Slip controller design and implementation in a Continuously Variable Transmission

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Abstract

Continuously Variable Transmissions (CVT) can be used to operate a combustion engine in a more optimal working point. Unfortunately, due to the relatively low efficiency of modern production CVT’s the total efficiency of the driveline is not increased significantly. This low efficiency is mainly caused by losses in the hydraulic actuation system and the variator. Decreasing the clamping forces in the variator greatly improves the efficiency of the CVT. However, lower clamping forces increase the risk of excessive belt slip, which can damage the system. In this paper a method is presented to measure and control slip in a CVT in order to minimize the clamping forces while preventing destructive belt slip. To ensure robustness of the system against torque peaks, a controller is designed with optimal load disturbance response. A synthesis method for robust P(D)-controller design is used to maximize the integral gain while making sure that the closed loop system remains stable. Experimental results prove the validity of the approach.
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1. Introduction

Fuel consumption and driveline efficiency are important issues in the automotive industry. Continuously Variable Transmissions (CVT) can cover a wide range of ratio’s, which makes it possible to operate a combustion engine in more efficient working points than stepped transmissions. This decreases the fuel consumption of the engine, but because of the relatively low efficiency of a CVT compared to manual transmissions the total driveline efficiency is not increased significantly.

The main reason for the low efficiency of modern production CVT’s are the high clamping forces in the variator necessary to prevent belt slip. Heavy belt slip can cause severe damage to the belt and pulleys of the variator, resulting in lower performance of the CVT in time. To prevent belt slip at all times, the clamping forces in modern production CVT’s are usually much higher (typically 30% or more) than needed for normal operation. Higher clamping forces result in additional losses in both the hydraulic and the mechanical system. This is due to increased pump losses and friction losses because of the extra mechanical load that is applied on all parts, particularly on the variator.

Studies have shown that reducing the clamping forces in a CVT result in a remarkable increase in efficiency [1], [2]. However, it also increases the risk of excessive belt slip when torque peaks act on the driveline. But belt slip does not always lead to damage to the belt and pulleys, as recent research shows [3]. No damage occurs as long as certain limits in belt slip speed and belt normal forces are not exceeded. In this paper, a method is presented to measure and control slip in a modern production CVT, namely the Jatco CK2 [4]. By using slip control it is possible to operate the CVT with minimal clamping forces, resulting in a higher efficiency, while preventing excessive belt slip. The most important requirement of the slip controller is that it has the ability to attenuate the load disturbances caused by torque peaks in the driveline. An additional problem in the controller design process is that the slip dynamics change for different values of ratio and speed. Therefore a robust gain-scheduled controller is desirable. To meet all requirements, a synthesis method for robust PI(D)-controllers with optimal load disturbance response is used [5]. The designed slip controller is simulated and subsequently tested on a test rig that is equipped with a 2.0-liter Internal Combustion Engine (ICE), a flywheel, an eddy-current brake, and a disc brake to simulate realistic road loads.
2. Continuously Variable Transmission

2.1 Working principle

In this research program the Jatco CK2 is used for testing, which is based on the metal push belt from Van Doorne's Transmission (VDT). Fig. 1 shows the layout of the variator, which is the key element of a CVT. The variator consists of a V-shaped metal belt between two sets of conical sheaves, also called pulleys. Both pulley sets have a fixed and a moveable pulley, opposed to each other. The moveable pulleys are actuated by hydraulic pressure cylinders. By adjusting the position of the pulleys the ratio of the variator is changed. The variator can cover any ratio between the two extremes shown in Fig. 1, low and overdrive.

The speed ratio of the variator is defined as:

\[ r_s = \frac{\omega_s}{\omega_p} \]  

(1)

Where \( \omega_s \) represents the angular speed of the secondary (output) shaft and \( \omega_p \) the angular speed of the primary (input) shaft.

Power is transmitted by means of friction between the belt and the pulleys. The torque that is transmitted through the variator can be calculated using the force balance on a pulley, according to [2]:

\[ T_{\text{ext},p,s} = \frac{2F_s R_{p,s} \mu}{\cos \alpha} \]  

(2)

Where \( T_{\text{ext},p,s} \) is the transmitted torque for respectively the primary and the secondary shaft, \( F_s \) is the secondary clamping force, which is the force applied by the pulleys onto the belt, \( R_{p,s} \) represent the primary and secondary running radius of the belt respectively, \( \alpha \) is the pulley wedge angle and \( \mu \) is the traction coefficient between belt and pulley. The traction coefficient \( \mu \) is not constant but depends on the relative slip between the belt and the pulleys, defined as [2]:

\[ \nu = 1 - \frac{r_s}{r_{s0}} \]  

(3)

Where \( r_{s0} \) is defined as the speed ratio when no torque is applied to the secondary variator shaft. The ratio
\( r_{so} \) can be reconstructed by measuring the position of one of the moveable pulleys using a linear displacement sensor. First the output of the displacement sensor is measured under no-load conditions for all ratios. The relationship between \( r_{so} \) and the output of the displacement sensor is approximated using a sixth order polynomial, which is then used to reconstruct the no-load ratio \( r_{so} \).

The relation between the traction coefficient \( \mu \) and the slip \( v \) is shown in Fig. 2 [6]. It can be seen that the slope of the curves and the maximum traction coefficients clearly depend on the ratio, but all curves show the same distinct shape. At first, for low slip values the traction coefficient increases with increasing slip, until a maximum value is reached. This region is called the microslip region. When the maximum value of the traction coefficient is reached, increasing slip will result in a slow decrease of the traction coefficient. This region is known as the macroslip region.

![Fig. 2. Traction coefficient \( \mu \) as a function of the relative slip measured with an input speed of 300 rad/s for ratios low (0.43), medium (1) and overdrive (2.25)](image)

### 2.2 Clamping force strategies

An increase in torque in the variator will lead to more slip. When the slip level remains in the microslip region, an increase in slip will also lead to an increasing traction coefficient, thus allowing the higher torque to be transmitted. In the macroslip region however, slip will increase drastically if no action is taken when the torque increases.

The majority of the current clamping force strategies are designed to keep the slip values within the microslip region at all times to prevent belt damage. This is achieved by applying a clamping force that is high enough to transmit the engine torque that is based on an estimation from the Engine Control Module (ECM). To make sure that torque shocks will not trigger excessive slip, this clamping force is multiplied by a safety factor of at least 1.3. Additionally a safety margin on the clamping force is added for low engine torques. Since the engine torque is in general relatively low in normal operation, the clamping forces are much too high most of the time. This contributes greatly to the low efficiency of modern CVT's mentioned earlier.

Using slip control, the clamping forces are actively controlled to maximize the efficiency of the CVT. This is achieved by maintaining an amount of slip, where the traction coefficient is near its maximum [2]. This means that the slip is controlled in the transition area of the micro- and macroslip regions. An increase in the torque level will lead to an increase in belt slip, but by adjusting the clamping force fast enough the slip will not reach destructive levels and therefore damage can be avoided.
3. Slip dynamics

3.1 Modeling slip dynamics

To be able to design a slip controller, the slip dynamics are modelled. The model is based on the CVT dynamics presented in Fig. 3, where \( T_e \) and \( J_e \) represent the engine torque and inertia respectively, \( T_d \) and \( J_d \) represent the driveline torque and inertia and \( T_{cvt,i,d} \) are given by (2).

