Schedulability analysis of synchronization protocols based on overrun without payback for hierarchical scheduling frameworks revisited

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Abstract—In this paper, we revisit global as well as local schedulability analysis of synchronization protocols based on the stack resource protocol (SRP) and overrun without payback for hierarchical scheduling frameworks based on fixed-priority pre-emptive scheduling (FPPS). We show that both the existing global and local schedulability analysis are pessimistic, present improved analysis, and illustrate the improvements by means of examples.

I. INTRODUCTION

A. Background

The Hierarchical Scheduling Framework (HSF) has been introduced to support hierarchical CPU sharing among applications under different scheduling services [1]. The HSF can be generally represented as a tree of nodes, where each node represents an application with its own scheduler for scheduling internal workloads (e.g. tasks), and resources are allocated from a parent node to its children nodes.

The HSF provides means for decomposing a complex system into well-defined parts called subsystems, which may share (so-called global) logical resources requiring mutual exclusive access. In essence, the HSF provides a mechanism for timing-predictable composition of course-grained subsystems. In the HSF a subsystem provides an introspective interface that specifies the timing properties of the subsystem precisely. This means that subsystems can be independently developed, analyzed and tested, and later assembled without introducing unwanted temporal interference. Temporal isolation between subsystems is provided through budgets which are allocated to subsystems.

As large extents of embedded systems are resource constrained, a tight analysis is instrumental in a successful deployment of HSF techniques in real applications. We therefore aim at reducing potential pessimism in existing schedulability analysis for HSFs. Looking further at existing industrial real-time systems, fixed priority pre-emptive scheduling (FPPS) is the de facto standard of task scheduling, hence we focus on an HSF with support for FPPS in the scheduling of tasks within a subsystem. Having such support will simplify migration to and integration of existing legacy applications into the HSF, avoiding a too big technology revolution for engineers.

Our current research efforts are directed towards the conception and realization of a two-level HSF that is based on (i) FPPS for both global scheduling of budgets (allocated to subsystems) and local scheduling of tasks (within a subsystem), (ii) the periodic resource model [1] for budgets, and (iii) the Stack Resource Protocol (SRP) [2] for both inter- and intra-subsystem resource sharing. For such an HSF, two mechanisms have been studied that prevent depletion of a budget during global resource access, i.e. skipping [3] and overrun [4]. Skipping prevents depletion by checking the remaining budget before granting resource access, and delaying access to a next budget period when the remaining budget is insufficient. Overrun prevents depletion by temporarily increasing the budget with a statically determined amount for the duration of that access. The overrun mechanism comes in two flavors, i.e. with payback and without payback, which determine whether or not the additional amount of budget has to be payed back during the next budget period.

B. Contributions

We show that existing global and local schedulability analysis of synchronization protocols based on SRP and overrun without payback for two-level hierarchical scheduling based on FPPS is pessimistic. We present improved global and local analysis assuming that the deadline of a subsystem holds for the sum of its normal budget and its overrun budget, and illustrate the improvements by means of examples. We briefly discuss further options for improvements.

C. Overview

This paper has the following structure. In Section II we present related work. A real-time scheduling model is the topic of Section III. The existing global and local schedulability analysis is recapitulated in Section IV, and improved global and local analysis is presented in Sections V and VI, respectively. Options for further improvements are briefly sketched in Section VII. The paper is concluded in Section VIII.
II. RELATED WORK

There has been a growing attention to hierarchical scheduling of real-time systems [5], [6], [7], [8], [1]. Deng and Liu [5] proposed a two-level HSF for open systems, where subsystems may be developed and validated independently. Kuo and Li [7] and Lipari and Baruah [8] presented schedulability analysis techniques for such a two-level framework with the FPPS global scheduler and the Earliest Deadline First (EDF) global scheduler, respectively. Shin and Lee [1] proposed the periodic resource model to specify guaranteed CPU allocations, an explicitly distinguishing a relative deadline $\Delta$ posed the explicit deadline periodic (EDP) resource model. We do therefore not consider local logical resources. For notational convenience, we assume that subsystems are given in order of decreasing priorities, i.e. $S_1$ has highest priority and $S_N$ has lowest priority.

