New Phasor Estimator in the Presence of Harmonics, DC Offset, and Interharmonics

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Abstract—This paper proposes the use of Artificial Neural Networks (ANN) to estimate the magnitude and phase of fundamental component of sinusoidal signals in the presence of harmonics, sub-harmonics and DC offset. The proposed methodology uses a preprocessing that is able to generate a signal that represents the influence of sub-harmonics and DC offset in the fundamental component. This signal is used as input of ANN, which estimate the influence of that signal in quadrature components. Using this information, corrections can be made in quadrature components then the real value of phasor of the fundamental component is estimated. The performance of the proposed algorithm was compared with classical methods such as DFT, using one and two cycles, and LES. The results showed that the proposed method is accurate and fast. The methodology can be used as a phasor estimator of system with poor Power Quality indices for monitoring, control and protection applications.

Index Terms—Phasor Estimation, Harmonics, Interharmonics, DC offset

I. INTRODUCTION

THE estimation of magnitude and phase angle, of current and voltage, of the fundamental component of the system, is essential in digital protection of electric power systems. It is vital that the phasor estimator can only estimate the component of interest, rejecting the unwanted components, such as harmonics, interharmonics (subharmonics), DC offset, and noise [1], [2].

Several studies have been conducted in order to propose estimators that are immune to the influence of harmonics and DC offset [3], [4], [5]. These components significantly affect the performance of the estimator, which introduces errors in several applications of power system such as power quality monitoring, control, and protection applications. This type of signal occurs during the faults in conventional transmission lines [2] in which there is no compensation.

During a fault in the compensated transmission lines [6], [7], the signals of voltage and current are composed by fundamental component, harmonics, DC offsets, subharmonics and noise. In this case, the presence of the subharmonics make the estimation of the fundamental component even more complex. Even in distribution systems, the presence of large nonlinear loads can introduce interharmonics close to the fundamental components, leading the conventional estimators to produce inaccurate estimations.

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In this work is present a methodology to estimate the phasor of the fundamental component in the simultaneous presence of harmonics and interharmonics close to the fundamental component.

II. PROPOSED METHOD

In Figure 1 is shown the block diagram of the methodology. First, two cycles of the input signal (J_1 e J_2) are modulated by sine and cosine, separately, and are then filtered by a full cycle moving average filter [8]. After this process, the first cycle of each signal (transient) is discarded, being used only the second cycle (J_2^C e J_2^S) of the modulated signals.

After finding J_2^C e J_2^S , two processes are performed in parallel. In the first, an arithmetic mean, denoted by $E\{\bullet\}$, is applied to J_2^C e J_2^S in order to help the future process of obtaining the real and imaginary parts, respectively, of the phasor. In the second, by using the windows J_2^C e J_2^S the characteristic signals are obtained and will be the inputs of artificial neural networks. The two processes are closed when the result of both is added and then the real and imaginary parts of the phasor are obtained. The real and imaginary parts of the estimation are obtained using J_2^C and J_2^S , respectively. Then, with the estimations of the real and imaginary parts, the magnitude and phase are generated. Let the signal of interest, presented in (1), composed of the fundamental component, harmonics, interharmonics and DC offset.

$$s(t) = A_1 \cos(\omega_1 t + \varphi_1) + \sum_{k=2}^{N_H} A_k \cos(k\omega_1 t + \varphi_k) + \sum_{i=1}^{N_I} D_i \cos(\omega_i t + \varphi_i) + I_0 e^{-t/\tau_0} \quad (1)$$

where A_1 , A_k , and D_i are the amplitudes of the fundamental signal, the k -th harmonic and the i -th interharmonic, respectively. ω_1 and ω_i are the angular frequencies of fundamental signal and the i -th interharmonic, respectively. The φ_1 , φ_i e φ_k are the phases of the fundamental signal, the i -th interharmonic and the k -th harmonic, respectively. And I_0 and τ_0 are the amplitude and time constant of the DC offset, respectively. The number of harmonics and interharmonics components are given by N_H and N_I , respectively.

