Can bad models give good controllers?

Mark Mutsaers
Department of Electrical Engineering
Eindhoven University of Technology
P.O. Box 513, 5600 MB Eindhoven
The Netherlands
E-mail: m.e.c.mutsaers@tue.nl

Siep Weiland
Department of Electrical Engineering
Eindhoven University of Technology
P.O. Box 513, 5600 MB Eindhoven
The Netherlands
E-mail: s.weiland@tue.nl

1 Introduction

In this abstract, we address the question whether it is necessary that models of dynamical systems, which are being used for the design of controllers, need to be of good quality. With a good quality, we mean that the most dominant dynamics of the system are captured in the model description. One would say that a good model, with a good quality, includes all phenomena of the dynamical system, while a bad model, with a bad quality, misses some main dynamics that one normally would like to include. We will discuss the question whether it is possible to synthesize good controllers, when using a bad model for the system. This question resulted from problems in model order reduction, where one tries to approximate a complex model, which has a good quality, with a reduced order model that will capture the dynamics of the original complex model with a high accuracy. When this approximation has this high accuracy, we say that the reduced order model is a good model for the original.

This is the case in most common used reduction strategies, however when one wants to do approximation for control purposes, we will show that this might not be the best approach. Depending on the control objective, it does not have to be the case that the dominant dynamics are relevant for control, and it can even happen that control relevant phenomena are truncated in the reduction step. In this presentation we consider the problem of control relevant model order reduction. This means that reduced order models are judged to be relevant in view of a control objective rather than in view of their open loop properties.

2 Control relevant model reduction

We will be interested in model reduction strategies that do not incur any degradation of optimal control performance. One control objective we will present is finding a controller $\Sigma_C$ that makes the mapping $d \mapsto z$ equal to zero. When $y$ is the full state vector, this is known as a disturbance decoupling problem (DDP). When the system $\Sigma_P$ has the representation

$$\Sigma_P := \{ \dot{x} = Ax + B_1 u + B_2 d, \quad z = H x, \quad y = x \},$$

we know that there exists a controller $\Sigma_C$, which even is a static feedback, that solves DDP if and only if

$$\mathcal{V}^*(A, B_2) \subset \ker H,$$

where $\mathcal{V}^*(A, B_2)$, shortened to $\mathcal{V}^*$, is the largest $(A, B_2)$-invariant subspace of the state space $\mathcal{X}$ [1, 2]. In many cases, this invariant subspace $\mathcal{V}^*$ has a lower dimension than the original state space $\mathcal{X}$. A possible method of reducing the complexity of the system is therefore a projection from $\mathcal{X}$ towards $\mathcal{X}_r = \mathcal{V}^*$, as depicted in Figure 2.

For this reduced system, DDP is still solvable and the resulting controller can be applied to the original system. However, this reduced model does not need to be a “good model” for the original model $\Sigma_P$.

In the presentation we will show that this can also be extended to other problems in geometric control theory as e.g. DDP with partial measurements and pole placement problems.

References