DISPERSION PROPERTIES OF AN ARRAY OF SLOTS ETCHED AT THE INTERFACE BETWEEN TWO INFINITE DIELECTRIC MATERIALS

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Abstract

An array of long slots, oriented along x, etched on a ground plane that separates two infinite dielectric materials constitutes the basic canonical problem investigated in this article. In a single slot structure, the direction of radiation and the attenuation constant could be directly linked to the dominant leaky wave pole \( k_{x_{lw}} \) that dominates the pertinent Green’s Function (GF) spectrum in the longitudinal direction, x. In a multi slot problem, the number of poles that appear in the GF is equal to the number \( N \) of slots composing the array and in view of this two problems arise. First, how to locate efficiently these \( N \) poles in the \( k_x \) spectrum of the GF parameters. Secondly to assess which of these poles are significant, i.e. dominate the behaviour of the array.

1 Introduction

An array of long slots, oriented along x, etched on a ground plane that separates two infinite dielectric materials constitutes the basic canonical problem to analyze the performances of a multi beam leaky wave array that has been recently proposed for use in a sub-mm wave imaging array system. Since the infinite dielectric materials simulate the presence of a dielectric lens it is very important to characterize the radiation from the slots in terms of direction of radiation and attenuation constant. In a single slot structure, these two fundamental parameters could be directly linked to the dominant leaky wave pole \( k_{x_{lw}} \) that dominates the pertinent Green’s Function (GF) spectrum in the longitudinal direction, x. In a multi slot problem, the number of poles that appear in the GF is equal to the number \( N \) of slots composing the array and in view of this two problems arise. First, how to locate efficiently these \( N \) poles in the \( k_x \) spectrum of the GF parameters. Secondly to assess which of these poles are significant, i.e. dominate the behaviour of the array.

A group of pioneers [1],[2] [and therein cited references] introduced a rather general formulation for the Green's function of a class of open waveguide problems based on the assumption of separability of the current functional dependence for the longitudinal and transverse coordinates. These GF’s are the solution of infinite strip lines excited by a given \( \delta \)-gap like impressed electric field. In [3] the theory was also extended to treat the case of two parallel microstrip lines, focusing on the crosstalk. In [4] and [5] a similar formulation was applied to a single infinite slot printed between two semi-infinite dielectric media. In that case the GF was the solution to an impressed \( \delta \)-gap electric current on the slot. This type of slot radiates a leaky-wave beam into the denser dielectric with a dispersion characteristic that is weakly frequency dependent, as suggested by the fact that the transmission line mode is quasi-TEM at low frequency. The theory in [4] and [5] was applied in [6] and [7] to obtain dielectric leaky lens antennas. Bread boards and relevant measurements confirmed, as predicted, the feasibility of an unusual broad-band leaky-wave antenna. A similar theory was also [8] to derive the GF of infinite array of infinitely long slots excited by periodically distributed \( \delta \)-gap sources.

Fig. 1. Slot array radiating in an infinite dielectric medium fed by impressed \( \delta \)-gap electric currents.

In this paper, the GF analysis is extended to array of slots printed between two infinite dielectric half spaces as in Fig. 1. The extension is achieved using a quasi-analytical spectral domain formulation, where the current spectra in the low-frequency regime are obtained by inversion of a closed form matrix, whose dimension is equal to the number of array elements. The paper differs from [8] in that only one current per slot is impressed. This renders continuous the longitudinal part of the spectrum. The propagation along each slot is dominated by the leaky wave contributions rather than by the periodicity of the excitation that would instead provide a dominant Floquet Wave (FW). Moreover, the case of a finite number of slots is also treated.

As illustrated next, the quasi non dispersive property found in
for the leaky-wave of a single slot is also found here for the array configurations, so that the antenna in Fig. 2 is a broad-band, non dispersive, phased array with potential of scanning in the E-plane. Such types of antennas are presently investigated by many industries and research centres for applications that include automotive positioning, ultra wide band point-to-point links, telecommunications, centimetre resolution synthetic aperture radars and through-the-wall imagers. All these applications will soon need increased performance with respect to the present state of the art. In particular, even if the different applications may differ for gain and scanning range requirements, all applications will benefit from the broad bandwidth that the prospected array promises.

