AN ANALYTICAL MODEL FOR INTRAVASCULAR MR ANTENNAS


* TNO Science and Industry
P.O. Box 6235
5600 HE Eindhoven, The Netherlands
E-mail: huib.j.visser@tno.nl

† Image Sciences Institute, University Medical Center Utrecht
Heidelberglaan 100
3584 CX Utrecht, The Netherlands

‡ Department of Electrical Engineering, Eindhoven University of Technology
P.O. Box 6235
5600 MB Eindhoven, The Netherlands

Keywords: Loop antennas, Blood vessels, Magnetic field, MRI, Sensitivity.

Abstract

For interventional Magnetic Resonance Imaging a need exists for intravascular MR antennas. These antennas need to be employed for the tracking of guide wires and catheters through blood vessels during surgery and/or for obtaining high resolution images of vessel walls. Such images cannot be obtained by conventional MRI operation due to the high Signal to Noise Ratio of the signals received with conventional receiver coils. By inserting receiver coils (antennas) into the blood vessels, the SNR can be improved up to a level that obtaining high resolution images becomes feasible.

Based on the fact that intravascular antennas will be much smaller than the applied RF wavelength, a static field, approximate, wire antenna model is developed. This model is thoroughly validated by comparing the analysis results with results obtained from dynamic models. It appears that especially for the large arteries (radius between 2mm and 3mm) and small diameter loop antennas (radius 0.5mm), sensitivities (magnetic field intensities) may be calculated using the static model that deviates 13% at maximum from results obtained by the dynamic model. Moreover, the static and dynamic sensitivities in the area of interest (in and around the vascular wall) show a similar behaviour as function of distance to the loop centre. Thus, under well-defined conditions the static model may be used to predict the absolute value of the sensitivity of a loop antenna with a reasonable accuracy. More important however is that the static model may be used to compare different designs with respect to sensitivity profiles.

1 Introduction

In Magnetic Resonance Imaging (MRI) a patient is positioned in a high intensity static magnetic field. The high intensity (main) magnetic field make the spin-possessing molecules in the body to align their magnetic moments with this field. When next a Radio Frequency (RF) pulse is emitted, causing the main magnetic field to deflect, the molecules will absorb energy that will be reradiated after the RF pulse has ceased to exist. This reradiation induces a current in a receiver coil and this received signal is a measure for the tissue being excited. By applying a position-dependent gradient in the main magnetic field, it is possible to identify the spatial location of re-emitted RF energy. As in a CT- or CAT-scan, image slices of patients are produced, but the details that can be obtained with MRI are far better. The radiation involved is – contrary to CT and CAT – non-ionising and, roughly, in the frequency range of 30MHz to 120MHz.

The resolution of an image to be obtained by MRI is directly related to the Signal-to-Noise Ratio (SNR). For obtaining detailed information of blood vessel walls for example, the commonly used receiver coils are not capable of producing the desired low SNR. Therefore, a logical next step in the evolution of MRI is the employment of intravascular receiver coils or antennas, to be inserted into the blood vessels for detecting areas of stenosis, dissection, aneurism or other vascular pathology. Intravascular MR antennas may also be used for tracking guide wires and catheters through blood vessels during surgery, i.e. during interventional MRI (iMRI).

To compare different antenna concepts quantitatively or to synthesise optimum antenna designs, an analytical antenna model is needed for a fast calculation of the magnetic field.
intensity (sensitivity) in the immediate surrounding of the antenna, [1].

2 Model development

The requirements for intravascular antennas intended for imaging differ from those intended for tracking. In the remaining we will describe antennas from a transmitting perspective. By virtue of reciprocity we will find the receiving characteristics. For arterial wall imaging a high magnetic field intensity is required outside the catheter boundaries up to and beyond the artery wall. Furthermore, the magnetic field needs to be radially homogeneous and preferably having a large longitudinal coverage. The latter characteristic will allow for multi-slice imaging.

For tracking purposes, the intravascular antenna needs to aid in visualising the catheter. Thereto, the antenna needs to locally disturb the main magnetic field. This disturbance gives rise to a local dephasing which becomes visible as a disturbance in the MRI image.

