Iterative SISO Feedback Design
For an
Active Vibration Isolation System

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Traineeship report
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Summary

Vibration isolation is accomplished basically with passive vibration isolation systems. The passive system has a weakness: the higher frequency vibrations are efficiently reduced but the lower frequency vibrations are not. Ground vibration isolation systems need to be even efficient both in the lower and in the higher frequencies band. A control system is therefore added to the passive system to counterbalance the lack of efficiency in the lower frequency band and at the natural frequency. The obtained system is called Active Vibration Isolation System.

In this work an active system of the firm IDE: the AVIS (Active Vibration Isolation System) is analyzed in order to design appropriate controllers.

The AVIS has six degrees of freedom. The modelling with Lagrange mechanics leads to a set of six coupled differential equation of motion. The AVIS is equipped with eight actuators coupled to the six degrees of freedom through the actuator matrix. Using the Laplace transformation, taking the actuators as input and the velocities in the six degrees of freedom as output, leads to a MIMO (Multiple Input Multiple Output) system with coupling (cross transfer functions).

Our goal is to design six controllers in the simplified SISO (Single Input Multiple Single) system, iteratively such that the effect of the omitted coupling terms is taken into account. To do this, the six transfer functions of the SISO system are measured and performances and robustness criteria are defined.

A set of controller types to be used is chosen based on the observation of the transfer functions.

Two iterations are sufficient to achieve a practical ground vibration reduction of 300% to 600%. In spite of a fine tuning effort there is a side effect of high frequency vibration amplification of 5%.

Nevertheless more reduction can be achieved if a MIMO approach is used to design the controllers
Chapter 1

Introduction

As device geometries go down and become highly sensitive to vibrations. Sensitive equipments such as submicron semiconductor production, interferometer, confocal optical imaging, scanning probe microscopy and others more, need to be isolated from vibration. Advanced methods of protecting vibration-sensitive tools from vibration included in site borne disturbances are developed.

Passive control methods can keep most of the vibrations under control, but low-frequency vibrations are still a problem. Passive damping treatments consist of an elastic spring and a damping unit. The combination of a mass of the system and the spring is known as a mechanical low-pass. The mechanical response of the spring-mass system decreases significantly for frequencies above the eigenfrequency, and the damper reduces the vibration amplitude especially within the resonance range. Because of the low-pass characteristic, passive vibration isolation systems for highly sensitive precision systems are designed with very low eigenfrequencies. Pneumatic anti-vibration mountings are normally used in these systems due to their low stiffness and high damping characteristics. Eigenfrequencies between 2 to 5 Hz are commonly achieved. Vibrations in this frequency range can be difficult to mitigate with conventional pneumatic tool isolation systems, since their resonance frequencies are in the same frequency range and may in fact amplify these vibrations.

Adding active vibration control to the system can cancel those problems. The principle is that signals acquired by extremely sensitive vibration detectors are analyzed by electronic circuitry driving (electro-dynamic) actuators which instantaneously produce a counter-force to compensate the vibration therefore to minimize the vibration transmissibility from the foundation to the table top. The figure 1.1 gives the Principle of Active Vibration Isolation.

![Figure 1.1 Principles of AVIS](image)

The AVIS has six degrees of freedom. Six sensors (geophones) allow real time measurement of absolute velocity of the table in the six-degrees-of-freedom and eight actuators convert the electric output signal of the controller-unit (voltage) into mechanical force acting between the frame and the table. The eight actuators and the six sensors are related to the velocities in the six degrees of freedom by the actuator matrix and the sensor matrix respectively.

The main goal of this work is tuning all the six controllers of the six-degrees-of-freedom iteratively and interactively to minimize (as small as possible) the transmissibility (transfer
function between the ground vibration and the table displacement) and trying to determine de limits of performance of the system.

For that purpose transfer function from the actuator force to the table velocity is determined experimentally (from frequency response measurements), objective criterions are defined on the sensitivity and the open loop system considering the performance (how well is isolation achieved) and the robustness (how well are uncertainties tolerated by the control system), and controllers are designed to achieve these requirements.

The report is organized as follow: in chapter 2 A brief description of the functionality and mathematic moulding of the active vibration isolation system (AVIS) are done.

In chapter 3 is described how the polarities of sensor and actuator are measured. The actuator matrix and the sensor matrix are defined from Laplace transformation of the Lagrange equations of motion.

Theoretical transfer function from the actuator force to the mass velocity, and theoretical transmissibility from the ground vibration velocity to the mass velocity of the model are definite in chapter 4. Measurements are made to compute the real transfer functions for a SISO model.

In chapter 5, performance and robustness criteria are determined, a set of filter types suitable to be used in order to fulfil those criteria is chosen. Two iterations of loop shaping tuning are done resulting in a set of six controllers satisfying the defined criteria.

Some conclusions and recommendations are given in chapter 6.
Chapter 2

Description and Modeling using Lagrange Mechanic of the AVIS

2.1 Description
The AVIS consists of a frame (massive and rigid), a table suspended on the frame through four isolator modules. The Isolator Module contains a pneumatic spring and two electromagnetic actuators (linear motors). The actuators are placed in horizontal and vertical direction in such a way that they assure actions in the six-degrees-of-freedom. The six geophones are fixed on three isolators in horizontal and vertical direction and give the corresponding information on the velocity. The Isolator Module is equipped with a mechano-pneumatic leveling system and a pre-amplifier for the geophones signal. The Controller Unit is a digital signal processor (DSP) capable of fast real-time signal control. Voltage amplifiers feed the actuators. Communications with external computers are assumed through serial ports. Figure 2.1 shows a picture of the AVIS and figure 2.2 gives a schematic representation of the isolator’s elements positioning with their positive directions.

Figure 2.1 Picture of the AVIS
2.2 Modeling of the AVIS using Lagrange Mechanics

Even though the AVIS is built and the controllers are implemented in a SISO way, it is necessary to get a complete mathematic model of the system. A comparison of the real working system with the complete model will surely give more explanations about the final result on the transmissibility or other transfer functions of the system.

The table is modeled as indicated in figure 2.3; the body fixed frame \((\mathcal{O}, \hat{\epsilon})\) is moving with respect to the reference frame \((\mathcal{E}, \hat{\epsilon})\). The position of any point is defined by the relative position in the relative frame and the position of the relative coordinate system center \(G\) the figure 2.4 gives a table point position description.

Considering the table as a rigid body six generalized coordinates are necessary to determine all the displacements included three translations and three rotations relative to the frame. Several combinations of generalized coordinates are possible. In this work the generalized coordinates \(x, y, z, \theta, \phi, \psi\) are chosen to describe the position of any point of the table in dimensional space: \(\mathbf{q}^T = [x, y, z, \theta, \phi, \psi]\) where \(x, y, z\) determine the location of the center of mass \(CM\) with \(\mathbf{r}_{CM} = [x, y, z]^{\mathcal{E}}\) and the Tait-Bryant angles \(\theta, \phi, \psi\) as indicates in figure 2.5 for general case and in figure 2.6 for small angles.

The generalized coordinates are disposed as given in the table 2.1. Table 2.2 gives the location of the actuators and the table 2.3 gives the location of the sensors (geophones).