Fig. 2. Realistic configuration of an array of N leaky-wave slots fed by microstrip-coupled radial stubs and covered by a wedge-shaped dielectric lens. In the inset: schematic direction of the beam in the H-plane.

2 Methodology for the finite array

In order to derive the equivalent magnetic currents, the width of each slot is considered small in terms of the wavelength so that the separation of variables can be invoked for the space dependence of the current in each slot.

\[ m_i(x',y') = v_i(x') m_i(y' - id_s) \]

The transverse function \( m_i \) is taken so that the edge singular conditions are verified in each slot, and it possesses a closed-form Fourier transform. Using this magnetic current representation and extending the formalism in [4] the CMFIE in spectral domain can be expressed by

\[ D(k_x) V(k_y) = I(k_y) \rightarrow V(k_y) = D^{-1}(k_y) I(k_y) \]

where \( V(k_y) \) is a vector of \( n \) element in which each \( v_i(k_y) \) is the Fourier transform of the unknown magnetic current on the \( i \)-slot, \( I(k_y) \) is the vector of the excitation magnetic fields on each slot and \( D(k_x) \) is a \( n \times n \) matrix that represent the Green’s function that provides the electric field from the electric sources accounting also for the presence of the slots. The generic \( D_{ij}(k_x) \) term of the matrix can be expressed by the convolution between the Green’s function of the layered stratification (in absence of the slots) and the transverse component of the magnetic current. The current in the \( i \)-slot \( v_i(x) \) can then be found as an anti Fourier transform. The advantage of such a procedure is that the slots do not have to be sub-gridded as in most numerical procedures, and the only inversion to be performed is of a matrix of dimension equal to the number of slots composing the array. The inversion is performed for each spectral component \( k_x \).

2 Infinite Array

The starting consideration is that if the array is constituted by an infinite array of long slots, periodically located in the transverse direction \( y \), and excited with progressive phase \( ky_0d_y \), the GF of the problem can be treated analytically and only one pole emerges. This pole can be referred to as the infinite array pole, \( k_{\infty}^{slw} \). The location of this pole depends on the transverse pointing angle of the array, \( \theta = \sin^{-1}(k_{\infty}^{slw}/k) \).

The active GF can then be obtained as extension of the one for finite arrays, where the summations over the \( N \) slots is extended to infinity and then expressed in the spectral domain via Poisson summation formula. The spectrum of the solution is continuous in \( x \) and discrete in \( y \). After the inversion of the Fourier transform, one can express the voltage at every slot by resorting to the periodicity condition.

\[ \text{Im}[k_x] \]

\[ \text{Re}[k_x] \]

Fig. 3. Complex plane topology and main singularities for the infinite array GF in (9). The steepest descent path (SDP) associated to an observation point on the array plane is also depicted.

The complex plane singularities pertinent to the spectrum in are shown in Fig. 3. The spectral Green’s function presents an infinite set of branch points associated to periodic images of the original branches. Fig. 3 refers to a case of periodicity \( d_y \) smaller than a wavelength in the dielectric and the array is pointing broadside; this implies that all branches are located on the imaginary axis. A leaky wave pole, \( k_{\infty}^{slw} \) is located in the bottom Riemann sheet for the branches in ±kS_y (gray area in Fig. 3, and in the top Riemann sheets of all the other branches. When deforming the integration path from the real
axis to the Steepest Descent Path (SDP) associated to the singularity in $k_2$ (shown in Fig.3 for the case of observation points lying on the array plane), the leaky wave pole is captured and its residue accounted as the dominant asymptotic contribution. The SDP deformation also leads to integrations around all the branch cuts associated to the sequence of branch points. The contributions of the branch points integrals represent diffracted cylindrical waves emerging from the $x=0$ axis and exponentially attenuated along $x$. Thus, they are practically negligible. Should the period $d_i$ be larger than $\lambda/2$ or the array be scanned beyond the scan blindness point, some of the further branches could become real, and more significant pole contributions could arise in the GF. These cases will not be considered in this paper since they are associated to arrays which are clearly badly spaced and the interest would be only theoretical. In conclusion, for well sampled arrays only one pole contribution dominates the infinite array GF.

