Crowdfunding for financing new ventures: consequences of the financial model on operational decisions

By
K.J.P.M. Voorbraak

BSc Biomedical Engineering — TU/e 2009
Student identity number 0566711

in partial fulfillment of the requirements for the degree of

Master of Science
in Operations Management and Logistics

Supervisors:
Dr. F. Tanrisever, TU/e, OPAC
Prof. Dr. R.J. Mahieu, TU/e, ITEM
Drs. J.T. Wieleman RC, Senior VP, ABN AMRO Bank
TUE. School of Industrial Engineering.
Series Master Theses Operations Management and Logistics

Subject headings: stochastic models, entrepreneurship, internet, finance
Abstract

Crowdfunding is getting more and more attention as an alternative venture financing technique. In this study, crowdfunding is investigated as the process of an entrepreneur requesting and receiving money and other resources from many parties for financing a project, in exchange for a monetary return. The financial model determines the cost of funding for the entrepreneur and the return investors will receive per period. This research aims to develop a quantitative framework to understand and evaluate the quantitative and qualitative implications of various crowdfunding models for the entrepreneur and his production decisions. The consequences of design of the crowdfunding process on the entrepreneur's operational decisions are explored for three financial models: debt structure, profit sharing structure and revenue sharing structure. For each model, the operational decisions made by the entrepreneur are determined. In addition, incentive problems that occur in the models are identified. Crowdfunding practitioners should ensure that the consequences of those problems are minimized and therefore guidelines are provided.
Acknowledgements

This report is the result of my Master Thesis project, which has been executed to obtain the degree of Master of Science in Operations Management and Logistics at Eindhoven University of Technology in Eindhoven. I have executed this project between March 2011 and August 2011 on behalf of Dialogues Incubator, wholly owned subsidiary of ABN AMRO Bank N.V. During this project, I learned much about a very new phenomenon: crowdfunding. I found out that this trend offers a valuable opportunity for entrepreneurs, but the ideal way of implementing it has not been discovered yet. I hope that my research will contribute to this.

From Dialogues Incubator, I would like to thank my colleagues from the Seeds team. It has been a pleasure to be a part of your team and to learn more from you about crowdfunding and its possibilities and threats. My special thanks go to Jos Wieleman, whose enthusiasm about my research and valuable feedback has brought my thesis to a higher level.

From Eindhoven University of Technology, my special thanks go to Fehmi Tanrisever, my first supervisor from the OPAC group. I enjoyed the discussions I had with you about my research. Besides, I appreciate that you have put me back on track every time I tended to bite off more than one can chew. Your enthusiasm about investigating something completely unknown has been a source of inspiration and I am very grateful for your support. In addition, I would like to thank my second supervisor, Ronald Mahieu from the ITEM group. I have really appreciated your valuable suggestions and feedback.

Finally, I would like to thank my boyfriend, parents and friends for their continuous support during this final phase of my studies. Thanks to them I got the chance to experience very enjoyable and valuable years as a TU/e-student, at which I will look back with good feelings.

Karen-Ann Voorbraak
Entrepreneurs seem to experience more and more problems obtaining funds nowadays. Banks have developed a reluctance to provide financing to entrepreneurs due to tightening regulation and financial crises. In order to overcome this problem, entrepreneurs are seeking alternative ways to obtain funding. One of the alternatives, crowdfunding is getting more and more attention and it is the scope of this research.

The idea of crowdfunding is to obtain funding from a large group of people, where each individual provides a small amount, instead of raising the money from a very small group of experienced investors. Additionally, crowdfunding is also used for acquiring information (Belleflamme et al. 2010). In that sense, it is an excellent tool for co-creation, in which both a firm and active customers create value through new forms of interaction, service and learning mechanisms (Prahalad and Ramaswamy, 2004). In this study, crowdfunding is investigated as the process of an entrepreneur requesting and receiving money and other resources from many parties for financing a project, in exchange for a monetary return.

This research is executed at Dialogues Incubator, wholly owned subsidiary of ABN AMRO Bank N.V., which is developing a platform that facilitates the crowdfunding process between entrepreneurs and investors. As a consequence, this research is mainly concerned with crowdfunding as a financing technique for new ventures or entrepreneurial projects. However, crowdfunding may also be used for funding art or books, or for philanthropic purposes although those alternatives are not considered in this research.

The financial model determines the cost of funding for the entrepreneur and the return investors will receive per period. A correct financial model is essential in order to keep all three stakeholders (entrepreneur, investors, and platform) involved in the long term. From the literature review, it can be concluded that there is a gap in literature in terms of a modeling framework to provide precise assessment of the financial payoff structure. Authors have mainly focused on the money-collection process and not on the period after the total amount has been collected and return should be paid to investors.

Information and incentive problems are of main importance in this context. Those problems occur when the interests of investors and entrepreneur are not aligned. On the one hand crowdfunding solves problems that occur in traditional financing methods. For instance, the entrepreneur gets more knowledge about customer demand, because investors could also be potential customers. On the other hand crowdfunding increases other information problems. For instance, the investors do not have a direct control mechanism, like detailed contracts or management influence, to control the incentives of the investors; they only have indirect control via social networks.

This research aims to develop a quantitative framework to understand and evaluate the quantitative and qualitative implications of various crowdfunding models for the entrepreneur and his production decisions. The consequences of design of the crowdfunding process on the entrepreneur's operational decisions are explored for three financial models: debt structure, profit sharing structure and revenue sharing structure. First, in the debt model, the entrepreneur borrows funds from investors and pays it back plus interest rate after maturity. Second, in the maturity based profit sharing model, the entrepreneur shares profit with investors until a particular moment in time. Finally, in the
maturity based revenue sharing model, the entrepreneur shares revenues with investors until a particular moment in time.

For each of the three model structures, a two period model is developed, to be able to examine the tradeoff between two periods. The entrepreneur aims to maximize personal two period profit. Hence, the entrepreneur’s production decisions do not aim to maximize the venture’s profit or the investor’s profit. In the first period, the entrepreneur starts running his business after he received funds for production from investors. In the second period, in case of no default the entrepreneur continues running his business. Two periods are considered to examine the following tradeoffs:

- **Debt model**: Optimal production quantity under bankruptcy risk.
- **Profit sharing model and revenue sharing model**: Optimal production quantity in the first period under the assumption that internal cash is used for production in the second period.

The entrepreneur can respond to these tradeoffs by creating an operational hedge with its production decisions. Here, operational hedging is defined as employing operational activities to mitigate risk exposure.

The variables that influence these three models are discovered and their influence tested, while taking into account demand uncertainty. Specifically, the effect of the size of the market is investigated. Furthermore, the consequences of increasing the amount that should be paid to investors are inspected. Subsequently, these models can be compared with respect to expected profit for the entrepreneur.

Numerical experiments have been executed to assess the models. Since there is no competition, if not exposed to risk the entrepreneur usually would produce the monopoly quantity, which maximizes firm value. Experiments with the debt model have been executed to investigate the consequences of bankruptcy risk on the optimal production quantity in the first period. In the second period, the entrepreneur optimally produces the monopoly quantity. The evaluation of the debt model has shown that increased bankruptcy risk causes the entrepreneur to adopt a conservative operating policy to protect against bankruptcy risk in the first period; he produces less than the monopoly quantity. This is in line with the findings of Tanrisever et al. (2009) and Xu and Birge (2004). For the provider of the loan and for the platform, survival of the firm is also of main importance. Therefore, this operational hedge against bankruptcy risk leads to minor incentive problems between the stakeholders.

The experiments in the profit sharing case aim to investigate the consequences of the assumption that internal cash should be used for production in the second period under profit sharing conditions. Like in the debt case, in the second period the monopoly quantity is the optimal production quantity. From the experiments with the profit sharing model follows that the entrepreneur behaves opportunistic in the first period, because he does not have to take care about the production costs, which are completely paid by investors. This causes incentive problems: if the entrepreneur produces such large amounts in the first period, the entrepreneur maximizes his personal benefits, but meanwhile firm profit may be zero. In the profit sharing case, as the payout to investors increases, the entrepreneur produces even more to ensure that the monopoly quantity can be produced in the second period: he creates an operational hedge to protect against risk of not being able to produce the optimal quantity in the second period. In the second period, incentive problems disappear. The entrepreneur produces the monopoly quantity in the second period because he does care about the costs then.
Finally, the revenue sharing case experiments aim to investigate the consequences for the first period production quantity of the assumption that internal cash should be used for production in the second period under revenue sharing conditions. In the first period, the entrepreneur behaves opportunistically like in the profit sharing case by overproducing to a large extent. In contrast to the other cases, in the second period the optimal production quantity is not the monopoly quantity, but it may be less. This is a consequence of the revenue sharing model in which the entrepreneur shares revenues: he does not share production costs. As a result, the optimal quantity in the second period is affected by increased payment to investors. Specifically, if the amount paid to investors increases, the optimal production quantity for the entrepreneur in the second period decreases. In addition, in the revenue sharing case, depending on the settings, the entrepreneur creates an operational hedge to decrease the probability of paying to investors. It is expected that this also occurs in the profit sharing case, but overproduction in the profit sharing case makes that this effect is negligible.

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Table 0: Main Findings

In conclusion, the debt model does lead to incentive problems due to underproduction of the entrepreneur in the first period. In the profit sharing model and the revenue sharing model, in the first period incentive problems occur as a result of significant overproduction of the entrepreneur who does not care about production costs, which are paid by investors. In the revenue sharing model incentive problems appear also in the second period where the entrepreneur produces less than the monopoly quantity, which is not beneficial for the investors.

Furthermore, the platform should set restrictions to control the entrepreneur’s behavior and to decrease incentive problems. To restrain opportunistic behavior of the entrepreneur in the first period, the amount that an entrepreneur can obtain should be constrained. Moreover, the parameters in both the profit sharing and revenue sharing model can be set such that the investor still receives a reasonable payoff. However, this is not a solution for the incentive problems that occur in the second period of the revenue sharing model. Namely, if those parameters are set such that the payout increases, the entrepreneur produces even less. Then, the maturity date of the contract should be enlarged to prevent incentive problems to occur. In addition, the platform should monitor the entrepreneur or obtain management influence to decrease incentive problems.
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List of Variables

\( q_t \)  
Production quantity in period \( t \)

\( c \)  
Production costs per unit

\( \theta \)  
Market size per period \( t \)

\( \bar{\epsilon}_t \)  
Normally distributed demand shock. The random variable \( \bar{\epsilon}_t \) is described by a normally distributed probability density function \( \varphi(\cdot) \), with mean \( \mu \) and variance \( \sigma^2 \), and cumulative distribution function \( \Phi(\cdot) \).

\( \bar{p}_t(q_t, \bar{\epsilon}_t) \)  
Price per unit in period \( t \)

\( \bar{R}_t(q_t) \)  
Revenues from sales in period \( t \)

\( \bar{n}_t(q_t) \)  
Profit by the entrepreneur in period \( t \) before extracting payment to investors

\( \bar{n} \)  
If \( \bar{n}_t(q_t) \geq \bar{n} \), the entrepreneur survives the first period in the debt case

\( r \)  
Interest rate

\( \bar{n} \)  
If \( \bar{n}_t(q_t) > \bar{n} \) the entrepreneur starts paying to investors

\( \beta \)  
If \( \bar{n}_t(q_t) > \bar{n} \) the entrepreneur pays out a part \( \beta \) of \( (\bar{n}_t(q_t) - \bar{n}) \) to investors

\( \bar{n}_t(q_t) \)  
Profit by the entrepreneur in period \( t \) after extracting payment to investors

\( q_m \)  
Monopoly quantity: production quantity that maximizes firm value.

\( q_2 \)  
Constrained second period production quantity

\( q_2^* \)  
Optimal production quantity in the second period 'Revenue Sharing Model'.

\( \bar{\epsilon}_t \)  
Entrepreneur pays out investors in period \( t \) if \( \bar{\epsilon}_t \geq \bar{\epsilon}_t \)

\( \bar{\epsilon}_1 \)  
Entrepreneur produces the monopoly quantity \( q_m \) in the second period only if \( \bar{\epsilon}_1 \geq \bar{\epsilon}_1 \)

\( q_s \)  
Intersection \( \bar{\epsilon}_1 \) and \( \bar{\epsilon}_1 \)
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1 Introduction

Entrepreneurs seem to experience more and more problems obtaining funds nowadays. Banks have developed a reluctance to provide financing to entrepreneurs due to tightening regulation and the financial crises (Het Financieele Dagblad, August 9th 2011, Appendix G). In order to overcome this problem, entrepreneurs are seeking alternative ways to obtain funding. One of the alternatives, crowdfunding is getting more and more attention in this context. Recently, the Dutch Minister of Economic Affairs, Agriculture and Innovation, Mr. M. Verhagen, promoted crowdfunding in his long term policy statement for the Dutch Parliament (MinEL&I, 2011). In addition, the European Commission has been encouraged to support crowdfunding and avoid the adoption of useless regulations on a national level (Eurada, 2011).

1.1 Definition of Crowdfunding

Crowdfunding is nothing new; it has been the backbone of the American political system for a long time (Howe, 2008; Kappel, 2009). According to Schwienbacher and Larralde (2010), raising funds by tapping a general public is the key element of crowdfunding. The idea of crowdfunding is to obtain funding from a large group of people, where each individual provides a small amount, instead of raising the money from a very small group of experienced investors. Entrepreneurs are not only motivated to use crowdfunding for obtaining funding; they also use it for acquiring information (Belleflamme et al. 2010). Hence, crowdfunding can be employed as a way of promoting the company, to support user-based improvement, or as a way for the producer to be better informed about the preferences of the consumer. In that sense, it is an excellent tool for co-creation, in which both firms and active customers create value through new forms of interaction, service and learning mechanisms. The process of value creation is rapidly shifting from a product- and firm-centric view to personalized consumer experiences (Prahalad and Ramaswamy, 2004). This is in line with Schwienbacher & Larralde (2010), who focus specifically on crowdfunding and state that investors do not only participate in crowdfunding projects because they have extrinsic motivations (monetary reward), but they also have intrinsic motivations (related to the pleasure or fun of doing the particular task i.e. investing). In this research, crowdfunding is defined as follows:

“Crowdfunding is the process of one party requesting and receiving money and other resources from many individuals for financing a project, in exchange for a monetary or non-monetary return on investment.”

Howe (2008) evaluates three typical examples of crowdfunding to explain the concept: peer-to-peer micro lending websites, funding art (books, music, etc.) and funding starting companies.

The first example is peer-to-peer micro lending websites like Kiva1 (see list of websites), where people can directly lend money to small businesses in the Third World. Kiva is a platform that uses the Internet to connect small businesses in the Third World with philanthropically minded lenders in the First World. Second, websites like TenPages2 are set up to support upcoming artists or writers by providing funding for the production of an album or publication of a book. Third, the crowd can fund a whole company or project executed by an entrepreneur. Entrepreneurs obtain money from the crowd to fund a concrete project with financial needs, via websites like Kickstarter3.
This research will focus on the third type of crowdfunding: funding new ventures. Schwienbacher & Larrañaga (2010) see crowdfunding as an interesting alternative for traditional financing methods like debt and equity financing. This research will elaborate on this observation.

1.2 Crowdfunding at Dialogues Incubator, ABN AMRO

This study is executed at Dialogues Incubator, wholly owned subsidiary of ABN Amro Bank N.V ("ABN AMRO"). The crowdfunding initiative developed by Dialogues Incubator is called “Seeds”. It has been established, following the observation of ABN AMRO that entrepreneurs have difficulties obtaining funding between €35,000 and €150,000. The Seeds-platform is a virtual environment where entrepreneurs request for funding and where the investors can commit funding. Hence, it does not intermediate and is not an electronic marketplace but it facilitates interaction between investors and entrepreneurs such that entrepreneurs can raise funds directly. As a result, three important stakeholders can be identified: the entrepreneur, the group of investors and the platform. Other stakeholders can be affected by the developments around crowdfunding, though. For instance, traditional financiers may become less attractive for entrepreneurs who desire to keep full control. However, those are out of scope of this research. Figure 1 illustrates the connection.

![Figure 1: Stakeholders](image)

The first stakeholder that can be defined is the entrepreneur. The entrepreneur makes an open call for funding a new venture or other project on the website. The entrepreneur offers the investors a return on their investment, which may be both a monetary return and/or a non-monetary return ("in-kind"). The monetary return is set beforehand and depends on the revenues of the venture. A contract is made up between entrepreneur and investors. The contract outlines what should be paid to the investors as a return on investment. Besides, the entrepreneur is contractually obliged to inform the investor about his progression and financial statements.

The second stakeholder is the group of investors. The group of investors is undefined beforehand, but consists of members of the crowd (online community). The investors receive a return on their investment, which depends on revenue made by the entrepreneur. The investor has the right to claim the assets in case the entrepreneur defaults, but needs to pay return to investors according to his revenues.

Third, the platform plays a reasonable role. The platform supports the crowdfunding process by facilitating entrepreneurs and investors to meet and to conclude an agreement. The platform also provides administrative support to the entrepreneur in the fulfillment of his contractual obligations and coordination of the cash flow from investor to entrepreneur via a blocked bank account (Escrow).
The crowdfunding process has been subdivided in three phases: the pre-crowdfunding-period, the crowdfunding-period, and the post-crowdfunding period. For more information and a schematic overview of the process, see Appendix A. In the pre-crowdfunding period, the platform screens entrepreneurs, they both sign a contract in which at least the fraction of revenues that should be paid to investors, date of maturity and the revenue level after which the entrepreneur should payout that fraction are captured.

Subsequently, the crowdfunding period starts. During the crowdfunding phase, investors pay into a third party account. Crowdfunding at this platform works with a minimum pledge: when the entrepreneur attains a threshold amount, he obtains his money, otherwise he does not and investors retrieve their investment. All payments are voided unless a minimal amount is reached before some deadline. On the other hand, if the entrepreneur has collected the threshold amount, he receives the amount minus a success fee. As a result, he sets up his business and the post-crowdfunding period begins. Within this phase, the entrepreneur yearly pays a particular return to the investors, which is described in earlier mentioned contract.

Along the crowdfunding process, three important decision making points for the platform can be identified. First, the platform needs to decide which entrepreneurs to accept on the platform and which not. To prevent swindlers from getting access to the platform, the platform has drawn up several objective criteria, which should be met by the entrepreneur. The entrepreneur should for instance be able to open a business bank account at ABN AMRO to perceive whether the entrepreneur passes BKR screening, which informs the platform about loans that the entrepreneur has obtained in the past. This problem is of legal nature and therefore it is not considered more thoroughly in this research.

Second, the platform limits the crowdfunding period and determines the exact duration of this period. The writer Craig Mod has analyzed the duration of the crowdfunding period using data of crowdfunding platform Kickstarter in an article on his website (Mod, 2010). He concludes that the duration of the crowdfunding period is not of much importance. The success rate of the crowdfunding period is mainly affected by the strength of the crowdfunding campaign set up by the entrepreneur. Other practitioners with whom this topic has been discussed, indicate similar findings. As a result, this topic seems to be less interesting to be explored in more detail from an operational perspective.

In contrast, the third decision on the financial model is very relevant and will be investigated in this study. One of the main decisions that should be made by a crowdfunding initiative is which financial payoff model to adopt. The financial model defines the capital structure of the venture and it determines both the cost of funding for
the entrepreneur and the return investors will receive per period. The common financial models are given in Table 1.

| **Equity based:** | the entrepreneur issues shares to obtain funding. |
| **Debt based:** | the entrepreneur borrows funds from investors and pays it back plus interest rate after maturity. |
| **Shared profit:** | the entrepreneur shares profit with investors. This may be maturity based (until a particular moment in time) or return based (up to a particular amount). |
| **Shared revenues:** | the entrepreneur shares revenues with investors. This may be maturity based (until a particular moment in time) or return based (up to a particular amount). |
| **Donation:** | the entrepreneur pays nothing in return. |

Table 1: Generic financial models for Crowdfunding

The current structure of the platform is an extraordinary structure based on revenue sharing. Appendix B elaborates on the details.

1.3 Research Objectives

Firms can use crowdfunding for financing their overall operations and growth. This research aims to develop a quantitative framework to understand and evaluate the quantitative and qualitative implications of various crowdfunding models for the entrepreneur and his production decisions if the firm’s capital structure consists of crowdfunding only. Through this end, a detailed understanding of crowdfunding is crucial. The focus will be on the financial model that defines the payout policy to investors. Determining a correct financial model is essential to restrain the negative consequences of asymmetrical information in order to keep all three stakeholders involved in the long term.

