Abstract—In this paper we apply the method of Proper Orthogonal Decomposition (POD) to identify a lower dimensional model of a benchmark problem representing an Industrial Glass Manufacturing Process (IGMP). In particular, we identify a reduced model by identifying the mapping from process inputs to POD modal coefficients by a subspace identification method. Reduced models obtained from POD are not well equipped to capture the process behavior under time varying uncertain process parameters. For this reason we propose a novel hybrid detection scheme which approximates the process (benchmark CFD model) exhibiting non-smooth geometric parameter dependence (corrosion and wear) by using lower dimensional models. Given state or output information this detection mechanism detects the process parameter operation regime and suggests a computationally faster lower dimensional model as an approximate for real process.

I. INTRODUCTION

Industrial Processes involving fluid flows are usually modeled by Navier-Stoke’s equations which are solved by some kind of spatial discretization. Due to this modeling approach they are referred to as Distributed Parameter Systems (DPS). Spatial discretization of Distributed Parameter Systems (DPS) is done by means of Finite Volume (FVM) or Finite Element methods (FEM) and Galerkin or Petro-Galerkin projection techniques and they are simulated in a Computational Fluid Dynamic (CFD) software environment. Although such discretizations approximate process behavior reasonably well, it leads to very high dimensional process models. Such process models can not easily be used for online plant optimization and control. Model Order Reduction (MOR) is therefore an important step before proceeding to control design [1]. The method of Proper Orthogonal Decomposition (POD) or Principle Component Analysis (PCA) is widely used for deriving lower dimensional models from the First Principle Model (FPM). The POD method searches for dominant patterns in the given process and defines an optimal, data-dependent basis, that is subsequently used as a projection space to infer reduced order models through Galerkin type of projections [2]. POD methods are empirical (data dependent) in nature and therefore these methods are susceptible to changes in process inputs and process parameters. Many processes show inherent non-linear dependence on process parameters. The reduced model obtained by POD techniques are not well equipped for such processes exhibiting nonsmooth dependence on process parameters i.e. bifurcation type of behavior [3]. The problem becomes worse when the physical boundaries of the process domain becomes uncertain. It is shown in this paper how the POD basis becomes obsolete over time when the throat wall of the glass manufacturing furnace (a geometric parameter) wears out over time. This wearing out of a throat wall causes initiation of back-flow i.e. inversion of the flow direction. This back-flow causes changes of critical process variable (Temperature, Pressure, Velocity...). Occurrence of back-flow is similar to bifurcation or trifurcation happening in many chemical processes exhibiting nonlinear parameter dependence.

This paper is organized in a few sections. The motivation - Industrial Glass Manufacturing Process (IGMP) is explained in the next section. A brief description of the POD technique is given in section III which is followed by a black-box type of system identification. We use a subspace identification technique coupled with POD modal coefficients to identify the low dimensional models. This is explained in the methodology section V along with the detection mechanism. Some results are presented in the section VI which is followed by future work and references.

II. INDUSTRIAL GLASS MANUFACTURING

Industrial Glass Manufacturing (IGM) is usually carried out in large furnaces which are very well designed in order to have a desired laminar flow pattern of the glass. A 2D view of a typical furnace is shown in Figure 1. The flow pattern of glass determines the residence time of the glass in the melting furnace which in turn determines the quality of the glass produced. The process is an example of very large scale integrated systems. Most of the process variables like temperature, velocity, pressure, viscosity are interacting
with each other. Due to this interacting nature the control of the furnace has to be done carefully. Usually pull rate (production rate), heat input and pressure valve positions are some of the control variables. Whereas variables of interest are temperature distribution and velocity profiles in the furnace. The product quality is determined by these two factors. The temperature maintained inside the furnace varies between 1400 – 1650 °C. The glass raw material enters from the left side (inlet) in the form of a batch blanket, it is melted by applying heat from the top. After circulating through the glass furnace for many hours glass passes through the throat and then leaves via the outlet. Based on the process operation there are roughly three regimes - glass melting (left side of hot spot), fining (on right side of hot-spot) to remove high concentration of dissolved gases from glass and refining (after throat) to remove all remaining nondissolved gases from the glass. The IGMP shows large variation in the time scales, from minutes to days. Process residence time shows combined behavior of an ideal Continuous Stirred Tank Reactor (CSTR) and an ideal Plug Flow Reactor (PFR). The transport of physical quantities in GMP can be approximated with reasonable accuracy by modeling it by a set of nonlinear Navier-Stokes equations of the form (1)