Fig. 3. CVT dynamics

The relative slip \( \nu \) is based on the no-load ratio \( r_{s0} \), which is experimentally obtained as mentioned earlier. Since this is not very suitable for modeling purposes, the geometric ratio \( r_g \) is used instead, which is a good approximation of \( r_{s0} \), defined as:

\[
    r_g = \frac{R_p}{R_s}
\]

(4)

In this model the geometric ratio is assumed quasi-stationary. This is possible because the geometric ratio has much slower dynamics than the slip dynamics during normal operation of a CVT. Based on this assumption, the slip dynamics can be derived using (1) and (3), resulting in:

\[
    \nu = -\frac{\dot{r}_g}{r_g}
\]

(5)

\[
    \dot{r}_g = \frac{\omega_p \omega_p - \omega_s \dot{\omega}_p}{\omega_p^2}
\]

(6)

With \( r_g \) quasi-static, the dynamics of the CVT can be described by:

\[
    \dot{\omega}_p = \frac{T_e - T_{cvt,p}}{J_e}
\]

(7)

\[
    \dot{\omega}_s = \frac{T_d - T_{cvt,d}}{J_d}
\]

(8)
Combining (1)-(8) results in the following expression for the slip dynamics:

\[ \dot{\nu} = \frac{1}{\alpha_p} \left( -\frac{2FR_0\mu(\nu)}{\cos(\alpha)J_d} + \frac{T_d}{J_d} \right) + \left(1 - \nu \right) \left( -\frac{2FR_0\mu(\nu)}{\cos(\alpha)J_e} + \frac{T_e}{J_e} \right). \]  

(9)

The derived dynamics for the slip are nonlinear. For controller design purposes, the system will be linearized around different operating points. With the linearized model a state space representation of the system will be defined.

For this purpose the traction coefficient is taken piecewise linear, to describe the micro- and macroslip region. Indicating the different regions with index \( i \), the traction coefficient can be written as:

\[ \mu = k_{1i}\nu + k_{2i}. \]  

(11)

Defining the state space as \( x = \nu \), and \( u = \begin{bmatrix} F & T_e & T_d \end{bmatrix} \), the system can be linearized around a certain working point \( x = \nu_0 \), resulting in the linear system:

\[ \dot{x} = Ax + Bu \]  

(12)

Where \( \dot{x} = x - x_0 \) and \( \ddot{u} = u - u_0 \). The linearized matrices \( A \) and \( B \) can now be derived:

\[ A = \frac{1}{\alpha_p} \begin{bmatrix} -\frac{T_{e0}}{J_e} & -\rho_0\psi_0F_0k_{1l} & \rho_0F_0r_0g_0k_{2l} \end{bmatrix} \]  

(13)

\[ B = \frac{1}{\alpha_p} \begin{bmatrix} \frac{1 - \nu_0}{J_e} \frac{\rho_0r_0g_0\nu_0k_{2l}}{J_e} \frac{1}{J_d} \frac{1}{J_d} \end{bmatrix} \]  

(14)

Where \( \rho_0 = \frac{2R_0}{\cos(\alpha)} \) and \( \psi_0 = \left( \frac{r_0}{J_e} + \frac{1}{J_d} \frac{1}{r_0} \right) \) are introduced for writing convenience. Using (2) \( T_{e0} \) in (13) is calculated from the other values to match the maximum torque that can be transmitted in the chosen working point, this results in:

\[ T_{e0} = \rho_0F_0r_0g_0\mu(\nu_0). \]  

(15)

The linearization process of (9) also produces a higher order term in both \( A \) and \( B \), but these are neglected because they are more than one order smaller than the other terms.

The derived linearized system will be used for controller design. This model has 3 inputs, but only the clamping force \( F \) can be controlled on implementation. The input torque \( T_e \) is controlled by the driver via the throttle pedal and the output torque \( T_d \) is determined by road conditions. Therefore they can be regarded as disturbances acting on the system.
3.2 FRF-measurements actuation system

The clamping force in the Jatco CK2 is applied using hydraulic pressure cylinders attached to the movable pulleys [4]. The oil pressure in the cylinders is regulated by a complex electro-hydraulic actuation system that is controlled by a PWM-based solenoid. The duty cycle of the PWM-signal determines the oil pressure that provides the clamping force, or the line pressure. The line pressure in the CK2 is limited between 0.66 and 4.2 MPa, between these values the pressure varies practically linear with the duty cycle. Modeling this electro-hydraulic system is a complex and time-consuming task. Therefore the system's dynamic response is determined using FRF-measurements. A good estimation of the system's response will then be used in the controller design process.

For the FRF-measurements, the duty cycle of the solenoid is taken as the input and the line pressure as the output. The measurements were performed at different pressures, ratios, and engine speeds. All measurements showed practically identical system responses, only with slightly different gains for low frequencies, but small enough to be neglected. Fig. 4 shows the result of one of the FRF-measurements. The system is estimated with a third order low-pass filter with a cut-off frequency of 6 Hz, which is also plotted in the figure. The frequency of the PWM signal is 50 Hz, this causes a peak in the FRF. Because the bandwidth of the system is much lower than 50 Hz it is not taken into account in the estimation.

![Fig. 4. Measured and estimated FRF of the line pressure circuit in the Jatco CK2](image-url)
4. Slip controller design

4.1 The control design problem

With the linearized model of the slip dynamics and the estimated transfer function of the actuation system, a slip controller can be designed. The slip dynamics are highly nonlinear and depend on many variables. Using (13) and (14), the variables that influence the slip dynamics the most can be found. There is a great difference in the system response between the micro- and macroslip region. For slip control design, attention is mainly focused on the macroslip region. In this region, ratio and primary speed have the largest influence on the dynamics. Because the dynamics depend on so many variables, it is practically impossible to design a controller that is stable in all situations and still has the desired performance.

To tackle this problem, a gain-scheduled linear controller will be designed. This is done by linearizing the slip dynamics in a number of working points and calculating the controller parameters for each working point. As mentioned earlier, the slip controller requires good load disturbance attenuation and must be robust to deal with model uncertainties. For this purpose, a gain-scheduling PID-controller was proposed. However, due to the large amount of measurement noise in automotive applications, the derivative term cannot be used. Therefore, a gain-scheduling PI-controller is proposed, as shown in Fig. 5. As can be seen, the gain is scheduled based on primary speed, ratio, and slip. Slip is used to determine whether the system is in the micro- or macroslip region. The setpoint also varies with the ratio, since the maximum traction coefficient is reached for different slip values, depending on the ratio. This is shown in Fig. 2.

![Fig. 5. Proposed slip controller](image)

4.2 Robust PI-controller synthesis method

To easily design controller parameters for multiple working points, while meeting both design requirements, a synthesis method for robust PI(D)-controllers with optimal load disturbance response is used [5]. The method is based on a constrained optimization problem that maximizes the integral gain of the PI(D)-controller while making sure that the maximum sensitivity is less than a specified value. Using the maximum sensitivity as the main design parameter, a trade-off can be made between load disturbance response and robustness with respect to model uncertainties.

The resulting controller parameters of this optimization process can be obtained graphically for a PI-controller. It produces a series of ellipses in the controller parameter space, called the $k$-$ki$ plane, for different frequencies of the system. These ellipses represent a boundary for the sensitivity constraint, and together they form a boundary surface in the $k$-$ki$ plane. Choosing combinations of $k$ and $ki$ below this surface ensures a stable and robust closed loop system. For optimal load disturbance attenuation, the maximum value of the integral gain is determined from the figure. The proportional gain is then determined graphically. Fig. 6 shows a typical result of the synthesis method.
Using this synthesis method for different ratios in the micro- and macroslip region, the gain-scheduling scheme presented in Table I is obtained. The differences between the micro- and macroslip region mentioned earlier, result in very different values for the controller parameters. This is because the system dynamics drastically change at the transition from the micro- to the macroslip region. The system matrix $A$ in (12) almost becomes zero in the macroslip region. This means that a part of the system dynamics disappear, resulting in great changes in the system’s gain.