B. Subsystem model

Each subsystem $S_s$ contains a set $T_s$ of $n_s$ periodic tasks $\tau_1, \tau_2, \ldots, \tau_{n_s}$ with fixed, unique priorities, which are scheduled by means of FPPS. For notational convenience, we assume that tasks are given in order of decreasing priorities, i.e. $\tau_1$ has highest priority and $\tau_{n_s}$ has lowest priority. The set $R_s$ denotes the subset of $M_s$ global resources accessed by subsystem $S_s$. The maximum time that a subsystem $S_s$ executes while accessing resource $R_l \in R_s$ is denoted by $X_{sl}$, where $X_{sl} \in \mathbb{R}^+ \cup \{0\}$ and $X_{sl} > 0 \iff R_l \in R_s$. The timing characteristics of $S_s$ are specified by means of a triple $< P_s, Q_s, X_s >$, where $P_s \in \mathbb{R}^+$ denotes its (budget) period, $Q_s \in \mathbb{R}^+$ its (normal) budget, and $X_s$ the set of maximum execution access times of $S_s$ to global resources. The maximum value in $X_s$ is denoted by $X_s$.

C. Task model

The timing characteristics of a task $\tau_{sl} \in T_s$ are specified by means of a quartet $< T_{sl}, C_{sl}, D_{sl}, C_{si} >$, where $T_{sl} \in \mathbb{R}^+$ denotes its minimum inter-arrival time, $C_{sl} \in \mathbb{R}^+$ its worst-case computation time, $D_{sl} \in \mathbb{R}^+$ its (relative) deadline, $C_{sl}$ a set of maximum execution times of $\tau_{sl}$ to global resources, where $C_{si} \leq D_{sl} \leq T_{sl}$. The maximum time that a task $\tau_{sl}$ executes while accessing resource $R_l \in R_s$ is denoted by $c_{sll}$, where $c_{sll} \in \mathbb{R}^+ \cup \{0\}$, $C_{sl} \geq c_{sl}$, and $c_{sl} > 0 \iff R_l \in R_s$.

D. Resource model

The CPU supply refers to the amount of CPU allocation that a virtual processor can provide. The supply bound function $\text{sbf}_\Omega(t)$ of the EDP resource model $\Omega(\Pi, \Theta, \Delta)$ that computes the minimum possible CPU supply for every interval length $t$ is given by

$$\text{sbf}_\Omega(t) = \begin{cases} t - (k+1)(\Pi - \Theta) + (\Pi - \Delta) & \text{if } t \in V^{(k)} \\ (k-1)\Theta & \text{otherwise,} \end{cases}$$

(1)

where $k = \max \left( \left\lceil \left( t - (\Delta - \Theta) / \Pi \right) \right\rceil, 1 \right)$ and $V^{(k)}$ denotes an interval $[k\Pi + \Delta - 2\Theta, k\Pi + \Delta - \Theta]$. The supply bound function $\text{sbf}_\Gamma(t)$ of the periodic resource model $\Gamma(\Pi, \Theta)$ is a special case of (1), i.e. with $\Delta = \Pi$.

1The focus of this paper is on synchronization protocols for global logical resources. We do therefore not consider local logical resources.

2In [11], it is required that $c_{sl} < C_{sl}$ and $c_{sl} < Q_s$. Moreover, it is observed that $c_{sl}$ will typically be much smaller than both $C_{sl}$ and $Q_s$. 

III. REAL-TIME SCHEDULING MODEL

We consider a two-level hierarchical FPPS model using the periodic resource model to specify guaranteed CPU allocations to tasks of subsystems and using a synchronization protocol for mutual exclusive resource access to global logical resources based on SRP and overrun without payback.

A. System model

A system $Sys$ contains a set $R$ of $M$ global logical resources $R_1, R_2, \ldots, R_M$, a set $S$ of $N$ subsystems $S_1, S_2, \ldots, S_N$, a set $B$ of $N$ budgets for which we assume a periodic resource model [1], and a single processor. Each subsystem $S_s$ has a dedicated budget associated to it. In the remainder of this paper, we leave budgets implicit, i.e. the timing characteristics of budgets are taken care of in the description of subsystems. Subsystems are scheduled by means of FPPS and have fixed, unique priorities. For notational convenience, we assume that subsystems are given in order of decreasing priorities, i.e. $S_1$ has highest priority and $S_N$ has lowest priority.
E. Synchronization protocol

Overrun without payback prevents depletion of a budget of a subsystem $S_i$ during access to a global resource $R_l$ by temporarily increasing the budget of $S_i$ with $X_{sl}$, the maximum time that $S_i$ executes while accessing $R_l$. To be able to use SRP in an HSF for synchronizing global resources, its associated ceiling terms need to be extended.