$$\frac{\partial (\rho \phi)}{\partial t} + \text{div} (\phi \rho v) = \text{div} (\Gamma \text{grad} \phi) + q_\phi \quad (1)$$

where $\rho$ is mass density, $v$ is velocity, $\text{div}$ is divergence operator, $\Gamma$ is diffusion coefficient, $q_\phi$ is source/sink term. when $\phi = 1$ we have continuity equation, for $\phi = v$ we have momentum equation and for $\phi = h$ we have energy balance equation. The source term brings the non-linearity. It is the term which gives contribution of physical effects like bubbling, boosting, reaction etc. Combination of different numerical schemes are used to obtain reliable simulation results. Some details about mathematical modeling of glass can be found in [4],[5],[6],[7]. Some more information about the CFD modeling software GTM-X can be found in [8]. Due to very high process temperature and due to the viscous nature of glass, the glass furnace is a hostile environment for sensor systems. Sensors are largely limited to temperature measurements in the bottom refractory of the melting furnace.

As 3 dimensional glass furnace model easily consist of $10^4 – 10^6$ finite elements. Simulating its steady and/or dynamic behavior takes days for a normal configured PC, it becomes very difficult to analyse its transient response to dynamic process changes. For this reason we are using an approximate 2D glass furnace which mimics the vertical cross section along the length of 3D glass furnace and it has only 2 grids cells in width direction. Currently we are trying to model the very slow geometric changes that take place in real 3D furnace in the form of throat or dam wall corrosion. Figure 1. This corrosion results into back-flow of molten glass from the working end to the melting furnace. Such back-flow behavior causes undesired changes in the temperature distribution in the fining and melting zone which ultimately leads to economic losses. For the 2D furnace we observe this bifurcation behavior i.e the occurrence of back-flow somewhere between throat height of 0.2m ($h_1$) and 0.3m ($h_2$). Figure 2 shows back-flow results into completely different velocity pattern in throat section.

![Fig. 2: Occurrence of back-flow](image)

**III. PROPER ORTHOGONAL DECOMPOSITION**

One of the most promising and significant techniques for an efficient reduction of large-scale nonlinear systems in fluid dynamics is the method of Proper Orthogonal Decompositions (POD) also known as the Karhunen-Loève method [9]. The method is based on the observation that flow characteristics reveal coherent structures or patterns in many processes in fluid dynamics. This has led to the idea that the solutions of model equations may be approximated by considering a small number of dominant coherent structures (called modes) that are inferred in an empirical manner from measurements or simulated data.

Given an ensemble of $K$ measurements $T^k(\cdot), k = 1, \ldots, K$ with each measurement defined on some spatial domain $\Omega$, the POD method amounts to assuming that each observation $T^k$ belongs to a Hilbert space $\mathcal{H}$ of functions defined on $\Omega$. With the inner product defined on $\mathcal{H}$, it then makes sense to call a collection $\{\varphi^j\}^\infty_{j=1}$ an orthonormal basis of $\mathcal{H}$ if any element, say $T \in \mathcal{H}$, admits a representation

$$T(x) = \sum_{j=1}^{\infty} a_j \varphi^j(x), \quad x \in \Omega.$$  

Here, the $a_j$’s are referred to as the coefficients and the $\varphi_j$’s are the modes of the expansion. The truncated expansion

$$T_n(x) = \sum_{j=1}^{n} a_j \varphi^j(x), \quad x \in \Omega$$

causes an approximation error $||T - T_n||$ in the norm of the Hilbert space. We will call $\{\varphi^j\}^\infty_{j=1}$ a POD basis of $\mathcal{H}$ whenever it is an orthonormal basis of $\mathcal{H}$ for which the total approximation error $\sum_{k=1}^{K} ||T^k - T^k_n||$ is minimal for all truncation levels $n$. This is an empirical basis in the sense that every POD basis depends on the data ensemble.

Using variational calculus, the solution to this optimization problem amounts to finding the normalized eigenfunctions $\varphi_j \in \mathcal{H}$ of a positive semi-definite operator $R : \mathcal{H} \to \mathcal{H}$ that
is defined as
\[ \langle \psi_1, R\psi_2 \rangle := \frac{1}{K} \sum_{k=1}^{K} \langle \psi_1, T_k^k \rangle \cdot \langle \psi_2, T_k^k \rangle \]
with \( \psi_1, \psi_2 \in \mathcal{H} \). \( R \) is well defined in this manner and corresponds to a positive semi-definite matrix whenever \( \mathcal{H} \) is finite dimensional. In that case, a POD basis is obtained from the normalized eigenvectors of \( R \) [2].