Another reason is that in the macroslip region the system’s gain becomes scalable by the primary speed $\omega_p$. This can be seen in (14), considering the fact that system matrix $A$ is practically zero. Therefore the gains in Table I for the macroslip region are scaled by the primary speed (in rad/s) in the controller.

### TABLE I

<table>
<thead>
<tr>
<th>Ratio</th>
<th>P-gain</th>
<th>I-gain</th>
<th>P-gain (x 100 rad/s)</th>
<th>I-gain (x 100 rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.43</td>
<td>1.7</td>
<td>30</td>
<td>0.435</td>
<td>1.61</td>
</tr>
<tr>
<td>1</td>
<td>1.9</td>
<td>53</td>
<td>0.294</td>
<td>1.087</td>
</tr>
<tr>
<td>2.25</td>
<td>3.6</td>
<td>110</td>
<td>0.21</td>
<td>0.772</td>
</tr>
</tbody>
</table>

Based on ratio, slip, and primary speed, the proper controller parameters are used. Between the working points shown in Table I interpolation will be used. To ensure the stability of the controller between these working points, several measures were taken. In the microslip region load disturbance response is not very important since slip will not cause any damage in this region. However, many model uncertainties are present, because the slip dynamics depend on many variables in this region. Therefore a maximum sensitivity of 1.2 is chosen in the controller synthesis method, which is relatively low. In the macroslip region a maximum sensitivity of 1.8 is chosen, this is much higher since there are less model uncertainties in this region and good load disturbance response is required. Additionally the worst-case values of the controller parameters were taken to ensure stability for every working point.

### 4.3 Controller implementation

In order to successfully implement the controller described in the previous section, an integral anti-windup is added. This is necessary because the output of the controller is limited between the minimum and maximum pressure level of the CK2. To prevent slip caused by the engine torque $T_e$, a feed forward term is added based on (2), which calculates the minimal clamping force to transmit the given engine torque. The engine torque is estimated using the engine speed and the throttle valve position. This feed forward is needed because the bandwidth of the slip controller is not sufficient to compensate for the fast dynamics of a combustion engine. With these additions the slip controller is ready for implementation.
5. Results

5.1 Test Setup

The developed slip controller will be implemented on a test rig. Fig. 7 shows a schematic representation of the test rig. It is designed to perform realistic drive train experiments, using a combustion engine as the power source and a flywheel, an eddy-current brake, and a disc brake to simulate road loads. The torques on the input and output shaft of the transmission are measured using telemetry systems. The Jatco CK2 can be controlled with the Transmission Control Module (TCM) that is used in a car or with the newly developed slip controller. This is very useful for efficiency comparison.

Slip is not measured directly, but is calculated from other signals, based on (3). The angular speeds of the primary and secondary shaft of the CVT are measured to determine the ratio $r$. The no-load ratio $r_{20}$ is determined using a LVDT to measure the displacement of the primary pulley, as described earlier. The system is accurate enough to detect 0.1% slip, which should be sufficient to implement the slip controller.

5.2 Efficiency measurements

Slip control is developed to improve the efficiency of CVT's, therefore efficiency measurements are carried out to proof what the benefits of the slip controller are in a production CVT. The efficiency when using the TCM is compared to the efficiency when using the slip controller. The efficiency comparison is carried out at fixed ratios and with a constant engine speed of 300 rad/s. The slip value is controlled between 0.5% for ratio 2.25 (overdrive) and 1.5% for ratio 0.43 (low). At these slip values the maximum efficiency of the CK2 is reached. The engine torque is gradually increased and plotted against the efficiency. Fig. 8 shows the result of an efficiency measurement in ratio 0.64.
Fig. 8. Comparison of efficiencies between TCM and slip control, measured for ratio 0.64, at an engine speed of 300 rad/s.

Fig. 8. shows that the efficiency improvement when using the slip controller is quite significant, especially for low engine torques. Since the average engine torque in normal drive cycles is usually relatively low, this is a very promising result. For ratios until ratio 1.4, the efficiency improvement is a little lower than for low ratios, but still in the order of 10 to 5%. For higher ratios than 1.4, the efficiency improvement becomes less. When driving in overdrive, there is hardly any improvement. This is caused by the minimum pressure level in the CK2 of 0.66 MPa, which results in a minimum clamping force of almost 10 kN. For normal engine torques hardly any slip will occur in overdrive with this clamping force level. Therefore the benefits of slip control cannot be fully exploited in the current CK2. Lower clamping forces are required for slip control in ratios near overdrive.

5.3 Load disturbance measurements

The previous section shows that slip control significantly increases the efficiency of a CVT. This was to be expected, based on previous studies. The next step is to perform experiments where torque peaks are introduced in the driveline, thus testing the performance of the slip controller with load disturbances. Also interesting in these tests are the amounts of slip that occur using slip control and whether this damages the belt and pulleys. Experiments were performed at fixed ratios and with a fixed engine speed of 200 rad/s. The slip was controlled at the same values that were used for the efficiency measurements. The eddy-current brake provided a constant torque high enough to reach a slip value at the transition between the micro- and macroslip region. Torque peaks were then introduced by suddenly engaging the disc brake. Fig. 9. Shows the result of one of these measurements.
The figure shows that the torque peaks cause belt slip, which was expected at the transition of the micro- and macro region. But instead of reaching destructive levels, the slip is quickly reduced to non-destructive levels because of the control action. The slip controller is able to deal with torque peaks of up to 1000 Nm in the drive shaft, although this causes the slip level to peak above 5% for short periods of time. Visual inspection however, showed that the belt was not damaged after such tests. This would mean that short peaks in slip do not cause belt damage. Additional tests should be performed to investigate if this is true for all operating points of the CVT. Another important aspect that should be considered is the long-term effect of slip control with respect to belt damage. If necessary, the bandwidth of the controller could be increased to get better load disturbance response, resulting in lower peak values of the belt slip. This can be achieved by improving the gain-scheduling scheme with more working points and using higher maximum sensitivities. If this is not sufficient, an alternative actuation system with a higher bandwidth should be used.
6. Conclusions and Recommendations

The developed slip controller shows efficiency improvements of the Jatco CK2 of up to 30% at low engine torques. Even larger improvements are expected if lower clamping forces could be applied, which makes it possible to use the benefits of slip control for higher ratios and even lower engine torques. This should be considered in future CVT research and design.

Using the slip controller it is possible to operate a CVT with minimal clamping forces, while preventing damage to the belt and pulleys. Relatively high slip levels (5-15%), which were present for very short periods of time during the tests, did not lead to damage to the system. The slip controller is able to attenuate load disturbances of up to 1000 Nm in the driveshaft and perhaps even more. This is true for the current test conditions, but more research is necessary to investigate the long-term effect of using slip control in a wide range of working points with respect to belt damage.

The next step is to implement the slip controller in a car to explore the possibilities of slip control even further.
References


Appendix A

Cross-section of the Jatco CK2

Fig. A.1. shows the cross-section and important part names of the Jatco CK2, which is used in this research program. For a complete and detailed description of the CK2 and its components the reader is referred to [7].

Fig. A.1. Cross-section of the Jatco CK2
Appendix B

Signal overview of the Jatco CK2

In order to implement the slip controller, the Transmission Control Module (TCM) of the Jatco CK2 is replaced by a controller implemented in dSPACE. To achieve this, all sensor and actuator signals of the CK2 had to be determined. A detailed description of all sensors, actuators, and signals used to control the CK2, can be found in [8]. Here a short overview of the signals is presented in Table B.1 and B.II.