Despite the different requirements for imaging and tracking purposes, most intravascular antennas reported in literature share that they may be regarded as derivatives of the single loop antenna. As we will see, the loop antenna is a logical choice, following from dimensional considerations.

We want to obtain analysis results for different antenna concepts very fast and possibly want to optimise antenna designs through repeated analyses. Therefore the need exists for an approximate modelling feasible for antennas that are small compared to the wavelength. In the remaining of this paragraph we will first consider the dimensional constraints. Then we will develop an approximate model for a loop antenna immersed in blood, that assumes a constant current and a static magnetic field. Finally, the validity of these two assumptions, constant current and static magnetic field, will be tested.

2.1 Dimensional constraints

Assuming an MRI scanner that produces a 1.5T strong main magnetic field results into a Larmor frequency of 63.87MHz. The Larmor frequency is the frequency with which atomic nuclei respond when interrogated by RF radiation. Due to the applied field gradient, the frequency will vary around this value and for convenience we will therefore assume a resonance frequency of 64MHz.

The medium that will surround the intravascular antenna will be mainly blood that is characterised by a relative permeability $\mu_r \approx 1$ and a relative permittivity $\varepsilon_r \approx 80$ at a temperature of 37° centigrade, [2,3]. The wavelength may thus be calculated as

$$\lambda = \frac{c_0}{\sqrt{\mu_r \varepsilon_r} \ f} = \frac{1}{f \sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}} = 0.52m \ (1)$$

The large blood vessels for which intravascular MRI antennas are needed have a diameter between 4mm and 6mm, [3]. Therefore an antenna diameter of about 2mm is anticipated, allowing the catheter to be manoeuvred through the vascular system without completely blocking the blood flow. For an antenna much smaller than the wavelength, the far field distance, $r_{ff}$ is given by, [4]

$$r_{ff} = \frac{\lambda}{2\pi}. \ (2)$$

With equation (1), we find that $r_{ff} = 82.8mm$. With the stated dimensions of intravascular antennas and vessels, the vessel wall position will be in the near field of the antenna.

2.2 Small loop antenna

The electromagnetic fields at position P, radiated by a small loop antenna of radius $a$ ($a \ll 0$), supporting a constant current $I_0$, see figure 1, are given by, [4]

$$E_\varphi = -\frac{j \omega \mu_0 a^2}{4} e^{-jkr} \left(\frac{j k}{r} + \frac{1}{r^2}\right) \sin(\varphi)$$

$$H_r = -\frac{j \omega \mu_0 a^2}{4} e^{-jkr} \left(\frac{2}{\eta r^2} + \frac{2}{j \omega \mu r^3}\right) \cos(\varphi)$$

$$H_\theta = -\frac{j \omega \mu_0 a^2}{4} e^{-jkr} \left(\frac{j \omega e}{r} - \frac{1}{j \omega \mu r} + \frac{1}{\eta r^2}\right) \sin(\varphi)$$

$$E_r = E_\theta = H_\varphi = 0 \ (3)$$

where $\omega = 2\pi f$ is the radial frequency, $\eta$ is the free space characteristic impedance, $\varepsilon_0 = \varepsilon_r \varepsilon_0$, $\mu_0 = \mu_r \mu_0$, and $k$ is the wave number.

Figure 1: Elementary loop antenna.
Far away from this elementary loop antenna, the \( r^1 \) terms dominate. Very close to the antenna the \( r^2 \) terms will be dominant. In the intermediate range, the \( r^2 \) terms will be dominant. If, for a very small but not elementary antenna, we can identify a region where the \( r^2 \) terms are clearly dominant and if this region is such that the artery wall will be in this region, it is likely that we may approximate the magnetic field in this region of interest by a static magnetic field.

Before we will verify this hypothesis, we will first have to assess the conditions for which the constant current approximation holds.

