Sequence-Decoupled Resonant Controller for Three-phase Grid-connected Inverters

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Abstract—This paper proposes a set of sequence-decoupled resonant controllers constructed with an easy-to-implement structure. It shows that either positive-sequence or negative-sequence components of three-phase unbalanced AC signals can be regulated individually in the stationary frame, while conventional and widely used resonant controllers cannot separately cope with these two sequences. Consequently, the extraction of the corresponding sequence component is not necessary for an individual sequence’s control. Moreover, controllers designed for positive and negative sequences can be assigned with different gains to meet practical applications. The principle of new sequence-decoupled resonant controllers is described based on a multi-state-variable structure. In addition, the relation with conventional resonant controllers is investigated. The controllers are found to be equivalent when dealing with both sequences together. Furthermore, two examples are presented to show the application of the proposed controller. The effectiveness of the proposed new controller and its performance in the two system applications are verified by experiments.

I. INTRODUCTION

For three-phase grid-connected inverters, proportional and integral (PI) controllers in the synchronous reference frame (SRF) are conventionally used for output current control. By transforming the feedback signals into the SRF, where they become dc quantities, steady-state errors can be effectively eliminated with a high DC gain. However, to deal with unbalanced grid conditions in the same manner, it is required to double controllers, cross decoupling, and the transformations between reference frames.

As a consequence, stationary frame proportional resonant (PR) controllers were introduced to regulate AC signals without any transformation to SRF. Also, PR controllers are able to achieve good reference tracking performance and control stability due to a very high gain around the resonance frequency [1] - [5]. In addition, a certain amount of frequency insensitivity can be brought by using a wider bandwidth of the resonant controllers. Compared with conventional PI controllers in the SRF, the complexity of the grid current control with PR controller is considerably reduced. It is a characteristic of resonant controllers to regulate positive-sequence and negative-sequence quantities jointly.

However, in the situations where only individual symmetrical sequence quantities need to be regulated, the currently applied resonant controllers lose their advantage since they cannot regulate positive- or negative-sequence components separately. For instance, in the system proposed in [6] and [7], a series connected inverter in between of the upstream grid and the downstream micro-grid is equipped with the function of eliminating negative-sequence currents. Instead of the PR controller, a PI controller in the negative SRF has to be employed, thereby getting back to the conventional method.

In this paper, further consideration reveals that better system performance may be achieved if the gains for each sequence quantity can be adjusted individually. Also, an easier implementation structure can facilitate the use of second-order resonant controllers when only positive sequence or negative sequence quantities need to be controlled. Therefore, this paper intends to propose a concept where the a-b-c quantities of a three-phase three-wire system can be controlled in terms of independent symmetrical positive and negative sequences in the stationary frame, not only in terms of two independent $\alpha$ and $\beta$ variables.

For this purpose, a sequence-decoupled resonant (SDR) controller is introduced, which can deal with each sequence component individually. It is demonstrated that the proposed controllers are equivalent to the conventional resonant controllers but can be constructed with a multi-state-variable structure, which is easier to implement. Furthermore, two practical systems are presented for the application of the SDR controllers. Finally, experiments are carried out to verify the effectiveness of the presented controller and its application.

II. STATIONARY FRAME SEQUENCE-DECOUPLED RESONANT CONTROLLERS

The principle of a stationary frame resonant controller is to achieve a compensator which has large gain and zero phase at the fundamental frequency of sinusoidal error signals. It can be derived from either the direct stationary frame integrator [3], or from the inverse transformation of the synchronous controller back to the stationary frame [1], [4]. This section starts with explaining the idea of the second method and then the SDR controller is derived.
A. Ideal SDR Controller

In general, a set of feedback error signals may consist of fundamental positive-sequence components, negative-sequence components, and harmonics, as expressed in the α-β frame and written with complex variables as

$$\mathcal{E}_{\alpha\beta}(t) = e_{\alpha}(t) + je_{\beta}(t)$$

$$= \sum_{n=1,3,\ldots}^{\infty} (E^+ e^{j\omega_1 t} + E^- e^{-j\omega_1 t}),$$

where $n$ is the harmonic number, $\omega_1$ the fundamental radian frequency; complex numbers are denoted with a bar subscript, and superscript $"^+"$ and $"^-"$ denote positive and negative sequences, respectively.

