On The Performance of Pre-Transformed Space-Time Block Coded OFDM Systems

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Abstract—In a previous work by Wu et al., it is shown that the performance of the pre-transformed space-time block coded orthogonal frequency division multiplexing (PT-STBC-OFDM) system has much superior performance compared to the normal STBC-OFDM system. In this paper, we present an analytical study on the bit error rate (BER) of the PT-STBC-OFDM system. We derive the noise distribution of PT-STBC-OFDM system and hence, obtain the BER numerically. We also derived two closed-form BER approximations for the PT-STBC-OFDM system at two different SNR regions. We show that by adding the PT operation, the system has better diversity compared to STBC-OFDM system at the medium SNR region. Using linear detection, the diversity of both systems at the high SNR regions are the same. Moreover, we demonstrate that at high SNR, a PT-STBC-OFDM system of transform size $N$ requires $5 \log_{10}(N)$ dB less SNR to achieve the same BER as without PT.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is well suited for high data rate applications in fading channels due to its high spectral efficiency and its ability to transform a frequency-selective fading channel into multiple flat fading channels on different subcarriers. In an OFDM system, data symbols on some subcarriers may be subject to strong attenuation and may not be detected correctly at the receiver. This has motivated research for a more robust transmission scheme combining coded division multiple access (CDMA) and OFDM, where the information symbols are spread across multiple subcarriers by some pre-transformation matrix. Here we refer to such systems as Pre-transformed (PT) OFDM systems or PT-OFDM systems in short. The PT-OFDM system demonstrates better BER performance compared to the conventional OFDM system in the high SNR region due to the spreading property of PT matrix [1]. In [2], it is further shown that the PT-OFDM system possesses other attractive properties such as low peak to average power ratio (PAPR), less spectral regrowth and superior packet error rate (PER) performance compared to a normal OFDM system.

Space time block coding (STBC) is an effective technique to exploit the spatial diversity in multiple input and multiple output (MIMO) systems [3] [4] [5]. For frequency selective fading channels, STBC can be combined with OFDM such that on each subcarrier, there exists effectively a flat fading STBC system [6].

Combining STBC and PT-OFDM makes the system performance more robust in fading channels. Through simulations, it has been shown that such a PT-STBC-OFDM system demonstrates superior BER performance compared to conventional STBC-OFDM system [7].

In this paper, we analyze the BER of a PT-STBC-OFDM system. We first derive the noise distribution of the PT-STBC-OFDM system, from which the BER is obtained through numerical methods. We show that PT-STBC-OFDM system demonstrates better BER performance than the conventional STBC-OFDM systems. The larger the transform size, the larger the performance gain achieved. We then introduce two methods to approximate the BER performance of PT-STBC-OFDM system in different SNR regions, and derive their corresponding closed-form BER expressions. We verified using computer simulations that the approximation matches the actual BER performance closely. From the approximations, we show that PT-STBC-OFDM system has better diversity at medium SNR regions. At high SNR region, the diversity of both systems are the same. However, the BER of the STBC-OFDM system can be reduced by $N$ times by using a size $N$ pre-transform. In terms of SNR gain, we show that the PT-STBC-OFDM system achieves an SNR gain of $5 \log_{10}(N)$ dB.

In this paper, we use the following notations: vectors and matrices are denoted as bold lower and upper case letters respectively. All vectors are column vectors. We use the superscripts $H$ and $*$ to denote matrix Hermitian and complex conjugation. The elements of vectors/matrices are denoted by letters with subscripted indices.

II. SYSTEM DESCRIPTION

Figure 1 shows the transceiver structure of a PT-STBC-OFDM system. The input data bits are mapped to the constellation symbols according to a mapping rule. The signal is then grouped into blocks of size $N$, denoted as $x = [x_1, x_2, \cdots, x_N]$, and pre-transformed to give

$$x_T = Tx,$$  \hspace{1cm} (1)

where $T$ is a unitary pre-transform matrix with constant amplitude elements such that $TT^H = I_N$ and $|t_{i,j}|^2 = 1/N$ for all $i$'s and $j$'s [8]. $I_N$ indicates identity matrix of size $N$. The transformed signal $x_T$ is fed to a symbol interleaver, where different symbols within the same PT block are assigned to different subcarriers and different OFDM symbols such that the channel responses in each PT-block are uncorrelated.
The interleaved data is then STBC coded, modulated using Inverse Fast Fourier Transform (IFFT) and transmitted through different transmit antennas. In this paper, we only consider the simple Alamouti coded 2 transmit and 1 receive antenna system. The results can be extended to other orthogonal space time block codes in a straightforward manner.