### 2.3 Constant current approximation

In [5] it was shown that the small-loop approximation of equation (3), \( a \ll \lambda \), may be obtained as a limiting case of the general exact series representation for a uniform current loop. Also demonstrated in [5] is that the \( H_z \) field component in the plane of the loop, at a distance of half a wavelength from the loop centre, as calculated by equation (3) is less than 5% in error with the exact solution for a uniform current for loop radii up to 0.11\( \lambda \) and less than 10% in error for loop radii up to 0.15\( \lambda \). Having thus established the validity of the small-loop antenna model for our purposes, we now need to find for what radii a loop antenna may be regarded as supporting a uniform current.

To start this assessment, first a circular loop antenna in air is considered. The input impedance of this antenna is analysed by expanding the current in a Fourier series, \([6-8]\). We take parameters beyond the scope of establishing the validity of our constant loop centre, as calculated by equation (3) is less than 5% in error for a uniform current for loop radii up to 0.11\( \lambda \) and less than 10% in error for loop radii up to 0.15\( \lambda \). Having thus established the validity of the small-loop antenna model for our purposes, we now need to find for what radii a loop antenna may be regarded as supporting a uniform current.

Next we need to verify if we can identify a near-field region of a loop antenna where we may approximate the radiated magnetic field by a static magnetic field.

### 2.4 Static magnetic field approximation

For a direct current element \( dl \) at position \( r_0 \), relative to a chosen origin, see figure 2, the induced magnetic field \( dH \) at position \( P=P(r) \), relative to the same origin, is given by, \([13]\)

\[
\frac{dH(r)}{4\pi R} = \frac{dI(r_0)}{4\pi R} = I(r_0) \frac{dl \times R}{4\pi R},
\]

(4)

where \( dI=I(r_0)dl \). Equation (4) is known as the Biot and Savart law.

The total magnetic field, \( H(r) \), of the current elements around a current path \( C \), is obtained by integrating equation (4) over the path

\[
H(r) = \frac{1}{C} \int \frac{dI(r_0)dl \times R}{4\pi R^3}.
\]

(5)
We recognise the \( r^2 \) dependence of the static magnetic field in equations (4) and (5), that we want to make use of in approximating the near-field of the loop antenna.

The integral in equation (5) is readily solved for observation positions on the axis of a circular loop, but off-axis and for shapes more complex than a circular loop this is difficult or impossible, [14]. To use the Biot and Savart law on more complicated wire structures, it is necessary to subdivide the structure into straight segments for which line integrals can be evaluated in closed form. To that end, we express the equation of such a line segment in terms of a single parameter \( s \), [14]

\[
I = l(s) = u_x x(s) + u_y y(s) + u_z z(s), \tag{6}
\]

where \( u_i, \ i = x,y,z, \) are unit vectors in, respectively, \( x-, y- \) and \( z- \) directions of a Cartesian coordinate system. The infinitesimal segment, \( dl \), in equation (5) is then

\[
dl = \frac{dl(s)}{ds} ds = \left[ u_x \frac{dx(s)}{ds} + u_y \frac{dy(s)}{ds} + u_z \frac{dz(s)}{ds} \right] ds \tag{7}
\]

The vector \( R \) is given by, see also figure 2

\[
R = u_x \left[ x(s) - x \right] + u_y \left[ y(s) - y \right] + u_z \left[ z(s) - z \right] \tag{8}
\]

where it is understood that \( P = P(x,y,z) \).

For a straight wire segment between the positions \( (x_i,y_i,z_i) \) and \( (x_j,y_j,z_j) \), supporting a unit current, the magnetic field at position \( P \) is then given by, [14]

\[
H(r)|_{t(0)=1} = u_x \left[ \int_{s=0}^{1} \frac{D_s}{(A + Bs + Cs^2)^{\frac{3}{2}}} ds \right] + u_y \left[ \int_{s=0}^{1} \frac{D_s}{(A + Bs + Cs^2)^{\frac{3}{2}}} ds \right] + u_z \left[ \int_{s=0}^{1} \frac{D_s}{(A + Bs + Cs^2)^{\frac{3}{2}}} ds \right] \tag{9}
\]

