time twice, we get the following relationship:

$$\frac{\partial^2 p}{\partial t^2} = \frac{1}{2\pi} \int_{-\infty}^{\infty} -\omega^2 \tilde{p}(\omega)e^{i\omega t} d\omega$$  \hspace{1cm} (2.4)

Substituting this relationship in the wave equation and using $\frac{1}{c^2} = \frac{k^2}{\omega^2}$ delivers:

$$\nabla^2 p + k^2 \tilde{p} = 0$$  \hspace{1cm} (2.5)

also known as the Helmholtz equation. The acoustic wave number, $k$ is defined as $\frac{\omega}{c}$.

Because it is allowed to process sound waves separately and adding them at the end to get the global solution, we will now take the Fourier transform of a sound wave with a single angular frequency $\omega$ and look at the spatial properties of this wave. The wavelength $\lambda$ can be measured in this way and is defined as:

$$\lambda = \frac{2\pi}{k} = \frac{c_0}{f}$$  \hspace{1cm} (2.6)

This is easy to understand. In order to describe, study and finally process a wave pattern far more complex we are going to use the wavenumber domain. For one wave, the determination of the wavenumber domain spectrum, of $k$-space is relatively easy. We are using two separate planes, the $x$- and $z$- plane, and determine the wavelength parallel to each of the axis, resulting in $\lambda_x$ and $\lambda_z$. 
In general the following relationship exists between the mentioned wavelengths:

\[ \lambda = \lambda_x \sin \theta = \lambda_z \cos \theta \]  \hspace{1cm} (2.7)

These separate waves also have a spatial frequency, called the wavenumber and can be derived as follows:

\[ k_x = \frac{2\pi}{\lambda_x} \sin \theta = k_x \sin \theta \] \hspace{1cm} \text{in x-direction} \hspace{1cm} (2.8)

\[ k_z = \frac{2\pi}{\lambda_z} \cos \theta = k_z \cos \theta \] \hspace{1cm} \text{in z-direction} \hspace{1cm} (2.9)

where \( k \) can be expressed as function of \( k_x \) and \( k_z \) as:

\[ k = \sqrt{k_x^2 + k_z^2} \] \hspace{1cm} (2.10)
By doing this, it is possible to draw \( \mathbf{k} \) in \((k_x, k_z)\)-space, which is shown in figure 2-1, but more important, due to the linearity property of sound waves, it is possible to add these results in the wavenumber domain.

This method is a nice tool, when you are studying situations like mentioned above. But when you want to determine the k-space of a more complex wave pattern, it is necessary to use the spatial Fourier transform.

When looking only at the x- direction, the integration variable \( t \) changes into \( x \):

\[
\hat{p}(k_x) = \int_{-\infty}^{\infty} p(x) e^{-j k_x x} \, dx \tag{2.11}
\]

Because we are dealing with two dimensional space, also the z- direction is included in the formula:

\[
\hat{p}(k_x, k_z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(x, z) e^{-j(k_x x + k_z z)} \, dx \, dz \tag{2.12}
\]

\( \hat{p} \) is a result of two different Fourier transforms: a one dimensional time domain and a two dimensional spatial-domain Fourier transform.
2.2 Evanescent waves

Sound waves obey the wave equation, see equation 2.1. The basis of NAH is a property of the radiation of these waves: near- and farfield of a sound emitting source. The difference between these two is the ability to observe evanescent waves in the nearfield and the disability of it in the farfield. The nearfield is the field, which extends from the source up to the point where the evanescent waves are still observable, after this point, the acoustic field is called the farfield. When we want to accomplish a high resolution, we must detect evanescent waves. In the farfield, evanescent waves are not observable anymore, because of the noise field, so it is necessary to be in the nearfield.

![Diagram showing nearfield and farfield with evanescent waves sensitive to noise](image)

Figure 2-3: Visualization of the definition of nearfield and farfield. Also explaining, why evanescent waves are very sensitive to noise

So the distance from the source is getting larger, it is more and more complicated to measure the evanescent waves. You detect a lot of noise, if the distance from the source is large, but when you calculate the inverse solution of the sound problem, your amplitude of the evanescent waves and the measured noise are blown up with the exponent. So a little bit of noise, can be a large disturbance at the source plane.
### 2.3 From the hologram plane back to the source

Herein the discussion of the derivation of three-dimensional extrapolation of acoustic properties is made. We need to have an infinitely large hologram plane and a source-free half space. There are not any sources allowed between the source and the hologram plane. When this is the case, a three-dimensional extrapolation of acoustic properties on an infinite number of parallel planes can be achieved.

When looking at the area between the source and the hologram plane, an unknown steady state pressure distribution has to meet the Helmholtz equation. Solving this equation, a solution is:

\[
\tilde{p} = (x, y, z, \omega) = A(\omega)e^{i(k_x x + k_y y + k_z z)}
\]  

(2.13)

The relation above is only valid for:

\[
k^2 = k_x^2 + k_y^2 + k_z^2
\]  

(2.14)

where \(k^2\) is a function of the angular frequency and the wave speed. Because, \(\omega\) is known due to frequency domain description of the sound pressure and \(k_x, k_y\), are known because of spatial Fourier transform, this leaves \(k_z\) to be the only variable.

Rewriting the equation above, leaves for \(k_z\):

\[
k_z = \pm \sqrt{k^2 - k_x^2 + k_y^2}
\]  

(2.15)

All waves of the source are traveling in positive z-direction, so all negative solutions for \(k_z\) can be ignored. \(k_z\) has three important solutions:

1) \(k_x^2 + k_y^2 = 0\) wave traveling only in z-direction; \(k_z = k\)
2) \(0 < k_x^2 + k_y^2 \leq k\) propagating wave: \(k_z\) is real 
3) \(k_x^2 + k_y^2 > k\) evanescent wave: \(k_z\) is complex

(2.16)
The first possibility is, when the wave component has only a value in z-direction, and the values in the (x,y)-plane equal zero. The second possibility is, that the real solutions to \( k_z \) are plane waves traveling in a direction away from the z=0 plane in the positive z-direction. When the last possibility is true, the waves are called evanescent waves, meaning, that their components in the (x,y)–plane are larger than the acoustic wavenumber, \( k \).