A. Input of ANN: Characteristic signal

The inputs of the neural network are found by making the process of modulation of the signal by cosine and sine, denoted by (2) e (3), respectively. Thus, through the modulation property, the frequency components of 60 Hz are relocated to the

Fig. 1. Schematic diagram of the proposed methodology.

zero frequency, becoming the DC components of modulated signals.

$$s_{MC}(t) = \cos(\omega_1 t) \cdot s(t) \quad (2)$$

$$s_{MS}(t) = \sin(\omega_1 t) \cdot s(t) \quad (3)$$

Replacing (1) in (2), and by using some trigonometric identities, we have the equation for the modulated signal by cosine ($s_{MC}(t)$):

$$\begin{aligned} s_{MC}(t) = & \frac{A_1}{2} \cos(\varphi_1) + \frac{A_1}{2} \cos(2\omega_1 t + \varphi_1) + \\ & + \left\{ \sum_{i=1}^{N_I} \frac{D_i}{2} (\cos[(\omega_i + \omega_1)t + \varphi_i] + \right. \\ & + \cos[(\omega_i - \omega_1)t + \varphi_i]) \} + \\ & + \left\{ \sum_{k=1}^{N_H} \frac{A_k}{2} (\cos[(k+1)\omega_1 t + \varphi_k] + \right. \\ & + \cos[(k-1)\omega_1 t + \varphi_k]) \} + I_0 e^{-t/\tau_0} \cos(\omega_1 t) \end{aligned} \quad (4)$$

Replacing (1) in (3), and by using some trigonometric identities we have the equation for the modulated signal by

sine ($s_{MS}(t)$):

$$\begin{aligned} s_{MS}(t) = & -\frac{A_1}{2} \sin(\varphi_1) + \frac{A_1}{2} \sin(2\omega_1 t + \varphi_1) + \\ & + \left\{ \sum_{i=1}^{N_I} \frac{D_i}{2} (\sin[(\omega_i + \omega_1)t + \varphi_i] + \right. \\ & - \sin[(\omega_i - \omega_1)t + \varphi_i]) \} + \\ & + \left\{ \sum_{k=1}^{N_H} \frac{A_k}{2} (\sin[(k+1)\omega_1 t + \varphi_k] + \right. \\ & - \sin[(k-1)\omega_1 t + \varphi_k]) \} + I_0 e^{-t/\tau_0} \sin(\omega_1 t) \end{aligned} \quad (5)$$

After the process of modulation, the signal is filtered using a moving average filter [8]. Through this process, the harmonic components of the signal are filtered and the interharmonics are attenuated. Thus, we find the equations for the Modulated Signal by Cosine Filtered (s_{MCF}) and the Modulated Signal by Sine Filtered (s_{MSF}), presented below:

$$\begin{aligned} s_{MCF}(t) = & \frac{A_1}{2} \cos(\varphi_1) + \\ & + \left\{ \sum_{i=1}^{N_I} \frac{D_i}{2} (B_{i1} \cdot \cos[(\omega_i + \omega_1)t + \varphi_i] + \right. \\ & + B_{i2} \cdot \cos[(\omega_i - \omega_1)t + \varphi_i]) \} + \\ & + I_0 C_1(t) e^{-t/\tau_0} \cos(\omega_1 t) \end{aligned} \quad (6)$$

$$\begin{aligned} s_{MSF}(t) = & -\frac{A_1}{2} \sin(\varphi_1) + \\ & + \left\{ \sum_{i=1}^{N_I} \frac{D_i}{2} (B_{i1} \cdot \sin[(\omega_i + \omega_1)t + \varphi_i] + \right. \\ & + B_{i2} \cdot \sin[(\omega_i - \omega_1)t + \varphi_i]) \} + \\ & + I_0 C_2(t) e^{-t/\tau_0} \sin(\omega_1 t) \end{aligned} \quad (7)$$

where B_{i1} and B_{i2} are constants of attenuation from the full-cycle moving average filter.

Using (6) and (7), we are able to find the Characteristic Signal of Cosine (s_{CC}) and the Characteristic Signal of Sine (s_{CS}), as shown by:

$$s_{CC}(t) = s_{MCF}(t) - E\{s_{MCF}(t)\} \quad (8)$$

$$s_{CS}(t) = s_{MSF}(t) - E\{s_{MSF}(t)\} \quad (9)$$

where $E\{s_{MCF}(t)\}$ and $E\{s_{MSF}(t)\}$ the arithmetic mean of $s_{MCF}(t)$ and $s_{MSF}(t)$, respectively.

Note that the values of s_{CC} and s_{CS} do not depend on the parameters of fundamental component, ie, A_1 and φ_1 . For this reason, they are named characteristic signals, because they represent only the influences of the imperfections in the input signal.

The signal s_{CC} will be used as the input of the neural network related to the signals modulated by cosine, and s_{CS} related to the signals modulated by sine.

B. Targets of ANN

In training of these neural networks, the following targets were considered :

$$Corr_{\cos} = \frac{A_1}{2} \cos(\phi_1) - E\{s_{MCF}(t)\} \quad (10)$$

$$Corr_{\sin} = -\frac{A_1}{2} \sin(\phi_1) - E\{s_{MSF}(t)\} \quad (11)$$

where $Corr_{\cos}$ and $Corr_{\sin}$ are the target of the neural networks related to the cosine and sine modulations, respectively.