It is well known in the literature that a proportional (P) and a resonant (R) controller are combined as a PR controller for the control of error feedback, where most of the high frequency or transient response of the regulator is determined by the proportional gain and the resonant controller gain determines the steady-state error. Therefore, the SDR controller in this paper also cooperate with a P controller. Since the P controller remains the same function without regarding the reference frame, only the SDR controller part is demonstrated below.

We transform the signal $\mathcal{E}_{\alpha\beta}(t)$ to the SRF by multiplying (1) with $e^{-j\omega_1 t}$ and $e^{j\omega_1 t}$, which corresponds to positive and negative SRF components, respectively. That is,

$$\mathcal{E}_{\alpha\beta}(t)e^{-j\omega_1 t} = E^+ + \sum_{n=2,4,\ldots}^{\infty} (E^+ e^{jn\omega_1 t} + E^- e^{-jn\omega_1 t}),$$

$$\mathcal{E}_{\alpha\beta}(t)e^{j\omega_1 t} = E^- + \sum_{n=2,4,\ldots}^{\infty} (E^- e^{jn\omega_1 t} + E^+ e^{-jn\omega_1 t}),$$

(2)

It can be seen that the fundamental positive- and negative-sequence error signals appear as DC quantities in the respective synchronous frames. In order to eliminate the error once the control loop is closed, an ideal integrator can be used to generate an infinite gain for the DC components in (2), as expressed in the frequency domain by

$$\mathcal{Y}^+_{\alpha\beta1}(s + j\omega_1) = \mathcal{E}_{\alpha\beta}(s + j\omega_1) \cdot \frac{K_I}{s},$$

$$\mathcal{Y}^-_{\alpha\beta1}(s - j\omega_1) = \mathcal{E}_{\alpha\beta}(s - j\omega_1) \cdot \frac{K_I}{s},$$

(3)

where $\mathcal{Y}^+_{\alpha\beta1}(s + j\omega_1)$ and $\mathcal{Y}^-_{\alpha\beta1}(s - j\omega_1)$ denote positive- and negative-sequence output signals, respectively, in corresponding synchronous frames, and $K_I$ is the integrator gain.

To transform the integrator back to the stationary frame, we substitute $s \leftarrow s - j\omega_1$ into the first equation of (3), and $s \leftarrow s + j\omega_1$ into the second one. It follows that

$$\mathcal{Y}^+_{\alpha\beta1}(s) = \mathcal{E}_{\alpha\beta}(s) \cdot \frac{K_I}{s - j\omega_1},$$

$$\mathcal{Y}^-_{\alpha\beta1}(s) = \mathcal{E}_{\alpha\beta}(s) \cdot \frac{K_I}{s + j\omega_1}.$$ 

(4)

By expanding the complex variables in (4), the following equations are derived:

$$s \cdot \mathcal{Y}^+_{\alpha\beta1}(s) + \omega_1 \cdot \mathcal{Y}^+_{\beta\beta1}(s) = K_I \cdot e_{\alpha}(s),$$

$$s \cdot \mathcal{Y}^-_{\alpha\beta1}(s) - \omega_1 \cdot \mathcal{Y}^-_{\beta\beta1}(s) = K_I \cdot e_{\beta}(s),$$

$$s \cdot \mathcal{Y}^-_{\alpha\beta1}(s) - \omega_1 \cdot \mathcal{Y}^-_{\beta\beta1}(s) = K_I \cdot e_{\alpha}(s),$$

$$s \cdot \mathcal{Y}^+_{\alpha\beta1}(s) + \omega_1 \cdot \mathcal{Y}^+_{\beta\beta1}(s) = K_I \cdot e_{\beta}(s).$$

(5)

(6)