1) Resource ceiling: With every global resource $R_l$, two types of resource ceilings are associated; an external resource ceiling $RC_l$ for global scheduling and an internal resource ceiling $rc_{sl}$ for local scheduling. According to SRP, these ceilings are defined as

$$RC_l = \min(N, \min\{s \mid R_l \in R_s\}), \quad (2)$$

$$rc_{sl} = \min(n_s, \min\{i \mid t_{i} > 0\}). \quad (3)$$

Note that we use the outermost min in (2) and (3) to define $RC_l$ and $rc_{sl}$ also in those situations where no subsystem uses $R_l$ and no task of $T_i$ uses $R_l$ respectively.

2) System/subsystem ceiling: The system/subsystem ceilings are dynamic parameters that change during the execution. The system/subsystem ceiling is equal to the highest external/internal resource ceiling of a currently locked resource in the system/subsystem.

Under SRP, a task $\tau_{sl}$ can only preempt the currently executing task $\tau_{sj}$ (even when accessing a global resource) if the priority of $\tau_{si}$ is greater (i.e. the index $i$ is lower) than $\delta_s$, its subsystem ceiling. A similar condition for preemption holds for subsystems.

3) Concluding remarks: The maximum time $X_{sl}$ that $S_i$ executes while accessing $R_l$ can be reduced by assigning a value to $rc_{sl}$ that is smaller than the value according to SRP. For HSRP [11], the internal resource ceiling is therefore set to the highest priority, i.e. $rc_{sl}^{HSRP} = 1$. Decreasing $rc_{sl}$ may cause a subsystem to become unfeasible for a given budget [16], however, because the tasks with a priority higher than the old ceiling and at most equal to the new ceiling may no longer be feasible.

The results in this paper apply for any internal resource ceiling $rc_{sl}^{'}$ where $rc_{sl} \leq rc_{sl}^{'} \leq rc_{sl}^{HSRP} = 1$.3

IV. RECAP OF EXISTING SCHEDULABILITY ANALYSIS

In this section, we briefly recapitulate the global schedulability analysis presented in [11] and the local schedulability analysis described in [15], [4]. Although the global schedulability analysis presented in [15], [4] looks different, it is based on the analysis described in [11] and therefore yields the same result.

For illustration purposes, we will use an example system $Sys_3$ containing two subsystems $S_1$ and $S_2$ sharing a global resource $R_l$. The characteristics of the subsystems are given in Table I.

<table>
<thead>
<tr>
<th>subsystem</th>
<th>$P_l$</th>
<th>$Q_l + X_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>$S_2$</td>
<td>7</td>
<td>$Q_2 + X_2$</td>
</tr>
</tbody>
</table>

Table I

SUBSYSTEM CHARACTERISTICS OF $Sys_3$.

A. Global analysis

The worst-case response time $WR_s$ of subsystem $S_s$ is given by the smallest $x \in \mathbb{R}^+$ satisfying

$$x = B_s + (Q_s + X_s) + \sum_{t < s} \left[ \frac{x}{P_t} \right] (Q_t + X_t), \quad (4)$$

where $B_s$ is the maximum blocking time of $S_s$ by lower priority subsystems, i.e.

$$B_s = \max(0, \max\{X_t \mid t > s \wedge X_t > 0 \wedge RC_l \leq s\}). \quad (5)$$

Note that we use the outermost max in (5) to define $B_s$, also in those situations where the set of values of the innermost max is empty. To calculate $WR_s$, we can use an iterative procedure based on recurrence relationships, starting with a lower bound, e.g. $B_s + \sum_{t \leq s} (Q_t + X_t)$. The condition for global schedulability is given by

$$\forall s \leq N \quad WR_s \leq P_s. \quad (6)$$

We merely observe that the global analysis is similar to basic analysis for FPPS with resource sharing, where the period $P_s$ of a subsystem $S_s$ serves as deadline for the sum of the normal budget $Q_s$ and the overrun budget $X_s$, and the interference of higher priority subsystems $S_j$ is based on the sum $Q_j + X_j$. We will therefore use a superscript $P$ to refer to this basic analysis for subsystems, e.g. $WR_s^P$.

In the sequel, we are not only interested in the worst-case response time of a subsystem $S_s$ for particular values of $B_s$, $Q_s$, and $X_s$, but in the value as a function of the sum of these three values. We will therefore use a functional notation when needed, e.g. $WR_s(B_s + Q_s + X_s)$.

