Modelling and identification
of the dynamic behavior
of a wire rope spring

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Preface

This report documents the Master Thesis of the author, which has been performed at the Dynamics and Control Division of the faculty Mechanical Engineering of the University of Technology Eindhoven and TNO-CMC, the Center of Mechanical and Maritime Structures, part of the Netherlands Organisation for applied Scientific research in Delft. During the Master thesis project TNO-CMC has been visited for a stay of four months, in which I have been able to carry out various experiments which were crucial for my research. Most of the other work has been performed at the DCT-lab at the University of Technology Eindhoven.

Eindhoven, February 2004
Summary

A shock and vibration absorber commonly used in a naval vessel is the wire rope spring. Wire rope springs consist of stranded wire rope held between rugged metal retainers ('bars'). Wire rope springs show a good damping performance due to rubbing and sliding friction between the wire rope strands. The wire rope spring adopts stranded wire rope as the elastic component and utilizes inherent friction damping between individual rope strands. Harmonic excitation of the spring leads to a so-called hysteresis loop resulting in energy dissipation. Therefore, the wire rope spring is an excellent damping device. In order to increase the knowledge about the dynamic behavior of the wire rope spring, the aim of this thesis is to model both the quasi-static and the dynamic behavior of the wire rope spring in the tension-compression mode.

The Bouc-Wen model is a hysteresis model on a phenomenological base, which contains four parameters \( \alpha, \beta, \gamma \) and \( n \). The quality \( n \) governs the smoothness from elastic to plastic behavior. The parameters \( \beta \) and \( \gamma \) mainly control the shape of the hysteresis loop. By a proper choice for the parameters \( \beta \) and \( \gamma \) it can represent hardening, softening and quasi-linear behavior. However, when the system exhibits softening behavior, various combinations for \( \beta \) and \( \gamma \) can lead to almost identical hysteretic curves. By adding a constraint for these two parameters this redundant parameter phenomenon can be avoided. A second solution is to adopt an altered version of the Bouc-Wen model in which the parameter \( \gamma \) is omitted.

With quasi-static cyclic loading tests it is shown that the wire rope spring, used in this thesis project, exhibits a kind of behavior which cannot be described by the Bouc-Wen model, because the wire rope spring possesses hardening behavior in tension and softening behavior in compression. Secondly, for small amplitudes the wire rope spring exhibits softening behavior, which via quasi-linear behavior gradually changes into hardening stiffness for increasing amplitudes. This phenomenon is called soft-hardening. Finally, under tension the loading path is the same for various amplitudes, which is called hardening overlap. In order to deal with these curiosities a modified version of the Bouc-Wen model is adopted to describe the behavior. An important consequence of the use of this modified Bouc-Wen model is that the set of parameters to be identified contains nine elements and is thus quite large. Accompanied with the redundant parameter phenomenon in the Bouc-Wen model, the identification has been a very difficult process. Various identification methods have been tried before a satisfying three stage identification procedure has been implemented. In the first stage only the stiffness behavior is taken into account. These parameters are set to the identified values in the second stage, in which the area of the hysteretic curve is described. In the last stage all parameters are released to optimally tune all values. Eventually the identified model response and the experimental results show good agreement. However, for small amplitude levels still a difference is present.

Simulations are carried out to gain insight into the dynamic behavior of the Bouc-Wen...
model and the modified Bouc-Wen model. A nonlinear dynamic response with an amplitude dependent resonance frequency and various superharmonic resonances has been observed. It has also been shown that the dynamic response differs for those combinations of $\beta$ and $\gamma$, which lead to almost identical hysteretic curves. This holds for the Bouc-Wen model as well as the modified Bouc-Wen model. This cannot be explained and certainly needs more attention in further research.

Shaker experiments result in frequency amplitude curves. An amplitude dependent resonance frequency moving to lower frequencies for increasing excitation amplitudes indicating softening behavior, has been found. Furthermore, a superharmonic resonance has been found, but it is very hard to distinguish. Shock experiments are carried out upon a shock table. These have confirmed that indeed wire rope springs are excellent shock absorbers since the difference between the amplitude of the excitation and the response is large. However, the shock simulations show that a discrepancy is present between both results. The difference in both amplitudes is probably caused by the frame, which has been needed during the experiments, because the experimental set-up was larger than the area of the shock-table. The excitation has been measured directly on the table, assuming the frame is infinite stiff and the damping can be neglected.

Theoretical frequency sweep simulations show that for larger amplitude levels the behavior of the wire rope spring and the modified Bouc-Wen model are approximately the same both qualitatively as quantitatively. The number of superharmonic resonances, found in simulations, indicate that the model is damped a bit too weakly. In contrast, for smaller amplitude levels the model seems to be damped too heavily. The resonance frequency as well as its amplitude is smaller than in the experiments. So similar as for the quasi-static behavior a difference is present. So the wire rope spring exhibits a kind of behavior for smaller amplitudes which cannot be described quantitatively by the modified Bouc-Wen model with the parameter set as identified in this thesis. Possible solutions are to use a second set of parameters for small amplitude levels, or to make the parameters amplitude dependent. It certainly is needed to gain more insight in the behavior of the wire rope spring for very small amplitude levels.

Finally, periodic solutions have been sought by means of the path-following method, in which the shooting method is implemented. At first some unstable solutions have been found, caused by numerical inaccuracies. This numerical problems are likely to be the result of the nonsmooth character of the Bouc-Wen model. This is indicated by the results of the path-following method for a smooth approximation of the Bouc-Wen model which have not resulted in unstable solutions when using the original accuracy of the integration scheme.
Chapter 1

Introduction

1.1 Vibration absorption using wire rope springs

On a naval vessel, shocks and vibrations cannot be avoided. As excitation sources can be mentioned: slamming, wave excitation, excitations caused by the drive or even underwater explosions. These vibrations may be undesirable or even unacceptable. Vibrations adversely effect the performance of machines, especially for radar positioning purposes, or may even cause damage to the machines. Secondly, vibrations with too high amplitude levels will probably lead to discomfort of the crew members. Therefore, the vibration levels on a ship are restricted according to ISO 6954.

In order to reduce the vibration level, there are numerous solutions, for example increasing the stiffness, using materials that have high damping characteristics, or by applying a control strategy. Three types of control can be distinguished: active, semi-active or passive control. Most of the time the latter is used on a ship. A vibration absorber is a typical example of a passive controller. It is placed below a deck to protect the equipment and the crew members on this deck.

A vibration absorber, which is often used, is the wire rope spring. Wire rope springs are a type of spring dampers, that consist of stranded wire ropes held between rugged metal retainers. Due to dry friction between the different layers hysteresis occurs. This is actually the reason why the wire rope spring is such a good damping device since energy is dissipated. Wire rope springs have the advantage that they do not suffer from aging as similar absorbers like LSM-springs. A second advantage is that the wire rope spring is suitable for attenuating heavy shocks as well as absorbing wide-band vibrations.

However, the presence of hysteresis is also the reason for a nonlinear response of the wire rope spring and the mass on top of it. The dynamic behavior of the wire rope spring is not fully understood yet. Since primarily the wire rope spring will be loaded in the tension-compression mode, it is logical to start the model development and analysis for this mode.

1.2 Practical importance of research on wire rope springs

As stated in section 1.1 the vibration level on a deck is restricted and therefore wire rope springs are used as a shock and vibration absorbing device. In this master thesis project it is tried to gain insight into the vibration reduction capabilities of the wire rope spring.

It may be expected that all springs exhibit a slightly different behavior due to geometric
1.3 Goal and outline of the thesis

The goal of this thesis is to model and analyze the behavior of a wire rope spring in the tension-compression mode. The quasi-static as well as the dynamic behavior of the wire rope spring will be considered. Experiments will be performed and the results will be compared with simulation results.

Chapter 2 will introduce the wire rope spring in general. A literature review will be given in which various items are discussed. First, a short introduction about hysteresis, which is present in the wire rope spring, will be given. Subsequently, an overview will be presented about earlier research involving wire rope springs. Chapter 3 will address the Bouc-Wen model, a hysteresis model. First, the model development will be presented. Secondly, a parametric study will be performed, which will address the influence of the various parameters on the quasi-static and dynamic behavior. Finally, a modified version of the Bouc-Wen model, that will be adopted to describe the behavior of the wire rope spring, will be discussed.

Chapter 4 will present the results of quasi-static experiments, that have been performed. Hence, a good impression about the "static" behavior of the wire rope spring will be obtained. Next, the experimental results will be used to identify the parameters of the modified Bouc-Wen model. Various identification procedures have been tried, but eventually a three stage identification process will be proposed in chapter 5 to identify the model.

Electronic shaker experiments have been performed to obtain frequency amplitude curves. In addition, shock experiments are carried out to learn how the wire rope spring behaves under shock excitations. The results of these experiments will be discussed in chapter 6. Chapter 7 describes the results of the various simulations of the dynamic behavior. First frequency sweeps have been performed in simulations as well. Secondly the shooting method in combination with path following has been used to find periodic solutions. Furthermore, the experimental excitation of the shock experiments is used to simulate the shock experiments. All experimental and simulation results will be be compared. Finally, in chapter 8 some conclusions will be drawn and recommendations for further research will be given.
Chapter 2

Wire rope springs

Prior to the experimental analysis and the modelling of the wire rope spring, several items have to be investigated. In section 2.1 the several types of wire rope springs are introduced and a closer look is taken at the working principle of a wire rope spring. Section 2.2 addresses the applications for the wire rope spring. The phenomenon of hysteresis is briefly described in section 2.4 as this occurs in wire rope springs. Finally, in section 2.5 a literature review is presented about the model development for wire rope springs.

2.1 Types of wire rope springs

Wire rope springs are an assembly of stranded wire rope held between rugged metal retainers ('bars'). These springs are also known as wire rope isolators, metal cable springs or steel cable springs. Roughly two types of wire rope springs can be distinguished, the helical type and the polycal type, shown in figure 2.1 and 2.2 respectively.

![Figure 2.1: A helical wire rope spring](image1)

![Figure 2.2: A polycal wire rope spring](image2)

Although an all-metal design, wire rope springs show a good damping performance due to rubbing and sliding friction between the wire rope strands. The wire rope spring adopts stranded wire rope as the elastic component and utilizes inherent friction damping between individual rope strands. It has the feature of both attenuating heavy shocks and absorbing wide-band vibrations. However, wire rope springs are not suitable for acoustic isolation.

Alternatives for wire rope springs are the Leaf Spring mountings. This kind of spring, shown in figure 2.3 consists of metal plates with elastomer between it. It is interchangeable
with the Y-series of the Socitec polycal wire rope spring, because the dimensions of the LSM-
spring and the Y-series of the Socitec polycal wire rope springs are the same. The advantage of
this kind of isolator is the capability of acoustic isolation. However, the main disadvantage is
that the elastomer between the metal suffers from aging, which results in brittleness and the
loss of its damping ability. Wire rope springs also provide a better shock isolation compared
to Leaf Spring mountings.

2.2 Applications of the wire rope springs

The main application of wire rope springs lies in the naval environment. A special application
is the isolation of a so-called Multi-Purpose Floating Floor, which is shown in figure 2.4 [4].
The Multi-Purpose Floating Floor consists of a frame based floating floor, that is supported
by wire ropes springs, which act in both vertical and horizontal directions plus some rubber
springs that only act in the horizontal direction. In that way an operational room and its
crew members are protected against underwater shock.

However, the presence of crew members also demands that the floating floor does not
invoke unacceptable amplification of desk vibrations. These design requirements are based
upon ISO 6954 ’ Mechanical vibration and shock - Guidelines for the overall evaluation of
vibrations in merchant ships’ [12]. It states that, for frequencies from 5 through 100 Hz,
vibration levels with velocities below 4 mm/s are unlikely to cause complaints whereas velocity
amplitudes higher than 9 mm/s will probably cause complaints.

The wire rope springs are also used as seismic protection for equipment in buildings [10].
Earthquake motions, when transmitted through conventionally constructed buildings, which
in strong excitation respond inelastically, reach the upper floor amplified and with their
frequency content spread over a wide range of frequencies. The seismic protection of a single
2. Wire rope springs

Figure 2.4: A Multi-Purpose Floating Floor

piece of equipment can be achieved by absorbing earthquake energy. Hence, wire rope springs are used to protect the equipment.

2.3 The loading directions for the wire rope spring

The wire rope spring can be used in different loading directions, also called modes. The following modes can be distinguished: the tension-compression, the roll and the shear mode. Figure 2.5 shows the various modes of a polycal wire rope spring. It is expected that the wire rope spring has different characteristics for the different modes.

Figure 2.5: The load directions for the polycal wire rope spring
2.4 Hysteresis

The phenomenon hysteresis occurs in several systems. It can for instance be found in systems with friction, such as wire rope springs, shape-memory alloys and ferromagnetic systems. Hysteresis leads to dissipation of energy.

![Hysteresis Diagram](image)

**Figure 2.6: A theoretical hysteresis loop**

However, before it is possible to model hysteresis it is first necessary to define its properties. This is possible by mapping the output, \( w(t) \) versus the input, \( u(t) \) of the hysteretic systems. In the case of the wire rope spring, \( w(t) \) represents the spring force and \( u(t) \) the spring deflection. In doing so, a hysteresis loop is created. By taking a closer look at figure 2.6, see [25] some properties can be derived.

Let \( u(t) \) increase from \( u_1 \) to \( u_2 \). This implies that the couple \((u(t), w(t))\) moves along the curve \( ADC \). On the other hand \( u(t) \) decreases from \( u_2 \) to \( u_1 \) along the unloading curve \( CBA \). In addition, the couple \((u(t), w(t))\) moves into the region \( S \) bounded by the major loop \( ABCDA \) when \( u(t) \) inverts its movement for \( u_1 < u(t) < u_2 \). The specified model must describe this behavior. By a suitable choice of the input \( u(t) \) the couple can attain any interior point of \( S \). Furthermore it should be noticed that whenever \( u_1 < u(t) < u_2 \) the value of \( w(t) \) is not determined by the value of \( u(t) \) at the same instant. Indeed does \( w(t) \) depend on the history of \( u(t) \). This is called the memory-effect. Additionally \( w(t) \) can also depend on the initial state of the system.

Another property of hysteresis is its rate-independence. It is required that the path of the couple \((u(t), w(t))\) is invariant with respect to any increasing time homeomorphism. This means that at any instant \( t \), \( w(t) \) only depends on the range of the restriction \( u(t) : [0, t] \rightarrow \mathbb{R} \) and on the order in which values have been attained. So it is independent of the derivatives of \( u(t) \). This condition is essential to give a graphical representation of hysteresis in the \((u(t), w(t))\)-plane. If it does not hold, the path of the couple \((u(t), v(t))\) depends on its velocity and a graphic representation in the \((u(t), w(t))\)-plane is not possible. Subsequently, hysteresis is a rate-independent memory-effect.
2.5 Development of models for the wire rope spring

The literature has been examined to make an inventory of models describing the dynamic behavior of the wire rope springs. Two basic types of models can be distinguished.

- A model on physical base
- A model on phenomenological base

2.5.1 A physical model

As mentioned earlier in this chapter dry friction occurs between the different wires. In order to describe the dynamic behavior of the wire rope spring it may be possible to develop a model by investigating the friction between the individual wires. Therefore it is suggested that the theory about steel wires, developed by Wiek [27], is a possible base for a physical model. In his work much attention is paid to the geometry of steel wires. A model is developed to calculate the changes in the curvature of strands in wires and ropes. These changes, caused by bending of a rope over a sheave, are important because they explain a part of the rope’s behavior due to forced bending. Furthermore a part of the material stresses can be calculated from the changes in the wire curvature. From this model it becomes clear that small changes of the rope geometry must be taken into account. Two equations for the wire rope cross section are derived. With these equations the various contact points between the wires in a strand can be calculated. The same equations can also be used to describe the contact problem between the different strands.

A second field of interest is the measurement of the contact forces between the rope and the sheave. Much attention is paid to the distribution of the contact forces in the available contact points. Also some calculation methods are proposed to approximate these contact forces. The importance of these stresses finds expression in the field of the rope endurance. In the case of forced bending the endurance can only be described on the base of endurance tests on wire ropes. Subsequently only when the endurance is expressed in such a way that each of the important stress component has its own term in the expression, it is possible to compare the theory better with experimental data.

Although a similar approach can give more insight in the stresses within the wire rope spring it is not immediately clear how to incorporate the dynamic behavior of the wire rope spring in a practical way. Additionally in Wiek’s work nothing is mentioned about the occurrence of dry friction. Hence the correct approach now seems to use a model on phenomenological basis. This is strengthened by existing literature about modelling of systems with hysteresis. Section 2.5.2 points out that many authors, which investigate different kinds of isolators use phenomenological models to describe the dynamic behavior of the isolator.

