The Expected Value of Wind Energy to Society
Allocating Wind-Driven Electricity Generation Capacity in ERCOT

by
L.S. Levij

BSc Industrial Engineering and Management Sciences – 2009
Student identity number Eindhoven University of Technology – 0609796
Student identity number Tilburg University - 478036

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Dr. F. Tanrisever, Eindhoven University of Technology
Prof.dr. B.J.M. Werker, Tilburg University
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Abstract

This master thesis proposes a model to gauge the expected value of wind energy to society, defined as the expected revenues of wind energy minus the expected ancillary service costs of wind energy. By deciding upon the allocation of wind-driven electricity generation capacity, the model, which applies to ERCOT, the closed electrical system serving most of the state of Texas, aims at maximizing the expected value of wind energy to society. Besides formulating the model, this master thesis also analyzes the model numerically in order to gain insight into the impact of each model input parameter on the maximum expected value of wind energy to society and the associated allocation of wind-driven electricity generation capacity.
Preface

This master thesis is concluding the master Operations Management and Logistics at Eindhoven University of Technology and the master Finance at Tilburg University. In this preface, I would like to explicitly thank those who supported me during my master thesis research.

First, I would like to thank my supervisor at Eindhoven University of Technology, Fehmi Tanrisever. Since the start of my research, Fehmi has been a continuous source of inspiration and guidance. Moreover, Fehmi has provided me with the great opportunity to carry out part of my research at the McCombs School of Business at the University of Texas at Austin.

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Laura Levij
Eindhoven, May 2012
Summary

Introduction
Electricity is one of the world’s most vital commodities; a world without it is hard to imagine. Unfortunately, the generation of electricity is one of human’s most polluting activities [Bosselman et al., 2010]. Nowadays, to counteract global warming, many countries are trying to reduce the emissions resulting from electricity generation by promoting renewable energy [IEA et al., 2010]. As there are too many types of renewable energy to consider, this master thesis focuses on one type only, wind energy.

Although wind energy is proclaimed to be environment-friendly, recent research did not find wind energy to have a positive impact on the environment [BENTEK, 2011, Liik et al., 2003]. Because of wind energy’s intermittent and unpredictable nature, differences between the amount of wind energy generated and the amount of wind energy planned for occur. As electricity cannot be stored, these differences are dealt with, on short notice, by traditional, but polluting, power plants. Deploying traditional power plants to make up for wind energy deviations does not only nullify wind energy’s positive environmental impact, but also results in costs, as traditional power plant owners demand compensation for altering their production on short notice [Krohn et al., 2009]. Since wind energy’s variability is driving the aforementioned pollution and costs, reducing this variability will benefit both society and the environment. One way to reduce wind energy’s variability is by geographical dispersion of wind farms, i.e., allocating wind farms such that their wind energy fluctuations do not overlap [Parsons et al., 2004].

In academia, there is a limited amount of quantitative research addressing geographical dispersion of wind farms. The goal of this master thesis is to complement the small strand of quantitative research concerning geographical dispersion of wind farms by considering an electrical system not yet considered by other researchers, ERCOT (Electricity Reliability Council of Texas).

Research
In this master thesis, wind energy is considered from a society viewpoint, as pollution and ancillary service costs, i.e., the costs of altering traditional power plant production in response to wind energy deviations, affect society as a whole and not one single entity or group of entities within society only. The focus lies on the value that society attributes to wind energy, where value refers to monetary value. As it is difficult to express environmental impacts in monetary terms, this master thesis only considers revenues and ancillary service costs. Specifically, the value of wind energy to society is defined as the revenues of wind energy minus the ancillary service costs of wind energy. The two research questions central in this master thesis are given below.
Research Question I: What is the expected value of wind energy to society, defined as the expected revenues of wind energy minus the expected ancillary service costs of wind energy, for a particular allocation of wind-driven electricity generation capacity in ERCOT?

Research Question II: What is the maximum expected value of wind energy to society, defined as the expected revenues of wind energy minus the expected ancillary service costs of wind energy, and the associated allocation of wind-driven electricity generation capacity in ERCOT for a particular level of wind-driven electricity generation capacity or a particular percentage of electricity to come from wind?

Research question I quantifies the effect of the positioning of wind farms, also referred to as wind-driven electricity generation capacity, in ERCOT, while research question II focuses on the optimal positioning of wind farms in ERCOT, i.e., the positioning that maximizes the value of wind energy to society. Another goal of research question II is to provide ERCOT policy makers with insights into feasible and desirable renewable portfolio standards (RPSs), where a RPS is specified as either a particular percentage of electricity to come from wind or a particular level of wind-driven electricity generation capacity. Note that, since the random variable wind speed is driving the revenues, ancillary service costs and value of wind energy to society, the actual wind energy generated in ERCOT cannot be determined in advance, therefore, the research questions refer to expected values.

To answer the research questions, two steps are taken, with the first step consisting of two parts. In the first part of the first step, a model capable of computing the expected value of wind energy to society for different allocations of wind-driven electricity generation capacity is build, hence, answering research question I. In the second part of the first step, the model is extended such that the maximum expected value of wind energy to society and the associated allocation of wind-driven electricity generation capacity can be retrieved for particular RPS formulations, hence, answering research question II. In the second step, the extended model is analyzed numerically. Specifically, a base case scenario is analyzed, in which the model input parameters are assigned probable values, and a sensitivity analysis is performed, in which the sensitivity of the maximum expected value of wind energy to society and the associated allocation of wind-driven electricity generation capacity to different model input parameters is established.

Results
The analysis of the base case scenario shows that the maximum expected value of wind energy to society in ERCOT is positive. Remarkable is that even though only ten locations are included in the base case scenario, the maximum expected value of wind energy to society is obtained when wind-driven electricity generation capacity is allocated to only six out of the ten locations. Unsurprisingly, this result is
entirely driven by correlations of wind energy generation; much capacity is allocated to locations with low correlations and no capacity is allocated to locations with high correlations.

The sensitivity analysis shows a substantial sensitivity for the following model input parameters: the expected wind energy generation at a particular location, the RPS, the price of wind energy, the ancillary service costs of over- and underproduction of wind energy, the annual discount rate and the considered timeframe. Other model input parameters that are considered in the sensitivity analysis, but for which no substantial sensitivity was found, are the variance of wind energy generation at a particular location, the correlation of wind energy generation between two locations and the number of considered locations.

Conclusion & Discussion

The following conclusions are drawn from the numerical analysis:

- **Improve the expected wind energy generation of one or a few wind farms:** There is a desirably large increase in the maximum expected value of wind energy to society when the expected wind energy generation of only one or a few wind farms is improved.

- **Allocate wind-driven electricity generation capacity to locations with low correlations of wind energy generation:** Slightly higher maximum expected values of wind energy to society are obtained when capacity is allocated to locations that have at least a few low correlations of wind energy generation.

- **Formulate ERCOT’s RPS such that the highest maximum expected value of wind energy to society is obtained:** There is an optimal level of electricity to come from wind or wind-driven electricity generation capacity, i.e., a level at which the maximum expected value of wind energy to society is highest. Hence, it is advisable to set ERCOT’s RPS equal to this optimal level.

- **Gather realistic values for the annual discount rate, the price of wind energy and the ancillary service costs of over- and underproduction of wind energy:** As the maximum expected value of wind energy to society and the associated allocation of wind-driven electricity generation capacity are sensitive these input parameters for which no probable values were obtained, realistic values for these parameters have to be gathered.

The limitations of this master thesis are: a lack of wind energy data for a reasonable number of locations in ERCOT (data could only be retrieved for ten locations in ERCOT), a model in which the expected value of wind energy to society is the same for all considered hours and a model that only takes into account revenues and ancillary service costs. Although the model’s application to ERCOT might intuitively be considered a limitation, the model can actually be applied to any electrical system around the world, as long as the model assumptions are met.
# Contents

List of Abbreviations 1

## 1 Introduction

1.1 Electricity and Electricity Markets 2
1.2 Wind Energy 3
1.3 Outline 5

## 2 Research Design

2.1 Research Questions 6
2.2 Research Method 8
  2.2.1 Step 1: Build and Extend the Model 9
  2.2.2 Step 2: Gather Data and Plug Data into the Model 10

## 3 The Model

3.1 The Expected Value of Wind Energy to Society 14
3.2 The Maximum Expected Value of Wind Energy to Society 17

## 4 Numerical Analysis

4.1 Hourly Wind Power Output per W of Capacity 21
4.2 Base Case Scenario 23
4.3 Sensitivity Analysis 24

## 5 Conclusion & Discussion

5.1 Conclusion 31
5.2 Discussion 32

References 34

A Appendix 1 - Locations in ERCOT 37
B Appendix 2 - Retrieving values for $p$, $s$ and $d$ 38
C Appendix 3 - Base Case Scenario in Mathematica® 39
D Appendix 4 - Graphs of Sensitivity Analysis 41
List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLT</td>
<td>Central Limit Theorem</td>
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<tr>
<td>CREZ</td>
<td>Competitive Renewable Energy Zone</td>
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<td>ERCOT</td>
<td>Electricity Reliability Council of Texas</td>
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<td>KKT</td>
<td>Karush-Kuhn-Tucker</td>
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<td>RPS</td>
<td>Renewable Portfolio Standard</td>
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<td>W</td>
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<td>Wh</td>
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1 Introduction

1.1 Electricity and Electricity Markets

Electricity is one of the world’s most vital commodities; a world without it is hard to imagine. Not only is electricity a vital commodity, it is also a unique commodity, as it is the only commodity that cannot be stored [Griffin and Puller, 2005]. At first sight, electricity’s non-storability feature may not look very impressive, but this feature turns out to significantly impact the handling of electricity. Because electricity cannot be stored, generation and consumption have to be balanced in real-time, i.e., at any moment of time the total amount of electricity withdrawn from the grid has to equal the total amount of electricity injected into the grid [Joskow, 2005]. This already daunting balancing task is further complicated by the fact that electricity consumption cannot be predicted accurately. Unsurprisingly, electricity markets, i.e., markets where the commodity electricity is traded both physically and financially, are complex and unique, in the sense that they are different from other commodity markets.