3 Finite Phased Array Dispersion

When the array is finite our formulation accounts for the finite number of slots via the inversion of a spectral matrix. Assuming $N$ the rank of the matrix, for each frequency point, each of the $N$ eigenvalues of the matrix vanishes in correspondence of a complex wave-number $kx$. These wavenumbers identify solutions which can provide a finite field also in absence of an excitation and thus can be referred to as natural modes of the finite array. Alternatively can be referred to as poles of the Green's function of the finite array. It is useful to look at the complex plane topology which can be derived almost analytically using the procedure described in section II.

1) Two elements array: The case in which the array is composed of two slots is considered first. This case is particularly interesting because it is the only case, except the one of the single slot and the infinite array case, in which the solution can be derived completely analytically without resorting to a numerical routine to invert a matrix. The complex topology for this case is shown in Fig. 4 which highlights the location of the two relevant zeroes of the determinant of the matrix. One pole, that we will refer to as the leaky pole, or even pole, $k_x^{\text{leaky}}$, presents a larger imaginary part. The other pole, that we will refer to as the propagating pole, $k_x^{\text{prop}}$, or odd pole, presents a very small imaginary part. The leaky and propagating poles are slightly shifted with respect to the pole characterizing a single slot of equal width in isolation. The shift of the leaky pole is toward larger imaginary values and lower propagation constant for the leaky pole, and in the opposite directions for the propagating pole. Associated to the eigen-values of the 2x2 matrix there are also two eigenvectors whose meaning is simple to interpret. The eigenvector associated to the leaky pole defines an even mode on the structure with both magnetic currents one equal to the other. The eigenvector associated to the propagation mode defines an odd mode with magnetic currents presenting opposite signs. If the array is excited at its two feed points with two equal electric currents the even mode is the only one excited with unitary amplitude and thus the contribution of the even mode on the solution $V_i$ is zero for both $i = 1, 2$. The amplitude of the spectrum in this case is shown in Fig. 4 high-lighting this behavior. The main reason why the two slot case is particularly interesting is that all other cases with $N$ slots can be seen as perturbations of this case.

![Fig. 4. Location of the poles in the complex $kx$ plane in the case of two slots](image)

2) 11 and 25 elements arrays: Fig. 5 presents a zoom of the complex domain plane for the case of an array composed of 11 and 25 slots. As expected we can observe 11 and 25 poles respectively. In both cases the poles are grouped in two sets, with each of these sets localized in a zone close to the one defined by the leaky and the propagation mode in the two slot case.

![Fig. 5. Location of the poles in the complex $kx$ plane in the case of 11 and 25 slots](image)

However it is important to note that all the radiating (leaky) poles are also located along the line, dashed, in the figure pertinent to the 11 slot case, that the unique poles, $k_x^{\text{thr}}$, in the infinite phased array configuration define in the complex plane as a function of the scanning angle, $\theta = \sin^{-1}(k_y/k)$. The poles in Fig 5 have been obtained by just mapping the...
determinant of the matrix in for a range of values of $k_x$. If one is interested in finding the location of these poles rapidly, and also to figure out which of these poles are more important, the previous results suggest that it is legitimate to assume systematically the infinite array solution as starting point in periodically excited finite but large array structures.

4 Concluding Remarks

In the specific problem that has been driving the present investigation, the dominant poles are all characterized by very similar wave-numbers, thus their dominance cannot be assessed only using their location, but only the amplitude of their residue becomes a critical parameter.

To this respect it appears that starting from the infinite array analysis the location of all the $N$ poles can be obtained very accurately. It also appears that in the case of finite array uniformly excited, only the poles localized close to the ones associated to the corresponding infinite array problem are significant and thus should be retained in the evaluation of the Green’s function. The proposed methodology is of general applicability to all those problems which involve multi transmission line problems, even when each of these transmission lines present radiation losses, and thus are more appropriately seen as leaky wave antennas.

REFERENCES