2.5.2 A phenomenological model

As it is clear from the previous section a physical model does not seem to be a pragmatic approach to describe the dynamic behavior of the wire rope spring. Therefore a closer look is taken to literature which uses a phenomenological model for describing the dynamics of a hysteretic system. Roughly this literature can be divided into two groups. The first group of researchers uses the Bouc-Wen model to describe the dynamic behavior of a wire rope spring, while the second group uses other phenomenological models.
The Bouc-Wen model

The Bouc-Wen model is originally proposed by Bouc [8], who investigated periodic motion of a hysteretic system. Wen [26] generalized the model and it evolved to a useful model to describe hysteretic behavior. He constructed a hereditary restoring force model that allows analytical treatment. This analytical model is versatile. Through proper choices for the parameters in the model it can represent a wide variety of hysteretic systems in combination with hardening and softening. A closer look at this Bouc-Wen model, including a parametric study, is taken in chapter 3.

Ko et al. [13] have experimentally analyzed a wire rope spring for the use of vibration isolation. They measured a nonlinear response, which is shown in figure 2.7. Furthermore it becomes clear that the dissipated energy increases progressively as the deformation of the damper increases.

A mathematical model representing restoring force versus displacement, with amplitude-dependent parameters has been established. The restoring force is decomposed into two parts, a "nonlinear nonhysteretic force" and a "pure" hysteretic force. The former is expressed by a single-valued nonlinear function in the displacement force plane, related to the hysteresis loop with amplitude dependent characteristics, shown in figure 2.8a. The hysteretic force, which is approximated by an ellipse with the same amplitude and the same area as the corresponding hysteresis loop, is replaced by a viscous damping function with amplitude- and frequency dependent coefficients (figure 2.8b).

Wong et al. [28] also adopt the Bouc-Wen model to describe the dynamic behavior of the wire rope spring. In this study multiharmonic steady state responses caused by an arbitrary periodic excitation are analyzed. Furthermore, it is shown that the Bouc-Wen model is capable of describing various hysteresis loops as the response characteristics of softening, hardening and quasilinear hysteretic systems are studied through numerical computations. Eventually Ni et al. [2] propose two modified versions of the Bouc-Wen model to describe the hysteresis behavior. These modifications are carried out because the Bouc-Wen model fails to describe soft-hardening hysteresis, which will be shown in chapter 3. One modified version is for the tension-compression mode, while the second model describes the behavior in both the roll and the shear mode. From this research it is clear that the modified Bouc-Wen models describe the observed hysteretic behavior very well.
Another source has investigated the use of wire rope isolators for seismic protection. Demetriades et al. [10] propose the Bouc-Wen model to describe the behavior of the wire rope isolator. The analytical predictions for the response of equipment, supported by wire ropes, are in good agreement with experimental results. Furthermore, a simplified analysis method is developed which is shown to be capable of providing reliable estimates of the peak response of the supported equipment.

Recently at Eindhoven University of Technology a start has been made with a first analysis of a polycal wire rope isolator. Leenen [14] uses the modified version of the Bouc-Wen model, proposed by Ni et al. [2], to describe the dynamic behavior of the wire rope isolator in the tension-compression mode. With a quasi-static cyclic loading test the characteristics of the hysteresis loops are obtained. A constrained optimization problem is formulated to identify the model parameters. Furthermore some simulations are carried out to predict the dynamic response. Although the used methods are not very elegant, as they are very time consuming, the results are very promising.

Models based on the presence of friction

The research on wire rope isolators has close connections with investigations on hysteretic dampers applied in the field of civil engineering. Therefore, a closer look is taken at so-called Stockbridge dampers. Sauter et Hagendorn [21] investigate the hysteresis in Stockbridge dampers (figure 2.9), an other type of hysteretic damper. Stockbridge dampers are widely used in overhead transmissions lines, for the damping of wind-excited oscillations of conductors. A wire cable dissipates energy in this damper.

It is stated that the damping mechanism is caused by statical hysteresis. This results from the Coulomb dry friction between the individual wires of the cable, when undergoing cable deformation. Jenkin elements are used to model the static hysteresis in the system. These Jenkin elements, arranged in parallel, form a Masing model, which consists of linear springs and Coulomb friction elements (figure 2.10).

Tinker and Cutchins [24] carry out a study of the dynamic characteristics of a wire rope vibration isolator, constructed with helical isolators. Emphasis has been placed on the analytical modelling of damping mechanics in the system. They describe a experimental investigation in which a static stiffness curve, hysteresis curves, phase trajectories and frequency response curves are obtained. A semi-empirical model having non-linear stiffness, n-th power velocity damping and variable Coulomb friction damping has been developed. The results of the experiments and this model have been compared. The authors find the fit of the model
2.5 Development of models for the wire rope spring

Figure 2.9: Stockbridge damper

Figure 2.10: (a) Masing model; (b) Jenkin element
2. Wire rope springs

upon the experimental data good.

In the past TNO has performed research on a polycal wire rope isolator [4] for the reduction of small amplitude vibrations. A model is developed to describe the dynamic behavior. The model consists of a spring with linear stiffness, $k$, in parallel with a modified Coulomb friction damper with dry friction force, $d$.

An exponential decay is added to the damping force. This is carried out in order to describe the measured loading cycle better. As a result the enhanced dry friction model becomes (2.1), where $u_e$ is a deflection constant that describes the rate of decay. For $u_e = 0$ $F_d$ equals the standard dry friction force. One advantage of this model is that it only consists out of three parameters, $k$, $d$, and $u_e$. Various springs are tested and a static fit is performed to match the model with the experimental hysteretic curves, (figure 2.11).

\[
F_s = k \cdot u \\
F_d = [d - u_e \cdot \frac{dF_d}{du}] \cdot sgn(\dot{u}) \\
F = F_s + F_d
\]

(2.1)

Shaker tests demonstrate that the enhanced friction model (2.1) describes the behavior of the wire-rope isolator well in a qualitative way. There is however a large quantitative mismatch. A much better simulation is obtained by increasing the friction force and the load reversal decay length considerably. The results of both the quasitatic as this 'dynamic' fit are presented in figure 2.5.2. Despite this quantitative mismatch the same non-linear dynamic response as in [13] is measured and simulated.

Figure 2.11: Measured hysteretic curve and model response for the enhanced dry friction model
Section 2.1 introduces the wire rope spring in general while its main application is presented in section 2.2. Due to friction between the different strands hysteresis occurs. In section 2.4 some characteristics of hysteresis have been derived, which proof that hysteresis is a rate-independent memory effect.

Finally, a literature review of the modelling of wire rope spring has been presented. A physical model based upon Wiek's theory does not seem a pragmatic approach. Hence, a phenomenological approach seems the best approach to describe the behavior of the wire rope spring. Subsequently, chapter 3 will address the Bouc-Wen model, which is often used to describe the behavior of the wire rope spring.

Figure 2.12: Results quasistatic fit and dynamic fit vs. experimental results

2.6 Results
Chapter 3

Bouc-Wen model

As is clear from subsection 2.5.2 the Bouc-Wen model is a commonly used model to describe hysteretic behavior. Hence, a modified version of the Bouc-Wen model is used in the present study to describe the dynamic behavior of the wire rope spring. Prior to applying this model to identify the parameters of an actual wire rope spring the Bouc-Wen model itself will be looked at in more detail. As a start a short review about its development is presented. Since it is important to understand the influence of the different parameters in the Bouc-Wen model a parametric study is carried out in section 3.2 to learn more about the physical meaning of the parameters and their influence on the hysteresis loop. Additionally, the dynamic behavior of the Bouc-Wen model is investigated in section 3.3.

3.1 Development of the Bouc-Wen model

The development of the Bouc-Wen model starts by the work of Bouc [8]. He considers forced vibrations of a nonlinear system with hysteresis under periodic excitation.

\[ \ddot{x} + z = p(t) \]  
\[ \dot{z} + \gamma |\dot{z}| \ddot{x} + \beta |z| = \alpha \dot{x} \]  

The equation of motion for this system is given by (3.1). In this equation \( z \) is the intrinsic force, \( x \) the displacement, \( t \) the time and \( p(t) \) the excitation force.

\[ \ddot{x}(t) + Q(x, \dot{x}) = p(t) \]  
\[ Q(x, \dot{x}) = g(x, \dot{x}) + z(x) \]  

\[ \dot{z} = -\beta |\dot{z}| z^n - \gamma |\dot{z}| z^m + \alpha \dot{x}, \quad n = 1, 3, 5, \ldots \]  
\[ \dot{z} = -\beta |\dot{z}| z^{n-1} |z| - \gamma |\dot{z}| z^n + \alpha \dot{x}, \quad n = 2, 4, 6, \ldots \]

Wen [26] presents a highly effective model for hysteretic systems. The equations of motion are given by equation (3.3)-(3.5). Wen states that the restoring force, \( Q \), in a nonlinear hysteretic model can be decomposed into two parts, \( g(x, \dot{x}) \) and \( z(x) \), where \( g \) is a (generally)
nonlinear nonhysteretic polynomial, which is a function of the instantaneous displacement and velocity and \( z(x) \) is a hysteretic component (3.4).

This model allows analytical treatment, for instance the possibility to calculate the Jacobian analytically. It can easily be seen that equation (3.2) is a special case of (3.5) when \( n = 1 \).

\[
\dot{z}(t) = \alpha \dot{z}(t) - \beta |\dot{z}(t)| z(t) |z(t)|^{n-1} - \gamma \dot{z}(t) |z(t)|^n
\]  

(3.6)

Nowadays the Bouc-Wen model, equation (3.6), which is equivalent to equation (3.5) for \( n \in N \) is a widely used model to describe nonlinear hysteretic systems due to its versatility and mathematical tractability.

### Table 3.1: The parameters of the Bouc-Wen model and their dimensions

<table>
<thead>
<tr>
<th>parameter</th>
<th>dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>([N^{1-m} m^{-1}])</td>
</tr>
<tr>
<td>( \beta )</td>
<td>([N^{1-n} m^{-1}])</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>([N^{1-n} m^{-1}])</td>
</tr>
<tr>
<td>( n )</td>
<td>[-]</td>
</tr>
</tbody>
</table>

Equation (3.6) contains four parameters, which are listed in table 3.1 with the corresponding dimensions. These parameters \( \alpha, \beta, \gamma \) and \( n \) are the loop parameters which control the shape and the magnitude of the hysteresis loops, \( \alpha, \beta \) and \( n \) are positive real numbers, and \( \gamma \) may be a positive or a negative real number. It must be noted that Wen states that \( n \in N \). However, it is now stated that \( n \) can be a real positive number. Unfortunately, the reason of this curiosity has not been found. For the case \( 0 < n < 1 \), the term \( |z(t)|^{n-1} \) will tend to infinity when \( z(t) \) approaches zero, which may lead to numerical errors. Hence (3.6) is rewritten in the following form:

\[
\dot{z}(t) = \dot{z}(t) \{ \alpha - [\gamma + \beta \text{sgn}(\dot{z}(t) \text{sgn}(z(t)))] |z(t)|^{n} \}
\]  

(3.7)

It should be noticed that the nonhysteretic term may be essential for describing the restoring force of actual hysteretic vibration isolators. From (3.7) it can easily be derived that

\[
\frac{\partial z}{\partial x} = \alpha - \left[ \gamma + \beta \text{sgn}(\dot{x}(t) \text{sgn}(z(t))) \right] |z(t)|^{n}.
\]  

(3.8)

The restoring force \( Q \) has been decomposed into \( g \) and \( z \), see equation (3.4), but if \( Q = z \), hence \( g = 0 \) it is clear from (3.8) that \( \frac{\partial Q}{\partial z} \), i.e. \( \frac{\partial z}{\partial x} \), varies only with \( z \) and the sign of \( \dot{z}(t) \). Hence, it is independent of \( x(t) \). Subsequently, \( \frac{\partial Q}{\partial z} |_{Q=0} = \frac{\partial z}{\partial x} |_{z=0} = \alpha \). This implies that for any specified value for \( Q = Q_z \) the hysteretic curves corresponding to various excitation and response levels have the same slope at these points where \( Q = Q_z \) and identical sign of \( \dot{z} \), which conflicts for instance with the experimental hysteresis loops obtained by Ko et al. [13]. This inconsistency can be avoided by including the non hysteretic term \( g \) in the restoring force expression. When \( g \) represents a linear spring, \( g = kx(t) \), this leads to

\[
\frac{\partial Q}{\partial x} = k + \frac{\partial z}{\partial x} |Q-kx = k + \alpha - [\gamma + \beta \text{sgn}(\dot{x}) \text{sgn}(Q-kx)] |Q-kx|^{n}.
\]  

(3.9)
3. Parameter influences on the static behavior of the Bouc-Wen model

The Bouc-Wen model, (3.7) contains four parameters, which have a strong influence on the hysteresis behavior. It is therefore very important to have a clear view on the influence of each parameter. Wong et al. [29] carry out a parametric study for the Bouc-Wen model. A similar approach is used here to gain a good understanding of the complete model for quasi-static behavior.

3.2.1 Influence of the parameters $\beta$ and $\gamma$

Table 3.2 [29] lists the slope $\frac{\partial z}{\partial x}$ at every stage of the hysteresis loop. It shows that the hysteresis loop is symmetric with respect to the origin, $x = 0, z = 0$. The stiffness difference in both loading-unloading and unloading-reloading is equal to $2\beta |z|^n$. Hence the parameter $\beta$ controls the stiffness change when the sign of $\dot{x}$ alters.

<table>
<thead>
<tr>
<th>$\dot{x}$</th>
<th>$\frac{\partial z}{\partial x}$</th>
<th>$z &gt; 0$</th>
<th>$z = 0$</th>
<th>$z &lt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{x} &gt; 0$</td>
<td>$\alpha - (\gamma + \beta)</td>
<td>z</td>
<td>^n$</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>$\dot{x} &lt; 0$</td>
<td>$\alpha - (\gamma - \beta)</td>
<td>z</td>
<td>^n$</td>
<td>$\alpha$</td>
</tr>
</tbody>
</table>

Table 3.2: The slope of the hysteresis loop

The shape of the hysteresis curve is mainly determined by the parameters $\beta$ and $\gamma$. Different combinations of $\beta$ and $\gamma$ lead to various hysteretic loops with different stiffness characteristics. A closer look is taken at the fashion of the slope in the case of loading, $\dot{x} > 0$ and unloading $\dot{x} < 0$ respectively. This leads to five types of hysteresis loops which are physically meaningful, corresponding to five combinations for $\beta$ and $\gamma$, which are summarized below.

- $(\beta + \gamma) > 0$ and $(\beta - \gamma) > 0$
- $(\beta + \gamma) > 0$ and $(\beta - \gamma) < 0$
- $(\beta + \gamma) > 0$ and $(\beta - \gamma) = 0$
- $(\beta + \gamma) < 0$ and $(\beta - \gamma) > 0$
- $(\beta + \gamma) = 0$ and $(\beta - \gamma) > 0$

More combinations for $\beta$ and $\gamma$ are possible. These are addressed in appendix B and it will be shown that these combinations do not yield a hysteretic curve.

For the above five cases simulations have been carried out in Matlab, using a Simulink model, see Appendix C. The hysteresis loops for these five cases are shown in figures 3.1-3.5, where the arrow indicates the direction of motion. The parameters $\alpha$ and $n$ are both kept constant at 1.0. The legend shows the values for the amplitude of the displacement. As mentioned in section 2.4 hysteresis is a rate-independent effect and therefore the velocity $\dot{x}$ has no influence on the shape of the hysteretic curve. In the first three cases softening nonlinear behavior occurs during loading due to the fact that $(\beta + \gamma) > 0$. In the fourth case, under loading, the system exhibits hardening non-linearity due to the fact that $(\beta + \gamma) < 0$, whereas in case five, where $(\beta + \gamma) = 0$ the system behaves quasi-linearly during loading.
3.2 Parameter influences on the static behavior of the Bouc-Wen model

Figure 3.1: Hysteresis loop for $(\beta + \gamma) > 0$ and $(\beta - \gamma) > 0$

Figure 3.2: Hysteresis loop for $(\beta + \gamma) > 0$ and $(\beta - \gamma) < 0$

Figure 3.3: Hysteresis loop for $(\beta + \gamma) > 0$ and $(\beta - \gamma) = 0$

Figure 3.4: Hysteresis loop for $(\beta + \gamma) < 0$ and $(\beta - \gamma) > 0$

Figure 3.5: Hysteresis loop for $(\beta + \gamma) = 0$ and $(\beta - \gamma) > 0$
It has to be noted that the figures 3.1 and 3.3 produce almost identical hysteretic curves. This rises the question whether a redundancy is present in the model and whether \( \gamma \) may not be omitted from the model. Figure 3.6 depicts the hysteretic curves for the combinations \( \beta = 0.8, \gamma = 0.2 \) and \( \beta = 0.5, \gamma = 0.5 \). Note that for both scenarios the following equation holds

\[ \beta + \gamma = 1 \]  

(3.10)

Indeed the hysteretic curves resemble each other, although a difference is present due to the different value of \( \beta \) since a higher value for \( \beta \) leads to a larger stiffness change when the sign of \( \dot{x} \) changes. A generalised form for equation (3.10) looks like

\[ \beta + \gamma = \text{constant} \]  

(3.11)

By substituting relation (3.11) into equation (3.8) a general conclusion can be drawn. If equation (3.11) holds and \( \beta \) and \( \gamma \) are both larger than zero, the slope of the hysteretic curve will be the same as long as \( \text{sgn}(\dot{x}) = \text{sgn}(\dot{z}) \). Therefore, the figures 3.1 and 3.3 produce almost the same hysteretic curve.