Subsequently, a Galerkin projection is used to obtain the reduced model as follows. Suppose that the system is governed by a PDE of the form
\[ \frac{\partial T_n}{\partial t} = \mathbf{A}(T_n) + \mathbf{B}(u) + \mathbf{F}(T_n, u, d) \] (2)
Let \( \mathcal{H}_n \) denote an \( n \) dimensional subspace of \( \mathcal{H} \) and let \( P_n : \mathcal{H} \to \mathcal{H}_n \) and \( I_n : \mathcal{H}_n \to \mathcal{H} \) denote the canonical projection and canonical injection maps. The reduced model is then given by
\[ P_n \frac{\partial T_n}{\partial t} = P_n \mathbf{A}(T_n) + P_n \mathbf{B}(u) + P_n \mathbf{F}(T_n, u, d) \] (3)
where \( T_n(t) = T_n(t) \) belongs to \( \mathcal{H}_n = P_n \mathcal{H} \) for all \( t \). In the specific case of a POD basis, the finite dimensional subspace \( \mathcal{H}_n = \text{span}(\varphi_1, \ldots, \varphi_n) \) where the \( \varphi_j \)’s denote POD basis functions. In that case, (3) becomes an ordinary differential equation in the coefficients \( a_j(t) \) in the expansion of \( T_n \).

IV. SUBSPACE IDENTIFICATION TECHNIQUE

Subspace Identification (SI) techniques got a strong momentum in the beginning of 1990s. It has origin and interpretation in terms of geometry, system theory and linear algebra. As most of the modern control theory uses a state space description of the system, subspace methods become a most favored identification technique. Some standard references include [10],[11]. This method is quite well applied for many MIMO LTI systems. Advantages of subspace method over prediction error method are compared in [12] for many industrial application. A general linear discrete time-invariant state space system is given as (4)
\[ x(k+1) = Ax(k) + Bu(k) \]
\[ y(k) = Cx(k) + Du(k) \] (4)
where \( x(k) \in \mathbb{R}^n \) is the state vector, \( u(k) \in \mathbb{R}^{n_u} \) is input vector, \( y(k) \in \mathbb{R}^{n_y} \) is the output vector. In subspace method the postulated model (4) is used to formulate 's' step ahead prediction (5),
\[ \hat{y}_k^{k+s-1}(k) = \mathcal{O}_s x(k) + T_s u_k^{k+s-1}(k), \] (5)
where \( T_s \) is a deterministic lower block triangular Toeplitz matrix and \( \mathcal{O}_s \) is the observability matrix, \( s \geq n \) and the state space model is observable when \( \text{rank}(\mathcal{O}_s) = n \) for \( k = 0, 1, \ldots, K-1 \) these equations in matrix form are (6)
\[ Y_{0,s,K-1} = \mathcal{O}_s X_{0,K-1} + T_s U_{0,s,K-1} \] (6)
\[ X_{0,K-1} \text{ is state vector, } Y_{0,s,K-1} \text{ and } U_{0,s,K-1} \text{ are output and input Hankel matrices respectively.} \]

System Identification (SI) by a subspace method is carried out in two steps in all the algorithms. First step always performs the (weighted) projection of the row space of the above mentioned Hankel matrices. From this projection the observability matrix \( \mathcal{O}_s \) and/or an estimate \( \hat{X}_{0,K-1} \) can be retrieved [12]. In the second step, the system matrices \( A, B, C, D \) and \( Q, S, R \) (when stochastic part is modeled as well) are determined.

Now we can find \( \mathcal{O}_s X_{0,K-1} \) by orthogonal projection by projecting the row space of \( Y_{0,s,K-1} \) onto the \( U_{0,s,K-1}^\perp \), where
\[ U_{0,s,K-1}^\perp = I_k - U_{0,s,K-1}(U_{0,s,K-1}U_{0,s,K-1}^T)^{-1}U_{0,s,K-1} \] (7)
with the property,
\[ U_{0,s,K-1}U_{0,s,K-1}^\perp = 0 \] (8)
This leads to
\[ Y_{0,s,K-1}U_{0,s,K-1}^\perp = \mathcal{O}_s X_{0,K-1}U_{0,s,K-1}^\perp \] (9)