The consequences of design of the crowdfunding process on the entrepreneur’s operational decisions will be analyzed for three models: the debt model, the maturity based profit sharing model and the maturity based revenue sharing model. Those models will be evaluated from an entrepreneur’s perspective. The variables that influence these models will be identified and their influence tested, while taking into account demand uncertainty. Subsequently, these models can be compared with respect to expected profit for the entrepreneur. As a result, the advantages and disadvantages for the entrepreneur of each payout policy will be evaluated, taking into account operational decisions made by the entrepreneur. Furthermore, the consequences of the entrepreneur’s decisions are identified for the investor. Namely, the entrepreneur always makes decisions that are most beneficial for his own expected profit. However, incentive problems may occur when the interests of investors and entrepreneur are not aligned. Therefore, insight in the consequences of those decisions is valuable for a crowdfunding platform, to be able to guarantee the interests of investors and make crowdfunding work as an alternative venture financing technique.
2 Review of Relevant Literature

This chapter will discuss existing literature that is relevant for this research. First, the origins of crowdfunding are discussed in Paragraph 2.1. Paragraph 2.2 explores literature that is available about modeling crowdfunding. Since crowdfunding is assessed in this research as a manner of financing entrepreneurial ventures, Paragraph 2.3 evaluates traditional entrepreneurial financing methods and these are compared to crowdfunding in Paragraph 2.4. Subsequently, in the remainder of this chapter, the benefits and risks of crowdfunding will be identified in relation to information asymmetry.

2.1 Causes of the trend

What causes the fact that crowdfunding seems to be a good alternative for entrepreneurs who are not able to acquire a bank loan? Crowdfunding finds its roots in crowdsourcing (Howe, 2008; Rubinton, 2011; Geerts 2009); the rise of crowdsourcing explains the popularity of crowdfunding. Crowdsourcing is the general phenomenon in which firms outsource tasks that are traditionally performed by their employees to people who use their own time to complete these tasks. Hence, people use crowdsourcing to obtain ideas, feedback and solutions from the “crowd”. The crowd is usually but not necessarily reached through the Internet or social media. Jeff Howe in June 2006 first mentioned the word “crowdsourcing” in Wired Magazine and in 2008 he wrote a book about crowdsourcing (Howe, 2008). Howe defines crowdsourcing as follows.

“Crowdsourcing is the act of a company or institution taking a job traditionally performed by a designated agent (usually an employee) and outsourcing it to an undefined, generally large group of people in the form of an open call.”

Howe (2008) states that crowdsourcing, and thus also crowdfunding, has emerged for four reasons. (1) As a result of the specialization of jobs, private individuals are interested in contributing to economic production in their spare time to do something different for a change or because they are willing to share their knowledge. (2) Dividing an overwhelming task into small enough chunks makes completing it not only feasible, but fun. See for example the open source software trend in the software industry (3) Increasing accessibility of information. (4) Emergence of online communities in which the online population is organized. The Internet allows for communication between amateurs and professionals. Where once professionals were in power, now a self-organizing community of amateurs takes on a large extent of the labor.

These four reasons seem to apply to crowdfunding as well. Other authors also endorse the importance of online communities. Shane and Cable (2002) show that social ties provide an important mechanism through which information asymmetry is overcome in venture finance in general. Wojciechowski (2009) identifies crowdfunding as an approach used to stream joined good will efforts of people; the option that community members have to invite each other to provide funding is an important trigger to donate. Ward and Ramachandran (2011) model the importance of peer effects on the contributions in crowdfunding. They claim that the number of investors is an indication for other investors of the probability of success of a certain project.

In conclusion, crowdfunding seems to be a subset of the more general concept of crowdsourcing. However, practically, crowdfunding is more difficult to implement because
of various legal, technical and social complexities (Rubinton, 2011; Schwienbacher and Larralde, 2010). Important legal limitations occur if equity is offered to the crowd, because making a widespread solicitation for equity offering is limited to publicly listed equity. In addition, problems may be caused by information asymmetry, a topic that will be evaluated later on. Investors are not specialists and may have access to less information about the industry, past performance of the entrepreneur and other relevant information. Protection of intellectual property may further be of concern, since the entrepreneur needs to disclose appropriate information to a wider audience than under traditional forms of fundraising. On the contrary, other information problems that exist in traditional venture finance are solved by crowdfunding. Paragraph 2.5 elaborates on asymmetrical information.

2.2 Modeling Crowdfunding

Not much research has been executed on modeling crowdfunding in particular. However, the scarce attempts of modeling crowdsourcing and crowdfunding are discussed subsequently.

Several models have been developed to illustrate crowdsourcing in which participants receive a return for executing a particular task. Those models are both with risky return (DiPalantion and Vojnovic, 2009; Archak and Sundararajan, 2009; Terwiesch and Xu, 2008) and guaranteed return (Horton and Chilton, 2010). When applying these models to crowdfunding, the risky return models seem to be more feasible, because the investors may not always get repaid their money. DiPalantion and Vojnovic (2009) and Archak and Sundararajan (2009) have modeled crowdsourcing as an auction, but this does not seem to be a useful instrument because the return provided to investors by crowdfunding is unknown in advance. The return does not only depend on the investor’s own efforts but to a larger extent on revenues influenced by the efforts of the entrepreneur.

DiPalantion and Vojnovic (2009) and Archak and Sundararajan (2009) have used game theoretic tools to model crowdsourcing. Furthermore, Rubinton (2011) has used those tools for modeling crowdfunding as well. He has developed a model in which interdependent agents operate in a dynamic, discrete setting. Potential investors decide whether to invest in passive investment, active investment, donation or whether they should wait for the next period. Moreover, he approaches the topic from investor’s perspective and focuses on the process of money collection: the crowdfunding period.

In addition, Belleflamme et al. (2010) identify a number of issues related to crowdfunding that are worth studying from an industrial organization perspective. Furthermore, they propose some preliminary efforts towards modeling crowdfunding. In their model, they associate crowdfunding with pre-ordering and price discrimination and they identify crowdfunding as an entrepreneur’s attempt to inform consumers of their product’s value. The trade-off that is explored is the following: with respect to other forms of external funding, crowdfunding has the disadvantage of delaying profit by one period (the crowdfunding period in which the entrepreneur collects the funds) and the advantage of offering an improved product to some consumers. In contrast, one would expect that investors invest mainly to receive financial return on investment and for the fun of being involved, and not especially to improve the final product.

In conclusion, there is a gap in literature in terms of a modeling framework to provide precise assessment of the post-crowdfunding period in particular.
2.3 Traditional Entrepreneurial Financing Methods

To be able to view crowdfunding in the broader concept of entrepreneurial finance and to define the financial risks and benefits of crowdfunding, first traditional financing methods are discussed. Financial capital is one of the necessary resources that a firm requires for startup and operations. Capital decisions and the use of both debt and equity have important implications for bankruptcy risk, operations, firm performance and growth potential (Cassar, 2004). In finance, capital structure refers to how a firm finances his assets by a mix of different securities (Brealey et al., 2008). Entrepreneurs finance their ventures usually by internal funds (funds provided by the start-up team), or external funds (like debt, equity financing or other funds provided by external parties) (Smith and Smith, 2004). An entrepreneur may not finance his firm with debt or equity only, but he may issue both. The ratio of debt to debt plus equity is called financial leverage. According to Modigliani and Miller (1958), in a perfect market the value of the firm is unaffected by its choice of capital structure. However, the market of startups is far from perfect, due to information asymmetries (Smith and Smith, 2004). Therefore, decisions with respect to capital structure are of main importance in this context.

The most important equity financing methods for small ventures are angel financing and venture capital financing. According to Berger and Udell (1998), angel finance and venture capital represent relatively small portions of small business finance. However, external private equity is very important, because a firm that receives this form of financing has an increased probability of success. This is caused by the fact that equity investors bring in knowledge because they are often successful entrepreneurs themselves or they add value by contributing their expertise and experience in a particular industry or geographical area. However, when the entrepreneur chooses for issuing equity, he loses a share of ownership. This results in upward potential for the investor and the investor usually receives dividend payments. Specifically, risk is shifted from the entrepreneur to the investor.

Angel finance and venture capital are both equity finance, but important differences can be identified. Angel finance differs from the other forms of external financing, because the angel market is not intermediated (Berger and Udell, 1998). The market for angel funding consists of a diverse set of high net worth individuals, also known as business angels, who directly invest in high-risk, high-return entrepreneurial ventures. These individual investors range from the successful, cashed-out entrepreneur on the one hand to individuals with little or no experience with venture investing on the other. In contrast to the angel market, the venture capital market is intermediated, since a venture capital firm invests on behalf of venture capital funds. The venture capitalist controls the fund's activities and usually has expertise in discovering and nurturing promising new ventures. Another important difference in the process between angels and venture capitalists is that venture capitalists perform more due diligence than angels.

In addition to equity, an entrepreneur may obtain external funding in the form of debt. Most of the external debt finance is provided to small businesses by financial institutions like banks (Berger and Udell, 1998). Collateral and guarantees are required to grant loans and to offer credit on favorable terms. In contrast to equity, in the debt case the entrepreneur carries most of the risk himself. In contrast to venture capitalists and angel financiers, traditional debt financiers play a passive role: they do not bring much knowledge and experience and do not ask for control.

According to the pecking order theory, entrepreneurs prefer to fund their venture by internal funding first. If internal financing is not sufficient and therefore external funding is
required, an entrepreneur prefers debt to equity because he remains full owner of the venture (Myers and Maljuf, 1984). In addition, debt financing brings tax benefits as the interest can be deducted from taxable operating income.

### 2.4 Crowdfunding vs. Traditional Entrepreneurial Financing Methods

In conclusion, the literature that is available allows to define the table as illustrated in Table 2. It gives a brief summary of crowdfunding in comparison to other traditional financing methods.

<table>
<thead>
<tr>
<th></th>
<th>Bank Loan</th>
<th>Angel Investor</th>
<th>Venture Capitalist</th>
<th>Crowdfunding</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Due Diligence</strong></td>
<td>Much</td>
<td>Much</td>
<td>Very Much</td>
<td>Limited</td>
</tr>
<tr>
<td><strong>Type Firm</strong></td>
<td>Low Risk</td>
<td>High Risk - High Return</td>
<td>High Risk High Return</td>
<td>All</td>
</tr>
<tr>
<td><strong>Firm Size</strong></td>
<td>Early Stage Firms</td>
<td>Early Stage Firms</td>
<td>Later Stage Firms</td>
<td>All</td>
</tr>
<tr>
<td><strong>Type Investment</strong></td>
<td>Debt</td>
<td>Equity</td>
<td>Equity</td>
<td>Debt, Equity, Profit sharing or Revenue sharing</td>
</tr>
<tr>
<td><strong>Intermediated</strong></td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td><strong>Active or Passive</strong></td>
<td>Passive</td>
<td>Relatively Active</td>
<td>Very Active</td>
<td>Passive/Relatively Active</td>
</tr>
<tr>
<td><strong>Investment Horizon</strong></td>
<td>5 years most common</td>
<td>5 to 10 years</td>
<td>3 to 7 years</td>
<td>5 to 10 years</td>
</tr>
<tr>
<td><strong>Exit</strong></td>
<td>Pay Back</td>
<td>Sales or IPO</td>
<td>Preferably IPO</td>
<td>No obligations</td>
</tr>
</tbody>
</table>

**Table 2: Traditional Financing Methods and Crowdfunding**

The two most important differences between traditional financing methods and crowdfunding are both the amount of due diligence executed and the possibly different financial model. First, in a crowdfunding situation the investors execute little due diligence, because they do not get access to detailed information about the entrepreneur and because the crowd does not consist of professionals who have the required knowledge. Second, crowdfunding can exist in the form of equity, debt, profit sharing or revenue sharing. Profit sharing and revenue sharing are financial models that share properties of both equity and debt. As a result, the financial benefits and risks of crowdfunding are summarized in Table 3.

<table>
<thead>
<tr>
<th></th>
<th>Entrepreneur</th>
<th>Investor</th>
<th>Platform</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Financial Benefits</strong></td>
<td>Increased access to funding</td>
<td>Ability to directly invest small amounts</td>
<td>Monetary return in the form of fees</td>
</tr>
<tr>
<td><strong>Financial Risks</strong></td>
<td>Less internal cash (in case of revenue sharing/profit sharing)</td>
<td>Financial return may be reduced given the importance of intrinsic motivation</td>
<td>Increased operational costs due to salaries, etc.</td>
</tr>
</tbody>
</table>

**Table 3: Financial benefits and risks of using crowdfunding instead of traditional external funding methods**

### 2.5 Asymmetric Information

Next to the financial benefits, crowdfunding might be an interesting addition to traditional financing methods, because information problems that occur in traditional financing methods are solved by crowdfunding. Traditional investors do have funds to finance ventures, but do not always have the knowledge what the crowd wants. On the other hand, the crowd has the knowledge about what kind of products will be successful
among the crowd, because all together they know what they like as a whole (“wisdom of
the crowd”). However, they do not have the funds. As a result, when every crowdmember
puts in a small amount, the crowd joins forces and has both the funds and the knowledge.
Therefore, this chapter elaborates on asymmetrical information.

The capital markets for small businesses are not perfect. Entrepreneurs are often not
able to gain sufficient funding due to uncertainties related to early start-up stage and
growth, which increase information problems between the entrepreneur and the resource
providers (Shane, 2003). In addition, the costs of funding may be extensive as a result of
information asymmetry, because the investor does not know whether he will retrieve his
money. Also Berger and Udell (1998) and Smith and Smith (2004) state that information
problems are a main difference between small business finance and large business
finance. In contrast to large firms, small businesses may have less extensive audited
financial statements, business assets and no information about repayment history or
historical profitability. As a consequence, as the small business develops, gains more
experience and builds on track record, the effect of asymmetric information is reduced
(Shane, 2003).

According to Smith and Smith (2004), information asymmetries occur because the
investor and the entrepreneur may have different expectations about venture success
and may have difficulty communicating their expectations to each other. They state that
information and incentive problems are at the core of negotiations between investors and
entrepreneur. As a result, entrepreneurs may not have sufficient access to external
capital and as a consequence internal and external capital sources are not perfectly
substitutable.

For dealing with information problems, external investors assess business quality
problems through screening, they set up contracts and they monitor firms (Smith and
Smith, 2004). On the one hand, external investors assess small business quality and
compliance of the entrepreneur through screening. Small businesses are screened by
conducting due diligence, including the collection of information about the business, the
market in which it operates, any collateral that may be pledged, and the entrepreneur or
start-up team (Berger and Udell, 1998). As a result, the reputation of the entrepreneur is
very important. External investors see the amount an entrepreneur invests of his own
money as a good indication to the real intentions of an entrepreneur (Avery et al. 1998).
Furthermore, private equity and debt markets offer entrepreneurs complex contracts to
deal with information problems (Berger and Udell, 1998; Smith and Smith, 2004). On the
other hand, postcontractual incentive problems may occur, which are also known as
moral hazard. Once a financial contract has been signed by both entrepreneur and
investor, the incentives of both parties can change. According to Smith and Smith (2004),
moral hazard occurs when both parties cannot monitor performance perfectly and when
contracts are incomplete. Information asymmetry encourages the entrepreneur to
undertake excessive risk with the money received from investors (Shane, 2003). In order
to keep the firm from engaging in exploitive activities or strategies, the external financiers
monitor the firm during the relationship to assess compliance and financial situation, and
to exert control, for instance by directly participating in managerial decision making by
venture capitalists or renegotiating waivers or loan covenants by commercial banks
(Berger and Udell, 1998).

In conclusion, traditional venture financiers try to overcome problems caused by
asymmetrical information by executing due diligence, signing contracts, monitoring
performance of the entrepreneur and executing management control. Crowdfunding
overcomes some of these problems, because demand risk may be decreased if the
entrepreneur is able to attract a large group of investors (Schwienbacher and Larralde, 2010). Moreover, the investors have an incentive to spread the information about the product if they participate in the profit of the venture; they benefit when their network buys the product. Capital is allocated in a democratic way: to attract investors, entrepreneurs should be very transparent about their plans and activities. Social networks seem to play a crucial role (Stuart and Sorenson, 2005; Shane and Cable, 2002): an entrepreneur needs to approach his current network to ask them for funding and in addition the entrepreneur should try to expand his network if his current network does not secure a sufficient amount of cash. This may work as a social control mechanism; people usually do not deceive their friends. However, time will learn whether this is indeed the case.

Information problems that occur in traditional financing methods might be solved by crowdfunding. However, the stakeholders also have to deal with additional risks. Informational benefits and risks are summarized in Table 4.

<table>
<thead>
<tr>
<th>Informational Benefits</th>
<th>Entrepreneur</th>
<th>Investor</th>
<th>Platform</th>
</tr>
</thead>
<tbody>
<tr>
<td>Improved knowledge about customer demand</td>
<td>Improved knowledge about performance of ventures</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Increased involvement with the company</td>
<td>No control mechanism, only indirect control via social network</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Informational Risks</td>
<td>Decreased competitive advantage due to disclosing ideas on the Internet</td>
<td>Reputation risk</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Informational Benefits and Risks of using crowdfunding instead of traditional funding methods
3 Methodology

From the previous chapter it can be concluded that there is a gap in literature in terms of a modeling framework to provide precise assessment of the post-crowdfunding period. Authors have mainly focused on the money-collection process, but not on the period after the total amount has been collected and return should be paid to investors. Nevertheless, insight in the various forms of financial models is of main importance for a crowdfunding platform that has to decide which financial model to support and an entrepreneur who needs to choose for a particular crowdfunding structure. This research addresses the gap by providing a quantitative framework, which evaluates the consequences of various crowdfunding models and several relevant market parameters.

In this chapter, the methodology and terminology are introduced. A list of variables is provided on page VII of this report. Several assumptions that are applicable to the framework are described and justified.

Assumption 3.1: The venture is completely financed by crowdfunding (debt, profit sharing or revenue sharing).

In order to investigate the crowdfunding financial model, three different models are compared: the debt model, the maturity based profit sharing model ("profit sharing model") and the maturity based revenue sharing model ("revenue sharing model"). The venture is completely financed by one of these crowdfunding types and does not have other sources of funding. Therefore the entrepreneur receives total production costs for the first period from investors. The equity model is out of the scope of this research, because this model suffers from legal restrictions. Namely, offering equity is limited to publicly listed equity. Furthermore, return based variants (i.e. payment until the investor received a particular amount) of profit sharing and revenue sharing are out of scope.

Assumption 3.2: The entrepreneur aims to maximize his personal profits over two periods.

For each of the three model structures, a two period model is developed. The entrepreneur aims to maximize his personal two period profit. In the first period, the entrepreneur starts running his business after he received funds for production from investors. In the second period, in case of no default the entrepreneur continues running his business. A model with two periods is considered, because this research aims to examine the following tradeoffs:

- **Debt model:** Optimal production quantity under bankruptcy risk.
- **Profit sharing model and revenue sharing model:** Optimal production quantity in the first period under the assumption that internal cash is used for production in the second period.

The entrepreneur can respond to these tradeoffs by creating an operational hedge with its production decisions. Here, operational hedging is defined as employing operational activities to mitigate risk exposure (Tanrisever et al., 2009).

Assumption 3.3: Fixed startup costs are zero.

For simplicity, fixed startup costs are disregarded, and therefore they are assumed to be zero. Consequentially, no bankruptcy occurs in the profit sharing and revenue sharing case wherefore the effects of using internal cash can be investigated in isolation.
Assumption 3.4: There are no corporate and personal taxes.

Taxes are not taken into account. In the debt case the entrepreneur may have tax benefits on interest paid, but tax is not a main issue in this research, and including them would make it unnecessarily complicated.

Assumption 3.5: The only variable that the entrepreneur can affect is the production quantity.

At the beginning of each period, the entrepreneur determines which quantity $q_t$ to produce, while taking into account market size $\theta$ and the production costs per unit $c$. This is the only decision that the entrepreneur makes, although in practice the entrepreneur can engage in a variety of activities, which generate profit and costs, like choices about markets, organization and innovation (Reid, 1999).

Assumption 3.6: The price of the product is defined by a linear inverse demand function (Varian, 2010).