This "redundant parameter" phenomenon has to be dealt with in the identification process. Imposing one of the constraints \( \beta > \gamma, \beta = \gamma \) and \( \beta < \gamma \) during the identification determines which softening scenario the final solution will be part of. For instance, imposing the second constraint \( \beta = \gamma \) leads to the third case, as depicted in figure 3.3. However, when the system is a priori known to display softening hysteresis loops, an altered version of the Bouc-Wen model omitting the parameter \( \gamma \) may be adopted for identification. If the parameter \( \gamma \) is omitted, the Bouc-Wen model reduces to the Ozdemir-model [19]. Bhatti and Pister [7] and Fujita et al. [11] have used the latter to model nonlinear damping devices with softening hysteretic behavior.

Except from the combination of the parameters \( \beta \) and \( \gamma \), also the influence of each parameter separately has to be investigated. Subsequently, simulations are performed with different values for \( \beta \) and \( \gamma \). Figure 3.7 shows the results of these simulations. The parameters \( \alpha \) and \( n \) are again set equal to 1.0. The values for \( \gamma \) in (a) and \( \beta \) in (b) are 0.2 and 0.8 respectively. The same trends can be seen in the case of varying \( \beta \) as well as the case of varying \( \gamma \) if another
3.2 Parameter influences on the static behavior of the Bouc-Wen model

Figure 3.7: The influence of the parameters $\beta$ (a) and $\gamma$ (b) on the hysteresis loop

value for $\gamma$ or $\beta$ is used, even when the system changes from hardening into softening when the value for $\beta$ or $\gamma$ changes.

A remark must be made. Earlier research shows that the wire rope spring possess a kind of hysteretic behavior which cannot be described by the Bouc-Wen model. Ni et al. [2] and Leenen [14] show that a wire rope spring possess softening stiffness for small amplitudes, which via quasi-linear behavior, gradually changes into hardening stiffness for increasing amplitudes. This is called soft-hardening and is shown in figure 3.8. Figure 3.8 has been constructed based upon simulation results with a modified version of the Bouc-Wen model as proposed by Ni et al. [2].

Figure 3.8: Soft-hardening hysteresis

It can be concluded from table 3.2 that for $z > 0$ the slope from the hysteretic curve cannot manifest itself first as decreasing and then as increasing, without changing the value for $\beta$ or $\gamma$ and therefore soft-hardening cannot be described.

Ni et al. [2] also observe that the loading path for various amplitudes is the same, as depicted in figure 3.9. Since the used wire rope spring possess hardening behavior in tension they call this phenomenon hardening overlap, which cannot be described by the Bouc-Wen model. This is confirmed by figure 3.4 as for increasing amplitudes the hysteretic curve becomes somewhat thicker and hence no hardening overlap is present. As we will see in
chapter 4 the used wire rope spring exhibits this hardening overlap behavior as well.

3.2.2 Influence of the parameters $\alpha$ and $n$

The parameter $\alpha$ controls the slope of the hysteresis loop at $z = 0$. So with other parameters kept constant different values of $\alpha$ influence the height and the thickness of the hysteresis loop, shown in figure 3.10.

The quantity $n$ governs the smoothness of the transition from linear to nonlinear range. Increasing values for $n$ lead to more elasto-plastic behavior. In the limiting case, where $n = \infty$, the system exhibits true elasto-plastic behavior. When $n = \infty$ equation (3.8) approximates infinity for $z > 0$ and for $z < 0$. However, as will be proven in section 3.4 $z$ has a maximum, $z_{\text{max}}$ which for $n = \infty$ is equal to one.

The values for $\beta$ and $\gamma$ in figure 3.10 are equal to 0.5 and $-0.4$ respectively. The parameter $\alpha$ is set equal to 1.0 when $n$ is varied. Similarly $n = 1.0$ when different values for $\alpha$ are applied to the system.

3.2.3 Synopsis

Summarizing, the Bouc-Wen model can describe various stiffness characteristics by a proper choice of $\beta$ and $\gamma$. When the system exhibits softening behavior different values for $\beta$ and $\gamma$ can lead to almost identical hysteresis loops. Therefore, a constraint for $\beta$ and $\gamma$ can be added or an altered version of the Bouc-Wen model, the Ozdemir model, can be used.

The Bouc-Wen model fails in describing soft-hardening hysteresis and hardening overlap, phenomena that are found in the hysteretic behavior of wire rope springs. To cope with these phenomena Ni et al. [2] propose a modified version of the Bouc-Wen model, which will be discussed in section 3.4. First however in section 3.3 the influence of the various parameters on the dynamic behavior is addressed.

3.3 The dynamic behavior of the Bouc-Wen model

In section 3.2 a parametric study has been carried out for the Bouc-Wen model. Since only the influence on the hysteretic curve is presented this section takes a closer look at the dynamic behavior.
3.3 The dynamic behavior of the Bouc-Wen model

behavior of the Bouc-Wen model. Because the Bouc-Wen model is a nonlinear model, various nonlinear dynamic characteristics may be expected, e.g. an amplitude dependent resonance frequency and various superharmonic resonances.

$$d(t) = x_T(t) - x_M(t)$$

which is equivalent to (3.1) and (3.3). Here $d(t)$ is defined as the difference between the displacement of the ground and of the mass, $M$. The mass is taken equal to 1 kg.

The ground motion is prescribed by an acceleration with increasing frequency which is called a frequency sweep-up. The amplitude is of the acceleration is kept constant at 9.811

$$\ddot{x}_T = a \cdot 9.81 \cos(2\pi ft)$$

Every minute the frequency, $f$, increases with a factor four, which implies a frequency sweep of two octaves per minute. At $f_0 = 0.1$ Hz the transient behavior, resulting from startup, is still present, which can be distinguished in the figures 3.12 - 3.16. Frequency sweeping will be discussed in more detail in section 6.1.

3.3.1 Influence of the parameters $\beta$ and $\gamma$ on the dynamic behavior.

Just like for the quasi-static behavior, the influence of the parameters $\beta$ and $\gamma$ on the dynamic behavior is observed. However, not all five cases as discussed in subsection 3.2.1 will be addressed. Instead a closer look is taken at the influence of $\beta$ and $\gamma$ separately. The values for $\alpha$ and $n$ are set equal to 1.

Figure 3.12 depicts the maximum acceleration $\ddot{x}_M$ against the excitation frequency $f$ for several values of $\beta$ and $\gamma = -0.4$. It can be seen that the amplitude of the resonance frequency decreases with an increase of $\beta$, because the hysteretic damping increases. Furthermore the resonance frequency shifts slightly to the left.
3. Bouc-Wen model

Figure 3.12: The frequency response curves for different values for $\beta$

Frequency amplitude curves are obtained for different values of $\gamma$ as well, which are depicted in figure 3.13. Similar as in the static case $\beta$ is equal to 0.8. Again due to the extra amount of damping the resonance frequency shifts to the left, accompanied with a decrease in amplitude.

Figure 3.13: The influence of $\gamma$ on the dynamic behavior

Figure 3.14 shows the maximum acceleration as function of the frequency for the combinations $\beta = 0.5$, $\gamma = 0.5$ and $\beta = 0.8$, $\gamma = 0.2$. Although these combinations lead to almost identical hysteretic curves, differences can be distinguished in the dynamic response. Not only the amplitude of the resonance frequency differs, but also a slight difference in the resonance frequency itself can be seen.

3.3.2 Influence of the parameters $\alpha$ and $n$ on the dynamic behavior.

The values for $\beta$ and $\gamma$ are 0.5 and -0.4 respectively when the influence of $\alpha$ and $n$ is investigated. The values for $\alpha$ is set to 1 when the influence of $n$ is investigated and vice versa.
3.3 The dynamic behavior of the Bouc-Wen model

Figure 3.14: Dynamic response for $\beta = 0.5$, $\gamma = 0.5$ (left) and $\beta = 0.8$, $\gamma = 0.2$ (right)

Figure 3.15 addresses the influence of $\alpha$ on the frequency amplitude curve of the Bouc-Wen model. It shows that the resonance frequency increases, which follows from equations (3.8) and (3.9), along with an increase in amplitude for higher values of $\alpha$.

Various frequency response curves for different values of $n$ are depicted in figure 3.16. For increasing values of $n$ an decrease in the resonance frequency is observed, along with an increase in amplitude.

Figure 3.16 shows that for $n = 10$ around 10 Hz a peak can be distinguished. Therefore, the steady state time history plot for an excitation frequency of 10 Hz is depicted in figure 3.17. It can be seen that for larger values for $n$ a subharmonic response is present. A possible explanation is that for an increasing $n$ the system behaves more elasto-plastic.
Figure 3.16: The influence of $n$ on the frequency response curve

Figure 3.17: Response on a periodic signal with $f = 10$ Hz for $n = 0.1$ and $n = 10$
3.4 Modified Bouc-Wen model

As follows from section 3.2 the Bouc-Wen model is a versatile, mathematically easy but physically hard to comprehend phenomenological model. Ni et al. [2] perform a cyclic loading test to experimentally obtain the hysteretic behavior of a wire rope isolator in the shear, roll and tension-compression mode. It is observed that symmetric soft-hardening behavior occurs in the shear and the roll mode, whereas asymmetric hysteretic behavior is found in the tension-compression mode. Therefore, two modified Bouc-Wen models are proposed to ensure that soft-hardening hysteretic behavior and hardening overlap in tension is described correctly.

\[ F(t) = F_2(t)[z(t) + F_1(t)] \]  
\[ F_1(t) = k_1x(t) + k_2 \text{sgn}(x(t))x(t)^2 + k_3 \ast x(t)^3 \]  
\[ F_2(t) = b\dot{x}(t) \]  
\[ \ddot{z}(t) = \dot{z}(t) \left\{ \alpha - [\gamma + \beta \text{sgn}(\dot{z}(t))\text{sgn}(z(t))] |z(t)|^n \right\} \]

While in the present study only the behavior in the tension-compression mode is studied, only a closer look is taken at this asymmetric model (3.15)-(3.18). This model is derived by modulating a symmetric hysteretic force with a nonlinear elastic stiffness. The function \( F_1(t) \) is stipulated to be an odd function with respect to \( x(t) \), meaning that \( F_1(-x) = -F_1(x) \). \( F_2(t) \) can be looked upon as a sort of modulating function to ensure the hardening overlap. Furthermore, \( F_2 \) introduces the asymmetry in the hysteretic curve.

The parameters of the Bouc-Wen model are already discussed in section 3.2. Therefore table 3.3 only depicts the additional parameters of the modified Bouc-Wen model. It can easily be seen that \( k_1, k_2, k_3 \) are stiffness parameters. As \( F_2 \) is just a dimensionless modulating function, the dimensions of \( b \) and \( c \) follow directly from (3.15) and (3.17).

<table>
<thead>
<tr>
<th>parameters</th>
<th>dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b )</td>
<td>[-]</td>
</tr>
<tr>
<td>( c )</td>
<td>[m(^{-1})]</td>
</tr>
<tr>
<td>( k_1 )</td>
<td>[Nm(^{-1})]</td>
</tr>
<tr>
<td>( k_2 )</td>
<td>[Nm(^{-2})]</td>
</tr>
<tr>
<td>( k_3 )</td>
<td>[Nm(^{-3})]</td>
</tr>
</tbody>
</table>

Table 3.3: The additional parameters of the modified Bouc-Wen model

The model parameters to be determined are simultaneously estimated by Ni et al. [3] and it turns out that the asymmetric Bouc-Wen model describes the observed hysteretic behavior very well. These values, summarized in table 3.4, are used here to simulate the modified Bouc-Wen model.

<table>
<thead>
<tr>
<th>( b )</th>
<th>( c )</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>( n )</th>
<th>( k_1 )</th>
<th>( k_2 )</th>
<th>( k_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8</td>
<td>0.2</td>
<td>147.8</td>
<td>44.7</td>
<td>-15.5</td>
<td>0.43</td>
<td>39.1</td>
<td>1.4</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Table 3.4: Arbitrary set of parameters
3.4.1 The static behavior of the modified Bouc-Wen model

In order to gain a better understanding of the modified Bouc-Wen model the quantities, \( z, F_1(t), F_2(t), z(t) + F_1(t) \) and \( F(t) \) are depicted in the figures 3.18 - 3.20, where the displacement \( x(t) = 4 \sin t \). It becomes clear that the hysteretic force \( z(t) \) has a 'finite' ultimate value \( z_{\text{max}} \) at the displacement \( x_{\text{max}} \). Analytically this maximum can be found by setting (3.8) equal to zero, i.e. \( \partial z / \partial x = 0 \), which leads to

\[
z_{\text{max}} = \left[ \frac{\alpha}{\gamma + \beta} \right]^{1/n}.
\]

(3.19)

Figure 3.18: The quantities \( z(t) \) on \( F_1(t) \) of the modified Bouc-Wen model

Figure 3.18 shows that \( F_1 \) is a nonlinear elastic spring with a positive value under tension and negative values in compression. Figure 3.19 depicts the extra component \( F_2 \) which is an asymmetric multiplication factor. The influence of \( F_2 \) on the hysteretic curve becomes clear from figure 3.20. By multiplying \( z(t) + F_1 \) with \( F_2 \) clearly an asymmetric hysteretic curve can be distinguished, represented by \( F(t) \).

3.4.2 Dynamic behavior of the modified Bouc-Wen model

Additional simulations are carried out to gain insight into the dynamic behavior of the modified Bouc-Wen model. The mass \( M \) which is placed upon the wire rope spring, is 3.5 kg. For the modified Bouc-Wen model the equation of motion changes into

\[
M \ddot{x}_M = F(d, \dot{d}, t, z)
\]

(3.20)

where \( F \) is the restoring force of the modified Bouc-Wen model. The system will be described in more detail in chapter 7. Again the ground is excited with an increasing frequency of two octaves per minute.
Figure 3.19: The component, $F_2(t)$, of the modified Bouc-Wen model

Figure 3.20: The influence of $F_2(t)$ on the hysteresis loop
Figure 3.21: Frequency amplitude curve of the modified Bouc-Wen model for different amplitude levels

Figure 3.21 clearly shows the nonlinear behavior of the modified Bouc-Wen model. The harmonic resonance frequency shifts to the left for increasing amplitudes of excitation, $A$, indicating softening behavior. Furthermore various superharmonic resonances can be distinguished.

Figure 3.22: Three steady state responses for the modified Bouc-Wen model

Figure 3.22 shows the time histories for the acceleration of the harmonic resonance, the second and the third order superharmonic resonance at 36.2, 18.2 and 12.2 Hz respectively. Clearly the influence of higher order superharmonic resonances is visible in figure 3.21, so the
system appears to be relatively weakly damped. Especially the odd resonance peaks can be seen very well.

3.5 Conclusion

This chapter has provided valuable insight in the Bouc-Wen model and the influence of the various parameters. One of the main conclusions is that the Bouc-Wen model can describe softening, hardening and quasi-linear behavior for a good choice of $\beta$ and $\gamma$. When the system exhibits softening behavior, a problem might occur since various combinations for $\beta$ and $\gamma$ can lead to almost identical hysteresis loops. This "redundancy" can lead to problems if the parameters of the Bouc-Wen model have to be identified. As a solution to this problem a constraint for $\beta$ and $\gamma$ can be added. If it is known a priori that the system exhibits softening stiffness the Ozdemir model, in which the parameter $\gamma$ is omitted, can be adopted.

Another important conclusion is that the Bouc-Wen model fails in describing soft-hardening hysteresis, since the slope of the hysteretic loop at the segment $z > 0$ cannot manifest itself as first decreasing and then increasing without changing a parameter value. Secondly, the Bouc-Wen model cannot describe hardening overlap. Both phenomena have experimentally been found for actual wire rope springs, similar to the type used in this thesis project. Therefore a modified version of the Bouc-Wen model is addressed in section 3.4 which is capable of describing these phenomena. In chapter 4 the results of the quasi-static experiments will be presented. Indeed soft-hardening hysteresis and hardening overlap will be observed and the modified version of the Bouc-Wen model will be adopted to describe the behavior of the wire rope spring. However, the identification, which is described in chapter 5, probably still will be a difficult process as the redundancy is still present.

The parameter influence on the dynamic behavior has been investigated as well. Frequency sweeps have been carried out to study the influence of each parameter. Obviously, the Bouc-Wen model has a nonlinear dynamic response. An amplitude dependant resonance frequency has been found. Furthermore superharmonic resonances are present. One main conclusion is that in general no subharmonic responses have been found due to the large amount of damping, except for large values of $n$. This holds both for the Bouc-Wen model as for the modified Bouc-Wen model. However, it can be expected $n$ will not become that large.