Electricity markets can also be characterized as dynamic, i.e., continuously changing [Macmillan, 2012]. Currently, two major trends can be distinguished in electricity markets: market restructuring and renewable energy. The first trend, market restructuring, refers to the transition from government-regulated monopoly markets to competitive markets. For a long time, electricity markets were dominated by vertically integrated utilities subject to governmental regulation. This market structure was deemed optimal, because of the economies of scale associated with electricity handling [Hunt and Shuttleworth, 1996]. However, in the last couple of decades, technology improvements diminished some of electricity handling’s economies of scale and, hence, also the necessity of vertically integrated utilities [Rothwell and Gómez, 2003]. These technology improvements, combined with dissatisfaction about the efficiency of government-regulated monopoly markets, started the worldwide restructuring of electricity markets; nowadays, many electricity markets are becoming, or have already become, more competitive. The second trend, renewable energy, can be seen as a response to the global warming concern. Scientists predict that due to excessive emissions of greenhouse gases, caused by human activity, the earth’s average temperature will keep rising steadily, which is likely to damage the earth’s climate and ecosystem [Bosselman, Eisen, Rossi, Spence, and Weaver, 2010, IPCC, 2010]. Unfortunately, electricity generation is one of the most polluting human activities. For example, in 2006, the U.S. electricity generation industry accounted for more than one-third of the nation’s CO₂ emissions. As many countries have committed themselves to stabilization or reduction of greenhouse gas emissions, environment-friendly substitutes for traditional electricity generation resources, such as coal and gas, are considered [Bosselman et al., 2010]. The most famous class of substitutes is renewable energy, which encompasses a wide range of renewable energy sources, such as wind, solar and biomass [IEA, NEA, and OECD, 2010].
The uniqueness, complexity and dynamics of electricity markets provide an interesting context for my master thesis. Although my master thesis will touch upon both of the trends described above, the focus will lie on the renewable energy trend. Specifically, within this trend, as there are too many types of renewable energy to consider, my master thesis will focus on wind energy. Presently, wind energy is the most advanced renewable energy; the costs of generating electricity from wind are significantly less than the generation costs associated with most other renewable energy sources, which makes wind energy the renewable energy most competitive with traditional electricity generation sources [Archer and Jacobson, 2003, Gielecki, Mayes, and Prete, 2001].

1.2 Wind Energy

Wind energy, i.e., electricity from wind, comes about when a wind turbine extracts kinetic energy from wind to drive an electric generator [Krohn, Morthorst, and Awerbuch, 2009]. The amount of kinetic energy that can be extracted at a given time at a particular site primarily depends on the local wind speed, although the air density and a wind turbine’s rotor blade area also have an impact [Ackermann, 2005]. In general, wind speed increases with height and the ability to capture wind energy increases with the rotor blade area [NAS, NAE, and NRC, 2010]. Over the last few decades, wind energy technology has greatly improved, resulting in taller wind turbines that can benefit from higher wind speeds and larger rotor blade areas that can extract more kinetic energy [Masters, 2004].

Even though many countries around the world are promoting wind energy, mainly because of its proclaimed positive environmental impact, recent research shows that wind energy positive’s environmental impact is actually negligible [BENTEK, 2011, Liik, Oidram, and Keel, 2003]. Whereas traditionally generated electricity contributes significantly to the emission of greenhouse gases and other air pollutants, wind energy is claimed to be emission-free [Hirst, 2002]. Even though wind energy does not directly contribute to air pollution, indirectly, wind energy does result in emissions [Ackermann and Söder, 2002]. Due to wind energy’s intermittent and unpredictable nature, the actual amount of wind energy generated may differ from the amount of wind energy planned for. These differences between actual and planned generation are dealt with by traditional power plants. As traditional power plants can be turned on and off as desired and are capable of quickly altering production levels, these plants are extremely suitable to use in combination with wind energy; when wind energy falls short, these plants produce more than planned, and when wind energy is above planned levels, these plants reduce production [Piwko, Osborn, Gramlich, Jordan, Hawkins, and Porter, 2005]. Unfortunately, frequent and substantial alterations of traditional power plants’ production levels are detrimental to the environment, as these alterations lead to the emission of air pollutants; even more air pollutants are emitted than during normal operations, i.e., when traditional power plant production is kept close to planned levels [BENTEK, 2011].
It is the pollution associated with changes in traditional power plants’ production levels that is driving wind energy’s indirect emissions and the nullification of wind energy’s positive environmental impact.

Not only are the changes in traditional power plant production related to wind energy detrimental to the environment, but they are also expensive [Krohn et al., 2009]. As the differences between actual and planned wind energy generation cannot be predicted well in advance, these differences have to be dealt with on short notice [Piwko et al., 2005]. Not surprisingly, traditional power plant owners that alter their production levels on short notice require compensation [ERCOT, 2011b]. In case of underproduction, i.e., actual wind energy generation is larger than the level of wind energy generation planned for, traditional power plants with a low start-up time are used to make up for the wind energy deficit. Unfortunately, plants with a low start-up time are extremely inefficient, in the sense that they require a lot of fuel to generate electricity [Bosselman et al., 2010, Krohn et al., 2009]. So, these plants have significant fuel costs. Besides being compensated for changing production, the fuel costs of these plants also have to be recovered, resulting in higher costs of underproduction than overproduction, i.e., when actual wind energy generation is larger than the level of wind energy generation planned for. Also, the inefficiency of plants with a low start-up time results in high emissions as well [BENTEK, 2011].

If wind energy is only a small portion of total electricity in a particular electrical system, as is currently the case in most electrical systems, total costs and emissions resulting from wind energy deviations might not be substantial. However, so far, wind energy does not turn out to be a cost-effective way of reducing greenhouse gas emissions and simply increasing the level of wind energy, as is envisioned in electrical systems around the world, is likely to result in high costs and high emissions [BENTEK, 2011]. Also, it is questionable whether there is sufficient traditional generation capacity available to cope with the potential increase in wind energy variability that the envisioned increase in wind energy might bring along [Archer and Jacobson, 2003, Weisser and Garcia, 2005].

As wind energy’s variability is driving the emissions, costs and concerns associated with wind energy, reducing this variability will be beneficial to both society and the environment. One way to reduce wind energy’s variability and, hence, improve wind energy’s cost-effectiveness, is by making use of the spatial variation of wind [Parsons, Milligan, Zavadil, Brooks, Kirby, Dragoon, and Caldwell, 2004]. Simply said, different geographical areas face different winds and by constructing wind farms, i.e., wind-driven electricity generation capacity, in different geographical areas, wind energy fluctuations in one area are unlikely to match wind energy fluctuations in other areas. As wind speeds, the prime determinant of wind energy, differ across areas, differences in wind energy across areas are the result [Ackermann, 2005]. Basic statistical theory shows that given imperfectly correlated wind speeds across geographical areas,
it might be detrimental for society and the environment to have all wind farms of a particular electrical system located in one area, e.g., the area with the highest average wind speed or the area with the lowest variability of wind speed. Having all wind farms located in one area might lead to either an undesirably high variability of wind energy in the system, which is accompanied by excessive emissions and costs, or an undesirably low average level of wind energy in the system, making wind energy’s positive environmental impact very small.

Even though a reduction in the variability of wind energy by means of geographical dispersion of wind farms is a strategy often mentioned, e.g., by Krohn et al. [2009], there is only a limited amount of research addressing this strategy in a quantitative fashion [Parsons et al., 2004]. To complement this small strand of research, my master thesis will quantify the effect of geographical dispersion of wind farms in a electrical system not yet considered by other researchers, ERCOT (Electricity Reliability Council of Texas). ERCOT is the electrical system serving most of the state of Texas, to be precise 85% of Texas’ electricity demand and 75% of Texas’ land area [ERCOT, 2010a]. Not only is ERCOT a system not yet considered by others, there are also two other motivations for focusing on this particular electrical system. First, ERCOT is a closed electrical system, i.e., it has no connections to other electrical systems. Since electricity cannot be retrieved from other systems, ERCOT has to balance electricity supply and demand within its own system. This aspect is of major relevance as wind energy’s variability also has to be managed within ERCOT itself. The closed grid aspect makes ERCOT quite unique, as most European and U.S. electrical systems have connections to neighboring systems [Weisser and Garcia, 2005]. Second, while the U.S. is the country with the largest amount of installed wind-driven electricity generation capacity, the part of Texas served by ERCOT has the most installed capacity in the country [DOE, 2009]. This makes ERCOT the world’s fifth region with respect to installed wind-driven electricity generation capacity [ERCOT, 2011a].

1.3 Outline

In this chapter, relevant literature is reviewed. In addition, the research gap that is motivating this master thesis, the absence of quantitative research concerning geographical dispersion of wind farms in ERCOT, is discussed. In Chapter 2, the research design that is used to fill the identified research gap is provided. Specifically, the research questions and research method are discussed. In order to answer the research questions, a model is proposed in Chapter 3. In Chapter 4, the model is analyzed numerically. First, a base case scenario, in which the model input parameters are assigned probable values, is analyzed. Second, a sensitivity analysis is performed in order to gain insight into the sensitivity of the model’s outcome to each of the different model input parameters. Finally, in Chapter 5, this master thesis is concluded and its limitations are discussed.
2 Research Design

Chapter 1 introduced the research gap that is motivating this master thesis; the absence of quantitative research concerning geographical dispersion of wind farms in ERCOT. This chapter complements the previous one by outlining the way in which the identified research gap will be filled. Specifically, this chapter will discuss the research questions and the research method used to answer the research questions.

2.1 Research Questions

Chapter 1 touched upon wind energy’s cost-effectiveness. First, wind energy turns out to have a negligible positive environmental impact, as the emissions abated by wind energy are set off by the emissions from traditional power plants coping with the differences between actual and planned wind energy [BENTEK, 2011]. Second, wind energy brings along significant costs, since traditional power plant owners have to be compensated for dealing with wind energy deviations. In accordance with literature, e.g., Krohn et al. [2009], I will refer to the costs of ramping up and down traditional power plants on short notice due to differences between actual and planned wind energy as ancillary service costs. The negligible positive environmental impact and the ancillary service costs associated with wind energy lead to the conclusion that, currently, wind energy is not a cost-effective way of reducing greenhouse gas emissions [BENTEK, 2011]. This conclusion, combined with the envisioned increase in wind energy in electrical systems around the world, calls for an improvement of wind energy’s cost-effectiveness. By reducing wind energy’s variability, which might be accomplished through geographical dispersion of wind farms, wind energy’s cost-effectiveness will improve.