The global feasibility area of the existing analysis is illustrated for our example system $Sys_3$ in Figure 1. Note that the $y$-axis is excluded, because we assume the capacity of subsystems to be positive, i.e. $Q_2 > 0$.

![Figure 1. Global feasibility area assuming FPPS.](image-url)
Figure 2 shows a timeline with a simultaneous activation of \( S_1 \) and \( S_2 \) for \( Q_2 = 3.0 \) and \( X_2 = 0 \), and a worst-case response time \( WR_2 \) of \( S_2 \) equal to 5.0. Note that even an infinitesimal increase of either \( Q_1 \) or \( Q_2 \) will make the system \( Sys \) unschedulable.

\[ \forall 1 \leq n, 0 < t < D_{st} \]
\[ b_{st} + C_{st} + \sum_{j < i} \frac{t}{T_j} \cdot C_{sj} \leq SBF_{\Gamma_i}(t), \quad \text{(7)} \]

where \( b_{st} \) is the maximum blocking time of \( \tau_{st} \) by lower priority tasks, i.e.

\[ b_{st} = \max(0, \max\{c_{sj} \mid j > i \land c_{sj} > 0 \land rc_{j} \leq \tau_{st} \}), \quad \text{(8)} \]

and \( SBF_{\Gamma_i}(t) \) is the supply bound function of the periodic resource model \( \Gamma_i(P_i, Q_i) \) for the subsystem \( S_i \) under consideration. Note that we use the outermost max in (8) to define \( b_{si} \) also in those situations where the set of values of the innermost max is empty.

The value for \( X_{di} \) depends on the local scheduler and the synchronization protocol. The maximum time that subsystem \( S_i \) executes while task \( \tau_{sil} \) accesses resource \( R_i \in R \) is denoted by \( X_{sil} \), where \( X_{sil} \in \mathbb{R}^+ \cup \{0\} \) and \( X_{sil} > 0 \Leftrightarrow c_{sil} > 0 \). For \( c_{sil} > 0 \), \( X_{sil} \) is given by [4]

\[ X_{sil} = c_{sil} + \sum_{j < rc_{sil}} C_{sj}. \quad \text{(9)} \]

The value for \( X_{di} \) is given by

\[ X_{di} = \max_{1 \leq i \leq n} X_{sil}. \quad \text{(10)} \]

V. IMPROVED GLOBAL ANALYSIS

As described in Section IV-A, the existing global schedulability analysis is based on FPS, where the period \( P_i \) serves as deadline for the sum of the normal budget \( Q_s \) and overrun budget \( X_s \).

A. Illustrating the improvement

In this section, we will present two steps that gradually improve the global analysis:

1) Limited pre-emption of overrun budget \( X_s \);
2) Blocking starts before the execution based on the overrun budget \( X_s \) starts;

1) Limited pre-emption of overrun budget \( X_s \): Subsystem \( S_1 \) can not preempt \( S_2 \) during those intervals of time when \( S_2 \) is accessing resource \( R_1 \) in general, and when \( S_2 \) is executing based on its overrun budget \( X_2 \) in particular. This limited preempt-ability of subsystem \( S_2 \) gives rise to improved schedulability of system \( Sys \), as illustrated in Figure 3. In this figure, it is assumed that \( X_2 \) can be executed without pre-emption. Note that \( X_2 \leq 3.0 \) and \( Q_2 \leq 3.0 \), because

Figure 3. Global feasibility area assuming limited pre-emption of \( X_s \).

\( S_1 \) and \( S_2 \) will otherwise miss their deadline, respectively. Further note that for \( Q_2 = 1.2 \) and \( X_2 = 3.0 \) the utilization of the system \( U = \frac{Q_1 + X_1}{P_1} + \frac{Q_2 + X_2}{P_2} = 1 \). Finally note that the feasibility area shown in Figure 3 would be identical when the global schedulability analysis would be based on fixed-priority scheduling with deferred pre-emption (FPDS) [17], [18], and each job of \( S_2 \) would consist of a sequence of two non-preemptable subjobs with computation times \( Q_2 \) and \( X_2 \), respectively.

We will briefly explain the anomalies in Figure 3 by means of timelines with a simultaneous release of \( S_1 \) and \( S_2 \) at time \( t = 0 \) and assuming that both \( S_1 \) and \( S_2 \) need their overrun budget for every activation.

