Post-Failure Modes for Steel Sheeting Subject to Concentrated Load (Web Crippling) and Bending Moment

H. Hofmeyer
J.G.M. Kerstens
H.H. Snijder
M.C.M. Bakker
Eindhoven University of Technology, Faculty of Architecture, Department of Structural Design.

Cold-formed trapezoidal sheeting of thin steel plate is a very popular product for building construction. It combines low weight and high strength and is economical in use. To increase the insight into the behaviour of the sheeting, this article presents new experiments in which first-generation sheeting behaviour is studied under combined concentrated load and bending. The experiments show that after ultimate load, three different post-failure modes occur. Mechanical models have been developed for the three post-failure modes. These models can help to explain why a certain post-failure mode occurs.

Keywords: steel sheeting, concentrated load, web crippling, bending moment, post-failure.

1 Introduction

Cold-formed trapezoidal sheeting of thin steel plate is a very popular product for building construction. It combines low weight and high strength and is economical in use. Figure 1 shows a photo of sheeting used for roof construction.

First generation sheeting is usually a rolled plate without any stiffeners. Second generation sheeting has stiffeners in the longitudinal direction only. Third generation sheeting also has stiffeners in the transverse direction. In this article, only first generation sheeting is investigated, although second and third generation sheetings are widely used. The reason for this limitation is that it is important to first understand the behaviour of the relatively simple first generation sheeting.

Next to trapezoidal sheeting, sheet sections will be introduced later in this article. Here, only their definitions are given (see figure 2). Trapezoidal sheeting is folded (rolled) plate with several webs and flanges. A sheet section is a combination of one bottom flange, two webs, and two half top flanges. Constraints are applied at the edges of the two half top flanges to let this sheet section behave like infinitely wide sheeting. A hat section is a sheet section without constraints at the edges of the two half top flanges.
Thus, it may behave differently from trapezoidal sheeting and sheet sections. At an interior support, sheeting is subjected to a concentrated load $F$ and a bending moment $M$ as shown in figure 3.

![Trapezoidal sheeting](image)

**Fig. 1. Sheeting used for roof construction.**

![Definitions of trapezoidal sheeting, sheet sections, and hat sections.](image)

**Fig. 2. Definitions of trapezoidal sheeting, sheet sections, and hat sections.**

At this moment, design rules do not provide sufficient insight in the failure behaviour of sheeting subjected to a concentrated load and a bending moment. To increase the insight in sheeting behaviour, in this article new experiments are presented in section 2. The experiments show three post-failure modes after ultimate load (a post-failure mode is a specific type of failure after ultimate load). In section 3, three mechanical models are presented to predict the ultimate load for every post-failure mode. The quality of the models is studied by comparing their predictions with experimental results (besides these models, also one very good model was developed for two post-failure modes at the same time, see [3]). Section 4 shows how the mechanical models increase the insight in sheeting behaviour. This by studying which mechanical model predicts the lowest load for changing sheeting variables. Section 5 presents the conclusions. In general, additional information to this article can be found in references [1, 2, 3, 4].
2 Experiments

In the experiments presented in this article, sheet sections are tested in a three-point bending test. The sheet sections are chosen to represent the situation in practice for two aspects: the sheet section cross-section variables (sheet section height for example) and the loading conditions \( \frac{M}{F} \) ratios and \( \frac{M}{F} \) ratios). Both should be similar for the experiments and the situation in practice. \( M_b \) is the ultimate bending moment and \( R_c \) the ultimate concentrated load the sheeting can bear.

All sheet sections, with their nominal values, are shown in figure 4. The variable \( L_{	ext{span}} \) stands for the span length, \( \theta_f \) for the angle between web and flange [deg.], \( b_f \) is the bottom flange width [mm]. The variable \( r_b \) stands for the bottom corner radius [mm] and \( L_p \) for the load-bearing plate width [mm]. The sheet sections having a length of 600 mm (test 1 to 15) were not part of the experiments originally. They were added later. Their aim was to be able to compare the experiments in this article (long span lengths) with the experiments of Bakker (short lengths) [5]. For the sheet sections the \( \eta \) ratio is 0.45 to 3.60 and the \( M/F \) ratio ranges from 125 to 575 mm. In figure 5, these \( M/F \)-ratios and \( \eta \) ratios are shown and the experimental values do represent the situation in practice.

The experimental research uses a three-point bending test configuration to test the sheet sections. Instead of a normal experimental set-up for three-point bending tests, an upside down set-up was used. This set-up made it possible to investigate the deformation of the cross-section by making castings of the inner cross-section during deformation. The test rig (mechanical properties) is shown in figure 6. A hydraulic jack is connected to a load-bearing plate, which loads the sheet section. The sheet section is supported by two support strips that are connected to four support rods by two support bars. The support rods are connected to a rigid beam that is the base of the test rig. Strips are fixed to the sheet section to avoid spreading of the webs and sway of the cross-section.
The experiments show three different yield line patterns after ultimate load (figure 7). Before ultimate load, no yield lines are visible. The differences for the three yield line patterns are as follows. Yield line pattern II shows yield lines -bold lines (a) and (b)- directly on both sides of the bottom corners. The other two patterns show no yield lines on the bottom corner, but one yield line in the web (a) and one in the flange (b). The third pattern is asymmetrical in the longitudinal direction. The other two patterns are symmetrical. Every yield line pattern is accompanied by a specific load versus web crippling deformation curve, see figure 7 at the bottom.
Fig. 7. Yield line patterns and accompanying load deformation curves.