This master thesis considers wind energy from the point of view of society. Acknowledging that emissions and ancillary service costs affect not one particular entity or group of entities, but society as whole, viewing wind energy from a society viewpoint seems a sensible choice. Presently, no entity in electricity markets is held accountable for emissions. Also, at least in ERCOT, no entity or group of entities is held fully accountable for the ancillary service costs associated with wind energy deviations. E.g., wind farm owners in ERCOT are exempted from charges when actual wind energy generation is less than 10% higher than the level of wind energy generation planned for. More extreme, wind farm owners in ERCOT are fully exempted from charges when actual wind energy generation is below the level of wind energy generation planned for [ERCOT, 2011b]. However, even though no particular entity or group of entities in ERCOT, such as wind farm owners, is held accountable for the emissions and ancillary service costs brought along by wind energy, these emissions and ancillary service costs are felt, by the ERCOT society. Hence, a society viewpoint is chosen.
Specifically, this master thesis focuses on the value that society attributes to wind energy, which, inspired by Krohn et al. [2009], I refer to as the value of wind energy to society. This value is the result of a tradeoff. On the one hand, wind energy brings benefits to society. Among others, every unit of traditionally generated electricity that is replaced by wind energy, without taking ancillary services into account, results in less emissions. On the other hand, wind energy also imposes costs on society. Examples are ancillary service costs and environmental costs, the indirect emissions resulting from wind energy deviations. In order to determine the value of wind energy to society as the result of wind energy’s benefits and costs, these benefits and costs have to be expressed in value terms as well. Unfortunately, value is an ill-defined concept, as it can refer to many different items, including intangible and immeasurable ones. However, most often, value is related to money and this is the interpretation I adhere to in this master thesis [Macmillan, 2012]. Opposed to other interpretations of the value concept, monetary value can be measured and it is the interpretation of the value concept that most people are familiar with. As it is hard to assign monetary values to environmental impacts, the value of wind energy to society will not include these impacts. Instead, benefits and costs that are easily expressed in monetary values are included, in particular, the value of wind energy to society encompasses the revenues of wind energy and the ancillary service costs of wind energy. For simplicity, the value of wind energy to society is defined as the revenues of wind energy minus the ancillary service costs of wind energy.

Having defined the value of wind energy to society, it makes sense to quantify the effect of geographical dispersion of wind farms as the change in the value of wind energy to society brought about by allocating wind farms across different geographical areas. Obviously, the exact change in the value of wind energy to society associated with geographical dispersion of wind farms depends on the formulation of the problem context, including the current allocation of wind farms and the set of areas across which wind farms are allowed to be distributed. Fortunately, one research question, research question I, is sufficient to incorporate all the different problem contexts.

Also, there is one stand-alone value that is of interest, the maximum value of wind energy to society that can be obtained by geographical dispersion of wind farms in ERCOT. Similar to many European countries and U.S. states, ERCOT has set a renewable portfolio standard [ERCOT, 2010a]. A renewable portfolio standard (RPS) usually takes one of the following two forms; either a specified percentage of electricity has to come from renewable energy sources or a specified amount of renewable energy generation capacity has to be installed [Bosselman et al., 2010]. ERCOT’s current RPS is formulated in the second form; it requires 5,880 MW of renewable energy generation capacity by 2015 and 10,000 MW by 2025 [PEW, 2007]. However, already in 2009, ERCOT surpassed the 2025 goal, with renewable energy generation capacity totaling 10,070 MW, of which 9,317 MW was wind-driven electricity generation.
capacity [ERCOT, 2010a]. Despite meeting its RPS ahead of schedule, ERCOT continues to promote renewable energy development and in June 2010 an additional 44,300 MW of wind generation capacity was under review [ERCOT, 2010a]. To provide ERCOT policy makers with preliminary insights into feasible and desirable RPSs, research question II is formulated. This research question can be used to determine the maximum value of wind energy to society as well as the associated allocation of wind farms in ERCOT under different RPS formulations.

Research question I and II are shown below and, unsurprisingly, both questions are focused on ERCOT. A striking feature of the questions is the use of the term "expected". As wind speed is a random variable, the actual level of wind energy generation, which, in its turn, affects the revenues of wind energy and the ancillary service costs of wind energy, cannot be determined in advance. Hence, the exact value of the revenues and ancillary service costs as well as the exact value of wind energy to society cannot be determined before actual wind energy generation has taken place, so, instead, expected values are used.

**Research Question I:** What is the expected value of wind energy to society, defined as the expected revenues of wind energy minus the expected ancillary service costs of wind energy, for a particular allocation of wind-driven electricity generation capacity in ERCOT?

**Research Question II:** What is the maximum expected value of wind energy to society, defined as the expected revenues of wind energy minus the expected ancillary service costs of wind energy, and the associated allocation of wind-driven electricity generation capacity in ERCOT for a particular level of wind-driven electricity generation capacity or a particular percentage of electricity to come from wind?

### 2.2 Research Method

To answer the research questions, two steps are taken. The first step consists of two parts. In the first part of the first step, a model capable of computing the value of wind energy to society for different allocations of wind-driven electricity generation capacity in ERCOT is build. In the second part of the first step, the model of the first part is extended such that the maximum value of wind energy to society and the associated allocation of wind-driven electricity generation capacity in ERCOT could be retrieved for different RPS formulations. In the second step, relevant data are gathered and plugged into the extended model of the first step. The research steps and the chapters of this master thesis that are associated with them are shown in Figure 1.
2.2.1 Step 1: Build and Extend the Model

In Section 2.1, already two important model aspects are mentioned. First, the model has to consist of two parts, one representing the revenues of wind energy and one representing the ancillary service costs of wind energy. Second, the key relationship in the model has to set equal the value of wind energy to society to the revenues of wind energy minus the ancillary service costs of wind energy.

In the model, the revenues of wind energy are defined as the price per unit of wind energy generated multiplied with the number of units of wind energy generated. In practice, however, there is not one single price per unit of wind energy. Not only are there different prices over time, there are also different prices for each market in which wind energy is offered. For example, ERCOT has two markets in which wind energy is offered, the day-ahead market and the real-time market, and, in general, both markets yield a different price [Baldick and Niu, 2005]. For simplicity, the model will abstain from complicated market operations, including the different markets that exist in ERCOT, and timing issues. Instead, the model will assume the existence of a single, time-independent price per unit of wind energy.

In the model, the ancillary service costs of wind energy are defined in a less straightforward manner; in my view, the model has to acknowledge that ancillary service costs are likely to differ depending on whether the difference between actual and planned wind energy relates to overproduction, i.e., actual generation is larger than planned generation, or underproduction, i.e., actual generation is less than planned generation. As explained before, both types of production result in compensation for traditional power plant owners, as both involve changes in traditional power plants’ production levels. However, the ancillary service costs related to overproduction only encompass compensation, while the ancillary service costs related to underproduction include compensation and additional costs, to be precise the fuel costs of the units of difference between actual and planned wind energy generation. To allow for different ancillary service costs, the model will distinguish between over- and underproduction. Specifically, the ancillary service costs of wind energy in case of overproduction are defined as the ancillary service cost per unit of overproduction of wind energy multiplied with the total number of units of overproduction.
of wind energy. The ancillary service costs of wind energy in case of underproduction are defined as the ancillary service cost per unit of underproduction of wind energy multiplied with the total number of units of underproduction of wind energy.

Chapter 1 explained that wind speeds, in particular the correlations between wind speeds in different geographical areas, are driving the potential positive effect of geographical dispersion of wind farms on society. As the model aims at providing insight into the effect of geographical dispersion of wind farms on the ERCOT society, wind speed obviously has to be included as one of the model’s input parameters. Unfortunately, wind speed is a random variable, which implies that the wind speed that will occur at a particular location in ERCOT at some time in the future cannot be determined. As wind speed affects wind energy generation, the amount of wind energy that will be generated at a particular location in ERCOT at some time in the future can also not be determined. In the end, the randomness of wind speed will also affect the revenues and ancillary service costs of wind energy as well as the value of wind energy to society. As future wind speeds cannot be assigned definite values, the model will work with expected values instead.

For clarity, the model details discussed above are also presented graphically; Figure 2 shows the structure of the model related to research question I. The model itself can be found in Section 3.1. In Figure 2, model inputs, for which data are gathered in the next step, are marked gray.

In the extended model, the model structure of Figure 2 is maintained, except for a few alterations. First, a constraint, representing a RPS, is added. Adding this constraint, formulated in the form of either a particular level of wind-driven electricity generation capacity or a particular percentage of electricity to come from wind, results in an additional input. The form of this input depends on the form of the RPS and will either be a specific level of wind-driven electricity generation capacity or a specific percentage of electricity to come from wind. Second, the number of units of wind-driven electricity generation capacity is no longer an input, as the extended model should provide the maximum expected value of wind energy to society and the associated allocation of wind-driven electricity generation capacity, i.e., the number of units of wind-driven electricity generation capacity in each different geographical area, as outcomes.

The extended model, which is used to answer research question II, can be found in Section 3.2.

2.2.2 Step 2: Gather Data and Plug Data into the Model

After building and extending the model, the model will be analyzed numerically. In order to do so, the model inputs discussed in the previous step have to assigned values. By gathering relevant data, a process described in the remainder of this section, model input values are estimated. In turn, these
estimations are used, see Chapter 4, to determine the maximum value of wind energy to society and the associated allocation of wind-driven electricity generation capacity in ERCOT under different RPSs.

Wind speed

Unsurprisingly, it is almost impossible to gather wind speed data for all different geographical areas in ERCOT; ERCOT namely covers an area of approximately 520,000 square kilometers [ERCOT, 2010a]. Fortunately, wind speed data for several locations in ERCOT could be retrieved. However, there are some issues associated with the data. First, there are only a limited number of locations in ERCOT, thirteen to be precise, for which wind speed time series are available [AEI, 2012]. Second, all the locations for which wind speed time series are available are located in the same part of Texas, the northwest (see colored dots in figure of Appendix 1). Third, for the locations for which wind speed time series are available, the overlap between the time series is low, e.g., when choosing a timeframe of one year at most ten locations can be included. Despite these three issues, the retrieved wind speed data will be used, as no other ERCOT wind speed data could be retrieved.