C. Estimation of Magnitude and Phase of the Fundamental Component of Signal

With the correction values found by the neural network (the neural network outputs, $\hat{C}_{corr_{cos}}$ e $\hat{C}_{corr_{sen}}$) the equations (10) and (11) are used to find the estimated value of A_1 and φ_1 . In other words, from the neural network outputs and the estimates of $E\{s_{MCF}(t)\}$ e $E\{s_{MSF}(t)\}$, is possible to estimate the parameters of the fundamental component:

$$\begin{aligned} & \left(\hat{C}_{corr_{cos}} + E\{s_{MCF}(t)\} \right)^2 + \\ & + \left(\hat{C}_{corr_{sen}} + E\{s_{MSF}(t)\} \right)^2 = \\ & = \left(\frac{\hat{A}_1}{2} \cos(\hat{\varphi}_0) \right)^2 + \left(-\frac{\hat{A}_1}{2} \sin(\hat{\varphi}_0) \right)^2 = K \end{aligned} \quad (12)$$

Thus, the estimates of amplitude and phase of fundamental component are respectively given by:

$$\begin{aligned} \hat{A}_1 &= 2\sqrt{K} \\ \hat{\varphi}_1 &= \text{tg}^{-1} \left\{ -\frac{\hat{C}_{corr_{sen}} + E\{s_{MSF}(t)\}}{\hat{C}_{corr_{cos}} + E\{s_{MCF}(t)\}} \right\} \end{aligned} \quad (13)$$

III. SIMULATIONS

This section presents the way that the NN was trained, tested and validate. The performance of the NN, in the training and testing phase, was measured by comparing the NN output with their respective target. In the validation phase were used several distorted signals in order to verify the final performance of the method. Two of these cases are shown in this section.

A. Training and Testing of Artificial Neural Networks

For both training and testing were used signals generated from (14), and all parameters were randomly generated, except the amplitude of the fundamental parameter that was kept constant. This is because the proposed method does not depend on the value of the fundamental components, once the corrections are made over the characteristic signals (8) and (9) that represent only the contribution of the all components, except the fundamental one. The sampling rate used was 64 samples per cycle. The sampling rate used was 64 samples per cycle of the fundamental component. Table I presents the variation range of each parameter.

$$\begin{aligned} s(t) &= A_1 \cos(\omega_1 t + \varphi_1) + \sum_{k=2}^7 A_k \cos(k\omega_1 t + \varphi_k) + \\ &+ D_1 \cos(\omega_i t + \varphi_i) + I_0 e^{-t/\tau_0}, \quad 0 \leq t \leq 2/60s \end{aligned} \quad (14)$$

TABLE I
RANGE OF EACH PARAMETER

Parameter	Range
Phases φ_1 e φ_i	0 to 2π rad
Frequencies f_i	10 to 50 Hz
Magnitudes A_i	0 to 30% of A_1
Magnitude I_0	0 to 100 ms
Time Constants τ_0	10 to 100 ms

Note that in (14) there are not harmonic components, because the moving average filter, that will be used in process

of obtaining the Characteristic Signal (neural network inputs), strongly attenuates these components.

Six hundred signals were used for training and another six hundred for testing. The neural network inputs were obtained as described in Subsection II-A.

The best performance of the neural networks were reached using multilayer neural networks [9]. Both were set with 16 neurons in input layer, 16 in the ridden layer and 1 in the output layer. Hyperbolic tangent sigmoid functions and linear functions were used as activation functions in the ridden layer and output layer, respectively. The training algorithm was the resilient backpropagation [10]. The performances were measured relative to the amount of outputs in the testing process, which were below a maximum percentage error, in relation to targets, of 5%. Neural networks related to the cosine and sine modulation obtained performances of 85% and 84%, respectively.

B. Validation Method

In validation, the estimations are made in a process of sliding-window in time using a window of 128 samples (2 cycles of the fundamental component of the signal), this process is shown in Figure 2 and was based on the methodology proposed in [11]. It is noteworthy that the neural networks were trained previously.

Fig. 2. Sliding-window estimation.

Two different cases of simulations will be presented. The first is composed by a signal defined as the sum of fundamental component, harmonics, DC offset, and just one interharmonic. The second similar to the first, but has two different interharmonics.