Yield line pattern II and the accompanying load deformation curve are defined as a rolling post-failure mode. This is because the yield lines at the bottom corner roll through the web and flange. Yield line pattern I and the accompanying load deformation curve are defined as an arc-like post-failure mode. This is because an arc-like shape develops in the web. Yield line pattern III and the accompa-
nying load deformation curve are defined as a yield eye post-failure mode, as an eye-like shape develops in the bottom flange.

In figure 8, post-failure modes are shown for all tests. The character "A" stands for a yield arc post-failure mode, the character "E" stands for a yield eye post-failure mode, "R" stands for a rolling post-failure mode, and "A>R" stands for a yield arc post-failure mode that is followed by a rolling post-failure mode. Finally, "A>E" stands for a yield arc post-failure mode that is followed by a yield eye post-failure mode.

<table>
<thead>
<tr>
<th>$L_{span}$</th>
<th>$\theta_{w}$</th>
<th>Post-failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>90 100</td>
<td>A&gt;R A&gt;E R* A&gt;E R</td>
</tr>
<tr>
<td>700</td>
<td>100 A</td>
<td>A&gt;E R</td>
</tr>
<tr>
<td>50 40</td>
<td>A&gt;R A&gt;E</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>90 100 A</td>
<td>A&gt;E R</td>
</tr>
<tr>
<td>90 40</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>50 100</td>
<td>A A A A A A</td>
<td></td>
</tr>
<tr>
<td>1200</td>
<td>90 100 A</td>
<td>A&gt;E A A A</td>
</tr>
<tr>
<td>70 40</td>
<td>A A A A A A</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>A A A A A A</td>
<td></td>
</tr>
<tr>
<td>50 40</td>
<td>A A A A A A</td>
<td></td>
</tr>
<tr>
<td>1400</td>
<td>50 40 A&gt;E</td>
<td>A&gt;E A A A A A A</td>
</tr>
<tr>
<td>1800</td>
<td>90 100 E</td>
<td>A A A A A A A A</td>
</tr>
<tr>
<td>70 40</td>
<td>A A A A A A</td>
<td></td>
</tr>
<tr>
<td>50 40</td>
<td>A A A A A A</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>A A A A A A</td>
<td></td>
</tr>
<tr>
<td>2400</td>
<td>90 100 A&gt;E</td>
<td>A A A A A A A A</td>
</tr>
<tr>
<td>70 100</td>
<td>A A A A A A</td>
<td></td>
</tr>
<tr>
<td>50 40</td>
<td>E A A A A A</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>A E A E A E</td>
<td></td>
</tr>
</tbody>
</table>

* More experiments in one field.

Fig. 8. Post-failure modes for experiments.

3 Post-failure models

The post-failure models are used to predict which post-failure mode occurs: all post-failure models in this section predict the mode initiation load $F_{mn}$. This is the intersection of curves describing elastic and plastic sheeting behaviour. For the yield arc and yield eye post-failure modes, load $F_{mn}$ is a prediction for the ultimate load, see figure 9. For the rolling post-failure mode, figure 9 shows that $F_{mn}$ is not a prediction of the ultimate load; it only predicts first plastic behaviour. The post-failure models are not used to predict the ultimate load. For this aim, an other model has been developed [3,4].

314
3.1 Mechanical model for the yield arc post-failure mode

**Elastic behaviour**

In 1995, Vaessen developed mechanical models to predict the elastic relationship between load and web crippling deformation for sheet sections [6,7]. A part of one of his models can be used to predict the elastic load $F_e$ on the load-bearing plate for a certain indentation of the cross-section (web crippling deformation $\Delta h_w$):

$$F_e = \frac{EI(3bf_f + 2b_w)\Delta h_w}{r_{bf}^2 \sin^2 (\theta_w) b_w \left(b_{bf} - \frac{4}{3} r_{bf} \sin (\theta_w) \right)} \quad \text{where} \quad I = \frac{1}{12} L_{bf} b_{bf}^3$$

The formula is valid for first-order elastic behaviour for a part of the sheet section (defined as "modelled cross-section"), as shown in figure 10. Flange or web buckling is not taken into account.

![Fig. 10. Modelled cross-section.](image)

**Plastic behaviour**

For the yield arc post-failure mode, the plastic behaviour of the cross-section is modelled as shown in figure 11. Making use of the principle of virtual displacements, the plastic load $F_p$ related to the web crippling deformation $\Delta h_w$ is as follows:
\[ F_p = \frac{2}{\sqrt{3}} \frac{f_y t^2}{L_{yw}} \left( \frac{\delta \phi_p}{\delta \Delta h_w} + \frac{\delta \phi_t}{\delta \Delta h_w} + \frac{\delta \phi_c}{\delta \Delta h_w} \right) \]  

(2)

\[ \frac{\delta \phi_p}{\delta \Delta h_w} = \frac{b_{yw} - L_{yw}}{2(b_{yw} - L_{yw})^2} + \frac{\text{sec} \theta_w}{b_{yw}^2} \left( \frac{1}{1 + \left( b_{yw} - L_{yw} \right)^2 \text{sec}^2 \theta_w} \right) \left( \frac{L_{yw}^2 - (b_{yw} - L_{yw})^2 - x^2}{\Delta h_w} \right) \]  

(3)

where

\[ x = \sqrt{(b_{yw} - \Delta h_w)^2 + b_{yw}^2 \cos^2 \theta_w} \]  

(4)

The factors \( \delta \phi_i / \delta \Delta h_w \) and \( \delta \phi_b / \delta \Delta h_w \) are just as complex as factor \( \delta \phi_t / \delta \Delta h_w \) and can be found in section 3.2 of report [2]. Distance \( L_{yw} \) is predicted by a method presented in section 4.3 of this report [2].