The demand function may not necessarily be linear for crowdfunding. However, until proven otherwise, linear inverse demand seems reasonable. The price of the product is defined by a linear inverse demand function $p_t(q_t, \tilde{\epsilon}_t) = \theta - q_t + \tilde{\epsilon}_t$ in which $\theta$ represents the market size (Varian, 2010). Variable $\theta$ is similar for both periods. To incorporate the implications of stochastic demand in the model, a demand shock $\tilde{\epsilon}_t$ is introduced in each period $t$, which likewise also affects the price. The demand shock $\tilde{\epsilon}_t$ has a normally distributed probability density function $\varphi(.)$, with mean $\mu$ and variance $\sigma^2$, and cumulative distribution function $\Phi(.)$. The demand shocks occur by the end of each period.

Assumption 3.7: The only risk that the entrepreneur faces is demand risk.

Although the entrepreneur may also be exposed to other uncertainty like technical uncertainty (Shane, 2003), only demand risk is taken into account. Other uncertainties are less important for the analysis in this research.

Assumption 3.8: The entrepreneur does not have any competitors.

In addition, it is assumed that the entrepreneur holds a monopoly position in both periods. This seems reasonable, as the entrepreneur identifies and initially exploits an opportunity (Shane, 2003). Consequently, the optimal production quantity that maximizes firm value is called the monopoly quantity. However, the model can be extended with the occurrence of competition.

At the beginning of each period, the entrepreneur determines which quantity $q_t$ to produce, while taking into account forecasted demand as market size $\theta$ and the production costs per unit $c$. In the first period, at that time the entrepreneur receives $cq_1$ from investors for production. By the end of each period, the entrepreneur realizes revenues $\overline{R}_t(q_t)$ from sales. For each of the models, revenues after each period are described by:

$$\overline{R}_t(q_t) = p_t(q_t, \tilde{\epsilon}_t)q_t = (\theta - q_t + \tilde{\epsilon}_t)q_t$$

The revenues minus the production costs $cq_t$, results in the profit for the firm in period $t$, given by $\pi_t(q_t)$:

$$\pi_t(q_t) = \overline{R}_t(q_t) - cq_t = (\theta - q_t + \tilde{\epsilon}_t)q_t - cq_t$$

This profit, however, does not take into account the costs of obtaining funding yet. The final profit for the firm is affected by those costs. The cost of funding differ per model, and after subtracting this cost, the venture makes profit.
This study aims to formulate a model that can be used to find the optimal production quantity $q_t$ that maximizes expected personal profit for the entrepreneur in the three cases. The entrepreneur aims to maximize personal two period profit (i.e. not the venture’s profit or the investor’s profit). The two period model is solved by backwards induction (Slikker, 2010). Hence, the second period optimal production quantity is determined first by solving a simple optimization problem. Using this information, one can determine what amount to produce in the first period.

In Chapter 4, the debt model will be evaluated. The model builds on a model developed by Tanrisever et al. (2009). The entrepreneur needs to make debt payments; he pays off the debt with a constant positive interest rate $r$. He should generate a pre-specified level of profit from its operations during the first period to be able to repay his debt and to ensure survival into the second period. Therefore a probabilistic survival constraint is introduced. The effects of use of internal cash should be eliminated from this model to isolate the bankruptcy problem. Therefore, in the second period, it is assumed that the entrepreneur obtains an additional loan to continue production. Tanrisever et al. (2009) showed that entrepreneurs can respond to bankruptcy risk by creating an operational hedge with its production decisions. For instance, he may produce less than the monopoly quantity in the first period to avoid bankruptcy (underproduction).

In Chapter 5, the profit sharing model will be considered, in which the entrepreneur shares profit with investors. Subsequently, in Chapter 6 the revenue sharing model is described, in which the entrepreneur shares revenues with investors, but not production costs. For both models, a two period model is drawn up, based on certain assumptions, while taking into account probabilistic demand. In contrast to the debt case, in the profit sharing case and revenue sharing case, the entrepreneur should use internal cash to continue production in the second period. This is likely, because the entrepreneur frequently did not pay back the entire amount after one period, and therefore obtaining additional funding is not so obvious. To isolate the internal-cash tradeoff, the assumption is made that fixed startup costs are zero. In particular is investigated whether the production decisions of the entrepreneur are affected by the fact that internal cash should be used in the second period.

For all three models, the expected two period profit is calculated for the entrepreneur. This is achieved by implementing the models in software package Wolfram Mathematica 8.0.1, and subsequently by executing numerical experiments. The results provide model-based guidelines for the financial repayment structure of a crowdfunding platform. For those models, the influence of the following variables is studied: $\theta$ market size, $r$ interest rate, $\beta$ percentage profit/revenues paid, $\pi/R$ threshold after which entrepreneur starts paying part of profit/revenues. In the next chapters is elaborated on those variables. Their individual impact and the managerial implications of their interactions on choosing for a particular crowdfunding platform are explored.
4 Debt Model

This chapter will introduce the debt model. An example of a crowdfunding platform that works with a debt structure is WeKomenErWel.nl\(^6\). The entrepreneur sells debt certificates and he pays back the loan after maturity with a particular interest rate.

We build on a model developed by Tanrisever et al. (2009). A two period model is created that aims to find the optimal production quantity that maximizes expected profit for the entrepreneur in the debt case under bankruptcy risk. The entrepreneur should generate a pre-specified level of profit from its operations during the first period to be able to repay his debt and to ensure survival into the second period. Therefore a probabilistic survival constraint is introduced.

Tanrisever et al. (2009) showed that entrepreneurs can respond to bankruptcy risk by creating an operational hedge with its production decisions. Here, operational hedging is defined as employing operational activities to mitigate risk exposure and reduce disadvantageous risk. For instance, he may produce less than the monopoly quantity in the first period to avoid bankruptcy (underproduction).

The mathematical model is drawn up in Paragraph 4.1, while taking into account certain assumptions. Subsequently, Paragraph 4.2 defines boundaries for solving the model. Paragraph 4.3 describes the final model, which will be used for executing numerical experiments.

4.1 Mathematical model

At the beginning of the first period, \( t = 1 \), the entrepreneur decides what quantity \( q_1 \) he will produce, according to his forecasted demand and the production costs per unit \( c \). Subsequently, he obtains an amount \( cq_1 \) from investors to produce \( q_1 \). In the first period, \( t = 1 \), the entrepreneur starts running his business. By the end of the first period, the entrepreneur pays back debt and interest. If the entrepreneur made sufficient revenues to pay back debt plus an interest rate, he proceeds to the second period, \( t = 2 \). Then, the entrepreneur decides again what quantity \( q_2 \) he will produce, according to his forecasted demand and the production costs per unit \( c \). Subsequently, he obtains an amount \( cq_2 \) from investors to produce \( q_2 \).

The price of the product is assumed to be defined by a linear inverse demand function for the entrepreneur’s product \( \tilde{p}_t(q_t, \tilde{\epsilon}_t) = (\theta - q_t + \tilde{\epsilon}_t) \geq 0 \). Here, \( \theta \) represents the market size and the random variable \( \tilde{\epsilon}_t \) is described by a normally distributed probability density function \( \varphi(\cdot) \), with mean \( \mu \) and variance \( \sigma^2 \), and cumulative distribution function \( \Phi(\cdot) \). By the end of each period, the entrepreneur realizes revenues from sales. Revenues after each period are described by:

\[
\tilde{R}_t(q_t) = p_t(q_t, \tilde{\epsilon}_t)q_t = (\theta - q_t + \tilde{\epsilon}_t)q_t
\]

As a result, profit in period \( t \), given by \( \tilde{\pi}_t(q_t) \), can be calculated:

\[
\tilde{\pi}_t(q_t) = \tilde{R}_t(q_t) - cq_t = (\theta - q_t + \tilde{\epsilon}_t)q_t - cq_t
\]

**Assumption 4.1:** Debt is issued at a constant interest rate and upon fully paying its previous debt the entrepreneur can borrow again in the second period to cover its production cost.
The entrepreneur makes debt payments after the first period; he pays off the debt with positive interest rate $r$. It is assumed that debt is fully repaid and a new loan is obtained in the second period, to isolate the consequences of probability of bankruptcy. In the next chapter is shown that using internal cash for production in the second period affects operational decisions of the entrepreneur. Hence, the costs of debt for the entrepreneur are given by $rcq_t$ in period $t$.

As a result, profit in period $t$, given by $\pi_t(q_t)$, can be calculated:

$$\pi_t(q_t) = \pi_t(q_t) - rcq_t = (\theta - q_t + \varepsilon_t)q_t - (1 + r)q_t$$

**Assumption 4.2:** The entrepreneur goes bankrupt and gets liquidated unless he pays debt and interest at the end of each period.

The entrepreneur must generate a particular level of profit during the first period to ensure survival into the second period. Therefore a probabilistic survival constraint is introduced: the entrepreneur only survives the first period if $\pi_1(q_t) > \bar{\pi}$. If the entrepreneur defaults, he goes bankrupt and gets liquidated (Reid, 1999). According to Tanrisever et al. (2009), entrepreneurs can respond to the bankruptcy risk, which is caused by demand uncertainty and the probabilistic survival constraint, by creating an operational hedge with its production decisions.

As a result, the value function $V(.)$ for the entrepreneur can be expressed as follows:

$$V(.) = \max_{q_{1 \geq 0}} E_{\varepsilon_1} [\pi_1(q_1) + V_2(q_1, \varepsilon_1)]$$

subject to $\pi_1(q_1) \geq \bar{\pi}$

where $V_2(q_1, \varepsilon_1) = \max_{q_{2 \geq 0}} E_{\varepsilon_2} [\pi_2(q_2)]$

The stochastic survival constraint indicates that the firm will go to the second period only if the firm survives the first period ($\pi_1(q_1) \geq \bar{\pi}$). In the model other assets of the firm are assumed to be zero: as a consequence the value of $\bar{\pi}$ is assumed to be zero. This assumption does not affect the analysis to a large extend and therefore this variable is not varied in the numerical experiments.

### 4.2 Solving second stage problem

Solving the two period model aims to identify the optimal production quantity that the entrepreneur will produce, based on maximizing expected profit. The problem can be solved by backwards induction, and therefore the second stage problem should be solved first. Consequentially, the solution for the second period can be filled out in the value function $V(.)$. Since there is assumed to be no competition in this situation, $q_2 = q_m = q^\ast$, where $q_m$ is the monopoly quantity.

**Proposition 4.1:** The optimal production level in the second period $q^\ast$ in the debt case is equal to the monopoly quantity $q_m = \frac{\theta-(1+r)c}{2}$. The optimal profit in the second period is given by $V_2(q_1, \varepsilon_1) = \frac{(\theta-(1+r)c)^2}{4}$. 

15
The optimal production quantity is found by setting $\frac{d}{dq_1} E_{\tilde{\varepsilon}_1} [\tilde{\pi}_2(q_2)] = 0$. See Appendix C for mathematical proof of propositions.

However, it has been assumed that the entrepreneur only goes to the second period if he survives the first period. Therefore, there are two demand scenarios that might occur. First, the possibility exists that the entrepreneur is not able to repay the entire debt; the default scenario. In the second scenario, the entrepreneur makes enough profit to survive the first period and make debt payments; the survival scenario. Two boundaries are defined that ascertain the threshold in which scenario $\tilde{\varepsilon}_1$ ends up.

**Proposition 4.2**: the entrepreneur survives the first period if

$$\tilde{\varepsilon}_1 \geq \tilde{\varepsilon}_1 = \frac{\pi + (1 + r) c q_1}{q_1} - \theta + q_1$$

As a result, the optimal profit in the second period is given by

$$V_2(q_1, \tilde{\varepsilon}_1) = \left( \frac{(\theta - (1 + r) c)^2}{4} | \tilde{\varepsilon}_1 \geq \tilde{\varepsilon}_1 \right)$$

Moreover the boundary $\tilde{\varepsilon}_1 = \tilde{\varepsilon}_1$ for which the venture survives the first period is obtained by setting $\tilde{\pi}_1(q_1) = (\theta - q_1 + \tilde{\varepsilon}_1) q_1 - (1 + r) c q_1 \geq \pi$. The following demand scenarios have been identified:

1. If $\tilde{\varepsilon}_1 < \tilde{\varepsilon}_1$ the entrepreneur does not survive the first period and he makes profit only in the first period.
2. If $\tilde{\varepsilon}_1 \geq \tilde{\varepsilon}_1$ the entrepreneur survives the first period and he makes profit in both the first and the second period.

### 4.3 Solving first stage problem

Subsequently, the solution of $V(\cdot)$ is obtained while taking into account the demand scenarios. According to backward induction, the second period value function $V_2(q_1, \tilde{\varepsilon}_1)$ is implemented in the two period value function $V(\cdot) = \max_{q_1>0} h(q_1)$ where

\[ h(q_1) = \pi_1(q_1) + \int_{\tilde{\varepsilon}_1}^{\infty} \pi_2(q_m) \varphi(\tilde{\varepsilon}_1) d\tilde{\varepsilon}_1 \]

where $\pi_1(q_1) = (\theta - q_1) q_1 - c q_1$, $q_m = \frac{\theta - (1 + r) c}{2}$ and $\tilde{\varepsilon}_1 = \frac{\pi + (1 + r) c q_1 - \theta + q_1}{q_1}$

**Proposition 4.3**: The optimal production quantity for the entrepreneur in the first period can be obtained by solving $\frac{dh(q_1)}{dq_1} = 0$.

The derivative of $h(q_1)$ is not easily algebraically solved while considering a normally distributed probability density function. Therefore numerical experiments will be executed using Wolfram Mathematica, which are discussed in Chapter 7.
5 Profit Sharing Model

In this chapter, the profit sharing model is introduced. The model can be used to find the optimal production quantity \( q_t \) that maximizes expected profit for the entrepreneur in the profit sharing case.

A two period model is drawn up in Paragraph 5.1, while taking into account certain assumptions. Subsequently, in Paragraph 5.2, boundaries are defined to solve the model. Paragraph 5.3 elaborates on the possible demand scenarios. Paragraph 5.4 describes the base model. However, this model is rather basic and therefore in Paragraph 5.5 and 5.6 several extensions will be discussed.

5.1 Mathematical model

At the beginning of the first period, \( t = 1 \), the entrepreneur decides what quantity \( q_t \) he will produce, according to his forecasted demand and the production costs per unit \( c \). Subsequently, he obtains an amount \( c q_t \) from investors to produce \( q_t \). In the second period, \( t = 2 \), the entrepreneur uses revenues made in the first period for production of \( q_2 \).

The price of the product is again assumed to be defined by a linear inverse demand function for the entrepreneur’s product \( p_t(q_t, \tilde{\epsilon}_t) = (\theta - q_t + \tilde{\epsilon}_t) \geq 0 \). Here, \( \theta \) represents the market size and the random variable \( \tilde{\epsilon}_t \) is described by a normally distributed probability density function \( \varphi(\cdot) \), with mean \( \mu \) and variance \( \sigma^2 \), and cumulative distribution function \( \Phi(\cdot) \). By the end of each period, the entrepreneur realizes revenues from sales. Revenues after each period are described by:

\[
R_t(q_t) = p_t(q_t, \tilde{\epsilon}_t)q_t = (\theta - q_t + \tilde{\epsilon}_t)q_t
\]

These revenues result in a profit level for the firm:

\[
\pi_t(q_t) = R_t(q_t) - cq_t = (\theta - q_t + \tilde{\epsilon}_t)q_t - cq_t
\]

Depending on the firm’s profit in the first period, the entrepreneur pays out investors.

**Assumption 5.1:** The entrepreneur uses internal cash (i.e. revenues made in the first period) to produce production quantity \( q_2 \).

Based on revenues, the entrepreneur determines the production quantity for the second period and uses internal cash for production. Hence, if internal cash is not sufficient to produce the optimal production quantity, the entrepreneur will produce less in the second period.

Furthermore, if profit level \( \pi_t(q_t) \) exceeds \( \pi \), the entrepreneur pays out a part \( \beta \) of the profit to investors. For now \( \pi \) is assumed to be zero, but this assumption is relaxed later on. Hence, if \( \pi_t(q_t) = (\theta - q_t + \tilde{\epsilon}_t)q_t - cq_t > 0 \), the entrepreneur pays out to investors; otherwise he pays nothing to the investors in that particular period. After subtracting this payment, the firm remains with profit

\[
\bar{\pi}_t(q_t) = \pi_t(q_t) - \beta \pi_t(q_t)I_{\{\pi_t(q_t)>0\}}
\]

in period \( t \). As \( \beta \) always lies between 0 and 1, the amount he owes to the investors is always smaller than the profit. Hence, the firm never defaults. Note that the entrepreneur received \( c q_t \) from investors for production in the first period. In contrast to the debt case, the entrepreneur does not necessarily pay back the entire amount in that same period. Therefore, the entrepreneur’s personal profits in the first period are given by:
Note that the assumption has been made that the entrepreneur aims to maximize his personal profits. Therefore, the value function for the entrepreneur can be expressed as follows:

\[
\hat{V}_1(q_1) = \hat{V}_1(q_1) + c q_1
\]

The model should be solved by backwards induction. Therefore, the second stage problem is solved first in the next chapter.

### 5.2 Solving second stage problem

Because the problem should be solved by backwards induction, the second stage problem is to be evaluated first. The solution for the second stage problem will be used to solve the first period. To accomplish that, the optimal production quantity in the second period \( q_2^* \) is required.

Like in the debt case, boundaries will be defined that enclose the different demand scenarios that may occur. For solving the value function \( V(\cdot, \cdot) \), two different boundaries for \( \tilde{\varepsilon}_t \) should be considered. First, \( \varepsilon_t \) specifies whether the entrepreneur pays out to investors or not in period \( t \). Thus, this boundary exists for both \( \tilde{\varepsilon}_t \) and \( \tilde{\varepsilon}_t \).

**Proposition 5.1:** In period \( t \), the entrepreneur makes positive profit and pays \( \beta \) percent of it to investors if \( \tilde{\varepsilon}_t > \varepsilon_t = q_t + c - \theta \).

Accordingly, the value function for the second period may also be expressed as

\[
V_2(q_1, \tilde{\varepsilon}_1) = \max_{q_2 \geq 0} E_{\tilde{\varepsilon}_2} [\hat{V}_2(q_2)] - \beta E_{\tilde{\varepsilon}_2} [\hat{V}_2(q_2) 1_{\{\tilde{\varepsilon}_2 > \varepsilon_2\}}]
\]

\[
= \max_{q_2 \geq 0} (\theta - q_2 - c) q_2 - \beta \int_{\varepsilon_2}^{\infty} \tilde{\varepsilon}_2 (q_2) \phi(\tilde{\varepsilon}_2) d\tilde{\varepsilon}_2
\]

From this value function, the optimal production quantity in the second period follows accordingly.

**Proposition 5.2:** If \( \pi = 0 \), in the profit sharing case, the optimal production level in the second period \( q_2^* \) is given by:

\[
q_2^* = \begin{cases} 
q_m & \text{if } q_m < q_2 \\
q_2 & \text{if } q_m \geq q_2 
\end{cases}
\]

Where \( q_m = \frac{(\theta - q_1 + \tilde{\varepsilon}_1) q_1 (1 - \beta 1_{\{\tilde{\varepsilon}_1 > \varepsilon_1\}})}{c} + q_1 \beta 1_{\{\tilde{\varepsilon}_1 > \varepsilon_1\}} \) and \( q_m = \frac{\theta - c}{2} \)
The optimal production quantity that results in maximum profit for the entrepreneur can be found by expressing \( \frac{d}{dq_2} V_2(q_1, \bar{e}_t) = 0 \). As a result, the optimal production quantity is the monopoly quantity \( q_m = \frac{\theta - c}{2} \). Notice that this applies as no competition is considered. However, earlier has been assumed that the production quantity in the second period is constrained by internal cash available. Basically, the entrepreneur desires to produce the monopoly quantity, but when this is not achievable as a result of insufficient revenues in the first period, he produces as much as possible, namely \( q_2 \).

Note that the assumption of positive price \( p_2(q_t, \bar{e}_t) = \theta - q_t + \bar{e}_t \geq 0 \) ensures that \( q_2 \) is always positive.

Likewise, a second boundary for \( \bar{e}_1 \) is defined that determines whether the entrepreneur produces more or less than the optimal production quantity in the second period:

\( \bar{e}_1 \). In contrast to the other boundary, this boundary exists for \( \bar{e}_2 \) only. It follows from the assumption as aforementioned that the production quantity in the second period is constrained by internal cash available \( c q_2 \leq \pi_{1}(q_1) + c q_1 \).