When we have a measured hologram, say at distance \( z_h \) of the source, it is possible to tell something about the source. The knowledge of \( k_z \) can be used when we have a measured hologram. This hologram contains information about \( k_x \) and \( k_y \) and thus \( \hat{p}(k_x,k_y) \). The relationship between the Fourier transform of the pressure in a plane at z=0 and the transform of any given plane at \( z=z_h \) is:

\[
\hat{p}(k_x, k_y, z, \omega) = \hat{p}(k_x, k_y, 0, \omega)e^{i k_z z} \quad (2.17)
\]

If we know the Fourier transform of the pressure at \( z=z_h \), we can write:

\[
\hat{p}(k_x, k_y, z, \omega) = \hat{p}(k_x, k_y, z_h, \omega)e^{i k_z (z-z_h)} \quad (2.18)
\]

where \( z \) is the distance in the half space ( \( z>0 \)). Here can be seen, why it is so difficult to measure evanescent waves. As said before, evanescent waves can be characterized with an imaginary \( k_z \). When filling in this value, it can be seen, that the exponent affects the amplitude: the imaginary \( j \) drops out of the exponent, resulting in \( e^{-|k_z| z} \). The amplitude of an evanescent wave is degrading by a negative exponent the further the distance to the source plane.

Now we can express the pressure as a function of position and time by doing an inverse Fourier transform on the static frequency domain pressure. In k-space extrapolating the pressure distribution is a relatively easy calculation, which is a big advantage of this domain. In spatial domain, the expression is a triple integral, according to [4]:

\[
p(x, y, z, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\omega_{	ext{max}}}^{\omega_{	ext{max}}} \hat{p}(k_x, k_y, z, \omega) e^{i (k_x x + k_y y - \omega t)} dk_x dk_y d\omega \quad (2.19)
\]

Figure 2-4: example of a pressure-measurement, in this case a trilling mode of a flat plate in 2-D (left) and 3-D (right).
2.4  **Rayleigh’s integral**

For numerical purposes the previously derived Fourier acoustics together with Rayleigh’s integral [5] can be used to determine near- and farfield properties of a known planar velocity distribution at the source plane. For solving radiation problems, where the normal velocity is known at the source plane, for example vibration plates or pistons, the most important and widely used formula is Rayleigh’s first integral:

\[
p(x, y, x) = -\frac{j\rho_0 c k}{2\pi} \int \int v_\xi(x', y', z') \frac{e^{j\sigma}}{\sqrt{-\sigma}} dx'dy'
\]

(2.20)
3 Digital Processing

3.1 Taking the next step

The last chapter was about the analogue acoustic part of the planar nearfield acoustic holography process. The remaining part, resulting in acoustic images, consists of digital acoustic signal processing. This takes several steps, explained in this chapter, but first the sampling theorem is introduced. This lays a firm basis for the rest of signal processing topics.

3.2 The sampling theorem

Sampling can be seen as a digital form of a continuous signal. It is thinkable, that there is a minimum in the number of digital points needed, to construct the measured signal again, without losing some valid information. When taking a closer look at this technique, two men played a large role in developing it, namely Nyquist and Shannon. They developed “The Nyquist Sampling Theorem” or also called “The Sampling Theory”.

Because the calculations described in the former chapter are so complex, they must be done by computer and so sampling is needed in the analogue to digital conversion. As said before, converting a analogue signal to a digital signal isn’t without complications and has several parameters. These parameters control the number of information and the resolution.

When sampling in the time domain, we need to know the maximum signal frequency $f_{\text{max}}$ that we want to process in the discrete time domain. Together with the minimum frequency we can calculate the bandwidth $W$. Than it is possible to determine the minimal sampling frequency in order to still detect $f_{\text{max}}$. It is defined as the Nyquist frequency:

$$f_{\text{nyquist}} = 2f_{\text{max}}$$  \hspace{1cm} (3.1)

In the spatial domain, microphones are doing the spatial sampling. They are set up in an array or can be moved along a certain area. The total numbers of microphones per meter, which all do a pressure measurement, is the sampling frequency in the spatial domain. Each measurement has its own place in a matrix, resulting in a full matrix, containing the whole pressure field in 2-D at a certain place. Because all signals are now digitalized, all digital image processing techniques can be used.
3.3 Propagator

The reason of switching between the spatial domain and the k-space domain is the form of the propagation. The propagation is a formula, which can allow you to calculate the Fourier transform of the pressure in any given point, given the solution on the hologram plane. The advantage of the k-space product, is the relatively easy product in comparison to the convolution integral that has to be solved in spatial domain. In formula, the propagation in k-space is (2.18)

So when we have discrete acoustical hologram data, we can simply multiply this matrix with the propagator matrix. This matrix contains three parts:
- propagating waves
- evanescent waves
- residual waves

The propagating waves just undergo a phase change ($k_z$ is real), whereas the evanescent waves are multiplied with an exponential power, which depends on wavenumber and $z - z_h$. The residual waves is everything outside the circle with radius $r = k_{\text{max}}$

Figure 3-1: Visualizing the three kinds of waves in the k-space; the red color represents the propagating waves, the blue color represents the evanescent waves and the grey color represents the residual waves.
3.4 Windows

One of the problems of digital signal processing is the discrete Fourier transform. This transformation keeps repeating its solution. Leakage can occur, because mostly this takes a lot of step differences along the way and step functions contain very high frequencies. So it is necessary to force the measured data at the edges to zero, which is the basic idea of a spatial window. To control these high wavenumbers, the window has a smooth tapering from one to zero. The low-frequency-data isn’t affected due to the maximal amplitude of the window equals 1. This can be seen in the following figures: the difference between a ‘normal set data’ and the windowed set data; resulting in a different 2D FFT of these two types of data.