**Fig. 11. Plastic behaviour for modelled cross-section.**

**Intersection of elastic and plastic curves**

Formulae 2, 3, and 4 can be simplified [2]. Making the simplified formulae equal to formula 1 (see introduction section 3) yields the predicted ultimate load of the modelled cross-section. This load is defined as \( F_{uw} \):
\[ F_{cst} = \frac{-\alpha - \beta + \sqrt{4A \alpha \left(b_{yw} - L_{w}\right) k + \left(\beta + \alpha\right)^2}}{2A \left(b_{yw} - L_{w}\right) k} \]  
\[ k = \frac{EI \left(b_{bf} + 2b_{w}\right)}{r_{bf}^2 \sin^2 \left(\theta_{w}\right) \left(b_{bf} - \frac{4}{3} r_{bf} \sin \left(\theta_{w}\right)\right)} \]  
\[ \alpha = f_{y} L_{bf}^2 \]  
\[ \beta = k L_{w} \left(C + B L_{w}\right) \left(b_{w} - L_{w}\right) \]  
\[ A = 0.0624 \]  
\[ B = -0.0101 \]  
\[ C = 0.5633 \]  

**Correction 1 of the ultimate load prediction**

Figure 13 shows that not only the modelled cross-section indents during loading, but also two parts adjacent to the modelled cross-section, over a length \( L_{w} \). The load needed to indent the cross-section equals \( F_{cst} / L_{b} \) per mm. Therefore, the load needed to indent a piece with width \( L_{w} \) equals \( F_{cst} L_{w} / L_{b} \). Because the indentation equals \( \Delta h_{w} \) at one end and zero at the other, it is estimated that only half the load is needed. Because there are two parts, the load to deform the two parts adjacent to the load-bearing plate, load \( F_{2p} \), simply equals:

\[ F_{2p} = F_{cst} \frac{L_{bf}}{L_{bf}} \]  

The ultimate load of the modelled cross-section \( F_{cst} \), can be corrected by adding the load \( F_{2p} \).

**Correction 2 of the ultimate load prediction**

Figure 13 shows that yield lines occur in the bottom flange of the sheet section. These yield lines dissipate energy, like the yield lines in the modelled cross-section. The yield lines in the bottom flange...
are shown in figure 12. For the moment, it is assumed that the bottom flange parts 1 and 3 do not rotate relative to each other.

![Diagram](image)

**Fig. 12. Simple model to predict the force $F_{bf}$ to deform the bottom flange.**

The extra force needed to form the yield lines in the bottom flange $F_{ybf}$ can be predicted as follows:

$$
\phi_d = \phi_e = \arcsin \frac{\Delta h_w}{L_{bf}}
$$

(13)

$$
\frac{\delta \phi_d}{\delta h_w} = \frac{\delta \phi_e}{\delta h_w} = \frac{1}{L_{bf} \left(1 - \left(\frac{\Delta h_w}{L_{bf}}\right)^2\right)^{\frac{3}{2}}} = \frac{1}{\sqrt{L_{bf}^2 - \Delta h_w^2}}
$$

(14)

$$
F_{ybf} = 2 \frac{2}{\sqrt{3}} \frac{f_y}{4} b y f \left(\frac{\delta \phi_d}{\delta h_w} + \frac{\delta \phi_e}{\delta h_w}\right)
$$

(15)

The ultimate load of the modelled cross-section $F_{ult}$ can be corrected by adding the load $F_{ybf}$.

**Correction 3 of the ultimate load prediction**

The load on the modelled cross-section $F_{ult}$ equals the load acting on the load-bearing plate $F$ plus an extra force $F_{ybf}$ due to indentation of the cross-section. Figure 13 illustrates this.

If the modelled cross-section deforms, yield lines develop in the bottom flange, which behave like hinges. Besides these yield lines, compressive forces develop in the bottom flange, due to the bending moment in the sheet section. These compressive forces, through the hinges, increase the force on the modelled cross-section. This increase of force strongly depends on the section length. Therefore, this effect will be defined as 'length effect'.

318
First, virtual displacements are used to predict the internal and external incremental energy. During an incremental change of the modelled cross-section indentation \( \Delta h_w \), the load \( F \) acting on the sheet section moves. Not only the distance \( \Delta h_w \) (which is only the case for the indented cross-section) but also for an extra displacement caused by the deflection of the sheet section. The incremental energy can be written as follows (using figure 13):

\[
\delta E_{e1} = F \delta h_w 
\]

\[
\delta E_{e2} = F \left( \delta h_w + \delta \phi \left( \frac{L_{\text{span}} - L_{bf}}{2} \right) \right) 
\]

\( \delta E_{e1} \) = incremental external energy cross-section only.

\( \delta E_{e2} \) = incremental external energy cross-section and sheet section deflection.

\( \delta h_w \) = incremental cross-section indentation.

\( \delta \phi \) = incremental sheet section rotation.

Influences of stress on yield line energy dissipation are neglected and it is assumed that the yield line pattern does not change geometrically during deformation. Then, because both mentioned external energy terms should equal the incremental internal energy and internal energy is equal for both cases, we can derive the following:

\[
\delta E_{e1} = \delta E_{e2} \Rightarrow
\]
\[ F_{cs} \delta h_w = F \left( \delta h_w + \delta p \left( \frac{L_{span} - L_{bf}}{2} \right) \right) \Rightarrow F = \frac{F_{cs} \delta h_w}{\delta h_w + \delta p \left( \frac{L_{span} - L_{bf}}{2} \right)} \Rightarrow \]

\[ F = F_{cs} \left( \frac{1}{1 + \delta p \left( \frac{L_{span} - L_{bf}}{2} \right)} \right) = F_{cs} \cdot f_1 \]

\[ f_1 = \text{length factor 1} \]

The factor \( \delta p / \delta h_w \) is complex and can only be predicted with complicated formulae (report [2], Appendix 3, section 3.3). The ultimate load of the modelled cross-section \( F_u \) is corrected by multiplying the load with factor \( f_1 \).

