The value creation potential of a new SCF product: Insured Pooled Receivables

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MANAGEMENT SUMMARY

The Royal Bank of Scotland (RBS) is introducing a new product: Insured Pooled Receivables (IPR). The product is meant for suppliers who have a pool of accounts receivable from different buyers. For risk management or liquidity reasons, the supplier decides to sell this pool of receivables to the bank. The bank can provide liquidity, but it generally has no appetite to take the credit risk of all these small buyers. Therefore it takes on insurance on these receivables.

The result of transferring the ownership of the accounts receivable and insuring them is an IPR deal. IPR can be positioned in the growing body of research to Supply Chain Finance (SCF). It has been shown in this field that coordinating financing functions of participants in the supply chain can create value, for example via reversed factoring. There are two main trends that make this type of financing, based on trade transactions, very promising.

The first is the fact that international trade has been growing fast in the last decade and this growth is predicted to continue: the world trade volume is expected to have quadrupled by 2040 (Buiter and Rahbari, 2011). The largest part of this growth will come from increasing trade between developing economies, where the availability of traditional funding is relatively low. Therefore SCF programs can be of great value in this area. Secondly, the regulatory trends in the financial sector (i.e. the installment of Basel III) force banks to revise their product portfolio. Since SCF products provide collateral, they are less risky and require less Risk-Weighted Assets (RWA). This is an important reason for the popularity of invoice financing during the recent financial crisis: despite European bank lending declining overall, this market has shown double digit growth since 2009 (Demica, 2012).

So, the market conditions for this product are rather promising, but an open question is if and in what circumstances IPR can actually be valuable. Therefore the research question is:

*What benefits does an Insured Pooled Receivables deal have for the involved parties and how does it affect the operations in the supply chain?*

To analyze this question, I create a model that is able to determine the risk adjusted present value of the cash flows for all different parties. If at least one party can increase its present value and no party is worse off by participating, there is value creation potential. By building up this model gradually, I am able to identify what elements make IPR valuable.

In the simplest model, there is a supplier who sells it goods to only one buyer, creating an account receivable with the normalized value of 1. There is no recourse, participants are risk neutral, and their goal is to maximize their expected payoff. The only unknown is whether the buyer will default or not. I show that under these circumstances, the present value of the cash flows for each party is equal to the present value in a situation without IPR. So for this simplified model with risk neutral investors, IPR has no value creation potential.
To make the model more realistic, it is very important to take risk aversion into account. There are several ways to do this, which essentially all stem from one general asset pricing formula. I will use expected utility theory, since concave utility functions are able to describe risk averse behavior, especially in the presence of market imperfections such as bankruptcy costs. Investors that are behaving according to a utility function are willing to pay a certainty premium to get rid of uncertainty in a cash flow.

I still use the simplified model with the transfer of only one receivable from supplier to insurer, but now these parties are risk averse. I show that if two parties have different utility functions, different risk aversion coefficients, different initial capital, or when informational asymmetry exists, this transaction can create value. This is actually a rather strong result: already in a situation where the role of the bank is ignored and there is only one buyer instead of a pool, there is value creation potential. This result can be validated though: this type of transaction is very similar to credit insurance, which is used extensively in practice.

The next important extension of the model is increasing the size of the pool to multiple receivables. Especially the correlation in such a portfolio of receivables will affect the variance and the risk of the product and therefore also its value. To incorporate this correlation, I use Moody’s Correlated Binomial Default Distribution. This model assumes that every obligor has the same size and default probability, and that each pair of obligors has the same constant correlation.

The lower the correlation between the obligors, the more the risk can be reduced. For a relatively high correlation, pooling decreases the risk a little, but a large part of the risk cannot be diversified away. The lower the correlation, the further the risk (and therefore also the certainty premium) can be reduced by using pooling. The impact of adding an extra receivable to the portfolio is largest when the initial portfolio is small.

The final extension of the model is to consider the timing of the cash flows and the role of the bank. It turns out however that according to this model, the bank is not able to improve the deal between the supplier and the insurer. This is because the value creation potential of IPR is mainly affected by parameters that influence the (relative) size of the risk, such as the default probability, the Loss Given Default (LGD), correlation, risk aversion, and initial capital. Since the bank immediately insures the risk it takes, it can only be compensated by the risk free rate.
This model however goes past the fact that providing liquidity can also be a valuable service. In a situation where the supplier is financially constrained, he already has so much debt that it is either impossible or very costly to raise more regular debt. IPR can offer a solution, because selling the accounts receivable can provide liquidity to the supplier, without further increasing the debt. An IPR deal can also help a supplier avoid the waiting time before claims are disbursed. Further the bank might be able to improve payment behavior of buyers, reduce the LGD or negotiate a better deal with the insurer. Therefore the bank is usually still able to claim some of the value creation potential.

These financial benefits of IPR can also positively affect the operations in the supply chain. Reduced financing costs can improve the supplier’s service level via reduced holding costs and increased liquidity can improve the supplier’s flexibility to make better operational decisions. The buyers’ service levels might be improved as well when the supplier extends the payment period. Finally, the interaction between the financial decision of using IPR and the operational aspects of a supplier’s pricing decision can optimize IPR’s benefits.

The recommendation for RBS is to continue the introduction of IPR and to make sure that is available in developing economies, where demand could grow very fast. The product is most valuable for suppliers with high risk portfolios that are also able to improve their operations with this financial product. In the implementation of the product, it is crucial to thoroughly assess the insurer’s creditworthiness, to carefully select an IT platform and to decide whether the deal should have recourse and should be disclosed. Future research should validate the results of this study by using a different method to account for risk, by using different default models, and by comparing discounting methods in a multiple period model. The most valuable extension of the model would be an explicit calculation of the value of liquidity.
PREFACE

In August 2007, I started studying Industrial Engineering and Management Sciences. In that same month the global financial crisis commenced with BNP Paribas freezing three of its hedge funds, because of its inability to value the underlying financial product. Just like the banking sector, I could not have anticipated the turbulence and changes that the next five and a half years would bring.

It were years with many new and unexpected experiences. Analogous to the financial sector, I feel I have improved myself in many ways, but also still have several opportunities to develop myself further. However, these five and a half years have not been a crisis for me, but a wonderful journey upon which I can always look back with much joy and probably some nostalgia.

I am grateful for the opportunity the Eindhoven University of Technology and Tilburg University gave me to follow the double degree Operational Finance: a combination of the masters Operations Management & Logistics and Finance. This thesis is the result of a combined graduation project for both masters. I am thankful to Eric Lemmens and my other colleagues at the Royal Bank of Scotland for the great opportunity to conduct my research within this bank. Further I owe a debt of gratitude to my supervisors Matthew Reindorp and Bertrand Melenberg, who always had time to provide me with highly valuable guidance and advice. Finally, I would like to thank my friends and family, who made the journey to becoming the person I am today both possible and very enjoyable.

Jesse Hoekstra

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INTRODUCTION

The global financial crisis that started in 2007 has had an enormous impact on the banking sector. Before the crisis, banks owned and sold many complex products of which the risks were not well understood. In combination with the bursting of the U.S. housing bubble and other economical factors, this led to heavy losses, government bailouts and even bankruptcies for banks.

The heavy risk taking of banks combined with trading scandals and excessive bonuses caused the public opinion on banks to plummet. Simultaneously, stricter rules for the financial sector are developed to prevent such a financial crisis from happening in the future. In this already tough environment, the Royal Bank of Scotland (RBS) also suffered from having to write down several billions of bad investment, mainly due to the acquisition of the Dutch bank ABN Amro.

To improve this situation, it is important for banks to regain the trust of its customers and of the society as a whole. Therefore it has to focus on offering services and products that provide sustainable value for its clients and can contribute to economical recovery. A good example of such a service is providing trade finance, since this enables companies to do business with each other, which stimulates the economy. A promising new area within trade finance is Supply Chain Finance (SCF). This thesis is conducted at the Trade Finance department of RBS in Amsterdam and focuses on a new SCF product: Insured Pooled Receivables.

1.1 Royal Bank of Scotland

The Royal Bank of Scotland was founded in 1727 in Edinburgh. It has had a long history of relative stability and growth, which made RBS one of the largest banks in the world. Shortly before and in the beginning of the recent financial crisis, RBS was even the world’s largest bank based on total assets. However, the bank experienced heavy trouble during the crisis. RBS had to make large write-downs on the value of past acquisitions (especially ABN Amro) and experienced heavy losses due to the global financial situation. When RBS announced in January 2009 that losses could accumulate to a record-high of 28 billion pounds, its stocks plummeted with 67% in one day. To prevent the bank from collapsing, the government took a large stake in the company, currently 82%.

RBS offers personal and business banking, private banking and corporate finance. The bank is divided in many divisions, of which Markets & International Business (M&IB) is of current interest. Two main sub departments in International Business are Transaction Services Product (focused on financial products) and Transaction Services Origination (focused on clients). RBS’ Trade Finance department is part of Transaction Services Product and focuses on financing (international) trade. Within this department, SCF solutions receive more and more attention lately. A specific example of a SCF product is Insured Pooled Receivables (IPR), a product that is currently in development at RBS. SCF is one of the fastest growing types of financing at RBS in
the past years. For the upcoming years further growth is expected, due to a changing regulatory and economical environment.

1.2 Supply Chain Finance

SCF solutions can be applied in trade transactions between buyer and seller: a typical supply chain setting. The supply chain has been studied extensively from an operational point of view under the term Supply Chain Management (SCM). In this field it has been proven that considering the supply chain as a whole, instead of each participant separately, can create substantial value. The emerging SCF field has recently shown that financing decisions in this setting can also be improved by incorporating this integrated way of thinking.

SCF is the optimization of financing of two or more companies in a supply chain by collaboration, often with help of a financial institution. The financial supply chain manages the financial functions that are induced by operational activities. SCF tries to coordinate the participants’ joint activities in order to create value (Hofmann, 2005).

The type of financing that has received most attention within SCF is reversed factoring. In this type of deals, there is often a large buyer doing business with a small supplier. The buyer helps the supplier with its financing needs in exchange for a discount or an extended payment period, or to improve the relationship with the supplier. Sometimes SCF is believed to consist only of this kind of deals where the buyer is leading (buyer-centric), but seller-centric deals can also fit within the definition of SCF perfectly. IPR is an example of a SCF product where the supplier is leading, i.e. the deal is seller-centric.

1.3 Insured Pooled Receivables

An IPR deal can provide a supplier with risk mitigation and liquidity. The typically large supplier sells its goods or services to multiple buyers. These buyers are granted a payment period, which means that they do not pay immediately but only after a certain amount of time. This creates accounts receivable on the asset side of the supplier’s balance sheet. These receivables need to be financed by the supplier and carry credit risk: the risk that the buyer will default on the payment.

To remove this risk and the financing need, the supplier can sell its pool of receivables to the bank. Since this provides additional services (risk mitigation and liquidity), the receivables are sold at a discount. The bank generally has no appetite to take the credit risk of all these small buyers, so it takes on insurance on these receivables, bought from an insurance company. The result of transferring the ownership of the accounts receivable and insuring them is an IPR deal.
1.4 Research question

It has been shown in the literature that SCF can create value for different parties. In an IPR deal there are three parties that bring something to the table: the supplier brings in the receivables, the insurer brings in risk capacity, and the bank brings in the capital. The question is if and how value can be created when these three parties participate in an IPR deal. Intuitively, banks should have cheaper access to capital than insurers, but insurers should have more information on the risk of companies, which would make them better able to take on the credit risk of receivables.

Especially the position of the supplier is interesting. What can be a reason for this supplier to sell its pool of receivables instead of taking on a regular bank loan? And how does this type of deal compare to the supplier taking on credit insurance himself? Further, Tanrisever et al. (2012) show that reversed factoring can have operational benefits in addition to the financial benefits. Thanks to improved financing the stocking levels of the supplier might increase, which would improve the service level to the customer and the efficiency of the supply chain. An interesting question is whether an IPR deal also has an operational impact, although in such a transaction the buyers do not play an (active) role. Therefore the research question is:

What benefits does an Insured Pooled Receivables deal have for the involved parties and how does it affect the operations in the supply chain?

The benefits of an IPR deal will be analyzed by creating a model that is able to determine the risk adjusted present value of the cash flows for all different parties. If at least one party can increase its present value and no party is worse off by participating, there is value creation potential.

1.5 Value creation potential

This thesis shows that if all parties are risk neutral and liquidity is not valued explicitly, an IPR deal has no value creation potential. This changes when investors are risk averse. In that case, investors are willing to pay a premium to exchange an uncertain cash flow for a certain one. This certainty premium differs across parties depending on individual parameters such as the risk aversion coefficient or initial wealth. If the insurer has a lower certainty premium than the supplier, it can be beneficial to transfer the risk of the accounts receivable from the supplier to the insurer.

This thesis also establishes when the value creation potential is highest. Risk premiums are higher when the pool of receivables is riskier, so more risk decreases the value of the receivables. The value creation potential of IPR however increases with risk, because higher risk also increases the difference between certainty premiums of different parties. Value can be created by transferring risk from a party with a relatively high certainty premium (the supplier)
to a party with a relatively low certainty premium (the insurer). Therefore the value creation potential increases in the supplier’s risk aversion, in the default probability and in correlation within the pool. The value creation potential decreases in the supplier’s initial capital and in the size of the pool.

The bank can also play a valuable role in an IPR deal, by relieving financial constraints of the supplier, mitigating the supplier’s risk during the waiting period of the insurance, and improving the buyer’s payment behavior. Besides financial benefits, an IPR deal can offer operational benefits to the supplier or to the supply chain as a whole. It can increase the service level of the supplier, but also improve the reliability of the supply chain. Further buyers’ deadweight cost of capital might be reduced and value creation potential might be optimized by aligning this financing decision with the operational aspects of pricing.

1.6 Approach

The body of this thesis commences by placing IPR in its scientific and practical context. The first part of chapter two focuses on the scientific research that has been done to topics related to IPR, such as SCF, trade credit and credit risk. The second part of chapter two highlights developments in trade and in the financial sector’s regulatory environment.

The third chapter deals with risk and the different methods to calculate a risky cash flow’s certainty equivalent. It also shows initial results for a simplified transaction of one receivable between the supplier and the insurer. Chapter 4 extends this framework to a model with a pool of receivables and considers the role of the bank. It shows the value creation potential of IPR in different circumstances. Subsequently the impact of an IPR deal on operations is considered in the fifth chapter. Chapter 6 addresses implementation and the final chapter shows conclusions, recommendations and limitations.
2  SCIENTIFIC AND PRACTICAL CONTEXT

The first section of this chapter reviews the literature to put IPR in its scientific context. In sections 2 and 3 the developments in trade and in the regulatory environment are described.

2.1  Literature review

This thesis about the value of IPR can be placed in the context of the growing body of research to Supply Chain Finance (SCF). After the large developments in Supply Chain Management for managing physical and informational flows in the supply chain, recently also the integrated approach of financial flows has received more attention. This chapter summarizes the research that has been done so far on trade credit, SCF, credit risk and pooling, products similar to IPR, and the operations finance interface.

2.1.1  Trade credit
Trade credit is of course a key aspect of IPR, since the supplier would not have accounts receivable to sell if it would not offer trade credit. An important reason for granting a payment period to one’s customers is the competitive reason: the stronger the competition in the product market, the larger the share of goods sold on credit (Fabbri and Klapper, 2009). Another argument is that in some situations a supplier has an advantage over financial institutions in financing its customers. This is because suppliers can anticipate capturing future business by financing, because they can gather information about buyers’ creditworthiness easier, and because the supplier often has a higher salvage value from the existing assets in case a buyer defaults (Peterson and Rajan, 1997).

Trade credit is a core aspect of global trading, since it is estimated that 80 to 90 percent of trade transactions include an extended payment period. Although trade credit is often an inter-firm transaction, a bank can still play an important role by offering liquidity or risk mitigation. Typical trade finance products are bank guarantees and letters of credit (providing risk mitigation) and factoring (providing liquidity). Due to the financial crisis the availability of trade finance becomes scarcer, which might hinder economic recovery. Therefore governments try to promote global trade by supporting trade finance (Chauffour and Farole, 2009).