<table>
<thead>
<tr>
<th>direction</th>
<th>X-translation</th>
<th>Y-translation</th>
<th>Z-translation</th>
<th>X-rotation</th>
<th>Y-rotation</th>
<th>Z-rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generalized coordinate</td>
<td>(x)</td>
<td>(y)</td>
<td>(z)</td>
<td>(\theta)</td>
<td>(\phi)</td>
<td>(\psi)</td>
</tr>
</tbody>
</table>

Table 2.1 Generalized coordinates choice
Figure 2.3 Absolute and relative frames

Figure 2.4 Table points’ positions definition

(a)  

(b)  

(c)

Figure 3.3 Elementary rotations \( \theta \) about \( \vec{e}_1 \) (a), \( \phi \) about \( \vec{e}_2 \) (b), and \( \varphi \) about \( \vec{e}_3 \) (c)

Figure 2.5 Generalized coordinates \( x, y, z, \dot{\theta}, \dot{\phi}, \varphi \) (For small angles)

Figure 2.6 Functional Dimensions
### Table 2.2 Actuators location

<table>
<thead>
<tr>
<th>Isolator Module</th>
<th>X-direction</th>
<th>Y-direction</th>
<th>Z-direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>Ay1</td>
<td>Az1</td>
</tr>
<tr>
<td>2</td>
<td>Ax2</td>
<td>-</td>
<td>Az2</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>Ay3</td>
<td>Az3</td>
</tr>
<tr>
<td>4</td>
<td>Ax4</td>
<td>-</td>
<td>Az4</td>
</tr>
</tbody>
</table>

### Table 2.3 Sensors location

<table>
<thead>
<tr>
<th>Isolator Module</th>
<th>X-direction</th>
<th>Y-direction</th>
<th>Z-direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>Sy1</td>
<td>Sz1</td>
</tr>
<tr>
<td>2</td>
<td>Sx2</td>
<td>-</td>
<td>Sz2</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>Sy3</td>
<td>Sz3</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Every position of the table can be defined as combination of the generalized coordinate. The most important points are the four corners and the center of the table. Their coordinates are described in the table 2.4 as function of the generalized coordinates.

In this modeling the motion is approximated as having small rotations. This simplification leads to the following positions depicted in the table 2.4.

<table>
<thead>
<tr>
<th>point 1</th>
<th>point 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>X₁ = x + 1/2d₁ -hφ + 1/2d₁φ</td>
<td>X₂ = x + 1/2d₁ -hφ + 1/2d₁φ</td>
</tr>
<tr>
<td>Y₁ = y + 1/2d₁ +hθ + 1/2d₁φ</td>
<td>Y₂ = y - 1/2d₁ +hθ + 1/2d₁φ</td>
</tr>
<tr>
<td>Z₁ = z -h + 1/2d₁θ - 1/2d₁φ</td>
<td>Z₂ = z -h - 1/2d₁θ - 1/2d₁φ</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>point 3</th>
<th>point 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>X₃ = x - 1/2d₁ -hφ + 1/2d₁φ</td>
<td>X₄ = x - 1/2d₁ -hφ - 1/2d₁φ</td>
</tr>
<tr>
<td>Y₃ = y - 1/2d₁ +hθ - 1/2d₁φ</td>
<td>Y₄ = y + 1/2d₁ +hθ - 1/2d₁φ</td>
</tr>
<tr>
<td>Z₃ = z -h - 1/2d₁θ + 1/2d₁φ</td>
<td>Z₄ = z -h + 1/2d₁θ + 1/2d₁φ</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Centere of Massa</th>
<th>Rotations (Tait Angles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X₅ = x</td>
<td>rtx = θ;</td>
</tr>
<tr>
<td>Y₅ = y</td>
<td>rty = φ;</td>
</tr>
<tr>
<td>Z₅ = z</td>
<td>rtz = φ</td>
</tr>
</tbody>
</table>

### Table 2.4 position coordinates

The mass may be considered as reduced at the center of the coordinate system. The principal inertial moments are \( J_x \), \( J_y \), and \( J_z \).

The excitation to the system can be considered as a forcing displacement on a spring and a forcing velocity on a damper.

It can be considered that the stiffness and the damping coefficients are the same for each direction with \( k_1, b_1 \) for X-direction; \( k_2, b_2 \) for Y-direction; \( k_3, b_3 \) for Z-direction.

A theoretical expression of the ground vibration as sinusoidal signal can be used for the purpose of modelling.
Xs = Re\{ Xo * exp(i * wx * t) \};
Ys = Re\{ Yo * exp(i * wy * t) \};
Zs = Re\{ Zo * exp(i * wz * t) \};

(2.1)

The damping force can then be calculated as.

\[ Fxf = - \sum_{i=1}^{4} b_{1i}(\dot{X}_i - \dot{X}_s) \]
\[ Fyf = - \sum_{i=1}^{4} b_{2i}(\dot{Y}_i - \dot{Y}_s) \]
\[ Fzf = - \sum_{i=1}^{4} b_{3i}(\dot{Z}_i - \dot{Z}_s) \]

(2.2)

And the potential energy in the hypothesis of mass reduced at the gravitational center G

\[ E_{p_x} = \frac{1}{2} \sum_{i=1}^{4} k_{xi}(X_i - X_s)^2 \]
\[ E_{p_y} = \frac{1}{2} \sum_{i=1}^{4} k_{yi}(Y_i - Y_s)^2 \]
\[ E_{p_z} = \frac{1}{2} \sum_{i=1}^{4} k_{zi}(Z_i - Z_s)^2 - mgZ_s \]

(2.3)

From a Matlab program using Lagrange’s mechanics is the differential equation of motion established

\[ \ddot{M} \ddot{q} + \dot{B} \ddot{q} + Kq = \dot{Q} \]

(2.4)

The matrices \( M(i,j), K(i,j), B(i,j) \) and \( Q(i) \) are given in the Appendix A.

If the system is identically symmetric, the stiffness and the damping coefficients are the same in each direction, the stiffness and the damping matrices have still cross terms because of the presence of \( h \) (height difference between CM and points \( P_1, P_2, P_3 \) and \( P_4 \)). There is then still coupling in the motion differential equations. The matrices of the system are given in (2.4), (2.5), and (2.6)

**Mass matrix**

\[
\begin{bmatrix}
  m & 0 & 0 & 0 & 0 & 0 \\
  0 & m & 0 & 0 & 0 & 0 \\
  0 & 0 & m & 0 & 0 & 0 \\
  0 & 0 & 0 & M(4,4) & M(5,4) & M(6,4) \\
  0 & 0 & 0 & M(4,5) & M(5,5) & M(6,5) \\
  0 & 0 & 0 & M(4,6) & M(5,6) & M(6,6)
\end{bmatrix}
\]

(2.5)

**Stiffness matrix**

\[
\begin{bmatrix}
  4k_1 & 0 & 0 & 0 & -4k_1h & 0 \\
  0 & 4k_2 & 0 & 4k_2h & 0 & 0 \\
  0 & 0 & 4k_3 & 0 & 0 & 0 \\
  0 & 4k_4h & 0 & 4k_4h^2 + k_1d_1^2 & 0 & 0 \\
  -4k_1h & 0 & 0 & 0 & 4k_1h^2 + k_2d_2^2 & 0 \\
  0 & 0 & 0 & 0 & k_1d_1^2 + k_2d_2^2 & 0
\end{bmatrix}
\]

(2.6)
**Damping matrix**

\[
\begin{bmatrix}
4b_1 & 0 & 0 & 0 & -4b_2 h & 0 \\
0 & 4b_2 & 0 & 4b_2 h & 0 & 0 \\
0 & 0 & 4b_3 & 0 & 0 & 0 \\
0 & 4b_4 h & 0 & 4b_2 h^2 + b_3 d_1^2 & 0 & 0 \\
4b_5 h & 0 & 0 & 0 & 4b_2 h^2 + b_5 d_2^2 & 0 \\
0 & 0 & 0 & 0 & 0 & b_3 d_1^2 + b_5 d_2^2
\end{bmatrix}
\]  