**Ultimate load \( F_u \)**

The ultimate load is now found as follows:

\[ F_u = (F_{cs} + F_{2p} + F_{ybf}) f_1 \]

\[ (19) \]

**Yield line distance \( L_{bf} \)**

The distance between yield lines \( L_{bf} \) is shown in figure 13. The distance \( L_{bf} \) can be determined by varying \( L_{bf} \) and finding the minimum value for the ultimate load \( F_u \) (see [1,2] for more information):

\[ L_{bf} = \frac{2f_p \mu^2 L_{bf} b_{pf} 2.601}{4F_{csu}} \]

\[ (20) \]

**Summary**

The ultimate load of a sheet section for the yield arc post-failure mode is predicted as follows:

Yield line distance \( L_{bf} \) is predicted by a mechanical model presented in section 4.3 of report [2].

The ultimate load for the modelled cross-section \( F_u \) is determined using formulae 5 to 11.

Distance \( L_{bf} \) is predicted by formula 20.

Load \( F_{2p} \) is predicted by formula 12.

Load \( F_{ybf} \) is predicted by formulae 13 to 15.

Factor \( f_1 \) is calculated by formula 18.

Load \( F_u \) is found by formula 19.
3.2 Mechanical model for the rolling post-failure mode

Elastic behaviour

Elastic behaviour is the same for sheet sections failing by the yield arc and rolling post-failure modes. Therefore, the formula presented in section 3.1 can be used.

Plastic behaviour

The plastic behaviour of the cross-section is modelled as shown in figure 14. Making use of the principle of virtual displacements, the plastic load \( F_p \) related to the cross-section indentation \( \Delta h_w \) equals:

\[
F_p = 2 \frac{\delta u_b}{\delta h_w} \frac{1}{\sqrt{3}} \frac{f y^2}{4} - L_{fb} + 2 \frac{\delta u_c}{\delta h_w} \frac{1}{\sqrt{3}} \frac{f y^2}{4} L_{fb} + 2 \frac{\delta u_b}{\delta h_w} \frac{1}{\sqrt{3}} \frac{f y^2}{4} - L_{fb}
\]

(21)

\[
\frac{\delta u_b}{\delta h_w} = \sin(\theta_w + \phi_c) \frac{\gamma f}{1 - \cos(\theta_w + \phi_c)}
\]

(22)

\[
\frac{\delta u_c}{\delta h_w} = \sin(\theta_w + \phi_c) \frac{\gamma f}{1 - \cos(\theta_w + \phi_c)} + \frac{\gamma f}{b_{ufl}} \Delta h_{ufl}
\]

(23)

\[
\Delta h_{ufl} = \frac{b_{ufl} (\cos \theta_w - \cos(\theta_w + \phi_c)) + \gamma f (\sin \theta_w + \phi_c - \sin(\theta_w + \phi_c))}{1 - \cos(\theta_w + \phi_c)}
\]

(24)

Calculating \( \phi_i \) as:

\[
\phi_c = -\frac{\Delta h_w \sin(\theta_w/2)}{\Delta h_w \cos(\theta_w/2) - b_w \sin(\theta_w/2)}
\]

(25)

\( \delta u \) = incremental movement of yield line \( i \).

\( r_f \) = corner radius bottom flange.

\( \Delta b \) = change of web width \( b_w \).

More information can be found in the report [2, Appendix 2, section 2.3.

Intersection of elastic and plastic curves

Formulae 1 and 21 can be set equal. The solution is shown by formula 26. The variable \( k \) is given by formula 6.

\[
F_{cul} = -k \left( L_{fb} \frac{2 f y^2}{\sqrt{3} \gamma f} \left( \frac{\theta_w}{2} \right) \sin \left( \frac{\theta_w}{2} \right) - b_w \left( L_{fb} \frac{2 f y^2}{\sqrt{3} \gamma f} \left( \frac{\theta_w}{2} \right) \cos \left( \frac{\theta_w}{2} \right) + b_w \cos(\theta_w) \right) \right)
\]

(26)
Corrections of the ultimate load prediction

The corrections of the ultimate load prediction for the rolling post-failure mode are the same as for the yield arc post-failure mode (section 3.1).

Summary

The ultimate load of a sheet section for the rolling post-failure mode is predicted as follows:

- The ultimate load for the modelled cross-section $F_{ult}$ is determined using formula 6 and 26.
- Distance $L_Y$ is predicted by formula 20.
- Load $F_{2a}$ is predicted by formula 12.
- Load $F_{2b}$ is predicted by formulae 13 to 15.
- Factor $f_2$ is calculated by formula 18.
- Load $F_{15}$ is found by formula 19.

3.3 Mechanical model for the yield eye post-failure mode

Elastic behaviour

Elastic behaviour is the same for sheet sections failing by the yield arc and yield eye post-failure modes. Therefore, the formula presented in section 3.1 can be used.

Plastic behaviour

The yield eye post-failure mode has an eye-like yield line pattern located on the bottom flange (see also section 2). In 1981, Murray and Khoo presented a paper that discussed some models to describe the behaviour of simple yield line patterns [8]. One of these patterns was called a flip-disc pattern and has a strong geometrical similarity to the eye-like yield line pattern of the yield eye post-failure mode. Figure 15 shows a thin-walled plate compressed by a force $F_p$. 