**Proposition 5.3:** If \( q_1 \leq q_m \), the entrepreneur produces the monopoly quantity \( q_m = \frac{\theta - c}{2} \) in the second period if \( \bar{e}_1 \geq \bar{e}_2 = \frac{c(q_m - q_1)}{q_1(1 - \beta)} - (\theta - q_1 - c) \). Otherwise, the entrepreneur produces the maximum possible amount \( q_2 = \frac{(\theta - q_1 + \bar{e}_1) q_1 (1 - \beta)}{c} q_1(1 - \beta) + q_1 \beta \).

Note that if assumed that \( q_1 \leq q_m \), where \( q_m \) is the point where \( \bar{e}_1 \) and \( \bar{e}_2 \) intersect, then \( \frac{c(q_m - q_1)}{q_1(1 - \beta)} \) is always positive, which leads to \( \bar{e}_1 > \bar{e}_2 \). As a result, \( \bar{e}_1 = \frac{c(q_m - q_1)}{q_1(1 - \beta)} - (\theta - q_1 - c) \). In Paragraph 5.5, the assumption \( q_1 \leq q_m \) will be relaxed. In conclusion, the optimal production quantity for the entrepreneur in the second period is given by corollary 5.1.

**Corollary 5.1:** In the profit sharing case, if \( q_1 \leq q_m \), then the optimal production quantity for the entrepreneur in the second period \( q_2^* \) is as follows:

\[
q_2^* = \begin{cases} 
q_2 & \text{if } \bar{e}_1 < \bar{e}_2 \\
q_2' & \text{if } \bar{e}_1 \leq \bar{e}_1 < \bar{e}_2 \\
q_m & \text{if } \bar{e}_1 \geq \bar{e}_2
\end{cases}
\]

Where \( q_2' = \left( \frac{(\theta - q_1 + \bar{e}_1) q_1}{c} \right) \), \( q_2'' = \left( \frac{(\theta - q_1 + \bar{e}_1) q_1 (1 - \beta)}{c} + q_1 \beta \right) \) and \( q_m = \frac{\theta - c}{2} \).

After inserting the optimal production quantity, the value function for the second period can be expressed as \( V_2(q_1, \bar{e}_1) = (\theta - q_1 - c) q_2 - \int_{\bar{e}_1}^{\infty} \beta \pi_2(q_2') \varphi(\bar{e}_2) d\bar{e}_2 \). According to backward induction, the second period value function \( V_2(q_1, \bar{e}_1) \) is implemented in the two period value function \( V(\cdot) \):

\[
V(\cdot) = \max_{q_1 \geq 0} h(q_1) = \max_{q_1 \geq 0} \mathbb{E}_{\bar{e}_1} \left[ \pi_1(q_1) + c q_1 + (\theta - q_1 - c) q_2^* - \int_{\bar{e}_1}^{\infty} \beta \pi_2(q_2') \varphi(\bar{e}_2) d\bar{e}_2 \right]
\]

where the two period objective function that should be maximized is defined as \( h(q_1) \). Subsequently \( V(\cdot) \) is solved, but first the demand scenarios will be described.
5.3 Demand Scenarios

For clarification, the three demand scenarios that appear according to the boundaries that have been defined in the previous paragraph are described. If \( q_1 < q_m \), the function \( h(q_1) \) should be considered in the following scenarios:

1. If \( \bar{e}_1 < e_1 < \bar{e}_2 \), then \( q_2^* = q_2' = \left( \frac{(\theta - q_1 + \bar{e}_1)q_1}{c} \right) \) and \( \beta I_{[\bar{e}_1, \bar{e}_2]} = 0 \)
   The second period production quantity is constrained by the revenues in the first period. In addition, \( \beta I_{[\bar{e}_1, \bar{e}_2]} = 0 \) implies that the entrepreneur does not pay to the investors in the first period.

2. If \( e_1 < \bar{e}_1 < e_2 \), then \( q_2^* = q_2'' = \left( \frac{q_1^2 - \theta (q_1 - c)}{c} \right) \) and \( \beta I_{[\bar{e}_1, \bar{e}_2]} = \beta \)
   The entrepreneur did not make enough profit in the first period to produce the monopoly quantity in the second period. In addition, \( \beta I_{[\bar{e}_1, \bar{e}_2]} = \beta \) implies that the entrepreneur pays to the investors in the first period.

3. If \( e_1 \geq \bar{e}_1 \), then \( q_2^* = q_m = \frac{\theta - c}{2} \) and \( \beta I_{[\bar{e}_1, \bar{e}_2]} = \beta \)
   The entrepreneur makes enough revenues in the first period to produce the monopoly quantity in the second period. In addition, \( \beta I_{[\bar{e}_1, \bar{e}_2]} = \beta \) implies that the entrepreneur pays to the investors in the first period.

5.4 Solving first stage problem

The entrepreneur’s value function \( V(.) \) has been defined as

\[
V(.) = \max_{q_1 \geq 0} \ h(q_1) = \max_{q_1 \geq 0} \ E_{\bar{e}_1} \left[ \bar{r}_1(q_1) + cq_1 + \left( \theta - q_2^* - c \right) q_2^* - \int_{\bar{e}_2}^{\infty} \beta \bar{r}_2(q_2^*) \varphi(\bar{e}_2) d\bar{e}_2 \right]
\]

Hence, taking into account the demand scenarios for \( q_1 \leq q_m \) results in the following objective function

\[
h(q_1) = \int_{-\infty}^{\bar{e}_1} \left( \bar{r}_1(q_1) + cq_1 + \pi_2 \left( q_2' \right) - \int_{\bar{e}_2}^{\infty} \beta \bar{r}_2 \left( q_2' \right) \varphi(\bar{e}_2) d\bar{e}_2 \right) \varphi(\bar{e}_1) d\bar{e}_1
\]

\[
+ \int_{\bar{e}_1}^{\bar{e}_2} \left( \bar{r}_1(q_1)(1 - \beta) + cq_1 + \pi_2 \left( q_2'' \right) - \int_{\bar{e}_2}^{\infty} \beta \bar{r}_2 \left( q_2'' \right) \varphi(\bar{e}_2) d\bar{e}_2 \right) \varphi(\bar{e}_1) d\bar{e}_1
\]

\[
+ \int_{\bar{e}_1}^{\infty} \left( \bar{r}_1(q_1)(1 - \beta) + cq_1 + \pi_2(q_m) - \int_{\bar{e}_2}^{\infty} \beta \bar{r}_2(q_m) \varphi(\bar{e}_2) d\bar{e}_2 \right) \varphi(\bar{e}_1) d\bar{e}_1
\]

Where \( \bar{r}_1(q_c) = (\theta - q_t + \bar{e}_t)q_t - cq_t \) and \( \pi_t(q_c) = (\theta - q_c)q_t - c q_t \)

\[
q_2' = \left( \frac{(\theta - q_1 + \bar{e}_1)q_1}{c} \right), \quad q_2'' = \left( \frac{(\theta - q_1 + \bar{e}_1)q_1(1 - \beta)}{c} + q_1 \beta \right), \quad q_m = \frac{\theta - c}{2}
\]

\[
\bar{e}_1 = q_t + \theta and \quad \bar{e}_2 = \frac{c(q_m - q_1)}{q_1(1 - \beta)} - (\theta - q_1 - c)
\]
Filling out this equation will provide with the expected total profit for the entrepreneur per production quantity $q_1$. Solving this equation for $\frac{dh(q_1)}{dq_1} = 0$ results in the optimal production quantity in the first period. However, an algebraic solution is not easily determinable. Therefore, numerical experiments have been executed in Wolfram Mathematica to investigate the model in Chapter 7.

The model discussed in this chapter so far is rather basic. Therefore, several extensions will be included. First, the assumption $q_1 \leq q_m$ will be relaxed. Second, the variable $\pi$ will be set larger than zero. Note that if the entrepreneur has generated a profit level $\pi(q_i)$ that exceeds $\pi$, the entrepreneur pays out a part $\beta$ of the surplus to the investors.

### 5.5 Extensions profit sharing model: $q_1 > q_m$

If the assumption $q_1 \leq q_m$ is relaxed, the boundaries $\overline{e}_1$ and $\overline{e}_1$ intersect if $q_1 = q_m$. After that point, $\overline{e}_1$ becomes the lower bound and $\overline{e}_1$ becomes the upper bound. The consequences of this conversion are twofold. On the one hand, it affects the boundary $\overline{e}_1$, because $\beta$ is ruled out in that function. On the other hand, it affects the demand scenarios.

**Proposition 5.4:** If $q_1 > q_m$, the entrepreneur produces the monopoly quantity $q_m = \frac{\theta - c}{2}$ in the second period if $\overline{e}_1 \geq \overline{e}_1 = \frac{c(q_m - q_1)}{q_1} - (\theta - q_1 - c)$. Otherwise, the entrepreneur produces the maximum possible amount $q_2^* = \frac{(\theta - q_1 + \overline{e}_1)q_1}{c}$.

**Corollary 5.2:** In the profit sharing case, if $q_1 > q_m$ and $\pi = 0$, then the optimal production quantity for the entrepreneur in the second period $q_2^*$ is as follows:

$$q_2^* = \begin{cases} q_2', & \text{if } \overline{e}_1 < \overline{e}_1 \\ q_m, & \text{if } \overline{e}_1 \geq \overline{e}_1 \end{cases}$$

where $q_2' = \left(\frac{(\theta - q_1 + \overline{e}_1)q_1}{c}\right)$ and $q_m = \frac{\theta - c}{2}$.

As a result, since the boundary $\overline{e}_1$ is smaller than $\overline{e}_1$ for $q_1 > q_m$, the demand scenarios are affected. The first period objective function is again defined as $h(q_1)$:

$$V(\cdot) = \max_{q_1 \geq 0} h(q_1) = \max_{q_1 \geq 0} E_{\overline{e}_1} \left[ \hat{\pi}_1(q_1) + (\theta - q_2^* - c)q_2^* - \int_{\overline{e}_2}^{\infty} \beta \hat{\pi}_2(q_2^*) \varphi(\hat{e}_2) d\hat{e}_2 \right]$$

The function $h(q_1)$ should be considered in the following scenarios:

1. **If $\overline{e}_1 \leq \overline{e}_1$, then $q_2^* = q_2' = \left(\frac{(\theta - q_1 + \overline{e}_1)q_1}{c}\right)$ and $\beta I_{[\overline{e}_1, \overline{e}_1]} = 0$**

   The second period production quantity is constrained by the revenues in the first period. In addition, $\beta I_{[\overline{e}_1, \overline{e}_1]} = 0$ implies that the entrepreneur does not pay to the investors in the first period.

2. **If $\overline{e}_1 < \overline{e}_1$, then $q_2^* = q_m = \frac{\theta - c}{2}$ and $\beta I_{[\overline{e}_1, \overline{e}_1]} = 0$**
The entrepreneur did make enough profit in the first period to produce the monopoly quantity in the second period. In addition, $\beta l_{[\hat{e}_1, \epsilon_1]} = 0$ implies that the entrepreneur does not pay to the investors in the first period.

3. If $\hat{e}_1 \geq \epsilon_1$, then $q^*_2 = q_m = \frac{\theta - c}{2}$ and $\beta l_{[\hat{e}_1, \epsilon_1]} = \beta$

The entrepreneur makes enough revenues in the first period to produce the monopoly quantity in the second period. In addition, $\beta l_{[\hat{e}_1, \epsilon_1]} = \beta$ implies that the entrepreneur pays to the investors in the first period.

Hence, if $q_1 > q_m$, taking into account those scenarios results in the following objective function:

$$
\begin{align*}
    h(q_1) &= \int_{-\infty}^{\hat{e}_1} \tilde{\pi}_1(q_1) + c q_1 + \pi_2(q_2^*) - \int_{\hat{e}_1}^{\infty} \beta \tilde{\pi}_2(q_2^*) \varphi(\tilde{e}_2) d\tilde{e}_2 \varphi(\tilde{e}_1) d\tilde{e}_1 \\
    &+ \int_{\hat{e}_1}^{\epsilon_1} \tilde{\pi}_1(q_1) + c q_1 + \pi_2(q_m) - \int_{\hat{e}_1}^{\infty} \beta \tilde{\pi}_2(q_m) \varphi(\tilde{e}_2) d\tilde{e}_2 \varphi(\tilde{e}_1) d\tilde{e}_1 \\
    &+ \int_{\epsilon_1}^{\infty} \tilde{\pi}_1(q_1)(1 - \beta) + c q_1 + \pi_2(q_m) - \int_{\hat{e}_1}^{\infty} \beta \tilde{\pi}_2(q_m) \varphi(\tilde{e}_2) d\tilde{e}_2 \varphi(\tilde{e}_1) d\tilde{e}_1
\end{align*}
$$

Where $\tilde{\pi}_1(q_t) = (\theta - q_t + \epsilon_t)q_t - c q_t$, $\pi_t(q_t) = (\theta - q_t)q_t - c q_t$

$q_2^* = (\frac{\theta - q_1 + \epsilon_1}{c})$, $q_m = \frac{\theta - c}{2}$

$\epsilon_1 = q_t + c - \theta$, $\hat{e}_1 = \frac{c}{q_1} - \theta + q_1$

Subsequently, the model will be extended for cases in which $\pi > 0$.

5.6 Extensions profit sharing model: $\pi > 0$

Notice that so far it was assumed that $\pi = 0$. However, it is desirable to include this variable in the profit sharing model as well. Subsequently this assumption will be relaxed, and $\pi$ will be introduced into the model. If the entrepreneur’s profit exceeds $\pi$, he starts paying to investors. Increasing $\pi$ has considerable consequences, which are gradually evaluated.

5.6.1 Profit function $\hat{\pi}_t(q_t)$

First, the modifications in the profit function $\hat{\pi}_t(q_t)$ will be considered. If the entrepreneur has generated a profit level $\tilde{\pi}_t(q_t)$ that exceeds $\pi$, the entrepreneur pays out a part $\beta$ of the surplus $\tilde{\pi}_t(q_t) - \pi$ to the investors. After subtracting this return for investors, the venture makes profit

$$
\hat{\pi}_t(q_t) = \tilde{\pi}_t(q_t) - \beta(\tilde{\pi}_t(q_t) - \pi) I_{[\pi_t(q_t) > \pi]}
$$
in period $t$. Again, it is assumed that the entrepreneur aims to maximize his two-period profit and the entrepreneur uses internal cash for production in the second period. As a result, the production quantity in the second period ideally is $q_m$, but $q_2$ is constrained by internal cash available. Evidently, $q_2$ is affected by $\pi$ as a result of changes in $\pi_1(q_1)$.

**Proposition 5.5:** If $\pi > 0$, in the profit sharing case, the optimal production level in the second period $q_2^*$ is given by:

$$ q_2^* = \begin{cases} q_m & \text{if } q_m < q_2 \\ \frac{q_2}{q_m} & \text{if } q_m \geq q_2 \end{cases} $$

Where $q_2 = \left(\frac{(\theta-q_1+e_t)q_1(1-\beta_1)}{\pi + cq_1}\right)$ and $q_m = \frac{\theta-c}{2}$

### 5.6.2 Second period solution

As a result of increasing $\pi$, both $\pi_t$ and $\pi_t^-$ are modified as well. First, $\pi_t$ is defined for $\pi > 0$. Remember that if $\pi_t$ exceeds $\pi_t^-$ the entrepreneur has to pay the investors, otherwise not.

**Proposition 5.6:** If $\pi > 0$, in period $t$ the entrepreneur makes positive profit and pays $\beta$ percent of it to investors if $\pi_t > \pi_t^- = q_t + c - \theta + \frac{\pi}{q_t}$.

Second, the boundary is modified which determines whether the entrepreneur produces more or less than the optimal production quantity in the second period: $\pi_t^-$.

**Proposition 5.7:** If $\pi > 0$, the entrepreneur produces the monopoly quantity $q_m = \frac{\theta-c}{2}$ in the second period if:

$$ q_2 = \left(\frac{(\theta-q_1+e_t)q_1(1-\beta_1)}{c}\right) $$

where $q_s$ is the value of $q_1$ at which the boundaries $\pi_t^-$ and $\pi_t^+$ intersect: $q_s = q_m - \frac{\pi}{c}$. Otherwise, the entrepreneur produces the maximum possible amount

$$ q_2 = \left(\frac{(\theta-q_1+e_t)q_1(1-\beta_1)}{c}\right) $$

The production quantity in the second period is constrained by internal cash available $c q_2 \leq \pi_1(q_1) + c q_1$. Note that $\pi_t^-$ and $\pi_t^+$ do no longer intersect in $q_m$, but in $q_s = q_m - \frac{\pi}{c}$. In conclusion, the optimal production quantity for the entrepreneur in the second period is given by Corollary 5.3.

**Corollary 5.3:** In the profit sharing case, if $q_1 \leq q_s$ and $\pi > 0$, then the optimal production quantity for the entrepreneur in the second period $q_2^*$ is as follows:

$$ q_2^* = \begin{cases} q_2^t & \text{if } \pi_t < \pi_t^- \\ q_2^r & \text{if } \pi_t \geq \pi_t^- \end{cases} $$

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If \( q_1 > q_s \) and \( \pi > 0 \), then the optimal production quantity for the entrepreneur in the second period is as follows:

\[
q^*_2 = \begin{cases} 
\frac{q_2'}{q_m} & \text{if } \hat{\varepsilon}_1 < \overline{\varepsilon}_1 \\
\frac{q_2''}{q_m} & \text{if } \hat{\varepsilon}_1 \geq \overline{\varepsilon}_1
\end{cases}
\]

Where \( q_2' = \frac{((\theta - q_1 + \overline{\varepsilon}_1)q_1)}{c} \), \( q_2'' = \frac{((\theta - q_1 + \overline{\varepsilon}_1)q_1(1 - \beta) + \beta(\pi + q_1))}{c} \) and \( q_m = \frac{\theta - c}{2} \).