(2.7)

**Generalized forces: \( Q \)**

\[
4(k_1 + b_i \omega_i) X_0 + Ax_2 - Ax_4 \\
4(k_2 + b_i \omega_i) Y_0 + Ay_1 - Ay_3 \\
4(k_3 + b_i \omega_i) Z_0 + Az_1 + Az_2 + Az_3 + Az_4 \\
4((k_2 + b_i \omega_i) Y_0 + Ay_1 - Ay_3) h + \frac{1}{2} (Az_1 - Az_2 - Az_3 + Az_4) d_1 \\
-4((k_1 + b_i \omega_i) X_0 + Ax_2 - Ax_4) h - \frac{1}{2} (Az_1 + Az_2 - Az_3 - Az_4) d_2 \\
\frac{1}{2} (Ax_2 + Ax_4) d_1 + \frac{1}{2} (Ay_1 + Ay_3) d_2
\]

(2.8)

Only when the height \( h \) is taken equal to zero that the system can be considered as completely decoupled. Further analysis may be then done on the base that directions are considered independent each other. A SISO approach in the analysis will then be justified.

When the ground vibrations are considered inexistent the generalized force contains only the disturbance forces and actuator forces.
Chapter 3

Actuators and Sensors Matrices and Polarities

3.1 Actuator matrix and polarity

The polarity is not given. It can not be computed. It can be then determined only experimentally. As the actuators are directly connected from the frame to the table, a short impulse excitation on the actuator will generate an in-phase-response in the first small time. The observation of this response signal determines the polarity of an actuator. If the input signal and the output are of the same or opposite sign the polarity is positive or negative. The table 3.2 gives the results of the experiment.

<table>
<thead>
<tr>
<th>Actuator</th>
<th>Direction</th>
<th>Actuator</th>
<th>Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ay1</td>
<td>Y+</td>
<td>Az1</td>
<td>z++</td>
</tr>
<tr>
<td>Ax2</td>
<td>x+</td>
<td>Az2</td>
<td>z+</td>
</tr>
<tr>
<td>Ay3</td>
<td>y-</td>
<td>Az3</td>
<td>z+</td>
</tr>
<tr>
<td>Ax4</td>
<td>x-</td>
<td>Az4</td>
<td>z+</td>
</tr>
</tbody>
</table>

Table 3.1 Actuators polarities

The Actuator matrix is easily deducted from the generalized force. Considering the vector of the Actuators, the actuator matrix is simply the Jacobean matrix of the field generalized force.

\[
\text{Act} = [A_{y1}, A_{z1}, A_{x2}, A_{z2}, A_{y3}, A_{z3}, A_{x4}]
\]

\[
\text{AcMax} = \begin{bmatrix}
\frac{\partial (Q_{nc})}{\partial (\text{Act})}
\end{bmatrix}_{{x_0,y_0,z_0=0 \text{ and } \text{Act}=0}}
\]

A unit value weighting is applied to each column to obtain the actuator matrix given in table 3.2

<table>
<thead>
<tr>
<th></th>
<th>(\dot{X})</th>
<th>(\dot{Y})</th>
<th>(\dot{Z})</th>
<th>(\dot{\theta})</th>
<th>(\dot{\phi})</th>
<th>(\dot{\phi})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ay1</td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.25</td>
</tr>
<tr>
<td>Az1</td>
<td></td>
<td></td>
<td></td>
<td>0.25</td>
<td></td>
<td>0.25</td>
</tr>
<tr>
<td>Ax2</td>
<td>0.5</td>
<td></td>
<td></td>
<td>-0.25</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>Az2</td>
<td></td>
<td></td>
<td></td>
<td>0.25</td>
<td>-0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Ay3</td>
<td>-0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.25</td>
</tr>
<tr>
<td>Az3</td>
<td></td>
<td></td>
<td></td>
<td>0.25</td>
<td>-0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Ax4</td>
<td>-0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.25</td>
</tr>
<tr>
<td>Az4</td>
<td></td>
<td></td>
<td></td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 3.2 Elements of actuators matrix

3.2 Sensor matrix and polarity

The polarity is determined experimentally. The table is excited with a light knock (impulse) in the axial direction where the concerning sensor is active and near his position. The observation of the sensor measurement sign determines the polarity of the sensor. If the knock is given in the positive direction and if the first step of the sensor measurement is also
in the positive direction then the sensor has positive polarity in the chosen axis system. The results are show in the table 3.3

<table>
<thead>
<tr>
<th>sensor</th>
<th>direction</th>
<th>sensor</th>
<th>direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sy1</td>
<td>y-</td>
<td>Sz1</td>
<td>Z+</td>
</tr>
<tr>
<td>Sx2</td>
<td>x-</td>
<td>Sz2</td>
<td>Z+</td>
</tr>
<tr>
<td>Sy3</td>
<td>y+</td>
<td>Sz3</td>
<td>Z+</td>
</tr>
</tbody>
</table>

Table 3.3 Sensors polarities

The sensor matrix is obtained by substituting in the vector position (Table 2.4) the corresponding projection (X, Y or Z) of a point by the equivalent sensors and their polarity. Then solving the obtained equations (3.4) in the generalized coordinates, taking the jacobian of the solution to the vector of the sensor (3.5) and applying a unit value weighting to each column leads to the sensor matrix given in the table 3.4

\[
Sensor = [S_{y1}, S_{x2}, S_{y3}, S_{z1}, S_{z2}, S_{z3}]
\]

\[
Sol = \text{solve}([Rc], [x_i, y_i, z_i, S_{x_i}, S_{y_i}, S_{z_i}])
\]  

\[
\text{SenMax} = \left[ \frac{\partial(Sol)}{\partial(Sensor)} \right]_\text{WEIGHTING=1}
\]  

<table>
<thead>
<tr>
<th></th>
<th>(\dot{X})</th>
<th>(\dot{Y})</th>
<th>(\dot{Z})</th>
<th>(\dot{\theta})</th>
<th>(\dot{\phi})</th>
<th>(\dot{\psi})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sy1</td>
<td>0.25</td>
<td>-0.5</td>
<td></td>
<td></td>
<td>-0.5</td>
<td></td>
</tr>
<tr>
<td>Sx2</td>
<td>-0.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sy3</td>
<td>0.25</td>
<td>0.5</td>
<td></td>
<td></td>
<td>-0.5</td>
<td></td>
</tr>
<tr>
<td>Sz1</td>
<td></td>
<td>0.5</td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sz2</td>
<td></td>
<td>-0.5</td>
<td>-0.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sz3</td>
<td></td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 3.4 Sensors matrix
Chapter 4

Transfer function (the frequency response function)

4.1 Definition from the model

![Mass Spring Damper Systems](image)

The differential equation of the system for one uncoupled degree of freedom (figure 4.1) can be given as:

\[ m\ddot{x} + b\dot{x} + kx = kx_0 + b\dot{x}_0 + f_{\text{actu}} + f_{\text{dist}} \]

\( x \): the mass position and \( X(s) \) its Laplace transformation

\( x_0 \): the ground vibration displacement and \( X_0(s) \) its Laplace transformation

\( f_{\text{actu}} \): the actuator force and \( F_{\text{actu}}(s) \) its Laplace transformation

\( f_{\text{dist}} \): the disturbance force and \( F_{\text{dist}}(s) \) its Laplace transformation

The Laplace transformation of the differential equation leads to three transfer functions:

1. The TRANSMISSIBILITY: The transfer function from the ground vibration \( x_0 \) to the displacement \( x \) of the mass

\[
T(s) = \frac{x}{x_0} = \frac{X(s)}{X_0(s)} = \frac{sX(s)}{sX_0(s)}
\]

\[
T(s) = \frac{bs + k}{ms^2 + bs + k}
\]

\[
T(s) = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}
\] (4.1)

The transmissibility is regarded as a rate of exchange for vibration from the ground to the mass. It is also an expression of the degree of the system isolation from the ground vibration. Its bode diagram is show in figure 4.2.a
2. The transfer function \( H(s) \) from the input \( f_{\text{act}} \) (actuator force) to the output \( \dot{x} \) (table velocity). Its Bode diagram is show in figure 4.2.b

\[
H(s) = \frac{\dot{x}}{f_{\text{act}}}
\]

\[
H(s) = \frac{\dot{X}(s)}{F_{\text{act}}(s)} = \frac{s}{ms^2 + bs + k}
\]

\[
H(s) = \frac{\omega_n^2 s}{s^2 + 2\zeta\omega_n s + \omega_n^2} \frac{1}{k} \tag{4.2}
\]

The transfer function \( D(s) \) from the input \( f_{\text{dist}} \) (disturbance forces) to the output \( \dot{x} \) (table velocity)

\[
D(s) = \frac{\dot{X}(s)}{F_{\text{disturb}}(s)}
\]

\[
D(s) = \frac{\omega_n^2 s}{s^2 + 2\zeta\omega_n s + \omega_n^2} \frac{1}{k} \tag{4.3}
\]

These transfer functions are implemented in the open-loop system figure 4.3. On the AVIS there are not sensors for the ground vibration measurement. The transmissibility is therefore not useful. The disturbance force can not be measured, and is also not useful.

There are two ways to implement a controller in this system: first as a feedback closed-loop system with \( H(s) \) as plant and to treat the ground vibration as equal to a certain disturbance with the advantage that the output \( \dot{X}(s) \) is measurable (available) and the input \( F_{\text{act}}(s) \) is controllable. Or implemented as a feedforward in combination with a feedback control. This is not possible because \( \dot{x}_0 \) the ground vibration velocity is not available in the concerning AVIS system of the IDE Company.
The feedback close-loop system will be implemented in this work for the above given reason. The closed-loop system is represented in the schema of the figure 4.4. The excitation \( x_{\text{dot \_ exc}} \) is zero in this case.

**Figure 4.3** Open-loop system

**Figure 4.4** Closed-loop system with plant \( H(s) \)

### 4.2 Measurements

To make the measurements of the transfer functions, real-time output data are necessary. Therefore two extern computers are connected to the controller unit through the external interface with two outputs Diag.0 and Diag.1 and the serial interface RS-232C.

On the one connected via the RS-232C is running the SAMBA software User Interface with the following settings for transfer function in X direction:
- Injection point: \( X \text{ trans output} \)
- Noise type: random/white noise
  - Gain: 0.2
  - Frequency: 0.9Hz

Resulting in approximately one coherence function.

Diagnostics: signal for Channel 0: \( X \text{ trans output} \)
- Signal for Channel 1: \( X \text{ trans row, input} \)

The injection point for the noise and the read output are indicated in the figure 4.4. On the second computer is running the Siglab (Matlab based software). The settings and coherence function plot are given in the Appendix C

### 4.3 Results

The resulting transfer functions obtained with the measurements are given in figure 4.5. As it can be seen from this figure, the resonance frequencies of the system are about 1.5 Hz regarding the horizontal motions and about 2.5 Hz regarding the vertical motion. The
resonance frequency in the horizontal axis rotation appears around 4 Hz and at about 2 Hz in the vertical axis rotation. High-frequency dynamic phenomena appeared from about 100Hz in all the six transfer functions.

4.4 Comments and observations

By comparison with the transfer function $H = \frac{\dot{X}}{F_d}$ of the reduced model (mass-spring-damper) of figure 4.2.b there are notable differences between the measurements and the reduced model.

1) In the lower frequencies the reduced model transfer function has a slope +1 whereas the real SISO measured transfer function shows a slope of about +1.5. The reason for this is the characteristic of sensor (geophone) at lower frequencies. The sensor ratio $\frac{\text{Voltage}}{\dot{x}}$ is not constant for lowers frequencies.

2) The appearance of MIMO behaviour around the resonance frequency that is justified by the presence of coupling terms in the real.

3) The high-frequency dynamic phenomena generated by the body deformation.

4) Unfortunately the system is more interactive through the coupling terms. The developed modelling in Appendix A shows that the SISO approach is far from the real MIMO system. Even when the MIMO system is modelled with small angles of rotation resulting in the mass, stiffness and damping matrices respectively (2.4), (2.5) and (2.6), the obtained transfer functions $H$ show a great change of characteristics for an slight change in the cross terms. More detail are given in Appendix B.

A SISO approach is necessary to get a first look but it is important to develop a MIMO model for further research.

The appearance of high-frequency dynamic phenomena has an impact on the stability of a system.
**Bode Plot FRF (Without Controllers)**

*Figure 4.5 Transfer functions \( \frac{\text{velocity}}{F_o} \) from measurements*
Chapter 5
The Controllers

5.1 General aspect and concept

As the transfer functions are obtained from measurements of SISO models, the design of the controller is done for each axis separately. Consider the feedback system of figure 4.4 the goal of the controller is to attenuate the floor vibrations.

The sensitivity $S$ defined by the ratio $S = \frac{x \_ \dot{d}}{d}$ is the primary measure of performance to the disturbance rejection problem as it relates the disturbance $d$ to the output $x \_ \dot{d}$. Therefore the sensitivity function is used as the main measurement instrument for the disturbance rejection in the design of the controllers. The sensitivity is expressed in terms of the system elements by

$$S(s) = \frac{1}{1 + C(s)H(s)}$$

with $H(s)$: Transfer function of Plant

and $C(s)$: The transfer function of the Controller

The main goal of the controller is to add active damping to the system to attenuate the resonance peak of the passive system. This technique is called “Skyhook damping”.

A proportional action $u = -K_p \dot{x}$ applied to the transfer function $H(s) = \frac{\tau s}{ms^2 + bs + k}$ in closed-loop leads to a damping of the peak as can be seen from the following expression

$$H_{cl}(s) = \frac{H(s)}{1 + K_pH(s)}$$

$$H_{cl}(s) = \frac{\tau s}{ms^2 + (b + \tau K_p)s + k}$$

(5.2)

A smaller gain in the proportional action generates a reduction of the sensitivity outside the attenuation bandwidth where $|S(s)|$ is higher or positive in $dB$; but at the cost of a sensible bandwidth reduction around the resonance frequency and a significant increasing of $|S(s)|$ at the resonance frequency. This trade-off is justified by the Bode’s Integral Theorem on Sensitivity (The area $A_1 (\Rightarrow |S(j\omega)| > 1)$ must be equal to the area $A_2 (\Rightarrow |S(j\omega)| < 1)$. This effect is currently called waterbed effect. In figure 5.1 an example is given with two values of $K_p$ ($K_p = 0.545$; and $0.0823$).

| $K_p$   | max($|S(s)|$) [dB] | COF$_{left}$ [Hz] | COF$_{right}$ [Hz] | $|S(s)|$ at $\omega_c$ (dB) |
|---------|--------------------|-------------------|-------------------|--------------------------|
| 0.545   | 20                 | 2.2               | 30.8              | -35                      |
| 0.0123  | 1.0                | 0.4               | 7                 | -7                       |

Table 5.1 max($|S(s)|$), COF$_{left}$, COF$_{right}$, and $|S(s)|$ at $\omega_c$ for $K_p$ variable
The sensitivity criteria has to be translated into the open-loop requirements in a relation of the form

\[ S_f(\max|S(s)|, |S(s)|_{at\omega}, COF_{left}, COF_{right}) = OL(G_m, P_m, B_w) \]  \hspace{1cm} (5.3) 

Finding this relation is beyond the scope of this work and may be subject of further research. Nevertheless, it has been taken into account in the main process of trial and error tuning with loop shaping.