![Fig. 14. Rolling post-failure mode, plastic behaviour.](image-url)
According to Murray and Khoo, the force $F_{bf}$ can be predicted using the following formula:

$$F_{bf} = \frac{f_{yb} t b}{6} \left[ 1 - \frac{2\Delta}{t} + \sqrt{\left(\frac{2\Delta}{t}\right)^2 + 1} \right] + 4 \left[ 1 - \frac{3\Delta}{2t(1 + 4a^2/b^2)} \right]$$

(27)

With:
- $a = 0.2b$,
- $F_{bf} =$ compressive force [N],
- $\Delta =$ flip-disc out-of-plane deflection [mm],
- $b =$ plate width [mm],
- $a =$ flip-disc half width [mm],
- $t =$ steel plate thickness [mm],
- $f_{yb} =$ steel yield strength [N/mm²].

**Intersection of elastic and plastic curves**

Formula 1 describes the relationship between the concentrated load $F$ acting on the sheet section and the sheet section web crippling deformation $\Delta_{w}$. Formula 27 defines the load $F_{bf}$ acting on the bottom flange needed to form a plastic mechanism for a certain flip-disc out-of-plane deflection $\Delta$. Thus, elastic and plastic formulae have different load and deformation variables. A relationship between the load on the sheet section $F$ and the load on the bottom flange $F_{bf}$ should be developed. Furthermore a relationship between the elastic cross-section deformation variable $\Delta_{w}$ and the plastic flip-disc deformation variable $\Delta$ should be developed.

**Cross-section deformation versus flip-disc deformation**

Figure 16 shows a possible relationship for this: for elastic behaviour, it is assumed that a certain width adjacent to the modelled cross-section will deform like the modelled cross-section. This certain width is set equal to the distance $2a$ between yield lines in the bottom flange during plastic deformation. This leads to the following derivation:
\[
\frac{2\Delta}{2a} = \frac{\Delta h_w}{2a} \Rightarrow \Delta = 0.5\Delta h_w
\] (28)

\(\Delta = \) flip-disc out-of-plane deflection [mm].
\(\Delta h_w = \) web crippling deformation [mm].

**Load at section versus load at bottom flange**

Looking at figure 16 it can be seen that the external bending moment in the section equals:

\[
M_e = \frac{FL\text{span}}{4}
\] (29)

\(M_e = \) external bending moment [Nmm]
\(F = \) concentrated load of support on section [N]
\(L\text{span} = \) span length [mm]

This gives the following assumptions:

* One concentrated load \(F\) models the load of the load-bearing plate.
* The flip-disc occurs in the position of this concentrated load, i.e., the location of highest bending moment.

The internal bending moment in the section can be derived as follows:

\[
\frac{Fx\text{bf}}{bx\text{bf} * s} = \frac{M_I}{I_s} \Rightarrow M_I = \frac{Fx\text{bf} I_s}{bx\text{bf} * s * t}
\] (30)

\(M_I = \) internal bending moment [Nmm].
\(bx\text{bf} = \) bottom flange width [mm].
\(I_s = \) moment of inertia [mm\(^4\)].
\(s = \) distance of bottom flange to centre of gravity sheet section [mm].

Because the internal and external bending moment should be equal, it can be derived that:

\[
\frac{FL\text{span}}{4} = \frac{Fx\text{bf} I_s}{bx\text{bf} * s * t} \Rightarrow F = \frac{4Fx\text{bf} I_s}{L\text{span} bx\text{bf} * s * t}
\] (31)
Fig. 16. Relationship between elastic cross-section deformation and plastic flip-disc deflection.

Intersection of elastic and plastic curves

Formula 28 can be substituted into formula 1 describing elastic sheet section behaviour. This results in the following:

\[
F_e = \frac{E I (y_{bf} + 2b_w) \Delta}{r_{bf}^2 \sin^2 (\theta) \left( b_{bf} - \frac{4}{3} r_{bf} \sin(\theta_w) \right)} \tag{32}
\]

Formula 27 can be substituted into formula 31. This results in:

\[
F_p = \frac{4F_{bf}^L}{L_{span} b_{bf} s + b'} \tag{33}
\]
If the elastic load $F_e$ and the plastic load $F_p$ are set as equal, the flip-disc out-of-plane displacement can be solved. Then the ultimate sheet section load $F_u$ can be calculated by using the value for $\Delta$ into formulae 32 or 33.

### 3.4 Verification of post-failure models

Experiments of section 2 are used failing by the yield arc post-failure mode. Table 1 shows the correlation, average, etc. for the experiments and Eurocode3 [9] predictions. Table 2 shows the experiments and the yield arc post-failure model predictions as presented in section 3.1.

**Table 1. Eurocode3 predictions for experiments failing by the yield arc post-failure mode.**

<table>
<thead>
<tr>
<th>Experiments</th>
<th>Correlation</th>
<th>Average</th>
<th>Standard deviation</th>
<th>Coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eurocode3</td>
<td>0.95</td>
<td>0.93</td>
<td>0.09</td>
<td>0.09</td>
</tr>
</tbody>
</table>

**Table 2. Post-failure model predictions for experiments failing by the yield arc post-failure mode.**

<table>
<thead>
<tr>
<th>Experiments</th>
<th>Correlation</th>
<th>Average</th>
<th>Standard deviation</th>
<th>Coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model MA1</td>
<td>0.70</td>
<td>1.32</td>
<td>0.36</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Eurocode3 predictions are better than the post-failure model predictions. Section 4 will suggest some differences between Eurocode3 and the models as a possible cause for the differences in performance.

Experiments of Bakker's thesis [5] are used that are failing by the rolling post-failure mode. Furthermore, the 7 experiments of section 2 (failing by the rolling post-failure mode) are used. Table 3 shows the correlation, average, etc. for the experiments and Eurocode3 predictions. The ultimate load of the experiments $F_{ult}$ is used for the comparison, not the mode initiation load $F_{init}$ because Eurocode3 predicts the ultimate load for sheet sections. In fact, almost all Bakker experiments do not satisfy the conditions for using Eurocode3 (see thesis [1] chapter 2 for more details). However, to have some possibilities to compare the Eurocode3 and the post-failure models, the experiments are still used.
Table 3. Eurocode3 predictions for experiments failing by the rolling post-failure mode.