![Figure 3-2: Difference between a set data and a windowed set data; It can be seen, that a window forces the data to zero at the edges.](image)

![Figure 3-3: Difference between a set data and a windowed set data. This time in k-space-domain, the right one contains far less higher wavenumbers, due to the application of a window.](image)
The red cross in the left figure is a result of the repetitive character of the discrete Fourier transform. The Fourier transform repeats the solution from figure 3-2, like sketched below:

Figure 3-4: illustration of the repetitive character of the Fourier transform in spatial domain

At the edges of each solution the data is fluctuating very much, resulting in high step-functions. These step functions contain a lot of high wavenumbers, which is well visualized in figure 3-3 (left). The red color is also present at very high values of k. The right picture of figure 3-3 does not have this problem. A window ‘forces’ the data to zero, to avoid problems like described above. If the Fourier-transform repeats itself with a set of windowed data, no large step functions occur and no high k-numbers are introduced.
There are two common used windows for PNAH: the Hanning- and Tukeywindow.

The Hanning window has the best smoothening effect, but the data is seriously affected. Sources near the midpoint of the circle (topview) are over-graded compared to sources near the edges. The Tukey window on the other hand has a cosine-tapered part near the edges of a percentage of the total width in size. The centre part has a plateau resulting in a large area of unaffected data.

For PNAH it is very important to know how much energy through leakage is introduced to the evanescent waves in the spectrum. This evanescent waves are multiplied with an exponential power, and considerable amounts of leakage, which can be referred as noise, will blow up if present in the evanescent wavenumbers. Because high resolution can only be achieved by evanescent wavenumbers, the choice of a window is very important. You need to get rid of leakage, but the data must be affected.

Figure 3-5: Hanning window (left) and Tukey window (right). Typical topview of the windows, Hanning (left) and Tukey (right).

Figure 3-6: This figure displays the amount of high wavenumbers introduced by applying a window. Applying a Hanning window (left) and a Tukey window (right).
Applying the Hanning window does not introduce leakage to the data. Therefore the Hanning window has less higher wavenumbers compared to the Tukey window, which can cause a bad solution if you apply the Hanning window. In most practical cases a Tukey window introduces no high wavenumbers so the Tukey window is the first choice.
3.5 **Spatial zero-padding**

Zero-padding is used in the spatial and the k-space domain. First zero-padding is explained in the spatial domain. Zero-padding is a technique to up sample the Fourier transform and minimize the wraparound error. On the edges of the data, we add zero-valued samples. After the 2D-FFT this results in a up sampled k-space. The wavenumber bins, $\Delta k_x$ and $\Delta k_y$ are smaller. By virtually expanding the measurement area, $\Delta k_x$ and $\Delta k_y$ decrease in size.

![Zeropadding visualized, adding of zero-valued samples to the data.](image)

Zeropadding is also used to smoothen the results in the k-space domain. By virtually expanding the measurement area, $\Delta k_x$ and $\Delta k_y$ decrease in size.

However, zero-padding can also be used in the k-space domain. This is a relative new technique, see the following experiment. The difference between a non-zeropadded solution and a solution including zero-padding is shown in the figure below, first in k-space, then in spatial domain:

![Applying of zero-padding in k-space: left picture (10x10), right picture (30x30). Adding zero valued points at the edges of the measurement grid.](image)

![Calculated solution without zeropadding (left) and with zeropadding (right). Less high energy is introduced on the right side (white spots on the left side)](image)
It can be seen, that the solution is more detailed with zero-padding. The zero-padding factor, the parameter, which states how much zeros you add, must always be a power of two and larger than the data size. In this case, you add zeros, both left and right of the k-spectrum at the end of your data.

### 3.6 K-space filter

As mentioned in section 2.2 high frequencies have a high sensitivity to noise. In order to eliminate these frequencies, a filter operation is needed. It has to be a filter, which doesn’t affect the low frequencies, but suppresses the noise. The application of a low pass-filter is just the same as applying a Tukey window with the only difference that the k-space filter is rotation symmetric around $(k_x, k_y) = 0$

![Figure 3-10: example of a k-space low pass filter.](image)

With the use of a low pass filter, a dilemma is found. The resolution of PNAH depends on how many evanescent waves can be measured, but on the other hand, these waves are sensitive for noise. So the cut-off of the filter used, is very important, because this parameter determines how much k-numbers you use to calculate your solution with. The filter cut-off depends on the signal to noise ratio of the measured data and is defined as the point, where the filter gain is half the maximum value. The formula of $k_{co}$ is:

$$k_{co} = \sqrt{\frac{D^{10} \log 10}{20 (z_h^2 - z_s^2)^2 + k^2}}$$

(3.2)
So in an ideal case, we want all frequencies below $k_{co}$ to pass through and all higher wavenumbers to be blocked. In formula-form:

$$
H_{k,low} = 1 \quad \text{for} \quad \sqrt{k_x^2 + k_y^2} \pi k_{co}
$$

$$
H_{k,low} = 0.5 \quad \text{for} \quad \sqrt{k_x^2 + k_y^2} = k_{co}
$$

$$
H_{k,low} = 0 \quad \text{for} \quad \sqrt{k_x^2 + k_y^2} \phi k_{co}
$$

(3.3)

### 3.7 Resolution

As stated in section 3.4 and 3.7, the resolution of PNAH depends on the post-processing (think of the dilemma of the k-space filter). But also on the way the measurements are done (think of the Nyquist-Shannon theorem). There are different parameters, within these two, on which the resolution really depends. One of them is the sensor array density.