The tuning process is made iteratively and interactively such that the uncertainty of the modelling of the system as SISO can be reduced.

In the first iteration a controller for each axis is designed individually based on the loop shaping and the stated criterions on the sensitivity. Even though more attention is paid to the achievement of the defined criterions, a real time measurement of the time response with Siglab allows a better verification of the designed controller.

In the second iteration the transfer function of a chosen axis is measured by setting out his controller; the remaining five axis have their controller on. Applying the same process as described in the first iteration a new set of controllers is design.
5.2 Criteria on the sensitivity

The main goal of the controller is to reduce the effect of disturbances. The sensitivity function $S$ is the primary measure of performance as far as it is related to disturbance rejection. Therefore the sensitivity function is the main tool in the trial and error tuning of the controllers.

1) As the action is basically to keep the absolute value of the sensitivity smaller than 5 dB, this indicates the first criterion. It is known from the theorem of Bode integral or waterbed effect that this costs a reduction of the deep height at the resonance frequency; and therefore amplifies the noise effect in the neighbourhood of the resonance frequency.

\[ |S|_{\text{max}} \leq 5 \text{ dB} \quad \text{For all frequencies} \quad \text{(Robustness)} \]

2) The bandwidth around the resonance frequency $(COF_{\text{left}}, COF_{\text{right}})$ has to be kept in certain limits. From the observations minima of $COF_{\text{left}} \geq 1 \text{ Hz}$, and $COF_{\text{right}} \geq 7 \text{ Hz}$ are chosen.

3) At the resonance frequency a sufficient suppressing of noise has to be provided by specifying an upper limit of the sensitivity. A maximum of $-25 dB$ is chosen from observations of time responses in this specific case.

All this considerations are summarized in figure 5.2

\[ \text{Figure 5.2: Criteria on the Sensitivity} \]

5.3 Procedure to design the controllers

5.3.1 Type of filters

1) Proportional action $K_p$

Recall that the ground vibration is here assimilated as noise. As indicated before the control action will be a pressing on the noise in the closed loop mainly at the frequencies where the sensitivity is positive in $\text{dB}$

2) The Notch filter and inverse Notch

is unavoidable to remove the resonance peak and the antiresonance.
3) **A Low Pass Filter**

Is necessary to eliminate higher frequencies noise up to the desired specified cut-off-frequency. A second-order filter will be used.

4) **A High Pass Filter**

is used to block unwanted low-frequency noise. A first order will be used as the slope in the lower frequencies is -1 for \( \frac{X}{F_a} \).

**Remark:** No Integral action \( K_i \) can be used as an integrator creates a saturation of the actuator at the lower frequencies resulting in an instability. The sensitivity \( |S(s)|_{s=\omega_c} \) tends to 1. That is not suitable to get noise reduction. More detail are given in the Appendix D

A quick view of the transfer functions of the filters used is depicted in the table 5.2.

<table>
<thead>
<tr>
<th>PID</th>
<th>PID = ( K_p )</th>
<th>( K_p ): Proportional gain [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Pass Filter 1</td>
<td>( HPF_1 = \frac{ks}{s+\omega_c} )</td>
<td>( k ): Proportional gain [-] ( \omega_c ): Cut-off-frequency [rad/sec]</td>
</tr>
<tr>
<td>Low Pass Filter 2</td>
<td>( LPF_2 = \frac{k\omega_c^2}{s^2+2\beta\omega_c s + \omega_c^2} )</td>
<td>( k ): Proportional gain [-] ( \omega_c ): Cut-off-frequency [rad/sec] ( \beta ): Damping constant [-]</td>
</tr>
<tr>
<td>Notch Filter</td>
<td>( Nch = \frac{k(s^2+\omega_c^2)}{s^2+Q\omega_c s + \omega_c^2} )</td>
<td>( k ): Proportional gain [-] ( \omega_c ): Center frequency [rad/sec] ( Q ): Damping constant [-]</td>
</tr>
</tbody>
</table>

**Table 5.2 Types of filters used**

### 5.3.2 Controller design.

As indicated earlier the design of the controller is done by loop shaping. A Matlab routine is used to achieve this process. The Matlab routine is built in three parts:

a) The first part reads the measured transfer function, defines the desired values of parameters in each kind of filter needed for the corresponding desired action.

b) The second part contains the parametric definition of all defined usable filters. They get the earlier values introduced as inputs.

c) The third part computes the controller, the open-loop transfer function, the sensitivity and the necessary functions in the analysis of the stability and disturbance rejection. The values of the parameters for the first iteration are given in the table 5.3.

The designed controller \( C \), the open-loop \( H_{ol} = CH \), the sensitivity \( S = \frac{1}{1+CH} \), and the closed loop \( H_{cl} = \frac{H}{1+CH} \) for \( Z\text{-translation} \) -axes are collected in figure 5.3 and figure 5.4.
Table 5.3: Filter parameters SISO design first iteration

Table 5.3: Filter parameters SISO design first iteration

<table>
<thead>
<tr>
<th>Axis</th>
<th>Controllers</th>
<th>$k$, or $K_p$</th>
<th>$f_c$ [Hz]</th>
<th>$\beta$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ztrans</td>
<td>Notch</td>
<td>1</td>
<td>90.6</td>
<td>–</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>$H_{opt}$</td>
<td>1</td>
<td>0.3</td>
<td>$f_1 = 0.3; f_2 = 0.15$</td>
<td>–</td>
</tr>
<tr>
<td>$PID = 0.1$</td>
<td>$HPF_1$</td>
<td>1</td>
<td>0.15</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>$LPF_2$</td>
<td>1</td>
<td>100</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Zrot</td>
<td>Notch</td>
<td>1</td>
<td>91.55</td>
<td>–</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>Notch</td>
<td>1</td>
<td>161</td>
<td>–</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>Notch</td>
<td>1</td>
<td>177</td>
<td>–</td>
<td>0.22</td>
</tr>
<tr>
<td>$PID = 0.04$</td>
<td>$HPF_1$</td>
<td>1</td>
<td>0.15</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>$LPF_2$</td>
<td>1</td>
<td>70</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

Figure 5.3 Z – translation 1st Iteration 5 Controllers off

The obtained controllers are implemented in the system; the stability of the system is kept as expected with Gain margin of 8.7 dB and 13.4 dB and Phase margin of 44° and 41° for $Z – translation$ -axes respectively. The observation of the sensitivity shows that the controller fulfills to the defined criteria with the parameters given in the table 5.4 except for the gain margin in $Z – translation$. 
Iteration
A major observation is made at this level. The system can be active with only one controller set on at a time; the system specifications are then known as computed here above. But in reality the system is active with all the six controllers set on. When this is done, the behaviour of the system does not correspond anymore to the single active controller system.