Figure 4 shows a timeline with \( Q_2 = 1.8 \) and \( X_2 = 2.4 \). Note that the second job of \( S_2 \) misses its deadline at time \( t = 14 \), because the third job of \( S_1 \) is allowed to start at time \( t = 10 \). An infinitesimal decrease of either \( Q_2 \) or \( X_2 \) will allow the execution of \( X_2 \) of the second job to start just before \( t = 10 \) and will allow the second job to meet its deadline.

Figure 4. Timeline for \( Q_2 = 1.8 \) and \( X_2 = 2.4 \) under limited pre-emption of \( X_2 \) with a deadline miss at \( t = 14 \).

Figure 5 shows a timeline with \( Q_2 = 2.0 \) and \( X_2 = 2.0 \). In this case, the second job of \( S_2 \) meets its deadline, because
the workload in the interval $[0, 14]$ is equal to the length of that interval. Note that the configurations of $S_2$ represented by the line segment of the line $2Q_2 + X_2 = 6.0$ between the points $<1.8, 2.4>$ and $<2.0, 2.0>$ are not feasible. Similarly, the configurations of $S_2$ represented by the line segment of the line $2Q_2 + X_2 = 6.0$ starting at $<1.8, 2.4>$ till point $<2.0, 2.0>$ are now feasible. Similarly, the configurations of $S_2$ represented by $Q_2 = 3.0$ and $0 \leq X_2 \leq 1.0$ are feasible as well. We will briefly explain the differences between Figures 3 and 7 by means of timelines.

Figure 8 shows a timeline with $Q_2 = 1.8$ and $X_2 = 2.4$. Because the second job of $S_2$ locks $R_1$ just before the activation of $S_1$ at $t = 10$, $S_2$ is allowed to execute $X_2$ at $t = 10$. As a result, the second job of $S_2$ does not miss its deadline at time $t = 14$.

Figure 9 shows a timeline with $Q_2 = 3.0$ and $X_2 = 1.0$. Similar to the previous case, because the first job of $S_2$ locks $R_1$ just before the activation of $S_1$ at $t = 5$, $S_2$ is allowed to execute $X_2$ at $t = 5$. As a result, the first job of $S_2$ does not miss its deadline at time $t = 7$.

B. Improving the global analysis

The improved global analysis is similar to the analysis for FPDS [17], [18] and FPPS with preemption thresholds [19] in the sense that we have to consider all jobs in a so-called level-$s$ active period to determine the worst-case response time $WR_s$ of subsystem $S_s$. Unlike the analysis described in [17], [18], [19], subsystems $S_{s-1}$ till $S_{RC_s}$ cannot preempt $S_s$ at the finalization time of $Q_s$ when $S_s$ is accessing $R_1$, as illustrated in Figures 8 and 9 for the times $t = 10$ and $t = 5$, respectively.

In the remainder of this section, we first present the analysis for the special case where every subsystem accesses at most one global resource, i.e. $M_s \leq 1$, and subsequently present the general case.

1) Access to a single global resource: The worst-case length $WL_s$ of a level-$s$ active period with $s \leq N$ is given by the smallest $x \in \mathbb{R}^+$ that satisfies

$$x = B_s + \sum_{t \leq s} \left( \frac{x}{P_t} \right) (Q_t + X_t).$$

(11)
To calculate \(WL_s\), we can use an iterative procedure based on recurrence relationships, starting with a lower bound, e.g. \(B_s + \sum_{i < s} (Q_s + X_i)\). The maximum number \(wl_s\) of jobs of \(S_s\) in a level-\(s\) active period is given by

\[
wl_s = \left\lfloor \frac{WL_s}{P_s} \right\rfloor. \tag{12}
\]

For a job \(t_{sk}\) of \(S_s\) with \(0 \leq k < wl_s\), we split the interval from the start of the level-\(s\) active period to the finalization of job \(t_{sk}\) in two sub-intervals: a first sub-interval including the execution of the normal budget \(Q_s\) by job \(t_{sk}\) and a second sub-interval from the finalization of \(Q_s\) by \(t_{sk}\) till the finalization of \(t_{sk}\).

