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An error on "A generating-function analysis of multiprogramming queues"

by

I.J.B.F. Adan
J. Wessels
W.H.M. Zijm

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AN ERROR NOTE ON
"A GENERATING-FUNCTION ANALYSIS OF MULTIPROGRAMMING QUEUES"

I.J.B.F. Adan*
J. Wessels *
W.H.M. Zijm†

University of Technology, Eindhoven

Abstract. In the paper entitled "A generating-function analysis of multiprogramming queues", Hofri derives explicit representations for some of the queue length probabilities of a multiprogramming model. These representations have the form of infinite sums of powers. In the present note it is demonstrated that these representations are not always correct. It is shown when these representations hold and when not.

I. Introduction

In [5] Hofri presented a generating function analysis of a queueing model of a multiprogramming system (see Figure 1).

![Figure 1: The queueing model for the multiprogramming system](image)

In the queueing model it is supposed that queue III of incoming jobs provides an infinite source of ever available jobs. The multiprogramming system consists of an input-output unit (IO) and a central processor (CP). Incoming jobs start at the IO with an exponentially distributed service
time with parameter $\mu'$. Subsequently, the job leaves the system with probability $p$ and proceeds to queue II at the CP with probability $1-p$. At the CP a job has an exponentially distributed service time with parameter $\mu$. Next the job is recycled to the IO unit where it joins queue I. The IO unit treats the jobs in queue I with nonpreemptive priority with respect to the new jobs in queue III.

The system may be represented by a continuous time Markov process with states $(i, j)$, $i = 0, 1, \ldots$ and $j = 1, 2, \ldots$ where $i$ and $j$ are the lengths of the queues II and I respectively (including the jobs being served). Let $(p_{i,j})$ be the equilibrium distribution of the Markov chain, which will undoubtedly exist if $(1-p)\mu' < \mu$ (see Appendix A in [5]). The determination of this distribution is the main topic of [5]. An important role in the analysis is played by the boundary probabilities $p_{0,j}$ and $p_{i,1}$ for which explicit representations are derived in the form of infinite sums of powers. These representations, however, are partly incorrect in the sense that they do not always hold for small $j$ and for small $i$. In the present note it will be shown where the derivations are incorrect and how more precise statements may be formulated and proved.

Hofri exploits the striking analogy of the Markov process of the multiprogramming system with the Markov process of the shortest queue problem. In doing so he can use a similar approach as Kingman used in [6] for the shortest queue problem. Actually, the incorrectness in Hofri's paper stems from a feature which does not occur in the shortest queue problem, at least not in the symmetric version as treated by Kingman. Recently, another approach has been developed for the shortest queue problem, leading to more explicit representations of the equilibrium probabilities (see [1]). The extension of this new approach to the asymmetric shortest queue problem (see [2]) encounters a similar complication as the one overlooked by Hofri. In fact, also for this new approach, the analogy can be exploited for the analysis of the multiprogramming system (see [3]).

In Section 2 Hofri’s analysis is sketched as far as needed. In Section 3 the analysis is repaired by proving an adapted statement.

2. The generating-function analysis

In this section we sketch the part of Hofri’s analysis which is crucial for our discussion. For the other parts and also for more details, the reader is referred to the extensive treatment by Hofri in [5].

Let $G(z, u)$ be the two-dimensional generating function of the equilibrium distribution $(p_{i,j})$:

$$G(z, u) = \sum_{i=0}^{\infty} \sum_{j=1}^{\infty} p_{i,j} z^i u^j.$$  

Application of the generating-function approach to the equilibrium equations of the Markov
process leads to a functional equation for $G(z, u)$, relating $G(z, u)$ to the boundary values $G(z, 1)$ and $G(0, u)$. Clearly, $G(0, u)$ is the (one-dimensional) generating function of the boundary probabilities $p_{0,j}$, $j = 1, 2, \ldots$. In Hofri's approach the explicit determination of $G(0, u)$ is crucial. He first proves that $G(0, u)$ can be continued to a meromorphic function with simple poles. Then he uses an incorrect version of Mittag-Leffler's Theorem to decompose $G(0, u)$ into partial fractions. From that decomposition he subsequently deduces an explicit representation for $p_{0,j}$ in the form of a series of powers. That representation is partly incorrect, in the sense that it does not always hold for small $j$. In the sequel of this section Hofri's determination of $G(0, u)$ will be outlined in more detail, and it is pointed out what precisely goes wrong in that derivation.