2.1.2  Supply Chain Finance
SCF is a relatively new area within trade finance which tries to coordinate the participants’ financing functions in order to create value. It is the optimization of financing of two or more companies in a supply chain by collaboration, often with help of a financial institution (Hofmann, 2005). One of the key methods of creating value is reducing the working capital inefficiencies. The conflict of interests between buyers that want to pay later and suppliers that want to collect earlier is often won by the strongest company that imposes its terms on the weaker partner. From a supply chain perspective this is suboptimal, since the company with the
highest financing costs now has a higher working capital. SCF can be a method to reduce these supply chain inefficiencies (Banomyong, 2005).

Another way for SCF to create value is reducing the deadweight cost of capital. Deadweight cost of capital is a premium on the charge of external finance that is caused by market imperfections, such as informational asymmetries, financial distress costs or monitoring costs (Froot et al., 1993). Larger companies often have a considerably lower deadweight cost of capital. When a smaller company supplies to a larger one and there is a SCF deal, the bank knows that the large buyer will make a payment to the smaller supplier. This can reduce a large part of the informational asymmetry and therefore decrease the deadweight cost of capital.

2.1.3 Credit risk and pooling
The main risk of the accounts receivable (that are a key part of an IPR deal) is credit risk: the risk that a borrower will fail to make the payments it is obligated to. Credit risk events do not occur too often, but when they occur, their impact is large: an investor might lose a substantial part or all of his investment. Bielecki and Rutkowski (2001) describe the two main quantitative models on the credit risk of counterparties: structural and reduced form models. A structural model tries to describe credit risk that is specific to a particular obligor: when the value of this obligor crosses a certain threshold, a credit event (e.g. default) occurs. In reduced-form models, the credit events are exogenously specified in terms of a jump process, without considering the value of a firm’s assets.

In the specific context of IPR, a bank finances not just one, but a pool of receivables. So it is important for the bank to model the possible losses on this pool of receivables. One of the key features that such a model has to deal with is correlation in the default of different buyers. You could try to capture all the correlations between different buyers in a correlation matrix. However, when the number of firms grows, this correlation matrix becomes enormous, requiring a lot of data and effort. A solution for this problem is the use of common factors. If you can find one or more common factors that explain why different firms are correlated, you only need to know how the firms are correlated with these factors. In this thesis, I will assume a homogeneous portfolio, where the correlation between each set of obligors is equal.

2.1.4 Similar products
Although no specific IPR related research has been done, research on other SCF products or products with similarities to IPR can already provide some insights. Factoring (not to be confused with finding common factors) is probably the financial service that is most closely related to IPR. Factoring accounts receivable is an economic decision whereby a specialized firm assumes the responsibility for the administration and control of a company’s debtor portfolio (Soufani, 2002). This product is a way for companies to raise short-term capital, to reduce or eliminate credit risk (depending on the level of recourse), and to outsource the process of collecting of accounts receivables. If a factoring deal includes recourse, the company will get its
money upfront, but is still responsible for the credit risk. It is also possible to have partial recourse.

Instead of a seller-centric factoring deal, it is also possible to make a deal from the buyer’s perspective: reverse factoring. Now, the buyer is the relatively larger company, with smaller suppliers. In such a deal the supplier can reduce its deadweight cost of capital by providing collateral of its customer, who has a higher credit rating. The reason for the buyer to initiate such a deal can be improving the relationship with the supplier (especially if it is a key supplier), or benefitting from a discount or extended payment period. For recent research on reverse factoring, see Tanrisever et al. (2012) and Van Laere (2012).

Collateralized Debt Obligations (CDOs) are not trade finance products, but do share the pooling property with IPR. CDOs are famous for their role in the current financial crisis, especially CDOs based on mortgages. They consist of a pool of mortgages that are packed together and later on sold in different tranches with different seniority. Although the receivables in an IPR deal are not divided in tranches or, there is also a pool of future cash flows with obligors that could possibly default. See Duffie and Gârleanu (2001) for a model on the risk and valuation of CDOs.

2.1.5 Operations finance interface
IPR is clearly a financial product, but it might also have an impact on the operations in the supply chain. Traditionally, many researchers have assumed that financial and operational decisions can be separated. A growing body of research criticizes this assumption and argues that there is a meaningful interaction between operations and finance.

Dotan and Ravid (1985) for example argue that debt-related and investment-related tax shields are substitutes for each other. So if an investment leads to relatively high depreciation costs, a debt-related tax shield is generally less efficient because the probability on an accounting loss is increased. Another example is asset-based financing, where companies use a certain percentage of their inventory or of their accounts receivable as collateral for their loan. This is especially interesting for small, but fast growing companies, who have trouble getting standard financing because of information asymmetries (Buzacott and Zhang, 2004).

Further, a financial perspective on the inventory holding costs can improve operational decision making. The inventory holding cost consists of two components: the material component (storage, obsolescence, insurance, etc.) and the financial component, i.e. the cost of capital dedicated to inventory. According to Serrano et al. (2010), it is not sufficient to use the WACC for the financial component, but the newsvendor fractile needs to be modified to make optimal decisions. Finally, Gupta and Wang (2009) argue that trade credit reduces the financial component of the holding costs for the buyer, so it increases service level down stream the supply chain.
2.2 Trends in trade

Because invoice-based financing of course has a strong intrinsic connection with trade, the trends in global trade are important to consider. Over the last years, the growth of international trade was generally higher than the GDP growth (figure 2.1). During the economic crisis however, trade also turned out to be quite vulnerable: it declined by 10% globally in 2009, compared to a GDP decline of only 0.6% (IMF, 2012). After a rebound in 2010, trade growth declined again as it suffered from the Eurozone crisis, very weak growth in the US and slower than expected growth in Asia. In 2013 the expansion of trade is expected to rise to 4.5%, provided that a breakup of the euro is averted and an agreement is reached to stabilize public finances in the US. In the longer term, trade is expected to grow by 6% until 2030 and 5% thereafter (Buiter and Rahbari, 2011). This means that according to these expectations the world trade volume will have quadrupled by 2040.

Currently Europe and North America are still the most dominant players in the world trade market, with 50% of total world exports from these continents (WTO, 2012). The third largest region is Asia, which is catching up quickly due to strong growth in trade with the US and Europe and growing maturity of intra-Asian trade. In the future, the share of developing economies will become larger and larger. Trade between developing countries has already increased from 13% in 2001 to 24% in 2011 and this number is expected to increase only further (UNCTAD, 2012).

While the US’ and Europe’s percentage of global GDP is declining, China and India are growing quickly. Their combined share in world GDP is expected to grow from 24% in 2010, via 39% in 2030, to 46% in 2060, making them world’s leading two economies (Johansson et al., 2012). Fast-ageing economic heavyweights like the US, Japan and the Eurozone will lose ground to emerging economies with a younger population (e.g. China, India, Brazil, and Indonesia).

SCF has already grown quite quickly over the last years and now accounts for 18% of the trade finance revenue (Pierron and Rajan, 2011). However, most of these revenues are made in developed economies like the US and Europe, while demand is especially high in developing economies. There are two main reasons for this: growing South-South trade and a limited
availability of funding for smaller companies in this area. When smaller companies have difficulty in finding financing, SCF programs can be of great value, since these can take the deadweight costs of capital away. Developing countries can therefore be an important target market for banks in the coming years. Still banks do need to account for the difficulties of facilitating trade in these countries due to the underdeveloped infrastructure.

2.3 Regulatory trends

The recent global financial crisis has had a big impact on the global economy and specifically on the banking industry. What started as the burst of the U.S. housing bubble quickly escalated into large downturns in stock markets, the bailout of banks by national governments and the threat of collapsing financial institutions. Due to the underestimation of the risk of complex (often collateralized) financial products and increased leveraging by financial institutions, bank solvency was (too) low and credit availability declined. The financial crisis spilled over to the real economy, resulting in a global recession and sharply rising unemployment rates.

One of the ways in which regulators hope to prevent such crises in the future is the installment of a new Basel Accord: Basel III. The Basel Accords have been composed by the Basel Committee on Banking Supervision since 1988 to improve the quality of banking supervision worldwide. These Accords are in principle only recommendations for local governments, but in practice they are enforced quite rigorously and therefore have a large impact. The goal of Basel III is to improve the banking sector’s ability to absorb shocks arising from financial and economic stress, to improve risk management and strengthen transparency (Basel Committee, 2011).

Basel III tries to accomplish this by imposing a minimum leverage and liquidity coverage ratio and by increasing the capital requirements. Capital requirements are formulated in percentages of Risk-Weighted Assets (RWA): assets or off-balance sheet exposures weighted according to risk (Jacques and Nigro, 1997). The weighting is done using a Credit Conversion Factor. This factor is a low percentage for low-risk products such as claims on governments or letters of credit, but is much higher for risky products, e.g. long term loans to companies.

Because of the financial crisis and the increasing reserve capital that is demanded from banks by Basel III, banks are more reluctant in offering traditional bank credit, which has a high Credit Conversion Factor. This suppressed liquidity leads to companies using their own cash reserves for financing (Demica, 2012). At some time however these reserves will be depleted and alternative lines of finance are required. One of the alternative approaches is finance based on trade receivables. Banks’ risk departments are keener to get loans collateralized and, being good risk mitigators, receivables can serve this purpose. Because of their lower risk, these products require lower RWA.

The regulatory environment makes invoice-based financing not only relatively more attractive for banks; it can also be a solution for companies. After the global credit crunch, recovery has
been uneven for different sized companies. Many large companies can raise money quite easily again in the capital markets, but small and medium-sized enterprises still have difficulties in raising external financing (The Economist, 2010). If suppliers have an IPR deal in place, they have the flexibility to offer their smaller customers an extended payment term (which serves as financing). Since the cost for this financing is lower to the large supplier than to the small buyer, both parties can benefit from such a transaction.

It is interesting to find out whether financing based on receivables has indeed grown in popularity since the beginning of the financial crisis. Demica, a company providing working capital solutions, has tried to determine the size of invoice-based financing and its trends by data collection and interviewing a selection of Europe’s leading banks. They consider three components within the invoice finance market: factoring, reversed factoring and Trade Receivables Securitization (TRS). TRS is closely related to IPR, but does not involve insurance.

Despite European bank lending declining overall, the invoice finance market has shown double digit growth since 2009. Its volume was over €1 trillion in 2011, 8% of the European GDP (Demica, 2012). An important reason for this growth is the shift from unsecured to secured funding, due to regulatory changes such as Basel II and Basel III. TRS is interesting for banks, since it is a safe and efficient way of providing liquidity that can reduce a company’s cost of funding. Since a bank is exposed to the underlying portfolio of receivables and not to the company itself, it can provide more funding to clients without increasing the risk on them.

Although trade financing becomes relatively more attractive in the new economic and regulatory environment, the increased capital requirements still force banks to significantly reduce their balance sheet. This deleveraging is likely to force some institutions out of the trade finance business (Trade Finance Magazine, 2012). Especially the banks in trade finance that are tight on capital and have a low market share will reconsider their activity in this area. Decreasing supply of trade finance combined with increasing demand might push the price of trade finance upwards in the future.

To conclude: both the scientific and practical context provide sufficient reasons to investigate the value creation potential of IPR. Research on other SCF products has been promising and the demand for SCF products is expected to rise due to regulatory and trade-related trends. To find out whether IPR can indeed be valuable, I will start developing a model that shows IPR’s value in the next chapter.
3 RISK

This chapter introduces a high level model for Insured Pooled Receivables (IPR) in the first section. Subsequently two simplified models, that assume risk neutral investors, are analyzed. Sections 3 to 5 deal with the introduction of risk aversion and its effects on valuation. In section 3.6 the value creation potential of a simplified IPR is shown and section 3.7 addresses conclusions and limitations.

3.1 High-level model

In short, IPR works as follows. First, a supplier sells its product or service to a variety of buyers, who are allowed to pay at a later stage. This creates accounts receivable on the balance of the supplier. The supplier wants to increase its liquidity or reduce the credit risk, so it sells the pool of receivables. The bank finances these receivables, but generally has no appetite to take on the risk of all these small buyers. Therefore it takes on insurance on these receivables, bought from an insurance company. The bank receives a fee from the supplier and has to pay a premium to the insurer. The bank’s net revenue is the difference between the two.

I will establish in which circumstances such a transaction can be valuable. I first present a high-level model for IPR. This model shows the parties that are involved in the deal and their interactions: solid arrows represent cash flows, dotted lines represent the transfer of goods or services. In a later stage, some of the important detailed aspects of the model are introduced, such as discounting the cash flows and accounting for risk.

![High-level model IPR](image)

Figure 3.1: High-level model IPR. Solid arrows represent cash flows, dotted arrows the transfer of goods or services. Numbers indicate the ordering of events in time.

In this high level model, I use the following variables and parameters:
\begin{align*}
N & \quad \text{number of buyers} \\
\delta_i & \in \{0,1\} \quad \text{default indicator, equals 1 in case of default} \\
D & \quad \text{number of defaults} \\
V & \quad \text{nominal value account receivable} \\
\pi_B & \quad \text{pricing bank} \\
\pi_I & \quad \text{premium insurer} \\
\theta & \in [0,1] \quad \text{recourse to the supplier} \\
l_i & \quad \text{nominal payment period} \\
\Omega & \quad \text{waiting period insurer} \\
\end{align*}

For an IPR deal, there are four relevant moments in time:

1. At time \( t = 0 \), the supplier delivers the product or service to the buyers. At this moment the accounts receivable for the value of \( NV \) are created on the balance sheet of the supplier. An initial assumption is that the sales to the different buyers are done at the same moment in time, with equal payment periods.

2. At time \( t = T (T < l_i) \), the supplier sells its receivables to the bank. The bank finances the supplier for the total amount, minus a premium: \( (1 - \pi_B)NV \). I assume that the bank immediately takes on insurance, to prevent being exposed to credit risk related to the buyers. The bank and the supplier should agree upon whether there is recourse in the contract. Recourse on the supplier means that the supplier still has to bear the credit risk of its buyers’ accounts receivable, while without recourse the bank takes over this credit risk. It is also possible that there is a partial recourse, where the supplier bears only part of the credit risk (typically \( \theta = 10\% \)). The bank does not need to insure this recourse, so the insured amount equals \( (1 - \theta)NV \). The insurer’s premium is therefore \( \pi_I (1 - \theta)NV \).

3. At time \( t = l_i \), the payment of the accounts receivables is due. Some of the buyers might not be able to fulfill their obligations. They will default on their payment, which makes the \( \delta_i \) related to these buyers equal to 1. The total number of defaults is \( D = \sum_{i=1}^{N} \delta_i \).

The buyers that have not defaulted will pay their receivables to the bank. For the buyers that have defaulted, the supplier needs to pay the recourse. So the net cash flow for the bank is \( (N - D)V + (D\theta)V = NV - (1 - \theta)DV \).

4. At time \( t = l_i + \Omega \), the payment of the insurer to the bank is due. This payment equals \( (1 - \theta)DV \).
To find out whether value can be created, these cash flows have to be compared to the situation where no receivables are sold, which is quite a bit easier. Again, the accounts receivable are created at time \( t = 0 \), but now the supplier only receives its payment at time \( s \). This payment equals \( (N - D)V \). The bank and the insurer play no role.

### 3.2 Simplified models

Models are always simplifications of reality. Generally there is a trade-off between insightfulness and realism: the further you simplify a model, the easier it is to draw conclusions from it, but the more unrealistic it becomes. To keep the IPR model tractable, the analysis starts with a highly simplified model. In such a model, it is not expected that value can be created by the use of IPR. In the rest of this chapter and in the following one, the model will be extended with more realistic elements gradually. In this way it can be made insightful what elements (if any) in the model allow for value creation.

#### 3.2.1 Base model

In the simplest model, there is a supplier who sells it goods to only one buyer, creating an account receivable with the normalized value \( V = 1 \). There is no recourse and the timing of the payments is considered irrelevant (i.e. \( l = 0 \) and \( \Omega = 0 \)). Participants are risk neutral and their goal is to maximize their expected payoff. The only unknown is whether the buyer will default or not. The probability of default equals \( P(\delta_i = 1) = p_i \), where \( \delta_i \) is a dummy variable that equals 1 when buyer \( i \) defaults.