The sensor distance in an array or the distance between two grid positions, make up a certain $\Delta x$ and a $\Delta y$ and can be seen as a spatial AD-conversion. Herein is the Nyquist criterion also valid, so the minimum distance between two grid points determines the maximum observable wavenumber, $k_{max}$ or $k_{nyquist}$. The minimal observable wavelength becomes:

$$
\lambda_{min} = 2\Delta x
$$

(3.4)

So the highest observable wavenumber is:

$$
k_{max} = \frac{2\pi}{\lambda_{min}}
$$

(3.5)

The maximal resolution of a technique can be described by the minimum distance in which a change in information can be detected. Combining the two formulas mentioned above, gives us the resolution of the acoustic image:

$$
R_{max} = \frac{\lambda_{min}}{2} = \frac{\pi}{k_{max}} = \Delta x
$$

(3.6)

So firstly the maximum resolution of the PNAH is determined by the sensor distance between two positions. So, if the sensor distance is too large compared to the wavenumbers one is interested in, the process will fail and the resolution will not be adequate. This was all on the assumption, that there was no noise, but when we want to actually measure something, there will always be noise. So we need to look at that.

When there are no evanescent waves available for measurement the minimum resolution is determined by the free field acoustic wavelength:

$$
R_{min} = \frac{\lambda}{2}
$$

(3.7)
In practice, the resolution $R$ is between $R_{\text{max}}$ en $R_{\text{min}}$. The real resolution depends on two factors, namely the amount of noise and the existence of evanescent waves. By PNAH the measurements are always done in the near field of the source.

When looking at a measured hologram, there is always some errors caused by noise. Noise is a collector’s item for errors in the microphone, background sound, electric noise and calibration noise. Not the noise itself concerns us, but the difference between the measured signal and the noise, also known as the signal to noise ratio or dynamic range $D$. The dynamic range is defined by:

$$D = 20^{10} \log \left( \frac{M}{E} \right)$$

(3.8)

In this formula, $M$ is the measured field amplitude and $E$ the noise amplitude. This is, besides $\Delta x$ another important quantity for the resolution, because the dynamic range $D$ influences the maximum measurable wavenumber as a function of the hologram distance $(z_h - z_s)$. Let us assume evanescent and propagating waves at the source plane at $z_s$ with equal amplitudes $A$. When we want to measure these evanescent waves, which amplitude decreases with the exponent of the hologram distance, $(z_h - z_s)$, the amplitude of these waves must be larger than the noise amplitude. So:

$$A e^{-(k_x^2 + k_y^2) k^2 (z_h - z_s)} \phi E$$

(3.9)

Since $k_{\text{nat}}$ in the hologram plane is determined by $\sqrt{k_x^2 + k_y^2}$, the upper boundary of the wavenumber with higher amplitude than the noise can be easily determined by rewriting the equation above as:

$$\left( k_x^2 + k_y^2 \right) \pi k^2 + \left[ \ln \frac{A}{E (z_h - z_s)} \right]^2$$

(3.10)

the maximum observable wavenumber is then:

$$k_{\text{max}} \pi \sqrt{k^2 + \frac{D \ln 10}{20(z_h - z_s)^2}}$$

(3.11)
Using $R = \frac{\pi}{k_{\text{max}}}$ we can write for the resolution $R$, according to [4]:

$$R = \frac{\pi}{\sqrt{k^2 + \left[ \frac{D \ln 10}{20(z_h - z_j)} \right]^2}}$$  \hspace{1cm} (3.12)

Thus the better the dynamic range $D$, the higher the resolution of the acoustic image when measuring in the nearfield.
### 3.8 Aliasing

As stated in the last section, the distance from the hologram to the source, \( z_h - z_s \), is very important for the resolution \( R \):

\[
R = \frac{\pi}{\sqrt{k^2 + \frac{D \ln 10}{2(20(z_h - z_s))}}} 
\]

(3.12)

So, one might think measuring as close as possible to the source, is the best thing, because the resolution will boost, if \( z_h - z_s \) decreases. But there is a main reason why there is a limit in decreasing the hologram distance.

When getting closer to the source, the spatial sampling, or \( \Delta x \) and \( \Delta y \), may cause aliasing. Aliasing is a result of under-sampling data. See the figure below:

![Figure 3-11: A high frequent sine wave is sampled at three points (arrows). The result is another sine wave with a lower frequency.](image)

A high frequent sine wave, illustrated in red, is sampled by \( \Delta x \), displayed by the arrows. This sampling results in the blue sine wave. Because this wave is the only one available after the acoustic measurements, we obtain a wave containing a larger wavelength and thus a lower frequency. So spatial aliasing is the inability to reconstruct the exact sound.
From the Nyquist-criterion we know, that \( 0.5k_{\text{sample}} \) is the maximum observable wavenumber. For the resolution in a noise-free environment, which implicates \( D \to \infty \), the resolution \( R \) (3.12) becomes approximately zero. This effect is limited by the ‘other’ formula for the resolution due to spatial sampling (3.6):

\[
R_{\text{max}} = \frac{\pi}{k_{\text{max}}} \quad ; \quad R = \frac{\pi}{\sqrt{k^2 + \left( \frac{D \ln 10}{20(z_h - z_s)} \right)^2}} 
\]

(3.13)

\[
k_{\text{max}} = \sqrt{k^2 + \left( \frac{D \ln 10}{20(z_h - z_s)} \right)^2} 
\]

(3.14)

So when the inequality (according to [4],

\[
0.5k_{\text{sample}} \phi \sqrt{k^2 + \left( \frac{D \ln 10}{20(z_h - z_s)} \right)^2} 
\]

(3.15)

is true, aliasing is the case. So there is a limit to the distance \( z_h - z_s \), at a chosen spatial sampling and at a certain \( D \). Below this limit, evanescent waves higher than the Nyquist wavenumber are observable.