<table>
<thead>
<tr>
<th>Bakker experiments (28) and experiments section 2 (7)</th>
<th>Correlation</th>
<th>Average</th>
<th>Standard deviation</th>
<th>Coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eurocode3</td>
<td>0.67</td>
<td>0.95</td>
<td>0.19</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Table 4 shows the experiments and the rolling post-failure model as presented in section 3.2. Now, the mode initiation load $F_{in}$ is used for the comparison, because the post-failure models predict the mode initiation load.

Table 4. Post-failure model predictions for experiments failing by the rolling post-failure mode.

<table>
<thead>
<tr>
<th>Bakker experiments (28) and experiments chapter 3 (7)</th>
<th>Correlation</th>
<th>Average</th>
<th>Standard deviation</th>
<th>Coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model MR1</td>
<td>0.85</td>
<td>0.83</td>
<td>0.18</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Model predictions are much better than the Eurocode3 predictions. However, Eurocode3 predicts the ultimate load, the post-failure models predict the mode initiation load.

Experiments of section 2 are used that failed by the yield eye post-failure mode. Table 5 shows the correlation, average, etc. for the experiments and Eurocode3 predictions. Table 6 shows the experiments and the yield eye post-failure model predictions as presented in section 3.3.

Table 5. Eurocode3 predictions for experiments failing by the yield eye post-failure mode.

<table>
<thead>
<tr>
<th>Experiments E (7)</th>
<th>Correlation</th>
<th>Average</th>
<th>Standard deviation</th>
<th>Coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eurocode3</td>
<td>0.87</td>
<td>0.91</td>
<td>0.10</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Table 6. Post-failure model predictions for experiments failing by the yield eye post-failure mode.

<table>
<thead>
<tr>
<th>Experiments E (7)</th>
<th>Correlation</th>
<th>Average</th>
<th>Standard deviation</th>
<th>Coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model ME1</td>
<td>0.91</td>
<td>1.03</td>
<td>0.37</td>
<td>0.36</td>
</tr>
</tbody>
</table>
Both the Eurocode3 and the yield eye post-failure model predict the experimental values well. The standard deviation of the post-failure model is significantly higher than for Eurocode3. However, these conclusions are based on 7 experiments only.

The post-failure models presented cannot be used as design rules because their performance is less than the current design rule Eurocode3. Another mechanical model, using the same principles, has been developed to be a future design rule [3]. Section 4 will show that the post-failure models are useful to increase the insight in sheeting behaviour.

4 Post-failure modes for changing variables

In section 3, a post-failure mechanical model was presented for each post-failure mode. Now, these three models are used for sheeting for which one variable value is changed. The mechanical model that predicts the lowest load is regarded as predicting the post-failure mode that is occurring. Every variable is presented in a separate section.

Section 3.4 shows that the averages of model predicted values divided by experimental values are not equal to 1.0. However, to compare predictions of the models, the averages had to be 1.0, which involves calibrating the models. The yield arc model predictions are divided by 1.32 (see table 2). The rolling model predictions are divided by 0.83 (see table 4) and the yield eye model predictions are divided by 1.03 (see table 6).

4.1 Corner radius
Experiments

Figures 4 and 8 in section 2 presented the post-failure mode for each experiment. Firstly, figures 4 and 8 indicate that for a larger corner radius (from 5 to 10 mm) a yield arc post-failure mode (A>R) changes into a rolling post-failure mode (R) (experiments 6 and 7-13). A yield arc post-failure mode followed by a rolling post-failure mode (A>R) was defined as a yield arc post-failure mode.

Secondly, the table indicates that a yield eye post-failure mode (E) changes into a yield arc post-failure mode (A) for larger corner radii. The changes of (A>E) into (A) point in the same direction: for small corner radii the yield arc post failure mode is still followed by a yield eye one (A>E), but for larger corner radii only a full yield arc post-failure mode occurs (A), experiments 55, 58, 59, and 68.

Post-failure mechanical models: yield arc into rolling

The three post-failure models are used for a sheet section with the following variable values. The corner radius $r_{bf}$ changes between 1 and 15 mm. Other variables are fixed: $b_1=100$ mm, $b_3=100$ mm, $b_5=100$ mm, $\theta_w=90$ degrees, $L_1=50$ mm, $L_{open}=600$ mm, $t=0.68$ mm, $f_f=340$ N/mm².
Figure 17 shows the results. On the left, the model predictions are shown. It can be seen that the yield arc model (MA1) gives the lowest values for all corner radii. Thus, it is predicted that the yield arc post-failure mode occurs for all corner radii. However, for an increasing corner radius, model MA1 predicts smaller distances $L_w$ (see figure 18). For corner radii larger than 8 mm, distance $L_w$ even equals to the corner radius. This is also shown in figure 18. A yield arc post-failure mode then is quite similar to the rolling post-failure mode. This is also indicated by the curve of the rolling model (MR1), the model for the rolling post-failure mode. This curve converges to the curve of the yield arc model.

Figure 17 on the right shows the experimental ultimate loads of tests 6 and 7-13 (see also figure 4). For the rolling post-failure modes (R) the experimentally found mode initiation load is used, not the ultimate load. The same trend (for ultimate load / mode initiation values and post-failure modes) is followed as the post-failure models indicate.

Post-failure mechanical models: yield eye into yield arc.