The buyer is in principle indifferent whether an IPR deal is done or not: whether he pays to the supplier or to the bank makes no difference for him. The only relevant cash flows are those of the supplier \( CF_s \), the bank \( CF_b \), and the insurer \( CF_i \). The subscript ‘ND’ will indicate a cash flow without an IPR deal, whereas the subscript ‘D’ indicates that there is an IPR deal. I assume for now that the insurer and the bank ask a fair premium, such that their expected value of the deal equals 0. Value is created for the supplier when \( E[CF_{S,D}] > E[CF_{S,ND}] \). I do not consider yet how this value should be distributed among the different parties.

Without selling its receivables, the expected cash flow to the supplier is:

\[
E[CF_{S,ND}] = E[(1 - \delta_i)V] = 1 - p_i.
\]

When there is an IPR deal without recourse, the insurer bears all the risk. Since all parties are risk neutral, the fair premium equals the expected payment: \( \pi_i = E[\delta_i V] = p_i \). This makes the expected cash flow to the insurer equal to:

\[
E[CF_{S,D}] = E[\delta_i V] = p_i.
\]
\[ E[CF_i] = E[\pi_i - \delta_i V] = p_i - p_i = 0. \] (3.2)

In this situation, the bank does not bear any risk or offers any discounting (since the timing is irrelevant), so it cannot ask an extra premium for its services. So the bank is not a relevant party now: it pays the premium to the insurer and charges the same premium to the supplier \((\pi_B = \pi_I = p_i)\). Further it receives payment \(V\) from the buyer (or from the insurer in case of default), but pays the same amount to the supplier. Therefore the expected cash flow equals zero:

\[ E[CF_B] = E[\delta_i V + (1-\delta_i) V -(1-\pi_B) V - \pi_I V = V -(1-p_i) V - p_i V = 0. \] (3.3)

Finally, the supplier pays the fee \(\pi_B = p_i\) to the bank and receives a certain payment of \(V\):

\[ E[CF_{S,D}] = V - \pi_B = 1 - p_i \] (3.4)

So the expected total cash flow of the bank and the insurer equals zero, while the expected cash flow of the supplier is equal to the situation without an IPR deal. This means that in this highly simplified model, the product cannot create any value for the supply chain.

### 3.2.2 Timing of cash flows

The first improvement to make the model more realistic is adding the timing of the cash flows. For now, the assumption still remains that all parties are risk neutral, but the time value of money is now acknowledged. Therefore the cash flows are discounted at the risk free rate.

If the supplier does not engage in the program, it will receive payment at time \(l_s\). The payment to the supplier is stochastic, since the buyer pays either \(V\) or 0. However, because all parties are risk neutral, the cash flow is discounted at the risk free rate, by multiplying formula (3.1) with the discount factor \(e^{-r_i l_s}\):

\[ PV(CF_{S,ND}) = \exp(-r_i l_s)(1 - p_i). \] (3.5)

The insurer can make its payment at time \(l_s + \Omega\), which decreases the present value of the expected cash outflow (it is now lower than \(p_i\), provided that \(l_s + \Omega > 0\)). The insurer has to charge a fair premium which makes its expected present value zero, so

\[ \pi_I = \exp(-r_I(l_s + \Omega)) p_i. \] (3.6)
Determining the premium that the bank should charge is the most involved calculation. It should pay the premium \( \pi_I \) to the insurer at time 0, but it also might receive a payment from the insurer at time \( l_s + \Omega \). The expected value of this payment is \( p_I \), so the cash outflow (\( \pi_I \)) is exactly offset by the present value of the expected cash inflow (\( \exp(-r_f l_s + \Omega) p_I \)).

To make the present value of the bank 0 (so that it asks a fair premium), the other two payments should also offset each other. The cash outflow to the supplier at time 0 equals \( 1 - \pi_B \), while the cash inflow from the buyer at time \( l_s \) equals \( 1 - \xi_I \). Therefore:

\[
1 - \pi_B = E[1 - \delta_s] \exp(-r_f l_s) \\
1 - \pi_B = (1 - p_I) \exp(-r_f l_s) \\
\pi_B = 1 - \exp(-r_f l_s) (1 - p_I). \tag{3.7}
\]

Now, the supplier receives \( 1 - \pi_B \) at time 0:

\[
PV(\pi_{S, B}) = (1 - \pi_B) = \exp(-r_f l_s)(1 - p_I). \tag{3.8}
\]

Again, the present value for the supplier is equal to the situation where it does not engage in the IPR deal. So also according to this simplified model with timed cash flows, IPR is not able to create value.

### 3.3 Risk aversion

One of the reasons that in the previous models the product cannot create value, is because all parties are risk neutral. In general, IPR can provide two kinds of main services: increasing liquidity for the supplier and mitigating its risks. In this simplified model that does not take financial constraints into account, providing liquidity on itself does not allow for value creation. The transfer of risks is in this situation also useless, since none of the parties cares about the risk. When risk aversion is introduced however, risk mitigation can be a valuable service and value might be created by using IPR.

To show this, I will first elaborate more on risk aversion. If two cash flows have the same expected payoff, a risk averse investor prefers a certain cash flow over a volatile one. This means that the investor not only considers the first moment of the payoff distribution (its expectation), but also its variability. Risk aversion is a well documented and accepted trait for investors. Investors in stock are compensated for the (systematic) risk they take, people pay a premium when taking on insurance to avoid risk, and many studies have shown that people on average prefer a sure gain over a random gamble, even if the latter has a (slightly) higher expected payoff (see for example Kahneman & Tversky, 1979).
3.3.1 Accounting for risk

Since people and investors in general are very often risk averse, it is not realistic to model IPR under the assumptions that all parties are risk neutral and only want to maximize the expected present value. For a risk averse party, a certain payment is worth more than an uncertain one with the same expectation. In the setting I introduced, the supplier has an uncertain cash flow (since buyer default is a stochastic variable), which can be made certain when he participates in the IPR deal. If the supplier is risk averse, mitigating the risk that a buyer defaults might have value. Therefore this risk has to be incorporated in the evaluation of the different alternatives.

In the rest of this section, I will use the standard work of Cochrane (2001) on asset pricing extensively. Asset pricing theory is used to account for the delay and the risk of an asset’s cash flows, where the correction for risk is often the most involved and important part. There are a few ways to account for this risk; examples are adjusting the discount factor, using risk neutral valuation or using the expected utility theory. According to asset pricing theory however, all these different methods are in essence the same.

The theory stems from one simple concept: price equals expected discounted payoff. Elaborating on this principle is possible in two main directions, namely absolute pricing and relative pricing. In absolute pricing (also called equilibrium pricing) each asset is priced by reference to its exposure to fundamental sources of macroeconomic risk. In relative pricing (or arbitrage pricing) the value of an asset is based on the price of other assets in the market. It uses the assumption that arbitrage opportunities should be absent in the market (except for the very short term). There are very few problems that are solved by these two pure extremes; it is usually a matter of finding a good combination.

The basics of asset pricing are summarized by the following equation:

\[
\pi_t = E(m_{t+1}x_{t+1})
\]

where \(m_{t+1} = f(\text{data, parameters})\)

\(\pi_t = \text{asset price, } m_{t+1} = \text{stochastic discount factor, } x_{t+1} = \text{asset payoff.}\)

The most difficult part of this formula is to determine the stochastic discount factor. One approach is to use utility functions. People generally do not value money linearly: $100 is usually worth more to someone who has an initial wealth of $10 than to someone with initial wealth of $1,000. Utility functions are a method to measure the usefulness of money. Since a simple monetary scale is often not appropriate, the utility function \(u(x)\) is used to assign a utility value to $x, which is typically the wealth of a decision maker.

Now, an investor considers how much to save and to consume, and what portfolio of assets to hold. The price of an asset should equal the expected discounted payoff, otherwise the investor will buy more or less of the asset until equilibrium is reached. This means that the marginal
utility loss of consuming a little less today and investing the result should equal the marginal utility gain of selling the investment at some point in the future and eating the proceeds.

The investor wants to maximize the utility of his consumption now \( c_t \) plus his consumption in the future \( c_{t+1} \). Investor’s impatience (or the time value of money) is reflected in the subjective discount factor \( \beta \). The original consumption level is denoted by \( e_t \), the price of the asset by \( \pi_t \), the payoff by \( x_{t+1} \), and the number of units of the asset to buy by \( \xi \). Therefore the current consumption is reduced by \( \pi_t \xi \), but the future consumption increased by \( x_{t+1} \xi \). If short selling is allowed, \( \xi \) can also take negative values. The investor has to solve the following problem by choosing the number of units of the asset to buy:

\[
\max_{\xi_t} u(c_t) + E \left[ \beta u(c_{t+1}) \right] \quad \text{s.t.}
\]

\[
c_t = e_t - \pi_t \xi_t
\]

\[
c_{t+1} = e_{t+1} + x_{t+1} \xi_t
\]

Setting the derivative with respect to \( \xi \) equal to 0 gives the standard pricing formula:

\[
\pi_t = E_t \left[ \beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1} \right].
\] (3.10)

The purpose of \( \beta \) in this formula is to account for the investor’s time preference. When \( \beta \) equals 1, the investor is indifferent between getting a cash flow now or in the next period. Usually, because of the time value of money, \( \beta \) is smaller than 1. By combining (3.9) and (3.10), one can conclude that for the stochastic discount factor holds that:

\[
m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}.
\] (3.11)

The general formula for asset pricing can now be adjusted for different situations. It is for example possible to use the definition of covariance to rewrite the general asset pricing formula to \( \pi = E(mx) = E(m)E(x) + \text{cov}(m, x) \). Further, since by definition \( E(mR^f) = 1 \), it should also hold that \( E(m) = 1 / R^f \). This means that the asset price can be written as:

\[
\pi = \frac{E(x)}{R^f} + \text{cov}(m, x).
\] (3.12)

The first part of this formula is the asset’s price in a risk neutral world (if utility would be linear), where the second part is a risk adjustment. The formula also implies that if the payoff of an
asset is uncorrelated with the discount factor and thus only has idiosyncratic risk, it can be discounted at the risk free rate.

If now a cash flow has no uncertainty, there is of course also no covariance with the stochastic discount factor. Therefore the cash flow can be discounted at the risk free rate $R_f$:

$$\pi_t = \frac{1}{R_f} x_{t+1}.$$  \hfill (3.13)

For a risky asset, there are multiple ways to go from the general asset pricing formula to a more specific method to calculate the price. If it is possible to calculate the covariance between $m$ and $x$, formula (3.12) can be used. Another possibility is to find an appropriate discount factor $1 / R^i$ for every cash flow the asset will produce:

$$\pi_t^i = \frac{1}{R^i} E_i[x_{t+1}^i].$$  \hfill (3.14)

The next two sections discuss two popular specifications of the general asset pricing formula: risk-neutral valuation and expected utility theory.

### 3.4 Risk-neutral valuation

Instead of using real probabilities $p$ to calculate the expected pay off and subsequently adjusting for the risk by using a covariance factor, it is also possible to use risk-neutral probabilities $p^*$. Suppose that one of $S$ possible states of nature can occur tomorrow. Individual states are denoted by $s$ and the probability of them occurring is $p(s)$. Each of these states leads to a certain marginal utility for the investor; the higher this is, the bigger the stochastic discount factor $m(s)$. Risk-neutral probabilities give greater weight to states with higher than average marginal utility:

$$p^*(s) = \frac{m(s)}{E(m)} p(s).$$  \hfill (3.15)

An interpretation of risk aversion is that it is equivalent to paying more attention to unpleasant states (which have a bigger impact on utility), compared to the actual probability of occurring. Denoting expectations that use risk-neutral probabilities by $E^*$, the price of an asset is:

$$\pi = \frac{E^*(x)}{R^i}.$$  \hfill (3.16)
In this valuation method all cash flows are discounted at the risk free rate, but risk is still accounted for via the risk neutral probabilities. It assumes a finite number of possible future states exists. Further there are securities traded in the market, of which is known what their price is and what their payoff is in every possible future state. In a complete market, enough securities are traded to calculate the state price of every state.

A state price is defined as the price at time zero of a claim that pays one dollar (or one unit of consumption) in one certain state and nothing in all other states. The risk neutral valuation method depends on the calculation of these state prices and the absence of arbitrage. An arbitrage opportunity arises when two securities are priced differently, while they are constructed in such a way that they have the same payoff. If such an opportunity arises, it is possible to make a sure gain. Many financial theories assume that if such an opportunity exists, it is immediately taken advantage of and disappears.

Risk neutral probabilities are equal to the state prices, multiplied by $R^t$. The risk neutral probabilities have the property that their sum always adds up to 1. It is important to notice that, although they are related, risk neutral probabilities are not equal to the real world probabilities that a certain state will occur. A difference between them is for example that risk-premiums implied by market prices are incorporated in the risk-neutral probabilities.

In practice, it is very difficult to find state prices for the large (possibly infinite) number of future states. Usually the state prices are obtained via the risk neutral probabilities, which can be obtained via probability theory in a different route. This somewhat complex route involves a continuous time framework, where the asset price is modeled as a Brownian motion. By rearranging the formula for the expected price, it is possible to find an arbitrage-free price with only an estimation of the volatility based on securities that depend on the same volatility parameter (Gisiger, 2010).

For IPR, the challenge would be to find suitable securities. In a simple model with a limited number of possible future states, one needs to find securities that can replicate the cash flows of IPR. The challenge is then to define these future states as realistically as possible. For a model with a stochastic process, one has to check whether the Brownian motion is a realistic assumption and find traded securities with similar underlying volatility.

### 3.5 Utility functions

Besides risk-neutral probabilities, another method to account for risk is expected utility theory. In this section, I will address the theoretical background of utility functions, their applicability to parties in IPR and two examples of utility functions.
3.5.1 Expected utility theory

As introduced before, utility functions are a method to measure the usefulness of money. For a risk averse investor, the two basic properties of a utility function $u(x)$ are that it is an increasing and concave function of $x$. So more money is better, but the more you have already, the lower the marginal value of an extra dollar.

Since a risk averse investor’s utility function is always concave, it is possible to apply Jensen’s inequality:

$$u(E[x]) \geq E[u(x)].$$  (3.17)

This means that utility functions intrinsically account for investors’ risk aversion: a sure cash flow is better than a random one. When this inequality is strict, investors are willing to pay a certainty premium to take away uncertainty: the certainty equivalent of an uncertain opportunity is lower than its expected value.

The certainty equivalent is defined as the guaranteed amount that an investor is willing to accept in exchange for giving up a stochastic cash flow. Put differently: it is the payoff amount we would accept in lieu of undergoing the uncertain situation (Lapin and Whisler, 2002). For example, it can be the amount someone is willing to pay to participate in a gamble or the premium he is willing to pay to insure himself against some risk.

If one uses expected value as a decision criterion, a business opportunity that pays $50 with probability 0.5 or $100 w.p. 0.5 has a value of $75. The utility of such an opportunity however depends on the risk aversion. When the risk aversion would be zero (and utility is linear), the value of this opportunity is still $75. However, for a positive risk aversion the certainty equivalent drops below $75. The larger the risk aversion, the closer the certainty equivalent gets to the lower bound (figure 3.2). Because of the concavity of $u(x)$, risk averse investors assign a greater negative value to the downside potential than a positive value to the upside potential.
So, a utility function has the desirable property that it punishes downside potential more than it rewards upside potential. This is typical for a risk averse investor: he does not fear the variability of a cash flow itself, but he fears the downside potential.

Different investors however can have very different risk aversion functions. Risk aversion is usually measured either in absolute terms or relative to the size of the cash flow \( x \). The absolute risk aversion function is defined as follows (Gerber and Pafumi, 1998):

\[
A(x) = -\frac{u''(x)}{u'(x)}.
\]  
(3.18)

The relative risk aversion function is very similar, but takes into account the size of the cash flow:

\[
R(x) = -\frac{xu''(x)}{u'(x)}.
\]  
(3.19)

First I will introduce the two classes of utility functions that are most common and exhibit constant risk aversion, either in absolute or in relative terms. The first class are exponential utility functions (with subscript EU):

\[
u_{EU}(x) = \frac{1}{\alpha} (1 - \exp(-\alpha x)).
\]  
(3.20)

Exponential utility functions are a single parameter function where \( \alpha \) represents the risk aversion. This parameter usually takes a value rather close to zero. The further away from zero, the more risk averse the investor and the more curvature in the utility function.