The worst-case finalization time \(WF_{sk}^{Q}\) of the normal budget \(Q_s\) of job \(t_{sk}\) with \(0 \leq k < wl_s\) relative to the start of the constituting level-\(s\) active period is given by

\[
WF_{sk}^{Q} = WR_{sk}^{P}(B_s + (k + 1)Q_s + kX_s), \tag{13}
\]

where \(WR_{sk}^{P}\) is the worst-case response time of a fictive subsystem \(S'_s\) with a period \(P'_s = (k + 1)T_s\), a normal budget \(Q'_s = (k + 1)(Q_s + X_s) - X_s\), and a maximum blocking time \(B_s\). Let \(S_s\) access \(R_l \in R_s\). When \(S_s\) starts to consume its overrun budget \(X_s\), the subsystems \(S_{s-1}\) till \(S_{RC_s}\) are already blocked. We only need to consider preemptions by subsystems with a priority higher than \(RC_s\) at and after the finalization of \(Q_s\), and therefore treat the preemptions by subsystems \(S_{s-1}\) till \(S_{RC_s}\) separately. The worst-case interference of the subsystems \(S_{s-1}\) till \(S_{RC_s}\) in the interval of length \(WF_{sk}^{Q}\) is denoted by \(WF_{RC_s}^{-1}\) and given by

\[
WF_{RC_s}^{-1} = \sum_{i = RC_s}^{s-1} \left\lfloor \frac{WF_{sk}^{Q}}{P_i} \right\rfloor (Q_i + X_i). \tag{14}
\]

Subsystems with a priority higher than \(RC_s\) can still pre-empt the execution of \(X_s\). Hence, the worst-case response time \(WR_{sk}\) of job \(t_{sk}\) of subsystem \(S_s\) is given by

\[
WR_{sk} = WR_{RC_s}^{P}(B_{RC_s} + (k + 1)(Q_s + X_s)) - kP_s, \tag{15}
\]

where \(WR_{RC_s}^{P}\) represents the worst-case response time of a fictive subsystem \(S'_{RC_s}\) with a (budget) period \(P_{RC_s}\) and a deadline equal to \((k + 1)P_s\), a normal budget \(Q'_{RC_s}\) equal to \((k + 1)(Q_s + X_s) - X_s\), an overrun budget \(X'_{RC_s}\) equal to \(X_s\), and a maximum blocking time \(B_{RC_s}\) given by

\[
B_{RC_s} = B_s + WF_{RC_s}^{-1}. \tag{16}
\]

Finally, the worst-case response time \(WR_s\) of subsystem \(S_s\) is given by

\[
WR_s = \max_{0 \leq k < wl_s} WR_{sk}. \tag{17}
\]

**Example: Sys1 with \(Q_2 = 3.0\) and \(X_2 = 1.0\).**

We determine \(WR_2\) using the analysis described above; see also Figure 9. Because \(S_2\) is the lowest priority subsystem, \(B_3 = 0\). We first determine \(wl_2\) using (11) and (12), and find \(WL_2 = 14\) and \(wl_2 = \left\lfloor \frac{WL_2}{T_2} \right\rfloor = \left\lfloor \frac{14}{7} \right\rfloor = 2\). Next we determine \(WR_{2,0}\) and \(WR_{2,1}\) using (13) till (16). Using (13), we find \(WF_{2,0}^{Q} = WR_{2,0}^{P}(B_2 + Q_2) = WR_{2,0}^{P}(3.0) = 5\). Because \(RC_s = 1\), \(WF_{1,0}^{Q} = \left\lfloor \frac{WF_{2,0}^{Q}}{P_1} \right\rfloor (Q_1 + X_1) = \left\lfloor \frac{5}{5} \right\rfloor = 2.0 = 2.0\). Using (16), we find \(B'_1 = B_2 + WF_{1,0}^{Q} = 2.0 = 2.0\). Using (15), we find \(WF_{2,0} = WR_{2,0}^{P}(B'_1 + (Q_2 + X_2)) = WR_{2,0}^{P}(6) = 6\). Similarly, we find \(WF_{2,1} = WR_{2,1}^{P}(7.0) = 13\), \(WF_{1,1} = \left\lfloor \frac{WF_{2,1}^{Q}}{P_1} \right\rfloor (Q_1 + X_1) = \left\lfloor \frac{13}{5} \right\rfloor = 2.0 = 6.0\), \(B'_1 = B_2 + WF_{1,1}^{Q} = 2.0 = 6.0\), and \(WF_{2,1} = WF_{2,1}^{P}(B'_1 + (Q_2 + X_2)) = WR_{2,1}^{P}(14) = 7\). Finally, using (17) we find \(WR_2 = \max(WF_{2,0}, WF_{2,1}) = \max(6, 7) = 7\).