where

\[
A = (x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2 \\
B = 2 \left[ (x_i - x)(x_j - x_i) + (y_i - y)(y_j - y_i) + (z_i - z)(z_j - z_i) \right] \\
C = (x_i - x_j)^2 + (y_j - y_i)^2 + (z_j - z_i)^2 \\
D_x = (y_j - y)(z_i - z) - (z_j - z_i)(y_i - y) \\
D_y = (z_j - z)(x_i - x) - (x_j - x_i)(z_i - z) \\
D_z = (x_j - x)(y_i - y) - (y_j - y_i)(x_i - x) \\n\]

The integrals can be calculated in closed form. The magnetic field at position \( P \), produced by multiple straight wire segments, is the sum of the contributions calculated for the isolated wire segments.

To validate the use of the Biot and Savart law, we will compare so called ‘sensitivities’ calculated with the above explained implementation of the Biot and Savart law and sensitivities calculated with the first two terms of the power series expansion of [11]. We define the sensitivity \( S \), as

\[
S = \frac{1}{I_0} \sqrt{B_x^2 + B_y^2} (TA^{-1}), \tag{11}
\]

where \( B_i = \mu H_i, \ i = x,y \). This definition is based on the reasoning that magnetic fields will be measured in the transverse \((xy)\)-plane only in a MR scanner wherein the main magnetic field is \( z \)-directed.

To verify the model, we place single loop antennas of radii 0.5mm, 1.0mm and 1.5mm at an angle \( \theta \), relative to the \( z \)-axis of a rectangular coordinate system. The sensitivity will be calculated as function of \( y \), i.e. the distance to the loop centre, see figure 3. In this figure, the \( z \)-directed circular cylinder indicates the vascular wall.

For these configurations we may equivalently keep the loop positioned parallel to and in the \( xy \)-plane and calculate the sensitivity for different rotation angles \( \theta \) and unit current \( I_0 \) as

\[
S = \sqrt{(B_x^2 + B_y^2)}, B_i = \sin(\theta)B_x + \cos(\theta)B_y, \text{ see figure 4.}
\]

For \( \theta = 0 \), the static (Biot and Savart law) and dynamic (power series expansion) sensitivities as function of distance to the loop centre are shown for the three loops in figure 5. In the same figure, also the relative difference \( \text{delta} \) between the two calculated results are shown. Note that both sensitivity and distance are displayed on a logarithmic scale.

The figure clearly shows that very close to the loop, where the fields show a \( r^2 \) dependence, the static sensitivity differs...
substantially from the dynamic one. This difference is larger for loops with a larger radius.

Figure 4: Alternative configuration for calculating the sensitivity of a single loop antenna, rotated an angle $\theta$ from the $z$-axis.

Far away from the loop, where the fields show a $r^{-1}$ dependence, the static sensitivity starts to differ substantially from the dynamic one again and very far away from the loop the difference becomes very large due also to the attenuation that is not accounted for in the static sensitivity calculation. In between these regions of substantially differing sensitivities, we observe a region where the difference between dynamic and static sensitivity is minimal. This region, where the fields are dominated by a $r^{-2}$ dependence, coincide with our region of interest, i.e. the position of the artery wall. The radii for medium and large arteries vary between 1.0mm and 3.0mm, [3].

Calculations of dynamic and static sensitivities as function of distance for rotated loops reveals that especially for the large arteries (radii between 2.0mm and 3.0mm, [3]) and small radius loop antennas ($a=0.5\text{mm}$), sensitivities may be calculated by the static method that deviate less than 13\% from the dynamic sensitivities.

Moreover, the static and dynamic sensitivities, in the region of interest (i.e., in and around the position of the vascular wall) show a similar behaviour as function of distance to the (tilted) loop antenna centre.

This means that, under well defined conditions, the static model may be used to predict the absolute value of the sensitivity of the loop antenna with a reasonable accuracy. More importantly is that the static model has the potential to be used to compare different intravascular antenna designs with respect to sensitivity profiles.