Power utility functions (with subscript PU) have one parameter for risk aversion as well, but now it is a different one (\( \gamma \)):

\[
u_{PU}(x) = \frac{x^{1-\gamma}}{1-\gamma}.
\]  
(3.21)

A main difference between the two types of functions is the characteristics of the risk aversion. Exponential utility functions exhibit Constant Absolute Risk Aversion (CARA), since

\[
A_{EU}(x) = -\frac{u''(x)}{u'(x)} = -\frac{-\alpha \exp(-\alpha x)}{\exp(-\alpha x)} = \alpha.
\]

This automatically implies that the relative risk aversion function is increasing in \( x \) for exponential utility. Investors who behave according to a power utility function on the other hand have absolute risk aversion that decreases in \( x \), but
have a Constant Relative Risk Aversion (CRRA): \[ A_{PU}(x) = \frac{-u''(x)}{u'(x)} = \frac{x^{-(\gamma+1)}}{\gamma x} = \frac{\gamma}{x}, \]

\( R_{PU}(x) = \gamma. \) When you want to describe a company’s or investor’s utility, it is important to choose an appropriate class of utility function and use the correct parameters.

### 3.5.2 Justification for the use of utility functions

Utility functions are used extensively in the economical literature to explain decisions of individuals in the presence of risk. Applying them to organizations or firms instead of individuals is slightly more unconventional. For smaller companies where the owner is also the manager, it is possible to use standard utility assessment methods, like letting the manager choose between numerous gambling examples (Pennings and Smidts, 2003). The preferences in these artificial situations are subsequently generalized to a utility function that is able to place a value on real life situations. For larger companies, owners and managers are in general not the same person and therefore can have different preferences. Therefore one should account for agency costs or assume they are absent. Still, utility functions are applied often to insurers (which are normally large companies), see for example Samson and Thomas (1983) or Tibletti (2006).

Using utility functions to value uncertain cash flows of a firm can also be justified by market imperfections. Since these functions are concave, they assign a higher marginal utility to lower values than to higher values. There are two reasons that a low realization of an uncertain cash flow can have more impact than a high one. First, there are financial distress costs. When the realization of a (sizeable) cash flow is very low, this can bring a company closer to bankruptcy. There are both direct and indirect costs associated to bankruptcy (so an additional downside disadvantage), while there are no additional upside benefits in case of a high realization. This is the first reason that a firm’s utility is non-linear and concave.

The second reason is the pecking order theory, which states that firms prefer internal financing over external financing. Due to informational asymmetries, external financing is often more expensive than internal funds. Therefore companies would like to fund new opportunities with internal funds. Not every opportunity has the same return, so normally the company prefers to first fund the opportunity with the highest return. The availability of funding is therefore most valuable for the most important project. The value of internal funding gradually decreases as only opportunities with lower returns are available. This shows again that marginal utility of cash flows is decreasing and a concave utility function is justified.

Because of this concavity, firms prefer certain cash flows over random ones and are prepared to pay a premium to avoid uncertainty. This certainty premium \( c \) can be found by solving the following equation:

\[ u(E[x] - c) = E[u(x)]. \]

(3.22)
The size of the certainty premium depends on the choice of the utility function, the risk aversion parameter and the distribution of the payoff $x$. In the next two subsections I will provide an example for both of the most used utility functions: the exponential and the power utility function. To discern between the two certainty equivalents, I will use the subscripts EU and PU again.

3.5.3 Exponential utility function

To find the certainty premium for exponential utility, I combine (3.20) and (3.22):

$$u(E[x] - c_{EU}) = E[u(x)]$$

$$\frac{1}{\alpha} (1 - \exp(-\alpha(E[x] - c_{EU}))) = E[u(x)]$$

$$\exp(-\alpha(E[x] - c_{EU})) = 1 - \alpha E[u(x)]$$

$$-\alpha(E[x] - c_{EU}) = \ln(1 - \alpha E[u(x)])$$

$$c_{EU} = E[x] + \frac{\ln(1 - \alpha E[u(x)])}{\alpha}. \quad (3.23)$$

In a simple example, the cash flow $x$ is either $50 or $0 (when the obligor defaults), with a default probability of 5 percent. This gives $E[x] = 47.5$. By assuming a risk aversion coefficient $\alpha = 0.02$, it is possible to calculate the expected utility and therefore also the certainty premium. The certainty premium equals $1.62 in this case, which is 3.2% of the nominal value. So in this simplified case, this would mean that a supplier is willing to pay a premium of 3.2% on the nominal value of the accounts receivable, to receive the expected value of the receivables without uncertainty. From now on I will refer to this percentage as the pricing $\kappa$. The pricing is quite sensitive to the parameters, as can be seen in table 3.1. In this table, it is shown what happens to $\kappa_{EU}$ when one of the parameters is halved or doubled (ceteris paribus).

Table 3.1: Sensitivity of pricing to parameters with an exponential utility function (CARA)

<table>
<thead>
<tr>
<th></th>
<th>Base case exp. utility</th>
<th>$\alpha = 0.01$</th>
<th>$\alpha = 0.04$</th>
<th>$p = 0.025$</th>
<th>$p = 0.1$</th>
<th>$V = 25$</th>
<th>$V = 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_{EU}$</td>
<td>3.2%</td>
<td>1.4%</td>
<td>8.9%</td>
<td>1.7%</td>
<td>5.9%</td>
<td>1.4%</td>
<td>8.9%</td>
</tr>
</tbody>
</table>

Interesting to see is that halving or doubling the risk aversion $\alpha$ or the nominal value $V$ has exactly the same effect on the pricing. This can be explained by taking a closer look at the pricing formula. I use that $x$ can be either 0 (with probability $p$) or $V$ (w.p. $1-p$) and that $u(0) = 0$: 

\[
\kappa_{EU} = \left( E[x] + \frac{\ln(1-\alpha E[u(x)])}{\alpha} \right)/V
\]

\[
\kappa_{EU} = \left( (1-p)V + \frac{\ln\left[1-\alpha(1-p)\frac{1}{\alpha}(1-\exp(-\alpha V))\right]}{\alpha} \right)/V
\]

\[
\kappa_{EU} = (1-p) + \frac{\ln\left[1-(1-p)(1-\exp(-\alpha V))\right]}{\alpha V}
\]

\[
\kappa_{EU} = (1-p) + \frac{\ln[p \exp(-\alpha V) - p - \exp(-\alpha V)]}{\alpha V}
\]  

(3.24)

In formula (3.24) there are three instances of \( \alpha \) and \( V \), every time in conjuncture with each other. Therefore it makes no difference whether \( \alpha \) or \( V \) is multiplied with a constant; the effect on pricing is the same.

### 3.5.4 Power utility function

To find the certainty premium for power utility functions, I combine (3.21) and (3.22):

\[
u(E[x]-c_{PU}) = E[u(x)]
\]

\[
\frac{(E[x]-c_{PU})^{1-y}}{1-y} = E[u(x)]
\]

\[
(E[x]-c_{PU}) = (E[u(x)](1-y))^{\frac{1}{1-y}}
\]

\[
c_{PU} = E[x] - (E[u(x)](1-y))^{\frac{1}{1-y}}.
\]  

(3.25)

Although the utility functions are different, I use the same parameters as for the exponential utility function. The only difference is the risk aversion parameter, which is now \( \gamma \) instead of \( \alpha \). If I assume a value of 0.3 for \( \gamma \), the pricing is 2.1%. The effect of changing the parameters can be found in table 3.2.

**Table 3.2: Sensitivity of pricing to parameters with a power utility function (CRRA)**

<table>
<thead>
<tr>
<th>Base case power utility</th>
<th>( \gamma = 0.15 )</th>
<th>( \gamma = 0.6 )</th>
<th>( p = 0.025 )</th>
<th>( p = 0.1 )</th>
<th>( V = 25 )</th>
<th>( V = 100 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa_{PU} )</td>
<td>2.1%</td>
<td>0.9%</td>
<td>7.0%</td>
<td>1.1%</td>
<td>4.0%</td>
<td>2.1%</td>
</tr>
</tbody>
</table>

It is immediately noticeable that the nominal value of a payment does not affect the pricing. The explanation for this is that power utility functions exhibit CRRA: if you correct for the size of the cash flow, risk aversion is constant. This means that if the nominal value doubles, the certainty premium also doubles and therefore pricing remains constant.
3.6 Value creation potential

So far, I have established that IPR is not able to create value when investors are risk neutral. Subsequently, I have shown that for a one period model, different methods to incorporate risk in the valuation of a cash flow are equivalent. Concave utility functions have shown to be an applicable way to incorporate risk into the valuation of an IPR deal. In this section, I will determine the value creation potential of a simplified model with risk averse investors. In this simplified model, I consider a transaction of one account receivable between the supplier and the insurer. Extensions such as the effect of pooling and the role of the bank are addressed in the next chapter.

In case the supplier and the insurer have the same utility function and all parameters are equal as well, they are willing to pay the same certainty premium. Therefore, transferring risk would not be useful. However, if the supplier’s certainty premium would be higher than that of the insurer, the transfer can be valuable. In that case the supplier can pay a premium that is lower than its maximum, while the insurer can receive a larger premium than its minimum. Consequently both parties are better off.

There are several reasons why the insurer’s certainty premium would be lower. The first reason is a difference in initial capital. So far I have implicitly assumed that initial capital \( w_0 \) is zero: utility is completely determined by the realization of one cash flow. If a party already has some initial capital, it is positioned further up the utility curve, which might impact the pricing of insurance. The certainty equivalents for both types of utility functions are adjusted in the following way:

\[
c_{EU} = E[x] + w_0 + \frac{\ln(1 - aE[u(x + w_0)])}{\alpha},
\]

\[
c_{PV} = E[x] + w_0 - \left( E[u(x + w_0)](1 - y) \right)^{\frac{1}{1 - y}}.
\]

Whether this impacts the pricing depends on the risk aversion function. If an investor satisfies CARA (so an exponential utility function), the initial capital should have no influence on the pricing, since the absolute value of the uncertain cash flow does not change. For CRRA aversion however (power utility function), the initial capital affects the relative size of the stochastic cash flow. This means that the pricing should depend on the initial capital.

These hypotheses are confirmed in table 3.3. It shows the potential for value creation for different levels of initial capital of the supplier \( w_{0,S} \) and insurer \( w_{0,I} \). I define the value creation potential \( \nu \) as the difference between the supplier’s pricing and the insurer’s pricing. For the exponential utility function the value creation potential \( \nu_{EU} \) is always 0, but for the power
utility function $\nu_{PU}$ can be positive, provided that the insurer has higher initial capital. Other parameters ($\alpha, \gamma, p, V$) are the same as before.

Table 3.3: Value creation potential versus initial capital

<table>
<thead>
<tr>
<th></th>
<th>$w_{0,s} = 0$</th>
<th>$w_{0,s} = 0$</th>
<th>$w_{0,s} = 0$</th>
<th>$w_{0,s} = 0$</th>
<th>$w_{0,s} = 50$</th>
<th>$w_{0,s} = 50$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$w_{0,i} = 0$</td>
<td>$w_{0,i} = 10$</td>
<td>$w_{0,i} = 50$</td>
<td>$w_{0,i} = 100$</td>
<td>$w_{0,i} = 50$</td>
<td>$w_{0,i} = 100$</td>
</tr>
<tr>
<td>$\nu_{EU}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\nu_{PU}$</td>
<td>0</td>
<td>1.0%</td>
<td>1.6%</td>
<td>1.8%</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

It can be seen that the impact of the insurer’s initial capital is largest when the supplier has none. Even if $w_{0,i}$ is relatively low (10, compared to a nominal value $V$ of 50), the difference between the supplier’s pricing (2.1%) and the insurer’s pricing (1.1%) is already one percent. Increasing the insurer’s capital brings down its pricing closer to 0%, so the value creation potential closer to 2.1%. If the supplier himself already has initial capital of 50, its pricing decreases to 0.5%, so the value creation potential is much lower.

Besides differences in initial capital, there are more ways in which value creation potential might originate. There might be an informational asymmetry between the supplier and the insurer. Due to being specialized in assessing credit risk and developing sophisticated models, an insurer is probably better in determining a probability that the buyer will default on its account payable. Normally, due to uncertainty and being risk averse, a supplier will overestimate the probability of default (to be on the safe side). Therefore he will be willing to pay more for the insurance than that it is actually worth.

The effect of informational asymmetry is illustrated in the following example. The supplier estimates the probability of default on a receivable with nominal value of 50 dollar to be 7%, while the insurer uses superior information and makes an estimation of 5%. Both parties have a power utility function with (except for default probability) the same parameters. Now, the reason for value creation potential is twofold. First, the certainty premium of the supplier is 2.9% (1.4 dollar), which is 0.8% higher than the insurer’s certainty premium. Further the supplier believes the expected value to be 46.5, while the insurer thinks it is 47.5. This means that the supplier has a certainty equivalent of the uncertain cash flow which equals $46.5 - 1.4 = 45.1$ dollar. The insurer’s certainty equivalent is $47.5 - 1.0 = 46.5$ dollar, which is a difference of 1.4 dollar with the supplier. This means that, relative to the nominal value $V$, the value creation potential is 2.8%.

Further, the insurer’s risk aversion for this transaction might be lower, which leads to lower pricing for both exponential and power utility functions. A difference in risk aversion can be explained by a difference in risk preferences. The main purpose of an insurance company is to take on this kind of risk, where the probability of a credit event is relatively low, but the
consequences of an event relatively high. An insurer is specialized in assessing this risk and usually also has a better diversified portfolio, so that it will be better able to diversify idiosyncratic risk away. Suppliers on the other hand are usually only willing to take some business risk, such as the risk that the demand for its products turns out lower than expected or the risk of production errors. In general it has no appetite for and is not specialized in dealing with credit risk, which increases its risk aversion for this type of risk.

3.7 Conclusion & limitations

If you simplify a model so far that all parties are risk neutral and have no credit constraints, Insured Pooled Receivables are not able to create value. To make the model more realistic, it is very important to take risk aversion into account. There are several ways to do this, which essentially all stem from the basic formula $p_t = E(m_{t+1} x_{t+1})$. Two specific ways are risk-neutral valuation and expected utility theory. Especially the latter is useful in this situation, since a concave utility function is realistic for firms that acknowledge pecking order theory and bankruptcy costs.

There are also some limitations to the use of expected utility theory. First, it is in practice very difficult to estimate risk aversion coefficients for all parties. However, the goal of this thesis is not to determine an exact price for IPR, but to make a proof of concept. This requires only substantiating that different parties have different risk aversion coefficients, which is easier to do.

Further, Kahneman and Tsversky (1979) criticize the axioms of utility theory and show that it is not compatible with people’s preferences in certain lotteries (i.e. most people choose irrationally according to utility theory). They claim that by establishing a correct utility, it is important to take into account a reference point and loss aversion. This critique is valid for individuals, although the authors use rather extreme examples to substantiate the results. For example, making people choose between 2,500 with probability 0.33 and 2,400 w.p. 0.34 is more a matter of behavioral psychology than it is about making economic decisions. I assume that the impact of these errors in corporate decisions is relatively small. This assumption is supported by findings of Pennings and Smidts (2003), who find no relation between loss aversion and organizational behavior and also find that a firm’s tactical decisions (such as using credit insurance / IPR) can be explained by the local shape of a decision maker’s utility function.

Finally, there is the Ellsberg paradox, which also suggests that expected utility theory does not properly describe actual human choices when probabilities are unknown. In the paradox, people are betting on drawing a ball with a certain colour from an urn. The urn contains 30 red balls and 60 other balls that are either black or yellow. It turns out that people prefer betting on a red ball over a black ball, but prefer the combination black or yellow over red or yellow. Ellsberg (1961) argues that these two options are preferred because their probability is known, while the
probabilities of the other two are unknown (but in expectation the same). This shows that many people exhibit ambiguity aversion: they prefer a known risk over an unknown risk. Standard expected utility theory is not able to account for this ambiguity aversion. There are ways to overcome this problem, such as the use of Choquet integrals (Chateauneuf & Cohen, 2008), but this is beyond the scope of this thesis and a topic for future research.