2) **Access to multiple global resources:** When a subsystem uses multiple global resources, we have to slightly adapt our analysis. In particular, when the resource ceiling \(RC_{sl}\) of resource \(R_l \in R_s\) is larger than \(RC_{sl'}\) of resource \(R_l' \in R_s\), i.e. more subsystems can pre-empt \(S_s\) during its access to \(R_l\) than to \(R_{l'}\), and the maximum execution access time \(X_{sl}\) of \(S_s\) to \(R_l\) is smaller than \(X_{sl'}\), the system may be schedulable for \(R_{l'}\) but not for \(R_l\). As an example consider a system containing 2 global resources \(R_1\) and \(R_2\) and 3 subsystems \(S_1\), \(S_2\), and \(S_3\), where the subsystems have timing characteristics as given in Table II. The schedulability of \(S_3\) for \(X_{3,1}\) follows immediately from the similarity of systems \(S_{sys1}\) and \(S_{sysII}\), and the feasibility area shown in Figure 7. Subsystem \(S_3\) just meets its deadline at \(t = 7\) for its overrun budget \(X_{3,2} = 0.4\) under worst-case conditions, i.e. a simultaneous release of all three subsystems at time \(t = 0\) and resources accesses by both \(S_1\) and \(S_2\) requiring the usage of their overrun budgets at every activation; see Figure 10. Note that subsystem \(S_3\) will miss its deadline at time \(t = 7\) for an infinitesimal increase \(e > 0\) of \(X_{3,2}\).

![Figure 10](image-url)

Figure 10. Subsystem \(S_3\) just meets its deadline at \(t = 7\) for \(X_{3,2} = 0.4\).
to use $X_i$ and $RC^i$ rather than $R_i$, where $RC^i$ is defined as
\[
RC^i = \max\{RC_l | R_l \in R_i\}.
\] (18)

Note that such an analytical approach would classify Example II as unschedulable, however.

Alternatively, we can determine the worst-case response time for each job of $S_i$ for individual global resources and subsequently take the maximum, i.e. we replace (15) by
\[
WR_{skl} = WR_{RC_l}(b'_{RC_l} + (k + 1)Q_s + kX_s + X_{skl}) - kP_s
\] (19)
and
\[
WR_{sk} = \max_{l} WR_{skl}.
\] (20)

Example: System $S_{sys}$
We (only) determine $WR_{3,0}$ using the analysis described above; see also Figure 10. Because $S_3$ is the lowest priority subsystem, $B_3 = 0$, and $WR^Q_3 = WR^P(B_3 + Q_3) = WR^P_3(3.0) = 5.0$. We first determine $WR_{3,0,1}$. For $R_1$ and $RC_l = 1$, we find $\Gamma^{Q}_{1,0} = \sum_{k=1}^{\infty} |WF^Q_{3,0}/T_0| (Q_s + X_s) = 2.0$ and $B'_2 = B_3 + WR^P_1(2.0) = 2.0$. Using (19), we find $WR_{3,0,1} = WR^P(B'_2 + Q_3 + X_{3,1}) = WR^P_1(6.0) = 6.0$. Next, we determine $WR_{3,0,2}$. For $R_2$ and $RC_2 = 2$, we find $\Gamma^{Q}_{2,0} = \sum_{k=2}^{\infty} |WF^Q_{3,0}/T_2| (Q_s + X_s) = 0.4$ and $B'_2 = B_3 + WR^P_2(0.4) = 0.4$. Using (19), we find $WR_{3,0,2} = WR^P(B'_2 + Q_3 + X_{3,2}) = WR^P_2(3.8) = 7.0$. Finally, using (20) we find $WR_{3,0} = \max(WR_{3,0,1}, WR_{3,0,2}) = \max(6.0, 7.0) = 7.0$.

VI. IMPROVED LOCAL ANALYSIS

Both the existing global schedulability analysis and the improved global schedulability analysis assume a deadline for a subsystem $S_i$ equal to its period $P_i$ for the sum of the normal budget $Q_s$, and the overrun budget $X_s$. The existing local schedulability analysis for the tasks of $S_i$ is exclusively based on $Q_s$, however. Hence, when a system is feasible from a global scheduling perspective, the latest finalization time of $Q_s$ is guaranteed to be at least $X_s$ before the next activation of $S_i$. Hence, we can use the supply bound function $sbf_e(t)$ of the EDP resource model $\Omega_e(P_s, Q_s, \Delta_s)$ for overrun without payback rather than $sbf_{\Gamma}(t)$ of $\Gamma(P_s, Q_s)$ in (7), where $\Delta_s = P_s - X_s$. Because $X_s \geq 0$ for all subsystems (by definition), $sbf_{\Gamma}(t) \leq sbf_e(t)$ for all subsystems. As a result, a subsystem may be schedulable according to the local analysis based on $sbf_e(t)$, but not be schedulable based on $sbf_{\Gamma}(t)$.