Price and ancillary service costs

Three other model inputs that have to be assigned values are the price per unit of wind energy generated, the ancillary service cost per unit of overproduction and the ancillary service cost per unit of underproduction. Values for these three inputs are estimated by means of the electricity supply curve,
also referred to as generation stack or power stack function [Geman and Roncoroni, 2006]. The electricity supply curve is a curve relating the price of electricity in a particular electrical system to the amount of electric power generated in the system. So, for a particular system and a specific amount of electric power generated, or a specific amount of electricity generated, as power and electricity are related (see Intermezzo), the electricity supply curve gives the associated price per unit of electricity as well as the marginal costs of decreasing and increasing generation. Once the amount of power or electricity generated in ERCOT is determined, the electricity supply curve could be used to estimate the price per unit of wind energy generated, by means of the price of electricity, and the ancillary service costs per unit of over- and underproduction, by means of the marginal costs of decreasing and increasing generation respectively.

Intermezzo

For clarification, two often-used terms in this master thesis are defined formally [Masters, 2004].

- **ELECTRICITY**: Electricity is a form of energy, electrical energy, flowing through a circuit (a closed electrical system). Similar to energy, electricity is measured in Joules or Watt-hours.

- **POWER**: Power refers to the rate at which energy is converted. Power is measured in Joules/second or Watts, with 1 Joule/second equal to 1 Watt.

Unfortunately, ERCOT’s electricity supply curve could not be retrieved and, therefore, the electricity supply curve of another U.S. electrical system, PJM, is used to assign values to the three model inputs. PJM’s electricity supply curve is depicted in Figure 3 and, as for most electricity supply curves, PJM’s electricity supply curve is increasing, implying that the marginal cost of decreasing generation will never exceed the marginal cost of increasing generation [Geman and Roncoroni, 2006, Krohn et al., 2009]. Hence, it seems sensible to estimate the ancillary service costs of over- and underproduction with the marginal costs of decreasing and increasing generation, as the ancillary service cost per unit of overproduction is unlikely to exceed the ancillary service cost per unit of underproduction.

![Figure 3: PJM’s electricity supply curve [CME, 2012](image)](image)
Other model inputs

Another model input that has to be assigned a value is the number of units of wind energy planned for. The value of this model input is set equal to the, yet to be determined, expected number of units of wind energy generated in ERCOT. Not only will this keep the model reasonably simple, it was also keep the model mathematically tractable. The last model input visible in Figure 2 is the number of units of wind-driven electricity generation capacity in ERCOT. As only the extended model is analyzed numerically, for which the number of units of wind-driven electricity generation capacity in ERCOT are no model input, no time is spent on finding a value for this model input. The last model input mentioned in research step 1 is the RPS, for which different values will be tried.

The power curve

All model elements and the relationships between them were explained before, except for the relationship between wind speed and number of units of wind energy generated. Using the power curve concept, wind speeds can be translated into units of power and, also, units of wind energy, as power and electricity are related (see Intermezzo). Even though power curves are turbine-specific, all power curves have a shape similar to the one of Figure 4 [Krohn et al., 2009, Richardson and McNerney, 1993]. In general, a wind turbine produces no power at low wind speeds, a rapidly increasing amount of power at medium wind speeds and a maximum amount of power at high wind speeds. Very high wind speeds, however, are cut out, implying that at very high wind speeds no power is generated. For simplicity, one power curve, that of a Vestas 112-3.0 MW wind turbine (see Figure 4), is used to relate wind speeds to the number of units of wind energy generated, irrespective of the specific type of turbine installed at each wind farm. To determine the number of units of wind energy generated from a power curve, besides wind speeds, also the rated capacity of wind farms, i.e., the maximum amount of power that can be obtained from wind farms, has to be taken into account. Specifically, in this master thesis, a wind farm’s power output will be expressed as a percentage of its rated capacity.

![Figure 4: Power curve of a Vestas 112-3.0 MW wind turbine](image)
3 The Model

In Section 3.1, the model capable of determining the expected value of wind energy to society for a particular allocation of wind-driven electricity generation capacity in ERCOT is formulated. Next, in Section 3.2, the model of Section 3.1 is extended such that the maximum expected value of wind energy to society and the associated allocation of wind-driven electricity generation capacity in ERCOT can be retrieved under a particular RPS.

3.1 The Expected Value of Wind Energy to Society

For reasons discussed in Chapter 2, the model’s key output is the expected value of wind energy to society. Specifically, in the model, the expected value of wind energy to society is defined as the expected revenues of wind energy minus the expected ancillary service costs of wind energy. While the expected revenues of wind energy are defined straightforwardly, the expected ancillary service costs of wind energy are not, as the model distinguishes between over- and underproduction of wind energy.

On top of the model structure outlined in Section 2.2, inspired by the shape of the electricity supply curve (see Figure 3), I decided to express the expected ancillary service costs of over- and underproduction of wind energy by means of quadratic terms. As most electrical systems, presumably including ERCOT, normally operate in the more steeply upward sloping part of the electricity supply curve, the inclusion of quadratic terms seems sensible. For simplicity, I decided to express the ancillary service costs of over- and underproduction by means of quadratic terms only, i.e., constants and linear terms are neglected.

Assumptions

1. The model applies to ERCOT, the closed electrical system serving most of the state of Texas.
2. Transmission issues, such as nonexistent or congested transmission lines, can be neglected.
3. All wind turbines within a single wind farm face the same wind speed.
4. All wind turbines are identical, i.e., they have the same rated capacity and power curve.
5. Wind speed only alters on an hourly basis.
6. The wind energy generated during one hour by a particular wind farm solely depends on the wind speed faced by that particular wind farm during the hour and the number of wind turbines within that particular wind farm. Also, the wind energy generated in ERCOT during one hour is simply the sum of the wind energy generated by all wind farms within ERCOT during the hour.
7. The planned level of wind energy is equal to the expected level of wind energy generated in ERCOT.
8. The wind energy generated in ERCOT during one hour is normally distributed.

9. The expected value of wind energy to society is the same for all considered hours.

Some assumptions require further clarification. Starting with assumption 2, which states that transmission issues can be neglected. In light of ERCOT’s competitive renewable energy zone (CREZ) project, which will significantly improve ERCOT’s transmission capability by expansion of existing transmission lines and construction of new transmission lines, this assumption seems reasonable [ERCOT, 2006].

Next, assumption 8, which is based on the central limit theorem (CLT). The CLT states that the sum of a large number of independent and identically distributed variables is normally distributed. However, the variables to which this assumption refers, the wind energy generated during one hour by wind farms in ERCOT, are neither independent nor identically distributed. Fortunately, the CLT also holds for variables that are neither independent nor identically distributed, as long as the degree of dependence and non-similarity is low, which is reasonably assumed. As the current number of wind farms in ERCOT is unknown, I assume the number of wind farms to be sufficient for the CLT to hold. Finally, according to assumption 9, as the model’s unit of measurement is one hour, the expected value of wind energy to society applies to one hour as well. For simplicity, however, it is assumed that the model’s input parameters are the same for every hour, such that also the expected value of wind energy to society is the same for all considered hours.

**Parameters**

- $c$ Rated capacity of a single wind turbine $W$
- $d$ Ancillary service cost per squared unit of downward deviation from the planned level of wind energy $$/ (Wh)^2$
- $k_i$ Number of wind turbines in wind farm $i$ with $i = 1, ..., n$
- $p$ Price per unit of wind energy $$/ Wh$
- $s$ Ancillary service cost per squared unit of upward deviation from the planned level of wind energy $$/ (Wh)^2$
- $\tilde{w}_i$ Wind speed faced by wind farm $i$ with $i = 1, ..., n$ $m/s$

**Definitions**

- Rated capacity of wind farm $i$
  
  \[ x_i = ck_i \]  
  \[ \text{(D.1)} \]

- Wind energy generated by wind farm $i$:
  
  \[ \tilde{g}_i = x_i f(\tilde{w}_i)^1 \text{, with } f(w_i) \text{ the power curve} \]  
  \[ \text{(D.2)} \]

\[1\text{In order to determine the wind energy generated by wind farm } i, \text{ expressed in Watt-hours, the right-hand side of the equation has to be multiplied with a unit of hours.} \]
• Wind energy generated in ERCOT:

\[ \tilde{g} = \sum_{i=1}^{n} \tilde{g}_i \]  

(D.3)

• Expected wind energy generated in ERCOT:

\[ \mu = E[\tilde{g}] = E[\sum_{i=1}^{n} \tilde{g}_i] = \sum_{i=1}^{n} E[\tilde{g}_i] \]  

(D.4)

• Variance of wind energy generated in ERCOT:

\[ \sigma^2 = \text{var}[\tilde{g}] = \sum_{i=1}^{n} \sum_{j=1}^{n} \text{cov}(\tilde{g}_i, \tilde{g}_j) \]  

(D.5)

• Covariance of wind energy generated between wind farm \( i \) and wind farm \( j \):

\[ \text{cov}(\tilde{g}_i, \tilde{g}_j) = E[(\tilde{g}_i - E[\tilde{g}_i])(\tilde{g}_j - E[\tilde{g}_j])] = E[\tilde{g}_i\tilde{g}_j] - E[\tilde{g}_i]E[\tilde{g}_j] \]  

(D.6)

Notation

• Row vector of rated capacity of wind farms in ERCOT

\[ \mathbf{x} = (x_1, \ldots, x_n) \]

Objective function

\[ V(\mathbf{x}) = p\mu - dE((\mu - \tilde{g})^+)^2 - sE((\tilde{g} - \mu)^+)^2 \]

The objective function expresses the expected value of wind energy to society, \( V(\mathbf{x}) \), as the expected revenues of wind energy, \( p\mu \), minus the expected ancillary service costs of wind energy, \( dE((\mu - \tilde{g})^+)^2 + sE((\tilde{g} - \mu)^+)^2 \), with \( a^+ = \max(a, 0) \). For reasons discussed before, the objective function distinguishes between over- and underproduction and includes quadratic terms only.