The combinations for $\beta$ and $\gamma$ which lead to almost identical hysteretic curves are addressed to learn whether the dynamic response resemble each other as well. However, it turns out that not only the amplitude of the resonance frequency changes but also the resonance frequency itself is slightly different. Again one can question what this means for the identification process, which will be carried out with the results of the quasi-static experiments. An important consequence seems that the resulting parameters have to be used in a frequency sweeping process to compare the resulting frequency amplitude curve with corresponding experimental results to know whether the correct values for $\beta$ and $\gamma$ have been obtained.
Chapter 4

Quasi-Static Experiments

A better understanding of the behavior of the wire rope spring is needed. This is achieved by carrying out cyclic quasi-static loading tests. During these tests different amplitudes are applied to have a clear idea how the wire rope spring behaves. Therefore both small and large amplitudes are applied. In total five springs are available, three springs have been received from Loggers, the supplier of the Socitec springs in the Netherlands; the other two were available from TNO-CMC. Hence, the five springs are labelled as log1, log2, log3, lab1 and lab2. All five springs are of the Socitec MP14-345 type which is designed to be loaded with 50 kilograms.

4.1 Experimental setup

The set-up for the quasi-static loading test is arranged on a tensile/compression loading bench. An axial load cell (10kN) is used to measure the tensile and compression forces. The wire rope spring has to be in fixed position. Therefore the clamps of the wire rope spring are bolted in metal blocks and clamped between the claws on the loading bench (figure 4.1). These clamps are used to fix the specimen in the testing bench.

Figure 4.1: The wire rope spring in the loading bench
The hysteretic behavior of friction-type dampers is heavily influenced by static preloading. It is therefore observed that all the wire rope springs have the same static preloading. In that way the different results can be compared better. Therefore, an initial length, $L_0$, is defined. Any deviation from this initial length, both in positive and negative direction, is called the deflection distance of the wire rope spring. The deflection, $x(t)$, is drawn on the horizontal axis of the loading cycle. The loading forces are measured in axial direction only. Sideward movement of the retainers is prevented. Therefore the resulting forces in the load cell, other than pure tensile and compression forces are neglected.

$$x(t) = L(t) - L_0$$  \hspace{1cm} (4.1)

All the loading tests are displacement controlled. It was experimentally found out that the only low velocities between 0 and 100 mm/s can be prescribed at the used loading bench. Therefore, periodic excitation levels, as shown in figure 4.2 are used.

### 4.2 Results

As already mentioned all five springs are tested. This section only addresses the results for one spring, log1. However, during the first experiments the initial load has not been measured correctly. This has not been noticed before the experiments on the shock table have been carried out, see section 6.2. Hence the possibility to carry out the quasi-static experiments for all springs has disappeared, as the wire rope springs, used in the shock experiments, have deformed plastically. So only the quasi-static results after the shock test for the spring labelled as log1 are discussed.

Figure 4.3 shows the results for the different amplitude levels. Clearly an asymmetric hysteretic curve can be distinguished similar as Ni et al. [2] observed and which they described.
by the modified Bouc-Wen model, equations (3.15)-(3.18). This asymmetric hysteretic curve is represented in figure 3.20 by \( F \). When looking at the hysteretic curves for the different amplitudes it can be seen that for very small amplitudes the spring has softening stiffness. For small amplitudes the system behaves quasi-linear. For larger amplitudes we see hardening stiffness in loading and softening stiffness in unloading. Furthermore we distinguish hardening overlap. The asymmetry in the hysteretic curve can be explained by means of figure 4.4, which shows the wire rope spring both in tension and compression. When the spring is in tension, the wires grab into one each other, leading to more contact points between the wires. This implies more dry friction. On the other hand, when the spring is compressed, less contact points are present, implying less dry friction. Secondly, a geometric effect plays a significant role. Under tension, the loading gradually changes into an axial load due to the elongation. Since in axial direction the rope is more stiffer than in other directions a greater force is needed. So, in general a greater force in tension is needed than in compression to achieve the same absolute displacement.

![Figure 4.4: The wire rope spring in tension and compression](image)

Some properties can be derived from these quasi-static tests, i.e. the effective stiffness, \( K_{eff} \), which is defined as the maximum force minus the minimum force divided by the difference between the maximum and minimum displacement.

\[
    K_{eff} = \frac{F_{max} - F_{min}}{x_{max} - x_{min}}
\]

(4.2)

The upper diagram of figure 4.5 depicts the effective stiffness, (4.2) as a function of the maximum displacement. The asterisks in the figure are the values obtained from the measurements. It can be seen that the effective stiffness becomes larger again as the displacement amplitudes reach larger levels.

Another interesting quantity is the energy loss. The energy loss percentage is calculated by dividing the surface area in the loop (energy loss) with the total amount of energy pumped in one cyclic loading. This is depicted in the lower diagram of figure 4.5.

4.2.1 Loading around other operation points

In section 2.2 the main application of wire rope springs, the multi-purpose floating floor, has been described. As the weight of a floating floor can be looked upon as a kind of static preloading, it is interesting to carry out some additional experiments around different working
4.3 Conclusions

By carrying out quasi-static periodic experiments many insights are obtained about the behavior of the wire rope spring. As we can conclude from the hysteretic curves the wire rope
spring exhibits hardening stiffness in tension and softening stiffness in compression. We can also distinguish hardening overlap and soft-hardening. As we saw in section 3.4 Ni et al. [3] have found the same behavior and suggested an asymmetric model, equations (3.15)-(3.18), to describe the hysteretic behavior. Hence, in chapter 5 the modified Bouc-Wen model is identified to describe the behavior of the wire rope spring.
4.3 Conclusions
Chapter 5

Identification of the model

As is clear from chapter 3 the Bouc-Wen model is a versatile model, that contains many parameters. Assuming the model is capable of describing the observed behavior, it is very important that the parameters are estimated correctly in order to describe the observed hysteretic behavior. Many efforts have been devoted to the development of identification procedures for nonlinear hysteretic systems, most of the time referring to the Bouc-Wen model. In section 5.1 these efforts are briefly discussed to obtain an overview of the various used methods. Section 5.2 addresses the identification procedure used in this study. Subsequently the results of the system identification are presented and the experimental hysteretic curves will be compared with those created by the model.

5.1 Identification of nonlinear hysteretic systems

Determination of the model parameters using experimental input and output data can be accomplished by system identification methods. Different procedures may be used to identify nonlinear hysteretic systems. These procedures include the time-domain least-squares method and the time-domain extended Kalman filtering technique.

A method for identification of degrading hysteretic restoring forces is proposed by Wen et al. [23]. This method is based on a classical, time-domain least-squares procedure where the parameters are obtained by solving a system of simultaneous linear equations. Based on applications for the parameter identification method, using experimental structural restoring force results, simple rules for determining the model parameters are suggested. The authors have successfully implemented this method. Yar and Hammond [30] stated that the estimation problem is a nonlinear optimization problem. Subsequently they use the Gauss-Newton method to set up a two stage iterative least-squares algorithm. The usefulness of the algorithm is validated through its application to various simulated time histories from the hysteretic model.

The time-domain extended Kalman filter is a well known technique for parameter estimation. Lin and Zhang [16] used this method to identify the parameters of a hysteretic SDOF-system defined by the Bouc-Wen model. A simulated earthquake input-response pair is used. In the absence of noise the extended Kalman filter with the global iteration procedure generally leads to a single set of results, independent of the initial conditions. The presence of non-white noise in the observations does not appear to be a problem. It is also found that a proper selection of the observation variables is important. Overall, the extended Kalman
5.1 Identification of nonlinear hysteretic systems

A filter with weighted global iteration appears to be a stable tool for this class of nonlinear identification. It is also possible to use a frequency-domain Kalman filtering algorithm. A recursive procedure for the identification is proposed by Bouc and Bellizi [6], that solves the model via iterative applications of a multiharmonic Galerkin method. During the recursive (online) identification the extended Kalman filter algorithm is used successfully.

Chassiakos et al. [9] used another identification technique. A method is presented for the online identification of hysteretic systems under arbitrary dynamic environments by using an adaptive estimation scheme. The availability of such an identification approach is crucial for the online control and monitoring of nonlinear structures to be actively controlled. It is shown that through the use of simulations and experimental results that the proposed approach can yield reliable results for the hysteretic restoring force under a wide range of excitation levels and response ranges.

Two approaches are followed in the mentioned identification procedures. In the first approach the exponential parameter in the Bouc-Wen model, \( n \), is taken as a known constant, [16]. As a result only the other model parameters have to be determined. This treatment can result in a linear estimation scheme or can avoid divergence in nonlinear iterative algorithms. However, a priori assumption about the exponential parameter certainly reduces, more or less, the accuracy of the identification. In the second approach a two-stage estimate scheme is introduced. In the first stage one or two parameters are fixed to assumed values while the others are estimated. The final step of the first stage is taken as the initial step of the second stage, in which all the model parameters are estimated simultaneously [30]. Although a multiple step algorithm can lead to good results, it is very time consuming.

To avoid these approaches Ni et al. [1] propose an alternative one stage estimate scheme. A frequency-domain method is developed to identify the model parameters from experimental data of periodic vibration tests of wire rope vibration isolators. Numerical simulations are carried out which show that the proposed method is insensitive to the noise in observation signals. The accuracy is verified by comparing the measured and identified components of the hysteretic restoring force. A similar approach is used to identify the two modified Bouc-Wen models [3]. By making use of the rate independent nature of the hysteretic behavior the measured quasi-periodic data sequences are modulated as periodic signals through modulation processing, from which the harmonic components of the displacement and the restoring force signals are obtained. The harmonic balance technique is then applied to lead to a frequency-domain least-squares estimate with an appropriate objective function. The Levenberg-Marquardt iteration algorithm is formulated for the parameter estimation in which a frequency-time-domain alternating scheme is introduced to perform the numerical calculations.

Similar as Yar and Hammond [30] Leenen [14] approaches the identification procedure as an optimization problem. Hence, a constrained optimization procedure is formulated to identify the parameters. To enhance the optimization process constraints are derived from the shape of the actual hysteresis curve. Additionally, a set of mesh points is defined, through which the model has to be fitted during the optimization loop. Eventually the identified hysteresis loop matches its experimental equivalent to a large extent. However, the method appears to be very time demanding and very sensitive for local optima.
5. Identification of the model

5.2 Proposed identification method

The present study addresses the identification of a wire rope spring, whose behavior is described by the modified Bouc-Wen model (3.15)-(3.18). The set of parameters, which has to be identified, \( y \), see equation (5.1), contains nine elements and is thus quite large. Therefore a good working identification method and algorithm is essential.

\[
y = \begin{bmatrix} b & c & \alpha & \beta & \gamma & n & k_1 & k_2 & k_3 \end{bmatrix}^T
\]  

(5.1)

5.2.1 Objective function

By substituting (3.15) to (3.17) into (3.18) an error function can be defined

\[
e(t) = \frac{F_2 \dot{F} - \dot{F}_2 F}{F_2^2} - \dot{F}_1 - \dot{x} \left\{ \alpha - \left[ \beta \text{sgn}(\dot{x}) \text{sgn} \left( \frac{F}{F_2} - F_1 \right) + \gamma \right] \left| \frac{F}{F_2} - F_1 \right|^n \right\}
\]  

(5.2)

If the actual measured hysteretic behavior coincides completely with the proposed model the error function \( e(t) \) is equal to zero. The identification process must tune the model parameters to yield a best fit between the measured hysteresis loops and the analytical model. For this purpose, a least squares estimate is defined with the solution \( y^* \) as

\[
\min_y \| E(y) \| = \| e(y) \|^2 = e^T(t)e(t)
\]  

(5.3)

where \( E \) is the norm of \( e(y)^2 \). The error actually is time-discrete because at each time sample of the measurement the error is calculated. Therefore the error must be defined as

\[
e(t) = [e(t_1), e(t_2), \ldots, e(t_m)]
\]  

(5.4)

where \( m \) is the number of samples.

5.2.2 Levenberg-Marquardt algorithm

In subsection 5.2.1 an objective function has been defined to fit the model on the experimental results. It is now important to find a good working algorithm which satisfies the descending condition,

\[
E_{y_{k+1}} < E_{y_k}.
\]  

(5.5)

where \( k \) equals the number of iterations. A general framework for such an algorithm looks like this

\[
k := 0 \quad y := y_0
\]

repeat

Find a descent direction \( h_k \)

\[
y_{k+1} := y_k + \alpha h_k;
\]

\[
k := k + 1
\]

until Stop
where $\alpha$ is found by line search. A descent direction satisfies

$$h_k^T E'_k(y) < 0$$

(5.6)

where $E'_k$ is found as

$$E'_k = J_k(y)^T e_k(y)$$

(5.7)

with the Jacobian matrix defined as

$$J_k = \frac{\partial e_i(y)}{\partial y} \bigg|_{y=y^k}$$

(5.8)

where the dimensions of $J_k$ and $e_k$ are $(9 \times m)$ and $(m \times 1)$. Such an algorithm is the Levenberg-Marquardt algorithm. If $\mu$ is small, it follows that $h \approx h_{GN}$, the step when using the Gauss-Newton algorithm, and if $\mu$ is large, the $h \approx -\frac{1}{\mu} E'$, a short step in the steepest descent direction. Thus, the choice for $\mu$ both influences the direction and the size of the step $h$. If $y$ is close to the solution $y^*$ the faster convergence of the Gauss-Newton method is needed, while the robustness of the steepest descent method is preferred when $y$ is far from $y^*$. So a suitable strategy for choosing the parameter $\mu$ is necessary for a robust algorithm. The update strategy for $\mu$ is discussed in appendix D.

### 5.2.3 Identification method

Various identification strategies have been attempted. Due to various reasons these methods did not perform satisfactorily. Appendix E addresses the various identification efforts. Finally, a three stage identification method is proposed to identify the parameterset $y$. One steady state period of the measured cyclic loading is used for evaluation of the model output. The amplitude of this loading cycle is 25 mm, because for smaller amplitudes the stiffness parameter $k_3$ probably does not have any influence. During all three stages the Levenberg-Marquardt algorithm is used.

#### Identified model

In the first stage the stiffness parameters, $k_1, k_2, k_3$, together with the modulating parameters, $b$ and $c$ are estimated while the hysteretic parameters, $\alpha, \beta, \gamma$ and $n$ are not taken into account. Hence, for the first stage the objective function changes into

$$e(t) = F(t) - F_1(t) \ast F_2(t).$$

(5.10)
5. Identification of the model

It is essential that the static parameters are fitted on a part of the curve where the hysteretic parameters hardly have any influence. Therefore the static fit is carried out on the part where $\dot{x}(t) > 0$. As $\beta$ controls the stiffness change, when the sign of $\dot{x}(t)$ alters, section 3.2, the beginning of that part is not taken into account. Figure 5.1 emphasizes the part of the hysteretic curve that is used for the static fit, where the arrows indicate the direction in which the hysteretic curve have been followed. This part is translated to the origin before the first stage of the fit is carried out. To take the asymmetry into account the value for $F(t)$ at $x(t) = 0$, $F(t)|_{x=x_0}$ is multiplied by $F_2(t)$ before it is subtracted from the measured force. In this way it is guaranteed that the slope of $F_{\text{stat}}$ is equal to the slope of the upper part of the hysteretic curve. Hence the static force $F_{\text{stat}}(t)$, equals

$$F_{\text{stat}}(t) = F(t) - b(\cos(t)) \cdot F|_{x=0}.$$  \hspace{1cm} (5.11)

and equation (5.10) changes into

$$c(t) = F_{\text{stat}}(t) - F_1(t) \cdot F_2(t).$$  \hspace{1cm} (5.12)

Figure 5.2 shows the result for the first identification stage. The resulting static fit has approximately the same slope as the hysteretic curve. This indicates that the identification has performed well.

Figure 5.1: The part of the hysteretic curve used for the static fit

Figure 5.2: Result of the static fit compared with the hysteretic curve
In the second stage the parameters $a$, $\beta$, $\gamma$ and $n$ are estimated. The parameters $k_1$, $k_2$, $k_3$, $b$ and $c$ are fixed to the values, estimated in the first stage. These are sorted in the parameterset $y_{hold}$. The objective function equals equation (5.2).

By identifying the hysteretic parameters the area of the hysteretic curve is obtained. A comparison between the experimental response and the model response after the second stage is depicted in figure 5.3. The experimental and the model response resemble each other well. Still, when $\dot{z}(t) < 0$ for $z(t) > 0$ as well as for $z(t) < 0$ a difference between both responses is present. The result might become better by adding an extra identification stage. Therefore an optional third stage may be added. In the third stage the parameter values obtained after the second stage are used as initial values and subsequently all the parameters are released. Hence, a better fit can be found. Under the assumption that the second stage has performed correctly the resulting hysteretic curve will probably not differ much from the curve obtained after two stages.