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Signal</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>PNP Switch N</td>
<td>Binary</td>
<td>0 or 12 V</td>
</tr>
<tr>
<td>PNP Switch R</td>
<td>Binary</td>
<td>0 or 12 V</td>
</tr>
<tr>
<td>PNP Switch D</td>
<td>Binary</td>
<td>0 or 12 V</td>
</tr>
<tr>
<td>PNP Switch L</td>
<td>Binary</td>
<td>0 or 12 V</td>
</tr>
<tr>
<td>Line pressure</td>
<td>Analog</td>
<td>1.0 - 4.0 V</td>
</tr>
<tr>
<td>Oil temperature</td>
<td>Analog</td>
<td>1.5 - 6.5 V</td>
</tr>
<tr>
<td>Primary pulley speed</td>
<td>Frequency</td>
<td>0 - 2400 Hz</td>
</tr>
<tr>
<td>Secondary pulley speed</td>
<td>Frequency</td>
<td>0 - 7700 Hz</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Actuator</th>
<th>Signal</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line pressure solenoid</td>
<td>50 Hz PWM</td>
<td>0 - 100% duty cycle</td>
</tr>
<tr>
<td>Lock up solenoid</td>
<td>50 Hz PWM</td>
<td>0 - 100% duty cycle</td>
</tr>
<tr>
<td>Stepper motor coil A</td>
<td>Binary</td>
<td>0 or 12 V</td>
</tr>
<tr>
<td>Stepper motor coil B</td>
<td>Binary</td>
<td>0 or 12 V</td>
</tr>
<tr>
<td>Stepper motor coil C</td>
<td>Binary</td>
<td>0 or 12 V</td>
</tr>
<tr>
<td>Stepper motor coil D</td>
<td>Binary</td>
<td>0 or 12 V</td>
</tr>
</tbody>
</table>
Appendix C

Overview actuation system Jatco CK2

The Jatco CK2 has a complex electro-hydraulic actuation system used to control the ratio, the clamping force, the torque converter lock-up clutch, the DNR-set, and the lubrication of all parts. A complete description of the entire hydraulic system is again found in [7]. For modeling purposes, especially the variator behavior is of interest. In order to create a good model, the actuation system of the variator is analyzed. The variator actuation system in the CK2 is based on the VDT hydraulic circuit which is shown schematically in Fig. C.1 [9].

The oil pump depicted in Fig. C.1 provides the pressure for the entire hydraulic actuation system. The secondary pressure $p_s$, also called the line pressure, is used to control the clamping force in the variator. The variator ratio is controlled by a 3-way valve, as shown in the figure. The variator is able to shift because the primary pressure surface $A_p$ is approximately twice the size of the secondary one $A_s$.

Although based on the VDT hydraulic system, the actual hydraulic actuation system in the CK2 is slightly different. The line pressure is not regulated using a single valve. Instead, a 2-stage valve system is used that can be controlled by a solenoid. The duty cycle of the PWM signal sent to the solenoid determines the line pressure. The line pressure circuit is shown schematically in Fig. C.2.
The actuation system for the ratio control in the CK2 is also quite different from the VDT approach. The system is shown schematically in fig. C.3. Using a mechanical link, the 3-way valve is connected to both the stepper motor and the pulley sensor, which is attached to the movable primary pulley. This results in a mechanical feedback system. The valve is actuated by the stepper motor, which takes steps of 0.1 mm. As a result the pulley will move to the desired position. This will also move the pulley sensor, which returns the valve to its equilibrium position (valve closed). Variations of the pulley position caused by leakage or line pressure changes are compensated automatically.
Appendix D

Modeling the CK2 in SIMULINK

D.1. Model structure

To safely experiment with different control strategies on the CK2, it was desirable to have a simulation model of the CK2. The model is created in SIMULINK using existing variator models that describe shifting behavior, torque transmission, and belt slip. Additionally, measured torque losses caused by the oil pump and the variator are incorporated in the model using look-up tables. And finally, the line pressure circuit and the ratio control circuit were modeled to complete the model. The structure of the total model is shown in fig. D.1. In the figure the solid lines represent internal connections, whereas the dashed and dashed-dotted lines represent the inputs and outputs from and to SIMULINK respectively.

Fig. D.1 Structural representation of the CK2 model in SIMULINK
Table D.1 gives an overview of the symbols used in figure D.1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value (unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_p )</td>
<td>Primary pulley pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>( p_s )</td>
<td>Secondary pulley pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>( \omega_p )</td>
<td>Primary pulley angular speed</td>
<td>rad/s</td>
</tr>
<tr>
<td>( \omega_s )</td>
<td>Secondary pulley angular speed</td>
<td>rad/s</td>
</tr>
<tr>
<td>( x_a )</td>
<td>Primary pulley axial position</td>
<td>m</td>
</tr>
<tr>
<td>( v_a )</td>
<td>Primary pulley axial speed</td>
<td>m/s</td>
</tr>
<tr>
<td>( F_c )</td>
<td>Secondary clamping force</td>
<td>N</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Geometric ratio</td>
<td>-</td>
</tr>
<tr>
<td>( T_{b,p} )</td>
<td>Primary pulley belt torque</td>
<td>Nm</td>
</tr>
<tr>
<td>( T_{b,s} )</td>
<td>Theoretical secondary pulley belt torque (no loss)</td>
<td>Nm</td>
</tr>
<tr>
<td>( T_{b,cm} )</td>
<td>Secondary pulley belt torque</td>
<td>Nm</td>
</tr>
<tr>
<td>( T_{b,km} )</td>
<td>Pump torque loss</td>
<td>Nm</td>
</tr>
</tbody>
</table>

As can be seen in the figure, the model only includes components that directly influence the variators behavior during normal operation of the CVT. Components like the torque converter and the oil pump were not modeled since they only influence the system at a vehicle launch situation, which is not considered here. Vehicle launch is, just as a kick down or an emergency brake situation considered to be a special operation mode of the CVT and should be treated separately. However, these operation modes are not included in this research program, since slip control is most interesting in normal (practically steady state) operation of the CVT. A more detailed description of the different submodels will be given in the next section.

### D.2.1. Line pressure circuit model

The line pressure circuit, shown in fig. C.2, is modeled with an estimated transfer function that is determined using FRF-measurements, as mentioned earlier in section III.B. This is much more convenient compared to creating a complete physical model that includes solenoid and valve models. Not only are the results practically equal to each other, the calculation time is also reduced when using a transfer function instead of a physical model. Taking into account the pressure limits of the line pressure circuit (section III.B), the system can be modeled in SIMULINK as shown in fig. D.2.