The two most famous utility functions are exponential utility and power utility. They mainly differ in their risk aversion characteristics: the former satisfies Constant Absolute Risk Aversion, the latter Constant Relative Risk Aversion. A party is willing to pay a certainty premium to get rid of uncertainty in a future cash flow. The value of this premium depends on the choice of utility function, the risk aversion coefficient and the other parameters. If two parties have different utility functions, different risk aversion coefficients, different initial capital, or when informational asymmetry exists, there can be potential to create value via credit insurance.

This is actually a rather strong result: already in a situation where the role of the bank is ignored and there is only one buyer instead of a pool, it is possible to create value. Therefore you would expect trade credit insurance to be used in practice and this is indeed the case: in 2011 the insured exposure of trade credit insurance was 1,800 billion euro for only ICISA members (ICISA, 2012), which is already more than 2.5% of the world GDP. The pricing of this type of insurance is relatively low, the last few years it was between 30 and 35 basis points for ICISA members. The goal of the next chapters is to find out what effect the specific aspects of IPR have on the value creation potential.
This chapter uses and extends the foundations of chapter 3 to make a more realistic model of Insured Pooled Receivables. The first important extension is increasing the number of buyers in the portfolio (which was assumed to be one in the previous chapter). In section 4.1 I will consider the effect of increasing this number, also depending on the correlation in the portfolio. Section 4.2 deals with the timing of the cash flows and the role of the bank and in the third section the assumption on the Loss Given Default is relaxed. The chapter concludes with a tangible example of IPR via a realistic case study, including a sensitivity analysis.

So far I have considered both the exponential utility function (exhibiting Constant Absolute Risk Aversion) and the power utility function (Constant Relative Risk Aversion). For this type of modeling the latter is the most suitable, since the relative size of the receivables should be more important to all parties than its absolute size. A receivable of one million euro can for example mean the difference between bankruptcy and solvency for a supplier with low initial capital, while on the other hand it can also be just one percent of an insurer’s total portfolio. Therefore I will mainly focus on the results of the power utility function, but for the sake of comparison and completeness I will also show exponential utility’s results.

In the previous chapter I used the zero utility premium principle to calculate the certainty premium. Since an investor is risk averse, he is willing to accept a certain amount that is lower than the expectation of his stochastic future cash flow. The amount he is willing to accept is the certainty equivalent and the difference between the expected value and the certainty equivalent is the certainty premium. Because the utility of the certainty equivalent equals the utility of the random cash flow (and therefore the increase in utility of such a transaction is zero), this principle is called the zero utility premium principle (Kaas et al., 2001).

4.1 Pool of buyers

The next step is applying the zero utility premium principle to the insurance of a pool of receivables instead of a single buyer. One of the key aspects of IPR is that it deals with multiple obligors simultaneously. The receivables of the different buyers will form a portfolio, which will affect the variance and the risk of the product and therefore also its value.

The receivables in a portfolio will often be correlated with each other. Suppliers tend to have different buyers operating in the same industry. Whether these buyers will default on their payments is related to some common factors, such as global economic factors and industry specific factors. If an industry is booming and the global economy is doing well, probably very few (or no) buyers will default. In opposite circumstances it is likely that multiple buyers will default on their payments simultaneously. There are however also idiosyncratic factors: one buyer might have a better strategy or management than the other, causing only the weaker buyers to default on their payments.
So typically the correlation in the portfolio will be between zero and one. For modeling purposes however, I will first investigate the two ‘extreme’ situations where correlation is either 0 or 1. Theoretically it is also possible that correlation is negative, but since this is in practice unlikely I will focus on positive correlations.

I am interested in the effect of pooling, but (in this instance) not in the effect of increasing the portfolio size. Therefore I will keep the total value of the portfolio constant at 100 and distribute this amount over the number of buyers, making the size of the individual receivables equal to $\frac{V_{tot}}{N}$. Further, I will use the utility functions defined in (3.20) and (3.21) and the following parameters: $\alpha = 0.02$, $\gamma = 0.6$, $p = 0.05$, and $w_0 = 0$. The risk aversion parameters are chosen based on graphical representations of the utility curve, to avoid extremely low or high risk aversion. At a later stage a sensitivity analysis will be performed on these and other parameters.

To find the certainty premium $c$, formulas (3.26) and (3.27) can still be used, but the calculation of the expectations becomes more involved with multiple buyers. The goal is to find the effect of the number of buyers in the portfolio $N$ on the pricing $\kappa$.

4.1.1 Extreme cases correlation

The first extreme situation I consider is where the correlation between buyers in the portfolio equals one. This means that the receivables are perfectly correlated and either all buyers make the payment or all buyers default. Since I keep the total portfolio value constant (irrespective of the number of buyers), the payoff equals either $V_{tot} (100)$ with probability $1-p$ or zero with probability $p$. That means that nothing changes compared to a situation with one buyer, which implies that pooling has no effect if the correlation is 1.

A correlation of 1 is something that will occur rarely in practice. The other extreme I consider is that the receivables are completely independent of each other and therefore the correlation is zero. Zero correlation is in this case also not very realistic (since buyers operate in the same industry), but it is more likely than perfect correlation and it has a much larger impact.

Since I now assume that receivables are independent and all have the same default probability, I can use the binomial distribution to model the number of defaults. The probability that the number of defaults $D$ equals $k$ is therefore:

$$P(D = k) = \frac{N!}{k!(N-k)!} p^k (1-p)^{n-k}.$$ (4.1)

If there is only one buyer, the only possible payoffs are 0 or 100. As the number of buyers in the portfolio grows, so does the number of possible payoffs. For two buyers it is 0, 50 or 100; for 4 buyers it is 0, 25, 50, 75 or 100; et cetera. Each of these possible payoffs gets assigned a utility
via formula (3.26) or (3.27) and a probability via (4.1). This makes it possible to find the expected payoff and expected utility and therefore also the certainty premium and pricing. Results for different values of N are shown in table 4.1.

Table 4.1: Sensitivity of pricing to number of buyers in portfolio for \( w_0 = 0 \)

<table>
<thead>
<tr>
<th>N</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>10</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa_{\text{EU}} )</td>
<td>8.9%</td>
<td>3.2%</td>
<td>1.9%</td>
<td>1.1%</td>
<td>0.5%</td>
<td>0.2%</td>
</tr>
<tr>
<td>( \kappa_{\text{PU}} )</td>
<td>7.0%</td>
<td>1.3%</td>
<td>0.6%</td>
<td>0.3%</td>
<td>0.2%</td>
<td>0.1%</td>
</tr>
</tbody>
</table>

It seems that the effect of the number of buyers in the portfolio on the pricing is very substantial. However, I have to distinguish between the exponential utility function (EU) and the power utility function (PU). It turns out that an investor behaving according to exponential utility only cares for the size of the individual receivables. The lower the size of each receivable, the less he values the insurance (i.e. the lower the pricing). Whether the portfolio contains 1 receivable with nominal value 100, or 15 of these receivables, the pricing is still 8.9%. If you keep the total value of the portfolio constant, increasing the number of buyers leads to lower individual nominal values and therefore lowers pricing. So it seems in table 4.1 that under exponential utility, pooling leads to lower pricing, but this effect can be attributed completely to the lower value of individual receivables.

For the power utility function, pooling does have a significant effect on the pricing. Since this type of utility exhibits CRRA, the nominal value of a single receivable does not affect the pricing (see chapter 3). Still, when the number of buyers in the portfolio increases, the pricing goes down. Especially the difference between one and two buyers is very big: if there is one buyer, the supplier is willing to pay 7 dollar to receive the expected value of 95 dollar for sure (i.e. its certainty equivalent is 88 dollar). However, when there are two buyers that each represent a receivable of 50 dollar, the supplier is only willing to pay 1.3 dollar to receive 95 dollar for sure.

A first explanation for this large difference is that the variance of the cash flow decreases with the number of buyers in the portfolio. The variance of a portfolio with zero correlation is:

\[
Var_{PF} = \frac{(V_{tot})^2 \cdot p(1-p)}{N} \tag{4.2}
\]

However, as can be seen in figure 4.1, the pricing decreases relatively faster than the portfolio’s variance. So there should be another cause for the large effect of adding an extra obligor to the portfolio. A possible explanation is that the marginal utility is very high around zero. With the current parameters of the power utility function, the utility of 0 dollar is 0, the utility of 50 dollar is 12.0 and the utility of 100 dollar is 15.8. So the difference in utility between 50 and 0 dollar is much larger than between 100 and 50 dollar. When there are two buyers instead of one, the probability of receiving 0 dollar is reduced significantly (from 0.05 to \( 0.05^2 = 0.0025 \)).
Therefore a large chunk of the risk is already reduced, so credit insurance is less valuable and the pricing drops.

The hypothesis that investors want to avoid very low utility is supported by the results of raising the initial capital. If you raise the initial capital, you are sure that even if every obligor defaults, you still have the initial capital and therefore your utility is not zero. Table 4.2 shows that this decreases the pricing for \( N = 1 \) a lot and that the impact of pooling is, absolutely as well as relatively, smaller. If you raise the initial capital even further (up to 100), the lower pricing can almost completely be explained by the reduced variance (see figure 4.2).

Table 4.2: Sensitivity of pricing to number of buyers in portfolio for \( w_0 = 10 \)

<table>
<thead>
<tr>
<th>( N )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>10</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa_{PU} )</td>
<td>3.3%</td>
<td>1.0%</td>
<td>0.6%</td>
<td>0.3%</td>
<td>0.1%</td>
<td>0.1%</td>
</tr>
</tbody>
</table>

4.1.2 Correlation between zero and one

In the extreme cases of correlation, the variance of the portfolio either remains constant or it goes to zero. It is a well known result in finance that if you increase the portfolio size, the idiosyncratic variance of individual assets becomes less important: only the covariance with the rest of the portfolio matters for the portfolio variance. If you assume a portfolio of \( N \) equally weighted assets with identically distributed payoffs \( x \) and common pair wise covariance \( \sigma_{12} \), this so called diversification effect can be summarized by the following formula:

\[
Var_{PF} = \frac{1}{N} Var(x) + \left(1 - \frac{1}{N}\right) \sigma_{12}.
\]  

(4.3)
Correlation can be incorporated in the model in many ways, for example via latent variable models. In these models, every individual obligor in the portfolio has a certain value (the latent variable) and whenever this value drops below a threshold (usually the firm’s liabilities), the obligor defaults. The dependence between default events is caused by dependence between the latent variables, usually via some common factors that affect all latent variables in a portfolio. These models are often used in practice; famous examples are the KMV model and CreditMetrics. A disadvantage of these models is that they hardly ever lead to closed form solutions and require (Monte Carlo) simulation (Frey et al., 2001).

Another option is to extend the binomial model (that was used for the zero correlation situation) in such a way that it can account for correlation. Moody’s has developed such a model under the name Moody’s Correlated Binomial Default Distribution, abbreviated MCB model (Witt, 2004). An advantage of a correlated binomial model is that it is easier to evaluate than more refined models. However, it requires the assumption that the portfolio is homogeneous: assets are interchangeable and have the same default probability $p$ and default correlation $\rho$. For the purposes of this thesis, this assumption is not too restrictive. If you want to model more realistic portfolios where different obligors have different characteristics, the model should to be extended (see for example Mori et al., 2009).

The first two assumptions of the MCB model are, as mentioned before, that (1) every asset has the same default probability $p$ and (2) each pair of assets has the same default correlation $\rho$. Now, $p_j^*$ is defined as the probability that asset $j$ defaults, given that assets 1 to $j-1$ have defaulted. Formally:

$$p_j^* = E(\delta_j \mid \delta_1 = 1, \ldots, \delta_{j-1} = 1). \quad (4.4)$$

For the first asset, the conditional part of the expectation has no information, so using assumption (1) it holds that $p_1^* = E(\delta_1) = p$. To find $p_2^*$, the conditional information that the first asset has defaulted should be taken into account: $p_2^* = E(\delta_2 \mid \delta_1 = 1)$. Since $\delta_2$ is binary, $E(\delta_2 \mid \delta_1 = 1) = P(\delta_2 = 1 \mid \delta_1 = 1)$. For ease of exposition I use $(\delta_1 = 1) = A$ and $(\delta_2 = 1) = B$. Using standard probability theory and the definition of correlation

$$\rho = \frac{E(A \cap B) - E(A)E(B)}{\sigma_A \sigma_B} = \frac{P_AB - P_A P_B}{\sqrt{P_A(1 - P_A)P_B(1 - P_B)}},$$

the following result is derived:
Would $\rho$ equal 0, the number of defaults is binomially distributed and $p_j^* = p$ for all $j$. If $\rho > 0$ however, the distribution is no longer binomial and for $N > 2$ an extra assumption is required to determine the joint probability distribution of $\delta_1, \ldots, \delta_N$. This third assumption is that the default correlation between asset $j+1$ and asset $j+2$ remains equal to $\rho$, regardless of the number of known defaults among the other $j$ assets:

$$p_{j+1}^* = p_j^* + (1 - p_j^*)\rho.$$  \hspace{1cm} (4.6)

Formula (4.6) implies that the default probability of asset $j+1$, conditional on $j$ defaults, is increasing in $j$. If the correlation is relatively high, then after a certain number of defaults it is almost sure that other assets will default as well. If you use $\rho = 0.5$ for example, after the default of the first 10 assets, the probability that other assets will also default is 99.9%. This induces the fat tail of the default distribution, which increases the risk for the investor in the portfolio.

The next step is to find the probabilities associated to a certain number of defaults. With these probabilities the expectation of $x$ and the expected utility of $x$ can be found, which enables the calculation of the certainty premium. I will first calculate the probability that no obligor will default. This requires every $\delta_j$ to be 0, so the probability equals the following expectation:

$$P(D = 0) = \mathbb{E} \left[ \prod_{i=1}^{N} (1 - \delta_i) \right] = \mathbb{E} \left[ (1 - \delta_1) \ldots (1 - \delta_N) \right].$$  \hspace{1cm} (4.7)

This can be verified by seeing that $\prod_{j=1}^{N} (1 - \delta_j)$ equals 1 when every $\delta_j = 0$ (i.e. there are no defaults) and 0 if at least one obligor defaults. So taking the expectation over this expression gives $\mathbb{E} \left[ \prod_{i=1}^{N} (1 - \delta_i) \right] = P(D = 0)*1 + P(D > 0)*0 = P(D = 0)$.

The following theorem is proposed:

**Theorem 4.1.** Let $\delta_1, \ldots, \delta_n, n \geq 1$ be identically distributed stochastic variables. Then it is true that
\[ E \left[ \prod_{j=1}^{N} (1 - \delta_j) \right] = \sum_{j=0}^{N} (-1)^j \binom{N}{j} E \left[ \prod_{j=1}^{j} \delta_j \right] \]

**Proof**

For a proof of theorem 4.1, see appendix B.

Using theorem 4.1 and the definition of \( p_j^* \) (formula 4.4), the probability that no obligor will default can be defined:

\[ P(D = 0) = 1 + \sum_{j=1}^{N} (-1)^j \binom{N}{j} \prod_{i=1}^{j} p_i^*, \tag{4.8} \]

Formula (4.8) covers the event that no obligor will default; the next step is to assign probabilities to states with one or more defaults. The probability that the first \( k \) obligors will default (with \( k > 0 \)) and the other \( N-k \) will survive is (using a similar derivation as in theorem 4.1):

\[ E[\delta_1 \cdots \delta_k (1 - \delta_{k+1}) \cdots (1 - \delta_N)] = E \left[ \prod_{j=1}^{k} \delta_j \prod_{j=k+1}^{N} (1 - \delta_j) \right] = \sum_{j=0}^{N-k} (-1)^j \binom{N-k}{j} \prod_{i=1}^{j+k} p_i^*. \tag{4.9} \]

However, this is only one configuration where \( k \) out of \( N \) obligors have defaulted, in total there are \( \binom{N}{k} \) possibilities. Since the portfolio is homogeneous, it is possible to multiply (4.9) with this number of combinations to find the probability that \( k \) obligors will default:

\[ P(D = k) = \binom{N}{k} \sum_{j=0}^{N-k} (-1)^j \binom{N-k}{j} \prod_{i=1}^{j+k} p_i^*. \tag{4.10} \]

The effect of the correlation is incorporated in this formula via the definition of \( p_i^* \). Figures 4.3 and 4.4 show that the effect of correlation on the default distribution is substantial. For the sake of illustration, I use a portfolio of 15 obligors for whom the default probability is increased from 0.05 to 0.25. If there is no correlation (figure 4.3), the distribution of the number of defaults has a typical binomial shape. Although the expectation of the number of defaults is the same when correlation is 0.5 (figure 4.4), its distribution is quite different. Due to the correlation there is a higher probability that no obligor defaults, but ‘tail risk’ events where more than half the portfolio’s obligors default are also more likely. This higher probability of extreme events illustrates why the variance of a portfolio with higher correlation is larger.
It is noticeable that figure 4.4 has a peak for \( k = N \), while it is 0 in \( k = N-1 \). This can be explained by the fact that for the MCB model the conditional probability of an obligor’s default approaches 1 if already many obligors before him have defaulted. Therefore it is very unlikely that all obligors will default, except for one or two. In reality one could expect that even in a very serious economic downturn, it would still be likely that at least a few companies would not default. However, in reality the portfolio would usually also not be homogeneous, explaining why a few exceptional companies could still survive. So this result is not completely realistic, but it is consistent with the assumptions.