Instead of solving (5) and (6) directly, they can be represented in a multi-state-variable structure as

$$\mathcal{Y}^+_{\alpha\beta1}(s) = \frac{1}{2} [K_I \cdot e_{\alpha}(s) - \omega_1 \cdot \mathcal{Y}^+_{\beta\beta1}(s)],$$

$$\mathcal{Y}^-_{\beta\beta1}(s) = \frac{1}{2} [K_I \cdot e_{\beta}(s) + \omega_1 \cdot \mathcal{Y}^+_{\beta\beta1}(s)],$$

(7)

$$\mathcal{Y}^+_{\beta\beta1}(s) = \frac{1}{2} [K_I \cdot e_{\beta}(s) + \omega_1 \cdot \mathcal{Y}^-_{\beta\beta1}(s)],$$

$$\mathcal{Y}^-_{\alpha\beta1}(s) = \frac{1}{2} [K_I \cdot e_{\alpha}(s) - \omega_1 \cdot \mathcal{Y}^-_{\beta\beta1}(s)].$$

(8)

In this manner, equations (7) and (8) can then be easily implemented in the α-β frame in the time domain by digital techniques. Fig. 1 shows the implementation structure diagrams of the controllers for regulating positive-sequence, negative-sequence components or for both sequence components. This controller is referred to as an ideal SDR controller, since it has infinite gain at the central frequency $\omega_1$ (see Fig. 2).

B. Practical SDR Controller

In practice, for improving the controller’s insensitivity to deviation from the central frequency, a non-ideal integrator instead of an ideal one should be used. Hence, the equations in (3) are rewritten as

$$\mathcal{Y}^+_{\alpha\beta1}(s + j\omega_1) = \mathcal{E}_{\alpha\beta}(s + j\omega_1) \cdot \frac{K_I \cdot \omega_1}{s + j\omega_1},$$

$$\mathcal{Y}^-_{\alpha\beta1}(s - j\omega_1) = \mathcal{E}_{\alpha\beta}(s - j\omega_1) \cdot \frac{K_I \cdot \omega_1}{s - j\omega_1}.$$ 

(9)

Fig. 1. Backward-coupled implementation structure diagrams of ideal SDR controllers for regulating (a) fundamental positive-sequence components, (b) fundamental negative-sequence components, and (c) both sequence components.
Fig. 3. Backward-coupled implementation structure diagrams of practical SDR controllers for regulating (a) fundamental positive-sequence components, (b) fundamental negative-sequence components, and (c) both sequence components.

Again, using the multi-state-variable structure, two groups of equations are obtained as

\[
\begin{align*}
    y_{a1}^+ (s) &= \frac{1}{\omega_0} \left[ \omega_0 (K_1 \cdot e_{a}(s) - y_{a1}^+ (s)) - \omega_1 \cdot y_{b1}^+ (s) \right], \\
    y_{b1}^+ (s) &= \frac{1}{\omega_0} \left[ \omega_0 (K_1 \cdot e_{b}(s) - y_{a1}^+ (s)) + \omega_1 \cdot y_{a1}^+ (s) \right], \\
    y_{a1}^- (s) &= \frac{1}{\omega_0} \left[ \omega_0 (K_1 \cdot e_{a}(s) - y_{a1}^- (s)) + \omega_1 \cdot y_{b1}^- (s) \right], \\
    y_{b1}^- (s) &= \frac{1}{\omega_0} \left[ \omega_0 (K_1 \cdot e_{b}(s) - y_{b1}^- (s)) - \omega_1 \cdot y_{a1}^- (s) \right].
\end{align*}
\]

The corresponding implementation structures are shown in Fig. 3. Because of the finite gain at the central frequency, as shown in Fig. 2, this is referred to as a practical or non-ideal SDR controller. The bandwidth of the controller around the resonance frequency is higher than for the ideal SDR controller, thereby being robust to small frequency variations. This point is similar to the conventional non-ideal R controller.