The received signal in the frequency domain after FFT can be written, in vector form, as

\[ [r_1, r_2] = \frac{1}{\sqrt{2}} [H_{1,1}, H_{1,2}] \begin{bmatrix} x_{T,1} & -x_{T,2}^* \\ x_{T,2} & x_{T,1}^* \end{bmatrix} + [n_1, n_2]. \]  

Here \( r_1 \) and \( r_2 \) are received signal vectors of length \( N \) at block intervals of 1 and 2 respectively; \( H_{i,j} \) is a diagonal matrix with diagonal element \( h_{i,j}(k) \) indicating the channel frequency response between transmit antenna \( j \) and receive antenna \( i \) at the \( k \)th symbol within the PT block; \( x_{T,i} \) is the \( i \)th transmitted blocks; \( n_i \) is the AWGN noise vector at the \( i \)th block with zero mean and variance \( \sigma^2 \) per dimension. The scaling of \( \frac{1}{\sqrt{2}} \) is used to normalize the transmission power such that the total transmission power from both antennas are maintained as 1.

After STBC combining, the signal is of the following form

\[ \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} |H_{1,1}|^2 + |H_{1,2}|^2 & 0 \\ 0 & |H_{1,1}|^2 + |H_{1,2}|^2 \end{bmatrix} \begin{bmatrix} x_{T,1} \\ x_{T,2} \end{bmatrix} + \begin{bmatrix} h_{1,1}^* \\ h_{1,2}^* \end{bmatrix} \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}. \]  

where \( |H_{i,j}|^2 \) is a diagonal matrix with the \( k \)th diagonal element equal to \( |h_{i,j}(k)|^2 \). As the noise present in \( y_1 \) and \( y_2 \) are statistically the same, we drop the block index in the rest of the paper to simplify the notation.

After equalization and performing the inverse pre-transform, the decision statistic of transmitted signal vector \( x \) can be written as

\[ d = T^H \frac{\sqrt{2}}{|H_{1,1}|^2 + |H_{1,2}|^2} \begin{bmatrix} H_{1,1}^* \\ H_{1,2} \end{bmatrix} \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}. \]  

The signal vector \( d \) is then passed through a hard decision device and a de-mapper to obtain the estimate of the transmitted bits, \( b \).

### III. Equivalent noise distribution

In this section, we derive the noise distributions for the STBC-OFDM system and the PT-STBC-OFDM system. This will help us in the derivation of the BER in the following sections.

#### A. STBC-OFDM system

After equalization, the noise for a conventional STBC-OFDM system, which is equivalent to a PT-STBC-OFDM system with PT matrix \( T = I_N \), can be expressed as

\[ \hat{n} = \frac{\sqrt{2}}{|H_{1,1}|^2 + |H_{1,2}|^2} \begin{bmatrix} H_{1,1}^* \\ H_{1,2} \end{bmatrix} \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}. \]  

The \( k \)th element of \( \hat{n} \) is given by

\[ \hat{n}(k) = \sqrt{2} \left( \frac{h_{1,1}(k)n_1(k) + h_{1,2}(k)n_2^*(k)}{|h_{1,1}(k)|^2 + |h_{1,2}(k)|^2} \right). \]  

In OFDM, the frequency domain channels on different subcarriers are correlated. However, here we assume sufficient symbol interleaving by using the symbol interleaver before STBC, such that the channels within each PT block are independent. In this case, \( h_{1,1}(k) \) and \( h_{1,2}(k) \) are identically independently distributed (i.i.d) for different values of \( k \). Therefore, the distribution of the noise \( \hat{n}(k) \) is identical for all the \( k \) values, and so we omit the index \( k \) in the following derivations.