Eq. (9) show that each column of \( Y_{0,s,K-1}U_{0,s,K-1}^\perp \) is linear combination of the columns of \( \mathcal{O}_s \), i.e. the column space of \( Y_{0,s,K-1}U_{0,s,K-1}^\perp \) is contained in column space of \( \mathcal{O}_s \). This is the first step of the all the subspace method numerical algorithms, now starts the second step. Here we have 2 classes. With the Singular Value Decomposition (SVD),
\[ Y_{0,s,K-1}U_{0,s,K-1}^\perp = (U_1 U_2) \begin{pmatrix} S_1 & 0 \\ 0 & V_1^T \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} \] (10)
\[ \text{rank}(Y_{0,s,K-1}U_{0,s,K-1}^\perp) = n_x \] (11)
\[ \mathcal{O}_s = U_1 S_1^{1/2} \] (12)
\[ X_{0,K-1}U_{0,s,K-1}^\perp = S_1^{1/2} V_1^T \] (13)
Left singular vectors contain information about observability \( \mathcal{O}_s \), where as right singular vectors contain info about states \( X_{0,K-1} \). State space matrices \( A \) and \( C \) can be determined from it. Algorithms like \( N4SID \) and \( CVA \) uses estimate of states whereas \( MOESP, IV - 4SID \) uses extended observability matrix. Once state space matrices \( A \) & \( C \) are determined, matrices \( B \) & \( D \) can be determined by solving parameter estimation problem.

V. METHODOLOGY

In this section we explain the methodology for identifying smaller dimensional, computationally faster models which can approximate the original non-linear, complex, very large process model showing non-smooth geometric parameter dependence. Furthermore this section will explain how to detect the operation regime of the geometric parameter (i.e.
below or above bifurcation value) from the obtained reduced model and some process measurements. In this paper a 2D benchmark CFD problem is considered as the original process.

The problem can be formulated as-

\[ x \in \Omega, \quad \Sigma : \dot{x} = f(x, u, h), \]  

identify the model \( \hat{\Sigma} \), such that the norm of the error between the solution of original CFD model and reduced identified model is minimum and is given by,

\[ \min_{k=0}^{K} \| T(spx, spy, k, h) - \tilde{T}(spx, spy, k, h) \|^2 \]  

Where,

- \( spx := \) spatial co-ordinate in x-direction,
- \( spy := \) spatial co-ordinate in y-direction, \( h := \) value of bifurcation parameter (geometric parameter - throat height),
- \( T(spx, spy, k, h) := \) the solution of the real process i.e. CFD model \( \Sigma \),
- \( \tilde{T}(spx, spy, k, h) := \) the solution of the identified model in reduced space \( \hat{\Sigma} \),

Solution to the problem can be found by using Proper Orthogonal Decomposition (POD) as a chosen method for Model Order Reduction (MOR), Subspace Identification (SI) to find state space matrices and Dynamic Detection Mechanism (DDM) to represent the geometric parameter variation in the process. This is explained below in detail.

For the input sequence \( \{ u(k) \}_{k=1}^{K} \), the glass furnace simulation model is excited. The snapshots of the temperature \( \{ T(k) \}_{k=1}^{K} \) (see POD details in section III) are collected as

\[ T_{\text{snap}} = \begin{bmatrix} T(1) & \ldots & T(K) \end{bmatrix}, \]

where \( T(k) \) is temperature snapshot at time instance \( k \). From this snapshot sequence optimal POD basis \( \phi_j \) and corresponding modal coefficients \( a_i(t) \) which characterize temporal evolution from the snapshot matrix we proceed to the model identification by the subspace method. By using the subspace method a mapping (state space matrices A,B,C,D) is found between the process inputs and first few POD modal coefficients corresponding to the dominant gain directions [13].

This process is repeated for two models; which correspond to two throat heights \( h = 0.2 \) i.e model M1 and \( h = 0.3 \) i.e. model M2. The general strategy is shown in Figure 3. The figure shows that the above said approach can be used for identifying the map between system inputs and velocity modal coefficients \( a_{ij}^T \) or for multi-variable \( a_{ij}^{TV} \) modal coefficients, which can be obtained by stacking temperature and velocity snapshots together. More on multidimensional/multivariable POD can be found in [14].

The identified model in general state space format is given by

\[ x(k + 1) = Ax(k) + Bu(k) \]

\[ \hat{a}_i(k) = C_{ai}x(k) \]  

We can reconstruct the estimated or reconstructed outputs from the estimated POD modal coefficients as

\[ \hat{y}_i(k) = C_{\Phi_n}C_{ai}x(k) = C_Tx(k) \]  

The last part of the strategy mentioned is the Dynamic Detection Mechanism (DDM) which is residual between either of the identified models, i.e. M1 or M2 with process (CFD software) at any time. DDM is shown in Figure 4.