III. RELATION TO CONVENTIONAL RESONANT CONTROLLERS

As derived in the previous section, SDR controllers make the independent control of positive-sequence and negative-sequence components possible in the stationary frame. The advantage of this individual sequence control by using SDR controllers will be further explained in the next section. Nevertheless, for dealing with both sequences, it is important to understand the relation between conventional R controllers and the proposed ones of Fig. 1 (c) and Fig. 3 (c). Note that (10) and (11) can be directly solved and the two groups of unknown variables are found to be

\[
\begin{align*}
    y_{a1}^+ (s) &= \frac{K_{1s} \omega_1 (s+\omega_c)}{(s+\omega_c)^2 + \omega_0^2} e_{a}(s) - \frac{K_{1s} \omega_0 \omega_c}{s^2 + 2\omega_0 s + \omega_0^2} e_{b}(s), \\
    y_{b1}^+ (s) &= \frac{K_{1s} \omega_1 (s+\omega_c)}{(s+\omega_c)^2 + \omega_0^2} e_{a}(s) - \frac{K_{1s} \omega_0 \omega_c}{s^2 + 2\omega_0 s + \omega_0^2} e_{b}(s), \\
    y_{a1}^- (s) &= \frac{K_{1s} \omega_1 (s+\omega_c)}{(s+\omega_c)^2 + \omega_0^2} e_{a}(s) + \frac{K_{1s} \omega_0 \omega_c}{s^2 + 2\omega_0 s + \omega_0^2} e_{b}(s), \\
    y_{b1}^- (s) &= \frac{K_{1s} \omega_1 (s+\omega_c)}{(s+\omega_c)^2 + \omega_0^2} e_{a}(s) + \frac{K_{1s} \omega_0 \omega_c}{s^2 + 2\omega_0 s + \omega_0^2} e_{b}(s).
\end{align*}
\]
where the approximations are allowed when $\omega_b \ll \omega_1$. Therefore, another implementation of the practical SDR controller is possible, as shown in Fig. 4 (a) and (b).

By combining (14) and (16), (15) and (17) we can have

$$y_{\alpha 1} (s) = y_{\alpha 1}^+ (s) + y_{\alpha 1}^- (s) \approx \frac{2K_1 \omega_1 s}{s^2 + 2\omega_b s + \omega_1^2} e_\alpha (s),$$

$$y_{\beta 1} (s) = y_{\beta 1}^+ (s) + y_{\beta 1}^- (s) \approx \frac{2K_1 \omega_1 s}{s^2 + 2\omega_b s + \omega_1^2} e_\beta (s).$$

(18)

(19)

It can be seen that the approximated SDR controllers in (18) and (19), as shown in Fig. 4 (c), are identical to the conventional resonant controllers [1]. However, it is worth noticing that SDR controllers used for both sequence components in Fig. 3 (c) provide the possibility of assigning different gains for the positive and negative sequences according to practical necessities while R controllers cannot.

In the same way, the ideal SDR controllers are studied and their corresponding implementation structure diagrams are shown in Fig. 5. Also, the controllers in Fig. 5(c) are identical to the conventional ideal resonant controllers.

IV. PRACTICAL APPLICATIONS

To illustrate the applicability of SDR controllers in practical situations and to highlight the controller design, two examples are discussed in detail in this section. Since the basic characteristics of SDR controllers are the same as for resonant controllers, the general control structure of the system in closed-loop control can be similar to designs with conventional PR controllers [8]. The main difference is that the SDR controllers will substitute the parts in which the R controllers are not convenient to handle individual sequences separately, aiming at giving another alternative to simplify the control.

A. Parallel Grid-connected Inverter

In general, most distributed generations are coupled to the grid through parallel grid-connected inverters, as shown in the structure diagram of Fig. 6. An LCL coupling filter is usually used for smoothing the output current. The primary source is represented by a dc source $V_{dc}$, from which the energy is derived from a upstream single-stage or multi-stage power converter system. Notice that isolation between the grid and the primary source is normally required either on the grid side...
or on the DC side, but this part is left out from the diagram for simplification.