Comparative plots for (Z-translation -axe) of $H$ the transfer function when all controllers are set out, and $H_d$ the transfer function when five controllers are set on and the Z-translation axis controller is set out in figure 5.4(a); and the corresponding sensitivity in figure 5.4(b) indicates that the observed axis becomes instable when five other controllers are implemented. There is a virtual change of the axis characteristics exhibited by a change of the transfer function (figure 5.4 (a) (red)). The first designed controller is not more able to assure the stability and the robustness as show in figure 5.4 (c) and (d) (red: a peak of $S(s) > 6dB$ at about 100Hz). This justifies the use of an iterative process to design a better controller.

![Comparison plots](image)

**Legend:**
- $H$: FRF measured with one axis the remains are set OFF.
- $H_d$: FRF measured with one axis the remains are set ON.
- $C$: Controller designed with one axis 5 remains are set OFF.
- $C_d$: Controller designed with one axis 5 remains are set ON.

*Figure 5.4 Compared Open-Loop and Sensitivity (Z-translation axis)*
In the second iteration the same procedure as described before is applied. The transfer function of a chosen axis is measured by setting out his controllers; the remaining five axes have all theirs controllers implemented; the obtained transfer function for $Z$–translation and $Z$–rotation–axes are show in the figure 5.5.

![Figure 5.5 Transfer Function $H_d$ (5 Controllers on)](image)

The sensitivity and the open-loop Bode diagram of $Z$–translation (figure 5.6) axes indicate clearly a distorted system in comparison of the preceding results. The table 5.5 gives a comparison of the parameter results from the system with one controller at time or with 5 controllers together.

|       | max($|S(s)|$)[dB] | COF$_{left}$[Hz] | COF$_{right}$[Hz] | $|S(s)|_{at_{max}}$ (dB) | Gain (Gm) [dB] | Phase (Pm) $^\circ$ |
|-------|-----------------|-----------------|------------------|-------------------------|----------------|-----------------|
| $Ztr$ | 4.45            | 1.1             | 14               | -25                     | 8.7            | 44              |
| $Zrot$ | 4.99           | 1.0             | 15               | -34.6                   | 13.4           | 41              |
| Crit  | $\leq 5$        | $\geq 1$        | $\geq 7$         | $\leq -25$              | $\geq 10$      | $\geq 45$      |

Table 5.5 Comparative Sensitivity parameters first iteration
The perturbation is more visible with the Gain and Phase margin. Gain margin of 
$-1.5\, dB$ and $2.6\, dB$ and Phase margin of $6.5^\circ$ and $7.8^\circ$ for $Z$ – translation and $Z$ – rotation -
axes respectively are obtained when five controllers are implemented. The system is not
anymore robust with respect to the one controller implemented system.
Applying the same criterion as described before a new set of controllers $C_d$ to the new
measured transfer function $H_d$ is designed. The results are collected in the table 5.6

<table>
<thead>
<tr>
<th>Axis and $K_p$</th>
<th>Controllers</th>
<th>$k$, or $K_p$</th>
<th>$f_c$ [Hz]</th>
<th>$\beta$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ztrans</td>
<td>Notch</td>
<td>1</td>
<td>91.1</td>
<td>–</td>
<td>1.5</td>
</tr>
<tr>
<td>$PID = 0.1$</td>
<td>Notch</td>
<td>1</td>
<td>107</td>
<td>–</td>
<td>1.52</td>
</tr>
<tr>
<td></td>
<td>LPF$_2$</td>
<td>1</td>
<td>117</td>
<td>1</td>
<td>–</td>
</tr>
<tr>
<td>Zrot</td>
<td>Notch</td>
<td>1</td>
<td>91.55</td>
<td>–</td>
<td>0.2</td>
</tr>
<tr>
<td>$PID = 0.06$</td>
<td>Notch</td>
<td>1</td>
<td>161</td>
<td>–</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>Notch</td>
<td>1</td>
<td>177</td>
<td>–</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>HPF$_1$</td>
<td>1</td>
<td>0.15</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>LPF$_2$</td>
<td>1</td>
<td>70</td>
<td>1</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 5.6: Filter parameters SISO design second iteration

A similar analysis can be done as in the first iteration. The designed controller $C_d$, the open-
loop $H_{dOL} = C_dH_d$, the closed loop $H_{dCL} = \frac{H_d}{1 + C_dH_d}$ and the sensitivity $S_d = \frac{1}{1 + C_dH_d}$ for
$Z$ – translation -axe are collected in the figure 5.6

The entire six controllers are implemented. The parameters of the controllers compared to
the criteria in the table 5.7

|               | max($|S(s)|$)[dB] | COF$_{left}$[Hz] | COF$_{right}$[Hz] | $|S(s)|$ at $\omega_c$ (dB) | Gain (Gm)[dB] | Phase (Pm)$^\circ$ |
|---------------|-----------------|-----------------|-----------------|----------------|--------------|----------------|
| Ztr d         | 4.45            | 2               | 10              | -17.5          | 9.4          | -57.8         |
| Zrot d        | 4.64            | 0.68            | 8.5             | -27.2          | 14.5         | 40            |
| Crit          | $\leq 5$        | $\geq 1$        | $\geq 7$        | $\leq -25$     | $\geq 10$    | $\geq 45$    |

Table 5.7 Sensitivity parameters second iteration

There is a violation of criteria in $COF_{left}$ and $|S(s)|$ at $\omega_c$ that can be avoided by a relaxation
on the $\max(|S(s)|)$ criteria from the $\max(|S(s)|) \leq 5dB$ to $\max(|S(s)|) \leq 6dB$

The second designed controller leads to a more robust system as can be seen in the
figure 5.7 (a) and (b)
Figure 5.6 Z-translation -axe

(a) Controller $C_d$

(b) open-loop $H_{OL} = C_d H_d$

(c) Closed loop $H_{dCL} = \frac{H_d}{1 + C_d H_d}$ and plant $H_d$

(d) sensitivity $S_d = \frac{1}{1 + C_d H_d}$

Figure 5.7 comparative plots for (Z-translation -axe)
5.4 Comments and recommendations.

A reduction of the amplitude of the low frequency disturbance accompanied by a negligible amplification (5%) of the high frequency noise is the remarkable outcome of the time response observation of $Z$-translation and $Z$-rotation axes figure 5.8. During the controller tuning it appears to be impossible to achieve a significant reduction of high frequency noise. That is a direct consequence of the positive values (in dB) of the sensitivity outside of the attenuation zone of the corresponding passive system (lower and higher frequencies around the resonance frequency).

The controlled time response obtained seems to be an unrefined result, even if the absolute amplitude reduction is about 600% in $Z_{\text{trans}}$ and 320% in $Z_{\text{rot}}$. The iterative tuning of the controller action seems to be limited; at the second iteration the process turns around and no improvement of controller is possible. It is therefore important to find other ways to conceive the Active Vibration Isolation System if a highly precision result is desired.

A MIMO analysis will certainly improve this performance as the missing cross terms are considered.

The performance is not easy to be defined because of the unpredictability of the ground vibration. Therefore the controller design is depending on the kind of the ground vibration
Figure 5.8 Time response Z-translation and Z-rotation axes
Chapter 6

Conclusions and recommendations

6.1 Conclusions

A few words are said in this work about the applications and the importance of an active vibrations isolation system.

A brief description of the functionality and mathematic modelling of the active vibration isolation system is done; leading to the derivation of the actuator matrix and the sensor matrix.

The derivation of theoretical transfer function from the actuator force to the mass velocity of the model is also done.

The AVIS system is more complex and the characteristics are not exactly defined; therefore measurements are made to define practical transfer functions for a SISO model. Transmissibility is not measured as the ground vibration measurement is not available.