Rewriting (3.15), we can experiment at certain \( D \), by changing the hologram distance \( z_h - z_s \), from a non-aliasing distance to an aliasing distance.

\[
(z_h - z_s) \phi \frac{D \ln(10)}{20 \sqrt{\frac{1}{4} (k_{\text{sample}})^2 - k^2}} 
\]

(3.16)

Plotting this curve, with a chosen spatial sampling of \( \Delta x = \Delta y = 0.3 \text{mm} \), we obtain:

Figure 3-12: Minimum hologram distance for 0.3 mm spatial sampling as a function of \( D \).
4 Results

4.1 Description of the experiment

To test the techniques from the last chapters we need to check it in practice. We have the following set up: a plate with three holes in it, each hole has a diameter of 2 mm and are placed 0.5 mm apart. A sound source is behind the plate. It is possible to determine the pressure field in normal direction by solving Rayleigh’s integral. This distribution serves as input for PNAH. It is impossible to determine the solution by hand, so a Matlab-script was written to calculate the pressure distribution at the source. (See Appendix A).

Figure 4-1: Experiment with a plate with three holes. Through each of them are coming sound waves. We measure at three mm of the plate.

With Rayleigh’s integral the sound pressure is calculated in x-z plane, using 50x50 points. The expectation was, that the hologram at the source should contain three red dots, symbolizing the three holes, but at only 3 mm away from the plate, the measured pressure was only one wide peak as can be seen in figure 4.2.

Figure 4-2: Measurement of the pressure at 3 mm of the plate with three holes. Topview (left) and 3-D plot (right).
Applying PNAH the result at three distances: at 3mm, at 1,5 mm and at the source itself, is:

\[
\text{hologram} \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad z_h = 3\text{mm}
\]

\[
z_h = 1.5\text{mm}
\]

\[
z_h = 0\text{mm}
\]

As can be seen, even at 1,5mm of the source it isn’t clear that there are three holes in the plate. The explanation of this fact, is given in chapter two (section 2.2). The amplitude of evanescent waves, which we need for high resolution PNAH, drops with the exponent, explaining why we only can see these waves extremely close to the source. That is when the exponent is visible.

There are a number of parameters which we can vary in this experiment:
- SNR, Signal To Noise Ratio
- Lead through of the K-space filter

In the next section the influence on the result of each parameter is visualized.
4.2.1 Signal to noise ratio

As stated in the theory in chapter two, noise is always present, when doing an experiment. The SNR, signal to noise ratio, is a parameter, which can simulate this noise in experiments. The dynamic range $D$ is set to 20 dB and the hologram distance is 1 mm.

This is the result:

![Figure 4-4: measured hologram (left) and pressure at the source (right), this time containing lots of noise.](image)

It is almost impossible to determine the three sources right now, because you can not say what real data is and what is blown up noise. In this particular case, we already know the solution, due to the simple set up, but in any other case, you can get a total different solution, because your SNR is too low.
4.2.2 Lead trough of the k-space filter

The last parameter which we vary in this experiment is the cutoff of the k-space filter. As stated in the theory in chapter three, in the section about the k-space filter, this is a very important operation. On one hand, we don’t want to let to much high k numbers in our processed hologram, but on the other hand, we want to have as much reliable data as possible. We must find a balance in these two. In this experiment, the dynamic range is 50 dB, the hologram distance is 1 mm and the cutoff is varied from a low value: 1000 rad/m, a medium value: 3000 rad/m and a high value:5000 rad/m. The results contain a figure of the form of the k-space filter and the consequents for the solution in spatial domain at the source:

K-co could be compared to the size of an open eye to look at the data with. If k-co is very small, it is comparable by looking with your eyes nearly closed. At very big values for k-co, you are looking with very big eyes.

The parameter k-co is depending on the signal to noise ratio and the distance of the hologram. At low values of k-co, you can see that only one source is visualized. At medium cutoff numbers, two more sources are revealed. Further increasing the cutoff will result in blown up noise, which makes it very difficult to see the three sources.
4.3 Influence of the hologram distance and the dynamic range; aliasing

For the verification of the curve of equation (3.16) or figure 3-12, two hologram distances are chosen; one distance in the “aliasing” area and the other one in the “no aliasing” area. Then the dynamic range is set to two different values: 20 dB and 50 dB and we are going to investigate the effect of aliasing in the k-space and at the source plane in the spatial domain.

First is the dynamic range 20 dB and we change the hologram distance, \( z_h - z_s \), from \( 5e^{-4} \) m, which causes no aliasing, to \( 5e^{-5} \) m, which causes aliasing.

The dynamic range is 20 dB and the distance \( z_h - z_s \) is \( 5e^{-4} \) m (no aliasing):

\[
[Pa]
\]

Figure 4-9: Minimum hologram distance for 0.3 mm spatial sampling as a function of D.

Figure 4-10: (a) k-space, (b) k-space windowed, (c) k-space filter with \( k_{co} = 4e3 \) rad/m, (d) spatial result at source plane
The dynamic range is 20 dB and the distance \( z_h - z_s \) is \( 5e^{-5} \) m (aliasing):

\[ [\text{Pa}] \]

Figure 4-11: (a) k-space, (b) k-space windowed, (c) k-space filter with \( k_c = 4e3 \) rad/m, (d) spatial result at source plane

It seems that the result containing aliasing is better at the source plane, than the result containing no aliasing. But there is an explanation for this result, see the next figure;

Figure 4-12: Picture of acoustic level amplitude versus the hologram distance, showing according to a noise level, the wrong or right interpolation to get the solution at the source plane

The amplitude of the evanescent waves is blowing up with the exponent, when getting closer to the source. With a dynamic range of 20 dB and a hologram distance of \( 5e^{-4} \) m (the right arrow), extrapolation is done from the noise level in stead of the amplitude of the evanescent waves, because the evanescent waves are not visible anymore, which gives the worse result.
When getting closer to the source, in this case, a hologram distance of $5e^{-5}$ m (the left arrow), extrapolation is done from the amplitude of the evanescent waves, which gives the right result at the source plane.