5.6.3 Model

Finally, the final profit sharing model, which will be used for numerical experiments, has been derived. Including \( \pi \) in the value function for the entrepreneur \( V(.) \) as aforementioned results in the following equation:

\[
V(.) = \max_{q_1 \geq 0} h(q_1) = \max_{q_1 \geq 0} E_{\hat{\varepsilon}_1} \left[ \pi_1(q_1) + cq_1 + \pi_2(q_2') - \int_{\varepsilon_2}^{\infty} \beta \left( \pi_2(q_2') - \pi \right) \varphi(\hat{\varepsilon}_2) d\hat{\varepsilon}_2 \right]
\]

Hence, taking into account the three demand scenarios for \( q_1 \leq q_s = q_m - \frac{\pi}{c} \) results in the following objective function for the entrepreneur:

\[
h(q_1) = \int_{-\infty}^{\varepsilon_1} \left( \pi_1(q_1) + cq_1 + \pi_2(q_2') - \int_{\varepsilon_2}^{\infty} \beta \left( \pi_2(q_2') - \pi \right) \varphi(\hat{\varepsilon}_2) d\hat{\varepsilon}_2 \right) \varphi(\hat{\varepsilon}_1) d\hat{\varepsilon}_1 + \int_{\varepsilon_1}^{\overline{\varepsilon}_1} \left( \pi_1(q_1) + cq_1 + \beta \pi + \pi_2(q_2'') - \int_{\varepsilon_2}^{\infty} \beta \left( \pi_2(q_2'') - \pi \right) \varphi(\hat{\varepsilon}_2) d\hat{\varepsilon}_2 \right) \varphi(\hat{\varepsilon}_1) d\hat{\varepsilon}_1 + \int_{\overline{\varepsilon}_1}^{\infty} \left( \pi_1(q_1) + cq_1 + \beta \pi + \pi_2(q_m) - \int_{\varepsilon_2}^{\infty} \beta \left( \pi_2(q_m) - \pi \right) \varphi(\hat{\varepsilon}_2) d\hat{\varepsilon}_2 \right) \varphi(\hat{\varepsilon}_1) d\hat{\varepsilon}_1
\]

Where \( \pi_t(q_t) = (\theta - q_t + \overline{\varepsilon}_t)q_t - c q_t \), \( \pi_t(q_t) = (\theta - q_t)q_t - c q_t \), \( q_2' = \frac{((\theta - q_1 + \overline{\varepsilon}_1)q_1)}{c} \), \( q_2'' = \frac{((\theta - q_1 + \overline{\varepsilon}_1)q_1(1 - \beta) + \beta(\pi + q_1))}{c} \), \( q_m = \frac{\theta - c}{2} \), \( \varepsilon_t = q_1 + c - \theta + \frac{\pi}{q_t} \) and \( \overline{\varepsilon}_1 = \frac{c(q_m - q_1) - \beta \pi}{q_1(1 - \beta)} - \theta + q_1 + c \).
Accordingly, taking into account the three demand scenarios for $q_1 > q_s = q_m - \frac{\pi}{c}$ results in the following objective function for the entrepreneur:

$$h(q_1) = \int_{-\infty}^{\infty} \left( \tilde{\pi}_1(q_1) + cq_1 + \pi_2(q_2) - \int_{-\infty}^{\infty} \beta \left( \tilde{\pi}_2(q_2) - \pi \right) \varphi(\tilde{\varepsilon}_2) d\tilde{\varepsilon}_2 \right) \varphi(\tilde{\varepsilon}_1) d\tilde{\varepsilon}_1$$

$$+ \int_{-\infty}^{\infty} \left( \tilde{\pi}_1(q_1) + cq_1 + \pi_2(q_m) - \int_{-\infty}^{\infty} \beta \left( \tilde{\pi}_2(q_m) - \pi \right) \varphi(\tilde{\varepsilon}_2) d\tilde{\varepsilon}_2 \right) \varphi(\tilde{\varepsilon}_1) d\tilde{\varepsilon}_1$$

$$+ \int_{-\infty}^{\infty} \left( \tilde{\pi}_1(q_1)(1 - \beta) + cq_1 + \beta \pi + \pi_2(q_m) - \int_{-\infty}^{\infty} \beta \left( \tilde{\pi}_2(q_m) - \pi \right) \varphi(\tilde{\varepsilon}_2) d\tilde{\varepsilon}_2 \right) \varphi(\tilde{\varepsilon}_1) d\tilde{\varepsilon}_1$$

Where $\tilde{\pi}_t(q_t) = (\theta - q_t + \tilde{\varepsilon}_t)q_t - cq_t$, $\pi_t(q_t) = (\theta - q_t)q_t - cq_t$

$$q_2 = \frac{(\theta - q_1 + \tilde{\varepsilon}_1)q_1}{c}$$

$$q_m = \frac{\theta - c}{2}$$

$$\tilde{\varepsilon}_t = q_1 + c - \theta + \frac{\pi}{q_t}, \quad \tilde{\varepsilon}_1 = \frac{cq_m}{q_2} - \theta + q_1$$

The profit sharing model will be implemented in Wolfram Mathematica in Chapter 7 to investigate the consequences of the different parameters.
6 Revenue Sharing

In this chapter, the revenue sharing model is introduced. This type is supported by the platform developed by Dialogues Incubator, ABN AMRO. The model can be used to find the optimal production quantity $q_t$ that maximizes expected profit for the entrepreneur in the revenue sharing case. Through this end, again a two period model is drawn up. However, the optimal production quantity for the second period, which was used in the previous chapters, cannot be obtained algebraically in the revenue sharing case. This increases the complexity of the model.

Paragraph 6.1 introduces the mathematical model. Subsequently, in Paragraph 6.2 the second stage solution is determined. Finally, Paragraph 6.3 describes the final revenue sharing model which will be used for numerical experiments in Chapter 7.

6.1 Mathematical model

The two period model in the revenue sharing case is rather similar to the model in the profit sharing case, however a particular fraction of revenues is shared with investors, instead of a part of the profit. At the beginning of each period $t$, the entrepreneur determines production quantity, $q_t$. In the first period, he receives amount $c q_t$ from investors, in the second period he uses internal cash for production.

Again the price $p_t(q_t, \tilde{\varepsilon}_t)$ is defined by a linear inverse demand function. Here, $\theta$ represents the market size and the random variable $\tilde{\varepsilon}_t$ is described by a normally distributed probability density function $\varphi(\cdot)$, with mean $\mu$ and variance $\sigma^2$, and cumulative distribution function $\Phi(\cdot)$. By the end of period $t$, the firm realizes revenues $\bar{R}_t(q_t)$ from sales, which are described by:

$$\bar{R}_t(q_t) = p_t(q_t, \tilde{\varepsilon}_t)q_t = (\theta - q_t + \tilde{\varepsilon}_t)q_t$$

The revenues result in a profit level for the firm of:

$$\bar{\pi}_t(q_t) = \bar{R}_t(q_t) - c q_t = (\theta - q_t + \tilde{\varepsilon}_t)q_t - c q_t$$

If the firm has generated a revenue level $\bar{R}_t(q_t)$ that exceeds $R$, the entrepreneur pays out a share of his revenues to investors. Hence, the entrepreneur shares the merits he receives, but he needs to pay for production himself. In this chapter, it is assumed that $R = 0$. However, in the numerical experiments the case when $R = 0$ is also considered. The mathematical model for that case is described in Appendix E. After subtracting payment to investors, in period $t$ the venture makes profit

$$\bar{\pi}_t(q_t) = \bar{\pi}_t(q_t) - \beta(\bar{\pi}_t(q_t) + c q_t)1_{(\bar{R}_t(q_t)>0)}$$

Assumption 6.1: The entrepreneur uses internal cash (i.e. revenues made in the first period) to produce production quantity $q_2$.

Like in the profit sharing case, the firm cannot default and the entrepreneur uses internal cash to produce production quantity $q_2$. Again, the entrepreneur’s personal profits in the first period are given by:

$$\hat{\pi}_1^E(q_1) = \hat{\pi}_1(q_1) + c q_1$$
Note that it is assumed that the entrepreneur aims to maximize his personal profits. Therefore, the value function for the entrepreneur can be expressed as follows:

\[
V(.) = \max_{q_1 \geq 0} E_{\bar{\epsilon}_1}[\hat{R}_1(q_1) + c q_1 + V_2(q_1, \bar{\epsilon}_1)]
\]

where

\[
V_2(q_1, \bar{\epsilon}_1) = \max_{q_2 \geq 0} E_{\bar{\epsilon}_2}[\hat{R}_2(q_2)]
\]

s.t. \( c q_2 \leq \hat{R}_1(q_1) + c q_1 \)

The model is again solved by backwards induction. Therefore, the second stage problem is solved first in the next chapter.

### 6.2 Solving second stage problem

Because the problem is solved by backwards induction, the second stage problem should be solved first. However, this is not as convenient as in the previous chapters. Subsequently is elaborated on the problem that occurs.

First, the different boundaries that enclose the demand scenarios are described. Like in the profit sharing case, \( \bar{\theta}_1 \) specifies per period whether the entrepreneur pays out to investors or not.

**Proposition 6.1:** In period \( t \), if \( R = 0 \) the entrepreneur makes positive revenues and pays \( \beta \) percent of it to investors if \( \bar{\epsilon}_1 > \epsilon_1 = q_1 - \bar{\theta} \).

Consequently, the value function for the second period may also be expressed as

\[
V_2(q_1, \bar{\epsilon}_1) = \max_{q_2 \geq 0}(\theta - q_2 - c)q_2 - \int_{\bar{\epsilon}_2}^{\infty} \beta(\hat{R}_2(q_2) + c q_2)\varphi(\bar{\epsilon}_2)d\bar{\epsilon}_2
\]

The optimal production quantity that results in maximum profit for the entrepreneur cannot be derived easily algebraically by expressing \( \frac{d}{dq_2} V_2(q_1, \bar{\epsilon}_1) = 0 \).

**Proposition 6.2:** In the revenue sharing case, the optimal production quantity in the second period is defined by \( \tilde{q}_2 \) and obtained by solving for \( q_2 \):

\[
q_2 = \frac{\theta}{2} - \frac{c}{2 \left(1 - \beta \left(1 - \Phi(\epsilon_2)\right)\right)}
\]

This equation is not algebraically solvable. Therefore, the value of \( \tilde{q}_2 \) is obtained numerically.

Still, several constraints have to be taken into account. Namely, the production quantity in the second period is constrained by internal cash available.

**Proposition 6.3:** If \( R = 0 \), in the revenue sharing case, the optimal production level in the second period \( q_2^* \) is given by:

\[
q_2^* = \begin{cases} 
q_2 & \text{if } \tilde{q}_2 \geq q_2 \\
\tilde{q}_2 & \text{if } \tilde{q}_2 < q_2 
\end{cases}
\]
where \( q_2 = \left( \frac{(\theta - q_1 + \tilde{\epsilon}_1)q_1(1 - \beta I_{\tilde{\epsilon}_1 > \tilde{\epsilon}_1})}{c} \right) \), \( \tilde{q}_2 \) is obtained by solving
\[
q_2 = \frac{\theta}{2} - \frac{c}{2(1-\beta(1-\Phi(\tilde{\epsilon}_2)))} \text{ for } q_2 \text{ and } \tilde{q}_2 = q_2 - \theta.
\]

Basically, \( q_2 \) defines the amount that the entrepreneur can produce at most in the second period, according to his revenues. Note that still the assumption of positive price \( p_t(q_t, \tilde{\epsilon}_t) = \theta - q_t + \tilde{\epsilon}_t \geq 0 \) ensures that \( q_2 \) is always positive.

Likewise, a second boundary for \( \tilde{\epsilon}_1 \) is defined that determines whether the entrepreneur produces more or less than the optimal production quantity in the second period: \( \tilde{\epsilon}_1 \). In contrast to the other boundary, this boundary exists for \( \tilde{\epsilon}_1 \) only. It follows from the assumption as aforementioned that the production quantity in the second period is constrained by internal cash available \( cq_2 \leq \pi_1(q_1) + cq_1 \).

**Proposition 6.4:** The entrepreneur produces \( \tilde{q}_2 \) in the second period if \( \tilde{\epsilon}_1 \geq \tilde{\epsilon}_1 = \frac{cq_2}{q_1(1-\beta)} - \theta + q_1 \). Otherwise, the entrepreneur produces the maximum possible amount
\[
q_2 = \left( \frac{(\theta - q_1 + \tilde{\epsilon}_1)q_1(1 - \beta I_{\tilde{\epsilon}_1 > \tilde{\epsilon}_1})}{c} \right).
\]

Note that the boundaries \( \tilde{\epsilon}_1 \) and \( \tilde{\epsilon}_1 \) never intersect and \( \tilde{\epsilon}_1 < \tilde{\epsilon}_1 \), as a result of the assumption \( R = 0 \). In conclusion, the optimal production quantity for the entrepreneur in the second period is given by Corollary 6.1.

**Corollary 6.1:** In the revenue sharing case, if \( R = 0 \), then the optimal production quantity for the entrepreneur in the second period \( q_2^* \) is as follows:
\[
q_2^* = \begin{cases} 
q_2' & \text{if } \tilde{\epsilon}_1 < \tilde{\epsilon}_1 \\
q_2'' & \text{if } \tilde{\epsilon}_1 \leq \tilde{\epsilon}_1 < \tilde{\epsilon}_1 \\
\tilde{q}_2 & \text{if } \tilde{\epsilon}_1 \geq \tilde{\epsilon}_1
\end{cases}
\]

Where \( q_2' = \left( \frac{(\theta-q_1+\tilde{\epsilon}_1)q_1}{c} \right) / \left( \frac{1-\beta(1-\Phi(\tilde{\epsilon}_2))}{c} \right) \) and \( \tilde{q}_2 \) is obtained by solving \( q_2 = \frac{\theta}{2} - \frac{c}{2(1-\beta(1-\Phi(\tilde{\epsilon}_2)))} \) for \( q_2 \).

As a result, after inserting the constraint in the value function, the first period objective function of the entrepreneur is defined as \( h(q_1) \):
\[
V(\cdot) = \max_{q_1 \geq 0} h(q_1) = \max_{q_1 \geq 0} E_{\tilde{\epsilon}_1} \left[ \pi_1(q_1) + cq_1 + \pi_2(q_2^*) - \int_{\tilde{\epsilon}_2}^{\infty} \beta(\pi_2(q_2^*) + cq_2^*) \varphi(\tilde{\epsilon}_2) d\tilde{\epsilon}_2 \right]
\]

This equation will provide the expected total profit for the entrepreneur per production quantity \( q_1 \) and \( q_2 \).
### 6.3 Solving first stage problem

In conclusion, the demand scenarios can be specified like in the profit sharing case. Subsequently $V(.)$ is solved while taking into account those demand scenarios.

The entrepreneurs value function $V(.)$ has been defined as:

$$V(.) = \max_{q_1 \geq 0} h(q_1) = \max_{q_1 \geq 0} \text{E}_{\tilde{e}_1} \left[ \tilde{\pi}_1(q_1) + cq_1 + \pi_2(q_2^\star) - \int_{\tilde{e}_2}^{\infty} \beta(\tilde{\pi}_2(q_2^\star) + c q_2^\star) \varphi(\tilde{e}_2) d\tilde{e}_2 \right]$$

However, as $q_2^\star$ is not obtained algebraically, this value is obtained numerically as well. As a result, both the production in the first and second period are derived numerically.

The objective function $h(q_1)$ can be specified as:

$$h(q_1) = \int_{-\infty}^{\tilde{e}_1} \left( \tilde{\pi}_1(q_1) + cq_1 + \pi_2(q_2') - \int_{\tilde{e}_2}^{\infty} \beta \left( \tilde{\pi}_2(q_2') + cq_2' \right) \varphi(\tilde{e}_2) d\tilde{e}_2 \right) \varphi(\tilde{e}_1) d\tilde{e}_1$$

$$+ \int_{\tilde{e}_1}^{\infty} \left( \tilde{\pi}_1(q_1) + cq_1 - \beta(\tilde{\pi}_1(q_1) + cq_1) + \pi_2(q_2'') - \int_{\tilde{e}_2}^{\infty} \beta \left( \tilde{\pi}_2(q_2'') + cq_2'' \right) \varphi(\tilde{e}_2) d\tilde{e}_2 \right) \varphi(\tilde{e}_1) d\tilde{e}_1$$

$$+ \int_{\tilde{e}_1}^{\infty} \left( \tilde{\pi}_1(q_1) + cq_1 - \beta(\tilde{\pi}_1(q_1) + cq_1) + \pi_2(q_2'') - \int_{\tilde{e}_2}^{\infty} \beta(\tilde{\pi}_2(q_2'') + cq_2'') \varphi(\tilde{e}_2) d\tilde{e}_2 \right) \varphi(\tilde{e}_1) d\tilde{e}_1$$

Where

$$\tilde{\pi}_1(q_1) = (\theta - q_1 + q_t - c)q_1, \quad \pi_2(q_2) = (\theta - q_2 - c)q_2$$

$$q_2' = \left( \frac{(\theta - q_1 + q_t)}{c} \right), \quad q_2'' = \left( \frac{(\theta - q_1 + q_t)q_1(1-\beta)}{c} \right)$$

$$\tilde{x}_1 = \frac{cq_2}{q_1(1-\beta)} - \theta + q_1, \quad \epsilon_1 = -\theta + q_t$$

$q_2'$ is obtained by solving

$$q_2 = \frac{\theta}{2} - \frac{c}{2(1-\beta(1-\phi(e_2)))}$$

The revenue sharing model will be implemented in Wolfram Mathematica in Chapter 7 to investigate the consequences of the different parameters. In Appendix E, the situation for $R > 0$ is described.
7 Numerical Experiments

Numerical experiments have been executed to assess the consequences of each of the crowdfunding models: debt model, profit sharing model and revenue sharing model. Subsequently, those experiments have been used to compare the models with each other. In this numerical analysis the optimal operating policy for the entrepreneur who has obtained funding by crowdfunding is investigated. The experiments aim to obtain the optimal production quantity that maximizes the payoff for the entrepreneur over both periods and the resulting optimal profit for the entrepreneur. The optimal operating policy is compared to the monopoly quantity \( q_m \), which is the profit maximizing production quantity when the firm would not be exposed to risk. This production quantity maximizes firm value.

In general, the random variable \( \varepsilon_1 \) is described by a normally distributed probability density function \( \varphi(\cdot) \), with mean \( \mu = 0 \) and standard deviation \( \sigma = 3 \). Experiments have been executed with increasing market size: \( \theta = 10, 20 \) and \( \theta = 30 \). Furthermore, production costs are given by \( c = 6 \). Furthermore, per model particular variables are varied.

A venture for which this example might be applicable is Qolors, an initiative funded by crowdfunding via Crowdaboutnow. The entrepreneur who runs Qolors imports t-shirts from China and sells them to women older than 30 years. Her fixed start-up costs are ignorable.

This chapter highlights the key results of the numerical experiments. In Appendix F, all results on the numerical experiments can be found. Paragraph 7.1 evaluates the numerical experiments with the debt model. Paragraph 7.2 discusses results of the profit sharing model and the revenue sharing model results are illustrated in Paragraph 7.3. Finally, Paragraph 7.4 compares the three models.

7.1 Debt Model

The experiments in the debt case aim to investigate the consequences of bankruptcy risk on the optimal production quantity in the first period. In addition, the expected two period profit is obtained for the entrepreneur. Bankruptcy risk is increased as the interest rate increases. Therefore, the influence of the interest rate is studied. Experiments have been executed with interest rate \( r = \{0.03, 0.09, 0.15, 0.2\} \). In addition, the effect of the market parameter is considered, and therefore market size is varied; \( \theta = \{10, 20, 30\} \). Their individual impact and the managerial implications of their interactions are explored. The experiments assume that the probabilistic survival constraint \( \bar{\pi} = 0 \) . Hence, the entrepreneur only survives the first period if the first period profits \( \bar{p}_1(q_1) \geq 0 \).

In Chapter 5, the objective function \( h(q_1) \) for the debt model has been defined, which should be maximized by the entrepreneur to gain maximum expected profit \( V(\cdot) = \max h(q_1) = h(q_1^*) \). First, the objective function is plotted against the production quantity \( q_1 \geq 0 \) in the first period \( q_1 \) with different interest rates. An example when \( \theta = 20 \) is illustrated in Figure 2.
Figure 2: Debt model: Expected profit $h(q_1)$ vs. Production quantity $q_1$ with different interest rate $r$

The results show that in the absence of interest the entrepreneur produces the monopoly quantity $q_m = \frac{\theta - (1+r)c}{2}$. However, the optimal production quantity in the first period $q_1^*$ decreases for increasing interest rate. Likewise, this is the case for other values of market size $\theta$. However, this effect decreases as $\theta$ increases, because the impact of the demand shock $\xi$ decreases as $\theta$ increases.

**Result 1:** As the probability of bankruptcy increases, the entrepreneur produces less than the monopoly quantity in the first period.

The results are demonstrated in Table 5. The numbers between brackets indicate the monopoly quantity $q_m$. In addition, the optimal profit has been obtained. The optimal profit is also illustrated in Table 5 with different interest rate $r$.

<table>
<thead>
<tr>
<th>$\theta$ = 10</th>
<th>$\theta$ = 20</th>
<th>$\theta$ = 30</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$q_1^*$ ($q_m$)</td>
<td>$\hat{\pi}$</td>
</tr>
<tr>
<td>$r = 0$</td>
<td>2 (2)</td>
<td>7.04</td>
</tr>
<tr>
<td>$r = 0.03$</td>
<td>1.75 (1.91)</td>
<td>6.41</td>
</tr>
<tr>
<td>$r = 0.09$</td>
<td>1.59 (1.73)</td>
<td>5.20</td>
</tr>
<tr>
<td>$r = 0.15$</td>
<td>1.43 (1.55)</td>
<td>4.12</td>
</tr>
<tr>
<td>$r = 0.2$</td>
<td>1.30 (1.4)</td>
<td>3.32</td>
</tr>
</tbody>
</table>

Table 5: Debt Model: Optimal production quantity $q_1^*$ (monopoly quantity $q_m$) and optimal profit $\hat{\pi}$

In conclusion, the evaluation of the debt model shows that increased bankruptcy risk promotes more conservative operational decisions of the entrepreneur. In the second period, the entrepreneur optimally produces the monopoly quantity $q_m$, because the monopoly quantity $q_m$ maximizes the value of the firm. Without bankruptcy risk, the monopoly quantity $q_m$ would also maximize profit in the first period. However, if an entrepreneur is exposed to bankruptcy risk in the first period, he produces less than the monopoly quantity: he creates an operational hedge to reduce bankruptcy risk. By doing so, the entrepreneur decreases the probability of default and as a consequence he increases the probability that he will make profits in the second period. As a result, the expected profit over two periods is enlarged.
7.2 Profit Sharing Model

The experiments in the profit sharing case aim to identify the optimal production quantity in the first period under the assumption that internal cash should be used for production in the second period. In addition, the expected two period profit is obtained by the entrepreneur. The optimal production quantity in the second period has been defined as the monopoly quantity $q_m$. The entrepreneur needs to make sufficient profit in the first period to be able to produce this optimal quantity in the second period. The amount he needs to pay out to investors reduces profit, which can be used for production in the second period. Therefore, the influence of the amount that should be paid to investors is investigated. Namely, this amount affects the risk that the optimal production quantity cannot be produced in the second period. Experiments have been executed both for $\beta = \{0, 0.2, 0.4, 0.6\}$ and $\pi = \{0, 50, 100\}$. In addition, the effects of the market parameter is considered, wherefore market size is varied; $\theta = \{10, 20, 30\}$. Their individual impact and the managerial implications of their interactions are explored.