Hofri first shows that $G(0, u)$ is regular in $|u| < \max \{1, \mu'/\mu\}$ and introduces the mapping

$$u = h(\zeta) = a + \phi(\zeta + \zeta^{-1}), \quad (1)$$

where $a$ and $\phi$, which are defined by (45) and (57) in [5], are positive constants. $h(\zeta)$ is a conformal mapping from $|\zeta| > 1$ on the whole $u$-plane, excluding the interval $[a-2\phi, a+2\phi]$. The unit circle $|\zeta| = 1$ is mapped two to one on the interval $[a-2\phi, a+2\phi]$. Next, the number $r > 1$ is determined as the largest number such that $h(\zeta)$ maps the annular $1/r < |\zeta| < r$ into the disk $|u| < \max \{1, \mu'/\mu\}$. By defining

$$\tilde{G}(\zeta) = G(0, h(\zeta)), \quad 1/r < |\zeta| < r,$$

Hofri proves that on the (nonempty) intersection of $1/r < |\zeta| < r$ and $1/r < |\zeta_1| < r$, the function $\tilde{G}(\zeta)$ satisfies

$$\frac{\tilde{G}(\alpha\zeta)}{\tilde{G}(\zeta)} = \frac{\alpha}{\beta} \frac{\zeta - \zeta_1/\alpha}{\zeta - 1/\zeta_1}, \quad (2)$$

where $\alpha$ and $1/\zeta_1$, which are defined by (51) and (66) in [5], are strictly larger than unity and $\beta$ is given by

$$\beta = \frac{\mu - \mu'\alpha}{\mu\alpha - \mu}.$$

Relationship (2) is deduced from the functional equation for $G(z, u)$, and is used to define $\tilde{G}(\zeta)$ over $|\zeta| > 1$ recursively as a regular function, except for simple poles at

$$\zeta = \overline{\zeta}_j = \frac{\alpha_{j-1}}{\zeta_1}, \quad j = 2, 3, \ldots \quad (3)$$

with corresponding residues $\overline{g}_j$. Denoting by $h^{-1}(u)$ the inverse of $h(\zeta)$ from the whole $u$-plane, excluding $[a-2\phi, a+2\phi]$, to $|\zeta| > 1$, it follows that $\tilde{G}(h^{-1}(u))$ is a regular function, except for simple poles at
\[ u = u_j = h(\tau_j) = a + \phi \left[ \frac{\alpha_j^{j-1} + \frac{\zeta_{j1}}{\alpha_j^{j-1}}}{\zeta_{j1}} \right], \quad j = 2, 3, ... \]  

with corresponding residues

\[ g_j = \phi \bar{g}_j \frac{\zeta_{j2} - 1}{\zeta_{j2}}, \quad j = 2, 3, ... \]

Since \( \bar{G}(h^w(u)) \) and \( G(0, u) \) coincide on the interior of the ellipse \( |h^w(u)| = r \), excluding \( [a-2\phi, a+2\phi] \), it follows that \( \bar{G}(h^w(u)) \) is the analytic continuation of \( G(0, u) \) over \( |u| \geq \max\{1, \mu'/\mu\} \). Now we come to the point where the analysis goes wrong.