Intuitively, increasing wind-driven electricity generation capacity will increase the expected revenues of wind energy. However, the effect of increasing wind-driven electricity generation capacity on the expected ancillary service costs of wind energy is not as clear-cut. For example, increasing wind-driven electricity generation capacity might result in an amplification of the total variability of wind energy in ERCOT. In turn, this might lead to an increase in expected ancillary service costs outweighing the increase in expected revenues, hence, destroying value. Using the objective function above, answers to research question I can be formulated. These answers can help ERCOT policy makers gain insight into the value-creating or value-destroying effect of increasing wind-driven electricity generation capacity at one or more locations in ERCOT.

\[ ^2 \text{In this master thesis, vectors and matrices are made bold.} \]
Rewriting the objective function

Given assumption 8, expressions for the expected ancillary service costs of over- and underproduction of wind energy can be derived (see Result 1 and Result 2). The intuition behind the results is rather straightforward. Acknowledging that the expected overproduction of wind energy is defined in a fashion similar to the variance of wind energy generated in ERCOT, \( \sigma^2 = E[(\tilde{g} - \mu)^2] \), and that overproduction of wind energy only relates to half of the symmetric normal distribution of wind energy generated in ERCOT due to the inclusion of a maximum operator, the expected overproduction of wind energy equals \( \frac{\sigma^2}{2} \). The exact same reasoning applies to the expected underproduction of wind energy, hence, the expected underproduction of wind energy equals \( \frac{\sigma^2}{2} \) as well.

**Result 1. Expected ancillary service costs of overproduction of wind energy**

\[
sE((\tilde{g} - \mu)^+)^2 = \frac{s\sigma^2}{2}
\]

**Result 2. Expected ancillary service costs of underproduction of wind energy**

\[
dE((\mu - \tilde{g})^+)^2 = \frac{d\sigma^2}{2}
\]

Inserting Result 1 and Result 2 into the objective function:

\[
V(x) = pm - \frac{(d+s)p\sigma^2}{2}
\]

Using D.4 - D.6:

\[
V(x) = p \sum_{i=1}^{n} E[\tilde{g}_i] - \frac{(d+s)p}{2} (\sum_{i=1}^{n} \sum_{j=1}^{n} \text{cov}(\tilde{g}_i, \tilde{g}_j))
\]

\[
= p \sum_{i=1}^{n} E[\tilde{g}_i] - \frac{(d+s)p}{2} (\sum_{i=1}^{n} \sum_{j=1}^{n} E[\tilde{g}_i\tilde{g}_j] - E[\tilde{g}_i]E[\tilde{g}_j])
\]

Linearity of the expectation operator implies: \(^3\)

- \( \sum_{i=1}^{n} E[\tilde{g}_i] = \sum_{i=1}^{n} E[x_if(\tilde{w}_i)] = \sum_{i=1}^{n} x_iE[f(\tilde{w}_i)] \)
- \( \sum_{i=1}^{n} \sum_{j=1}^{n} E[\tilde{g}_i\tilde{g}_j] - E[\tilde{g}_i]E[\tilde{g}_j] = \sum_{i=1}^{n} \sum_{j=1}^{n} (E[x_if(\tilde{w}_i)x_jf(\tilde{w}_j)] - E[x_if(\tilde{w}_i)]E[x_jf(\tilde{w}_j)]) = \sum_{i=1}^{n} \sum_{j=1}^{n} (x_i, x_j \{ E[f(\tilde{w}_i)f(\tilde{w}_j)] - E[f(\tilde{w}_i)]E[f(\tilde{w}_j)] \}) \)

3.2 The Maximum Expected Value of Wind Energy to Society

Unsurprisingly, the modeling information provided in the previous section also applies to this section. New in this section is the formulation of a maximization problem, which includes a constraint corresponding to a RPS. To ease readability and interpretation, this section makes use of matrix notation.

\(^3\)As in footnote 1, in order to determine the wind energy generated by wind farm \( i \) or wind farm \( j \), the right-hand sides of the equations have to be multiplied with a unit of hours.
Additional parameters

\( \alpha \)  Percentage of total electricity to come from wind or percentage of total electricity generation capacity to be wind-driven electricity generation capacity

\( K \)  Total electricity or total electricity generation capacity\(^4\) \( W \)

Additional notations\(^5\)

- Row vector of \( n \) ones
  \[ e = (1, \ldots, 1) \]
- Row vector of expected wind power output per \( W \) of capacity at wind farms in ERCOT
  \[ \theta = (\theta_1, \ldots, \theta_n), \text{ with } \theta_i = E[f(\tilde{w}_i)] \]
- Matrix of covariances of wind power output per \( W \) of capacity between wind farms in ERCOT
  \[ \Sigma = \begin{bmatrix} \sigma_{11} & \ldots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \ldots & \sigma_{nn} \end{bmatrix}, \text{ with } \sigma_{ij} = \text{cov}(f(\tilde{w}_i), f(\tilde{w}_j)) \text{ and } \sigma_{ij} = \sigma_{ji} \]

Maximization problem\(^6\)

\[
\begin{align*}
\max & \quad V(x) = px\theta^T - \frac{1}{2} x\Sigma x^T \\
\text{s.t.} & \quad ex^T = \alpha K \\
& \quad x \geq 0
\end{align*}
\]

In the maximization problem, a quadratic objective function is combined with two linear constraints. Hence, the problem can be classified as a quadratic programming problem [Jensen and Bard, 2003]. For the maximization problem to have a solution, the objective function has to be convex, a condition that is met when \( \Sigma \) is positive semidefinite, which, by definition, is true for all covariance matrices. However, only when \( \Sigma \) is positive definite, which is the case when all eigenvalues of \( \Sigma \) are positive, the extended model has a unique solution, i.e., a local maximum that is equal to a global maximum [Jensen and Bard, 2003]. Eigenvalues of \( \Sigma \) are positive when there are no full dependencies between wind energy generation at wind farms in ERCOT. As it is highly unlikely that there are full dependencies between wind energy generation at wind farms in ERCOT, I assume \( \Sigma \) to be positive definite.

\(^4\)On the next page, it is explained why \( K \) has a unit of Watt, irrespective of whether \( K \) refers to total electricity (generally expressed in Watt-hours) or total electricity generation capacity (generally expressed in Watt).

\(^5\)The transpose of a vector or matrix is denoted with a T and the inverse of a matrix is denoted with a \(-1\).

\(^6\)In order to determine the expected value of wind energy to society, the expected revenues have to be multiplied with a unit of hours and the expected ancillary service costs have to be multiplied with a unit of hours\(^2\).
The objective function, $V(x)$, is the same as the rewritten objective function of Section 3.1, only now denoted using matrix notation. The first linear constraint in the maximization problem describes a RPS, but this single constraint has to accommodate two different types of RPS formulations. The first type, a particular level of wind-driven electricity generation capacity, is easily accommodated by the constraint, as long as the level of wind-driven electricity generation capacity is set equal to a percentage, $\alpha$, of total electricity generation capacity, $K$. The second type, a particular percentage of electricity has to come from wind, can also be accommodated, although this requires some explanation. Electricity is generally denoted in Watt-hours implying that the right-hand side of the constraint, if $\alpha$ and $K$ are chosen such that they refer to electricity, would be denoted in Watt-hours. Hence, the left-hand side of the constraint should have the same unit, Watt-hours, so the rated capacity of wind farms in ERCOT, denoted in Watt, has to be multiplied with a time unit of hours. As the maximization problem is time-independent, i.e., the expected value of wind energy to society is the same for all hours, both sides of the constraint refer to one hour and it is possible to divide both sides by one hour resulting in a constraint where total electricity, $K$, is unconventionally denoted in Watt. The second linear constraint in the maximization problem enforces the rated capacity of all wind farms in ERCOT to be nonnegative. This constraint seems sensible, as the minimum attainable rated capacity of a wind farm is obviously zero.

Using the maximization problem above, answers to research question II can be formulated. These answers can help ERCOT policy makers gain insight into the optimal allocation of wind-driven electricity generation capacity in ERCOT as well as feasible and desirable RPSs.

**Karush-Kuhn-Tucker conditions**

Before the Karush-Kuhn-Tucker (KKT) conditions corresponding to the maximization problem can be derived, the method of Lagrange multipliers has to be applied. This method involves the incorporation of the linear constraints and the objective function into one new function, the Lagrangian function $\Lambda$, and the introduction of new variables, the Lagrange multipliers $\lambda$ and $\nu$ (a n-dimensional row vector).

The Lagrangian function:

$$\Lambda(x, \lambda, \nu) = V(x) + \lambda(\text{e}x^T - \alpha K) + \nu x^T$$

$$= p x^T \theta - \frac{d+s}{2} x^T \Sigma x + \lambda(\text{e}x^T - \alpha K) + \nu x^T$$

Next, the KKT conditions are determined. These conditions are sufficient for a global maximum when $\Sigma$ is positive definite [Jensen and Bard, 2003].

$$\frac{\delta \Lambda(x, \lambda, \nu)}{\delta x} = 0$$

$$0 = p\theta - (d+s)x\Sigma + \lambda e + \nu$$

(Stationarity)
Using the KKT conditions above, the optimal solution \( x^* \) for a given \( n \) can be found. As finding \( x^* \) by hand is a tedious job, the computing language Mathematica®, capable of finding \( x^* \) within reasonable time, will be used.
4 Numerical Analysis

In this chapter, the maximization problem of Section 3.2 is analyzed numerically. In order to do so, the inputs of the maximization problem are assigned values, which is done in Section 4.2. However, first, in Section 4.1, the maximization problem’s inputs related to the hourly wind power output per W of capacity are estimated. Specifically, the averages, sample variances, sample covariances and sample correlations of hourly wind power output per W of capacity are estimated.

4.1 Hourly Wind Power Output per W of Capacity

To estimate inputs related to the hourly wind power output per W of capacity, hourly wind speeds have to be combined with the power curve of Figure 4. As the exact formula of the power curve is unknown, the power curve is approximated by the s-shaped function \( f(w_i) \) given below (also see Figure 5).

\[
f(w_i) = \begin{cases} 
  \frac{(w_i)^a}{b + (w_i)^a} & \text{if } 0 < w_i \leq 25 \text{ m/s} \\
  0 & \text{otherwise}
\end{cases}
\]

with \( a = 5.94 \) and \( b = 80,498.31 \).