A feedback control system is chosen to design the controller with as target to reduce the ground vibration considered as noise. The sensitivity function is therefore used as main tool because the ground vibration is assimilated to a disturbance of the feedback system. The controller design is then a disturbance rejection problem. A set of criteria is defined based on the Nyquist Stability Criteria and the robustness.

As the ground vibration frequencies range is large, the Bode integral creates a trade-off between the reduction of the noise in the positive sensitivity frequency zone and the amplification of the noise in the negative sensitivity frequency zone noise and vice versa. This trade-off limits drastically the performance of the designed controllers.

Controllers are tuned based on the Transfer function measurements of a single axis. However, when all the controllers are turned on simultaneously, it appears that robustness and performance criteria are not anymore satisfied. The reason for this is the missing coupling between the individual axes omitted in the SISO modelling.

An iterative tuning process leading to a better disturbance rejection solves the problem of the omitted coupling.

6.2 recommendations

In this report the feedbacks controllers are manually tuned in a loop shaping procedure. A better set of controllers can be obtained by using an optimization process in the design of the controller instead of a manually loop shaping tuning. The optimum controllers’ parameters are then obtained satisfying the specified criteria.

This research is focussed on feedback control design. The development of a feedforward control in combination with the feedback control may also lead to a significant improvement of performance. Beforehand a ground vibration real time measurement must be added to the AVIS equipment.
In this study the AVIS is considered as a combination of six SISO systems. However in practice the AVIS behaves as MIMO system. The most important consideration is the modelling of the system as a MIMO model and the design of controller based on the MIMO model. This will certainly improve the performance of the control action on the ground vibration.
References


Bernstein Denis, S. “A Student Guide to Classical Control”, University of Michigan, Ann Arbor, Michigan

Appendix
Appendix A: Coefficients of Mass, Stiffness, Damping Matrix and Generalized Force

Mass matrix

\[
\begin{bmatrix}
m & 0 & 0 & 0 & 0 & 0 \\
0 & m & 0 & 0 & 0 & 0 \\
0 & 0 & m & 0 & 0 & 0 \\
0 & 0 & 0 & M(4,4) & M(5,4) & M(6,4) \\
0 & 0 & 0 & M(4,5) & M(5,5) & M(6,5) \\
0 & 0 & 0 & M(4,6) & M(5,6) & M(6,6)
\end{bmatrix}
\]  
(A.1)

\[M(4,4) = J_{xx} + m(y^2 + z^2)\]
\[M(4,5) = -m(xy + xz \phi)\]
\[M(4,6) = J_{xx} \phi + m((y^2 + z^2) \phi + x(y-z \theta))\]
\[M(5,4) = M(4,5)\]
\[M(5,5) = J_{yy} + J_{zz} \alpha^2 + m(x^2 + z^2 + (x^2 + y^2) \alpha^2 - 2yz \theta)\]
\[M(5,6) = (J_{xx} - J_{yy}) \phi + m(\theta(yz - x) + yz(\theta^2 - 1) + (y^2 - z^2))\]
\[M(6,4) = M(4,6)\]
\[M(6,5) = M(5,6)\]
\[M(6,6) = \phi^2 J_{xx} + \theta^2 J_{yy} + J_{zz} + m((y^2 + z^2) \phi^2 + (x^2 + z^2) \theta^2 + x^2 + y^2 + 2(\theta xy - xz) \phi + 2yz \theta)\]

Stiffness matrix

\[
\begin{bmatrix}
K(1,1) & 0 & 0 & 0 & K(5,1) & K(6,1) \\
0 & K(2,2) & 0 & K(4,2) & 0 & K(6,2) \\
0 & 0 & K(3,3) & K(4,3) & K(5,3) & 0 \\
0 & K(2,4) & K(3,4) & K(4,4) & K(5,4) & K(6,4) \\
K(1,5) & 0 & K(3,5) & K(4,5) & K(5,5) & K(6,5) \\
K(1,6) & K(2,6) & 0 & K(4,6) & K(5,6) & K(6,6)
\end{bmatrix}
\]  
(A.2)

\[K(1,1) = k_{11} + k_{12} + k_{13} + k_{14}\]
\[K(1,5) = -(k_{11} + k_{12} + k_{13} + k_{14}) h\]
\[K(1,6) = -\frac{1}{2} (k_{11} - k_{12} - k_{13} + k_{14}) d_1\]
\[K(2,2) = k_{21} + k_{22} + k_{23} + k_{24}\]
\[K(2,4) = (k_{21} + k_{22} + k_{23} + k_{24}) h\]
\[K(2,6) = \frac{1}{2} (k_{21} + k_{22} - k_{23} - k_{24}) d_2\]
\[K(3,3) = k_{31} + k_{32} + k_{33} + k_{34}\]
\[ K(3,4) = \frac{1}{2}(k_{31} - k_{32} - k_{33} + k_{34})d_1 \]
\[ K(3,5) = -\frac{1}{2}(k_{31} + k_{32} - k_{33} - k_{34})d_2 \]
\[ K(4,4) = (k_{21} + k_{22} + k_{23} + k_{24})h^2 + \frac{1}{4}(k_{31} + k_{32} + k_{33} + k_{34})d_1^2 \]
\[ K(4,5) = -\frac{1}{4}(k_{31} - k_{32} + k_{33} - k_{34})d_1d_2 \]
\[ K(4,6) = \frac{1}{2}(k_{21} + k_{22} - k_{23} - k_{24})hd_2 \]
\[ K(5,5) = (k_{11} + k_{12} + k_{13} + k_{14})h^2 + \frac{1}{4}(k_{31} + k_{32} + k_{33} + k_{34})d_2^2 \]
\[ K(5,6) = \frac{1}{2}(k_{11} - k_{12} - k_{13} + k_{14})hd_1 \]
\[ K(6,6) = \frac{1}{4}((k_{11} + k_{12} + k_{13} + k_{14})d_1^2 + (k_{21} + k_{22} + k_{23} + k_{24})d_2^2) \]

**Damping matrix**

\[
\begin{bmatrix}
B(1,1) & 0 & 0 & 0 & B(5,1) & B(6,1) \\
0 & B(2,2) & 0 & B(4,2) & 0 & B(6,2) \\
0 & 0 & B(3,3) & B(4,3) & B(5,3) & 0 \\
0 & B(2,4) & B(3,4) & B(4,4) & B(5,4) & B(6,4) \\
B(1,5) & 0 & B(3,5) & B(4,5) & B(5,5) & B(6,5) \\
B(1,6) & B(2,6) & 0 & B(4,6) & B(5,6) & B(6,6)
\end{bmatrix}
\]

\[
\begin{align*}
B(1,1) &= b_{11} + b_{12} + b_{13} + b_{14} \\
B(1,5) &= -(b_{11} + b_{12} + b_{13} + b_{14})h \\
B(1,6) &= -\frac{1}{2}(b_{11} - b_{12} - b_{13} + b_{14})d_1 \\
B(2,2) &= b_{21} + b_{22} + b_{23} + b_{24} \\
B(2,4) &= (b_{21} + b_{22} + b_{23} + b_{24})h \\
B(2,6) &= \frac{1}{2}(b_{21} + b_{22} - b_{23} - b_{24})d_2
\end{align*}
\]
\[ B(3, 3) = b_{31} + b_{32} + b_{33} + b_{34} \]
\[ B(3, 4) = \frac{1}{2} (b_{31} - b_{32} - b_{33} + b_{34})d_1 \]
\[ B(3, 5) = -\frac{1}{2} (b_{31} + b_{32} - b_{33} - b_{34})d_2 \]
\[ B(4, 4) = (b_{21} + b_{22} + b_{23} + b_{24})h^2 + \frac{1}{4} (b_{31} + b_{32} + b_{33} + b_{34})d_1^2 \]
\[ B(4, 5) = -\frac{1}{4} (b_{31} - b_{32} + b_{33} - b_{34})d_1d_2 \]
\[ B(4, 6) = \frac{1}{2} (b_{21} + b_{22} - b_{23} - b_{24})hd_2 \]
\[ B(5, 5) = (b_{11} + b_{12} + b_{13} + b_{14})h^2 + \frac{1}{4} (b_{31} + b_{32} + b_{33} + b_{34})d_2^2 \]
\[ B(5, 6) = \frac{1}{2} (b_{11} - b_{12} - b_{13} + b_{14})hd_1 \]
\[ B(6, 6) = \frac{1}{4} ((b_{11} + b_{12} + b_{13} + b_{14})d_1^2 + (b_{21} + b_{22} + b_{23} + b_{24})d_2^2) \]

