A-POSTERIORI ERROR ESTIMATION AND OPTIMAL ADAPTIVITY FOR FLUID-STRUCTURE INTERACTION

* Harald van Brummelen, Kris van der Zee, Peter Fick and René de Borst

1 Delft University of Technology
PO Box 5058, 2600GB Delft, Netherlands
Email: {e.h.vanbrummelen, k.g.vanderzее, p.w.fick}@tudelft.nl

2 Eindhoven University of Technology
PO Box 513, 5600MB Eindhoven, Netherlands
Email: r.d.borst@tue.nl

ABSTRACT

Numerical simulations of fluid-structure interaction generally require vast computational resources. An interesting paradox in fluid-structure-interaction computations is that the computational cost is typically dominated by the subsystem that is of least practical interest, viz., the fluid. For realistic applications the fluid often consumes nearly all the computational resources, while practical engineering interest is restricted to the structural response. For instance, in [3] Farhat reports that for the investigation of the aeroelastic response of an F16 aircraft more than 98% of the total simulation time is spent inside the fluid solver and the mesh update algorithm.

Goal-oriented a-posteriori error estimation and optimal adaptive refinement strategies provide an approach to bypass this paradox. Based on the solution of an appropriate dual problem, the contribution of local errors in the fluid solution to a particular functional (observation) of the structural displacement can be established. Only the regions in the fluid domain that have a pronounced influence on the error in the functional of interest need to be refined in the discretization.

The general approach for goal-oriented a-posteriori error estimation has been developed by Becker and Rannacher [1] and Oden and Prudhomme [7,8]. The approach can be summarized as follows: Suppose that we are interested in the observation \( J(u) \) of the solution \( u \in U \) of the linear variational problem:

\[
a(u, v) = b(v) \quad \forall v \in V,
\]

with \( U \) and \( V \) certain Hilbert spaces, \( a : U \times V \to \mathbb{R} \) a bounded bilinear form and \( b : V \to \mathbb{R} \) and \( J : U \to \mathbb{R} \) bounded functionals. Let \( \tilde{U} \subset U \) and \( \tilde{V} \subset V \) represent conforming finite-element spaces. The corresponding finite-element approximation of \( J(u) \) is \( J(\tilde{u}) \) with \( \tilde{u} \in \tilde{U} \) according to:

\[
a(\tilde{u}, \tilde{v}) = b(\tilde{v}) \quad \forall \tilde{v} \in \tilde{V},
\]

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Let \( z \in V \) now denote the solution of the dual problem:

\[
a(w, z) = J(w) \quad \forall w \in U.
\]  

(3)

Then the error in the functional of interest can be expressed as

\[
J(u) - J(\hat{u}) = a(u, z) - a(\hat{u}, z) = b(z) - a(\hat{u}, z) = \langle r(\hat{u}), z \rangle_{V' \times V} = \langle r(\hat{u}), z - \hat{z} \rangle_{V' \times V},
\]  

(4)

for any \( \hat{z} \in \hat{V} \), where \( r : U \to V' \) represents the residual functional, \( V' \) is the dual of \( V \) and \( \langle \cdot, \cdot \rangle_{V' \times V} \) denotes the duality pairing on \( V' \times V \). The final identity in (4) follows from Galerkin orthogonality. The final expression in (4) conveys that a large contribution to the error in the quantity of interest can occur whenever the product of the residual and the interpolation error in dual solution is locally large. Hence, in such areas, the finite-element space must be refined, either by increasing the order of approximation (\( p \) refinement) or by refining the mesh (\( h \) refinement).

The a-posteriori error estimation framework applies generically to variational formulations of linear (initial-)boundary-value problems and linear observables \( J \). The methodology can be extended to nonlinear functionals \( J(\cdot) \) and semi-linear functionals \( a(\cdot, \cdot) \) by means of linearization. For fluid-structure-interaction problems, however, several complications arise. First, the interface at which the fluid and structure interact constitutes a free-boundary, i.e., its position is unknown a priori and is to be determined as part of the solution. This induces a complicated nonlinear interconnection between the governing initial-boundary-value problems, and the domains on which these are defined. Consequently, the formulation of an appropriate linearized dual problem is nontrivial on account of the occurrence of shape derivatives. For fully Eulerian formulations [2], the shape dependence manifests itself differently but still results in complex derivatives. Second, the treatment of the interface conditions in the primal formulation of the fluid-structure-interaction problem, including the enforcement of boundary conditions and the evaluation of load functionals, generally has nontrivial consequences for the well-posedness of the dual problem. Two distinct formulations that appear to be equivalent for the primal problem, can behave very differently for the dual problem.

To illustrate the a-posteriori error estimation and optimal adaptive refinement procedures, we present numerical results for the model problem in [4] pertaining to steady incompressible Stokes flow in a channel with a backward-facing step and a flexible bottom composed of a string (see Figure 1), and for the panel problem of [6] pertaining to an unsteady compressible flow over a beam [5].

**Key words:** Fluid-Structure Interaction, Free-Boundary Problems, A-Posteriori Error Estimation, Goal-Oriented Adaptivity, Optimal-Adaptive Methods
Figure 1: Illustration of optimal-adaptive refinement for FSI: horizontal velocity (color) and optimal $h$-refined mesh for a steady incompressible Stokes flow in a channel with a backward-facing step and a partly flexible boundary composed of a string. The quantity of interest is the average displacement of the structure.
REFERENCES