The three post-failure models are used for a sheet section with the following variable values. The corner radius $r_c$ changed between 1 and 15 mm. Other variables are fixed: $b_r=100$ mm, $b_t=100$ mm, $b_f=100$ mm, $\theta_w=90$ degrees, $L_{bf}=100$ mm, $L_{posc}=2400$ mm, $t=0.68$ mm, $f_s=340$ N/mm$^2$.

Fig. 18. A yield arc post-failure mode with a small distance $L_w$ is quite similar to a rolling post-failure mode. Distance $L_w$ cannot be smaller than the corner radius.
Figure 19 shows the results. On the left, the model predictions are shown. It can be seen that first the yield eye model gives the lowest values, followed by the yield arc model. Thus, it is predicted that first the yield eye post-failure mode occurs and then the yield arc post-failure mode. This is in accordance with the experimental findings.

Figure 19 on the right shows the experimental ultimate loads of tests 55, 58, 59, and 68 (see also figures 4 and 8). The same trend (for ultimate load values and post-failure modes) is followed as the post-failure models indicate.

**Behavioural insight**

Figures 17 and 19 show why a specific post-failure mode occurs for a certain value of the corner radius. However, to really understand why this is the case, it is important to know why the lines in the figure increase, decrease, or remain the same for different corner radii.

It was shown that for an increasing corner radius, a yield arc post-failure mode changes into a rolling post-failure mode. This because distance $L_e$ (figure 18) decreases for an increasing corner radius. The mechanical model in [2], Appendix 4, section 4.3.1, explains why distance $L_e$ decreases. The explanation is quite elaborate and will not be repeated here.

It was also shown that for an increasing corner radius, a yield eye post-failure mode changes into a yield arc post-failure mode. This is explained below.

All three models predict decreasing strength for an increasing corner radius. This can be explained because for a larger corner radius the elastic web crippling stiffness decreases. Because the ultimate load is predicted by intersection of the elastic and plastic curve, and the plastic curves are decreasing, the predicted ultimate load will decrease too.

It cannot be easily determined why the strength according to the yield arc model decreases slightly more strongly than the strength predicted by the yield eye model (the reason why for large corner radii the yield arc post-failure occurs).
Conclusions

Experiments show that for an increasing corner radius, a yield arc post-failure mode is followed by a rolling post-failure mode. Furthermore, they show that a yield eye post-failure mode is followed by a yield arc post-failure mode.

The post-failure mechanical models show that for an increasing corner radius, a yield arc post-failure mode is followed by a rolling post-failure mode. This because the yield arc model predicts a lower distance $L_w$. Why this model predicts a lower distance $L_w$ is explained in [2], Appendix 4, section 4.3.1.

The post-failure mechanical models show that for an increasing corner radius, a yield eye post-failure mode is followed by a yield arc post-failure mode.

4.2 Span length

Experiments

Figures 4 and 8 show that for a larger span length ($A>R$) changes into ($A$), experiments 1, 3, 22, and 24. This means that for a small span length a yield arc post-failure mode is followed by a rolling post-failure mode. For a larger span length a pure yield arc post-failure mode occurs. For even larger span lengths ($A$) changes into ($A>E$), for instance experiments 43 and 55. This means that for larger span lengths, a pure yield arc post-failure mode changes into a yield arc post-failure mode followed by a yield eye post-failure mode. Sometimes, a pure yield eye post-failure mode occurs for long span lengths.

Post-failure mechanical models: rolling into yield arc

Unfortunately, it was not possible to find a set of sheet section variable values to show a change of the rolling into a yield arc post-failure mode for increasing span length.

Post-failure mechanical models: yield arc into yield eye

The three post-failure models are used for a sheet section with the following variable values. The span length $L_w$ is changed between 600 and 2400 mm. Other variables are fixed: $b_p=100$ mm, $t_p=3$ mm, $b_r=100$ mm, $b_p=100$ mm, $\theta_p=90$ degrees, $L_w=100$ mm, $t=0.68$ mm, $f_s=340$ N/mm². Figure 20 shows the results. On the left, the model predictions are shown. It can be seen that first the yield arc model (MA1) produces the lowest values, then the yield eye model (ME1). Thus, it is predicted that for increasing span length, the yield arc post-failure mode changes into a yield eye post-failure mode. This is in accordance with the findings of the experiments.
Figure 20 on the right shows the experimental ultimate loads of tests 43 and 55 (see also figures 4 and 8). The same trend for ultimate load values and post-failure modes is observed as the post-failure models indicate.

**Behavioural insight**

For long span lengths, the stresses in the bottom flange increase strongly. Therefore the plastic load for the yield eye model is reached more easily. Although the rolling and yield arc post-failure mechanical models decrease in strength (due to the length factor), this effect is not as strong as the influence of increasing stress for the yield eye model.

**Conclusions**

Experiments show that for an increasing span length, a yield arc post-failure mode is followed by a yield eye post-failure mode. Furthermore, they suggest that a rolling post-failure mode is followed by a yield arc post-failure mode. The post-failure mechanical models show results in accordance with the experiments.

4.3 **Angle between web and flange**

**Experiments**

Figures 4 and 8 show that for an increasing angle between web and flange (A>R) changes into (R), experiments 7-13 and 14. Furthermore, (A) changes into (A>E) and (E), experiments 40, 41, 56, and 57. This means that, for an increasing angle, the yield arc post-failure mode changes into a yield eye or a rolling post-failure mode.

**Post-failure mechanical models: yield arc into yield eye**

Unfortunately, it was not possible to find a set of sheet section variable values to show a change of a yield arc into a yield eye post-failure mode for increasing angle between web and flange.