So in order to see the effects of aliasing it is necessary to increase the value of the dynamic range D. To avoid the effect mentioned above, a value is chosen for the dynamic range of 50 dB.

![Figure 4-13: Minimum hologram distance for 0.3 mm spatial sampling as a function of D.](image)

First is the dynamic range 50 dB and we change the hologram distance, $z_{h} - z_{s}$, from $1e^{-3}$ m, which causes no aliasing, to $1e^{-3}$ m, which causes aliasing.

The dynamic range is 50 dB and the distance $z_{h} - z_{s}$ is $1e^{-3}$ m (no aliasing):

![Figure 4-14: (a) k-space, (b) k-space windowed, (c) k-space filter with $\hat{k}_{co}=3e5$ rad/m, (d) spatial result at source plane.](image)
The dynamic range is 50 dB and the distance \( z_h - z_s \) is \( 1e^{-4} \) m (aliasing): [Pa]

![Image](image-url)

**Figure 4-15:** (a) k-space, (b) k-space windowed, (c) k-space filter with \( k = 3e5 \) rad/m, (d) spatial result at source plane

In this result aliasing is well visualized. There are major differences in the k-space of both experiments. In the aliasing case, there are a lot of strips in the k-space, displaying the folding back of high frequencies to low frequencies, which are not really there. The reason why this can not be seen in the spatial result at the source is the following: The amplitude of the wave carried out by the three sound sources is that big, that the small amplitude of higher wavenumbers, which fall back due to aliasing, just is not visible. But in the k-space it is clearly, that if a too small value is taken for the hologram distance, \( z_h - z_s \), 'ghost' frequencies will appear in your result by aliasing.

In this example aliasing is well-seen in the k-space domain, but the corresponding results in the spatial domain are not visible due to the high amplitude of the pressure by each of the three sound sources. The next experiment is to show, that although it is not visible in the spatial domain, aliasing has its effects there. We are going to do that in two different ways. First the solution of Rayleigh’s integral is calculated at \( \frac{1}{2} z_h \) using a spatial sample rate of 0.3 mm, which serves as reference. Then two pressure fields are calculated using PNAH, also at \( \frac{1}{2} z_h \), one using a spatial sample rate of 0.3mm and the other using a spatial sample rate of 3mm. Those two pressure fields are compared to the correct solution of Rayleigh’s integral at \( \frac{1}{2} z_h \). The reason why the comparison is done at half the hologram distance and not at the source plane, is that the amplitude of evanescent waves and also noise is blown up with the exponent of the hologram distance. See section 2.2 figure 2-2.
The hologram distance must be chosen in such a way, that the first time, using the spatial sampling of 0.3mm is not causing aliasing and the second time, using the spatial sampling of 3mm, is causing aliasing:

Figure 4-16: Minimum hologram distance for 0.3 mm and 3mm spatial sampling as a function of D.

For this experiment a hologram distance of 1e-3m is chosen.
The difference between the pressure field using PNAH and the solution of Rayleigh’s integral is the error:

\[ P_{\text{wellsampled, PNAH}} - P_{\text{Rayleigh}} \] and \[ P_{\text{undersampled, PNAH}} - P_{\text{Rayleigh}} \], where well sampled means a spatial sampling of 0.3mm and under sampled means a spatial sampling of 3mm.

It is important to compare the right points of the matrices \( P_{\text{wellsampled, PNAH}} \) and \( P_{\text{Rayleigh}} \) with \( P_{\text{undersampled, PNAH}} \), because the dimensions of \( P_{\text{wellsampled, PNAH}} \) and \( P_{\text{Rayleigh}} \) is (50x50) and the dimension of \( P_{\text{undersampled, PNAH}} \) is (5x5). After the comparison is made, we can plot the mean square error:

![Figure 4-17: The error in dB, left in case of spatial sampling 0.3mm and right in case of spatial sampling 3mm.](image)

It is very obvious, that the mean square error of the non-aliasing case is much smaller than in case we have aliasing. To get a single value for the error in comparison to the solution from Rayleigh’s integral, we calculate the MSE of the two cases:

The RMS value of the error is:

\[
MSE = \frac{\| \text{abs}(\text{error}_{\text{Rayleigh}}) - \text{abs}(\text{error}_{\text{non-aliasing}}) \|_2}{\| \text{abs}(\text{error}_{\text{Rayleigh}}) \|_2} = 0.18 \rightarrow 18\%
\]

\[
MSE = \frac{\| \text{abs}(\text{error}_{\text{Rayleigh}}) - \text{abs}(\text{error}_{\text{aliasing}}) \|_2}{\| \text{abs}(\text{error}_{\text{Rayleigh}}) \|_2} = 0.87 \rightarrow 87\%
\]

So the MSE in case of aliasing is much larger than in case in which we don’t have aliasing. In the spatial results at the source plane, aliasing was not spotted, but in this experiment aliasing was well visualized.
Note

The same experiment is performed with spatial sampling 3mm as done with the spatial sampling of 0.3mm, namely changing the hologram distance from a non-aliasing distance, in this case, 1e-2m, to a aliasing distance 1e-3. The problem, that occurred, is, that that at the non-aliasing distance of 1e-2m, that a lot of noise was detected. Because, the hologram distance is far away from the our reference plane, in this case, 0.2mm from the source, the noise was blown up by the exponent, resulting in a terrible result at that reference plane, with a pressure of 120dB or more. This result is not used, because it is impossible to compare these results, on one hand the pressure at the plane 0.2mm of the source with a hologram distance of 1e-2 m (no aliasing) and a hologram distance 1e-3m (aliasing) without changing the filter cutoff. If the filter cutoff is changed, the comparison between the error using a fine spatial sampling can not be compared with the result of a bad spatial sampling.
Conclusion

There are a number of conclusions we can make in this report. Aliasing can be a source of many mistakes, using PNAH. At a fixed dynamic range, there are limits to the hologram distance, so the closer to the source the measurement is, the better, is not true and can cause aliasing. There are two kinds of aliasing which are investigated in this report. On one hand, we have the aliasing due to the hologram distance at a fixed spatial sampling and on the other hand we have the aliasing due to the difference in spatial sampling at a fixed hologram distance. The experiment was not ideal to detect the two forms of aliasing, because of the high amplitude of the wave caused by the three sound sources. Therefore the folding back of high frequencies is not well visualized in the spatial domain at the source plane. In the case of the 20 dB dynamic range, the result at the source plane is even better when we measure at an aliasing distance than at a non aliasing distance. But the explanation of this phenomenon is very simple, at a large distance from the source, the wrong interpolation is made at a relative low dynamic range.