Next, the retrieved windspeeds of Subsection 2.2.2, that, luckily, are already measured on an hourly basis, are combined with the approximated power curve \( f(w_i) \) to estimate both univariate and joint probability distributions of hourly wind power output per W of capacity. Due to limitations of the retrieved wind speed data, univariate probability distributions could only be estimated for thirteen locations. However, as the maximum number of locations that had wind speeds available for the same year, the year 2008, was limited to ten, joint probability distributions could only be estimated for pairs of ten locations. For consistency, I limited the number of univariate probability distributions to ten; specifically, only locations for which hourly wind speeds for the year 2008 were available were included. However, I did base the univariate probability distributions on the entire wind speed time series available for the ten locations, whereas the joint probability distributions are solely based on the year 2008.

![Figure 5: Approximation of power curve of Figure 4 using \( f(w_i) \) as defined above](image-url)
Given the univariate probability distributions of hourly wind power output per W of capacity, averages and sample variances of hourly wind power output per W of capacity for the ten included locations are estimated, see Table 1 and 2. Given the joint probability distributions of hourly wind power output per W of capacity, sample covariances of hourly wind power output per W of capacity for pairs of the ten included locations are estimated. As it is difficult to interpret covariances, the dependency of wind power output per W of capacity for each pair of locations is expressed in another way as well, using the concept of correlation. Correlation measures the linear dependency between two variables and is, therefore, only meaningful when two variables are linearly related. Fortunately, scatterplots of the wind speed data for the year 2008 for all pairs of locations show linear relationships, implying that the calculation of correlations is meaningful. The sample correlations are shown in Table 3.

<table>
<thead>
<tr>
<th>Location</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expectation</td>
<td>0.46</td>
<td>0.45</td>
<td>0.60</td>
<td>0.46</td>
<td>0.34</td>
<td>0.46</td>
<td>0.58</td>
<td>0.42</td>
<td>0.60</td>
<td>0.54</td>
</tr>
</tbody>
</table>

Table 1: Averages of hourly wind power output per W of capacity

<table>
<thead>
<tr>
<th>Location</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance</td>
<td>0.10</td>
<td>0.11</td>
<td>0.17</td>
<td>0.15</td>
<td>0.14</td>
<td>0.14</td>
<td>0.15</td>
<td>0.12</td>
<td>0.14</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Table 2: Sample variances of hourly wind power output per W of capacity

Unsurprisingly, as the ten included locations are situated in the same part of Texas, sample correlations between the locations are positive (see Table 3). Also unsurprisingly, pairs of locations that are close to one another, e.g., location 1 and 2, have the highest sample correlations.

<table>
<thead>
<tr>
<th>Location</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.00</td>
<td>0.89</td>
<td>0.39</td>
<td>0.69</td>
<td>0.58</td>
<td>0.66</td>
<td>0.52</td>
<td>0.43</td>
<td>0.67</td>
<td>0.36</td>
</tr>
<tr>
<td>2</td>
<td>0.89</td>
<td>1.00</td>
<td>0.41</td>
<td>0.68</td>
<td>0.54</td>
<td>0.65</td>
<td>0.53</td>
<td>0.44</td>
<td>0.75</td>
<td>0.37</td>
</tr>
<tr>
<td>3</td>
<td>0.39</td>
<td>0.41</td>
<td>1.00</td>
<td>0.59</td>
<td>0.48</td>
<td>0.52</td>
<td>0.74</td>
<td>0.54</td>
<td>0.59</td>
<td>0.83</td>
</tr>
<tr>
<td>4</td>
<td>0.69</td>
<td>0.68</td>
<td>0.59</td>
<td>1.00</td>
<td>0.75</td>
<td>0.87</td>
<td>0.69</td>
<td>0.54</td>
<td>0.77</td>
<td>0.52</td>
</tr>
<tr>
<td>5</td>
<td>0.58</td>
<td>0.54</td>
<td>0.48</td>
<td>0.75</td>
<td>1.00</td>
<td>0.64</td>
<td>0.55</td>
<td>0.45</td>
<td>0.57</td>
<td>0.39</td>
</tr>
<tr>
<td>6</td>
<td>0.66</td>
<td>0.65</td>
<td>0.52</td>
<td>0.87</td>
<td>0.64</td>
<td>1.00</td>
<td>0.62</td>
<td>0.49</td>
<td>0.71</td>
<td>0.46</td>
</tr>
<tr>
<td>7</td>
<td>0.52</td>
<td>0.53</td>
<td>0.74</td>
<td>0.69</td>
<td>0.55</td>
<td>0.62</td>
<td>1.00</td>
<td>0.62</td>
<td>0.68</td>
<td>0.68</td>
</tr>
<tr>
<td>8</td>
<td>0.43</td>
<td>0.44</td>
<td>0.54</td>
<td>0.54</td>
<td>0.54</td>
<td>0.45</td>
<td>0.49</td>
<td>0.62</td>
<td>1.00</td>
<td>0.48</td>
</tr>
<tr>
<td>9</td>
<td>0.67</td>
<td>0.75</td>
<td>0.59</td>
<td>0.77</td>
<td>0.57</td>
<td>0.71</td>
<td>0.68</td>
<td>0.48</td>
<td>1.00</td>
<td>0.51</td>
</tr>
<tr>
<td>10</td>
<td>0.36</td>
<td>0.37</td>
<td>0.83</td>
<td>0.52</td>
<td>0.39</td>
<td>0.46</td>
<td>0.68</td>
<td>0.45</td>
<td>0.51</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 3: Sample correlations of hourly wind power output per W of capacity

More information regarding the ten locations is provided in Appendix 1.
4.2 Base Case Scenario

In the base case scenario, ten locations are considered. The expected hourly wind power output per W of capacity for all ten locations is set equal to the average of the averages of hourly wind power output per W of capacity of Table 1. Similarly, the variance of hourly wind power output per W of capacity for all ten locations is set equal to the average of the variances of hourly wind power output per W of capacity of Table 2. The covariances, excluding the variances, of wind power output per W of capacity between pairs of locations are set equal to those found before. As discussed in Subsection 2.2.2, the price per unit of wind energy and the ancillary service costs per squared unit of over- and underproduction can be assigned values by means of PJM’s electricity supply curve (see Figure 3). Using this curve and an assumption concerning the total electricity generation capacity in ERCOT of 140 GW, values for $p$, $s$ and $d$ are retrieved (see Appendix 2).

As Chapter 3 maximizes the expected value of wind energy to society for one hour, the base case scenario would also refer to one hour. However, as it is unrealistic to change the allocation of wind-driven electricity generation capacity on an hourly basis, a more realistic timeframe of twenty years is chosen for the base case scenario [Krohn et al., 2009]. Specifically, the expected value of wind energy to society for twenty years is calculated as the present value of a twenty-year annuity, see formula below. In the formula below, \( r_{hour} \) is defined as the annual discount rate \( r \) divided by the number of hours/year, which, for simplicity, is set equal to 8760.

\[
V(x)_{20 \text{ years}} = V(x)\left[1 - \left(1 + \frac{r_{hour}}{r_{hour}}\right)^{-20 \times 8760}\right]
\]

The inputs of the base case scenario, including most inputs described above, are summarized in Table 4.

<table>
<thead>
<tr>
<th>Input</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.1000</td>
<td></td>
</tr>
<tr>
<td>$d$</td>
<td>5000</td>
<td>$(GWh)^2$</td>
</tr>
<tr>
<td>$\theta_i, \forall i$</td>
<td>0.4900</td>
<td></td>
</tr>
<tr>
<td>$K$</td>
<td>140</td>
<td>GW</td>
</tr>
<tr>
<td>$n$</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>115000</td>
<td>$/GWh$</td>
</tr>
<tr>
<td>$s$</td>
<td>3000</td>
<td>$(GWh)^2$</td>
</tr>
<tr>
<td>$\sigma_{ii}, \forall i$</td>
<td>0.1387</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Inputs of base case scenario

The maximum expected value of wind energy to society in the base case scenario, which is obtained using Mathematica® (see Appendix 3), equals $55 billion. The associated optimal allocation of wind-driven electricity generation capacity across the ten locations, which equal the ten locations in ERCOT discussed in the previous section, except with respect to expectations and variances, is given in Table 5.
Remarkable in Table 5 is the allocation of wind-driven electricity generation capacity to only six locations. As the ten locations in the base case scenario have the same expectations and variances of hourly wind power output per W of capacity, the allocation of capacity is entirely driven by the correlations between the locations. Unsurprisingly, much capacity is allocated to locations that have low positive correlations with other locations, such as locations 8 and 10 (see Table 3), and no capacity is allocated to locations that have high positive correlations with other locations, such as locations 4 and 9 (see Table 3).

The wind-driven electricity generation capacity allocated to each location can easily be translated into the number of wind turbines to be constructed at each location, as long as the rated capacity of a single wind turbine is known (using D.1 of Section 3.1). Obviously, in practice, it is impossible to construct a non-integer number of wind turbines. However, for ease of calculations, I abstained from practical issues concerning wind-driven electricity generation capacity, such as an integer number of wind turbines.

### 4.3 Sensitivity Analysis

In this section, the inputs of the base case scenario are changed one by one, which is referred to as sensitivity analysis, in order to gain insight into the impact of each input on the maximum value of wind energy to society and the associated allocation of wind-driven electricity generation capacity. Obviously, the most important graphs related to the sensitivity analysis are included in this section. For completeness, less important graphs are included in Appendix 4.