![SIMULINK model of the line pressure circuit](image)

### D.2.2. Ratio control circuit model

For the ratio control circuit FRF-measurements could not be used to determine the systems dynamic response. This is because the relationship between the stepper motor position (input) and the ratio (output) is non-linear. Therefore a physical model is created of the 3-way valve with the mechanical link to the stepper motor and the primary pulley sensor, as depicted in fig. C.3. To achieve this, the 3-way valve is modeled using a model for turbulent orifice flow \[^8^,\[^9^\]. The orifice depends on the position of the valve, which in turn depends on the position of the stepper motor and the pulley sensor. The flow through an orifice is given by:
\[ Q_v = \text{sign}(\Delta p) c_k A_o \sqrt{\Delta p} \]  
(D.1)

Where \( Q_v \) represents the flow through the orifice, \( \Delta p \) the pressure difference, \( c_k \) the orifice resistance coefficient and \( A_o \) the orifice surface. The pressure difference is given by:

\[
\Delta p = p_s - p_p \quad \text{for} \quad x_v > 0
\]
(D.2)

\[
\Delta p = p_{\text{drain}} - p_p \quad \text{for} \quad x_v < 0
\]
(D.3)

Where \( p_{\text{drain}} \) is the drain pressure of the CK2 and \( x_v \) is the valve position with \( x_v = 0 \) when the valve is in its equilibrium position (closed). The orifice resistance coefficient \( c_k \) is assumed to be constant. The orifice surface however, depends on the valve position. The valve position can be written as:

\[
x_v = \frac{x_{\text{stp}} + x_{pp}}{2}
\]
(D.4)

Where \( x_{\text{stp}} \) represents the position of the stepper motor. The orifice surface as a function of the valve position was determined by measuring the geometry of the valve and the valve openings. The result is depicted in fig. D.3.

![Fig. D.3. Relationship between valve orifice surface area and valve position](image)

Besides the orifice flow into the primary cylinder, there is a leak flow from the cylinder. This flow depends on the primary pressure \( p_p \) and a leakage coefficient \( c_{pl} \) resulting in:

\[
Q_l = p_p c_{pl}
\]
(D.5)

Together with (D.1), the total flow to the primary cylinder is given by:

\[
Q_t = Q_v - Q_l
\]
(D.6)

The total flow that is obtained from (D.6) is used to calculate the primary pulley pressure. This is achieved by using a model for the cylinder pressurization of the primary pulley, given by:

\[
p_p = \int \left( \frac{(Q_t - x_{pp} A_p \beta)}{x_{pp} A_p} \right) dt
\]
(D.7)
Where $\beta$ represents the bulk modulus of the fluid. The entire system is modeled in SIMULINK as shown in Fig. D.4.

**Fig. D.4. SIMULINK model of the ratio control circuit**

### D.2.3. Shifting model

The shifting model used for the CK2 is based on the Shafai model, which is applied to the dynamics of the movable primary pulley. The dynamics of the pulley are given by:

$$a_{pp} = \left( F^*_p - F_p - c_s v_{pp} \right) / M_p \tag{D.8}$$

Where $a_{pp}$ represents the acceleration of the pulley, $M_p$ the mass of the pulley, $c_s$ the Shafai damping factor, $F_p$ the primary clamping force and $F^*_p$ the force applied to the pulley by the belt. $F^*_p$ is reconstructed using experimental KpKs-data that depends on the ratio $r$ and the safety factor, which is calculated by:

$$Sf = T_{p,ck} \cos \alpha / 2 \mu_{max} F_z R_p \tag{D.9}$$

The primary and secondary clamping forces are calculated using the following expression:

$$F_{p,s} = p_{p,s} \omega_{p,s} + \omega_{p,s}^2 f_{c,p,s} \tag{D.10}$$

Where $f_{c,p,s}$ represents the centrifugal coefficients for the primary and the secondary pulley respectively. The geometric ratio is calculated from the axial position $x_{pp}$ of the primary pulley using the geometric relations between the belt and the pulleys [6]. With these equations combined, the shifting model can be constructed in SIMULINK, resulting in the model shown in Fig. D.5.

**Fig. D.5. Shifting model used in SIMULINK**

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D.2.4. Belt model

The belt model is relatively simple and is based on measured traction curves that are used to reconstruct the torque transmitted through the variator, depending on slip and ratio. Slip was defined in (3) and the transmitted torque can be calculated using (2). The belt model used for the CK2 model is shown in fig. D.6.

![Fig. D.6. Belt model used in SIMULNIK](image)

D.2.5. Measured torque losses

Finally, the measured torque losses in the CK2 due to pump losses and variator losses are added to model using look-up tables. The data that is used in the simulation model is presented in tables D.II and D.III. The completed model of the CK2 is included in a simple driveline model of the TR3 test rig, which is based on the CVT dynamics depicted in fig. 3. The engine, torque converter and the brakes are not modeled for model simplicity. The model is validated by comparing the simulation results with experiments performed on the TR3, this is presented in appendix G.

**TABLE D.II**

<table>
<thead>
<tr>
<th>Line pressure (MPa)</th>
<th>Primary speed (rad/s)</th>
<th>155</th>
<th>210</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.66</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>19</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE D.III**

<table>
<thead>
<tr>
<th>Line pressure (MPa)</th>
<th>Ratio (-)</th>
<th>0.43</th>
<th>1</th>
<th>2</th>
<th>2.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.66</td>
<td>5</td>
<td>6</td>
<td>9</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>7</td>
<td>15</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>10</td>
<td>23</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>15</td>
<td>29</td>
<td>31</td>
<td></td>
</tr>
</tbody>
</table>
Appendix E

Measurement and calibration of the no-load ratio

E.1. LVDT pulley position sensor

In order to determine belt slip the no-load ratio $r_{o}$ has to be measured. As mentioned before, it is not measured directly, but it is reconstructed from measurement data. This section will describe how this is done in the Jatco CK2.

The no-load ratio is reconstructed from the axial position of the primary pulley, which is measured using an LVDT connected to the existing pulley position sensor in the CK2 shown in fig. C.3. More information about the construction of the LVDT in the CK2 can be found in [10]. Fig. E.1 schematically shows the placement of the LVDT position sensor in the CK2.

![LVDT pulley position sensor](image)

The core of the LVDT should be positioned in its central position when the ratio is near medium ($r_{o} \approx 1$). This is necessary to reach the highest possible resolution with the LVDT by utilizing the full range of the sensor. The easiest way to achieve this is to make sure that the variator is in low when the LVDT is built in. Since the maximal stroke of the primary pulley is 17.4mm, the core should be positioned $17.4/2 = 8.7$mm out of the centre of the LVDT housing. Then the core will be approximately at its central position in ratio medium. This does not have to be extremely precise, since the sensor still needs to be calibrated.
E.2. Calibration and measurement

Before measurements can be performed to determine the no-load ratio, the sensor and its amplifier must be calibrated to utilize the maximal range of 0-10V that the amplifier can provide. This is achieved by using the following procedure:

NOTE: All measurements should be performed with no load applied to the secondary pulley side!

- First, warm up the CK2 until the working temperature of 85°C is reached.
- Shift the ratio to low and overdrive and write down the voltage the amplifier indicates. Since the extreme values of the variator (low and overdrive) vary with the line pressure level, low should be measured at the maximal pressure level (4.2 MPa) and overdrive at the minimal pressure level (0.66 MPa).
- By varying the "zero" and "span" settings on the amplifier, the voltage in low should indicate 9.95V (at maximum pressure) and in overdrive the amplifier should indicate 0.05V (at minimum pressure). The reason that these values are just outside the full range of the amplifier is because the LVDT has a small voltage drift with the oil temperature, which is discussed later on.

Now that the full range of the sensor can be utilized, the relationship between the no-load ratio \( r_{no} \) and the amplifier output voltage can be determined. This relationship is then approximated using a 6th order polynomial, which can be used to reconstruct the no-load ratio as described earlier. This relationship is obtained by shifting the CK2 slowly from low to overdrive and plot the ratio vs. the output voltage. Since no load is applied to the secondary axle, the measured ratio represents the no-load ratio. The result of such a measurement is shown in fig. E.2.

![Fig. E.2. Measurement of no-load ratio vs. amplifier voltage at line pressure level 0.7 MPa](image)

The figure shows that the 6th order polynomial provides a good approximation of the no-load ratio. This measurement has to be repeated for different line pressure levels since the relationship between the no-load ratio and the amplifier output varies a little with the line pressure due to deformation of the pulleys and extension of the belt. The result is a set of 6th order polynomials that is used to reconstruct the no-load ratio as a function of the amplifiers voltage and the line pressure level.