After changing the dynamic range to 50 dB, aliasing was well visualized in the k-space domain. However, a difference between non aliasing and aliasing was not visible at the source plane.

At a chosen distance we changed the spatial sampling from a fine mesh, 0.3mm to a bad mesh, 3mm. In this case, aliasing occurred and because the error was calculated at $1 \over 2 z_h$ between the fine mesh and the bad mesh, the difference between the non-aliasing spatial sampling and the aliasing spatial sampling was significant. (RMS=0.27)

It must be said, that in this simple experiment the solution at the source plane and the place of the three sound sources was already known. This makes it easy to say if a measurement mesh is fine or bad. In practical cases, the place and sometimes even the number of sources is not known, which makes it even more difficult to obtain the right measurement array which does not cause aliasing.
Appendix A: Matlab file to calculate the inverse solution to the sound problem

```matlab
noised_p_v_rayleigh;

pause;
%E=matrixdeling(p_h_noise);
plate_mode=p_h_noise;

% load(’plate_mode.mat’,’-mat’); % load function

set(figure,’Name’,[’measured sound pressure of a plate mode’]); % generating picture with set name
% with supplot two different imaging functions are displayed in a single figure.
% imshow() is a standard display function for image processing that is used % acoustic imaging. Since we have complex information the absolute values % with abs() are taken from the input matrix ”plate_mode”.
% surf() is a standard display function that generates a surface plot of % the acoustic data, again the absolute value is processed.
subplot(121); imshow(abs(plate_mode),[]); title(’2D imshow’);
subplot(122); surf(abs(plate_mode),’edgecolor’,’none’); title(’3D surface plot’);
axis tight; grid off; colormap(jet);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%
N = size(plate_mode,1); % rows
M = size(plate_mode,2); % columns
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%
% Border-padding
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%
NB = 128;
MB = 128;
plate_mode_bp =borderpadding(plate_mode,NB,MB);
% subplot(321); imshow(abs(plate_mode),[]); title(’nonborderpadded data’); colormap(jet);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%
% Windowing; fill in ’tukey’ or ’hanning’ for ”type”
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

type = 'tukey';
switch type  % switch statements for processing different type of window
case {'tukey'}
    % a Tukey window has a cosine tapered part at the edges and a
    % constant part in the middle. The ratio of taper to constant can
    % be changed, when this ratio is 1 we have a Hanning window.
    NR = 0.3;   % ratio of taper to constant sections (choose from range 0 to 1) for the rows
    MR = 0.3;   % ratio of taper to constant sections (choose from range 0 to 1) for the columns
    NR = NB/N;
    MR = MB/M;
    window1dm = tukeywin(NB,NR); window1dn = tukeywin(MB,MR);
end;

case {'hanning'}
    window1dm = hann(NB); window1dn = hann(MB);
    otherwise,  % no window chosen and no window applied (a window with only ones does not
    % affect data)
    window1dm = ones(NB,1); window1dn = ones(MB,1);
end;

window2d = ones(NB,MB);  % generating standard window with all matrix cells set to 1

% Following code first sets all the rows in the 2D window to the
% appropriate values.
for (n = 1:NB),
    window2d(n,:) = window2d(n,:) .* window1dn(:)';
end;
% And finally sets the columns of the window to the right values
for (m = 1:MB),
    window2d(:,m) = window2d(:,m) .* window1dm(:);
end;

% The windowing procedure is a straightforward multiplication of the
% matrix cells of the plate_mode with the 2D window.
plate_mode_wndwd = plate_mode_bp .* window2d;  % windowed version

% Plot function that displays both window and data and the resulting
% windowed data.
set(figure,'Name',['windowing procedure illustrated']);
subplot(221); plot(window1dm); title('window shape');
axis tight; grid off;
subplot(222); surf(abs(fftshift(fft2(plate_mode_wndwd))),'edgecolor','none'); title('3D surface
plot');
axis tight; grid off; colormap(jet);
subplot(223); imshow(abs(plate_mode),[]); title('plate mode with border-padding');

% Fast Fourier Transform in 2D

ZP = 128;     % zero-padding size used for the Fast Fourier Transform. For fast processing,
% this value has to be set to a power of two and larger then
% the data size.

% The 2D Fast Fourier Transform has a standard processing function called
% fft2(). fftshift() is used for acoustic holography purposes and only shifts the
% resulting spectrum, see what happens if you leave fftshift() out..
% PLATE_MODE_WNDWD = fftshift(fft2(plate_mode wndwd));
PLATE_MODE = fftshift(fft2(plate_mode,ZP,ZP));

% Plot function that displays the windowed and non-windowed plate mode in
% both x,y-space and k-space (2D FFT-ed version of the input)
set(figure,'Name','[difference in spectra between windowed and non-windowed data']);
subplot(221); imshow(abs(plate_mode),[]); title('plate mode'); colorbar;
% log10() in the following plot is used for display purposes, see what happens to the image if