#### Expectation of hourly wind power output per W of capacity

An increase in the expectation of hourly wind power output per W of capacity, \( \theta_i \), \( \forall i \), results in a substantial increase in the maximum expected value of wind energy to society. For example, for \( \theta_i \), \( \forall i \), equal to 0.30 a value of $32 billion is attained, while for \( \theta_i \), \( \forall i \), equal to 0.70 a value of $81 billion is attained. Even more interesting is that the maximum expected value of wind energy to society already increases when only one wind farm has a high expected hourly wind power output per W of capacity. Figure 6, in which \( \theta_i \) for \( i = 1, 2, 3 \) is varied, shows that as soon as the expectation of location \( i \) outgrows the other locations’ expectations, the maximum expected value of wind energy to society increases steeply, which is caused by the allocation of an increasing share of total wind-driven electricity generation capacity to location \( i \). Also, not unexpectedly, when the expected hourly wind power output per W of capacity of a few wind farms is increased, instead of only one, the maximum expected value of wind energy to society increases even more steeply, as can be seen in Figure 7.

<table>
<thead>
<tr>
<th>Location</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity (GW)</td>
<td>2.47</td>
<td>2.25</td>
<td>0.00</td>
<td>0.00</td>
<td>2.39</td>
<td>0.35</td>
<td>0.00</td>
<td>3.31</td>
<td>0.00</td>
<td>3.23</td>
<td>14.00</td>
</tr>
</tbody>
</table>

Table 5: Optimal allocation of wind-driven electricity generation capacity in the base case scenario
Variance of hourly wind power output per W of capacity

Intuitively, one expects a higher variance of hourly wind power output per W of capacity, $\sigma_{ii}, \forall i$, to result in a lower maximum expected value of wind energy to society. Although this intuition is confirmed, the impact of the variance of hourly wind power output per W of capacity on the maximum value of wind energy to society is limited, i.e., far less substantial than the impact of the expectation of hourly wind power output per W of capacity. For example, an extremely high $\sigma_{ii}, \forall i$, of one results in a value of $50$ billion, while an extremely low $\sigma_{ii}, \forall i$, of zero only results in a value of $60$ billion. Also, as can be seen in Figure 8 and 9, high maximum expected values of wind energy to society are only obtained if the variance of hourly wind power output per W of capacity is very, perhaps unrealistically, low.

Correlation of hourly wind power output per W of capacity $^8$

Similar to the impact of the variance of hourly wind power output per W of capacity, the impact of the correlation of hourly wind power output per W of capacity on the maximum expected value of wind energy to society is less substantial than the impact of the expectation of hourly wind power output per

$^8$The correlation of hourly wind power output per W of capacity between location $i$ and location $j$ is denoted by $\rho_{ij}$, with $\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}$.
W of capacity. Obviously, low correlations lead to higher maximum expected values of wind energy to society than high correlations. Figure 10 and 11 show that the inclusion of only a few low correlations already increases the maximum expected value of wind energy to society by several billion dollars.

Figure 8: The maximum expected value of wind energy to society when varying $\sigma_{ii}$ for $i = 1, 2, 3$

Figure 9: The maximum expected value of wind energy to society when varying $\sigma_{11}$ till $\sigma_{55}$

Figure 10: The maximum expected value of wind energy to society when varying $\rho_{11}$ till $\rho_{110}$
Total electricity to come from wind or total wind-driven electricity generation capacity

Figure 12 clearly shows the existence of an optimal level of total electricity to come from wind or total wind-driven electricity generation capacity; at the optimal level of $\alpha K$, the maximum expected value of wind energy to society is highest. Even though the expected revenues of wind energy are higher for higher levels of $\alpha K$, so are the expected ancillary service costs. Unfortunately, as Figure 13 shows, the expected revenues of wind energy increase linearly, while the expected ancillary service costs increase exponentially, therefore, higher levels of $\alpha K$ do not always result in higher maximum expected values of wind energy to society. Unsurprisingly, in order to meet higher levels of $\alpha K$, more and more wind-driven electricity generation capacity is allocated to locations with the lowest correlations (see Figure 14).

Price of wind energy and ancillary service costs of over- and underproduction

The price of wind energy, $p$, and the ancillary service costs of over- and underproduction, $s$ and $d$ respectively, have a substantial impact on the maximum expected value of wind energy to society (see Appendix 4). As it is uncertain whether the chosen values for $p$, $s$ and $d$ are realistic, the results of the base case scenario and sensitivity analysis should be interpreted with caution.
Annual discount rate and timeframe
Also the impact of the annual discount rate and the timeframe on the maximum expected value of wind energy to society are substantial (see Appendix 4). When increasing the annual discount rate, first, the maximum expected value of wind energy to society falls rapidly, but after a while, the maximum expected value of wind energy to society stabilizes. Opposedly, when increasing the timeframe, first, the maximum expected value of wind energy to society rises quickly, but after a while, the maximum expected value of wind energy to society stabilizes. Given the uncertainty associated with the annual discount rate in particular, once again, the results of the numerical analysis should be viewed with care.

Number of locations
To generate Figure 15, the first location was chosen to equal location 1 of the base case scenario. Next, the number of locations was increased by one at a time, starting by adding location 2 of the base case scenario to the first location and ending by adding location 10 of the base case scenario to the previously considered nine locations, location 1 till location 9.
Figure 15: The maximum expected value of wind energy to society when varying \( n \) from 1 to 10

The first value of Figure 16 equals the maximum expected value of wind energy to society in the base case scenario. As the base case scenario only considers ten locations, increasing the number of locations to more than ten requires the introduction of new locations. Based on the ten locations of Section 4.1, a relationship between the distance associated with each pair of locations and the correlation associated with each pair of locations could be established, see formula below. This relationship, in its turn, could be used to estimate the correlation between randomly generated wind farm locations in ERCOT, assuming ERCOT to be a square with sides of length 1122 km. The expectation and variance of hourly wind power output per W of capacity for each randomly generated location were set equal to the expectation and variance of hourly wind power output per W of capacity of Table 4. Similar to Figure 15, to generate Figure 16, the number of locations was increased by ten at a time, starting with the ten locations of the base case scenario and each time adding ten randomly generated locations.

\[
\text{correlation} = \begin{cases} 
1 & \text{if distance} = 0 \\
0.806 - 0.009 \text{distance} & \text{otherwise}
\end{cases}
\]

Figure 16: The maximum expected value of wind energy to society when varying \( n \) from 10 to 70
It can be concluded from Figure 15 and 16 that the impact of the number of locations on the maximum expected value of wind energy to society is limited. The largest increase in the maximum expected value of wind energy to society occurs when the number of locations is increased starting from a small number of locations, $n < 20$. For a large number of locations, $n > 20$, the size of the increase in the maximum expected value of wind energy to society decreases. It is important to note that the order of inclusion of the locations is important; the displayed maximum expected values of wind energy to society are dependent on the specific order in which the locations are included.
5 Conclusion & Discussion

5.1 Conclusion

In this master thesis, a model to gauge the expected value of wind energy to society, defined as the expected revenues of wind energy minus the expected ancillary service costs of wind energy, is proposed. By deciding upon the allocation of wind-driven electricity generation capacity, the model, which applies to ERCOT, the closed electrical system serving most of the state of Texas, aims at maximizing the expected value of wind energy to society. In this master thesis, besides formulating the model, the model is also analyzed numerically, resulting in the following conclusions:

- **Improve the expected hourly wind power output of one or a few wind farms:** Evidently, improving the expected hourly wind power output per W of capacity increases the maximum expected value of wind energy to society. However, the sensitivity analysis of Section 4.3 shows that even when the expected hourly wind power output per W of capacity of only one or a few wind farms in ERCOT could be improved, already a substantial increase in the maximum expected value of wind energy to society would result. Also, the sensitivity analysis shows that it is far more beneficial to improve the expected hourly wind power output per W of capacity than lowering the variance of hourly wind power output per W of capacity.

- **Allocate wind-driven electricity generation capacity to locations with low correlations of hourly wind power output:** The numerical analysis shows that the allocation of wind-driven electricity generation capacity to locations in ERCOT that have at least a few low correlations with other locations results in slightly higher maximum expected values of wind energy to society. The numerical analysis also shows that the distribution of wind-driven electricity generation capacity across more locations is not necessarily beneficial, in the sense that it does not always increase the maximum expected value of wind energy to society substantially. The explanation for this phenomenon might be found in the base case scenario which shows that, irrespective of the number of locations considered, the maximum expected value of wind energy to society is attained when wind-driven electricity generation capacity is allocated to the locations with low correlations.

- **Formulate ERCOT’s RPS such that the highest maximum expected value of wind energy to society is obtained:** According to the sensitivity analysis of Section 4.3, there is an optimal level of total electricity to come from wind or total wind-driven electricity generation capacity, i.e., a level at which the maximum expected value of wind energy to society is highest. Therefore, it is recommendable for ERCOT to set its RPS equal to the found optimal level.
• Gather realistic values for the annual discount rate, the price of wind energy and the ancillary service costs of over- and underproduction: There is a substantial sensitivity of the maximum expected value of wind energy to society to input parameters for which no realistic values were obtained. Specifically, the input parameters referred to are the annual discount rate, the price of wind energy and the ancillary service costs of over- and underproduction. Given the sensitivity to these input parameters for which no realistic values were obtained, the results of the numerical analysis, and the conclusions drawn from them, should be interpreted with care. To obtain a more realistic estimate of the maximum expected value of wind energy to society, realistic values for these input parameters have to be gathered.

5.2 Discussion

This master thesis has three limitations: a lack of wind speed data for a reasonable number of locations in ERCOT, a model in which the expected value of wind energy to society is the same for all considered hours and a model that only takes into account revenues and ancillary service costs. These limitations, and the future research that should take place to overcome them, are discussed below.

Unfortunately, wind speed data were retrieved for only thirteen locations in ERCOT. On top of that, the overlap between the retrieved wind speed time series was low. Therefore, only ten locations were included in the numerical analysis. To provide ERCOT policy makers with a more realistic estimate of the maximum expected value of wind energy to society and the associated allocation of wind-driven electricity generation capacity, future research should gather wind speed data for many more locations in ERCOT. When considering more locations in ERCOT, the concern related to violation of assumption 8 of the model, stating that the wind energy generated in ERCOT during one hour is normally distributed, decreases. Besides gathering more wind speed data, as mentioned in Section 5.1, realistic estimates of the annual discount rate, the price of wind energy and the ancillary service costs of over- and underproduction are also required for a more realistic estimate of the maximum expected value of wind energy to society and the associated allocation of wind-driven electricity generation capacity in ERCOT.