Since this method is experimental, the procedure has to be performed for every combination of CK2, LVDT and amplifier.

As mentioned before, the LVDT has a small voltage drift for different operating temperatures. Since the LVDT is submerged in the oil reservoir of the CK2, the LVDT takes on the temperature of the oil. Especially at startup, the temperature is quite different from the normal operating temperature. This temperature drift can lead to slip offset values of up to 1.5% in overdrive. This is of course undesirable.
when slip is controlled at 0.5%. Therefore this temperature drift should be compensated. This is done using a rough approximation of the temperature drift at different ratios (in V/°C), based on measurements. Between these ratios, interpolation is used. To determine the temperature drift, the CK2 is simply warmed up at a fixed ratio. The result of such a measurement is shown in fig. E.3. The resulting approximated temperature drift for ratios low, medium and overdrive is shown in table E.1.

### Table E.1

<table>
<thead>
<tr>
<th>Ratio (-)</th>
<th>Temperature drift (V/°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.43 (low)</td>
<td>0.00094</td>
</tr>
<tr>
<td>1 (medium)</td>
<td>0</td>
</tr>
<tr>
<td>2.25 (overdrive)</td>
<td>-0.0015</td>
</tr>
</tbody>
</table>

![Fig. E.3. Temperature drift of the LVDT measured in ratio low](image)

Table E.1 shows that in ratio medium no temperature compensation is needed. This is because the core of the LVDT is in its central position here. The further the core moves out of its center, the larger the effect of the temperature drift seems to get. The output voltage is compensated using the temperature drift values from table E.1, the actual temperature and the ratio. With this rough temperature compensation the maximal slip offset error caused by the temperature drift becomes smaller than 0.5%, which is acceptable for implementing the slip controller.
Appendix F

The TR3 test rig

F.1. Controller component overview

The TR3 is a test rig designed to perform experiments on a real drive line. In this research program it was used to test the slip controller on both drive line efficiency as robustness to torque peaks. A detailed description of the TR3 test rig and its components is found in [11] and [12]. A schematic overview of the main components is shown in fig. 7.

For this research program all components of the test rig are controlled by a dSPACE system to perform the slip control measurements. On the test rig the throttle valve of the eddy-current brake are controlled by sending analog signals to the existing Schenck controllers, which are also described in [12]. The signals that should be send to the Schenck controllers, along with the connector pinning is shown in table F.1 and F.11. The signal names are in German, this is done to avoid misunderstandings when reading the Schenck manual, which is also written in German.

<table>
<thead>
<tr>
<th>TABLE F.1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Signal Names and Pinning for Connector X18 on the Schenck LSG 2010</strong></td>
</tr>
<tr>
<td><strong>Signal</strong></td>
</tr>
<tr>
<td>n-soll (0-10V analoge)</td>
</tr>
<tr>
<td>Analoge GND</td>
</tr>
<tr>
<td>M-soll (0-10V analoge)</td>
</tr>
<tr>
<td>n-ist (0-10V analoge)</td>
</tr>
<tr>
<td>Analoge GND</td>
</tr>
<tr>
<td>M-ist (0-10V analoge)</td>
</tr>
<tr>
<td>+24V</td>
</tr>
<tr>
<td>0V</td>
</tr>
<tr>
<td>0V</td>
</tr>
<tr>
<td>Betriebsart 1 (0-24V dig.)</td>
</tr>
<tr>
<td>Betriebsart 2 (0-24V dig.)</td>
</tr>
<tr>
<td>Betriebsart 3 (0-24V dig.)</td>
</tr>
<tr>
<td>Betriebsart 4 (0-24V dig.)</td>
</tr>
<tr>
<td>Anschleppen (0-24V dig.)</td>
</tr>
<tr>
<td>Rechnervorrang Bremse (0-24V dig.)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE F.11</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Signal Names and Pinning for Connector XI1 on the Schenck LSR 2000</strong></td>
</tr>
<tr>
<td><strong>Signal</strong></td>
</tr>
<tr>
<td>Stellgroesse Motor (0-10V analoge)</td>
</tr>
<tr>
<td>0V</td>
</tr>
<tr>
<td>Rechnervorrang Stellg. (0-24V dig.)</td>
</tr>
<tr>
<td>+24V</td>
</tr>
</tbody>
</table>
The CK2 can be controlled either with the TCM that is used in a car or with dSPACE. To switch between the TCM and dSPACE, a dedicated electronic circuit was designed. The electronic circuit also buffers all measured signals from the CK2, so that the measured signals can always be used in dSPACE. With this option it is easy to compare different clamping force strategies with the one that is used in the TCM. Fig. F.1 shows a schematic overview of the electric circuit and all signals used to control the TR3 test rig. Between the signals in dSPACE and the CK2, signal conditioning and amplification is needed. This is also shown in fig. F.1. A detailed description of how to control the CK2 with dSPACE is given in [13].

To connect all signals from the test rig to the dSPACE board, a separate connection box was build. This box was designed to collect all signals from and to the test rig and connect them to the right connectors on the dSPACE connection panel.

With the current system all components of the TR3, except for the disc brake and the shift lever, can be controlled using dSPACE. The eddy-current brake can be operated with torque- or speed control. And of course the CK2 can be controlled (line pressure, torque converter and ratio) by dSPACE or by the TCM.

F.2. Improved input torque measurement shaft

The input torque measurement shaft that was designed in [11] proved to be useless for the chosen application. The shaft was designed to be very stiff and light, resulting in a thin walled, hollow shaft. This caused the shaft to have a very small angular displacement, thus a very large amplification of the signal was necessary. And furthermore, hollow shafts show much more hysteresis than solid shafts, which is an undesirable effect for torque measurement.

Therefore a new measurement shaft was designed with the same strength and torque capacity as the previous one (the same strength criteria were used as in [11]). But this time a solid shaft was designed. This results in the shaft that is depicted in fig. F.3, which is a relatively thin solid shaft. This shaft has a much larger angular displacement than the previous one when torque is applied to it, and hardly any hysteresis. To increase the sensitivity of the shaft even more and to have a better compensation against bending of the shaft, a double strain gage bridge is used. The wiring diagram of the double strain gage bridge is shown in fig. F.4.

The new torque measurement shaft was calibrated for ± 600 Nm, although the maximum torque of the ICE is only 180 Nm. This is done because this maximum torque value ICE is an average torque, but the torque peaks generated by the ICE can be more than twice the average maximum torque.

The calibration results in a practically linear relationship between the measured torque and the output of the torque measurement shaft and is found to be:

\[ T_m = 63.98V_{out} + 10 \]  \hspace{1cm} (F.1)

Where \( T_m \) is the measured torque and \( V_{out} \) the amplifier voltage output, which ranges between ±10V.
Fig. F.1. Schematic overview of electronic circuit and signals used to control the TR3 with dSPACE and the CK2 with dSPACE or the TCM.
Fig. F.2. Schematic overview of the signals and connections between the TR3 and dSPACE
Fig. F.3. Drawing of the newly designed input torque measurement shaft

Fig. F.4. Wiring diagram of a double strain gage bridge
Appendix G

Experiments and model validation

The model of the CK2 presented is appendix D, together with a drive line model of the TR3 test rig, is used to simulate new controller strategies or features (like anti-windup strategies or feed forward signals) before it is tested on the test rig itself. This saves time and can provide good starting values for new control parameters or features. But before the model is used, it is validated by comparing it by experimental data. First the shifting behavior and the pressure levels are validated, to check if the model of the CK2 gives a good approximation of the reality.