So far, it has been proved that \( G(0, u) \) can be continued to a meromorphic function over the whole \( u \)-plane with simple poles at the points \( u = u_j \) and corresponding residues \( g_j \), \( j = 2, 3, ... \). In order to obtain expressions for the boundary probabilities \( p_{0,i} \) the meromorphic function \( G(0, u) \) is decomposed into partial fractions. To deduce the partial fraction decomposition, Hofri uses the following (incorrect) special version of Mittag-Leffler's Theorem (see [5], pp. 140):

**If** \( f(z) \) **is a meromorphic function, with a countable set of simple poles** \( a_j \quad (a_j \neq 0) \) **with corresponding residues** \( c_j \), \( j = 1, 2, ... \), **and if for some nonnegative integer** \( n \) **we have that**

\[ \sum_{j=1}^{\infty} \frac{|c_j|}{|a_j|^{n+1}} < \infty, \]  

**then** \( f(z) \) **can be written in the form**

\[ f(z) = f_0 + \sum_{j=1}^{\infty} \frac{c_j}{z-a_j} \]  

It can be easily seen that the version as stated above is incorrect, since, if the integer \( n \) is strictly positive, then condition (6) does not guarantee (although it should do) the convergence of the series in (7) (take \( z = 0 \)). We now outline how Hofri deduces the desired representation for \( p_{0,i} \) from that version of Mittag-Leffler's Theorem (see [5], (118)-(128)). Indeed, Hofri shows that there exists a nonnegative integer \( n \) such that condition (6) is satisfied for \( G(0, u) \), so \( G(0, u) \) can be written in the form

\[ G(0, u) = g + \sum_{j=2}^{\infty} \frac{g_j}{u-u_j} \]  

and since \( G(0, 0) = 0 \), representation (8) can be rewritten as

\[ G(0, u) = \sum_{j=2}^{\infty} \frac{g_j u}{u_j(u-u_j)}. \]
For all \( |u| < 1 \) that representation can be expanded as

\[
G(0, u) = \sum_{j=2}^{\infty} \frac{-g_j u^{j-1}}{u^j (1 - u / u_j)} = \sum_{i=1}^{\infty} u^i \sum_{j=2}^{\infty} \frac{-g_j}{u_j^{i+1}},
\]

from which it can be concluded that for all \( i \geq 1 \)

\[
p_{0,i} = \sum_{j=2}^{\infty} \frac{-g_j}{u_j^{i+1}}.
\]

The partial fraction decomposition (8) is incorrect. As a consequence, representation (9) for the boundary probabilities \( p_{0,i} \) is partly incorrect, in the sense that representation (9) does not always hold for small \( i \).

In the next section a correct version of Mittag-Leffler's Theorem is stated, and it is indicated how that result can be used to derive the partial fraction decomposition. However, we will follow a slightly different approach to decompose \( G(0, u) \) into partial fractions. That approach is based on the Theorem of Residues. Then it is shown when precisely representation (9) for \( p_{0,i} \) holds and when not.

3. How to repair the analysis

We first state a correct version of Mittag-Leffler's Theorem.
According to Carathéodory [4], pp. 225-226, it holds that:

\[
\text{If } f(z) \text{ is a meromorphic function, with a countable set of simple poles } a_j (a_j \neq 0) \text{ with corresponding residues } c_j, j = 1, 2, ..., \text{ and if for some nonnegative integer } n \text{ condition (6) is satisfied, i.e.}
\]

\[
\sum_{j=1}^{\infty} \frac{|c_j|}{|a_j|^{n+1}} < \infty,
\]

\[
\text{then } f(z) \text{ can be written in the form}
\]

\[
f(z) = g(z) + \sum_{j=1}^{\infty} \frac{c_j z^n}{a_j^j (z - a_j)}
\]

where \( g(z) \) is an entire function.