It can be shown that \( \hat{n} \) has the same distribution as \( \hat{n}^s = \sqrt{2}yn_s \), where \( y = \sqrt{\frac{1}{|h_{1,1}|^2 + |h_{1,2}|^2}} \), and \( n_s \) is an AWGN having the same distribution as \( n_1 \) and \( n_2 \) in (6). We use the superscript \( s \) to indicate STBC system. As the real and imaginary parts of the noise have the same statistical property, in the following study, we will derive the noise distribution in only one dimension. The distribution of \( |h_{1,1}|^2 + |h_{1,2}|^2 \), i.e. \( \frac{1}{\Delta} \) is chi-square with 4 degrees of freedom. Assuming the variance of \( h_{1,1} \) and \( h_{1,2} \) is 1, the probability density function (PDF) of \( y \) can be worked out as

\[ f_y(y) = \frac{1}{2y^5} \exp\left(-\frac{1}{2y^2}\right) u(y), \]  

where \( u(y) \) is the unit step function. The PDF of \( \hat{n}^s \), i.e. the noise of the decision statistic for STBC-OFDM system, can be worked out using the distribution of \( y \) as in (7):

\[ f_{\hat{n}^s}(x) = \frac{3\sigma^4}{(x^2 + 2\sigma^2)^{3.5}} \]  

where \( \sigma^2 \) is the variance of the AWGN noise \( n_1 \) and \( n_2 \) per dimension.
B. PT-STBC-OFDM system

For PT-STBC-OFDM system, the noise term in (4) for \(d(k)\) is given by

\[
\tilde{n}_{pt}^{t} (k) = t_k^H \begin{bmatrix} \sqrt{2} \left[ H_{1,1}^* \right] \end{bmatrix} \left[ \begin{array}{c} n_{1,t}^* \n_{2,t}^* \end{array} \right] \]

\[
= \sum_{i=1}^{N} \sqrt{2} \left( t_{k,i}^* \sigma_{n1,i}^* n_{1,i} + h_{1,i}^* n_{2,i} \right) \]

\[
= \sum_{i=1}^{N} \sqrt{2} \left( \begin{array}{c} \sum_{i=1}^{N} \left( \frac{\tilde{n}_{i}^*}{\sqrt{N}} \right) \end{array} \right), \tag{9}
\]

where \(t_k\) is the \(k\)th column of PT matrix \(T\), \(\phi_{k,i}\) is the phase of \(t_{k,i}\), which does not affect the distribution of \(n_i\), and superscript \(pt\) indicates pre-transform. Define \(n_{sab}^{t} = \frac{\tilde{n}_{i}^*}{\sqrt{N}}\), the noise of PT-STBC system is, therefore, a sum of \(N\) random variables, with the PDF given by

\[
f_{n_{sab}^{t}} (x) = \sqrt{N} f_{\tilde{n}_{i}^*} (\sqrt{N} x) = \frac{3\sqrt{N} \sigma^4}{N e^{-2} + 2 \sigma^2 x^2}. \tag{10}
\]

If we assume sufficient symbol interleaving such that the channels within one block are independent, the PDF of \(\tilde{n}_{pt}^{t}\) is the convolution of the PDF’s of \(n_{i}^{sab}\)’s on all the symbols within the block.

Calculation of the PDF for \(\tilde{n}_{pt}^{t}\) by convolving the PDF’s of all the \(n_{i}^{sab}\)’s does not result in a closed-form expression. It is simpler to obtain the characteristic function of \(\tilde{n}_{pt}^{t}\), which is the product of the characteristic functions of all \(n_{i}^{sab}\)’, as follows:

\[
\Phi_{\tilde{n}_{pt}^{t}} (\omega) = \int_{-\infty}^{\infty} f_{\tilde{n}_{pt}^{t}} (x) \exp (j\omega x) dx
\]

\[
= \prod_{i=1}^{N} \left[ \int_{-\infty}^{\infty} \sqrt{N} f_{\tilde{n}_{i}^*} (\sqrt{N} x_i) \exp (j\omega x_i) dx_i \right]
\]

\[
= \prod_{i=1}^{N} \left[ \int_{-\infty}^{\infty} \frac{3\sqrt{N} \sigma^4 \exp (j\omega x_i)}{N (x_i)^2 + 2 \sigma^2 x_i^2} dx_i \right]. \tag{11}
\]

Direct evaluation of (11) does not lead to a closed-form solution. We, therefore, resort to numerical method to obtain the characteristic function of the noise PDF for PT-STBC-OFDM system. The PDF of the noise can then be obtained by inverse Fourier transform of the characteristic function.