The control part consists of an outer-loop grid current control and an additional inner loop if necessary for improving system performance. Since this section focuses on the current regulation by the SDR controller, neither the inner-loop control, nor the dc-link voltage control is discussed, supposed to be regulated by the upstream converter system or by the inverter itself. As shown in Fig. 6, the outer loop is illustrated with a proportional (P) controller and a SDR controller. Specifically, either the positive SDR controller or the positive plus negative SDR controller is chosen according to different grid conditions.

For a balanced utility grid, only a positive-sequence SDR is needed so that the whole system becomes easier for digital implementation, compared with conventional resonant controllers. For an unbalanced utility grid, then a positive-sequence plus negative-sequence SDR can be employed. This is equivalent to a resonant controller in this situation, just providing an alternative implementation structure. However, as already said, it is possible to assign different weights for the positive and negative sequences when using two individual SDR controllers.

It is remarked that the dead-time effects of inverter bridges constructed with IGBTs, usually introduce harmonic distortion into the output voltage of the inverter when the dead time is large to a certain extent (e.g., more than 2% of the switching period). Since either the resonant controllers or SDR controllers, only produce a good performance at the fundamental frequency level, and harmonic components cannot be compensated with the closed-loop control. Although the technique of multiple resonant controllers for selective low-order harmonics compensation can be employed to mitigate dead-time effects [5] [9], other simpler solutions are possible, unless a significantly distorted grid is dealt with. Therefore, another dead-time compensation technique [10] is added to the control in this paper, and only fundamental-frequency SDR controllers are used in the experiment.

B. Series Grid-connected Inverter

The second type of application is a series grid-connected inverter, which usually controls the grid current to help improving power quality of the grid. As shown in Fig. 7, either unbalanced grid or unbalanced loads (in general, they could be users, suppliers, or micro-grids) may cause asymmetric three-phase grid currents. An inverter connected in series with the grid and the loads through isolation transformers can work as an virtual voltage source to eliminate the asymmetrical currents, i.e., negative-sequence currents in three-phase three-wire systems.

Therefore, instead of regulating the total grid current, only negative-sequence currents are eliminated by the series inverter [6] [7]. As already mentioned, a SRF PI controller has to be used since conventional resonant controllers cannot separately control individual sequences. Consequently, more computation is required and the dynamics of the outer loop are relatively slowed down due to removing of positive-sequence components. However, with a SDR controller, the complexity can be reduced, leading to similar final results. For this purpose, as shown in Fig. 7, grid currents are fed back and compared with reference currents set to zero. Further, the output errors are regulated by a proportional plus negative-sequence SDR controller, which forces only the negative-sequence component to vanish. Afterwards, the outputs from the controller are dealt with by an inner loop before sending command signals to the pulse width modulation (PWM) inverter.

Obviously, the errors between feedback signals and zero references involve positive-sequence and negative-sequence components. For the negative SDR controller, only negative-sequence components are regulated and output, while the positive-sequence components are damped. Note that a P controller still introduces a fraction of positive-sequence error signals due to the proportional gain, named as $K_p$, although it has little influence on the output of the inverter. In other words, a small $K_p$ leads to negligible influence of positive-sequence signals.

V. Experimental Results

Experiments were carried out to verify the effectiveness of the SDR controllers. The two applications in Section IV were tested by using a same type of three-phase grid-connected inverter. The controllers were built with a dSPACE DS1104 setup. A programmable power source was used to emulate the grid, and a dc machine was employed as a distributed primary source for the dc bus of the inverters.

System parameters are shown in Table I. The grid impedances $Z_{ga,b,c}$ are assumed to be combined with the output inductors. Due to computation time requirements, the inverter works with a sampling frequency of 8kHz, half the switching
frequency. In the experiments, a proportional plus practical SDR controllers are tested with following control parameters: \( \omega_1 \) is set to 314 rad/s, \( \omega_0 \) to 10 rad/s, the gain coefficient \( K_I \) is 50, and the proportional gain \( K_p \) is set to 1 and 0.2 for application A and B of Section IV, respectively.