Notice that condition (6) now guarantees the convergence of the series in (10). This condition can of course be satisfied by many choices of \( n \). In particular, if condition (6) holds for \( n \), then it also holds for any integer larger than \( n \). But it is desirable to keep \( n \) as small as possible. The smallest nonnegative integer \( n \) such that condition (6) holds for \( G(0, u) \), can be obtained
from the asymptotic behaviour of the poles $u_j$ and the residues $g_j$ as $j \to \infty$. From (4), we obtain that as $j \to \infty$,

$$u_j = \frac{\phi}{\zeta \alpha} \alpha^j.$$

The asymptotic behaviour of $g_j$ can be obtained from formula (105) in [5], which should read as

$$g_{j+1} = g_j \left( \frac{\alpha^2}{\beta} \frac{\alpha^{j-1} / \zeta - \zeta / \alpha}{\alpha^{j-1} / \zeta - 1 / \zeta} \right). \quad j = 2, 3, \ldots$$

From that equality and (5) it is easy to show that as $j \to \infty$,

$$g_j = C \frac{\alpha^2}{\beta} g_j \left( \frac{\alpha^2}{\beta} \right)^j,$$

where $C$ is given by

$$C = \prod_{k=2}^{\infty} \frac{\alpha^{k-1} / \zeta - \zeta / \alpha}{\alpha^{k-1} / \zeta - 1 / \zeta} > 0.$$

Hence, for any nonnegative integer $n$, as $j \to \infty$,

$$\frac{g_j}{(u_j)^{n+1}} = C \frac{\alpha^2}{\beta} \left( \frac{\zeta \alpha}{\phi} \right)^{n+1} \left( \frac{1}{\beta \alpha^{n-1}} \right)^j \quad (11)$$

So, define

**Definition.**

*Let $m$ be the smallest nonnegative integer such that $1/|\beta| \alpha^{m-1} < 1$.*

**Remark 1.**

It can be shown that $m$ is always strictly positive, and that $m$ is possibly larger than unity. For instance, for $\mu' = 1$, $\mu = 2$ and $p = 3/25$ we obtain from (51) in [5] that $\alpha = 11/4$ and $\beta = -1/6$, so in that case $m = 3$.

Then, from (11), the smallest nonnegative integer $n$ such that

$$\sum_{j=2}^{\infty} \frac{|g_j|}{|u_j|^{n+1}} < \infty,$$

is given by $n = m$, and thus the representation of $G(0, u)$ which is found by Mittag-Leffler's Theorem, is a modification of (8):
\[ G(0, u) = g(u) + \sum_{j=2}^{\infty} \frac{c_j u^m}{u_j^m(u-u_j)}, \]  

(12)

where \( g(u) \) is an entire function. It remains however, to determine \( g(u) \), for which an additional condition is required. If, for instance, it can be shown that \( \lim_{|u| \to \infty} \frac{|g(u)|}{|u|^m} = 0 \),

(13)

then it can be concluded from a simple generalization of Liouville's Theorem (see e.g. Carathéodory [4], pp. 164-166) that \( g(u) \) is a polynomial of degree at most \( m-1 \), whence by representation (12) (notice that \( p_{0,0} = 0 \)),

\[ g(u) = \sum_{k=1}^{m-1} p_{0,k} u^k. \]  

(14)

Instead of trying to establish (13), we will use a slightly different approach to reach (12) and (14). That approach is based on the analysis in § 7.4 in Whittaker and Watson [7], and in fact proves that a slightly weaker condition than (13) is sufficient to conclude to (14).

Let \( E_l \) be the ellipse in the \( u \)-plane corresponding to \( |h^-(u)| = |\xi| = (1+\alpha)\bar{\xi}_l/2 \) for \( l = 2, 3, \ldots \). Since

\[ \bar{\xi}_l < \frac{1+\alpha}{2} \bar{\xi}_l < \bar{\xi}_{l+1}, \]

no ellipse \( E_l \) passes through any poles of \( G(0, u) \).