IV. BER PERFORMANCES OF STBC-OFDM AND PT-STBC-OFDM SYSTEMS

Based on the noise distribution of STBC-OFDM system in (8), we can derive the error probability for a QPSK modulated STBC-OFDM system. The BER is given by

\[
P_e (\gamma) = \frac{1}{2} \left( 1 - \sqrt{\gamma (\gamma + 3)} \right) \tag{12}
\]

where \(\gamma = \frac{E_s}{2\sigma^2}\) indicates the signal to noise ratio (SNR) per receive antenna. Here, we have adopted a different approach in deriving the BER of STBC-OFDM systems compared to the conventional method. In the conventional method, the BER of a particular channel realization is first calculated and then such BER is averaged over the distribution of the channel. In our approach, we first calculate the effective noise distribution of the decision statistic, which incorporated the channel distribution, and then obtain the BER from this effective noise distribution directly. It can be shown that the BER expression obtained in (12) is equal to \(\frac{1}{2} \left( \frac{1}{\sqrt{\gamma}} - 1 \right)^2 \left( 2 + \frac{1}{\sqrt{\gamma}} \right)\), which is the BER obtained using conventional method. The reason we adopt this BER derivation approach is that it simplifies the BER and BER approximation derivations for PT-STBC-OFDM systems subsequently. Moreover, it also gives more insight on the performance behavior of PT-STBC-OFDM systems.

Similarly, the BER of PT-STBC-OFDM system for different transform sizes \(N\) can also be obtained from the noise PDF through numerical methods. Figure 2 shows the BER performance of PT-STBC-OFDM systems for different transform sizes in comparison with the STBC-OFDM systems. We can see that in the medium SNR region, PT-STBC-OFDM system demonstrates much better performance compared to STBC-OFDM system. Moreover, at medium SNR, the slope of BER against SNR curve of PT-STBC-OFDM is also steeper than the STBC-OFDM. It is observed that the larger the transform size, the larger the slope, and hence the larger the diversity gain. The performance gain over STBC are 2.7dB, 5.5dB and 8.1dB for transform sizes of 4, 16 and 64 at BER=10^-5. At high SNR region, the diversity gain of both STBC-OFDM and PT-STBC-OFDM system becomes the same, which is determined by spatial diversity of the system. However, the PT-STBC-OFDM system still performs better in terms of SNR gain. For all SNR regions, better performance is observed for larger transform sizes.

![Fig. 2. BER of PT-STBC-OFDM system for different transform sizes (QPSK modulation).](image)
From Figure 2, we can see that the BER of a PT-STBC-OFDM system consists two regions. In medium SNR region, the slope of the BER versus SNR curve increases with increasing PT size. At high SNR region, the slope becomes parallel with the slope of normal STBC system. We will, in the following, derive the BER approximation for these two SNR regions separately.

From (9), the noise in PT-STBC-OFDM system is a sum of \( n_{\text{sub}} \)'s. The mean of \( n_{\text{sub}} \) can be straight-forwardly shown to be 0. The variance of \( n_{\text{sub}} \) can be calculated by

\[
\sigma^2_{n_{\text{sub}}} = \int_{-\infty}^{\infty} x^2 \frac{3\sqrt{N}\sigma^4}{(N x^2 + 2\sigma^2)^2} \, dx
\]

where \( \sigma^2 \) is the variance of the AWGN noise in (2). Let \( x = N\sigma \tan \theta \), then \( dx = N\sigma \sec^3 \theta \, d\theta \). Substituting variable \( x \) with variable \( \theta \), we transform the original integration problem to trigonometry integration and the variance of \( n_{\text{sub}} \) can be obtained as

\[
\sigma^2_{n_{\text{sub}}} = \frac{3\sigma^2}{2N} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\tan^2 \theta}{\sec^3 \theta} \, d\theta = \frac{1}{N} \sigma^2. \quad (14)
\]

According to Central Limit Theorem, the distribution of \( \tilde{n}_{\text{pt}} \), which is a sum of \( N \) i.i.d. random variables with finite variance, is well approximated by a Gaussian distribution with zero mean and variance equal to \( \sigma^2 \) when \( N \) is sufficiently large.