The final model response is depicted in figure 5.4. It can be concluded that the model response for the larger part matches its experimental equivalent, although still small differences can be distinguished. Figure 5.5 depicts hysteresis loops for various amplitudes, $A$. When comparing this figure with figure 4.3 it can be seen that the model response and the experimental response show quite good agreement, although especially for small amplitude levels a slight difference occurs. When using the identified parameter set to describe the dynamic behavior this might lead to differences since only small amplitude levels have been used to excite the system in the dynamic experiments.

Table 5.1 lists the parameters of the modified Bouc-Wen model and their corresponding values after three identification stages.

5.3 Conclusion

Various identification efforts have been carried out to estimate the parameters of the modified Bouc-Wen model. Eventually a satisfying identification method is proposed and implemented.
5. Identification of the model

Figure 5.4: Identified model versus measurement

Figure 5.5: Hysteretic curves produced by the identified model

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>1.1241</td>
<td>[-]</td>
</tr>
<tr>
<td>$c$</td>
<td>$0.1557 \times 10^3$</td>
<td>[m/s]</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$295.4195 \times 10^3$</td>
<td>[Nm$^{-1}$]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$73.3932 \times 10^3$</td>
<td>[Nm$^{-1}$]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$45.4206 \times 10^3$</td>
<td>[Nm$^{-1}$]</td>
</tr>
<tr>
<td>$n$</td>
<td>0.1833</td>
<td>[-]</td>
</tr>
<tr>
<td>$k_1$</td>
<td>$39.9015 \times 10^3$</td>
<td>[Nm$^{-1}$]</td>
</tr>
<tr>
<td>$k_2$</td>
<td>$-0.5355 \times 10^6$</td>
<td>[Nm$^{-2}$]</td>
</tr>
<tr>
<td>$k_3$</td>
<td>$0.0290 \times 10^9$</td>
<td>[Nm$^{-3}$]</td>
</tr>
</tbody>
</table>

Table 5.1: Identified model parameters
This three stage estimation scheme can be regarded as a time dependent weighted least squares estimation. In the first stage the static parameters are estimated so the weight fully lies on these parameters. After convergence has been reached in the first stage, the hysteretic parameters are identified in the second stage. During the last stage all parameters are released to optimally tune all parameters. This results in an identified model, which shows good resemblance with its experimental equivalent.

In chapter 6 the results of the dynamic experiments will be presented. In chapter 7 the identified parameters are used to analyse the dynamic behavior of the wire rope spring. Various simulations are carried out to obtain frequency amplitude curves. Eventually, these simulations will be compared with the experimental results to find out whether the modified Bouc-Wen model describes the dynamic behavior of the wire rope spring correctly.
Chapter 6

Dynamic Experiments

In chapter 4 the static experiments have been described. Since much interest lies in the dynamic behavior of the wire rope spring various experiments have been performed at the laboratory of TNO-CMC. First, experiments are carried out upon a electronic shaker to acquire frequency response curves, which will be presented in section 6.1. The wire rope isolator is also designed for shock absorption and hence shock experiments have been performed. For all experiments the springs labeled as log2, lab1 and lab2 have been used.

6.1 Experiments on the shaker

Electronic shaker experiments have been carried out to learn more about the vibrational behavior. The shaker excites the system with a frequency sweep signal. By measuring the response a frequency amplitude curve is obtained.

6.1.1 Experimental Setup

Figure 6.1: The experimental setup on the shaker

Figure 6.2: A wire rope spring on the shaker
Three springs are placed together on the shaker table. A dummy mass, $M$, of 148.4 kg, equally distributed over the three springs, is placed on top of the wire rope springs. Hence, the center of mass of the dummy mass lies exactly above the geometrical center of the equilateral frame formed by the three springs, which is schematically depicted in figure 6.3.

Subsequently, the system can be regarded as a single degree of freedom system and it is sufficient to measure the response directly above the center of mass. The excitation acceleration is measured on the shaker table. For both measurements SCADAS-systems have been used to measure the acceleration. The sample frequency, $f_s$, equals 333.33 Hz. At the maximum frequency, $f_{\text{max}}$, which is 30 Hz, still at least ten points are available to describe one excitation period. Additional experiments with a lower dummy mass are carried out as well.

The frequency sweep is divided into two parts: a sweep-up and a sweep-down. It has been decided to carry out both sweep-ups as sweep-downs to find out whether the system behaves the same during a sweep-up and a sweep-down. During a sweep-up the excitation frequency increases, while the frequency decreases during a sweep-down. A slow frequency sweep of 1 octave/minute is used, implying that during a sweep-up the excitation frequency doubles per minute, see equation (6.1), where $f_0 = 1.6$ Hz for a sweep-up and $f_0 = 30$ Hz for a sweep-down.

$$f = f_0 \times 2^{\frac{t}{60}} \text{ for a sweep-up}$$

$$f = f_0 \times 2^{\frac{t}{30}} \text{ for a sweep-down}$$

While the main application of the wire rope springs lies in the naval field, see section 2.2, the shaker table excites the system with constant velocity amplitudes according to standard regulations [12]. Every experiment is carried out three times to get an indication about the repeatability. Furthermore accidental failures during one sweep can be corrected with the results from another sweep. So if during one sweep something unusual happens or a failure is made, the results from two other sweeps can be used to compare the data and if necessary correct the measurement. It can be concluded that the repeatability seems pretty good since the various responses resemble each other well.

6.1.2 Results of the shaker experiments

A fifth-order low-pass filter with a cut-off frequency, $f_c$, of 35 Hz has been used to filter the collected data, because a signal with a frequency of 50 Hz is superimposed on the measure-
6. Dynamic Experiments

Figure 6.4 shows the absolute magnitude of the FFT, $X(f)$, for a measured response before and after filtering. It confirms that the 50 Hz signal is present and has vanished after filtering.

![Magnitude of the FFT a measured response before and after filtering](image)

Various velocity amplitudes have been used as excitation leading to frequency amplitude curves for the wire rope spring system, shown in figure 6.5.

![Maximum experimental acceleration, $\ddot{x}_M(t)$ for different excitation levels](image)

Figure 6.5 shows the nonlinear dynamic behavior of the wire rope spring. Clearly an amplitude dependent harmonic resonance can be seen. Furthermore, a second peak occurring at a slightly higher frequency, can be distinguished. Possibly, this peak is a second harmonic resonance since all the springs have a slight different behavior. However, this does not seem very reasonable as the difference between both frequencies is quite large. A second explanation can be that the springs have not been placed exactly straight under the mass and it is a
resonance peak of the shear or roll mode, but this does not seem logical given that the stiffness in those directions is smaller than in the tension-compression mode and hence the resonance frequency is expected to occur at a lower frequency, which is also shown by Ni et al. [2]. Another option can be the presence of a $1/k^{th}$ subharmonic response implying that the period time is $k$ times the period time of the excitation. By inspecting the period time of the excitation and the response it has been determined that both period times are the same, which is also confirmed by figure 6.6.

Figure 6.6: Comparison between the shaker excitation and response around 13.6 Hz

Figure 6.7 shows the excitation velocity for $v_{amp} = 20$ mm/s. Although this signal ought to have a constant amplitude it is clear that between the frequencies 8 and 17 Hz this is not true, probably due to the interaction between the mass and the shaker excitation. This might be the reason for the second peak.

Figure 6.7: The excitation signal for $v_{amp} = 20$ mm/s.

Figure 6.6 compares the response and the excitation around 13.6 Hz. The excitation even after filtering still has a strange form. Figure 6.8 depicts the power spectral density, $S_{xx}$, of the excitation signal to see which frequencies are present in the signal. It can be seen that a
6. Dynamic Experiments

A frequency of 27.5 Hz is superimposed on the signal. This presence of this frequency cannot be explained.

For weakly damped nonlinear systems, as e.g. the Duffing system, it is known that there can exist more than one solution at the same frequency. In this case three solutions are found, two stable at the points a and c(-), and one unstable solution (- -), see figure 6.9. Two cyclic-fold bifurcations lead to jumps which implies that the responses for an increasing or decreasing frequency are different.

Next, a closer look is taken at the frequency range from 1.6 till 5 Hz to find out whether superharmonic resonances can be found. The remark must be made that it is very hard to
distinguish a superharmonic resonance. At approximately 5 Hz a peak can be distinguished in figure 6.5, which might be a superharmonic resonance. Figure 6.11, which shows the excitation and response signal around 4.8 Hz, indicates that the small peak can be caused by the excitation. However, it can also be the case that the peak in the excitation is caused by the superharmonic resonance.

Figure 6.11 shows two small resonances at respectively 2.15 and 1.70 Hz, which are both measured during a sweep-down which assures that no transient behavior is present. These might be two superharmonic resonances.

Finally, a comparison is made between the excitation and response signal for \( v_{\text{amp}} = 20 \) mm/s to learn whether the wire rope spring damps the signal. Figure 6.13 shows the maximum values of both signals against the excitation frequency. It can be seen that for frequencies above 15 Hz the wire rope springs show a very good damping performance when loaded with a mass of 50 kg per spring. Due to the resonance frequency the amplitude of the excitation is larger for frequencies below 15 Hz.

Additional measurements are carried out with less mass attached to the system. Figure 6.14 shows the response for three different excitation levels, which qualitatively do not differ.
6. Dynamic Experiments

Figure 6.12: Experimental responses around two different frequencies

Figure 6.13: Comparison between the excitation and response signal
very much from the frequency amplitude curves in 6.5. Again an amplitude dependent harmonic resonance can be distinguished, but the resonance frequency as well as its amplitude has changed in comparison with the original setup. Furthermore the second peak is present as well.

![Frequency Amplitude Curves](image)

Figure 6.14: Experimental shaker response with less mass attached to the wire rope springs

A strange peak can be seen just before the resonance peak. Figure 6.15 shows the excitation and response signal around 11 Hz for $v_{\text{amp}} = 4$ mm/s. Nothing unusual can be seen for the response and the excitation.

![Excitation and Response Signals](image)

Figure 6.15: Measured excitation and response for mass 2 around 11 Hz

A second small peak can be distinguished around 6.1 Hz which may be caused by the excitation as well. However comparing the excitation and response signals does not lead to a result as has been shown in figure 6.11. Both signals are shown in figure 6.16 but nothing unusual can be distinguished for the excitation. So perhaps this might be a superharmonic resonance but it cannot be assured, because figure 6.16 shows that nothing unusual can be distinguished for the response as well.
6. Dynamic Experiments

6.2 Shock experiments

It has been mentioned in section 2.1 that wire rope springs perform as excellent shock absorbers. Therefore also various shock experiments are carried out on a shock table, present at the laboratory of TNO-CMC, to learn more about the shock response behavior of the wire rope spring.

6.2.1 Experimental Setup

The setup of the experiments shows much resemblance with the shaker experiments. The same triangular frame and blocks are used to load the springs. However figure 6.17 makes clear that the separate dummy masses are attached to each other in a different way, because they are not allowed to move with respect to each other, or even worse loose contact with each other. Therefore, the total mass above the springs, $M = 151$ kg, is slightly more than the mass used in the shaker experiments.

Figure 6.16: Excitation and response signal around 6.1 Hz

Figure 6.17: The setup on the shock table

Figure 6.18: A wire rope spring on the shock table

Figure 6.17 shows that the springs are not directly attached to the table, but are placed on
6.2 Shock experiments

a frame, because the area of the table is not large enough. Again it is assured that the center of mass lies exactly in the geometrical center the three springs. The excitation is measured directly on the shock table. It is assumed that the damping in the frame is negligible and the frame is infinitely stiff. The same sensors are used again to measure the acceleration of the table and the dummy mass with a sample frequency of $10^4$ Hz. The whole setup is schematically depicted in figure 6.19.

![Schematic sideview of the experimental shocktable set-up](image)

Figure 6.19: Schematic sideview of the experimental shocktable set-up

6.2.2 Results

Before the data are analyzed a low-pass filter has been used. The cut-off frequency is 100 Hz. In total four shocks are carried out. Figure 6.20 depicts the input for the applied shock. Figure 6.21 depicts the corresponding response. The difference in the amplitude of both vibrations is approximately a factor 38 so it can be concluded that the wire rope spring is indeed an excellent shock absorber.

![Experimental excitation during a shock](image)

Figure 6.20: Experimental excitation during a shock

![Response for a shock experiment](image)

Figure 6.21: Response for a shock experiment

More shocks are carried out but essentially they do not give more information. Figure 6.21 shows that the transient behavior disappears very fast. The resulting frequency is considered. It can easily be seen that this frequency is approximately 10 Hz but gradually increases for a decreasing amplitude.
6. Dynamic Experiments

6.3 Conclusion

Both shock and shaker experiments have been carried out to learn more about the dynamic behavior of the wire rope springs. As expected nonlinear dynamic behavior has been found for the wire rope spring system, since the frequency amplitude curves display a superharmonic resonance and an amplitude dependent harmonic resonance. However, the presence of a second peak, that occurs at a little higher frequency than the resonance frequency cannot be explained yet. Furthermore it can be concluded that above approximately 15 Hz the wire rope spring performs as a good damping device when the wire rope spring is loaded with a dummy mass of approximately 50 kg.

The shock experiments show that wire rope springs are indeed very good shock absorbers, because the difference in amplitude between the excitation and the response is quite large. The first peak of the excitation is approximately a factor 38 larger than the corresponding response.

The experimental results will be compared with simulation results in chapter 7, because simulations have been performed with the modified Bouc-Wen model with the parameter set as identified in chapter 5 to gain insight in the dynamic behavior.
Chapter 7
Simulation results

In chapter 5 the parameters of the modified Bouc-Wen model have been identified. Furthermore, chapter 6 has shown the results of the dynamic experiments as carried out on the electronic shaker and the shock table. In this chapter the simulation results for the dynamic behavior are presented and the results will be compared with the experimental results.

7.1 Setup of the model

![Diagram of a wire rope spring system]

Figure 7.1: Schematic representation of a wire rope spring system

The setup of the dynamic experiments, which is schematically depicted in figure 7.1, has been described in chapter 6. The system consists of a wire rope spring with restoring force $F$ and a mass $M$ on top of it. The system of motion equals

$$M \ddot{x}_M(t) = F(d, \dot{d}, z, t)$$ (7.1)

where the deflection of the spring, $d(t)$, is determined as the difference between the base displacement, $x_T$, and the position of the mass, $x_M$,

$$d(t) = x_T(t) - x_M(t)$$ (7.2)
Again constant velocity amplitude levels have been used to excite the system. Therefore the base velocity, \( \dot{x}_T(t) \) is prescribed according to:

\[
\dot{x}_T = v_{\text{amp}} \cos(\omega t)
\]  

(7.3)

and equation (7.2) and its time derivative change into

\[
d(t) = \frac{v_{\text{amp}}}{\omega} \sin(\omega t) - x_M(t)
\]

\[
\dot{d}(t) = v_{\text{amp}} \cos(\omega t) - \dot{x}_M(t)
\]  

(7.4)

The equation of motion (7.1) can be rewritten as a set of three first order ordinary differential equations with the three state variables defined as \( x_1 = x_M(t) \), \( x_2 = \dot{x}_M(t) \) and \( x_3 = z(t) \).

\[
\dot{x} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} x_2 \\ \frac{1}{M} \left[ b \left( \frac{v_{\text{amp}}}{\omega} \sin(\omega t) - x_1 \right) \right] x_3 + k_1 \left( \frac{v_{\text{amp}}}{\omega} \sin(\omega t) - x_1 \right) + \ldots \\
\ldots k_2 \text{sgn} \left( \frac{v_{\text{amp}}}{\omega} \sin(\omega t) - x_1 \right) \left( \frac{v_{\text{amp}}}{\omega} \sin(\omega t) - x_1 \right)^2 + \ldots \\
\ldots k_3 \frac{v_{\text{amp}}}{\omega} (\sin(\omega t) - x_1)^3 \right] \\
\left( v_{\text{amp}} \cos(\omega t) - x_2 \right) \left[ \alpha - [\beta \text{sgn}(v_{\text{amp}} \cos(\omega t) - x_2)] \text{sgn}(x_3) + \gamma \right] |x_3|^n \]
\]  

(7.5)

This system is equal to the Simulink model as presented in appendix C. Equation (7.5) explicitly depends on time due to the prescribed excitation and therefore it is a non-autonomous system. Because the available software for calculating periodic solutions is for autonomous systems, the non-autonomous system (7.5) has to be transformed into an autonomous system. For this purpose an autonomous oscillator has been used:

\[
\dot{x}_4 = x_4 + \omega x_5 - x_4 \left( \frac{x_4^2}{v_{\text{amp}}^2} + \frac{x_5^2}{v_{\text{amp}}^2} \right)
\]

\[
\dot{x}_5 = -\omega x_4 + x_5 - x_5 \left( \frac{x_4^2}{v_{\text{amp}}^2} + \frac{x_5^2}{v_{\text{amp}}^2} \right)
\]  

(7.6)

When this oscillator is used to replace the excitation, \( x_T \), two extra state variables are introduced, \( x_4 \) and \( x_5 \). The oscillator has an asymptotically stable solution \( x_4 = v_{\text{amp}} \sin(\omega t) \), \( x_5 = v_{\text{amp}} \cos(\omega t) \). Therefore, disregarding transients of (7.6) equation (7.4) can be rewritten as

\[
d(t) = \frac{x_4(t)}{\omega} - x_1(t)
\]

\[
\dot{d}(t) = x_5(t) - x_2(t)
\]  

(7.7)

and the non-autonomous system (7.5) changes into
7. Simulation results

7.2 Long term behavior

In chapter 6 the results of the frequency sweep experiments are presented. Some nonlinear characteristics as an amplitude dependent harmonic resonance are found. In this section two different approaches for obtaining frequency amplitude curves are presented, frequency sweeping using low sweep rates in 7.2.1 and the shooting method in combination with path following techniques (sections 7.2.2 and 7.2.3).