Both identified models represent the two extreme cases-back-flow or no-back-flow.

Let's denote 1 : M1 i.e no back-flow, 2 : M2 i.e back-flow, and \( i : \) sensor index, then we have residuals (error) at any time at some sensor location as,

\[ \varepsilon_i(t) = ||(y_i(k) - \hat{y}_i^1(k))||^2 - ||(y_i(k) - \hat{y}_i^2(k))||^2. \]
y-axis position and different x-axis position). The set of sensors is given by \( \{ S_i \}_{i=1}^{ny} \). Where \( ny = 4 \), and the four sensors at those locations are [Figure 1], S6 = \( Y(x = \text{before left end of the throat}) \), S7 = \( Y(x = \text{left end of the throat}) \), S8 = \( Y(x = \text{middle of the throat}) \), S9 = \( Y(x = \text{right end of the throat}) \).

Sensors at location S1 to S5 are located in the melting and fining section which are far from the throat section, and hence are not of interest in this work. We now proceed by defining residuals at the above sensor locations as defined in (18). We can now confirm occurrence of the back-flow and make a decision about choice of identified model. If all elements of set \( \{ \varepsilon_i(t) \}_{S_S}^{S_6} > 0 \): we are in regime 2 i.e we have back-flow and if all elements of set \( \{ \varepsilon_i(t) \}_{S_S}^{S_6} < 0 \): we are in regime 1 i.e we have no back-flow. After confirming the regime we can select the correct subspace identified (reduced) model from either of the models. Occurrence of back-flow first starts at right hand side of throat (side close to the working end) and then it proceeds to the left side i.e. towards the inlet. As we are interested in occurrence of back-flow at left hand side entrance of throat (side close to fining section, S7) [Figure 1] we have certain range of throat height \( h^1 < h^* < h^2 \) such that at certain critical height \( h^* \) we observe back-flow at the left end of the throat i.e. at S7. As the corrosion phenomenon is very slow (timescale of years), occurrence of back-flow is very slow process. From this process knowledge we can predict its occurrence if each element of above mentioned set of residual starts changing their sign from negative to positive, one after another. In short, we are identifying reduced models corresponding to corrosion or no corrosion case and comparing both of them at every time instance with real process. Model which gives small error represents the real process.

VI. RESULTS AND DISCUSSION

Simulation experiments were performed on the 2D glass furnace model for two different throat heights for a time horizon of 60 hr. The data was extracted and imported in MATLAB environment for carrying out POD and subspace identification. As identified models are tremendously fast (> 1000 times faster than the CFD simulation), the computational performance of the identified models is not compared with the original CFD models. The lower dimensional maps were identified for single input - glass pull rate/production rate and three outputs which are first three POD modal coefficients for temperature (which is similar to process inputs and the POD modal coefficients for temperature were identified only). Identification of the map between POD modal coefficients for velocity or for multi-variable modal coefficient is not done in this work. Moreover, as we are interested in the reconstructed outputs (eq. 17), the parameter fit of the identified models to the given data is not shown anywhere. We have, S6 at \( x = 25 \text{ m} \), i.e. 3m left of the throat entrance, S7 at \( x = 28 \text{ m} \), i.e. at the throat entrance, S8 at \( x = 29 \text{ m} \), i.e. at the middle of the throat, S9 at \( x = 30 \text{ m} \), i.e. right end of the throat.

Figure 7 compares the performance of the original and the
VII. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

We conclude this paper with the remark that for a slow geometric parameter variation one can use the mentioned hybrid type of detection mechanism which employs subspace method with POD to identify lower dimensional models which are computationally faster than the CFD simulations. While implementing the strategy mentioned in section V one needs to take care of a proper excitation signal and a corresponding collection of snapshots such that POD captures the interesting dynamics. The suggested detection mechanism is easy to implement and can be used to detect the uncertain value of bifurcating parameter (throat height). This detection mechanism can predict the occurrence of back-flow in advance as well.

B. Future Works

Immediate extension to the current research in near future could be the system identification for MIMO case, derivation of a better mathematical framework for the switching mechanism, implementation of a parameter dependent Luenberger observer to observe the geometric parameter variation and model identification in a closed loop setting. Other tasks required to carry out above said future work is to create an interface between MATLAB and GTM-X (CFD) software.

REFERENCES