Intermittency and Structure Functions in Channel Flow Turbulence

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We present a study of intermittency in a turbulent channel flow. Scaling exponents of longitudinal streamwise structure functions, $\xi_p/\xi_1$, are used as quantitative indicators of intermittency. We find that near the center of the channel the values of $\xi_p/\xi_1$ up to $p = 7$ are consistent with the assumption of homogeneous and isotropic turbulence. Moving towards the boundaries, we observe a growth of intermittency which appears to be related to an intensified presence of ordered vortical structures. We argue that the clear transition in the nature of intermittency appearing in the region close to the wall is related to a new length scale which becomes the relevant one for scaling in high shear flows.

We have performed a direct numerical simulation achieving a high statistical accuracy (about $10^4$ in time units $U_0/h$, where $U_0$ is the centerline velocity and $h$ is the channel half-width). Numerical simulations have been performed on a massively parallel machine using a LBE (lattice Boltzmann equation) code. The spatial resolution of the simulation was $256 \times 128 \times 128$ grid points. Periodic boundary conditions were imposed along the streamwise ($x$) and spanwise ($z$) directions, whereas no-slip boundary conditions were applied at the top and bottom planes (normal to wall direction, $y$). The Reynolds number is $Re = 3000$. Further details about the numerical scheme can be found in [11] and references therein. In the following we use wall units defined as $y^+ = y \cdot u^*/v$ and $r^+ = r/v^*$ where $v^*$ is the friction velocity [12]. In these units, the channel is $640$ long, $320$ wide, and $320$ high.

To study intermittency in the channel, we introduce the following $y$-dependent longitudinal streamwise structure functions:

$$S_p(r^+,y^+) = \langle |v_x(x^++r^+,y^+,z^+)-v_x(x^+,y^+,z^+)|^p \rangle. \quad (1)$$

The average is taken at a fixed $y^+$ value (the normal to wall coordinate). The quantities $S_p(r^+,y^+)$ have been...
measured for each value of \( y^+ \). Our data set allows enough statistical accuracy to estimate \( S_p(r^+, y^+) \) for \( p \leq 7 \). Because of the low Reynolds number, we use extended self-similarity (ESS) [13] in order to extract \( \zeta_p \) values. We remind that ESS consists of measuring structure functions as a function of \( S_3 \) rather than in terms of space separation \( r \). This procedure allows a much better accuracy for the evaluation of the scaling exponents, although it does not provide any estimate of the \( y^+ \) dependence of \( \zeta_3 \). In order to compute the scaling exponents \( \zeta_p(y^+) \) we have analyzed the ESS local slopes \( D_{p,q}(r^+, y^+) = d \log [S_p(r^+, y^+)]/d \log [S_q(r^+, y^+)] \) for each value of the \( y^+ \) coordinate. We have found two regions in \( y^+ \), hereafter referred to as region H (“homogeneous”) and region B (“boundary”), respectively, where well defined constant local slopes for the scaling exponents can be detected. Region H is close to the center of the channel \( (y^+ \geq 100) \) while region B is close to the viscous sublayer \( (20 \leq y^+ \leq 50) \). In region H, the scaling exponents \( \zeta_p(H) \) are found to be approximately the same as the ones measured in homogeneous and isotropic turbulence. On the other hand, in region B the scaling exponents \( \zeta_p(B) \) have been found to be much smaller than \( \zeta_p(H) \). Moreover, while in region H the scaling range starts at \( r^+ \geq 25 \), in region B the scaling range starts at \( r^+ \geq 50 \), consistently with previous findings [14]. In the intermediate region between region H and region B, it is difficult to identify a range in \( r \) where a scaling exponent can be defined with enough confidence.

In order to clarify the discussion, we show in Figs. 1 and 2 the local slopes \( D_{6,3}(r^+, y^+) \) and \( D_{4,2}(r^+, y^+) \), respectively, for \( y^+ = 30, 70, 80, 150 \). In the intermediate region (i.e., \( y^+ = 70, 80 \)), the analysis in terms of local slope does not provide a well defined scaling exponent since the plateau in \( r^+ \) is very short. In Figs. 3 and 4 we show \( \zeta_6/\zeta_3 \) and \( \zeta_4/\zeta_2 \), respectively, as a function of \( y^+ \) with the associated error bars. The large error bars in the region \( 50 \leq y^+ \leq 100 \) indicate that scaling exponents defined through \( D_{p,q}(r, y^+) \) are poorly defined and should be considered just as effective exponents obtained by the power law fit of the ESS analysis. The situation described in Figs. 1 and 3 is similar for all the scaling exponents \( \zeta_p \) computed in our analysis. Finally, in Table I, we list the numerical values of the scaling exponents for region B, for region H, and for homogeneous isotropic turbulence [15].