![Figure 4.3: Default distribution N=15, p=0.25, rho=0](image)

![Figure 4.4: Default distribution N=15, p=0.25, rho=0.5](image)

The default distribution, based on formula (4.10), has a significant effect on the value of IPR. This is because the default distribution influences the variability of the cash flow. Risk averse parties are willing to pay a certainty premium to get rid of the variability of the cash flow. The higher the variability of the cash flow, the higher the risk and the certainty premium. The main cause of the cash flow’s variability in this model is the distribution of the number of defaults.

I established before that if the correlation is 1, the risk is not reduced by pooling and therefore the pricing is constant. For a correlation of 0 on the other hand, the effect of increasing the number of obligors in a portfolio is very significant. As expected, for correlation between 0 and 1, the effect of pooling on pricing is intermediate. For a relatively high correlation, pooling can decrease the pricing a little, but only down to a certain level. The lower the correlation, the

![Figure 4.5: Effect of pooling & correlation with power utility, w0 = 0, γ = 0.6, p = 0.05](image)
further the initial pricing (for one receivable) can be reduced by using pooling. Similar to the result established for zero correlation, the impact of adding an extra receivable to the portfolio is largest when the initial portfolio is small. As can be seen in figure 4.5, increasing \( N \) from 1 to 2 has a huge effect on pricing. Adding a third receivable still has a moderate impact, but the marginal contribution of adding extra obligors diminishes quickly.

The effect of correlation on pricing is quite robust against changes in the parameter settings or choice of utility function. Appendix C shows how correlation affects pricing when the exponential utility function is used, the risk aversion coefficient is changed, or the initial capital is increased. It turns out that these changes mainly have an impact on the pricing for \( N = 1 \), which ranges between 1.6% and 35.1%. However, the relative impact of correlation and pooling is very similar: lower correlation leads to lower pricing and the marginal benefits of adding an extra obligor are decreasing quickly.

Slight differences occur for the settings where risk aversion is smaller (figure A.4) or initial capital is higher (figure A.5). In these situations, it is less important for the investor to avoid a cash flow of 0, making the relative difference in pricing between \( N = 1 \) and \( N = 2 \) smaller. This makes the graph less steep initially, but it keeps on decreasing longer, also for higher levels of \( N \). This same effect can be noticed when the exponential utility function is used instead of the power utility function (figure A.2). Increasing the risk aversion coefficient (figure A.3) has an opposite effect: it is very important to avoid a utility of zero; increasing \( N \) from 1 to 2 decreases the probability that \( x \) is zero significantly, so the graph is very steep initially.

**4.2 Role of the bank**

Thus far, I have ignored the role that the bank can play in an IPR deal and focused on the transaction between supplier and insurer. A transaction between those two parties can mainly be valuable if there are differences in risk preferences. The size of the value creation potential depends on the initial capital, the number of obligors, their correlation, et cetera. This section introduces the timing of the cash flows in the model. I will show whether involving the bank increases the value creation potential, for example by providing liquidity.

**4.2.1 Timing of risky cash flows**

To account for the timing of cash flows I will incorporate the nominal payment period \( l \), and the waiting period \( \Omega \) into the model. The bank’s cash outflows occur at time 0, the buyers that have not defaulted fulfill their obligations at \( t = l \), and \( \Omega \) months thereafter the insurer has to disburse the claims for the defaulted buyers (see figure 4.6). Because of the time value of money, receiving a cash flow today is more valuable than receiving it in a few months.
When the timing of cash flows is incorporated, the model changes from a static one period model to a dynamic multiple period model. There are multiple ways to deal with risk and discounting in such an intertemporal setting. A first option would be to use an intertemporal utility function $u(x_1, x_2, x_3, ...)$, where the investor’s preferences on the timing of the cash flows is explicitly taken into account in the utility function itself. An alternative is to first calculate certainty equivalents of all future cash flows using a ‘normal’ utility function and to subsequently discount these certainty equivalents at the risk free rate. An extensive comparison to make a substantiated choice between these two methods is outside the scope of this thesis and a topic for future research. I assume that investors’ preferences on risk and timing of cash flows are constant in time, which allows that certainty equivalents are discounted at the risk free rate.

Calculating the present value of a cash flow involves the following steps. First, all possible payoff states are identified. Subsequently, every payoff state is assigned a utility via the utility function and a probability via formula (4.8). With the values for the utility and probability it is possible to calculate the expected utility. Since utility functions are concave, the expected utility can be converted into a certainty equivalent (CE) via the inverse of the utility function. For the power utility function, the certainty equivalent can be calculated as follows:

$$CE_{pu}(x) = \left( E[u(x)](1 - \gamma) \right)^{-\frac{1}{\gamma}}.$$  \hspace{1cm} (4.11)

The final step is discounting this certainty equivalent to $t = 0$. Since I already account for the risk of the cash flows via the use of utility functions, I can discount the certainty equivalent of the receivables at the risk free rate $r_f$. I assume that interest is compounded continuously, so the risk adjusted present value of a cash flow occurring at time $t$ (with $t$ in years) is determined by the following formula:
\[ PV(x_t) = \exp(-r_f t)CE(x_t). \]  

(4.12)

If there is no IPR deal, the present value of the bank and insurer is of course zero and the present value of the supplier is straightforward to calculate via (4.11). The central question of this thesis is whether the present value of all three parties can be increased by setting up an IPR deal. For the supplier it is of course beneficial to receive its cash flow earlier and without uncertainty, but whether its present value increases depends on the height of the premium it pays to the bank.

Assuming that the insurer does not go bankrupt, the bank has no risk: it always receives the nominal value of the receivables. However, it has to pay the supplier and insurer early (at \( t = 0 \)) and receives the cash inflow later. This is costly because of the time value of money, so the bank asks a premium to compensate for this.

The question whether the insurer’s present value is positive is most straightforward: the premium it receives should be higher than the certainty equivalent of its cash outflow. Therefore I first calculate the minimum premium that the insurer demands: \( \pi_{I,\text{min}} \). This minimum premium is the difference between the present value of the insurer’s wealth without and with an IPR deal:

\[ \pi_{I,\text{min}} = w_{0,I} - \exp(-r_f (l_s + \Omega))CE(w_{l_s+\Omega,I}), \]  

(4.13)

where

\[ w_{l_s+\Omega,I} = \exp(r_f (l_s + \Omega))w_{0,I} - DV. \]  

(4.14)

Without IPR, the insurer’s wealth is equal to its initial wealth. When there is an IPR deal, the wealth is lower, since there is a positive probability that there will be a cash outflow at \( t = l_s + \Omega \) (due to the default of one or more buyers). The premium that the insurer demands is therefore positive and consists of compensation for the expected value of the loss and a compensation for the risk.

Subsequently the minimum premium of the bank \( \pi_{B,\text{min}} \) can be determined. This is at least equal to \( \pi_{I,\text{min}} \), since the bank pays the premium to the insurer. On top of this, the bank should be compensated for the risk it takes and for providing liquidity. The bank’s risk is minimal however, because in the end it always receives \( V_{tot} \). The only small risk is that it receives part of this amount later (from the insurer instead of the buyer), so the bank loses interest on this amount for \( \Omega \) months. The liquidity part of the premium is usually more substantial and depends on \( l_s, \Omega \) and \( r_f \). The minimum premium of the bank is:
\[ \pi_{B,\text{min}} = \pi_{I,\text{min}} + V_{t_0} + w_0 \exp(-r_f (l_s + \Omega))CE(w_{l_f+\Omega,B}), \]  

(4.15)

where \( w_{l_f+\Omega,B} = \exp(r_f (l_s + \Omega))(w_{0,B} - V_{t_0}) + \exp(r_f \Omega) (N - D)V + DV. \)  

(4.16)

Finally, the maximum premium the supplier is willing to pay (\( \pi_{S,\text{max}} \)) can be calculated. On the basis of the default probability, the risk aversion, the initial capital, and many other variables, it is possible to determine the risk adjusted present value of the receivables without an IPR deal. If the supplier participates in the IPR deal it receives \( V_{t_0} \) minus the premium at \( t = 0 \), so its maximum premium is simply the difference between this risk adjusted present value and \( V_{t_0} \):

\[ \pi_{S,\text{max}} = V_{t_0} - \exp(-r_f l_s)CE(\exp(r_f l_s)w_{0,S} + (N - D)V) - w_{0,S}. \]  

(4.17)

The value creation potential of IPR is the difference between the bank’s minimum premium and the supplier’s maximum premium:

\[ V = \pi_{S,\text{max}} - \pi_{B,\text{min}}. \]  

(4.18)

If the minimum is lower than the maximum, value can be created if the supplier sells its pool of receivables to the bank, who subsequently insures this pool. Section 4.4 gives a tangible example of IPR’s value creation potential.

4.2.2 Value creation potential bank

Previously I have shown that value can be created by a transaction between the supplier and the insurer, if they have different risk preferences. The supplier should either have a higher risk aversion coefficient or have lower initial capital, which makes the relative risk of the receivables larger. The size of the value creation potential depends on the size of these differences, the default probability, portfolio size and correlation, et cetera.

The bank however takes virtually no risk; its main role is setting up the deal and providing liquidity. In this model, providing liquidity does not create any extra value: certain cash flows (or certainty equivalents) should be discounted at the risk free rate, no matter who the owner of the cash flow is. Transferring ownership of a certain cash flow therefore does not create value from a liquidity point of view.

So incorporating the bank in the model makes it more realistic, but does not increase the value creation potential. However, there are a few reasons why involving a bank in the deal in practice can still be beneficial and increase the value creation potential. The first and most important reason is to relieve the possible financial constraints of a supplier. In a situation where the supplier is financially constrained, he already has so much debt that it is either impossible or very costly to raise more regular debt. It is therefore also costly to offer its buyers extended
payment, since this requires (very costly) financing. IPR can offer a solution, because selling the accounts receivable can provide liquidity to the supplier, without further increasing the debt or deteriorating the payment term for the customers. So IPR can serve as an alternative and, because of the collateral it provides, much cheaper form of financing for the supplier.

There are more reasons why the involvement of a bank can increase the value of IPR. For a financially constrained firm it can be important to avoid the waiting period before the insurance company disburses the claims. If the supplier insures the credit risk of its buyers itself (so not via the bank), it has to finance the waiting period if buyers have defaulted. Financial constraints can make this financing very costly, so even though the supplier has insurance, there is still a risk when buyers default. When the supplier has an IPR deal (including the involvement of the bank), the supplier’s risk is completely eliminated.

Further, the role of the bank can provide some benefits. It can be difficult for a supplier to discipline its buyers when they are paying too late or not at all, since the supplier wants to generate future revenues from these buyers and therefore wants to maintain the relationship. The bank generally has no direct relationship with the buyers, which increases its disciplinary power and therefore also the payment behavior of the buyers. If this payment behavior is better, the minimum premium that the insurer asks can be reduced and therefore more value can be created.

So although in the model the bank is not able to add to the value creation potential, involving the bank in the IPR deal can still have quite some benefits. Therefore the bank is in a good position to claim some of the value creation potential in an IPR deal. A good opportunity for future research would be to improve the model in such a way that the benefits of liquidity can also be quantified. Then it would also be possible to make a good argument how much of the value created should be distributed to the bank and how much to the supplier and insurer.

Besides increasing the total value creation potential, the bank can also create value for itself. Since providing and redistributing liquidity is one of the core tasks of a bank, it should be able to do this efficiently. Put differently: banks make money by profiting from the difference between saving and lending interest rates. In that sense, an IPR deal should not be different than other ways of generating revenue and therefore is a way for the bank to grow its business.

Finally, the bank might be able to negotiate a better deal with the insurer, because it has more experience, better information and/or higher volumes than suppliers. This does not create value for the deal as a whole, but it allocates some of the value creation potential from the insurer to the bank.

There is one other reason why it might be beneficial to involve the bank in the deal: a possible reduction of the Loss Given Default. Since I have not introduced this parameter before, this will be explained in more detail in the next section.
4.3 Loss Given Default

In this section I will relax the assumption that in case of default, the complete nominal value of the receivable is always lost. In practice, a lender will usually be able to recover a part of the loan and he therefore loses less than 100% of the nominal value in a default event. To model this, banks use the Loss Given Default (LGD): the credit loss incurred if an obligor of the bank defaults (Schuermann, 2004). The LGD is usually modeled as a percentage of the Exposure At Default.

There is (additional) value creation potential if the Loss Given Default is lower for the bank than for a supplier. One of the main determinants of LGD is the seniority or the degree to which the claim is subordinated. According to the Absolute Priority Rule (APR), in case of bankruptcy senior claimholders should be fully satisfied before any distributions are made to more junior creditors. Therefore the LGD is lower for more senior claimholders. The seniority of an account receivable does not change when it is sold from the supplier to the bank. However, several studies (e.g. Eberhart and Weiss, 1998) show that the APR is often violated in practice, to speed up resolution. Banks usually have a stronger position in these negotiations than suppliers, because they have more capital, better lawyers, et cetera. Therefore a bank can be able to recover more of their loss than a supplier would, so its LGD can be lower.

The LGD can have a twofold impact on the value of a receivable: it affects both the certainty premium and the expected value of a receivable. If you would be able to reduce the LGD of a receivable with a default probability of 10% from 100% to 50%, you increase the expected value with 5 percentage points. Besides that, the variance of your cash flow shrinks and therefore also the certainty premium. As an example I use the situation where $V_{tot} = 100$, $N = 1$, $\gamma = 0.6$, and $w_0 = 50$. In such circumstances the pricing (certainty premium divided by nominal value) drops from 3.0% if LGD = 100% to 0.6% if LGD = 50%. So if a supplier has an LGD of 100%, its certainty equivalent is $100 - 10 - 3 = 87$, while it would be $100 - 10 * 50% - 0.6 = 94.4$. The difference between these two certainty equivalents of the same cash flow (7.4) creates a significant value creation potential for IPR.

The improvement of decreasing the LGD on the expected value of a cash flow is straightforward: it equals the reduction in LGD times the default probability. The effect on the certainty premium (pricing) is less straightforward, so some results are presented in table 4.3. These figures show that, besides affecting the expected value, lowering the LGD also reduces the risk and the certainty premium substantially.
4.4 Illustrative case

To illustrate the idea behind IPR and its value creation potential, I will treat a realistic case extensively. I will first describe the case’s characteristics, followed by the calculation of the value creation potential and a sensitivity analysis.

I consider a large oil company which sells its product to 10 different retailers. These buyers are all in the same industry and quite similar, so the portfolio is homogeneous with correlation 0.5. A realistic assumption on the probability of default for each obligor would be 2%. The total value of the receivables is 50 million euro and these receivables are due in 3 months, with an annual risk free rate of 5%. Would a buyer default, the Loss Given Default to the supplier (LGD) is 80%.

The supplier is offered to sell the receivables to the bank and receive a certain cash flow at time $t = 0$. This cash flow equals 50 million minus the premium that the bank charges for the service. If the supplier sells its receivables, the bank immediately insures them at an external insurer and pays a premium for this. In case a buyer defaults on its payment, the insurer disburses the claim 3 months later. Due to its better bargaining position, the LGD for the bank (LGD$_B$) is only 70%.