The validation experiment is performed by taking steps in the ratio setpoint at a constant vehicle speed of 60 km/h. The same experiment was performed in the simulation model and the results are compared in fig. G.1 and G.2. It is seen that there are only slight variations between the dynamic behavior of the model and the experimental data, which means that the model of the CK2 gives a good approximation of the shifting behavior and the steady state pressure levels.

![Fig. G.1. Validation of the shifting behavior of the CK2 SIMULINK model](image1)

![Fig. G.2. Validation of the pressure levels in the CK2 SIMULINK model](image2)
For slip control it is of course important that the simulated slip in the variator resembles the reality good enough. This is validated by an experiment where torque steps are applied to the drive line at a fixed ratio and a constant vehicle speed. In this experiment the slip was controlled at 1.5%, both the slip value and the secondary pressure are compared. The result is shown in fig. G.3 and G.4.

![Simulated slip vs Measured slip](image1)

**G.3.** Validation of the slip values in the CK2 SIMULINK model.

![Simulated secondary pressure vs Measured secondary pressure](image2)

**G.4.** Validation of the secondary pressures using slip control in the CK2 SIMULINK model.

Fig. G.3. shows that the modeled slip resembles the measured slip quite accurately. The secondary pressure when using slip control however, shows a larger difference. This is probably because of the fact that the drive line model is still very simple. No accurate model for the engine, the torque converter or the brakes was included yet. But it still gives a reasonable indication of the pressure levels when using slip control. The model is useful to get more insight in the response of the system to external influences. It is also used to make a good initial guess for system parameters before implementing it on the test rig, but it is not accurate enough to use it for controller tuning.
Appendix H

Efficiency improvement measurements

The efficiency improvement experiments mentioned earlier were performed at different ratios. To keep a fixed ratio when controlling the CK2 using the TCM, the manual shift mode was chosen that is available on the TR3 test rig. In this mode 6 predefined ratios can be chosen. Fig. H.1 shows the results of the efficiency improvements at 4 different ratios. All tests were done with a constant ICE speed of 300 rad/s. The figure shows that the efficiency improvement is best seen for low ratios, this is a result of the fact the optimal slip value cannot be obtained for higher ratios because of the minimal pressure level of the CK2.

![Graphs showing efficiency improvement measurements for different ratios at a constant ICE speed of 300 rad/s](image)

Fig. H.1. Efficiency improvement measurements for different ratios at a constant ICE speed of 300 rad/s
Appendix I

PI-controller optimization

The synthesis method for robust PI-controllers mentioned before has been implemented in Matlab for the CK2 slip dynamics. With the m-file presented below the constrained optimization problem is solved for a chosen working point, resulting in a figure comparable to fig. 6. From the obtained figure the controller parameters can be determined, as mentioned before. In the m-file two functions are used (r2Rp.m and ellips.m), which are also presented below.

Synthesis method for robust PI-controller based on constrained optimization

clear all
warning off

% Defining constant parameters of the slip dynamics
L = 0.7028; % Belt Length
a = 0.168; % Pulley centre distance
A_sec = 0.014113; % Secondary cylinder surface area
Jc = 0.356; % Engine and transmission inertia
Jd = 4.97; % Drive line inertia
cosphi = cos(2*pi*(11/360)); % Pulley wedge angle

% Defining the working point
wp0 = 100; % Primary speed
rg0 = 0.43; % Ratio
k1 = 0; % Traction coefficient slope
k2 = 0.09; % Traction coefficient offset

slip0 = 0.015; % slip
mu0 = 0.09; % Traction coefficient
F0 = 1e4; % Clamping force
Te0 = (2*F0*Rp0*mu0)/(cosphi); % Input torque

% Defining linearized Slip Dynamics
% x=[mu] and u=[Td,Te,F]

rho=(2*Rp0*cosphi); % Variables introduced for
psi=(rg0/s)+1/(rg0*kd)); % writing convenience

% State space representation of the slip dynamics
A=[(-Te0)0 -rho*F0*psi*k1 +((rho*F0*rg0*k2*j)/wp0];
B=[1/(d*rg0) (1/te-slip0/3e) +rho*psi*k2 -rho*psi*slip0*k1 +((rho*rg0*slip0*k2)/2)/wp0];
C=[1];
D=[0 0 0];

% Determining the transfer function with respect to the clamping force
[NUMs,DENs]=ss2tf(A,B,C,D,s,3);

[nums]=size(NUMs);
syms s
numG1=0;denG1=0;
for i=1:n
    coeffnum=NUMs(l,i)*s^(m-i);
    coeffden=DENs(l,i)*s^(m-i);
    numGl=coeffnum+numGl;
    denGl=coeffden+denGl;
end

g1=numGl/denGl;

% Estimated transfer function of the Line pressure circuit
[NUMb,DENb]=butter(3,5*2*pi,'s?);

for i=1:m
    coeffnum=-42e5*Asec*NUMb(l,i)*s^(m-i);
    coeffden=DENb(l,i)*s^(m-i);
    numG2=coeffnum+numG2;
    denG2=coeffden+denG2;
end

g2=numG2/denG2;

save sysvar g1 g2

clear s g1 g2 G1 G2 i

load sysvar

% Defining empty ellipse matrices
ellipstot=[];

% Constrained optimization problem (From Panagopoulos 2002)
for omega=0.1:0.1:1.50
    s=i*omega;
    C=1;

    % Robustness parameter (1/max. sensitivity)
    Rs=1/1.5;

    % Define the subsystems
    G1=subs(g1); % Linearized slip dynamics
    G2=subs(g2); % Actuator transfer function

    % Define total Plant G = G1 * G2
    G=G1*G2;

    % Sensitivity criterion
    A=real(G);B=imag(G);
    r=sqrt(A^2+B^2);
    b=(A*B)(r^-2);
    k=(omega*B*C)(r^-2);
    a=Rs; r;
    b=(omega*Rs)/*;
    M=[a^2 0 0 b^2];
    m=[h;k];
    c=1;
    k=100;

    [el]=ellips(M,m,c,k); % create constraint ellips
    ellipstot=[ellipstot; el]; % collect all ellipses
plot(ellipstot(:,1),ellipstot(:,2)) % plot all ellipses in k-ki plane

title('Sensitivity contrast in k-ki plane')
xlabel('k1'); ylabel('k2');

grid

% end of m-file.