2.5 Model generalisation

With the validity of the static model shown for single loop antennas, the next question to be answered concerns the validity of employing this model for multiple loops, or more generally, wire antennas extending the length of a single loop of radius $a=1.7\text{mm}$. For convenience we restrict ourselves to multiple loop antennas and consider two issues: A uniform current assumption and mutual coupling between closely spaced turns.

In a multi-turn loop, wherein every single loop satisfies the radius restriction, every loop may be regarded as supporting a uniform current. However, phase differences exist between the different turns. The turn spacing, in terms of wavelengths, is so small though that applying array theory to the multi-turn loop will result in effectively having $N$ turns at the same position. $N$ is the number of turns. Of course this reasoning is only valid if mutual coupling between the turns may be neglected, which, in air, especially for closely packed turns is not true, [10,15].

For loops immersed in blood the situation is different. Current will flow into the medium surrounding the loops, increasing the coupling of closely spaced loops. So, for bare wire antennas exceeding the single loop length limit and being comprised into a small volume the static approach will fail. In practice, however, insulated wire will be used, making the conducting medium act as a shielding, thus reducing the...
mutual coupling between the wires or turns of a multi-turn loop, [16]. In [17] it is shown that for a thin insulation layer, the uniform current flow is maintained and the impedance of the insulated loop is equal to that of the bare loop immersed into the conducting medium. So the uniform current approximation holds and mutual coupling should be less severe than in air.

Finally, based on [17] we assess the maximum wire length of an intravascular antenna to be approximately 120mm and based on a comparison of self and mutual inductance, [13,18], between different loops immersed in blood we advice a circular loop separation of one loop radius.

3 Intravascular MR antenna evaluation

Now that we have demonstrated the validity of the static model, we may employ this model to compare different antenna concepts quantitatively. The comparison is performed on basis of ‘sensitivity profiles or patterns’, i.e. two-dimensional cuts of the three-dimensional sensitivity patterns calculated for antennas positioned along the MR main magnetic field direction. All antennas evaluated are intended for being mounted on circular cylinders of radius 1mm and height 10mm. The radius was chosen with a view to a practical implementation for in vitro testing.

Figure 6 shows some intravascular antenna concepts.

Figure 6: Intravascular antenna concepts. (a) Antiparallel wire, [19]. (b) Double helix wire, [19]. (c) Opposed double helix wire, [19]. (d) Single loop, [20]. (e) Double loop. (f) Triple loop. (g) Solenoid, [21]. (h) Dual opposed solenoids, [21]. (i) Saddle coil. (j) Four-wire centre return. (k) Four-wire birdcage. (l) Quadrature coil, [22].

3.1 Antennas for tracking

For tracking purposes, the antenna – mounted on top of a catheter – needs to be detectable with a high degree of positional accuracy. Therefore, the antenna needs to have a very inhomogeneous sensitivity pattern with the peak values at or very near the antenna position. Candidate antennas are: The ‘antiparallel wire antenna’, the ‘double helix antenna’, the ‘opposed double helix antenna’, the ‘centre return antenna’, see figure 6 and the ‘perpendicular coils antenna’. The perpendicular coils antenna is shown in figure 7.

Figure 7: Perpendicular coils antenna.

Figure 8 shows two sensitivity profiles for a perpendicular coils antenna. The sensitivity profile in the yz-plane is identical to that in the xz-plane.

Figure 8: Sensitivity profiles. Height 3mm, inner radius 0.8mm, outer radius 0.9mm. 15 turns up and down, 8 segments per circumference. Coils tilted 45° relative to z. Top. xz-plane, y=0.05mm. Bottom. xy-plane, z=1.5mm.
The figure clearly demonstrates the localised magnetic field. To compare the different antenna concepts, cross sections of the sensitivity patterns are shown in figure 9.

Figure 9: Cross section of antenna sensitivity patterns at antenna half height along the x-axis.

The cross sections along the y-axis reveal that only the centre return antenna and the perpendicular coils antenna exhibit a rotationally symmetric sensitivity pattern. Although the centre return antenna shows a better localised sensitivity, the perpendicular coils antenna is easier to construct since it does not need a wire through the cylinder-axis. Therefore this antenna is considered as best choice.