The three parties use a power utility function to account for the risk of the cash flows. The risk aversion coefficient of such a function ($\gamma$) varies between 0 and 1. The supplier is most risk averse with $\gamma_S = 0.6$, while $\gamma_B = 0.4$ and $\gamma_I = 0.3$. Further the initial capital of the bank and the insurer is 100 million euro and the supplier’s initial capital equals 10 million euro.

The first step in calculating the value creation potential is to find the value of the receivables to the supplier without an IPR deal. The certainty equivalent of the risky cash flow is 49.1 million euro 3 months from now, which implies a present value of 48.5 million euro. The maximum premium the supplier is willing to pay ($\pi_{S,max}$) is therefore 1.5 million, or 3.07% of the nominal value of the receivables.

If the insurer participates in the IPR deal, there is a positive probability that it has to pay for claims. Although the probability that the total claim size is 0 is quite high (93.7%), there is for
instance also a 2.7% probability that the insurer has to pay more than 10 million in 6 months. The expected value of the loss is 0.70 million, but due to the insurer’s risk aversion the certainty equivalent is slightly higher: 0.72 million. Since the payment is due 6 months from now, the present value equals 0.70 million, or 1.41% of the nominal value of the receivables.

The minimum premium of the bank ($\pi_{B,\text{min}}$) is determined in a similar way. If the bank participates in the deal, the present value of its wealth is 98.67 million euro, which is 1.33 million less than its initial wealth. Therefore the minimum premium that the bank requires is 2.67%, composed of 1.41% premium for the insurer and 1.26% for the added services of the bank.

The value creation potential $\nu$ equals $\pi_{S,\text{max}} - \pi_{B,\text{min}} = 0.20$ million, or 40 basis points of the nominal value of the receivables. Although the values of the parameters in this case study are quite realistic, they are still arbitrary. Therefore a sensitivity analysis shows how the value creation potential changes when the parameter values are lower or higher. The parameters for the sensitivity analysis are shown in table 4.4 and the results in figure 4.7.

Table 4.4: Parameters sensitivity analysis

<table>
<thead>
<tr>
<th></th>
<th>$p$</th>
<th>LGD$_S$</th>
<th>LGD$_B$</th>
<th>$y_S$</th>
<th>$w_{0,S}$</th>
<th>$\rho$</th>
<th>$y_I$</th>
<th>$w_{0,I}$</th>
<th>$V_{\text{tot}}$</th>
<th>$N$</th>
<th>$l_s$</th>
<th>$r_f$</th>
<th>$\Omega$</th>
<th>$w_{0,B}$</th>
<th>$\nu_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>1%</td>
<td>70%</td>
<td>60%</td>
<td>0.3</td>
<td>0</td>
<td>0.2</td>
<td>0</td>
<td>50</td>
<td>20</td>
<td>5</td>
<td>0</td>
<td>1%</td>
<td>0</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>Base</td>
<td>2%</td>
<td>80%</td>
<td>70%</td>
<td>0.6</td>
<td>10</td>
<td>0.5</td>
<td>0.3</td>
<td>100</td>
<td>50</td>
<td>10</td>
<td>3</td>
<td>5%</td>
<td>3</td>
<td>100</td>
<td>0.4</td>
</tr>
<tr>
<td>High</td>
<td>4%</td>
<td>90%</td>
<td>80%</td>
<td>0.9</td>
<td>100</td>
<td>0.8</td>
<td>0.6</td>
<td>150</td>
<td>80</td>
<td>20</td>
<td>6</td>
<td>10%</td>
<td>6</td>
<td>150</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Figure 4.7: Sensitivity analysis value creation potential
The sensitivity analysis shows that the value creation potential $\nu$ is quite robust against changes in one parameter. There are a few parameters, related to risk preferences of different parties, that can explain the value creation potential. If the supplier has a higher risk aversion coefficient, a higher LGD and lower initial capital (and therefore higher relative risk), it turns out to be profitable to transfer these risks by using IPR. If one of these differences disappears (e.g. $LGD_s = LGD_B$), the value creation potential is reduced, but it is still positive. Only if all three differences are reduced significantly, the value creation potential disappears.

The value creation potential is also affected by the size of the risk: the higher the risk, the higher the potential savings by using IPR. This explains the impact of $p$ and $\rho$ in figure 4.6 (and to a lesser extent, also $V_{tot}$ and $N$). Providing liquidity is not a service that creates value in this model, which explains why $l_s$, $r_f$ and $\Omega$ have (almost) no effect on the value creation potential $\nu$. The sensitivity analysis also shows that the role of the bank is negligible in this model: the bank hardly takes any risk, so $\gamma_B$ and $w_{0,B}$ do not affect $\nu$.

The fact that the value creation potential is consistently positive means that IPR can be quite valuable for the involved parties. The final step would be to find a fair distribution of the created profit. Since the role of the bank is negligible in this model, one would expect that the bank is not able to claim a part of the created value. If the bank claims value, the division of profits would be unstable, since the insurer and supplier could increase their profits by forming a coalition on their own. However, the bank might be able to provide valuable services (such as liquidity) that are not captured in the model. For these reasons (as mentioned in section 4.2.2), the bank might still be able to claim some of the value creation potential. The actual division of profits depends on the strength of each of the parties in the negotiations. The next chapter will consider whether IPR can also affect the operations in the supply chain.
5 POTENTIAL IMPACT ON OPERATIONS

I demonstrated in the previous chapter that in the right circumstances, Insured Pooled Receivables can have financial benefits for all the involved parties. A recent study of Tanrisever et al. (2012) shows that reversed factoring, another variation within Supply Chain Finance, has operational benefits on top of these financial benefits. The goal of this chapter is to find out the impact of IPR on the operations in the supply chain.

Although the opportunity of using an IPR deal can have a positive NPV and provides liquidity, it is not immediately clear that it actually reduces the supplier’s financing costs. There are at least two reasons however to assume that it does. First, the supplier discounts its random cash flows using a utility function for a reason: a negative realization of the cash flow can increase the probability of bankruptcy and reduce the availability of internal financing. Because of market imperfections such as bankruptcy costs and information asymmetries, this can increase the external financing costs. This is why these states have a particularly low utility. Therefore, improving utility (which IPR in the right circumstances does) is likely to also reduce financing costs.

Secondly, IPR reduces the variability in availability of financing. For production settings, uncertainty in demand requires a safety stock, since the costs of having too little inventory are higher than the costs of having too much inventory. Arguably, uncertainty in the availability of financing also leads to the supplier having a ‘safety stock’ of financing. Too much financing can lead to a costly excess of cash, but too little financing may lead to pass upon positive NPV projects, which is usually more costly. By using IPR, the supplier can free up the financing that is used as ‘safety stock’ and therefore reduce the financing costs.

Supplier
To find the impact on the supplier’s operations, it is first important to define in what kind of operational environment the supplier is working. Tanrisever et al. (2012) discern between a Make To Order (MTO) and a Make To Stock (MTS) business setup. Since their paper is written in the context of reversed factoring, it is quite plausible to also consider a MTO model. This is because in a reversed factoring arrangement, there is generally a large buyer with many smaller suppliers. The orders from the large buyer therefore constitute a relatively large part of the supplier’s revenue, which might justify a MTO business setup. For IPR however, I consider a situation where a large supplier has many smaller buyers. For such a supplier it is quite complex and probably inefficient to make the production decisions based on individual orders. Therefore I believe it is more realistic to assume that the supplier uses a MTS production strategy.

Now, there are at least two ways in which IPR might affect operations in the short term: by the supplier’s reduced financing costs or by its increased liquidity. I will first consider what impact reduced financing costs can have on the operational decisions. In the MTS model, one of the most important operational decisions is the stocking decision. This decision is usually a trade off
between the holding costs of inventory and the (back-) ordering costs. The simplest way to make this trade off is the EOQ model, but this is only valid for non-stochastic demand. When demand is random and the stocking decision precedes the demand realization, a newsvendor model is the preferred model.

Although the newsvendor model is more complex than the EOQ model, the essence is still the same: the optimal inventory level increases with the penalty or backordering cost and decreases with the holding cost. The use of IPR should not affect the penalty cost, but it can have an impact on the holding cost. This holding cost consists of two components: a physical component (for storage, obsolescence, etcetera) and a financial one (the cost of financing the inventory) (Serrano et al., 2010).

In the previous chapter I established that if parties have different risk preferences, there can be value creation potential for IPR. If the minimum premium the bank asks is lower than the maximum premium the supplier is willing to pay to break even, all parties can be better off by implementing IPR with a pricing somewhere in the range between \( \pi_{B,\text{min}} \) and \( \pi_{S,\text{max}} \). If such a pricing is established, the financing costs for the supplier can be decreased and therefore also the financial component of its holding cost. With a lower holding cost, the newsvendor fractile increases and the optimal inventory level is higher. This enhances the supplier’s service level, which is beneficial for the buyers and for the reliability of the supply chain as a whole.

The model I use only describes the value creation potential of IPR via differences in risk preferences. However, the product also provides the supplier with liquidity, which can be valuable too. Since the supplier now has the option to convert its receivables into cash via IPR at any time, it always has the flexibility to extend the buyer’s payment period if this is necessary or beneficial from a competitive point of view. Would a financially constrained supplier not have the IPR option, it might not be able to do this, since it would increase the probability that this already troubled supplier will default. If the supplier’s competitors are able to provide better conditions to the buyers, the supplier might lose business because of its inflexibility.

For suppliers with very tight financial constraints, it might not even be possible to accept positive NPV business opportunities, since the investments these opportunities require cannot be funded. IPR can resolve this problem. Therefore the product might also indirectly create value on the longer term by allowing the supplier to make better operational decisions (in this case accepting the positive NPV business opportunity).

Buyers

Even though the buyers are not directly involved in the IPR deal, there also might be operational enhancements for them. If the supplier decides to extend the payment term, this is not only financially beneficial for the buyer, it might also positively affect the buyer’s operations. To see this, it is important to discern between physical and financed inventory. Physical inventory is the
number of physical units on a location, while financed inventory equals the units of inventory that have been paid for minus those for which payment has been collected (Song and Tong, 2011). To find the total costs of inventory, these two types of inventory have to be multiplied with the physical component of the holding cost and the finance component, respectively.

A longer payment period does not affect the physical inventory, but it reduces the financed inventory. This makes it possible for the buyer to increase the inventory, without suffering from higher costs. Higher inventory increases the service level to companies downstream or to customers.

Even if the supplier is not willing to share the extra value created by IPR with its buyers, it can still be possible to create some benefits. This has to do with the deadweight cost of capital. Deadweight cost of capital is a premium on the charge of external finance that is caused by market imperfections, such as informational asymmetries, financial distress costs or monitoring costs (Froot et al., 1993). Usually the deadweight cost of capital is larger for smaller companies, because they suffer more from informational asymmetries and other market imperfections.

This means that in the IPR setup, where a large supplier serves smaller buyers, the buyers often have a higher deadweight cost of capital. Therefore it is more costly for them to finance their purchases than it is for a supplier to finance its sales via accounts receivable. So for the supply chain as a whole, it would be better if the supplier extends its payment period and covers these financing costs by increasing the price slightly. In this way the supplier breaks even while the buyer benefits more from the reduced financing costs than that he suffers from the increased price.

A short numerical example can serve as an explanation for the above argument. Imagine a large supplier who sells its product for €1 million to a smaller buyer. The supplier can get financing at a rate of 8% per year, while the buyer pays 10% per year. The buyer takes half a year to convert this purchase into sales, so without a payment term it would need to finance €1 million for six months. With the financing costs of 10% per year, this would cost the buyer €50,000, adding up to a total cost (including purchases) of €1.05 million.

If the supplier is willing to grant an extended payment term, but not make any extra costs, it should charge its financing costs to the buyer via an increased price. For a payment term of 3 months, the supplier has financing costs of €20,000. When it charges those to the buyer, he has to pay €1.02 million, but only needs financing for 3 months. This costs $ \frac{2.5\% \times 1.02}{1.02} = 0.0255 \text{ million.}$ Therefore the total costs of the buyer add up to €1,045,500, a reduction of €4,500 euro. If the supplier finances the whole six months, its price increases to €1.04 million. Since the buyer now has no financing costs, its benefits increase to €10,000.

These benefits can only be reaped however if the difference in financing rates between the buyer and supplier are caused by information asymmetries. If the extra 2 percent the buyer pays
for its financing is not deadweight cost of capital, but a premium for risk, there is no arbitrage opportunity. Otherwise, considering these financial and operational decisions jointly can create value in the supply chain.

**Pooling**

As I have established before, pooling is beneficial from a financial point of view. If I apply the realistic parameter settings from the case study in section 4.4 for example, 10 separately considered receivables of 5 million are worth 48.33 million to a supplier, but a pool of these receivables is worth 48.46 million, 0.27% more. It is interesting to find out whether pooling has also operational benefits.

Although 0.27% does not seem like very much, it can have quite an operational impact. If the risk free rate is 5%, a value of 48.33 million now is equal to 48.46 million three weeks from now. This means that by pooling, the supplier can without costs extend its payment period of currently three months by three extra weeks, an increase of almost 25%. This could improve the competitiveness of the supplier significantly and simultaneously reduce the buyer’s working capital.

**Increasing the benefits**

The literature has shown that making operational and financial decisions jointly can be valuable (see for example Xu and Birge, 2004 or Turcic, 2008). This might also be true if the financial decision of using IPR is combined with the operational aspects of the supplier’s pricing decision. The hypothesis is that if IPR allows for value creation potential, this potential can be enlarged by using the reduced financing costs to lower the price, which would increase the demand. If then the revenue increases, so does the nominal amount of the receivables, which in turn increases the (absolute) value creation potential of IPR.

If the supplier’s goal is to maximize its profit, its pricing decision basically depends on two parameters: the price elasticity of demand and the variable costs per unit. The profit is the revenues minus the costs and revenue equals price multiplied by demand. The relation between price and demand is described by the price elasticity $E_d$: the higher the absolute value of $E_d$, the stronger the demand for a product responds to a change in its price. The price elasticity depends on many factors: the current price, the product’s necessity, competition, brand loyalty, etcetera. Price elasticity is defined as (Tellis, 1988):

$$E_d = \frac{P}{S} \times \frac{dS}{dP}, \quad (5.1)$$

where $P$ is the price and $S$ the sales.
For almost all products $E_d$ is negative: when the price increases, sales decrease. As long as $-1 < E_d < 0$, demand for a product is inelastic: the percentage change in sales is smaller than that in price. This means that when the price is raised, the total revenue increases. Changing the price does not affect revenue for $E_d = -1$ and for elastic demand ($E_d < -1$), revenue will decrease if the price is raised.

When the price of a product is very low, demand will be inelastic: most customers would still be willing to buy the product when the price would be increased. The higher the price, the more elastic the demand will be. When the price becomes too high, customers will choose to buy the product from a competitor, buy a substitute product, or not buy this type of product at all. From a revenue point of view, the price should be increased until $E_d = -1$.

The supplier should not be optimizing its revenues however, but its profit. If the price is now increased even further, demand will become elastic. Sales will therefore decrease a little faster than the price increases, so the revenue decreases slightly. On the other hand, the variable costs of the sales will also decrease. As long as the revenue drop is small, it might be good to increase the price even though demand is elastic, since the profit will still grow. I assume a company will increase the price until the decrease in variable costs no longer offsets the decrease in revenue.

Now, when a supplier decides to use IPR and is able to reduce its financing costs, this means that its variable costs will decrease. If the supplier now decreases its price, its revenues will grow, since in the equilibrium price was elastic. Because the variable costs have been reduced, this revenue increase will be bigger than the increase in costs. A higher revenue also means that in absolute sense, the value creation potential of IPR increases (because the pool of receivables is bigger). Appendix D shows mathematically that combining this financial and operational decision is beneficial.
The previous chapters have shown that there is theoretical support for the value creation potential of IPR. Since the trends in trade finance and the financial regulatory environment also positively affect the potential to use IPR, RBS should consider implementing the product in practice. This final chapter covers some of the issues related to implementation.