1) *First Case:* The signal of equation (1) will be used for simulations. Where the fundamental frequency component ($\omega_1 = 60\text{Hz}$) will be considered $A_1 = 5$ pu and $\varphi_1 = \pi/3$. This values were chosen to show that the method is independent of A_1 . Remember that the ANN was trained using $A_1 = 1$ pu. For the interharmonics, will be considered $N_I = 1$, and its parameters are $\omega_{i=1} = 2\pi 18$ rad/s, $D_1 = 1.5$ pu and $\varphi_{i=1} = \pi/4$ rad. For the harmonics will be considered $N_H = 6$, where its parameters are $A_{k=2} = 0,75$ pu, $A_{k=3} = 0.50$ pu, $A_{k=4} = 0.35$ pu, $A_{k=5} = 0.25$ pu, $A_{k=6} = 0.15$ pu, $A_{k=7} = 0.10$ pu and phases chosen randomly between 0 and 2π rad. For the DC offset will be considered $I_0 = 5$ pu and $\tau_0 = 50$ ms. The sampling rate is 64 samples per cycle of fundamental component (60 Hz).

The performances of the proposed method were evaluated against three classical non-parametric methods. They are the one cycle DFT, two cycles DFT, and Least Squares Error (LES). The estimates of amplitude and phase of fundamental

components are shown in Figures 5 and 6, respectively. As can be seen, after the point of convergence (2 cycles), the proposed method has the best performance.

Fig. 3. Magnitude Estimation of the Fundamental Component - First Case.

Fig. 4. Phase Estimation of the Fundamental Component - First Case.

The mean square error of the methods were calculated using the estimated values between the third and seventh cycles, i.e., disregarding the transient (first 2 cycles). These performances are shown in Table II. The results show that the performance of the proposed algorithm is the best among the other methods, both for the estimation of the magnitude as for the phase. It is noteworthy that the errors of the magnitude of the proposed algorithm is about 60 times smaller than the algorithm LES, that presented the second best performance. The worst results were obtained with the use of DFT filters, which illustrates that these filters are severely affected by DC components and interharmonics.

TABLE II
ROOT MEAN SQUARE OF THE MAGNITUDE AND PHASE ESTIMATED - FIRST CASE.

	Magnitude (%)	Phase (%)
Proposed Method	0.179	2.11
One Cycle DFT	42.1	490
Two Cycles DFT	27.2	299
LES	11.0	127

2) *Second Case:* The signal considered in this case is the same as in the first case with the exception of the interharmonic. In this case, two interharmonic components will be considered, i.e., $N_I = 2$. Its parameters are $\omega_{i=1} = 2\pi 20$ rad/s, $\omega_{i=2} = 2\pi 40$ rad/s, $D_1 = 1.5$ pu, $D_2 = 1.5$ pu, $\varphi_{i=1} = \pi/4$ and $\varphi_{i=2} = \pi/6$ rad.

The estimates of amplitude and phase of the fundamental components are shown in Figures 5 and 6, respectively. As can be seen, after the point of convergence (2 cycles), the proposed method has the best performance.

Fig. 5. Magnitude Estimation of the Fundamental Component.

Fig. 6. Phase Estimation of the Fundamental Component.

The mean square error of the methods were calculated using the estimated values between the third and seventh cycles, i.e., disregarding the transient (first 2 cycles). These performances are shown in Table III. The results show that the performance of the proposed algorithm is the best among the other methods, both for the estimation of the magnitude as for the phase. It is noteworthy that the errors of the magnitude of the proposed algorithm is about 20 times smaller than the two cycle DFT, that presented the second best performance. The worst results were obtained using the one cycle DFT.

Is important to note that the performance of the proposed method in the second case was lower than the first. This result is because the proposed method was trained to be immune to the presence of only one interhamonic component.

TABLE III
ROOT MEAN SQUARE OF THE MAGNITUDE AND PHASE ESTIMATED -
SECOND CASE.

	Magnitude (%)	Phase (%)
Proposed Method	2.47	24.80
One Cycle DFT	110.52	1177
Two Cycles DFT	53.73	580
LES	61.11	660

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IV. CONCLUSION

This paper presented a new phasor estimator based on artificial neural networks. The convergence of this estimator is two cycles. The estimator is able to estimate the fundamental component of the signal in presence of harmonics, interharmonics, and DC offset. The proposed method was compared with three classical methods, they are DFT, of one and two cycles, and LES of a cycle. The results showed that the proposed method is efficient when compared with such methods.

The results presented here came from studies that are under development in the field of phasor estimation in presence of harmonics, interhamonics and DC offset distortions. The methodology is being improved to be applied in protection, control and PQ areas. The time of convergence is being investigated to reduce to a single cycle. The method is being investigated to estimate the interharmonic parameters, frequency and magnitude as well.

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