7.2.1 Frequency sweeping

To gain insight into the frequency amplitude curves and the presence of periodic solutions frequency sweeps with constant velocity amplitudes have been performed, which makes it easy to compare the experimental and simulation results. The non-autonomous system (7.5) has been used. It must be noticed that the response is only an approximation of the steady-state behavior. Because the excitation frequency is always changing a periodic solution is never reached, only approximated.

\[
\dot{x} = \begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6 
\end{pmatrix} = \begin{pmatrix}
x_2 \\
\frac{1}{M} [k_1 (\frac{x_4}{\omega} - x_1) x_3 + k_2 (\frac{x_4}{\omega} - x_1)^2 + \frac{k_3}{\omega} (\frac{x_4}{\omega} - x_1)^3] \\
\ldots k_2 \text{sgn}(\frac{x_4}{\omega} - x_1)(\frac{x_4}{\omega} - x_1)^2 + k_3 (\frac{x_4}{\omega} - x_1)^3] \\
(x_5 - x_2) [\alpha - \beta \text{sgn}(x_5 - x_2) \text{sgn}(x_3) + \gamma] |x_3|^\eta \\
x_4 + \omega x_5 - x_4 \left( \frac{x_4^2}{v_{\text{amp}}^2} + \frac{x_3^2}{v_{\text{amp}}^2} \right) \\
-\omega x_4 + x_5 - x_3 \left( \frac{x_4^2}{v_{\text{amp}}^2} + \frac{x_3^2}{v_{\text{amp}}^2} \right)
\end{pmatrix}
\]  
(7.8)

7.2 Absolute acceleration of the mass for various excitation levels

Figure 7.2 shows the absolute acceleration levels for various excitation levels, where at startup the transient behavior is still present. In comparison with figure 6.5 it can be seen that qualitatively the behavior is the same. For larger amplitude levels both responses also
quantitatively match pretty well. However, for smaller amplitudes a large difference occurs between the experimental and the simulation response. The harmonic resonance occurs at a higher frequency in the experiments than in the simulations. Furthermore, the amplitude of the harmonic resonance in the simulations is smaller compared to its experimental equivalent.

![Graph showing excitation and response signals](image1)

**Figure 7.3:** Experimental input and corresponding simulation result

The described sweeps have been performed with a theoretical excitation signal. A measured experimental excitation, see the upper diagram of figure 7.3, has been used in the simulations as well. The diagram also shows that the amplitude of the excitation is not constant between 8 and 17 Hz. The lower diagram of figure 7.3 depicts the corresponding response. Around 16 Hz a second peak is visible, which is enlarged in figure 7.4. From these figures it is clear this is caused by the excitation since this peak is present in the excitation as well as.

![Graph showing excitation and response signals around 16 Hz](image2)

**Figure 7.4:** The excitation and response signal around 16 Hz
7. Simulation results

Still, it is not completely clear why for smaller amplitudes the model does not describe the observed behavior correctly. It seems like the model is damped too heavily for small amplitude levels. Hence, the wire rope spring exhibits a kind of behavior for small amplitudes that cannot be described quantitatively by the modified Bouc-Wen model with the parameter set as identified in chapter 5. Perhaps a separate set of parameters will describe the behavior for very small amplitude levels more accurately. A second solution might be to use an amplitude dependent set of parameters. In the worst case, it may also be that the modified Bouc-Wen model cannot accurately describe the behavior of the wire rope spring for small amplitude levels.

7.2.2 Shooting method

An elegant and fast periodic solution solver is the shooting method as described for instance by Parker and Chua [20] and Leine and Van de Wouw [15]. At each iteration step the shooting method shoots forward to the initial conditions corresponding to the periodic solution. The shooting method finds periodic solutions of the system by solving a two-point boundary value problem in which solutions are sought of

\[ H(x_0, T) = x_T - x_0 = 0 \]  \hspace{1cm} (7.9)

where \( T \) is the period time of the periodic solution and \( x_0 \) is a state on the periodic solution. The zero of \( H(x_0, T) \) is found by a Newton-Raphson procedure. For the details behind the shooting method the reader is referred to Parker and Chua [20].

The final set of equations which has to be solved is

\[
\begin{bmatrix}
\Phi_T(x_0^i) - I & f(x_T^i)
\end{bmatrix}
\begin{bmatrix}
\Delta x_0^i
\end{bmatrix}
= \begin{bmatrix}
x_0^i - x_T^i
\end{bmatrix} \hspace{1cm} (7.10)
\]

where \( \Phi_T \) is the monodromy matrix and \( f(x) \) is the 5-dimensional function (7.8). The shooting method solves in each step the set of equations (7.10) and then updates

\[
\begin{bmatrix}
x_0^{i+1} \\
T_0^{i+1}
\end{bmatrix}
= \begin{bmatrix}
x_0^i \\
T_0^i
\end{bmatrix}
+ \begin{bmatrix}
\Delta x_0^i \\
\Delta T_0^i
\end{bmatrix} \hspace{1cm} (7.11)
\]

The superscripts indicate the iteration count. During the iterations the error must decrease monotonically

\[ e^{i+1} < e^i \] \hspace{1cm} (7.12)

where the error is defined as the norm of the residue

\[ e^i = \|x_T^i - x_0^i\| \] \hspace{1cm} (7.13)

7.2.3 Path following

A disadvantage of the shooting method is that the initial guess has to be close enough to the periodic solution. So one can say that the shooting method does not find a periodic solution but it refines an already good guess of the periodic solution. This more or less also holds for other periodic solution solvers, [15]. However, these solvers can very well be used in combination with continuation techniques to determine branches of periodic solutions.
Once a periodic solution is found, it is interesting how the periodic solution changes when a parameter of the system is varied. The variation of the parameter leads to a branch of solutions in the solutions-parameter space. A commonly used parameter to vary is the frequency. The parameter of interest is called the bifurcation parameter. This process is also called path following.

The most simple form of path following is the sequential continuation method. This method stepwise increases the bifurcation parameter \( \omega \) with small steps \( \Delta \omega \). When the periodic solution at \( \omega = \omega^* \) is known, the periodic solution at \( \omega = \omega^* + \Delta \omega \) may be found by using the solution at \( \omega = \omega^* \) as initial guess. However, problems occur when a branch turns around. Then the solution at \( \omega = \omega^* \) is not a good guess for the next solution at \( \omega = \omega^* + \Delta \omega \). Therefore the shooting method might not converge. Even if the periodic solver converges, a part of the branch is generally not followed, meaning that the shooting method does not follow the solution branch.

An improved method to overcome this problem is the arclength continuation method, which is based upon a prediction step tangent to the branch of solutions, where the tangent is described by \( p \). In the predictor step \( i \), a neighbor solution \( x_{p,i}, \omega_{p,i} \) is predicted on the tangent of the solution path, starting from the known solution \( x_{s,i}, \omega_{s,i} \),

\[
\begin{bmatrix}
  x_{p,i} \\
  \omega_{p,i}
\end{bmatrix} = \begin{bmatrix}
  x_{s,i} \\
  \omega_{s,i}
\end{bmatrix} + \sigma_i \begin{bmatrix}
  p_i^x \\
  p_i^\omega
\end{bmatrix} \tag{7.14}
\]

where \( \sigma \) is a well chosen step size. Subsequently, an iterative correction is used orthogonal to the prediction step.

\[
\begin{bmatrix}
  x_{c,i,m+1} \\
  \omega_{c,i,m+1}
\end{bmatrix} = \begin{bmatrix}
  x_{c,i,m} \\
  \omega_{c,i,m}
\end{bmatrix} - \left[ \left( \frac{\partial H}{\partial x} + \frac{\partial H}{\partial \omega} \left( \frac{\partial H^{-1}}{\partial x} \frac{\partial H}{\partial \omega} \right)^T \right)^{-1} x \right]_c \tag{7.15}
\]

During the correction process the norm of the residue must decrease monotonically. If this condition is violated a new prediction is calculated with a new stepsize \( \sigma_i \). A graphical representation of the predictor-corrector mechanism is depicted is schematically depicted in figure 7.5.

For a full description on path following with arclength continuation the reader is referred to Nayfeh and Balachandran [17] or Leine and Van de Wouw [15]. In this thesis a non-autonomous path following method with arclength continuation is used, where the bifurcation parameter is \( \omega \), the radial frequency. Hence, the non-autonomous system (7.5) has been used.

### 7.2.4 Local stability of periodic solutions.

This section deals with the local stability of periodic solutions, which have been obtained with for instance the shooting method. The stability analysis which is used is called after Floquet. The evolution in time of an infinitesimal small perturbation is considered, thereby making use of a set of linearised differential equations. These linear differential equations are obtained by linearising the nonlinear differential equations around the periodic solution. It can be proven, see e.g. [15] that the eigenvalues, \( \lambda_i \) of the monodromy matrix \( \Phi_T \), which are often called Floquet Multipliers, all have to lie within the unit circle for a stable periodic solution when
dealing with a non-autonomous system. If the system of interest is an autonomous system one Floquet Multiplier is equal to one.

If more than one Floquet multipliers lie outside the unit circle the periodic solution is unstable. Finally, if one or more Floquet multipliers lie on the unit circle and the rest within the unit circle one cannot conclude whether the solution is stable or unstable, since the higher order terms, which have been neglected become more important. The type of bifurcation that occurs depends on the way the Floquet multipliers leave the unit circle. There are three possible scenarios, depicted in figure 7.6. First, a Floquet multiplier leaves the unit circle through +1, which results in a transcritical, a symmetry-breaking or a cyclic-fold bifurcation. Secondly, a Floquet multiplier can leave the unit circle through -1, resulting in a period-doubling bifurcation. Finally, a Neimark bifurcation occurs when two complex conjugate Floquet multipliers leave the unit circle.
7.2.5 Periodic solutions for the wire rope spring

First, periodic solutions are sought by means of the autonomous shooting method, thereby making use of (7.8), to gain insight in the working principle of the method. Furthermore an impression is obtained whether the final periodic solutions are found easily. One result, for \( v_{\text{amp}} = 20 \text{ mm/s} \) at \( f = 3.6 \text{ Hz} \) will be discussed in more detail. At this frequency a peak can be distinguished in figure 7.2. The Floquet multipliers of the periodic solution are equal to 0.0381, 0.4605 + 0.1701i, 0.4605 − 0.1701i, 0.5719 and 0.9998. So the last Floquet multiplier is approximately equal to one because the solution is found for the autonomous system. It can be concluded that the solution is stable. Figure 7.7 depicts the time history plot of the three states \( x_1, x_2 \) and \( x_3 \). Figure 7.8 depicts the various phase projections.

![Time history plot for \( f = 3.6 \text{ Hz} \) and \( v_{\text{amp}} = 20 \text{ mm/s} \)](image)

Figure 7.7: Time history plot for \( f = 3.6 \text{ Hz} \) and \( v_{\text{amp}} = 20 \text{ mm/s} \)

![Phase projections for \( f = 3.6 \text{ Hz} \)](image)

Figure 7.8: calculated phase projections for \( f = 3.6 \text{ Hz} \)

Secondly, the non-autonomous shooting method in combination with path following has been implemented, so periodic solutions are calculated for the non-autonomous system (7.5).
7. Simulation results

The radial frequency, \( \omega = 2\pi f \), is used as bifurcation parameter. For each found periodic solution the maximum displacement of the mass, \( x_1 \), is calculated and depicted. During the process a Runge-Kutta integration scheme has been used. For less accurate calculations, \( \text{tol} = 10^{-6} \), some of the periodic solutions seemed to be unstable. Very small changes in \( \omega \) caused very large changes in the Floquet multipliers, so the accuracy of the solutions was doubted. Therefore, the accuracy of the Runge-Kutta integration scheme is increased by decreasing its absolute tolerance, \( \text{tol} \), from \( 10^{-6} \) to \( 10^{-10} \). After increasing the accuracy of the integration scheme the same simulation is carried out again. Figure 7.9 shows that all unstable solutions have disappeared.

![Figure 7.9: Bifurcation diagram with only stable solutions for the modified Bouc-Wen model for \( \text{tol} = 10^{-10} \)](image)

The unstable solutions can be the result of the nonsmooth character of (7.1). Hence, the sgn-functions in (7.5) are replaced by

\[
\text{sgn}(x) = 1 - \frac{1}{e^{2ax} + 1} \quad \text{(7.16)}
\]

where the quantity \( a \) governs the slope around zero. Figure 7.10 depicts the result for the approximation of \( \text{sgn}(x) \) when \( x = 0.02 \sin(2t) \). The quantity \( a \) is set equal to 1000. It can be seen that the system is smoothed around zero. By substituting (7.16) into (7.5) a smooth approximation of the modified Bouc-Wen model is obtained. Subsequently, path following has been used again to obtain a bifurcation diagram where the initial tolerance of \( 10^{-6} \) for the integration scheme has been used.

![Figure 7.10: Approximation of \( \text{sgn}(x) \)](image)

The resulting bifurcation diagram is depicted in figure 7.11. It shows that for the smooth model only stable solutions have been
found. Subsequently, it can be concluded that indeed the nonsmooth character of the modified Bouc-Wen model causes the numerical inaccuracies which make it necessary to enlarge the accuracy of the numerical integration scheme. However, decreasing the tolerance of the integration scheme certainly increases the computational time so one has to question what is more important: a faster simulation with the smooth approximation of the Bouc-Wen model or the exact calculation, demanding significantly more computational time.

Finally, a comparison is made between frequency sweeping and the shooting method in combination with path following. Both methods have to yield the same result since the same system has been used. Subsequently, the comparison between the experiments and frequency sweeping as made in section 7.2.1 should hold for the shooting method in combination with path following as well. Figure 7.12 shows the maximum response $x_1$ for frequency sweeping.
and for the shooting method in combination with path following for the modified Bouc-Wen model. It confirms that the two responses are the same.

7.3 Shock simulations

In section 6.2 the shock experiments are discussed. The experimental input is used to simulate the shock experiments with the same Simulink model, that has been used for the frequency sweeping. The results of the experiments and the simulations will be compared with each other.

7.3.1 Results and comparison with the experiments

Figure 7.13 depicts the results of the experimental and simulated shock response. The amplitude of the first peak in the simulation is higher than the experimental amplitude. This can be explained as the excitation has been measured directly on the table while the setup has been standing on the frame. Probably the frame has not been stiff enough and the damping cannot be totally neglected.

![Comparison between experimental and simulated shock response](image)

Figure 7.13: Comparison between experimental and simulated shock response

Figure 7.13 shows that the frequencies of the free vibration resemble each other well. They are almost equal and a simple calculation shows that the steady state frequency of the simulated response is around 9.5 Hz. It has been calculated in 6.2 that the experimental frequency is approximately to 10 Hz. To retrieve more information about both signals the magnitude of the FFT for the experimental and simulation response are depicted in figure 7.14.

The difference between the experimental and simulation frequencies with the large magnitude can be distinguished very well. In the simulation these occur at a little higher frequency. Furthermore for higher frequencies the magnitude for the FFT of the simulations is larger, which indicates that the experimental response is damped heavier.

Finally, figure 7.15 depicts the three state variables during the applied shock.
Figure 7.14: Magnitude of the FFT for the experimental and simulation response

Figure 7.15: The three state variables during the shock simulation
7.4 Conclusions

In this chapter various methods have been used to examine the long term behavior of the system. To start frequency sweeps have been performed to obtain frequency amplitude curves, which have been compared with the experimental results. The modified Bouc-Wen model and the experimental results show qualitatively the same behavior. For larger amplitude levels both responses agree well in a quantitative way too, but for smaller amplitudes quite a difference can be distinguished. However, it is not clear whether it is needed to identify a separate set of parameters for very small amplitude levels or that the Bouc-Wen model just cannot describe this behavior. Another approach might be to establish an amplitude dependent set of parameters, although this seems very difficult since not all parameters have a physical meaning.