In Chapter 6, the objective function $h(q_1)$ for the profit sharing model has been defined, which should be maximized by the entrepreneur to gain maximum expected profit $V(.) = \max_{q_1 \geq 0} h(q_1) = h(q_1^{*})$. First, the objective function is plotted against the production quantity in the first period $q_1$ for different payout percentages $\beta$. An example for $\theta = 20$ is illustrated in Figure 3.

![Figure 3: Profit Sharing model, Expected profit $h(q_1)$ vs. Production quantity $q_1$ with different pay out fraction $\beta$](image)

Let us first investigate the consequences of varying $\beta$, which indicates the percentage of profit that the entrepreneur needs to pay to investors. If the entrepreneur does not have to pay to investors, there is no increased risk that the optimal production quantity cannot be produced in the second period. Hence, the case $\beta = 0$ can be seen as the benchmark. In that case, the entrepreneur produces around $\frac{\theta}{2}$ in the first period, which is significantly larger than the monopoly quantity $\frac{\theta - c}{2}$. This means that the entrepreneur overproduces in the first period (i.e. produces more than the monopoly quantity).

**Result 2:** In the profit sharing case without risk, the entrepreneur produces around $\frac{\theta}{2}$ in the first period.
However, the figure illustrates that in this model the optimal production quantity in the first period $q_1^*$ rises for increasing $\beta$. Likewise, this is the case for larger values of market size $\theta$. Furthermore, the optimal profit has been obtained.

**Result 3**: *As the amount paid to investors increases as a result of larger $\beta$, the entrepreneur produces more than $\frac{\theta}{2}$ in the first period.*

The consequences of varying market size for optimal production quantity $q_1$ and optimal profit $\bar{\pi}$ are demonstrated in Table 6.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\beta = 0$</th>
<th>$\beta = 0.2$</th>
<th>$\beta = 0.4$</th>
<th>$\beta = 0.6$</th>
<th>$\beta = 0.8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi = 10$</td>
<td>5.07 (2)</td>
<td>5.22 (2)</td>
<td>5.38 (2)</td>
<td>5.54 (2)</td>
<td>5.71 (2)</td>
</tr>
<tr>
<td>$\pi = 20$</td>
<td>9.94 (7)</td>
<td>10.49 (7)</td>
<td>11.20 (7)</td>
<td>11.88 (7)</td>
<td>12.59 (7)</td>
</tr>
<tr>
<td>$\pi = 30$</td>
<td>148.84</td>
<td>131.02</td>
<td>113.90</td>
<td>97.67</td>
<td>82.45</td>
</tr>
</tbody>
</table>

**Table 6**: Profit Sharing Model: Optimal production quantity $q_1^*$ (monopoly quantity $q_m$) and optimal profit $\bar{\pi}$ with different market size $\theta$.

Second, the implications of increasing the threshold $\pi$ are investigated. Note that if profit level $\bar{\pi}(q_1)$ exceeds $\bar{\pi}$, the entrepreneur pays a fraction $\beta$ of $(\bar{\pi}(q_1) - \bar{\pi})$ to the investors. As $\pi$ increases, the entrepreneur pays less to investors, and therefore the risk that he is not able to produce the optimal quantity in the second period decreases. As a result, there is less reason for overproduction as $\pi$ increases. However, still the entrepreneur produces around $\frac{\theta}{2}$ in the case $\beta = 0$, which is overproduction itself.

**Result 4**: *As the amount paid to investors decreases as a result of larger $\pi$, the entrepreneur tends to overproduce less in the first period for increasing $\beta$.*

The results of the numerical experiments support this statement, which can be seen in Table 7 with $\theta = 30$.

<table>
<thead>
<tr>
<th>$\pi$</th>
<th>$\beta = 0$</th>
<th>$\beta = 0.2$</th>
<th>$\beta = 0.4$</th>
<th>$\beta = 0.6$</th>
<th>$\beta = 0.8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi = 0$</td>
<td>15.01 (12)</td>
<td>15.74 (12)</td>
<td>16.94 (12)</td>
<td>18.82 (12)</td>
<td>20.71 (12)</td>
</tr>
<tr>
<td>$\pi = 50$</td>
<td>369.00</td>
<td>313.64</td>
<td>259.72</td>
<td>208.89</td>
<td>163.14</td>
</tr>
<tr>
<td>$\pi = 100$</td>
<td>369.00</td>
<td>333.46</td>
<td>299.04</td>
<td>236.14</td>
<td>163.14</td>
</tr>
</tbody>
</table>

**Table 7**: Profit Sharing Model: Optimal production quantity $q_1^*$ (monopoly quantity $q_m$) and optimal profit $\bar{\pi}$ with different $\pi$ and $\theta = 30$.

Consequently, the optimal profit is given in figure 4, plotted against the threshold $\pi$ with different $\beta$. This figure illustrates another consequence of increasing the threshold $\pi$. At some point, the threshold becomes so large that the probability of reaching this threshold is very small. As a consequence, the curve of $\beta = 0$ acts as an asymptote for the curves of $\beta > 0$ as $\pi$ goes to infinity. This phenomenon can be clearly distinguished in the figure with $\theta = 20$. 
In this paragraph, the experiments in the profit sharing case have been discussed, which aim to investigate the consequences of the assumption that internal cash should be used for production in the second period under profit sharing conditions. Like in the debt case, in the second period the monopoly quantity $q_m$ is the optimal production quantity. If the entrepreneur does not have to pay to investors, there is no increased risk that the optimal production quantity cannot be produced in the second period. In that case, the entrepreneur produces approximately $\frac{\theta}{2}$ in the first period. In contrast to the debt case, this is not the monopoly quantity. This is more than the monopoly quantity $q_m$, because the entrepreneur does not care about production costs, which are completely paid by the investors.

Furthermore, the results show that the result of higher payment to investors from higher $\beta$ or lower $\pi$ promotes increased overproduction of the entrepreneur. In that case, the entrepreneur produces more than $\frac{\theta}{2}$ in the first period: he creates an operational hedge to protect against risk of not being able to produce the optimal quantity in the second period. Hence, by producing more in the first period, the entrepreneur increases the probability of making sufficient revenues to make optimal profit in the second period.

7.3 Revenue Sharing Model

Like in the profit sharing case, the experiments in the revenue sharing case aim to investigate the consequences of the assumption that internal cash should be used for production in the second period under revenue sharing conditions. In addition, the expected two period profit is obtained for the entrepreneur. However, in contrast to the profit sharing case, the optimal production quantity in the second period $q_2$ is not the monopoly quantity, but is obtained by solving $q_2 = \frac{\theta}{2} - \frac{c}{2(1-\beta(1-\phi(\epsilon_2)))}$ with $\epsilon_2 = -\theta + q_2$ for $q_2$. The entrepreneur needs to make sufficient profit in the first period to be able to produce this optimal quantity in the second period. The payout to investors reduces profit, which can be used for production in the second period. Hence, the influence of the
amount that should be paid to investors is investigated, because it affects the risk that the optimal production quantity cannot be produced in the second period. Experiments have been executed both for payout percentage \( \beta = \{0, 0.2, 0.4, 0.6\} \) and threshold \( R = \{0, 50, 100\} \). In addition, the effect of the market parameter is considered, wherefore market size is varied; \( \theta = \{10, 20, 30\} \). Their individual impact and the managerial implications of their interactions are explored.

In Chapter 7, the objective function \( h(q_1) \) for the revenue sharing model has been defined, which should be maximized by the entrepreneur to gain maximum expected profit \( V(.) = \max_{q_1 \geq 0} h(q_1) = h(q_1^*) \). First, the objective function is plotted against the production quantity in the first period \( q_1 \) with different payout percentages \( \beta \). An example with \( \theta = 20 \) and \( R = 0 \) is illustrated in Figure 5.

![Figure 5: Revenue Sharing model: Expected profit \( h(q_1) \) vs. Production quantity \( q_1 \) with different payout fraction \( \beta \)](image)

Again, first the consequences of varying \( \beta \) are explored. Figure 5 illustrates that the entrepreneur produces approximately \( \frac{\theta}{2} \) in the first period for any value of \( \beta \).

**Result 5:** In general, if \( R = 0 \) the entrepreneur produces around \( \frac{\theta}{2} \) in the first period and the production quantity increases slightly for increasing \( \beta \).

Table 8 shows the optimal production quantity in the first period \( q_1^* \) with different values of \( \theta \). Between brackets, the monopoly quantity is given.

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( q_1^* (q_{m}) )</th>
<th>( \hat{R} )</th>
<th>( q_1^* (q_{m}) )</th>
<th>( \hat{R} )</th>
<th>( q_1^* (q_{m}) )</th>
<th>( \hat{R} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta = 0 )</td>
<td>5.07 (2)</td>
<td>28.96</td>
<td>9.94 (7)</td>
<td>148.84</td>
<td>14.97 (12)</td>
<td>369.00</td>
</tr>
<tr>
<td>( \beta = 0.2 )</td>
<td>5.13 (2)</td>
<td>21.41</td>
<td>9.95 (7)</td>
<td>111.12</td>
<td>15.00 (12)</td>
<td>281.25</td>
</tr>
<tr>
<td>( \beta = 0.4 )</td>
<td>5.17 (2)</td>
<td>15.19</td>
<td>9.96 (7)</td>
<td>74.93</td>
<td>15.00 (12)</td>
<td>194.99</td>
</tr>
<tr>
<td>( \beta = 0.6 )</td>
<td>5.17 (2)</td>
<td>10.13</td>
<td>10.00 (7)</td>
<td>42.49</td>
<td>15.00 (12)</td>
<td>112.50</td>
</tr>
<tr>
<td>( \beta = 0.8 )</td>
<td>5.17 (2)</td>
<td>5.06</td>
<td>10.00 (7)</td>
<td>20.00</td>
<td>15.00 (12)</td>
<td>45.00</td>
</tr>
</tbody>
</table>

Table 8: Revenue Sharing Model: Optimal production quantity \( q_1^* \) (monopoly quantity \( q_{m} \)) and optimal profit \( \hat{R} \) with different \( \theta \)

Notice that, in contrast to the profit sharing case, increasing \( \beta \) does not affect the optimal production quantity in the first period to a large extent; \( q_1^* \) increases very little for increasing \( \beta \). This is caused by the fact that \( \hat{q}_{1} \) is different in the revenue sharing case;
namely not the monopoly quantity. The values for $q_2$ are given in Table 9. It can be seen that $q_2$ declines as $\beta$ increases.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\beta = 0$</th>
<th>$\beta = 0.2$</th>
<th>$\beta = 0.4$</th>
<th>$\beta = 0.6$</th>
<th>$\beta = 0.8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 = 10</td>
<td>2.00 (2) 7.00 (7) 12.00 (12)</td>
<td>1.25 (2) 6.25 (7) 11.25 (12)</td>
<td>0.00 (2) 5.00 (7) 10.00 (12)</td>
<td>0.00 (2) 2.50 (7) 7.50 (12)</td>
<td>0.00 (2) 0.00 (7) 0.00 (12)</td>
</tr>
<tr>
<td>0 = 20</td>
<td>2.00 (2) 7.00 (7) 12.00 (12)</td>
<td>1.25 (2) 6.25 (7) 11.25 (12)</td>
<td>0.00 (2) 5.00 (7) 10.00 (12)</td>
<td>0.00 (2) 2.50 (7) 7.50 (12)</td>
<td>0.00 (2) 0.00 (7) 0.00 (12)</td>
</tr>
<tr>
<td>0 = 30</td>
<td>2.00 (2) 7.00 (7) 12.00 (12)</td>
<td>1.25 (2) 6.25 (7) 11.25 (12)</td>
<td>0.00 (2) 5.00 (7) 10.00 (12)</td>
<td>0.00 (2) 2.50 (7) 7.50 (12)</td>
<td>0.00 (2) 0.00 (7) 0.00 (12)</td>
</tr>
</tbody>
</table>

Table 9: Revenue Sharing Model: $q_2$ (and $q_m$) for varying $\theta$

Furthermore, the implications of increasing the threshold $R$ are investigated. Note that if revenue level $\tilde{r}(q_t) + cq_t$ exceeds $R$, the entrepreneur pays a fraction $\beta$ of $(\tilde{r}(q_t) + cq_t - R)$ to the investors. As $R$ increases, the entrepreneur pays less to investors. If $R$ becomes positive and fraction $\beta$ is increased, this does affect the first period production quantity with $\theta = 20$ notably. This can be seen in Table 10. The entrepreneur decreases production for increasing $\beta$ in that case. With $\theta = 10$ and $\theta = 30$, increasing $R$ has reduced effect on the optimal production quantity in the first period. Those results can be found in Appendix F.

<table>
<thead>
<tr>
<th>$R$</th>
<th>$q_1^*(q_m)$</th>
<th>$\tilde{r}$</th>
<th>$q_1^*(q_m)$</th>
<th>$\tilde{r}$</th>
<th>$q_1^*(q_m)$</th>
<th>$\tilde{r}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9.94 (7) 148.84</td>
<td>9.94 (7) 148.84</td>
<td>9.94 (7) 148.84</td>
<td>9.94 (7) 148.84</td>
<td>9.94 (7) 148.84</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>9.95 (7) 111.12</td>
<td>9.92 (7) 131.00</td>
<td>9.81 (7) 145.66</td>
<td>9.65 (7) 142.67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>9.96 (7) 74.93</td>
<td>9.87 (7) 114.60</td>
<td>9.77 (7) 100.92</td>
<td>9.44 (7) 139.88</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>10.00 (7) 42.49</td>
<td>9.77 (7) 95.33</td>
<td>9.77 (7) 95.33</td>
<td>9.77 (7) 95.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>10.00 (7) 20.00</td>
<td>9.77 (7) 95.33</td>
<td>9.77 (7) 95.33</td>
<td>9.77 (7) 95.33</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 10: Revenue Sharing Model: Optimal Production Quantity $q_1^*$ with different $R$ and $\theta = 20$

Moreover, increasing $R$ does influence $q_2$. However, after a particular point the production quantity in the second period does not decrease for higher $\beta$, because the threshold is not reached anymore. This can be seen in Table 11, which displays $q_2$ ($q_m$) for increasing values of $R$.

<table>
<thead>
<tr>
<th>$R$</th>
<th>$q_1^*(q_m)$</th>
<th>$\tilde{r}$</th>
<th>$q_1^*(q_m)$</th>
<th>$\tilde{r}$</th>
<th>$q_1^*(q_m)$</th>
<th>$\tilde{r}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7.00 (7) 7.00 (7)</td>
<td>7.00 (7) 7.00 (7)</td>
<td>7.00 (7) 7.00 (7)</td>
<td>7.00 (7) 7.00 (7)</td>
<td>7.00 (7) 7.00 (7)</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>6.25 (7) 6.28 (7)</td>
<td>6.28 (7) 6.28 (7)</td>
<td>6.28 (7) 6.28 (7)</td>
<td>6.28 (7) 6.28 (7)</td>
<td>6.28 (7) 6.28 (7)</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>5.00 (7) 5.14 (7)</td>
<td>5.14 (7) 5.14 (7)</td>
<td>5.14 (7) 5.14 (7)</td>
<td>5.14 (7) 5.14 (7)</td>
<td>5.14 (7) 5.14 (7)</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>2.50 (7) 3.85 (7)</td>
<td>3.85 (7) 3.85 (7)</td>
<td>3.85 (7) 3.85 (7)</td>
<td>3.85 (7) 3.85 (7)</td>
<td>3.85 (7) 3.85 (7)</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>0.00 (7) 3.29 (7)</td>
<td>0.00 (7) 3.29 (7)</td>
<td>0.00 (7) 3.29 (7)</td>
<td>0.00 (7) 3.29 (7)</td>
<td>0.00 (7) 3.29 (7)</td>
<td></td>
</tr>
</tbody>
</table>

Table 11: Revenue Sharing Model: Optimal Production Quantity $q_2$ (and $q_m$) for varying $R$ and $\theta = 20$

Result 6: In general, the entrepreneur produces around $q_2$ in the first period and depending on the market size $\theta$ the production quantity decreases slightly for increasing $R$.

The optimal profit is affected by increasing the threshold $R$. In figure 6, the optimal profit is illustrated against the threshold $R$ with different $\beta$. The curves show a similar asymptotical behavior as in the profit sharing case. Namely, the curve of $\beta = 0$ acts as an asymptote for the curves of $\beta > 0$ as $R$ goes to infinity.
The experiments in this paragraph were focused on investigation of the consequences of the assumption that internal cash should be used for production in the second period under revenue sharing conditions. In contrast to the other cases, the optimal production quantity in the second period \( q_2 \) is not the monopoly quantity, but obtained by solving \( q_2 = \frac{\theta}{2} - \frac{c}{2(1-\beta)(1-\Phi(z_2))} \) for \( q_2 \). This is caused by the fact that the entrepreneur shares revenues only; he does not share production costs. As a consequence, \( q_2 \) decreases with increasing payout to investors (higher \( \beta \) or lower \( R \)).

In the first period, in most cases the entrepreneur produces around \( \frac{\theta}{2} \) in the first period. However, changes in the amount that should be paid to investors (in the value of \( \beta \) and \( R \)) have a minor effect on the first period production quantity. Because the optimal production quantity in the second period decreases, the entrepreneur does not tend to overproduce to a large extent contrary to the profit sharing case in the first period, because he makes sufficient revenues to produce \( q_2 \) in the second period. However, for \( R = 0 \) a minor increase in production quantity can be distinguished for increasing \( \beta \).

Furthermore, if \( \theta = 20 \), the first period production quantity decreases when \( R \) increases. With \( \theta = 10 \) and \( \theta = 30 \), increasing \( R \) has reduced effect on the optimal production quantity in the first period. Specifically, this phenomenon can be declared by the boundaries that shift. This only occurs with \( \theta = 20 \), because then ending up in scenario 2 (no payment to investors in the first period) or 3 (payment to investors payment to investors in the first period) is both likely. If \( \theta = 10 \), the entrepreneur usually ends up in scenario 2 and if \( \theta = 10 \), the entrepreneur usually ends up in scenario 3. As a result, the entrepreneur decreases the probability that he needs to pay to investors in the first period by decreasing his production quantity. Hence, he creates an operational hedge to decrease the probability of paying to investors. This phenomenon has not been identified in the profit sharing case, because overproduction causes that its effect is negligible in that case. However, it might be expected that it exists in the profit sharing case as well.
Hence, the minor changes in the first period production quantity for different values of \( R \) and \( \beta \) do have minor effects in comparison a production quantity of \( \frac{\theta}{2} \). Therefore, the major result is that the first period production quantity is relatively stable around \( \frac{\theta}{2} \).

### 7.4 Comparison of the Models

Three different models have been evaluated: the debt model, the profit sharing model and the revenue sharing model. First, the findings will be compared for each model regarding operational decisions. If looking at operational decisions made by the entrepreneur, the experiments have provided several interesting findings. The findings are summarized in table 12. Subsequently, the financial models can be compared with respect to expected profit.

<table>
<thead>
<tr>
<th></th>
<th>Period 1</th>
<th>Period 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Debt Model</strong></td>
<td>Monopoly quantity ( q_m = \frac{\theta - (1+r)c}{2} ) or less to decrease the probability of default</td>
<td>Monopoly quantity ( q_m = \frac{\theta - (1+r)c}{2} )</td>
</tr>
<tr>
<td><strong>Profit Sharing Model</strong></td>
<td>Around ( \frac{\theta}{2} ) or more with high payments to investors, to increase the probability of making sufficient revenues to realize optimal profit in the second period.</td>
<td>Monopoly quantity ( q_m = \frac{\theta - c}{2} )</td>
</tr>
<tr>
<td><strong>Revenue Sharing Model</strong></td>
<td>Around ( \frac{\theta}{2} )</td>
<td>Less than the monopoly quantity, namely the solution to ( q_2 = \frac{\theta}{2} - \frac{c}{2(1-\beta(1-\phi(q_2)))} ) for ( q_2 ), which depends on payment to investors.</td>
</tr>
</tbody>
</table>

Table 12: Summary of the optimal production quantity for the entrepreneur in both periods for the debt model, the profit sharing model and the revenue sharing model.