---

**Function r2Rp**

```
function [Rp,Rs,phi]=r2Rp(r-new,l-new,a-new)
%
% (c) 2002 TU/e
% auteur : ir. Bram Bonsen
% datum : 11 april 2002
% doel : bereken de straal van de pulleys (Rp[m],Rs[m]) aan de
% hand van een overbrengingsverhouding (r=ws/wp=Rp/Rs),
% de bandlengte L[m] en de pulleyafstand a[m].
%
% INPUT:
% r : ratio (TUE)
% L : lengte band
% a : afstand tussen de assen van de pulleys
%
% OUTPUT:
% Rp: straal primaire as
% Rs: straal secundaire as
%
% L-2*a*cos(\phi)=Rp*(pi/2*phi)+Rs*(pi/2*phi)
% phi=asin((Rp-Rs)/a) % linearisatie: phi=(Rp-Rs)/a;
% r=Rp/Rs
% taylor: cos(\phi)=1-(1/2)*\phi^2;
%

% als r = 1, dan is de relatie singulier (a=0), dus aparte berekening voor r=1:
% a1=(1/a_new)*(r_new-1)^2;
% a2=pi*(r_new+1);
% a3=2*a_new-1

for p=1:length(r-new)
    if a1(p)=0
        Rs(p)=a3/a2(p);
    else %abc formule
        Rs(p)=(-a2(p)+sqrt(a2(p)^2-4*a1(p)*a3))/(2*a1(p));
    end;
end;

Rp=Rs.*r_new;
phi=asin((Rp-Rs)/a_new);

% end of r2Rp
```
function [x] = ellips(A, m, c, k);
% input: A  positive definite symmetric matrix of dimension 2 by 2
% m  column vector of length 2, center of ellipse
% c  positive constant
% k  number of points on ellipse (must be at least 2)

% output: x  a (k by 2) matrix whose rows are the coordinates of
%         k points on the ellipse \((y-m)'*\text{inv}(A)'*(y-m) = c^2\)

r = A(1,2) / sqrt(A(1,1) * A(2,2));
Q = zeros(2,2);
Q(1,1) = sqrt(A(1,1) * (1+r) / 2);  \% construct matrix Q for
Q(1,2) = -sqrt(A(1,1) * (1-r) / 2);  \% transformation of circle
Q(2,1) = sqrt(A(2,2) * (1+r) / 2);  \% to ellipse
Q(2,2) = sqrt(A(2,2) * (1-r) / 2);
alfa = linspace(0, 2*pi, k);        \% define angles
z = [cos(alfa); sin(alfa)];        \% points on unit circle
x = (m*ones(1, k) + c*Q*z)';        \% coordinates of points on ellipse

plot(x(:, 1), x(:, 2))

% end of procedure ELLIPS
Nomenclature and Acronyms

Acronyms

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CVT</td>
<td>Continuously Variable Transmission</td>
</tr>
<tr>
<td>ICE</td>
<td>Internal Combustion Engine</td>
</tr>
<tr>
<td>VDT</td>
<td>Van Doorne's Transmissie</td>
</tr>
<tr>
<td>OD</td>
<td>Overdrive</td>
</tr>
<tr>
<td>ECM</td>
<td>Engine Control Module</td>
</tr>
<tr>
<td>TCM</td>
<td>Transmission Control Module</td>
</tr>
<tr>
<td>PWM</td>
<td>Pulse Width Modulation</td>
</tr>
<tr>
<td>FRF</td>
<td>Frequency Response Function</td>
</tr>
<tr>
<td>LVDT</td>
<td>Linear Variable Differential Transformer</td>
</tr>
<tr>
<td>TR3</td>
<td>Test Rig 3</td>
</tr>
</tbody>
</table>

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value [Unit]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_p$</td>
<td>Primary cylinder surface area</td>
<td>$301.92\times10^{-4}$ [m$^2$]</td>
</tr>
<tr>
<td>$A_s$</td>
<td>Secondary cylinder surface area</td>
<td>$141.13\times10^{-4}$ [m$^2$]</td>
</tr>
<tr>
<td>$A_o$</td>
<td>Orifice surface area</td>
<td>[m$^2$]</td>
</tr>
<tr>
<td>$F_p$</td>
<td>Primary clamping force</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_s$</td>
<td>Secondary clamping force</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_p^*$</td>
<td>Belt force exerted on primary pulley</td>
<td>[N]</td>
</tr>
<tr>
<td>$J_p$</td>
<td>Engine inertia</td>
<td>0.356 [kgm$^2$]</td>
</tr>
<tr>
<td>$J_d$</td>
<td>Driveline inertia</td>
<td>4.96 [kgm$^2$]</td>
</tr>
<tr>
<td>$M_p$</td>
<td>Primary pulley mass</td>
<td>2 [kg]</td>
</tr>
<tr>
<td>$Q_l$</td>
<td>Primary cylinder leak flow</td>
<td>[m$^3$/s]</td>
</tr>
<tr>
<td>$Q_o$</td>
<td>Orifice flow</td>
<td>[m$^3$/s]</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>Total primary cylinder flow</td>
<td>[m$^3$/s]</td>
</tr>
<tr>
<td>$R_p$</td>
<td>Primary belt radius</td>
<td>[m]</td>
</tr>
<tr>
<td>$R_s$</td>
<td>Secondary belt radius</td>
<td>[m]</td>
</tr>
<tr>
<td>$T_e$</td>
<td>Engine torque</td>
<td>[Nm]</td>
</tr>
<tr>
<td>$T_d$</td>
<td>Driveline torque</td>
<td>[Nm]</td>
</tr>
<tr>
<td>$T_{p,ext}$</td>
<td>Primary pulley belt torque</td>
<td>[Nm]</td>
</tr>
<tr>
<td>$T_{s,ext}$</td>
<td>Theoretical secondary pulley belt torque (no loss)</td>
<td>[Nm]</td>
</tr>
<tr>
<td>$T_{s,ext}$</td>
<td>Secondary pulley belt torque</td>
<td>[Nm]</td>
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<tr>
<td>$T_{p,loss}$</td>
<td>Pump torque loss</td>
<td>[Nm]</td>
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<tr>
<td>$S_f$</td>
<td>Safety factor</td>
<td>[-]</td>
</tr>
<tr>
<td>$a_{pp}$</td>
<td>Primary pulley axial acceleration</td>
<td>[m/s$^2$]</td>
</tr>
<tr>
<td>$c_k$</td>
<td>Orifice resistance coefficient</td>
<td>0.2 [-]</td>
</tr>
<tr>
<td>$c_{pl}$</td>
<td>Primary cylinder leakage coefficient</td>
<td>$3.4\times10^{-2}$ [m$^3$/Pa s]</td>
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<tr>
<td>$c_s$</td>
<td>Shafai damping coefficient</td>
<td>$1.25\times10^{-6}$ [Ns/m]</td>
</tr>
<tr>
<td>$f_{cp}$</td>
<td>Primary pulley centrifugal coefficient</td>
<td>$45\times10^{-2}$ [Ns$^2$/rad$^2$]</td>
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<tr>
<td>$f_{cp}$</td>
<td>Primary pulley centrifugal coefficient</td>
<td>$5.4\times10^{-2}$ [Ns$^2$/rad$^2$]</td>
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<tr>
<td>$p_p$</td>
<td>Primary pulley pressure</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$p_s$</td>
<td>Secondary pulley pressure</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$r_g$</td>
<td>Geometric ratio</td>
<td>[-]</td>
</tr>
<tr>
<td>$r_s$</td>
<td>Speed ratio</td>
<td>[-]</td>
</tr>
<tr>
<td>$r_{\theta}$</td>
<td>No-load speed ratio</td>
<td>[-]</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Value [Unit]</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------</td>
<td>--------------</td>
</tr>
<tr>
<td>$v_{pp}$</td>
<td>Primary pulley axial speed</td>
<td>[m/s]</td>
</tr>
<tr>
<td>$x_{pp}$</td>
<td>Primary pulley axial position</td>
<td>[m]</td>
</tr>
<tr>
<td>$x_{step}$</td>
<td>Stepper motor axial position</td>
<td>[m]</td>
</tr>
<tr>
<td>$x_v$</td>
<td>Primary valve axial position</td>
<td>[m]</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Pulley wedge angle</td>
<td>11[°]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Oil bulk modulus</td>
<td>$1.5e^{9}$ [N/m²]</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Traction coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Relative belt slip</td>
<td>[-]</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>Primary pulley angular speed</td>
<td>[rad/s]</td>
</tr>
<tr>
<td>$\omega_s$</td>
<td>Secondary pulley angular speed</td>
<td>[rad/s]</td>
</tr>
</tbody>
</table>
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