% you leave it out.
subplot(222); imshow(log10(abs(PLATE_MODE)),[-10 1]); title('2D fft of plate mode'); colorbar;
subplot(223); imshow(abs(plate_mode_wndwd),[]); title('windowed plate mode'); colorbar;
subplot(224); imshow(log10(abs(PLATE_MODE_WNDWD)),[-10 1]); title('2D fft of windowed
plate mode'); colorbar;
colormap(jet);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%
% Inverse solution in k-space
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%

% f = 2362.5;                                % frequency of excited sound [Hz]
zh = dz;                              % hologram distance z [m]
z = [0 zh/2 zh];                      % distances we want to calculate [m]
d_x = dx;                            % spatial sampling spacing [m]
c0 = 343;                               % propagation speed of soundwaves in air [m/s]
rho0 = 1.2;                             % density of air
k = 2*pi*f*c0;                          % wavenumber of incident wave in air [rad/s]

Nk = size(PLATE_MODE_WNDWD,1);          % rows in k-space
Mk = size(PLATE_MODE_WNDWD,2);          % columns in k-space
P = zeros(Nk,Mk,length(z));             % initialising matrix

dkn = 2*pi/(d_x*(Nk-1));                % dkn - wavenumber samples (delta k)
dkm = 2*pi/(d_x*(Mk-1));                % dkm - wavenumber samples
for (iz = 1:length(z)),  
% for every distance z
for (n_ky = 1:Nk),  
% on every wavenumber in row-direction
for (m_kx = 1:Mk),  
% on every wavenumber in column-direction
% a k_x and k_y can be determined
kx = (m_kx - Mk/2 - 1)*dkm;  
ky = (n_ky - Nk/2 - 1)*dkn;  
% with this k_x and k_y we can determine the unknown part of
% the vector: k_z
kz(n_ky,m_kx) = sqrt(k^2 - kx^2 - ky^2);  
% k_z is used it the propagator exponential below that is
% multiplied with the Fourier transformed, windowed plate mode,
% which results in the k-space sound pressure at a distance z:
P(n_ky,m_kx,iz) = exp(j*kz(n_ky,m_kx)*(z(iz) - zh)) *  
PLATE_MODE_WNDWD(n_ky,m_kx);  
end;  
end;  
end;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
% Low-pass filtering in k-space  
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
% following this inverse solution we need to process the data in such
% a way that we don't blow up data and make it suitable for inverse
% 2D FFT

D = 30; %SNR;  % Dynamice range of the measurement (signal-to-noise ratio)
Ed = 1;  % noise level amplitude
Md = 10^(D/20);  % maximum field amplitude

% determination of the low-pass filter ct-off depending on the dynamic
% range and z-distances
[k_co,R] = ksfcutoff(Md,Ed,zh,z(1),k);

zpf = 2 ;  % zero-padding factor in k-space (try factors of 1,4,8 and 16 instead)

Nk_zp=zpf*Nk;  % resulting row-length after zero-padding
Mk_zp=zpf*Mk;  % resulting column-length after zero-padding
ksftype = 'costapered';  % type of filter used
ksfparam = 0.3;  % taper ratio for used filter (see filter function for more info)

nstart = floor(zpf*(NB-N)/2) + 1;  % start of signal region
nstop = Nk_zp - ceil(zpf*(NB-N)/2);  % end of signal region
mstart = floor(zpf*(MB-M)/2) + 1;  % start of signal region
mstop = Mk_zp - ceil(zpf*(MB-M)/2);  % end of signal region
% all calculated planes are now filtered, zero-padded and inverse Fourier
% transformed in order to get the desired results
for (ind_z = 1:length(z))
    P_ksf(:,:,ind_z) = ksfilter(P(:,:,ind_z),k_co,dkn,ksftype,ksfparam); % k-space low-pass filter
    P_zp(:,:,ind_z) = zeropadding(P_ksf(:,:,ind_z),Nk_zp,Mk_zp); % zero-padding in k-space
    p_tmp(:,:,ind_z) = ifft2(ifftshift(P_zp(:,:,ind_z))); % inverse 2D FFT
    p(:,:,ind_z) = (Nk_zp/Nk)*(Mk_zp/Mk)*p_tmp(nstart:nstop,mstart:mstop,ind_z); % cut-out of the proper areas and amplitude correction following zero-padding end;

% plots of results...
set(figure,'Name','[k-space processing and results']);
subplot(221); imshow(log10(abs(P_ksf(:,:,1)) + 1e-6),[]); title('Low-pass filter is applied');
subplot(222); imshow(log10(abs(P_zp(:,:,1)) + 1e-6),[]); title('Zero-padding in k-space');
subplot(223); imshow(abs(p_tmp(:,:,1)),[]); title('Resulting sound pressure at the source plane with padding'); colorbar;
subplot(224); imshow(abs(p(:,:,1)),[]); title('Resulting sound pressure at the source plane'); colorbar;
colormap(jet);

set(figure,'Name','[measured hologram and the 3 processed planes']);
subplot(221); imshow(abs(plate_mode),[]); %title('Measured hologram at ',num2str(z(1)) ' m'); colorbar;
subplot(222); imshow(abs(p(:,:,3)),[]); %title('Processed hologram ',num2str(z(1)) ' m'); colorbar;
subplot(223); imshow(abs(p(:,:,2)),[]); %title('Plane at ',num2str(z(2)) ' m'); colorbar;
subplot(224); imshow(abs(p(:,:,1)),[]); %title('Source plane ',num2str(z(3)) ' m'); colorbar;
colormap(jet);
hold on;
References

## Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_0$</td>
<td>speed of sound</td>
<td>$\frac{m}{s}$</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>density</td>
<td>$\frac{kg}{m^3}$</td>
</tr>
<tr>
<td>$k$</td>
<td>Wavenumber</td>
<td>$\frac{rad}{m}$</td>
</tr>
<tr>
<td>$F(...)$</td>
<td>Fourier transform</td>
<td>-</td>
</tr>
<tr>
<td>$F^{-1}(...)$</td>
<td>Inverse Fourier transform</td>
<td>-</td>
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