The model has shown that the debt model is a relatively conservative model compared to the other two: the entrepreneur produces the monopoly quantity or less in both periods. In contrast, in the first period of both the profit sharing model and the revenue sharing model, the entrepreneur acts opportunistically by producing much more than the monopoly quantity without worrying about costs. In the second period, the entrepreneur cools down because he needs to use internal cash for production then. In the profit sharing model, the entrepreneur always produces the monopoly quantity. In the revenue sharing model, the entrepreneur produces even less, because he shares only revenues and not costs.

In addition to earlier mentioned findings, the models can be used for comparing the financial models with each other. This can be achieved by filling out market parameters, equalizing expected profit for the entrepreneur and obtaining values for \( r \), \( \pi/R \) and \( \beta \). For instance, in the example that is considered in this chapter with \( \theta = 30 \), debt model with interest rate \( r = 0.2 \) and profit sharing model with \( \pi = 0 \) and \( \beta = 0.4 \) result in similar expected profit for the entrepreneur. Likewise, profit sharing model with \( \pi = 50 \) and \( \beta = 0.2 \) and revenue sharing with \( R = 100 \) and \( \beta = 0.6 \) are comparable with \( \theta = 30 \).

In conclusion, from an entrepreneur’s perspective the profit sharing model and the revenue sharing model seem to be more beneficial than debt, as the entrepreneur does not need to pay back the entire loan if he did not make sufficient profits/revenues. However, those two models may lead to incentive problems that have significant consequences for the investors.
8 Discussion

In this discussion, this study will be evaluated in hindsight. The main findings are discussed in Paragraph 8.1. Subsequently, Paragraph 8.2 reviews the limitations of this research and finally Paragraph 8.3 discusses the implications for theory and practice.

8.1 Main Findings

This study has made a first start at modeling the various crowdfunding models that are available to subdivide the benefits of a crowdfunded venture between entrepreneur and investors. The results from this study consist of a quantitative framework, which evaluates the consequences of various crowdfunding models and several relevant market parameters for the entrepreneur. The results provide model-based guidelines for the financial repayment structure of a crowdfunding platform. Three different models have been evaluated: the debt model, the profit sharing model and the revenue sharing model.

The evaluation of the debt model has shown that increased bankruptcy risk causes the entrepreneur to adopt a conservative operating policy to protect against bankruptcy risk in the first period. This is in line with the findings of Tanrisever et al. (2009) and Xu and Birge (2004). The operational hedge to decrease bankruptcy risk does lead to incentive problems, because producing less than the monopoly quantity does not maximize firm value. The investor’s main aim is to retrieve his investment after one period, and he does not worry about a second period. However, these incentive problems are minor in comparison to the incentive problems that occur in the other two models.

In the profit sharing model in the second period, the entrepreneur produces the monopoly quantity. In the second period of the revenue sharing case however, the optimal production quantity for the entrepreneur in the second period decreases with increasing payout to investors. The entrepreneur maximizes profit by lower production, because he should pay for production costs in the second period himself, while revenues are shared. Incentive problems occur: the entrepreneur’s behavior in the second period does not represent the investor’s interests. Once the entrepreneur maximizes his own profit by deciding about the production quantity in the second period, profit of the investor is decreased by underproduction. Platform and investors should be aware of underproduction by the entrepreneur in the revenue sharing model in the periods after the first period. This problem is not solved by rising parameters that increase the return on investment. Namely, as those parameters get higher, the entrepreneur produces even less. A solution for this problem can be to extend the maturity date of contracts, to monitor the entrepreneur’s decisions and venture performance or insure management control.

From the experiments with the profit sharing model and the revenue sharing model in the first period follows that the entrepreneur behaves opportunistic in the first period in both cases, because he does not care about the production costs, which are completely paid by investors. Since the investor receives the money without sharing the assets of the firm like with equity, the entrepreneur’s main goal is to obtain as much cash as possible. This causes incentive problems to occur. The entrepreneur produces a very large amount in the first period to maximize his personal benefits, but meanwhile firm profit may be zero. In the profit sharing case, this leads to incentive problems between investors and entrepreneur, because the investor does not get any return on investment when firm profit is very low. In contrast, in the revenue sharing model the entrepreneur maximizes revenues in the first period. As the investor obtains a part of those revenues, the
entrepreneur’s behavior also maximizes profit for the entrepreneur. The interests of both investors and entrepreneur are aligned, but this behavior is disadvantageous for the firm value. To restrain such opportunistic behavior of the entrepreneur in the first period, the platform should limit the amount that can be obtained by entrepreneurs for protecting investors. The entrepreneur should not be allowed to decide which amount to ask from investors all alone. Therefore thorough screening of the entrepreneur is essential for crowdfunding. The crowd is not able to do that themselves because they are no professionals, so this may be a task of a crowdfunding platform.

Additionally, in the first period of the profit sharing model, as the payout to investors increases, the entrepreneur produces even more to ensure that the monopoly quantity can be produced in the second period: he creates an operational hedge to protect against risk of not being able to produce the optimal quantity in the second period. As a consequence, incentive problems also increase in the first period of the profit sharing model. Limiting the amount an entrepreneur can ask from investors is again the solution for this problem. Contrary to the profit sharing case, in the first period of the revenue sharing model the entrepreneur does not tend to overproduce significantly, because the optimal production quantity in the second period decreases. The entrepreneur already makes sufficient revenues to produce the optimal production quantity in the second period.

Furthermore, in the first period of the revenue sharing case, depending on the settings, the entrepreneur creates an operational hedge to decrease the probability of paying to investors. It is expected that this also occurs in the profit sharing case, but overproduction in the profit sharing case makes that this effect is negligible. This is an important point to note, but its effects seem to be minor. In conclusion, the main findings are summarized in Table 13.

<table>
<thead>
<tr>
<th>Model</th>
<th>Impact of higher costs of funding on production decisions ( (\pi/\beta \uparrow, \pi/R \downarrow) )</th>
<th>Impact of larger market size on production decisions ( (\theta \uparrow) )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Debt Model</strong></td>
<td>Increased underproduction in the first period to protect against bankruptcy risk</td>
<td>Decreased underproduction in the first period to protect against bankruptcy risk</td>
</tr>
<tr>
<td><strong>Profit Sharing Model</strong></td>
<td>Opportunistic behavior in first period</td>
<td></td>
</tr>
<tr>
<td><strong>Revenue Sharing Model</strong></td>
<td>Increased overproduction in the first period to ensure production of the monopoly quantity in the second period</td>
<td>Increased overproduction in the first period to ensure production of the monopoly quantity in the second period</td>
</tr>
<tr>
<td></td>
<td>Increased underproduction in the second period because production costs are not shared</td>
<td>Increased underproduction in the second period because production costs are not shared</td>
</tr>
</tbody>
</table>

Table 13: Main Findings
8.2 Implications

This study provides important information for both academics and practitioners. It is valuable for operations management literature, because this study considers the influence of capital structure in a crowdfunding environment on operational decisions. The results confirm findings of authors like Tanrisever et al. (2009) and Xu and Birge (2004), who investigate production decisions in a debt environment under demand uncertainty and market imperfections. In addition, this study has examined the impact of profit sharing and revenue sharing on production decisions. Besides, this research is a valuable contribution to literature that is available about crowdfunding. By our knowledge, no author has compared the various financial payoff structures possible for crowdfunding so far. As a result, this study is a first effort of modeling it and determining the consequences.

Furthermore, the mathematical framework that has been developed is a useful tool that can be used by practitioners. The models can be used to set parameters that fix the payoff structure for individual entrepreneurs. In addition, the framework can be used for comparing the financial models with each other and to determine the expected profit for a particular entrepreneur who desires to obtain funding by crowdfunding. An interview with a business professional has brought forward that extending the practical applicability of the models contributes to increasing value for practitioners. For comparing the expected profits for different models with each other in practice, the model should be expanded with parameters for tax and fixed costs. Besides, investigation of the influence of the number of investors on market size would be valuable for future research.

8.3 Limitations and Recommendations

This research has some limitations that should be mentioned. First, in this research has been assumed that the price of the product follows a linear pricing model. However, it might be that a linear pricing model is not applicable to successfully crowdfunded projects, because the ability to create a “hype” determines success or failure of money collection by crowdfunding. Nevertheless, not much historical data is available to keep track of this and therefore this assumption seems reasonable until proven otherwise. Future research should provide a decisive answer.

Moreover, in this research, the models are never exposed to bankruptcy risk simultaneously with the need of using internal cash for production in the second period. However, in practice coexistence of both may occur. Assumptions have been made to be able to investigate the consequences of bankruptcy and use of internal cash in isolation. In the debt model is assumed that the entrepreneur receives an additional loan in the second period, if he is able to repay his debt in the first period. Therefore, there is no need to use internal cash. Besides, fixed startup costs are assumed to be zero. Therefore, the entrepreneur does not face bankruptcy risk in the profit sharing model and revenue sharing model.

Furthermore, only two periods have been taken into account. However, in the revenue sharing model, the entrepreneur may go bankrupt in the second period and later periods, because the percentage of the revenues that should be paid to investors may exceed the profits made by the entrepreneur. Therefore, it would be interesting to extend the two period model to more periods in the revenue sharing case. However, for the analysis in this research a model consisting of two periods is sufficient.
Hence, assumptions have been made which cause bankruptcy risk simultaneously with the need of using internal cash for production in the second period not to occur. The properties that have been investigated cannot be eliminated by assumptions though. Therefore, those seemed to be most relevant for this research. However, in future research it would be interesting to relax the assumptions, to combine both phenomena in one model and to determine how both bankruptcy risk and use of internal cash interact.

In addition, the models only consider the production decisions in the crowdfunding models while ignoring the effects of competition in the industry. It might be interesting for future research to investigate the financial models for a company to compete with other firms in the industry.

Another recommendation for future research would be to include equity in the analysis. This research aims to compare the various possible crowdfunding models, and accordingly debt, profit sharing and revenue sharing models are compared. However, the models are not compared to equity in this research. It has been decided to not include equity, because offering equity via crowdfunding brings complex legal issues. Therefore, investigating equity in a crowdfunding environment should be brought forward as a suggestion for future research.


9 Conclusion

This research has been conducted at Dialogues Incubator, ABN AMRO Bank N.V., which has developed an online crowdfunding platform that may be used by entrepreneurs to finance new ventures. To assist them and other practitioners with choosing a particular financial payoff structure, this research has put three different financial models side by side. The debt model, the maturity based profit sharing model and the maturity based revenue sharing model have been taken into account in this comparison; for each of them a mathematical model has been developed. Executing numerical experiments on the mathematical framework has assessed the implications of those crowdfunding models, and resulted in the following conclusions:

- **Debt model**: increased bankruptcy risk causes the entrepreneur to adopt a conservative operating policy to protect against bankruptcy risk in the first period.
- **Revenue sharing model and profit sharing model**: the entrepreneur behaves opportunistic in the first period of both cases, because he does not care about production costs. This leads to severe incentive problems between investor and entrepreneur in the first period, because it is disadvantageous for firm profit. To restrain such opportunistic behavior of the entrepreneur, the platform should limit the amount that can be obtained by entrepreneurs to protect investors and execute screening of entrepreneurs to assess their reliability.
- **Revenue sharing model**: in the second period, the optimal production quantity for the entrepreneur decreases with increasing payout to investors. This results in incentive problems between investor and entrepreneur in the second period of the revenue sharing model. Stakeholders should be aware of underproduction by the entrepreneur in the revenue sharing model in the second period. This problem is not solved by rising parameters that increase the return on investment, because as those parameters get higher, the entrepreneur produces even less. This problem can be solved by monitoring the entrepreneur or extending the maturity date of contracts.
- **Profit sharing model**: in the first period, as the payout to investors increases, the entrepreneur creates an operational hedge to protect against risk of not being able to produce the optimal quantity in the second period. As a consequence the entrepreneur produces more than the monopoly quantity in the first period. This problem can be solved by limitation of the amount that entrepreneurs can obtain.

In conclusion, from an investor's perspective, the debt model turns out to be the most beneficial one, because fewer incentive problems occur. However, for the entrepreneur this model is less desirable because he needs to pay back the entire debt after one period. This is not the case with the profit sharing model and revenue sharing model. However, they both have their advantages and disadvantages. The profit sharing model is preferred above the revenue sharing model because the incentive problems only occur in the first period. However, in practice an entrepreneur may mask profit to prevent payout to investors. This is not the case with the revenue sharing model. Nevertheless, when implementing the revenue sharing model in practice, much needs to be achieved to overcome incentive problems after the first period. The main conclusion of this research is that it is of main importance to take care of incentive problems when setting parameters and determining maturity date. The framework that has been developed may be a useful tool to investigate the effects of those parameters. Overall crowdfunding platforms should be careful with implementing profit sharing- or revenue sharing structures.
10 References


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## 11 Websites

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Figure 7: Crowdfunding Process Dialogues Incubator, ABN AMRO Bank N.V.
Appendix B

The current crowdfunding structure supported by the platform of Dialogues Incubator, which is based on revenue sharing, is elaborated on in this appendix. To this end, the structure is mathematically evaluated and a significant consequence is emphasized. The price of the product is assumed to be defined by a linear inverse demand function for the entrepreneur’s product

\[ p_t(q_t, \tilde{e}_t) = (\theta - q_t + \tilde{e}_t) \geq 0 \] (Varian, 2010). Here, \( \theta \) represents the market size, \( q_t \) the production quantity and \( \tilde{e}_t \) is a normally distributed demand shock with mean value \( \mu \), standard deviation \( \sigma \) and probability density function \( \phi(\tilde{e}_t) \). As a result, the entrepreneur’s profit is given by

\[ \pi_t(q_t) = (\theta - q_t + \tilde{e}_t)q_t - c q_t \]

where the production costs per unit are defined as \( c \).

The current model has one eye catcher as a result of moral hazard that occurs once the entrepreneur has received his money. Let us explore this problem in more detail by evaluating the current structure for one period.

At the beginning of the period the entrepreneur decides what quantity \( q_t \) he will produce, according to his forecasted demand and the production costs per unit \( c \). The optimal production quantity turns out to be the monopoly quantity

\[ q_m = \frac{\theta - c}{2} \]

Subsequently, via crowdfunding he obtains an amount \( c q_m \) from investors to produce \( q_m \). After receiving funds by crowdfunding, the entrepreneur’s profit turn out to be

\[ \pi_t(q_t) = \pi_t(q_t) - acq_t l_{(a < \tilde{e}_t(q_t) \leq b)} - \beta c q_t l_{(b < \tilde{e}_t(q_t) \leq g)} - \gamma c q_t l_{(g < \tilde{e}_t(q_t))} \]

where \( a \) illustrates the percentage that the entrepreneur should pay to investors if revenue lies between \( a \) and \( b \), \( \beta \) illustrates the percentage that the entrepreneur should pay to investors if revenue lies between \( b \) and \( c \) and \( \gamma \) illustrates the percentage that the entrepreneur should pay to investors if revenue exceeds \( g \). The entrepreneur starts paying \( a \) percent if \( \tilde{e}_t \geq \frac{a}{q_1} = \frac{a}{q_t} - \theta + q_t \), etc.

According to the crowdfunding payment scheme, if the revenues of the entrepreneur reach a particular threshold, the entrepreneur starts paying return to investors. The amount that he starts paying is a particular fixed percentage of the amount \( c q_m \) he received. As a result, the profit for the entrepreneur decreases abruptly. This causes the incentives of the entrepreneur to change, once he received the money. With this paying scheme, the entrepreneur no longer aims to produce the monopoly quantity, because this no longer maximizes his personal benefits. This, while for the investors profit maximization is of main importance to increase their return on investment. The more the thresholds are incorporated, the more the effects of incentive problems diminish. If the number of thresholds goes to infinity, the consequences of incentive problems go away, and is basically dealt with a direct revenue sharing model. Therefore, this crowdfunding model technically can be considered as revenue sharing, although they are legally completely different.
Appendix C Mathematical Proof

Proposition 4.1

The boundary $\bar{\epsilon}_1 = \bar{\pi}_1$ for which the venture survives the first period is obtained by setting $\bar{\pi}_1(q_1) = (\theta - q_1 + \bar{\epsilon}_1)q_1 - (1 + r) cq_1 \geq \bar{\pi}$.

$$\bar{\epsilon}_1 \geq \bar{\pi}_1 = \frac{\bar{\pi} + (1 + r) cq_1 - \theta + q_1}{q_1}$$

The optimal production quantity in the second period is found by setting

$$\frac{d}{dq_2} E[\bar{\pi}_2(q_2)] = \int_{\bar{\pi}_1}^{\infty} \frac{d\bar{\pi}_2(q_2, \bar{\epsilon}_2)}{dq_2} \varphi(\bar{\epsilon}_2)d\bar{\epsilon}_2 - \bar{\pi}_2(q_2, \bar{\pi}_1) \frac{d\bar{\pi}_1}{dq_2} = (\theta - 2q_2 + (1 + r)c)(1 - (1 - \Phi(\bar{\epsilon}_1))) = 0$$

As $1 - \beta \left(1 - \Phi(\bar{\epsilon}_2)\right)$ is always positive, $(\theta - 2q_2 - (1 + r)c)$ should be zero. Hence, the optimal production quantity is the monopoly quantity $q_m = \frac{\theta - (1 + r)c}{2}$. Notice that this applies as no competition is considered.

Proposition 4.2

Substituting the optimal second period solution into the profit function results in the optimal profit. As a result, the value function for the entrepreneur $V(.)$ may also be written as:

$$V(.) = \max_{q_1 \geq 0} E[\bar{\pi}_1(q_1) + V_2(q_1, \bar{\epsilon}_1)]$$

$$= \max_{q_1 \geq 0} E[\bar{\pi}_1(\theta - q_1 + \bar{\epsilon}_1)q_1 - (1 + r) cq_1 + \bar{\pi}_1 \left(\frac{\theta - (1 + r)c^2}{4}\right) | \bar{\epsilon}_1 \geq \bar{\pi}_1]$$

$$= \max_{q_1 \geq 0} \int_{\bar{\pi}_1}^{\infty} \left(\theta - q_1 + \bar{\epsilon}_1\right)q_1 - (1 + r) cq_1 \varphi(\bar{\epsilon}_1)d\bar{\epsilon}_1 + \int_{\bar{\pi}_1}^{\infty} \left(\theta - q_1 + \bar{\epsilon}_1\right)q_1 - (1 + r) cq_1 + \frac{\left(\theta - (1 + r)c^2\right)}{4} \varphi(\bar{\epsilon}_1)d\bar{\epsilon}_1$$

Proposition 5.1

If $\bar{\epsilon}_1$ exceeds $\bar{\pi}_1$, the entrepreneur should payout to investors, otherwise not. Given that the entrepreneur pays investors if $\bar{\pi}_1(q_t) = (\theta - q_t + \bar{\epsilon}_1)q_t - c q_t > 0$, follows that the entrepreneur pays out investors if $\bar{\epsilon}_1 > q_1 + c - \theta$.

Proposition 5.2
The optimal production quantity that results in maximum profit for the entrepreneur can be found by expressing \( \frac{d}{dq_2} V_2(q_1, \bar{\epsilon}_1) = 0 \).

\[
\frac{d}{dq_2} V_2(q_1, \bar{\epsilon}_1) = (\theta - 2q_2 - c) - \beta \int_{\bar{\epsilon}_2}^{\infty} \frac{d\bar{\pi}_2(q_2, \bar{\epsilon}_2)}{dq_2} \varphi(\bar{\epsilon}_2) d\bar{\epsilon}_2 + \beta \bar{\pi}_2(q_2, \bar{\epsilon}_2) \frac{d\bar{\epsilon}_2}{dq_2}
\]

\[
= (\theta - 2q_2 - c) - \beta(\theta - 2q_2 - c) \int_{\bar{\epsilon}_2}^{\infty} \varphi(\bar{\epsilon}_2) d\bar{\epsilon}_2 = (\theta - 2q_2 - c) \left(1 - \beta \left(1 - \Phi(\bar{\epsilon}_2)\right)\right) = 0
\]

As \(1 - \beta \left(1 - \Phi(\bar{\epsilon}_2)\right)\) is always positive, \((\theta - 2q_2 - c)\) should be zero. Hence, the optimal production quantity is the monopoly quantity \(q_m = \frac{\theta - c}{2}\). Notice that this applies as no competition is considered.