**Generalized forces**

\[ [Q(1), Q(2), Q(3), Q(4), Q(5), Q(6)]' \]

\[ Q(1) = (k_{11} + k_{12} + k_{13} + k_{14} + (b_{11} + b_{12} + b_{13} + b_{14})i\omega_x)X_0 + Ax_2 - Ax_4 \]
\[ Q(2) = (k_{21} + k_{22} + k_{23} + k_{24} + (b_{21} + b_{22} + b_{23} + b_{24})i\omega_y)Y_0 + Ay_1 - Ay_3 \]
\[ Q(3) = (k_{31} + k_{32} + k_{33} + k_{34} + (b_{31} + b_{32} + b_{33} + b_{34})i\omega_z)Z_0 + Az_1 + Az_2 + Az_3 + Az_4 \]
\[ Q(4) = ((k_{41} + k_{42} + k_{43} + k_{44} + (b_{41} + b_{42} + b_{43} + b_{44})i\omega_x)X_0 + Ax_2 - Ax_4)h \]
\[ \frac{1}{2} ((k_{31} - k_{32} - k_{33} + k_{34} + (b_{31} - b_{32} - b_{33} + b_{34})i\omega_z)Z_0 + Az_1 - Az_2 - Az_3 + Az_4)d_1 \]
\[ Q(5) = -((k_{11} + k_{12} + k_{13} + k_{14} + (b_{11} + b_{12} + b_{13} + b_{14})i\omega_x)X_0 + Ax_2 - Ax_4)h \]
\[ \frac{1}{2} ((-k_{31} - k_{32} + k_{33} + k_{34} + (b_{31} - b_{32} + b_{33} + b_{34})i\omega_z)Z_0 - Az_1 - Az_2 + Az_3 + Az_4)d_2 \]
\[ Q(6) = \frac{1}{2} (((-k_{11} + k_{12} + k_{13} - k_{14} + (b_{11} + b_{12} + b_{13} - b_{14})i\omega_x)X_0 + Ax_2 + Ax_4)d_1 \]
\[ \frac{1}{2} ((k_{21} + k_{22} - k_{23} - k_{24} + (b_{21} + b_{22} - b_{23} - b_{24})i\omega_y)Y_0 + Ay_1 + Ay_3)d_2 \]
Appendix B: *Transfer function and Sensitivity with System Parameters Variation (Theoretical Model Simulation)*

*Transfer Function H(s) for x-translation and y-rotation*
Symmetric System when

\[
\begin{align*}
    k_{x,i} & \text{ for } i=1,2,3,4 = C_{kx} \\
    k_{y,i} & \text{ for } i=1,2,3,4 = C_{ky} \\
    k_{z,i} & \text{ for } i=1,2,3,4 = C_{kz} \\
    b_{x,i} & \text{ for } i=1,2,3,4 = C_{bx} \\
    b_{y,i} & \text{ for } i=1,2,3,4 = C_{by} \\
    b_{z,i} & \text{ for } i=1,2,3,4 = C_{bz}
\end{align*}
\]

with \( C_{kx}, C_{ky}, C_{kz}, C_{bx}, C_{by}, C_{bz} \) Constants

\[\Delta k_x = 4.8\%\]
\[\Delta k_y = 10.3\%\]
\[\Delta k_z = 7.3\%\]
\[\Delta b_x = 4.8\%\]
\[\Delta b_y = 8.2\%\]
\[\Delta b_z = 7.3\%\]

The non-symmetric system is built with difference of order

\[\Delta k_x, \Delta k_y, \Delta k_z, \Delta b_x, \Delta b_y, \Delta b_z\] are obtained from

\[
\Delta k_{x,y,z} = \frac{\max k_{x,y,z,i} - \min k_{x,y,z,i}}{\max k_{x,y,z,i}}
\]

\[
\Delta b_{x,y,z} = \frac{\max b_{x,y,z,i} - \min b_{x,y,z,i}}{\max b_{x,y,z,i}}
\]
Appendix C: **Siglab Settings and Coherence Function**

(a) Siglab Settings

(b) Coherence
Appendix D: Why no use of Integrator action in the Controller.

No Integral action I can be taken as the transfer function has a zero, the sensitivity $S$ will becomes consequently to an I action

$$S(s) = \frac{s^2 + 2\xi\omega_n s + \omega_n^2}{s^2 + 2\xi\omega_n s + (1 + \frac{KI}{k})\omega_n^2}$$

The following observations on $S$ are made:

$$|S| \rightarrow \frac{1}{(1 + \frac{KI}{k})} \text{ if } s \rightarrow 0 \text{ and } |S| \rightarrow 1 \text{ if } s \rightarrow \infty$$

$$|S| \rightarrow \frac{-\omega_n^2 + 2\xi\omega_n^2 + \omega_n^2}{-\omega_n^2 + 2\xi\omega_n^2 + (1 + \frac{KI}{k})\omega_n^2} \text{ if } s \rightarrow \omega_n$$

$$|S| \rightarrow \frac{2\xi}{-1 + 2\xi + (1 + \frac{KI}{k})} \text{ if } k \gg KI$$

$$|S| \rightarrow 0 \text{ if } k \ll KI$$

That is not suitable to get noise reduction; quite the contrary happens with a stronger amplification of noise at the resonance frequency $\omega_n$ as is showed in figure (a), (b) and (c)
Appendix E.1: *Filter parameters SISO design first iteration.*

<table>
<thead>
<tr>
<th>Axis</th>
<th>Controllers</th>
<th>$k$, or $K_p$</th>
<th>$f_c [Hz]$</th>
<th>$\beta$</th>
<th>$Q$</th>
</tr>
</thead>
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</tr>
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## Appendix E.2: *Filter parameters SISO design second iteration.*

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Appendix F: **Open Loop Transfer Function Second Iteration**
(Six Controllers On)
Appendix G: Sensitivity Second Iteration (Six Controllers On)
Appendix H: *Gain and Phase Margin Measurements (X and Y axes) (Second iteration)*

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</tr>
<tr>
<td>$Y_{rot}$</td>
<td>10.1</td>
<td>-69.6</td>
</tr>
<tr>
<td>Crit</td>
<td>$\geq 10$</td>
<td>$\leq 45$</td>
</tr>
</tbody>
</table>
Appendix J: *Time response Measurements (X and Y axes)*

![Graph showing time response measurements for X-translation non-controlled.](image)

![Graph showing time response measurements for X-translation controlled.](image)

![Graph showing time response measurements for X-rotation non-controlled.](image)

![Graph showing time response measurements for X-rotation controlled.](image)
Y-translation non controlled

Y-translation controlled

Y-rotation non controlled

Y-rotation controlled