Figure 3 depicts the distribution of the noise \( \tilde{n}_{\text{pt}} \) of PT-STBC-OFDM system for different transform sizes \( N \). Also shown in the figure is the PDF of Gaussian distributed random variable with the same variance. It could be seen from the figure that when the transform size \( N \) increases, the distribution of the noise of PT-STBC-OFDM system becomes more and more Gaussian-like. It is interesting to look at the noise distribution of PT-OFDM system where STBC is absent in comparison. In PT-OFDM system, the noise is also a sum of \( N \) i.i.d. random variables \( n_{\text{stbc}} \), where \( n_{\text{stbc}} \) denotes noise in OFDM system. However, the variance of each term in the summation is infinite because of the use of zero-forcing detection. As a result, central limit theorem is no longer applicable.

Using the Gaussian assumption, for QPSK modulated signal, the BER of PT-STBC-OFDM system can be worked as

\[
P_e(\gamma) = Q \left( \sqrt{\frac{E_x}{2\sigma^2}} \right), \quad (15)
\]

where

\[
Q(y) = \frac{1}{\sqrt{2\pi}} \int_{y}^{\infty} \exp(-\frac{x^2}{2}) \, dx. \quad (16)
\]

Figure 4 shows the accuracy of Gaussian approximation in predicting the BER of PT-STBC-OFDM system. We can see that the BER obtained by Gaussian approximation provides a good estimate of the actual BER of PT-STBC-OFDM system in medium SNR regions, especially for systems with large transform sizes. However, the Gaussian approximation becomes invalid at the high SNR region. This could be explained by looking at the mini-figure in Figure 3. The mini plot shows the tail portion, which affects the performance in high SNR regions, of noise distribution in PT-STBC-OFDM system in comparison with Gaussian distribution. We can see the noise in PT-STBC system significantly differs from Gaussian distribution, especially for small \( N \). This explains the invalidity of Gaussian approximation in high SNR regions.

To get a good approximation of the BER of PT-STBC-OFDM system in high SNR region, we make the following assumption. When SNR is high, the total noise is dominated by the largest noise in the summation, i.e.

\[
\tilde{n}_{\text{pt}} = \sum_{i=1}^{N} \sqrt{2n_i} \\sqrt{\frac{1}{N} \sqrt{\frac{1}{N} [h_{1,1}(i)]^2 + [h_{1,2}(i)]^2}} \approx \sqrt{2n_{m_1}} \\sqrt{\frac{1}{N} \sqrt{[h_{1,1}(m_1)]^2 + [h_{1,2}(m_1)]^2}} = n_{\text{sub}} \quad (17)
\]

where we rank the noise in one PT block such that

\[
|n_{m_1}| \geq |n_{m_2}| \geq \cdots \geq |n_{m_N}|.
\]

and \( m_i(i=1,2,\cdots,N) \) is the subcarrier index.

Define \( n_{LB} = n_{m_1} \), obviously the noise power of \( \tilde{n}_{\text{pt}} \) is lower-bounded by \( n_{LB} \), i.e. \( |n_{LB}|^2 \leq |\tilde{n}_{\text{pt}}|^2 \). Using order statistics, the PDF of \( n_{LB} \) can be obtained as [9]

\[
f_{[n_{LB}]}(x) = \frac{N!}{(N-1)!} F^{-1}_{[n_{LB}]}(x) f_{[\xi_{+}]}(x), \quad (19)
\]

where \( f_{[\xi_{+}]}(x) \) is PDF of \( |\xi_{+}| \) which is given by

\[
f_{[\xi_{+}]}(x) = \frac{6\sqrt{N}\sigma^4}{(N x^2 + 2\sigma^2)^{2.5}}, \quad (20)
\]

and \( F_{[\xi_{+}]}(x) \) denotes the cumulative distribution function (CDF) of \( |\xi_{+}| \) and can be worked out to be

\[
F_{[\xi_{+}]}(x) = \frac{\sqrt{N} x (N x^2 + 3\sigma^2)}{(N x^2 + 2\sigma^2)^{1.5}}. \quad (21)
\]
The PDF of $|n_{LB}|$ can thus be worked out as
\[
f_{|n_{LB}|}(x) = \frac{6\sigma^4 N^{2+1} N^{-1} (x(Nx^2 + 3\sigma^2))}{(N^2 + 2\sigma^2)^{1.5N^2+1}}.
\] (22)