If \( u \) is not a pole of \( G(0, z) \) and if \( l \) is sufficiently large such that \( E_l \) encloses \( u \), then, since the only poles of the integrand are the poles of \( G(0, z) \) and the point \( z = u \), we have by the Theorem of Residues that

\[ \frac{1}{2\pi i} \int_{E_l} \frac{G(0, z)}{z-u} \, dz = G(0, u) + \sum_{k=2}^{l} \frac{g_k}{u_k-u}. \]  

(15)

But inserting

\[ \frac{1}{z-u} = \frac{1}{z} + \frac{u}{z^2} + \cdots + \frac{u^{m-1}}{z^m} + \frac{u^m}{z^m(z-u)}, \]

yields that

\[ \frac{1}{2\pi i} \int_{E_l} \frac{G(0, z)}{z-u} \, dz = \sum_{k=0}^{m-1} \frac{1}{2\pi i} \int_{E_l} \frac{G(0, z)u^k}{z^{k+1}} \, dz + \frac{u^m}{2\pi i} \int_{E_l} \frac{G(0, z)}{z^m(z-u)} \, dz \]

(16)

\[ = \sum_{k=1}^{m-1} p_{0,k} u^k + \sum_{k=0}^{m-1} \sum_{j=2}^{l} \frac{g_j u^k}{u_j^{k+1}} + \frac{u^m}{2\pi i} \int_{E_l} \frac{G(0, z)}{z^m(z-u)} \, dz. \]

Hence, by (15) and (16),
\[ G(0, u) = \sum_{k=1}^{m-1} p_{0,k} u^k + \sum_{j=2}^{l} \frac{g_j u^m}{u_j^m (u-u_j)} + \frac{u^m}{2\pi i} \int_{E_l} G(0, z) \frac{dz}{z^m(z-u)}. \]  

We now prove that as \( l \to \infty \),

\[ \int_{E_l} \frac{G(0, z)}{z^m(z-u)} \, dz \to 0. \]

**Lemma.** (cf. Lemma 4 in [6])

\[ G_l = \sup_{u \in E_l} \frac{|G(0, u)|}{|u^m|} \to 0 \quad \text{as} \quad l \to \infty. \]

**Proof.**

Since by (1), \( u = k(\zeta) - \phi \zeta \) as \( |\zeta| \to \infty \), it is sufficient to prove that

\[ \bar{G}_l = \sup_{|\zeta|=(1+\alpha)\zeta_l/2} \frac{|\bar{G}(\zeta)|}{|\zeta^m|} = \sup_{0 \leq \theta < 2\pi} \frac{|\bar{G}((1+\alpha)\zeta_l e^{i\theta}/2)|}{|((1+\alpha)\zeta_l e^{i\theta}/2)^m|} \to 0 \quad \text{as} \quad l \to \infty. \]

It holds that

\[ \frac{\bar{G}_l}{G_{l-1}} \leq \frac{\zeta_{l-1}^{m-1}}{\zeta_l^{m}} \sup_{0 \leq \theta < 2\pi} \frac{|\bar{G}((1+\alpha)\zeta_l e^{i\theta}/2)|}{|G((1+\alpha)\zeta_{l-1} e^{i\theta}/2)|}. \]

Inserting that \( \zeta_l = \alpha \zeta_{l-1} \), by (3), and then applying relation (2), yields

\[ \frac{\bar{G}_l}{G_{l-1}} \leq \frac{1}{1 + |\beta| |\alpha^{m-1}|} \frac{(1+\alpha)\zeta_{l-1}/2 + \zeta_l/\alpha}{(1+\alpha)\zeta_{l-1}/2 - 1/\zeta_l}. \]

Hence, since \( 1/|\beta| |\alpha^{m-1}| < 1 \) and \( \zeta_{l-1} \to \infty \) as \( l \to \infty \), there exists a positive number \( R \), strictly less than unity, such that for all \( l \) sufficiently large,

\[ \frac{\bar{G}_l}{G_{l-1}} \leq R, \]

which proves that \( \bar{G}_l \) tends to zero as \( l \) tends to infinity.

**Remark 2.**

In fact, as will be shown below, the Lemma states a weak version of condition (13).