Fig. 8 shows the experimental results related to a parallel grid-connected inverter. In the situation of a balanced grid, grid currents are regulated with a single positive SDR controller, as shown in Fig. 8 (b). Due to the dead-time effect, the practical output currents are slightly distorted although a dead-time compensation is used, but the phase tracking works well at the fundamental frequency. Further comparing, another negative SDR controller was added in parallel, and the output currents present a small additional improvement, as shown in Fig. 8 (c). In general, the results show a good performance of the positive SDR controller under balanced grid condition.

For the parallel inverter working with an unbalanced grid, the corresponding experimental results are shown in Fig. 9. The unbalanced grid voltages for test are given in Table I (Case I). In this grid condition, the positive and negative SDR controllers achieve the same results as a conventional resonant controller, as illustrated in Fig. 9 (b) and (c).

Fig. 10 shows the results of a second application, namely negative-sequence current elimination with a series connected inverter. In this case, more unbalanced grid voltages are given in Table I (Case II) in order to observe unbalanced currents more clearly. The loads are equipped with three resistors of 34\( \Omega \), 30\( \Omega \), and 30\( \Omega \) for phase A to C, respectively. Without using the series inverter, the three-phase unbalanced grid currents are delivered from the grid to the loads, as shown in Fig. 10 (b). When using the series inverter with a negative-sequence SDR controller as described in Fig. 7, the elimination of negative-sequence currents are achieved. The balanced grid currents are shown in Fig. 10 (c).

**VI. Conclusion**

This paper has introduced a sequence-decoupled resonant controllers constructed with a multi-state-variable structure, which make independent control of positive-sequence and negative-sequence AC signals possible in a stationary frame and provide an easier implementation method. The relation to conventional resonant controllers was also discussed. It showed that the combination of positive and negative SDR controllers are approximated to conventional R controllers, but the advantage of SDR controllers with backward-coupled implementation structure was pointed out, that is, the gain for either sequence can be assigned separately. In addition, two examples were given to highlight the implementation of SDR controllers and to show their simplicities for individual sequence control. Finally, the effectiveness of the proposed controllers were verified by experiments.

**TABLE I**

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unbalanced Grid voltage (Case I)</td>
<td>( V_{ga} )</td>
<td>198V 0°</td>
</tr>
<tr>
<td></td>
<td>( V_{gb} )</td>
<td>171.71V 125.21°</td>
</tr>
<tr>
<td></td>
<td>( V_{gc} )</td>
<td>171.71V 125.21°</td>
</tr>
<tr>
<td>Unbalanced Grid voltage (Case II)</td>
<td>( V_{ga} )</td>
<td>180V 0°</td>
</tr>
<tr>
<td></td>
<td>( V_{gb} )</td>
<td>137.5V 130.9°</td>
</tr>
<tr>
<td></td>
<td>( V_{gc} )</td>
<td>137.5V 130.9°</td>
</tr>
<tr>
<td>Filter inductor</td>
<td>( L_{a,b,c} )</td>
<td>2mH, 0.03Ω</td>
</tr>
<tr>
<td>Filter inductor</td>
<td>( L_{a,b,c} )</td>
<td>2mH, 0.03Ω</td>
</tr>
<tr>
<td>Filter capacitor</td>
<td>( C_{a,b,c} )</td>
<td>6μF</td>
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<tr>
<td>DC-bus</td>
<td>( V_{DC} )</td>
<td>750V</td>
</tr>
<tr>
<td>Switching freq.</td>
<td>( f_{sw} )</td>
<td>10kHz</td>
</tr>
<tr>
<td>Dead time</td>
<td>( \Delta t )</td>
<td>5μs</td>
</tr>
</tbody>
</table>

Fig. 8. Experimental results when using the practical SDR controller under balanced utility grid, where (a) shows three-phase grid voltages, (b) three-phase grid currents regulated by positive SDR controller only, and (c) the three-phase grid currents regulated by both positive and negative SDR controllers.
Fig. 9. Experimental results under unbalanced utility grid, where (a) shows three-phase grid voltages, (b) three-phase grid currents regulated by new positive and negative SDR controllers, and (c) the three-phase grid currents regulated by conventional resonant controllers.

REFERENCES