Direct evaluation of low bound for BER of PT-STBC system by integrating (22) is difficult. Since we are only interested to evaluate the BER of PT-STBC system in high SNR regions, it is reasonable to assume $F_{|n_{LB}|}(x) \approx 1$ in (19). Therefore (19) can be simplified to
\[
f_{|n_{LB}|}(x) \approx \frac{6N^{2.5}}{2^{2.5}} \frac{\sqrt{N\sigma^4}}{(Nx^2 + 2\sigma^2)^2.5}.
\] (23)

Therefore, the BER of PT-STBC system of transform size $N$ at high SNR region can be approximated by
\[
P_e(\gamma, N) \approx \frac{N}{2} \left( 1 - \frac{\sqrt{N\gamma(N\gamma + 3)}}{(N\gamma + 2)^{1.5}} \right)
\] (24)

To get some insight on the SNR, BER relationship and the transform size $N$ and BER relationship, we use Maclaurin series with argument $\frac{1}{\gamma}$ to expand the BER expression in (24),
\[
P_e\left(\frac{1}{\gamma}, N\right) = \frac{3}{4N\gamma^2} - \frac{5}{2N^2\gamma^3} + \frac{105}{16N^3\gamma^4} + \cdots
\] (25)

Under the high SNR assumption, we can drop the terms with orders high than 2, and have the final BER approximation given by
\[
P_e(\gamma, N) \approx \frac{3}{4} \left( \frac{1}{\sqrt{N\gamma}} \right)^2
\] (26)

Similarly we can do the same to the BER for STBC-OFDM systems given in (12). We get the BER of STBC-OFDM system at high SNR regions can be approximated by
\[
P_e(\gamma) = \frac{3}{4\gamma^2} - \frac{5}{2\gamma^3} + \frac{105}{16\gamma^4} + \cdots \approx \frac{3}{4\gamma^2}
\] (27)

Comparing the two BER expressions in (26) and (27) for PT-STBC-OFDM and STBC-OFDM systems at high SNR regions, we can see that the diversity of both systems are equal to 2, which is from the Alamouti code. However, the BER of PT-STBC-OFDM system is only $\frac{1}{N}$ of the BER of STBC-OFDM system. In terms of SNR advantage, comparing to STBC-OFDM system, the PT-STBC-OFDM system has an SNR gain of $5\log_{10}(N)$dB at the high SNR regions.

Figure 4 shows comparison of the BER of the PT-STBC-OFDM system for different transform sizes $N$ with the BER obtained by both Gaussian approximation and the approximation given in (24). We can see that the BER of PT-STBC-OFDM system can be very closely approximated using two methods above. In the medium SNR region, the larger the transform size, the larger the slope of BER versus SNR curve. In the high SNR region, the diversity of PT-STBC-OFDM for all transform sizes becomes the same as normal STBC system. This is consistent with the analysis. It also verifies that the SNR gain of PT-STBC-OFDM systems is $5\log_{10}(N)$dB at high SNR regions.

VI. CONCLUSION

In this paper, we presented a pre-transformed STBC-OFDM system, where a linear unitary and constant modulus transform is applied before the STBC encoding. We analyzed the noise distribution for PT-STBC-OFDM and STBC-OFDM systems, and obtained their BER performances. We also introduced two approximation methods to obtain an accurate closed-form approximation of the BER of PT-STBC-OFDM system in all the SNR regions. We demonstrate that in the low to medium SNR region, the diversity achieved by PT-STBC-OFDM systems is much higher. In the high SNR region, the diversity of both PT-STBC-OFDM system and conventional STBC-OFDM system becomes the same. By using a size $N$ pre-transform, we can achieve an SNR gain of $5\log_{10}(N)$dB compared to STBC-OFDM systems in the high SNR region.

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