3.2 Antennas for imaging

For intravascular imaging the antenna should show a sensitivity that extends from the antenna body to the vascular wall and that is, preferably, homogeneous along the direction of the main magnetic field. Furthermore, the sensitivity should be independent from the observation angle in the transverse plane. Candidate antennas are: The ‘single loop antenna’, the ‘double loop antenna’, the triple loop antenna’, the ‘dual opposed solenoids antenna’, the ‘saddle coil antenna’ and the ‘bird cage antenna’, see figure 6.

Figure 10 shows a dual opposed solenoids antenna.

Figure 10: Dual opposed solenoids antenna.

Figure 11 shows two sensitivity profiles for a dual opposed solenoids antenna. The sensitivity profile in the yz-plane is nearly identical to that in the xz-plane.

Figure 11: Sensitivity profiles. Height per coil 3mm, gap between coils 3mm, radius 1mm. 15 turns per circumference. Top. xz-plane, y=0.05mm. Bottom. xy-plane, z=4.5mm.

Comparing the sensitivity pattern in the xy-plane with the corresponding one for the perpendicular coils antenna, figure 8, clearly demonstrates the extended sensitivity characteristic of the dual opposed solenoids antenna.

Again, different antenna concepts have been compared by looking at cross sectional sensitivity patterns, see figure 12. The patterns in the y-plane look similar.

Figure 12: Cross section of antenna sensitivity patterns at antenna half height along the x-axis.

At distances from the centre where we may expect the vessel wall (2mm to 3mm for the large arteries), we see that the best
antenna – judging from the magnetic field amplitude – is the dual opposed solenoids antenna.

4 In vitro testing

The final validation of the analytical model developed will be delivered by comparing calculated sensitivity patterns for the different antennas with images created by a MR system, wherein the intravascular antennas are being used for active tracking. Thereto a number of prototype antennas have been constructed, see figure 13.

As an example the calculated sensitivity patterns in the xy-plane are shown together with the images created with a MR system for a triple loop antenna. For the results shown in figure 14, the main magnetic field is lined up with the antenna.

Figure 13: Realised intravascular antenna prototypes.

Figure 14: Calculated sensitivity profile (top) and MR image (bottom) for main magnetic field rotated 0°.

Figure 15: Calculated sensitivity profile (top) and MR image (bottom) for main magnetic field rotated 45°.

Figure 16: Calculated sensitivity profile (top) and MR image (bottom) for main magnetic field rotated 90°.
A rotation of the main magnetic field, shown in figures 15 and 16, is equivalent to having the antenna axis rotated with respect to this field. The rotation results give information on the applicability of the antenna for use in arteries that are not in line with the main magnetic field. In figure 15 the results are shown for the main magnetic field rotated 45° and in figure 16 the results are shown for the main magnetic field rotated 90°.

We have to stress that the prototype antenna has been hand-made and that the lumen of the antenna has been filled with silicone gel, not accounted for in the model. Furthermore, although the MR images are directly related to the sensitivity, the exact values have been lost in the signal processing. The calculated sensitivity profiles have been scaled for a visual match with the MR images. Taking all this into account, the calculations and measurements show a fair agreement.

5 Conclusions

For intravascular antennas, being much smaller than the applied RF wavelength, a static field, approximate model is developed. Especially for the large arteries (radii between 2mm and 3mm) and small diameter loop antennas (radius 0.5mm), sensitivities (magnetic field intensities) may be calculated using the static model that deviates 13% at maximum from results obtained by the dynamic model. Moreover, the static and dynamic sensitivities in the area of interest (in and around the vascular wall) show a similar behaviour as function of distance to the loop centre.

Before the intravascular antennas may be used in practice, some safety issues need to be addressed. One important issue concerns the transmission line connected to the antenna that may heat up in excess of 40° centigrade due to resonance effects. Measures need to be taken to prevent transmission line resonances.

References