The optimal production level in the second period \(q_2^*\) in the profit sharing case is equal to the monopoly quantity \(q_m = \frac{\theta - c}{2}\), but if not sufficient internal cash is available, the entrepreneur produces as much as possible.

The constraint states that \(cq_2 \leq \bar{\pi}_1(q_1) + cq_1\), thus: \(cq_m = c \left(\frac{\theta - c}{2}\right) \leq ((\theta - q_1 + \bar{\epsilon}_1 - c)q_1) \left(1 - \beta I(\bar{\epsilon}_1 > \bar{\epsilon}_1)\right) + cq_1 = c \bar{\epsilon}_2\). As a result:

\[
q_2 = \left(\frac{(\theta - q_1 + \bar{\epsilon}_1)q_1 \left(1 - \beta I(\bar{\epsilon}_1 > \bar{\epsilon}_1)\right)}{c} + q_1 \beta I(\bar{\epsilon}_1 > \bar{\epsilon}_1)\right)
\]

**Proposition 5.3**

The production quantity in the second period is constrained by internal cash available \(cq_2 \leq \bar{\pi}_1(q_1) + cq_1\). As a result, the entrepreneur produces the monopoly quantity in the second period only if \(c \left(\frac{\theta - c}{2}\right) \leq ((\theta - q_1 + \bar{\epsilon}_1 - c)q_1) \left(1 - \beta I(\bar{\epsilon}_1 > \bar{\epsilon}_1)\right) + cq_1\).

Consequently, \(q_2^* = q_m\) only if \(\bar{\epsilon}_1 \geq \frac{c(q_m - q_1)}{q_1 \left(1 - \beta I(\bar{\epsilon}_1 > \bar{\epsilon}_1)\right)} - (\theta - q_1 - c) = \bar{\epsilon}_1^*\).

**Proposition 5.4**

\[
\bar{\epsilon}_1 = \frac{c(q_m - q_1)}{q_1 \left(1 - \beta I(\bar{\epsilon}_1 > \bar{\epsilon}_1)\right)} - (\theta - q_1 - c)
\]

If \(q_1 > q_m\), then \(\bar{\epsilon}_1^* < \bar{\epsilon}_1 < \bar{\epsilon}_1 = c - \theta + q_1\) and thus \(I(\bar{\epsilon}_1^* > \bar{\epsilon}_1) = 0\).

**Proposition 5.5**

The constraint states that \(cq_2 \leq \bar{\pi}_1(q_1) + cq_1\), thus:
\[ c q_m = c\left(\frac{\theta - c}{2}\right) \leq ((\theta - q_1 + \tilde{\epsilon}_t - c)q_t) \left(1 - \beta I_{[\tilde{\epsilon}_1 > \epsilon_{1_1}]}\right) + cq_t + \beta \pi I_{[\tilde{\epsilon}_1 > \epsilon_{1_1}]} = cq_t \]

As a result: 
\[ q_2 = \left(\frac{(\theta - q_1 + \tilde{\epsilon}_t + \pi)q_t(1 - \beta I_{[\tilde{\epsilon}_1 > \epsilon_{1_1}]}) + \beta I_{[\tilde{\epsilon}_1 > \epsilon_{1_1}]}(\pi + cq_t)}{c}\right) \]

### Proposition 5.6

The entrepreneur pays investors if \( \tilde{n}_t(q_t) = (\theta - q_t + \tilde{\epsilon}_t - c)q_t > \pi \). Hence, if \( \tilde{\epsilon}_t > q_t + c - \theta + \frac{\pi}{q_t} \).

### Proposition 5.7

As a result, the entrepreneur produces the monopoly quantity in the second period only if \( c \left(\frac{\theta - c}{2}\right) \leq ((\theta - q_1 + \tilde{\epsilon}_t - c)q_t) \left(1 - \beta I_{[\tilde{\epsilon}_1 > \epsilon_{1_1}]}\right) + cq_t + \pi \beta I_{[\tilde{\epsilon}_1 > \epsilon_{1_1}]} \). Consequently, \( q_2^* = q_m \) only if \( \tilde{\epsilon}_1 \geq \frac{c(q_m - q_1) - \pi \beta I_{[\tilde{\epsilon}_1 > \epsilon_{1_1}]} + \pi}{q_1(1 - \beta I_{[\tilde{\epsilon}_1 > \epsilon_{1_1}]} - (\theta - q_1 - c) = \tilde{\epsilon}_1} \).

### Corollary 5.3

To figure out whether \( \epsilon_{1_1} > \tilde{\epsilon}_1 \) if \( q_1 > q_s, q_s + 1 \) is filled out in both \( \epsilon_{1_1} \) and \( \tilde{\epsilon}_1 \). If \( \epsilon_{1_1}(q_s + 1) > \tilde{\epsilon}_1(q_s + 1) \) then is concluded that \( \epsilon_{1_1} > \tilde{\epsilon}_1 \) for \( q_1 > q_s \).

### Proposition 6.1

Given that the investors are only paid in the first period when \( \tilde{n}_t(q_t) = (\theta - q_t + \tilde{\epsilon}_t)q_t > R \), it may be concluded that the entrepreneur pays out investors if \( \tilde{\epsilon}_t > -\theta + q_t + \frac{R}{q_t} \).

### Proposition 6.2

The optimal production quantity that results in maximum profit for the entrepreneur cannot be derived easily algebraically by expressing \( \frac{d}{dq_2} V_2(q_1, \tilde{\epsilon}) = 0 \).

\[ \frac{d}{dq_2} V_2(q_1, \tilde{\epsilon}) = (\theta - 2q_2 - c) - \beta \int_{\tilde{\epsilon}_2}^{\infty} \frac{d(\tilde{n}_2(q_2) + cq_2 - R)}{dq_2} \varphi(\tilde{\epsilon}_2) d\tilde{\epsilon}_2 \]

\[ = (\theta - 2q_2 - c) - \beta (\theta - 2q_2) \int_{\tilde{\epsilon}_2}^{\infty} \varphi(\tilde{\epsilon}_2) d\tilde{\epsilon}_2 \]

\[ = (\theta - 2q_2) \left(1 - \beta \left(1 - \Phi(\tilde{\epsilon}_2)\right)\right) - c = 0 \]
Proposition 6.3

The constraint states that \( cq_2 \leq \tilde{r}_1(q_1) + c q_1 \), thus: \( cq_2 \leq ((\theta - q_1 + \tilde{e}_1)q_1)(1 - \beta I_{\{e_1 > e_1\}}) + \beta I_{\{e_1 > e_1\}}(R) = cq_2 \)

As a result:

\[
q_2 = \left( \frac{(\theta - q_1 + \tilde{e}_1)q_1}{c} \right)
\]

Note that \( q_2 \) is always larger than zero as a consequence of the assumption that the price is equal to or larger than 0.

Proposition 6.4

The entrepreneur produces \( q_2 \) in the second period if revenues in the first period do not exceed the amount of cash required to produce \( q_2 \) in the second period. Hence,

\[
((\theta - q_1 + \tilde{e}_1 - c)q_1)(1 - \beta I_{\{e_1 > e_1\}}) + cq_1 + \beta (cq_1 + R)I_{\{e_1 > e_1\}} \geq cq_2
\]

Consequently, \( q_2 \) is in the feasible set if \( \tilde{e}_1 \geq \frac{cq_2 - R \beta I_{\{e_1 > e_1\}}}{q_1(1 - \beta I_{\{e_1 > e_1\}})} - \theta + q_1 = \overline{e}_1 \).

Note that the boundaries \( e_1 \) and \( \overline{e}_1 \) intersect for \( R = cq_2 \), and \( e_1 \leq \overline{e}_1 \) if \( R \leq cq_2 \).

\[
e_1 = -\theta + q_1 + \frac{R}{q_t} = \frac{cq_2 - R \beta I_{\{e_1 > e_1\}}}{q_1(1 - \beta I_{\{e_1 > e_1\}})} - \theta + q_1 = \overline{e}_1
\]

\[
R = \frac{cq_2 - R \beta I_{\{e_1 > e_1\}}}{(1 - \beta I_{\{e_1 > e_1\}})}
\]

\[
R \left( 1 - \beta I_{\{e_1 > e_1\}} \right) + R \beta I_{\{e_1 > e_1\}} = \frac{cq_2}{(1 - \beta I_{\{e_1 > e_1\}})}
\]

As soon as \( cq_2 - R \beta < 0 \), then \( e_1 > \overline{e}_1 \). However, on the domain \( R \leq cq_2 \), that term will never become negative. Therefore, \( e_1 \) is only smaller than \( \overline{e}_1 \) if \( R \leq cq_2 \). As a result, the indicator function is only 1 if those conditions hold.
Appendix E Revenue Sharing Model for $R > 0$

In this appendix, the adjustments are discussed that should be made in the revenue sharing model propositions as introduced in chapter 6, when increasing $R$.

**Proposition 6.1:** In the revenue sharing case, the entrepreneur shares his revenues in the first period with investors if $\bar{\varepsilon}_1 > \varepsilon_1 = q_1 - \theta + \frac{R}{q_1}$.

Consequently, the value function for the second period may also be expressed as

$$V_2(q_1, \bar{\varepsilon}_1) = \max_{q_2 \geq 0} (\theta - q_2 - c)q_2 - \int_{\bar{\varepsilon}_2}^{\infty} \beta\left(\tilde{\pi}_2(q_2) + cq_2 - R\right)\varphi(\bar{\varepsilon}_2)d\bar{\varepsilon}_2$$

The optimal production quantity that results in maximum profit for the entrepreneur cannot be derived easily algebraically by expressing $\frac{d}{dq_2}V_2(q_1, \bar{\varepsilon}_1) = 0$.

**Proposition 6.2:** The optimal production quantity in the second period considering revenue sharing is defined by $\bar{q}_2$ and obtained by solving:

$$q_2 = \frac{\theta}{2} - \frac{c}{2\left(1 - \beta\left(1 - \Phi(\bar{\varepsilon}_2)\right)\right)}$$

for $q_2$. If the amount paid to investors increases as a result of larger $\beta$ or smaller $R$, the optimal production quantity for the entrepreneur in the second period decreases.

This equation is not algebraically solvable. Therefore, the expected profit will be enumerated for every combination of $q_1$ and $q_2$, to find the optimal quantity for both periods.

Still, several constraints have to be taken into account. Namely, the production quantity in the second period is constrained by internal cash available.

**Proposition 6.3:** In the revenue sharing case, the optimal production quantity in the second period is described by:

$$q_2^* = \begin{cases} \frac{q_2}{\bar{q}_2} & \text{if} \quad \bar{q}_2 \geq q_2 \\ \frac{q_2}{\bar{q}_2} & \text{if} \quad \bar{q}_2 < q_2 \end{cases}$$

where $q_2 = \left(\frac{\theta - q_1 + \bar{\varepsilon}_1}{c}q_1\left(1 - \beta\left(1 - \Phi(\varepsilon_2)\right)\right) + R\beta\left(1 - \Phi(\varepsilon_2)\right)\right)$ and $\bar{q}_2$ is obtained by solving

$$q_2 = \frac{\theta}{2} - \frac{c}{2\left(1 - \beta(1 - \Phi(\bar{\varepsilon}_2))\right)}$$

for $q_2$.

Basically, $q_2$ defines the amount that the entrepreneur can produce at most in the second period, according to his revenues. Note that still the assumption of positive price $p_t(q_t, \bar{\varepsilon}_t) = \theta - q_t + \bar{\varepsilon}_t \geq 0$ ensures that $q_2$ is always positive.

Likewise, a second boundary for $\bar{\varepsilon}_1$ is defined that determines whether the entrepreneur produces more or less than the optimal production quantity in the second
period: $\varepsilon^-$ In contrast to the other boundary, this boundary exists for $\bar{\varepsilon}_1$ only. It follows from the assumption as aforementioned that the production quantity in the second period is constrained by internal cash available $cq_2 \leq \hat{r}_1(q_1) + cq_1$.

**Proposition 6.4:** The entrepreneur produces $\tilde{q}_2$ in the second period if $\bar{\varepsilon}_1 \geq \varepsilon^-$, where $\varepsilon^-$ is defined by:

$$
\varepsilon^- = \begin{cases} 
\frac{cq_2 - R \beta}{q_1(1-\beta)} - \theta + q_1 & \text{if } R \leq cq_2 \\
\frac{cq_2}{q_1} - \theta + q_1 & \text{if } R > cq_2 
\end{cases}
$$

Note that the boundaries $\varepsilon^-$ and $\varepsilon^+$ intersect for $R = cq_2$, and $\varepsilon^+ \leq \varepsilon^-$ if $R \leq cq_2$. In conclusion, the optimal production quantity for the entrepreneur in the second period is given by corollary 6.1.

**Corollary 6.1:** The optimal production quantity in the second period is given by $q_2^*$. If $R \leq cq_2^*$ :

$$
q_2^* = \begin{cases} 
q_2' & \text{if } \bar{\varepsilon}_1 < \varepsilon^- \\
q_2'' & \text{if } \varepsilon^- \leq \bar{\varepsilon}_1 < \varepsilon^-
\end{cases}
$$

and if $R > cq_2^*$ :

$$
q_2^* = \begin{cases} 
q_2' & \text{if } \bar{q}_2 \geq q_2 \\
q_2'' & \text{if } \bar{q}_2 < q_2
\end{cases}
$$

Where $q_2_{[\varepsilon^- \leq \varepsilon^+]} = q_2' = \frac{(\theta - q_1 + \bar{\varepsilon}_1)q_1}{c}$, $q_2_{[\varepsilon^+ > \varepsilon^+]} = q_2'' = \frac{(\theta - q_1 + \bar{\varepsilon}_1)q_1(1-\beta) + \beta R}{c}$ and $\bar{q}_2$ is obtained by solving $q_2 = \frac{\theta}{2} - \frac{c}{2(1-\beta(1-\phi(\bar{\varepsilon}_2)))}$ for $q_2$.

As a result, after inserting the constraint in the value function, the first period objective function is defined as $h(q_1)$:

$$
V(.) = \max_{q_1 \geq 0} h(q_1) = \max_{q_1 \geq 0} E_{\bar{\varepsilon}_1} \left[ \hat{r}_1(q_1) + \pi_2(q_2^*) - \int_{\varepsilon^+}^{\infty} \beta(\pi_2(q_2^*) + cq_2^* - R)\varphi(\bar{\varepsilon}_2)d\bar{\varepsilon}_2 \right]
$$

This equation will provide the expected total profit for the entrepreneur per production quantity $q_1$ and $q_2$.

**Solving first stage problem**

In conclusion, the demand scenarios can be specified like in the profit sharing case. Subsequently $V(.)$ is solved while taking into account those demand scenarios.

The entrepreneurs value function $V(.)$ has been defined as:

$$
V(.) = \max_{q_1 \geq 0} h(q_1) = \max_{q_1 \geq 0} E_{\bar{\varepsilon}_1} \left[ \hat{r}_1(q_1) + \pi_2(q_2^*) - \int_{\varepsilon^+}^{\infty} \beta(\pi_2(q_2^*) + cq_2^* - R)\varphi(\bar{\varepsilon}_2)d\bar{\varepsilon}_2 \right]
$$
However, as $\bar{q}_2^\prime$ is not obtained algebraically, this value is obtained numerically as well. To achieve that, every possible $\bar{q}_2^\prime$ is filled out, and the total profit is derived. As a result, both the production in the first and second period are derived numerically.

The objective function $h(q_1)$ can be specified if $R \leq c\bar{q}_2^\prime$ as:

$$h(q_1) = \int_{e_1}^{\bar{e}_1} \left( \bar{\pi}_1(q_1) + c q_1 + \pi_2(\bar{q}_2) - f_{x_2}^{\infty} \beta \left( \bar{\pi}_2 \left( q_2 \right) + c q_2 \right) - R \right) \phi(\bar{e}_2)d\bar{e}_2 \phi(\bar{\epsilon}_1)d\bar{\epsilon}_1$$

$$+ \int_{\bar{e}_1}^{e_1} \left( \bar{\pi}_1(q_1) + c q_1 - \beta(\bar{\pi}_1(q_1) + c q_1 - R) + \pi_2(\bar{q}_2) - f_{x_2}^{\infty} \beta \left( \bar{\pi}_2 \left( q_2 \right) + c q_2 \right) - R \right) \phi(\bar{e}_2)d\bar{e}_2 \phi(\bar{\epsilon}_1)d\bar{\epsilon}_1$$

$$+ \int_{e_1}^{\bar{e}_1} \left( \bar{\pi}_1(q_1) + c q_1 - \beta(\bar{\pi}_1(q_1) + c q_1 - R) + \pi_2(\bar{q}_2) - f_{x_2}^{\infty} \beta \left( \bar{\pi}_2 \left( q_2 \right) + c q_2 \right) - R \right) \phi(\bar{e}_2)d\bar{e}_2 \phi(\bar{\epsilon}_1)d\bar{\epsilon}_1$$

Where

\[ \bar{\pi}_1(q_1) = (\theta - q_t + \bar{\epsilon}_t - c)q_t, \pi_2(q_2) = (\theta - q_2 - c)q_2 \]

\[ q_2^\prime = \frac{(\theta - q_t + \bar{\epsilon}_t)q_t}{c}, q_2^\prime = \frac{(\theta - q_t + \bar{\epsilon}_t)q_t}{c} \]

\[ \bar{e}_1 = \frac{c q_2 - (R) \beta}{a_1(1 - \beta)} - \theta + q_t, \bar{\epsilon}_t = -\theta + q_t + \frac{R}{q_t} \]

\[ \bar{q}_2^\prime \text{ is obtained by solving } q_2 = \frac{\theta - z}{z (1 - \beta (1 - \phi(\bar{q}_2)))} \]

Furthermore, if $R > c\bar{q}_2^\prime$:

$$h(q_1) = \int_{\bar{e}_1}^{e_1} \left( \bar{\pi}_1(q_1) + c q_1 + \pi_2(\bar{q}_2) - f_{x_2}^{\infty} \beta \left( \bar{\pi}_2 \left( q_2 \right) + c q_2 - R \right) \phi(\bar{e}_2)d\bar{e}_2 \phi(\bar{\epsilon}_1)d\bar{\epsilon}_1$$

$$+ \int_{\bar{e}_1}^{e_1} \left( \bar{\pi}_1(q_1) + c q_1 + \pi_2(\bar{q}_2) - f_{x_2}^{\infty} \beta \left( \bar{\pi}_2 \left( q_2 \right) + c q_2 - R \right) \phi(\bar{e}_2)d\bar{e}_2 \phi(\bar{\epsilon}_1)d\bar{\epsilon}_1$$

$$+ \int_{e_1}^{\bar{e}_1} \left( \bar{\pi}_1(q_1) + c q_1 - \beta(\bar{\pi}_1(q_1) + c q_1 - R) + \pi_2(\bar{q}_2) - f_{x_2}^{\infty} \beta \left( \bar{\pi}_2 \left( q_2 \right) + c q_2 - R \right) \phi(\bar{e}_2)d\bar{e}_2 \phi(\bar{\epsilon}_1)d\bar{\epsilon}_1$$

Where

\[ \bar{\pi}_1(q_1) = (\theta - q_t + \bar{\epsilon}_t - c)q_t, \pi_2(q_2) = (\theta - q_2 - c)q_2 \]

\[ q_2^\prime = \frac{(\theta - q_t + \bar{\epsilon}_t)q_t}{c} \]

\[ \bar{e}_1 = \frac{c q_2}{a_1} - \theta + q_t, \bar{\epsilon}_t = -\theta + q_t + \frac{R}{q_t} \]

\[ \bar{q}_2^\prime \text{ is obtained by solving } q_2 = \frac{\theta - z}{z (1 - \beta (1 - \phi(\bar{q}_2)))} \]

The revenue sharing model will be implemented in Wolfram Mathematica in Chapter 7 to investigate the consequences of the different parameters.
## Appendix F Results

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Banken draaien abrupt de geldkraan dicht

Buitengewoon overval van de bedrijfsfinanciering

Paniekmeter schiet verder de hoogte in

Het Financieele dagblad, August 9th 2011, nr 185