Figure 11 shows an example of the supply bound functions $sbf_e(t)$ and $sbf_{\Gamma}(t)$ for subsystem $S_2$ of system $S_{sys}$ with $Q_2 = 1.8$ and $X_2 = 2.4$.

VII. DISCUSSION

In this section, we consider directions for further improvements.

A. Decreasing external resource ceilings

Figure 10 showed a timeline where subsystem $S_3$ just meets its deadline at $t = 7$ for $X_{3,2} = 0.4$. By decreasing the external resource ceiling $RC_2$ of resource $R_2$ from 2 to 1, subsystem $S_1$ can no longer pre-empt the execution of $S_3$. As a result, the resource holding time $\{t\}$ of $R_2$ by $S_3$ is reduced from $Q_1 + X_{1,1} + X_{3,2} = 2.4$ to $X_{3,2} = 0.4$.

For this particular example, it immediately follows from the similarity with system $S_{sys}$ that we can even increase $X_{3,2}$ to 1.0 when we decrease $RC_2$ from 2 to 1 without making the system unschedulable. In general, decreasing a resource ceiling $RC_i$ from $u$ to $v$ may improve the schedulability of subsystems $S_u$, with $s \geq u \geq v$, and worsen the schedulability of subsystems $S_v$ with $u > w \geq v$. Hence, given the improved global schedulability presented in Section V, we may further improve the schedulability of a system by decreasing external resource ceilings of global resources. Note that this improvement is only possible because of the limited preemptability of the overrun budget on the one hand and the fact that the overrun budget is executed as last budget.

B. Further global analysis improvements

We briefly consider two further improvements of the global analysis, which we also illustrate by means of system $S_{sys}$, i.e.

- the deadline $P_i$ holds for $Q_s$, only;
- discarding the remainder of $X_s$ upon a replenishment.

Because the deadline $P_i$ only holds for $Q_s$, the improvement of the local schedulability analysis described in Section VI does no longer apply for these two further improvements of the global analysis.

1) Deadline only for normal budget: The overrun budget is needed if and only if the normal budget $Q_s$ of a subsystem $S_i$ becomes depleted whilst $S_i$ holds a global resource. As soon as the normal budget is replenished, there is no need to use the overrun budget. Hence, the deadline of a subsystem $S_i$ only holds for its normal budget. The resulting improvement is illustrated in Figure 13.

2) Overrun ends upon replenishment: The last improvement results from the observation that the remainder of the overrun budget $X_s$ of a subsystem $S_i$ can be discarded upon replenishment of its normal budget $Q_s$. As a result, the
utilization $U$ of the subsystems expressed as $\sum_{i=1}^{N} \frac{Q_i + X_i}{P_i}$ can become larger than 1. The resulting improvement is illustrated in Figure 14.

![Figure 14](image)

Figure 14. Feasibility area assuming overrun ends upon replenishment.

Figure 12 shows a timeline for $Q_2 = 2.8$ and $X_2 = 3.0$ with a simultaneous activation of $S_1$ and $S_2$ at $t = 0$. The figure illustrates that 0.8 of the overrun budget $X_2$ is lost at times $t \in \{7, 21, 35\}$ and that 2.8 is lost at times $t \in \{14, 28\}$.

VIII. CONCLUSION

We showed that existing global and local schedulability analysis of synchronization protocols based on SRP and overrun without payback for two-level hierarchical scheduling based on FPPSs is pessimistic. We presented improved global and local analysis assuming that the deadline of a subsystem holds for the sum of its normal budget and its overrun budget, and illustrated the improvements by means of examples. We briefly discussed further options for improvements, i.e. (i) to decrease external resource ceilings and (ii) to assume that the deadline $P_i$ only holds for $Q_i$ and that $X_i$ can be discarded upon a replenishment of the budget of $S_i$. For improvement (ii), the improved local analysis can not be applied, however.

The evaluation of the improvements through simulation, the consequences of decreasing resource ceilings, the sustainability of the analysis [20], and the applicability of the improvements identified for the other flavor of the overrun mechanism, i.e. with payback, are left as topics of future work.

ACKNOWLEDGEMENTS

We thank Martijn M.H.P. van den Heuvel from the TU/e for his comments on an earlier version of this paper.

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