In Chapter 3, it was assumed that the expected value of wind energy to society is the same for all considered hours. However, in reality, amongst others, seasonal and annual patterns in energy generation and consumption are present. Hence, the expected value of wind energy to society is likely to differ over time, as the expected wind energy generated in ERCOT, the price of wind energy and the ancillary service costs per squared unit of over- and underproduction are likely to differ over time [Richardson and McNerney, 1993]. In order to obtain a more realistic estimate of the maximum expected value of wind energy to society and the associated allocation of wind-driven electricity generation capacity in ERCOT,
both of which might be significantly different from the base case scenario of Section 4.2, future research should allow the model input parameters to vary over time.

The model proposed in this master thesis only takes into account one particular class of benefits and costs related to wind energy, revenues and ancillary service costs. Unsurprisingly, as touched upon in Chapter 2, there are more classes of benefits and costs that are interesting to consider, e.g., environmental benefits and costs. However, future research extending the model proposed in this master thesis should give priority to the inclusion of investment and transmission costs. Investment costs are costs associated with the construction of wind farms, such as grid connection and wind turbine costs [Krohn et al., 2009]. Transmission costs are costs associated with the transportation of electricity across the grid, such as congestion costs, i.e., costs arising from the use of other power plants due to a lack of transmission capacity running from wind farms [ERCOT, 2010b]. Both classes of costs are likely to have a substantial impact on the optimal allocation of wind-driven electricity generation capacity in ERCOT, i.e., the allocation that maximizes the maximum expected value of wind energy to society. First, as investment costs are highest for new wind farms, since foundations and grid connections are not yet in place, expanding existing wind farms might turn out to be the most cost-effective way of achieving higher levels of wind energy or wind-driven electricity generation capacity. Second, regarding transmission costs, it might be more cost-effective to allocate wind-driven electricity generation capacity to locations with sufficient transmission capacity even though these locations have inferior wind energy characteristics.

Although the model of Chapter 3 applies to ERCOT, the model is not limited to ERCOT. As long as the model assumptions denoted in Chapter 3 are met by a particular electrical system, except for the first assumption, the model can be applied to that system. Hence, opposed to the three points discussed before and perhaps counterintuitively, the focus on ERCOT is not a limitation of this master thesis.
References


## A Appendix 1 - Locations in ERCOT

Sources: AEI [2012] and OWNO [2012]

<table>
<thead>
<tr>
<th>Identifier</th>
<th>Name</th>
<th>Position</th>
<th>Latitude</th>
<th>Longitude</th>
</tr>
</thead>
<tbody>
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<td>AEI Wind Test Center</td>
<td>South of Amarillo</td>
<td>N 34.95</td>
<td>W 101.80</td>
</tr>
<tr>
<td>0014</td>
<td>White Deer</td>
<td>Northeast of Amarillo</td>
<td>N 35.00</td>
<td>W 101.00</td>
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<tr>
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</tr>
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<tr>
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</tr>
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<td>Southeast of Amarillo</td>
<td>N 34.65</td>
<td>W 100.55</td>
</tr>
<tr>
<td>0051</td>
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<td>West of Abilene</td>
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<tr>
<td>5940</td>
<td>Cochran County</td>
<td>Northwest of Lubbock</td>
<td>N 32.73</td>
<td>W 102.76</td>
</tr>
</tbody>
</table>
B Appendix 2 - Retrieving values for $p$, $s$ and $d$

Retrieving $p$

Using the figure above, a value for $p$, the price of wind energy, is retrieved by determining the price associated with a total electricity generation capacity of 140 GW (black vertical and horizontal line in figure). The associated price equals $115/MWh, hence, $p$ is set equal to $115000/GWh$.

Retrieving $s$

In case of overproduction of wind energy, ERCOT has to deploy less traditional electricity generation capacity than originally planned for (blue line in figure). Therefore, using the figure above, a value for $s$, the ancillary service costs of underproduction, is retrieved by substracting the price associated with a total electricity generation capacity of 139 GW from the price associated with a total electricity generation capacity of 140 GW. This results in a $s$ of $3000/(GWh)^2$.

Retrieving $d$

Opposely, in case of underproduction of wind energy, ERCOT has to deploy more traditional electricity generation capacity than originally planned for (red line in figure). A value for $d$ is retrieved by substracting the price associated with a total electricity generation capacity of 140 GW from the price associated with a total electricity generation capacity of 141 GW. This results in a $d$ of $5000/(GWh)^2$. 
Input - General

Price of electricity ($/GWh)
\( p = 115000; \)

Ancillary service cost of upward deviation ($/GW^2h^2)
\( s = 3000; \)

Ancillary service cost of downward deviation ($/GW^2h^2)
\( d = 5000; \)

Number of wind farms
\( n = 10; \)

Percentage of total electricity to come from wind / Percentage of total electricity generation capacity to be wind-driven electricity generation capacity
\( \alpha = 0.10; \)

Total electricity / Total electricity generation capacity (GW)
\( k = 140; \)

Discount rate
\( r = 0.10; \)

Number of years to consider
\( y = 20; \)

Input - Ten locations in ERCOT

Expected hourly wind energy generation at 10 locations in ERCOT (h)
\( \theta_{\text{ten}} = \{0.4612, 0.4456, 0.5983, 0.4605, 0.3395, 0.4623, 0.5779, 0.4203, 0.5972, 0.5375\}; \)

Variance of hourly wind energy generation at 10 locations in ERCOT (h^2)
\( \sigma_{\text{ten}} = \{0.1044, 0.1084, 0.1543, 0.1359, 0.1361, 0.1509, 0.1200, 0.1352, 0.1728\}; \)

Covariance of hourly wind energy generation at 10 locations in ERCOT (h^2)
\( \sigma_{\text{ten}} = \{0.1044, 0.0951, 0.0513, 0.0870, 0.0689, 0.0781, 0.0652, 0.0482, 0.0801, 0.0478\}, \{0.0951, 0.1084, 0.0550, 0.0884, 0.0658, 0.0789, 0.0683, 0.0503, 0.0904, 0.0509\}, \{0.0513, 0.0550, 0.1685, 0.0946, 0.0723, 0.0787, 0.1185, 0.0767, 0.0895, 0.1422\}, \{0.0870, 0.0884, 0.0946, 0.1543, 0.1087, 0.1257, 0.1051, 0.0735, 0.1118, 0.0844\}, \{0.0689, 0.0658, 0.0723, 0.1087, 0.1359, 0.0869, 0.0789, 0.0580, 0.0775, 0.0590\}, \{0.0781, 0.0789, 0.0787, 0.1257, 0.0869, 0.1362, 0.0890, 0.0628, 0.0966, 0.0705\}, \{0.0652, 0.0683, 0.1185, 0.1051, 0.0789, 0.0890, 0.1510, 0.0843, 0.0965, 0.1099\}, \{0.0482, 0.0503, 0.0767, 0.0735, 0.0580, 0.0628, 0.0843, 0.1200, 0.0611, 0.0652\}, \{0.0801, 0.0904, 0.0895, 0.1118, 0.0775, 0.0966, 0.0965, 0.0611, 0.1353, 0.0781\}, \{0.0478, 0.0509, 0.1422, 0.0844, 0.0590, 0.0705, 0.1099, 0.0652, 0.0781, 0.1729\}; \)
**Simplification**

*Averages*

\[ \text{averagetheta} = \text{Mean}[\text{thetaten}[[1]]] \]
\[ \text{averagevar} = \text{Mean}[\text{varten}[[1]]] \]

*Replace vectors and matrices with averages*

\[ \text{thetan} = \text{ConstantArray}[\text{averagetheta}, n] \]
\[ \text{varn} = \text{ConstantArray}[\text{averagevar}, n] \]
\[ \text{sigman} = \text{ReplacePart}[\text{sigmaten}, \{i, i\} \rightarrow \text{averagevar}] \]

**Solution**

*Rated capacity of wind farms (GW)*

\[ \text{decisionvariables} = \text{Array}[x, n] \]

\[ \text{listofn} = \text{Range}[1, n, 1] \]

*Expected value of wind energy to society (one hour)*

\[ \text{solutionhour} = \text{Maximize}[\{p*\text{thetan}.\text{decisionvariables} - 0.5*(d + s)*\text{decisionvariables}.\text{sigman}.\text{decisionvariables},\]
\[ \text{Sum}[\text{decisionvariables}[i], \{i, 1, n\}] == \alpha k, \text{decisionvariables}[[#]] > 0 \& \& /\& \text{listofn}] \]

*Expected value of wind energy to society (y years)*

\[ \text{rhour} = \frac{r}{8760} \]
\[ \text{solutionyyyears} = \text{solutionhour}[1]^*\left((1 - (1 + \text{rhour})^{-y*8760})/\text{rhour}\right) \]
D Appendix 4 - Graphs of Sensitivity Analysis

Expectation of hourly wind power output per W of capacity

The maximum expected value of wind energy to society when varying $\theta_i$ for $i = 1, 2, 3$ (close-up)

The allocation of wind-driven electricity generation capacity to location $i$ when varying $\theta_i$ for $i = 1, 2, 3$

The allocation of wind-driven electricity generation capacity when varying $\theta_1$ till $\theta_5$
Variance of hourly wind power output per W of capacity

The maximum expected value of wind energy to society when varying $\sigma_{ii}$ for $i = 1, 2, 3$ (close-up)

The allocation of wind-driven electricity generation capacity to location $i$ when varying $\sigma_{ii}$ for $i = 1, 2, 3$

The allocation of wind-driven electricity generation capacity when varying $\sigma_{11}$ till $\sigma_{55}$
Correlation of hourly wind power output per W of capacity

The allocation of wind-driven electricity generation when varying $\rho_{1i}$ for $i = 2, ..., 10$

The allocation of wind-driven electricity generation capacity when varying $\rho_{11}$ till $\rho_{55}$
Price of wind energy and ancillary service costs of over- and underproduction

The maximum expected value of wind energy to society when varying $p$

The maximum expected value of wind energy to society when varying $s$

The maximum expected value of wind energy to society when varying $d$
Annual discount rate and timeframe

The maximum expected value of wind energy to society when varying $r$

![Graph](image1)

The maximum expected value of wind energy to society when varying the timeframe

![Graph](image2)