Our results indicate that there is a transition in the nature of intermittency between region H and region B. While in region H intermittency is close to what has been observed in isotropic and homogeneous turbulence, in region B much stronger intermittency is observed, which reflects in lower values of the scaling exponents \( \zeta_p \) for \( p \geq 3 \) and larger ones for \( p < 3 \). Between the two regions, a competition between the two types of intermittency should take place, leading to a poorly defined scaling law.

An important question to be addressed concerns the physical mechanisms which produce much stronger inter-
From Fig. 3, we can argue that the increase of intermittency should be related to the increase of momentum flux and, therefore, to the mean (local) shear.

Moreover, it is well known that turbulent flows near the wall are characterized by well defined coherent structures. In Fig. 5 we show the rms helicity $h$ ($h_{\text{rms}} = \langle (\hat{\omega} \cdot \hat{v}) \rangle_{\text{rms}}$) as a function of $y^+$ which is again peaked in region B. Coherent structures carry a significant amount of helicity while dissipation is found to peak in the interstitial region between helicity-carrying structures [16]. Indeed, a clear-cut anticorrelation between helicity fluctuations and dissipation is systematically detected in our numerical simulations. Thus, the alternate presence of regions of high helicity and regions of high dissipation may be responsible for the enhancement of intermittency in region B.

A more quantitative way to investigate the increase of intermittency in region B can be achieved by the following argument. According to the Howarth–Von Karman–Kolmogorov equation for homogeneous shear flows turbulence (see Hinze [17]), one can define a length scale $L_{\lambda}(y)$ in terms of the mean energy dissipation $\epsilon(y)$ and the mean shear $\Sigma(y)$, as follows:

$$ L_{\lambda}(y) = \left( \frac{\epsilon(y)}{\Sigma(y)^3} \right)^{1/2}. \tag{3} $$

In the presence of mean shear $\Sigma$, for any scale $r$ we can define two characteristic time scales, namely, the eddy turnover time $r/\delta v(r)$ and $1/\Sigma$. We expect that when the mean shear is large enough, the eddy turnover time is not the relevant time scale for energy transfer from large to small scales. The inequality $r/\delta v(r) < 1/\Sigma$ gives the range of scales $r$ where the effect of shear should not be relevant to small scale statistics. By using the
Kolmogorov estimate $\delta v(r) \sim \epsilon^{1/3} r^{1/3}$, we find that the above inequality can be written as $r \leq L_s$. Thus, for $r \leq L_s$ one expects that the scaling properties of turbulence are not affected by the mean shear. On the other hand, for $r \gg L_s$ one expects that the mean shear may significantly change the amount of intermittency. In our numerical simulation $L_s(y)$ becomes small only in the region B, i.e., where an increase of intermittency is observed (see Fig. 6). This result confirms our finding of a rather clear transition in the physical nature of intermittency, somehow similar to the Bolgiano scaling appearing in the thermal turbulence. Hence, this preliminary analysis seems to indicate that the change of scaling exponents cannot be reduced to a perturbative effect in terms of the mean shear. As a final observation, by inspecting the peak value approached by the $\zeta_p(y)$ exponents near the wall, we find an interesting similarity with the values, $\zeta_{p\text{PS}}$, pertaining to a passive scalar advected by a turbulent 3D homogeneous and isotropic velocity field [18]. Values of passive scalar $\zeta_{p\text{PS}}$ are shown together with our present data in Table I.

Table I suggests that the passive scalar behavior can be traced to these helicity-carrying coherent structures being passively advected by the flow. This observation is in qualitative agreement with the results reported by Pumir and Shraiman [19].

In conclusion, we have rather clear evidence that, in wall bounded turbulence, the increase of intermittency near the wall is strongly related to the increase of the mean shear. We have introduced a characteristic length scale $L_s$ induced by the mean shear whose physical meaning is equivalent to the Bolgiano scale for natural convection. Velocity fluctuations at scales $r \gg L_s$ are observed to be more intermittent than in homogeneous and isotropic turbulence.

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[18] Passive scalar values from S. Ciliberto (private communication).