Since \( |u| = \phi(1+\alpha)\zeta_l/2 \) as \( u \) on the ellipse \( E_l \) and \( l \to \infty \), and \( u_i = \phi \zeta_i \) as \( i \to \infty \), it can be shown that for all points \( u \) on the ellipse \( E_l \).
\[
\left| \frac{|u|}{u_i} - 1 \right| \geq \delta > 0,
\]
where \(\delta\) is independent of \(l\) and \(i\). Hence, for all \(l\),
\[
\sup_{u \in E_l} \sum_{i=2}^{\infty} \frac{|c_i|}{u_i^n |u - u_i|} \leq \sum_{i=2}^{\infty} \frac{|c_i|}{u_i^{n+1}} \frac{1}{\delta} < \infty,
\]
so by Lebesgue's Dominated Convergence Theorem,
\[
\sup_{u \in E_l} \sum_{i=2}^{\infty} \frac{c_i}{u_i^n(u - u_i)} \to 0 \quad \text{as} \quad l \to \infty.
\]
Then, by representation (12), the Lemma reduces to
\[
\sup_{u \in E_l} \frac{|g(u)|}{|u|^m} \to 0 \quad \text{as} \quad l \to \infty,
\]
which is slightly weaker than condition (13).

Now as \(l \to \infty\),
\[
\left[ \frac{G(0, z)}{z^m(z - u)} \right] dz = O(G_l),
\]
and so by the Lemma mentioned above, this integral tends to zero as \(l\) tends to infinity. Therefore, letting \(l \to \infty\) in (17), leads to

**Theorem.**

\[
G(0, u) = \sum_{k=1}^{m-1} p_{0,k} u^k + \sum_{j=2}^{\infty} \frac{g_j u^m}{u_j^n(u - u_j)}. \tag{18}
\]

We now show how the partial fraction decomposition of \(G(0, u)\) leads to the desired representation for \(p_{0,i}\). For \(|u| < 1\) we obtain that (notice that \(|u_j| \geq 1\) for all \(j\))
\[
\sum_{j=2}^{\infty} \frac{g_j u^m}{u_j^n(u - u_j)} = -\sum_{j=2}^{\infty} \frac{g_j u^m}{u_j^{m+1}} \sum_{i=0}^{\infty} \frac{u^i}{u_j^i} \\
= \sum_{i=0}^{\infty} u^{m+i} \sum_{j=2}^{\infty} \frac{g_j}{u_j^{m+1+i}},
\]
where changing summations is allowed, since
\[
\sum_{j=2}^{\infty} \sum_{i=0}^{\infty} \frac{|g_j u^{m+i}|}{|u_j|^{m+1+i}} \leq \sum_{j=2}^{\infty} \frac{|g_j|}{|u_j|^{m+1}} \frac{1}{1 - |u|} < \infty.
\]
From (18) and the Theorem, it follows that

**Corollary.**

\[ p_{0,i} = - \sum_{j=2}^{\infty} \frac{g_j}{u_j^{i+1}} \quad \text{for all } i \geq m. \]

The Corollary states that the series (9) (or (130) in [5]) for the boundary probabilities \( p_{0,i} \) holds for all \( i \geq m \) (instead of all \( i \geq 1 \)), and by (11), the series (9) is **divergent** for \( i < m \), and thus cannot represent \( p_{0,i} \) for \( i < m \). A similar remark holds for series (134) in [5] for the boundary probabilities \( p_{i,1} \), in the sense that the series (134) holds for \( i + 1 \geq m \) and not for the smaller \( i \). We do not pursue here to deduce the partial fraction decomposition for the two-dimensional generating function \( G(z, u) \), providing series expressions for \( p_{i,j} \). That analysis is much more complicated than the one for the one-dimensional generating functions \( G(0, u) \) and \( G(z, 1) \), and leads to cumbersome expressions for \( p_{i,j} \). However, in [3] it is shown that explicit expressions for \( p_{i,j} \) can be obtained by using a compensation approach, which is not based on generating-function analysis, but directly applies to the equilibrium equations. In particular, the expressions for \( p_{i,j} \) are valid for those \( i \) and \( j \) satisfying \( i + j \geq m \), and not for smaller \( i \) and \( j \).

**References**

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