**Post-failure mechanical models: yield arc into rolling**

Post-failure models MA1, MR1, and ME1 are used for a sheet section with the following variable values. The angle between web and flange $\theta_0$ changes between 50 and 90 degrees. Other variables
are fixed: \(b_w=100\, \text{mm}, \ r=10\, \text{mm}, \ b_s=100\, \text{mm}, \ L_a=50\, \text{mm}, \ L_{sep}=50\, \text{mm}, \ t=0.68\, \text{mm},\)

\(f_y=340\, \text{N/mm}^2\). Figure 21 shows the results. On the left, the model predictions are shown. It can be seen that the yield arc model (MA1) always gives the lowest values. However, distance \(L_{w1}\) in the yield arc model decreases until the corner radius value is reached. Similar to the situation for the changing corner radius, it is then likely that the yield arc model predicts a rolling post-failure mode. Thus, the sequence is a yield arc followed by a rolling post-failure mode. This is in accordance with the findings of the experiments. Figure 21 on the right shows the experimental ultimate loads of tests 7-13 and 14 (see also figures 4 and 8). The same trend for ultimate load values and post-failure modes is followed as the post-failure models indicate.

![Graph showing model predictions for changing angle between web and flange.](image)

**Fig. 21. Model predictions for changing angle between web and flange.**

**Behavioural insight**

For an increasing angle between web and flange, the stiffness of the cross-section decreases (see formula 1) which results in lower ultimate loads. This is true for both the yield arc and rolling models.

However, for the yield arc model, formulae in [2], Appendix 4, section 4.3.1 can be used to show that distance \(L_{w1}\) reduces, resulting in higher ultimate loads. As figure 21 shows, the decrease of cross-section stiffness and reduction of distance \(L_{w1}\) more or less compensate each other with a constant ultimate load.

For the rolling model there is another explanation for decreasing ultimate loads for increasing angle. Figure 22 shows that if the angle between web and flange \(\theta_s < 90\) degrees, deformation of the cross-section leads to the web pushing the bottom flange downwards. This downward movement is in the opposite direction to the direction in which the force is working. Thus, smaller angles increase the load.
Fig. 22. For the rolling post-failure mode, an angle between web and flange causes the web pushing the bottom flange downwards, thus increasing the load.

Conclusions

Experiments show that for an increasing angle between web and flange, a yield arc post-failure mode is followed by a rolling post-failure mode. Furthermore, they show that a yield arc post-failure mode is followed by a yield eye post-failure mode.

The post-failure mechanical models also show that a yield arc post-failure mode is followed by a rolling post-failure mode. This because the distance $L_*$ will reduce for larger angles between web and flange. An explanation why this distance shows reduction is too elaborate to explain here, but formulae in [2], Appendix 4, section 4.3.1 can be used to show this.

4.4 Load-bearing plate width

Experiments

Figures 4 and 8 show that for an increasing load-bearing plate width a yield arc post-failure mode (A) changes into a yield arc followed by a yield eye post-failure mode (A->E), experiment 71 and 72. This indicates that for increasing load-bearing plate width, a yield arc post-failure mode is followed by a yield eye post-failure mode.

Post-failure mechanical models: yield arc into yield eye

The three post-failure models are used for a sheet section with the following variable values. The load-bearing plate width $L_*$ is changed between 50 and 150 mm. Other variables are fixed: $h_1=100$ mm, $r_1=10$ mm, $h_0=100$ mm, $r_0=100$ mm, $\theta_0=50$ degrees, $L_m=2400$ mm, $l=0.68$ mm, $f_0=340$ N/mm².

Figure 23 shows the results. On the left, the model predictions are shown. It can be seen that first the yield arc model (MA1), then the yield eye model (ME1) gives the lowest values. Thus, the sequence is a yield arc followed by a yield eye post-failure mode. This is in accordance with the findings of the experiments.
Figure 23 on the right shows the experimental ultimate loads of tests 71 and 72 (see also figures 4 and 8). The same trend for ultimate load values and post-failure modes is observed as the post-failure models indicate.

![Graph showing load-bearing plate width vs. load](image)

**Fig. 23. Model predictions for changing load-bearing plate width.**

**Behavioural insight**

Figure 23 on the left shows that for a decreasing load-bearing plate width, the rolling model values decrease more than the yield arc model values. If the lines were to cross, this would suggest a rolling to a yield arc post-failure mode sequence. Unfortunately, there are no experiments that can prove this suggestion.

If the load-bearing plate width increases, the rolling and yield arc models increase their values, because the modelled cross section (see figure 9) is wider. But the yield eye model (ME1) is not sensitive to increasing the load-bearing plate width, see formulae 32 and 33. Finally, the yield eye model predicts lower values than the yield arc model.

**Conclusions**

Experiments suggest that for an increasing load-bearing plate width, a yield arc post-failure mode is followed by a yield eye post-failure mode. The post-failure mechanical models also show that a yield arc post-failure mode is followed by a yield eye post-failure mode. This is because for the yield eye post-failure mode, only the bending moment in the section reduces due to the larger load-bearing plate width. For the yield arc post-failure mode, the modelled cross-section is larger.

5 **Conclusions**

New experiments have been carried out in which sheet sections were loaded by concentrated load and bending moment. The experiments showed that after ultimate load, three post-failure modes occur.

Mechanical models have been developed for all three post-failure modes. The models do not perform as good as Eurocode3, but they can be used to increase insight in sheeting behaviour.
For some sheet section variables, namely the corner radius, span length, angle between web and flange, and the load-bearing plate width, experiments and the post-failure mechanical models predict the same sequence of post-failure modes for a changing variable. The mechanical models provide insight into why a specific post-failure mode occurs.

Acknowledgements

This research was supported by the Technology Foundation STW, applied science division of NWO and the technology programme of the Ministry of Economic Affairs in The Netherlands.

Literature