Next, the shooting method in combination with path following has been used to follow branches of periodic solutions. The Floquet theory is used to investigate the local stability of the final periodic solutions. Due to numerical errors some unstable solutions have been found. By increasing the accuracy of the numerical integration scheme all unstable solutions vanish. The numerical inaccuracies are likely caused by the nonsmooth character of the modified Bouc-Wen model.

The input of the shock experiments has been used to simulate these experiments. It has turned out that although the frequencies of the resulting free vibration show quite good agreement, the peak of both responses differs quite a lot. An explanation can be that the acceleration is measured directly on the shock table while the springs have been placed on a frame, whose flexibility and damping cannot be neglected.
Chapter 8

Conclusions and recommendations

8.1 Conclusions

The goal of this thesis has been to model the quasi-static and dynamic behavior of a wire rope spring in the tension-compression mode. In order to achieve this goal a number of preliminary steps had to be taken.

The wire rope spring exhibits asymmetric hysteretic behavior with softening behavior in compression and hardening stiffness in tension. For small amplitudes the wire rope spring possesses softening behavior which via quasi-linear behavior gradually changes into hardening behavior for increasing amplitudes. Furthermore, hardening overlap in tension has been found. Some typical nonlinear characteristics have been found for the dynamic behavior: an amplitude dependent resonance frequency and various superharmonic resonances. However, wire rope springs are suitable for damping vibrations with frequencies higher than 15 Hz if a wire rope spring is loaded with a dummy mass of 50 kg. Due to the resonance frequency the amplitude of vibrations below 15 Hz is enlarged. Furthermore, shock experiments have confirmed that the wire rope springs are very good shock absorbers.

A modified version of Bouc-Wen model has been used to describe the behavior of the wire rope spring because the Bouc-Wen model itself cannot represent soft-hardening and hardening overlap. Although the Bouc-Wen model is a commonly used model to describe hysteretic behavior since it can represent softening, hardening and quasi-linear behavior, it is not an easily applicable model. First, it is really difficult to understand the physical meaning of some of the parameters. Secondly, a redundant parameter phenomenon is present since different values for $\beta$ and $\gamma$ can lead to almost identical hysteretic curves. Two possible solutions can avoid this redundancy: (1) adding a constraint for $\beta$ and $\gamma$ restricts the possible combinations drastically or (2) adopting an altered version of the Bouc-Wen model in which the parameter $\gamma$ is omitted. However, these scenarios for $\beta$ and $\gamma$ do not yield the same dynamic response. An explanation for this has not been found yet. The dynamic response of the Bouc-Wen model exhibits no subharmonic responses except for large values of $n$.

The identification of all parameters has been a difficult task, because the weight on the various parameters varies a lot during the process. Eventually, a three stage identification strategy has led to satisfying results. The model response and the experimental results are in good agreement, but for small amplitude levels still a difference occurs.

The shock simulations and the experimental shock responses agree qualitatively well. However, the amplitude of the vibration differs. The flexibility and the damping of the frame,
which has been needed to put the total system on the shock table, may be responsible for this, because the excitation, which is also used in the simulations, has been measured directly on the table. The theoretical and experimental frequencies of the resulting free vibration are in good agreement.

Frequency sweeping has provided a first impression about the vibration behavior of the modified Bouc-Wen model with the identified set of parameters. Qualitatively the model response and the experimental results show good agreement. For larger amplitudes both responses resemble each other well in a quantitative way, although it seems that the model response is damped a bit weaker since the number of superharmonic resonances is larger. For small amplitudes both responses quantitatively differ since the model response is damped too heavily. The shooting method in combination with path following has been used to calculate periodic solutions of the modified Bouc-Wen model. The local stability has been determined using Floquet theory. By decreasing the tolerance of the integration scheme all solutions are stable.

For small amplitude levels, both for quasi-static as well as for dynamic behavior, the wire rope spring exhibits a behavior which differs significantly from the behavior for larger amplitude levels. Hence, it seems not possible to describe quantitatively the behavior for all amplitude levels with one set of parameters. The quasi-static experiments and the identification method are routines which have to be implemented in the design process to calculate the response of the wire rope spring system in a certain application or environment. If a priori it is known that the wire rope spring is loaded with large amplitude or small amplitude levels only, the described method will likely lead to good results and will be useful in the design process.

8.2 Recommendations for further research

For further research it is recommended to analyse the small amplitude level behavior of the wire rope spring in more detail since it is yet not totally understood what happens. It cannot be expected that the modified Bouc-Wen model will be able to describe the behavior for all amplitude levels with one set of parameters. Possible solutions are to identify a second set of parameters for small amplitudes only or to try to establish a amplitude dependent relationship for one or more parameters. The latter will be a very difficult task and it is not known whether this yields better results. A third solution might be to develop a model on physical grounds, although it can be questioned whether this is feasible.

The Bouc-Wen model needs to be investigated more in detail as well, because the interaction between \( \beta \) and \( \gamma \) still gives rise to various questions. The main issue might be why two scenarios for \( \beta \) and \( \gamma \) lead to almost identical hysteretic curves but do not yield identical dynamic responses. Additionally the Ozdemir model has to be addressed as well, because omitting the parameter \( \gamma \) means that the redundant parameter phenomenon will vanish too.

During this master thesis project only the behavior in the tension-compression mode has been considered. Since wire rope springs are used in the shear and roll mode as well, it is very logical to analyze the behavior in these modes. A more symmetric hysteretic behavior can be expected.

Eventually, it will be the challenge to analyze the behavior of the wire rope spring when the motion is not restricted to one mode only. This will certainly be more comparable to the real world because it can never be guaranteed that the excitation is in one direction only. A
8. Conclusions and recommendations

Consequence seems that interaction between the different directions will play a role, leading to a serious increase of the calculation time, which really can be a disadvantage. However, if in all directions the model yields good results the calculation tool can be implemented into the design process and it will be possible to calculate beforehand the behavior of the spring, that will be used, to find out whether it is suitable for the task.
8.2 Recommendations for further research
Appendix A

Dimensions of the wire rope spring

The figure below as the data table A.1 are taken from the datasheet for the Socitec MP 14 series polycal wire rope spring [22].

Figure A.1: The dimensions of the wire rope spring

<table>
<thead>
<tr>
<th>Height H [mm]</th>
<th>Width U [mm]</th>
<th>Width V [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>128</td>
<td>128</td>
<td>152</td>
</tr>
</tbody>
</table>

Table A.1: The dimensions of the wire rope spring
Appendix B

Possible combinations for $\beta$ and $\gamma$

In this appendix the combinations for $\beta$ and $\gamma$ that are not addressed in chapter 3 are discussed. In total eight combinations are possible. Since five possible combinations for $\beta$ and $\gamma$ have been discussed, only three combinations are left:

- $\beta + \gamma = 0$ and $\beta - \gamma < 0$
- $\beta + \gamma < 0$ and $\beta - \gamma < 0$
- $\beta + \gamma < 0$ and $\beta - \gamma = 0$

With these combinations for $\beta$ and $\gamma$ simulations have been carried out. The values for $\alpha$ and $n$ are set equal to 1.0, similar as in the other cases. The results are depicted in the figures below. They clearly show that these various combinations do not lead to a hysteretic curve.

![Figure B.1: Response for $(\beta + \gamma) = 0$ and $(\beta - \gamma) < 0$](image)
Figure B.2: Response for $(\beta + \gamma) < 0$ and $(\beta - \gamma) < 0$

Figure B.3: Response for $(\beta + \gamma) < 0$ and $(\beta - \gamma) = 0$
Appendix C

Simulink models

Figure C.1: The model used to simulate the static experiments
Figure C.2: The Simulink model to simulate the dynamic behavior
Appendix D

The Levenberg Marquardt algorithm

In section 5.2 a brief outline is given about the Levenberg-Marquardt algorithm and the update strategy for $\mu$. More details will be discussed in this appendix.

D.1 Comparison of various optimization algorithms

A general framework for an algorithm looks like

\begin{equation}
\begin{aligned}
k &:= 0 \quad y := y_0 \\
\text{repeat} &\\
\text{Find a descent direction } h &\\
y &:= y + \alpha h; \\
k &:= k + 1
\end{aligned}
\end{equation}

until Stop

The simplest method in framework (D.1) is based on the steepest descent direction,

\begin{equation}
h = -E'(y),
\end{equation}

and computing $\alpha$ by line search. This method is robust when $y$ is far away from the solution $y^*$, but has poor final convergence.

In Newton’s method the step is found as the solution to

\begin{equation}
E''(x)h = -E'(y)
\end{equation}

where

\begin{equation}
E''(y) = J_k^T(y)J_k(y) + \sum_{i=1}^{m} e_i e''_i(y).
\end{equation}

This method has quadratic final convergence but it is not robust. Furthermore it requires the implementation of second order derivatives. The Gauss-Newton method is based on the model of the Taylor expansion of $e$. 
The matrix $A = J_k^T J_k$ is symmetric and positive semidefinite. If $J_k$ has full rank, then $A$ is positive definite which implies that $h$ satisfies the descent direction $h^T E'(y) < 0$. By comparison with (D.3) it is shown that if $e(y^*) = 0$, then $A \approx E''$ for $y$ close to $y^*$ and the Gauss-Newton method also has quadratic final convergence, but in general the final convergence is linear. However, the lack of robustness is shared with Newton's method.

The choice for the Levenberg-Marquardt algorithm, (D.6) is then a good choice as it combines the robustness of the steepest descent method with the final convergence of the Gauss-Newton method.

$$
e(y + h) \approx l(h) = e(y) + J_k(y) h,$$
$$E(y + h) \approx L(h) \equiv \frac{1}{2} l(h)^T l(h). \quad \text{(D.5)}$$

D.2 Update strategy for the Marquardt parameter

The update strategy for the Marquardt parameter $\mu$ is presented in subsection 5.2.2. The background behind this update strategy will be presented below as described by Nielsen [18]. First however the initial value for $\mu$ must be determined. Therefore it is reasonable to relate the initial value for $\mu_0$ to the size of the eigenvalues. The maximum of the diagonal elements in the initial Jacobian, $J_0^T J_0$ has the same order of magnitude as $\max(\lambda_i)$, so a simple strategy for choosing $\mu_0$ is given by

$$\mu_0 = \tau * \max \{ J_0(y_0)^T J_0(y_0)_{ii} \} = \tau * \max \{ (A_0)_{ii} \} \quad \text{(D.7)}$$

where $(A_0)_{ii}$ represents the $(i,i)^{th}$ element of the matrix $A_0$. A small value for $\tau\mu$ is used if it is believed that $y_0$ is close to the solution $y^*$.

Basically there are two classes of strategies for updating the Marquardt parameter. One class is related to (D.6). Find $\alpha$ by line search and use information from this to update $\mu$. This update strategy is used in the MatLab Optimization Toolbox implementation leastsq.

The other class of strategies is based on the observation that by the choice for $\mu$ both the direction and the size $h$ are influenced, and the method can be implemented without a proper line search. If $E(y + h) < E(y)$, then $y_{k+1} = y_k + h$, corresponding to $\alpha = 1$ in (D.6). Otherwise, $y_{k+1} = y_k$ and $\mu$ is increased.

However $\mu$ is decreased when $y$ is close to $y^*$. This is decided by the step being so small that $L(h)$ is a good approximation to $E(x + h)$. This can be expressed in terms of the "gain factor"
D. The Levenberg Marquardt algorithm

\[
\rho = \frac{E(y) - E(y + h)}{L(0) - L(h)} \tag{D.8}
\]

From (D.5) and (D.6) it follows that

\[
L(0) - L(h) = -h^T J^T e - \frac{1}{2} h^T J^T J h
\]

\[
= \frac{1}{2} J^T \left[ 2E' + (J^T J + \mu I - \mu I) h \right] \tag{D.9}
\]

\[
= \frac{1}{2} h^T (\mu h - E') .
\]

Since both \( \mu h^T h \) and \(-h^T E'\) are positive it follows that the nominator of (D.8) is positive implying that \( \rho > 0 \) is equivalent with descending condition being satisfied with \( y_k = y, \ y_{k+1} = y + h \). If \( \rho \) is large, the \( \mu \) can be decreased in order to get closer to the Gauss-Newton direction in the next iteration step.

If \( \rho \leq 0 \), then \( y \) is not changed, but \( \mu \) is increased with the twofold purpose of getting closer to the steepest descent direction and reduce the step length. Also if \( \rho > 0 \) but small, it might be better to use a larger damping parameter in the next iteration step. So a simple implementation of this simple update strategy has the following form (D.10), where \( 0 < \rho_1 < \rho_2 \) and \( \beta, \gamma > 1 \).

\[
\begin{align*}
\text{if } \rho < \rho_1 & \text{ then } \mu := \beta \ast \mu \\
\text{if } \rho > \rho_2 & \text{ then } \mu := \frac{\mu}{\beta} \\
\text{if } \rho > 0 & \text{ then } y := y + h
\end{align*} \tag{D.10}
\]

Nielsen [18] proves, although this method with \( \rho_1 = 0.25, \rho_2 = 0.75, \beta = 2 \) ans \( \gamma = 3 \) is good in many cases, that each decrease in \( \mu \) is immediately followed by an increase and the norm of the gradient has a rugged behavior, figure D.1. Hence a new update-strategy is proposed, which will be used during the identification routine, that has the same simplicity as (D.10), but avoids the jumps in \( \mu_{\text{new}} / \mu \) across the thresholds \( \rho_1 \) and \( \rho_2 \). Furtermore, if \( \rho < 0 \) in consecutive steps, then \( \mu \) grows faster.

\[
\begin{align*}
\text{if } \rho > 0 & \text{ then } \\
& y := y + h \\
& \mu := \mu \ast \max \left\{ \frac{1}{\gamma} \left( 1 - (\beta - 1)(2\rho - 1)^p \right) \right\}, \ \nu := \beta \\
\text{else } & \mu := \mu \ast \nu; \ \nu := 2 \ast \nu
\end{align*} \tag{D.11}
\]

with \( \nu \) initialized to \( \beta \) and \( p \) being an odd integer. Table D.1 shows the values for all parameters in the updating strategy.

A comparison between the updating strategies is shown in the figures D.1 and D.2 [18] below. Nielsen shows that not only for this example but in most cases updating strategy (D.11) does indeed perform better.
D.2 Update strategy for the Marquardt parameter

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Table D.1: The values for the parameters in the update strategy for $\mu$

Figure D.1: Marquardt’s method for update strategy D.10 [18]

Figure D.2: Marquardt’s method for update strategy D.11 [18]
Appendix E

Identification method

Section 5.2 addresses the three stage identification procedure used to identify the parameters of the modified Bouc-Wen model. However, the choice for a three stage identification procedure does not seem the most logical one, as Ni et al. [3] show that it is possible to identify all parameters in one stage. Therefore various one stage identification methods have been investigated and tried. However, several problems occurred during the implementation which led to the described identification procedure. The other investigated procedures and the corresponding problems are addressed in this appendix. The Levenberg-Marquardt algorithm is used during all identification loops.

E.1 One stage frequency domain estimation scheme

Ni et al. [1] develop a frequency domain method to estimate the parameters of the Bouc-Wen model. Eventually they use this method to estimate the parameters of the modified Bouc-Wen model [3]. The same approach has been followed.

The input, the displacement, \( x(t) \), is a periodic motion. As a result, the restoring force contains multiple harmonics and can be expressed as equation (E.1), where \( N \) is equal to the number of harmonic components taken into account and \( T \) is the period time [5].

\[
F(t) = \frac{f_0}{2} + \sum_{i=1}^{N} f_i \cos \left( \frac{2\pi it}{T} \right) + \sum_{i=1}^{N} f_i^* \sin \left( \frac{2\pi it}{T} \right)
\]  

(E.1)

The harmonic components \( f_i \) and \( f_i^* \) can be calculated from

\[
f_i = \frac{2}{N} \sum_{i=1}^{N} F_i \cos \left( \frac{2\pi it}{T} \right)
\]

(E.2)

\[
f_i^* = \frac{2}{N} \sum_{i=1}^{N} F_i \sin \left( \frac{2\pi it}{T} \right)
\]

Figure E.1 shows the measured response and the approximation by equation (E.1) when 12 harmonic components are taken into account, as figure E.2 shows the result when 32 harmonic components are taken into account.
E.1 One stage frequency domain estimation scheme

For $N = 12$ this does not lead to good results. Still a large difference between both hysteresis loops can be distinguished, especially around $F(t) = F_{\text{max}}$. The choice $N = 32$ appears to have good resemblance. Figure E.3 takes a closer look around $F(t) = F_{\text{max}}$ for $N = 32$. It can be seen that still a difference between both curves is present and that the approximation has a slight rugged behavior. So $N$ needs to be quite large before a good approximation of $F(t)$ is obtained. Furthermore the calculation of the harmonic components is quite time-consuming.

The algorithm is based upon the Fourier series as it alternates between the frequency and time-domain. The time-domain values for $x(t)$, $F(t)$, $\dot{z}(t)$ and $\dot{F}(t)$ over the period time $T$, are first obtained by the inverse FFT of $F_{\text{FFT}}$ and $x_{\text{FFT}}$. Then the time discrete value for $e(t)_k$, corresponding to $y_k$ is computed. The forward FFT of the time-domain discrete values for $e(t)_k$ yields the values for $e_{\text{FFT}}^k$. Similarly the values for of $J_{\text{FFT}}^k$ are evaluated from the from the forward FFT to the time domain discrete values of $J_k$. Subsequently, the Levenberg-Marquardt algorithm calculates $y_{k+1}$.

This method has been implemented in MatLab. Convergence has not been reached as the parameters stayed very close to their initial values. Figure E.4 shows the values for $\alpha$ during the iteration problem. Clearly the value for $\alpha$ does not change very much, even after 161 iteration steps. Figure E.4 also depicts that also the value for $k_1$ does not change over the iterations. Hence, the method does not seem robust enough for this problem as the values for the various parameters do not vary much from their initial values, even after many iterations steps.
E. Identification method

E.2 One stage identification procedure in the time domain

Since the frequency domain method has not led to good results a one stage time domain least squares estimate scheme is proposed. This method shows very much resemblance with the frequency domain method, only each step is calculated in the time domain. Again a good agreement between the experimental and model response has not been reached. During the iteration process the condition number for \( h = (J_k^T J_k + \mu \ast I)^{-1} (E') \) becomes very large, which indicates a nearly singular matrix. The obtained parameters return a bad response, as in tension a large discrepancy is present and it fails to describe the memory effect, which is a characteristic property of hysteresis.

To retrieve the cause of these accuracy problems each equation of the modified Bouc-Wen model as well as the various combinations have been identified separately. The values, identified by Leenen [14], have been used to simulate the modified Bouc Wen model. The results are used to identify the various parameters. No accuracy problems occurred during the identification for \( z, F_1 \) and \( F_2 \). For various initial guesses convergence has been reached, although more steps have been needed for the identification of the hysteretic parameters.
E.2 One stage identification procedure in the time domain

One possible reason can be that various combinations for \( b \) and \( c \) in equation (3.17) produce the same response. However no accuracy problems occurred during the identification for \( F_2(t), F_2(t) \ast z(t) \) or \( F_2(t) \ast F_1(t) \). Convergence has been reached in all cases although the final values for \( b \) and \( c \) may vary. Figure E.6 depicts the identification result for \( F_2(t) \ast z(t) \). A good agreement can be seen between both hysteretic curves, although very small differences can be distinguished. The value for the parameters during the identification process are shown in figure E.7. It can be seen that \( b \) and \( c \) converge very fast, as the value for the hysteretic parameters \( \alpha, \beta \) and \( \gamma \) hardly changes during the first 25 steps.

Figure E.6: Model response for \( F_2 \ast z \)

Only the case \( F_1(t) + z(t) \) does not lead to good results. Various initial guesses, \( y_0 \), are
taken. A correct $y^*$ can be obtained without accuracy problems. Sometimes correct end values are obtained with accuracy problems. So during the identification stage the condition number becomes too large, but eventually it succeeds to find the correct parameter values. If no correct solution is obtained, the accuracy problems are always present. Figure E.8 shows the model response for two different parameter sets, $y^*$.

![Figure E.8: Two different model responses for $F = F_1 + z$](image)

The left picture clearly shows two identical hysteretic curves. It also shows that the memory effect is described correctly. During the identification process no accuracy problems occurred. However a large difference can be distinguished in the right figure. Furthermore the memory effect is not described correctly. The accuracy problem is present during the identification. The final parameters are listed in table E.1, where the original values are taken from [14]

<table>
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<tr>
<td>$k_3$</td>
<td>0.0022</td>
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Table E.1: Parameter sets for $F = F_1 + z$ that return a correct (left) and an incorrect model response (right).

The values of the parameters during the identification stage are investigated to find the reason for the accuracy problem. Hence, figure E.9 depicts the parameter values during the identification process when no accuracy problems are present. During the first 20 steps only the values for $n$, $k_1$, $k_2$ and $k_3$ change. The stiffness parameters, $k_1$, $k_2$ and $k_3$ approximate their endvalues within 10 percent. However the hysteretic parameters, $\alpha$, $\beta$, $\gamma$ hardly change their value in this stage of the identification process. They start to converge when the stiffness parameters do not change very much anymore. Hence, it seems like the weight upon the parameters changes during the iteration process. First almost all the weight lies upon the stiffness parameters. When $k_1$, $k_2$ and $k_3$ has almost reached their final value the weight is
shifted towards the hysteretic parameters which start to converge towards their final values.

![Graph showing parameter values over iterations](image)

Figure E.9: The parameters value during the identification of $F = F_1 + z$

It also happens that the condition number becomes too large during the identification process but eventually the estimated parameter values are correct. A similar behavior of the various parameters during the identification process can be distinguished in figure E.10. Roughly, accuracy problems are present between iteration steps 150 and 650 and the parameters $\beta$ and $\gamma$ tend towards zero. However the condition number becomes smaller again and eventually the parameters converge towards the correct final value.

![Graph showing parameter values over iterations](image)

Figure E.10: Value for the parameters for $F(t) = F_1(t) + z(t)$ when accuracy problems are present
E. Identification method

Figure E.11 depicts the parameter values of figure E.10 between iteration steps 600 and 700. Indeed it shows that the hysteretic parameters, $\alpha$, $\beta$ and $\gamma$ do not start to change until the stiffness parameters have reached their final value within 10 to 15 percent.

![Graph showing parameter values between iteration steps 500 and 600.](image)

Figure E.11: Parameter value for $F(t) = F_1(t) + z(t)$ between iteration steps 500 and 600

So it can be concluded that in the various cases the hysteretic parameters $\alpha$, $\beta$ and $\gamma$ do not change very much until the stiffness parameters are almost equal to their final values. Hence, it seems a correct approach to use a three stage identification procedure. Other methods might work too but due to time limitations the three stage identification procedure, as described in section 5.2, has been used to identify the parameters of the Bouc-Wen model.
E.2 One stage identification procedure in the time domain
Appendix F

The influence of $\beta$ and $\gamma$ on the modified Bouc-Wen model

Section 3.2.1 addresses the influence of the parameters $\beta$ and $\gamma$ on the hysteretic curve obtained by the Bouc-Wen model. It has been shown that different values for these parameters lead to almost identical hysteresis loops. Since nothing is mentioned how the various combinations for $\beta$ and $\gamma$ influence the modified Bouc-Wen model, two possible combinations for $\beta$ and $\gamma$, listed in table F.1, are investigated.

\[ \beta + \gamma = \text{constant} \quad (F.1) \]

Equation F.1 describes the relationship between both combinations. It must be noticed that scenario I uses the values as identified in chapter 5.

<table>
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<tr>
<td>scenario II</td>
<td>$118.8138 \times 10^3$</td>
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Table F.1: Two combinations for $\beta$ and $\gamma$

From (3.19) it can be concluded that $z$ has a maximum value, independent of $x$. For both scenarios this maximum is equal to 144 N. The slope $\frac{\partial F_1}{\partial x}$ is equal to

\[ \frac{\partial F_1}{\partial x} = k_1 + 2k_2 \text{sgn}(x)x + 3k_3 x^2 \quad (F.2) \]

It can be proven that (F.2) is never equal to zero for any $x \in R$ and that equation (F.2) is monotonically increasing. Hence, for smaller amplitudes the described scenarios can lead to different hysteretic curves if the hysteretic force $z$ is of the same order as $F_1$. The figures F.1 and F.2 depict the hysteretic force $z$ for $x = 25$ and $x = 5$ mm respectively. The corresponding stiffness force $F_1$ can be seen in the figures F.3 and F.4.

These figures confirm that $z$ and $F_1$ have approximately the same value for small amplitudes, $x = 5$ mm and that for larger amplitudes, $x = 25$ mm $F_1$ is much larger than $z$. The resulting hysteretic curves are shown in the figures F.5 and F.6. Indeed for small amplitudes differences can be distinguished as for larger amplitudes the difference between both curves has dissapeared.
Figure F.1: Hysteretic force $z$ for two combinations of $\beta$ en $\gamma$ for $x = 25$ mm

Figure F.2: Hysteretic force $z$ voor twee combinations of $\beta$ en $\gamma$ for $x = 5$ mm

Figure F.3: $F_1$ for $x = 25$ mm

Figure F.4: $F_1$ for $x = 5$ mm

Figure F.5: Hysteretic curve for two combinations of $\beta$ en $\gamma$ for $x = 25$ mm

Figure F.6: Hysteretic curve for two combinations of $\beta$ en $\gamma$ for $x = 5$ mm
Secondly, the influence of $\beta$ and $\gamma$ on the dynamic behavior is investigated in section 3.3. Now, a closer look is taken at the influence of the combination for $\beta$ and $\gamma$ on the dynamic response of the modified Bouc-Wen model. Therefore, frequency sweeps are carried out with a velocity amplitude of 20 mm/s and 4 mm/s.

Figure F.7: Dynamic response for $\dot{x}_T = 20 \cdot 10^{-3} \sin(\omega t)$ for both scenarios

Figure F.7 shows the dynamic response for both scenarios when $v_{amp} = 20 \cdot 10^{-3}$ mm/s. For the second scenario more damping is present as the amplitude of the resonance frequency has decreased. Differences between both responses might be expected because the wire rope spring is excited with small amplitude levels. The same trend has been found for the Bouc-Wen model, because in figure 3.14 the second scenario with a larger $\beta$ and a smaller $\gamma$ leads to a decrease in the amplitude of the resonance frequency.

Figure F.8: The dynamic responses for $\dot{x}_T = 4 \cdot 10^{-3} \sin(\omega t)$

Again the difference in the amplitude of the resonance frequency can be seen. However, in the second case the resonance frequency occurs at a bit higher frequency. So an increase in $\beta$ accompanied with an decrease in $\gamma$ leads to an increase for the resonance frequency and
a decrease in the corresponding amplitude. Due to time limitations the reason for this has not been investigated in more detail.
Appendix G

Time history plots and phase portraits

In section 7.2 the results of the path following method are presented. Numerical errors are present, resulting in unstable solutions. Especially around \( f = 3.0831 \) Hz many unstable solutions have been found with Floquet multipliers lying outside the unit circle.

\[
\begin{array}{ll}
0.0567 + 0.1610i & 0.1583 + 0.2798i \\
0.0567 - 0.1610i & 0.1583 - 0.2798i \\
-1.4857 & 1.1168
\end{array}
\]

Table G.1: Floquet multipliers for two unstable solutions at \( f = 3.0381 \) Hz

Figure G.1: Steady state time plot for \( f = 3.30831 \) Hz

Table G.1 lists the Floquet multipliers for two unstable solutions, both found at \( f = 3.0381 \) Hz. A Floquet multiplier first leaves the unit circle through \(-1\) and at the following
unstable solution the Floquet multiplier leaves the unitcircle through 1.

The time history plots of the three state variables are depicted in figure G.1. Nothing indicates an unstable solution. This is confirmed by figure G.2, which shows the phase projections of the various states.

Figure G.2: The phase projections at $f = 3.0381$ Hz
Bibliography


## Nomenclature

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<td>$J$</td>
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Samenvatting

In dit rapport wordt gekeken naar de modellering van het quasi-statische en dynamische gedrag van een staaldraadveer in de trek- en drukrichting. Een staaldraadveer is een niet-lineaire trillingsdemper, wiens voornaamste toepassing in de scheepvaart ligt. Wanneer de veer belast wordt, wordt er ten gevolge van het langs elkaar heen bewegen van de draden energie opgenomen. Dit verschijnsel wordt hysterese genoemd. Hieraan ontleent de staaldraadveer zijn dempende werking.

Het Bouc-Wen model is een zeer gangbaar model, dat frequent wordt toegepast om systemen met hysterese te beschrijven. Dit model bezit vier parameters, α, β, γ en n. Door een juiste keuze voor de parameters β en γ is het model in staat om softening, quasi-lineair of hardening gedrag te beschrijven. Echter, voor softening gedrag kunnen verschillende combinatories van waarden voor deze twee parameters tot bijna identieke hysterese curves leiden. Door het aanleggen van een constraint kan dit redundant karakter worden verminderd. Een tweede oplossing kan zijn om een aangepaste versie van het Bouc-Wen model te gebruiken waarin de parameter γ is weggelaten.

Door middel van het aanbrengen van quasi-statische cyclische belastingen is echter aangetoond dat het resulterende hysterese karakter niet door het Bouc-Wen model beschreven kan worden. De resulterende hysterese curve heeft namelijk een asymmetrisch karakter met toenemende stijfheid onder trek- en afnemende stijfheid onder drukbelasting. Verder blijkt dat voor kleine amplitudes softening gedrag wordt gevonden dat via quasi-lineair gedrag overgaat in hardening gedrag voor toenemende uitwijkingen. Dit wordt soft-hardening genoemd. Een laatste karakteristieke eigenschap van de hysterese curve is dat voor toenemende amplitudes de afgelegde weg hetzelfde is. Dit wordt hardening overlap genoemd.

Om de waargenomen fenomenen te kunnen beschrijven is een aangepaste versie van het Bouc-Wen model aangenomen. Dit heeft echter tot gevolg dat de set parameters die geïdentificeerd moet worden uit negen elementen bestaat en dus vrij groot is. Samen met het redundant karakter van het Bouc-Wen model zelf heeft dit tot veel problemen geleid tijdens de identificatie van de parameters. Uiteindelijk zorgde een drietraps identificatiedemethode voor bevredigende resultaten. Voor grote uitwijkingen beschrijft het model het werkelijke gedrag vrij goed, maar voor kleine uitwijkingen blijft een afwijking aanwezig.

Een volgende stap is het uitvoeren van simulaties om zo een indruk te krijgen van het dynamische gedrag van zowel het Bouc-Wen model als ook het aangepaste Bouc-Wen model. Hieruit blijkt dat het model een niet-lineaire responsie bezit met een resonantie frequentie, die amplitude afhankelijk is. Daarnaast zijn er diverse superharmonische resonanties gevonden. Zeer opvallend is dat de responsies voor verschillende combinatories van β en γ nu wel duidelijk zichtbare verschillen vertonen. Een verklaring hiervoor is helaas niet gevonden en dit punt verdient zeker meer aandacht.

Aangezien de meeste interesse uitgaat naar het dynamisch gedrag van de staaldraad-
veer zijn er diverse frequentie sweeps uitgevoerd, die resulteren in frequentie amplitude curves. Hieruit volgt dat de staaldraadveer een aantal kenmerkende niet-lineaire dynamische eigenschappen bezit: een amplitude afhankelijke resonantie frequentie, die voor toenemende uitwijkingen naar een lagere frequentie verschuift wat op softening gedrag duidt, en een aantal superharmonische resonantiepieken. Door de grote hoeveelheid hysterese demping zijn er geen subharmonische responsies aanwezig. Verder zijn er diverse schokexperimenten uitgevoerd die aantonen dat de staaldraadveer zeer geschikt is om schokken te dempen. Dezelfde schok simulaties zijn uitgevoerd om het model te vergelijken met de werkelijkheid. De frequenties van de resulterende trilling komen redelijk goed overeen maar het inschakelverschijnsel van beide bewegingen verschilt duidelijk. Een mogelijke reden is het gebruik van een frame om de opstelling op de schokbank geplaatst te krijgen. Er is namelijk vanuit gegaan dat de damping hierin verwaarloosd kan worden en daarom is het excitatiesignaal gemeten op de schoktafel zelf.

Het uitvoeren van frequentie sweeps voor het gemodificeerde Bouc-Wen model met de geidentificeerde parameter set toont aan dat kwalitatief het gedrag hetzelfde is. Voor grotere uitwijkingen komt het gedrag ook kwantitatief vrij goed overeen, alhoewel het lijkt alsof het model iets te licht gedempt is. Voor kleine uitwijkingen zijn er echter duidelijke kwantitatieve verschillen tussen de simulaties en de experimenten. Nu is het model te veel gedempt. Het lijkt er dus op dat de staaldraadveer een gedrag vertoont dat niet door het gemodificeerde Bouc-Wen model kan worden beschreven met maar één set parameters. Mogelijke oplossingen hiervoor zijn het gebruik van een tweede set parameters voor kleine amplitudes of het amplitude afhankelijk maken van één of meerdere parameters. Hieraan dient duidelijk meer aandacht besteed te worden in een mogelijk vervolgonderzoek.

De shooting methode in combinatie met path following is gebruikt om periodieke oplossingen te berekenen. In eerste instantie leidde dit tot diverse instabiele oplossingen ten gevolge numerieke onnauwkeurigheden. Deze worden waarschijnlijk veroorzaakt door het niet gladde karakter van het Bouc-Wen model, aangezien er geen instabiele oplossingen zijn gevonden wanneer een gladde benadering van het Bouc-Wen model wordt gebruikt.
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