First, a crucial assumption in the model is that the insurer will not default. In practice, the number of large trade credit insurers is limited. The most important players on this market are Euler Hermes, Coface, Atradius, and AIG. Before implementing the product, the bank should make sure how realistic this assumption is that these companies cannot default. If this assumption is not completely realistic, it should assess how big the risks and consequences are when insurer default occurs. These might be big, since if a credit insurer defaults, probably many buyers have defaulted on their receivables, causing high claims for the insurer. So if the insurer defaults, it is likely that also many buyers in the pool have defaulted, which in the case of IPR leads to a large loss for the bank. On the other hand, some of the insurers might be considered “too big to fail”, meaning that the government will protect these institutions from default because else the whole financial system will become unstable. This happened for example with AIG during the 2008 financial crisis. Government protection decreases the risk of insurer default.

Further, the implementation of the product requires an IT platform. To be able to handle the invoices from many different buyers without manually checking all of them, a rather sophisticated IT system is needed. This requires a relatively high upfront investment cost, which has to be recovered over a longer period of time. This creates a risk for the bank: if for some reason there is little demand for IPR, the bank will lose money due to the relatively high investment costs.

A way to reduce this risk is to invest in a system that is not only applicable to IPR, but also capable to handle for example e-invoicing, direct debiting and SEPA (Single Euro Payments Area). A direct debit is a financial transaction in which one party withdraws funds from another party’s bank account directly. SEPA is a payment-integration initiative to improve the efficiency of cross-border payments in Europe. According to a trade advisor within RBS who was involved in setting up IPR deals, the integration of these functions in one system is one of the most important requirements of clients. This view is supported by research of Celent (Pierron and Rajan, 2011), which shows that companies are demanding systems that integrate trade finance, payments, cash management, FX, and derivatives into one view.

For reversed factoring an important operational issue is to onboard suppliers. This can be difficult, because they might not see the value or have the right technology. IPR does not have such an issue, because the buyers do not have to know about the arrangement. Still, the parties should make an agreement about disclosing the deal to the buyers or not. The supplier might
want to keep the deal undisclosed, because of competition or negotiation reasons. Not disclosing requires however that the supplier remains the collection agent. This leads to a commingling risk: payments that are designated to the bank are mixed with a supplier’s own funds, which might be troublesome in case the supplier defaults.

Another trade-off is the choice between having a partial recourse on the supplier or insuring the full amount. If there is partial recourse, the supplier still keeps some of the credit risk. Without recourse, a supplier has no incentive to select proper buyers or monitor their payments, which might cause the quality of receivables to deteriorate. So a partial recourse prevents a supplier from selecting clients that are not trustworthy. However, it might also have the consequence that regulators no longer view the transaction of selling the receivables as a true sale. In that case the financing for the supplier is on-balance instead of off-balance, which has a detrimental effect on the supplier’s leverage ratio and possibly even on its rating. The impact and optimal level of recourse is an interesting topic for future research.

Finally, for IPR in an international setting, trade barriers and a lack of regulatory harmonization are operational issues that hinder the implementation. These issues are particularly relevant in developing economies, since the trade-related infrastructure in these countries lags behind that of developed countries considerably (Pierron and Rajan, 2011). The Free Trade Agreements for emerging economies are for instance less consistent with the World Trade Organization and other global rules. The impact and possible mitigation of these issues should be considered by the bank before closing an international IPR deal.
7 CONCLUSIONS AND RECOMMENDATIONS

The goal of this thesis is to find the benefits of Insured Pooled Receivables (IPR) for the involved parties, both on a financial and an operational level. The first section of this final chapter shows the findings and results. Subsequently, these results will be translated into recommendations for RBS. The last section discusses the limitations of this study and topics for future research.

7.1 Conclusions

IPR can be positioned in the growing body of research to Supply Chain Finance. It has been shown in this field that coordinating financing functions of participants in the supply chain can create value, for example via reversed factoring. There are two main trends that make this type of financing, based on trade transactions, very promising.

The first is the fact that international trade has been growing quicker than the world GDP in the last decade and this growth is predicted to continue: the world trade volume is expected to have quadrupled by 2040 (Buiter and Rahbari, 2011). The largest part of this growth will come from increasing trade between emerging economies, where the availability of traditional funding is relatively low. Therefore SCF programs can be of great value in this area. Secondly, the regulatory trends in the financial sector (i.e. the installment of Basel III) force banks to revise their product portfolio. Since SCF products provide collateral, they are less risky and require less RWA. This is an important reason for the popularity of invoice financing during the recent financial crisis: despite European bank lending declining overall, this market has shown double digit growth since 2009 (Demica, 2012).

The research question is whether IPR, in these favorable circumstances, is able to create value. I have established that in a simplified model where investors are risk neutral, IPR has no value creation potential. In reality, most investors are risk averse. There are different methods to account for risk, but I have shown that they all stem from one basic asset pricing formula and are in essence the same. Since the use of utility functions can also be justified by financial distress costs and the pecking order theory, I have used utility functions to account for risk throughout this thesis.

Since utility functions for risk averse firms are concave, these firms are willing to pay a certainty premium to avoid uncertainty. IPR is comparable with paying a certainty premium: the supplier receives a certain cash flow now instead of an uncertain one later, but has to pay a premium for this. However, this certainty premium needs not to be the same for all parties: different risk preferences lead to different risk premiums. I show that even in a simplified model with the transfer of only one receivable from supplier to insurer, the supplier is likely to have a higher certainty premium than the insurer. This means that IPR has value creation potential. This potential increases with the size of the risk and with larger differences in risk aversion or initial capital between insurer and supplier.
When the number of buyers in the pool is increased from one to multiple, correlation among obligors will play an important role. If the correlation is one, nothing will change compared to a situation with one buyer. For a correlation of zero, the certainty premium decreases in the pool size, because the variance of the cash flow is reduced via diversification. In practice correlations between zero and one are more common, but these are more difficult to model. Therefore I introduce the MCB model, which shows that the certainty premium is, as expected, decreasing in the correlation between obligors.

The implication of these findings is slightly less intuitive however: a lower risk on the pool of cash flows reduces the value of IPR. This can be explained by the fact that IPR’s main value creation potential stems from transferring a risk to a party with different risk preferences. Therefore IPR can create the most value when the risk on the pool of receivables is relatively high. The sensitivity analysis confirms this conclusion: the value creation potential of IPR is mainly affected by parameters that influence the (relative) size of the risk, such as the default probability, the Loss Given Default, correlation and the supplier’s risk aversion and initial capital.

The most important contribution of the bank is providing liquidity to the supplier, which is especially valuable when the supplier is financially constrained. Further, the bank might be able to improve payment behavior of buyers, reduce the LGD or negotiate a better deal with the insurer. In the end, an IPR deal can have a positive NPV, provide liquidity, and reduce the supplier’s financing costs. Financing costs can be reduced if IPR can avoid market imperfections such as bankruptcy costs or information asymmetries, or if it can free up funds that are used as ‘safety stock’.

These financial benefits of IPR can also positively affect the operations in the supply chain. Reduced financing costs can improve the supplier’s service level via reduced holding costs and increased liquidity can improve the supplier’s flexibility to make better operational decisions. The buyers’ service levels might be improved as well when the supplier extends the payment period. Finally, the interaction between the financial decision of using IPR and the operational aspects of a supplier’s pricing decision can optimize IPR’s benefits.

7.2 Recommendations

The findings of this thesis lead to the following recommendations for RBS:

- Continue the introduction of IPR at RBS. The product can create value in circumstances that are not uncommon, so it can be a valuable extension to the current product portfolio in many situations. IPR also fits well in the regulatory trend which requires banks to take less risk.

- Make sure RBS is able to use IPR and other SCF products in developing economies. Trade is expected to grow relatively fast in the upcoming decades, especially in developing
economies. Firms in these economies usually have difficulties in getting traditional financing, so the demand for SCF solutions can grow rapidly in these areas. However, RBS has to take into account that the infrastructure for trade financing in these countries in generally less developed.

- Target suppliers with relatively high risk portfolios of accounts receivable. In general this means smaller pools with high correlation, where the probability of default and LGD is relatively high. The value creation potential is highest for suppliers with such pools, while the insurer takes on (almost) all the risk.

- Convince suppliers to use IPR by showing that it has both financial and operational benefits. The product cannot only provide liquidity and reduce financing costs, but it can also improve the supplier’s service level or even operations in the whole supply chain.

- Thoroughly assess the probability that the insurer will default. Although this probability will usually be very low, the insurer’s creditworthiness is crucial to be able to use IPR as a low-risk product for the bank.

- Carefully select an IT platform for IPR. Investing in a system that is only able to do IPR transactions might be too risky. Therefore it would be good to invest in a multifunctional system, which is for instance also capable to handle e-invoicing, direct debiting and SEPA.

- Consider whether the deal should be disclosed and whether recourse should be used. Not disclosing might be preferable for the supplier, but it can lead to a commingling risk. Partial recourse might increase the quality of the receivables, but it can also impede the possibility to use IPR as off-balance sheet financing.

7.3 Limitations and future research

In this final section I will address some limitations to this study and suggest some possibilities for future research. First, the use of utility functions has some drawbacks. The core of the criticism is that it is in practice difficult to establish a risk aversion coefficient, that some axioms of utility theory are not consistent with people’s behavior (Kahnemann and Tsversky, 1979), and that expected utility theory cannot account for people’s ambiguity aversion (i.e. the preference of known risks over unknown ones). Although not all components of this criticism are relevant for IPR, it would be good to validate the results of this study using a different method that accounts for risk, such as risk neutral probabilities.

A second limitation is the use of Moody’s Correlated Binomial Default Distribution. Although this model quite realistically describes the default correlation in a portfolio, it has to make some simplifying assumptions. It assumes for instance that all obligors in the portfolio have the same
size and default probability. Another assumption is that conditional correlation is constant, which implies that if the first half of obligors in a portfolio defaults, the probability is almost 1 that they all default. In practice there are usually some stronger or more flexible firms that would survive such a crisis. Because each default model has its limitations, it would again be good to validate the results using a range of different credit risk models.

Thirdly, a valuable extension of the model would be to consider the value of liquidity explicitly. In this research, I have made some qualitative arguments why increased liquidity can be valuable for the supplier. If these benefits can be quantified, the value of IPR can be estimated more precisely. It can also help the bank to determine the extra value it brings to the table in an IPR deal, which could give an indication on how to distribute the created value among the different parties.

Finally, in a dynamic multiple period model there are different ways to deal with risk and discounting. I have chosen to calculate certainty equivalents for each period and discount these at the risk free rate, but it is also possible to use an intertemporal utility function that addresses an investor’s time preferences more explicitly. A comparison between these methods would be a good topic for future research.
REFERENCES


APPENDICES

A List of abbreviations

APR Absolute Priority Rule
CARA Constant Absolute Risk Aversion
CDO Collateralized Debt Obligation
CE Certainty equivalent
CF Cash flow
CRRA Constant Relative Risk Aversion
EOQ Economic Order Quantity
EU Exponential Utility
FX Foreign Exchange
GDP Gross Domestic Product
IPR Insured Pooled Receivables
LGD Loss Given Default
MCB Moody’s Correlated Binomial
MTO Make To Order
MTS Make To Stock
NPV Net Present Value
PU Power Utility
RBS Royal Bank of Scotland
RWA Risk Weighted Assets
SCF Supply Chain Finance
SCM Supply Chain Management
SEPA Single Euro Payments Area
WACC Weighted Average Cost of Capital
This appendix aims to prove the following theorem.

**Theorem 1.** Let \( X_1, ..., X_n, n \geq 1 \) be identically distributed stochastic variables. Then we have that

\[
E \left[ \prod_{i=1}^{n} (1 - X_i) \right] = \sum_{j=0}^{n} (-1)^j \binom{n}{j} E \left[ \prod_{i=1}^{j} X_i \right]^2.
\]

We will prove this through a series of lemmas.

**Lemma 1.** Let \( X_1, ..., X_n, n \geq 1 \) be a set of commuting variables (i.e. \( x_i x_j = x_j x_i \)). Then we have that

\[
\prod_{i=1}^{n} (1 - x_i) = \sum_{j=0}^{n} (-1)^j \sum_{I:|I|=j} \prod_{i \in I} x_i.
\]

**Proof** We shall prove this through induction. First consider the case \( n = 1 \). Then clearly the left hand side reads \( 1 - x_1 \), as does the right hand side, because the only subset \( I \subseteq \{1\} \) such that \( |I| = 1 \) is \( \{1\} \) itself. Now assume the statement is true for \( n = m \) and consider the case \( n = m + 1 \). We then have that

\[
\prod_{i=1}^{m} (1 - x_i) = \prod_{i=1}^{m} (1 - x_i)(1 - x_{m+1}) = \prod_{i=1}^{m} (1 - x_i) - \prod_{i=1}^{m} (1 - x_i)x_{m+1}.
\]

We can now apply the induction hypothesis to both terms on the right hand side. Doing this clearly gives

\[
\prod_{i=1}^{m} (1 - x_i) = \sum_{j=0}^{m} (-1)^j \sum_{I:|I|=j} \prod_{i \in I} x_i \quad \text{and} \quad \prod_{i=1}^{m} (1 - x_i)x_{m+1} = \sum_{j=0}^{m} (-1)^j \sum_{|I|=j} \prod_{i \in I} x_i x_{m+1}.
\]

---

1 I would like to thank Ralph Klaasse, BSc Applied Mathematics and BSc Applied Physics, for his help on deriving this formula.

2 The right hand side is clearly equivalent to the expression \( 1 + \sum_{j=1}^{n} (-1)^j \binom{n}{j} E \left[ \prod_{i=1}^{j} X_i \right] \), as \( \binom{n}{0} = 1 \) and empty products are equal to unity by definition.
For each index set $I$ in the second expression, define a new index set $J := I \cup \{m + 1\} \subseteq \{1, \ldots, m + 1\}$. Since $I$ is a subset of $\{1, \ldots, m\}$, it is clearly also a subset of $\{1, \ldots, m + 1\}$. In the first expression the index set $J$ never contains $m + 1$, in the second expression it always does, so by conditioning we can rewrite the above to

$$\prod_{i=1}^{m} (1 - x_i) = \sum_{j=0}^{m} (-1)^j \sum_{\substack{J \subseteq \{1, \ldots, m + 1\} \quad \text{s.t.} \quad |J| = m + 1 \quad \text{and} \quad \forall i \in J \quad i \notin \{m + 1\}} } \prod_{i \in J} x_i \quad \text{and} \quad \prod_{i=1}^{m} (1 - x_i) x_{m+1} = \sum_{j=0}^{m} (-1)^j \sum_{\substack{J \subseteq \{1, \ldots, m + 1\} \quad \text{s.t.} \quad \forall i \in J \quad i \notin \{m + 1\}} } \prod_{i \in J} x_i$$

We now subtract the above to expressions to obtain

$$\prod_{i=1}^{n} (1 - x_i) = \sum_{j=0}^{m} (-1)^j \sum_{\substack{J \subseteq \{1, \ldots, m + 1\} \quad \text{s.t.} \quad \forall i \in J \quad i \notin \{m + 1\}} } \prod_{i \in J} x_i + \sum_{j=0}^{m} (-1)^{j+1} \sum_{\substack{J \subseteq \{1, \ldots, m + 1\} \quad \text{s.t.} \quad \forall i \in J \quad i \notin \{m + 1\}} } \prod_{i \in J} x_i$$

$$= 1 + \sum_{j=1}^{m} (-1)^j \sum_{\substack{J \subseteq \{1, \ldots, m + 1\} \quad \text{s.t.} \quad \forall i \in J \quad i \notin \{m + 1\}} } \prod_{i \in J} x_i$$

$$= 1 + \sum_{j=1}^{m} (-1)^j \sum_{\substack{J \subseteq \{1, \ldots, m + 1\} \quad \text{s.t.} \quad \forall i \in J \quad i \notin \{m + 1\}} } \prod_{i \in J} x_i + \sum_{j=0}^{m} (-1)^{j+1} \sum_{\substack{J \subseteq \{1, \ldots, m + 1\} \quad \text{s.t.} \quad \forall i \in J \quad i \notin \{m + 1\}} } \prod_{i \in J} x_i$$

$$= 1 + \sum_{j=1}^{m} (-1)^j \sum_{\substack{J \subseteq \{1, \ldots, m + 1\} \quad \text{s.t.} \quad \forall i \in J \quad i \notin \{m + 1\}} } \prod_{i \in J} x_i$$

$$= 1 + \sum_{j=1}^{m} (-1)^j \sum_{\substack{J \subseteq \{1, \ldots, m + 1\} \quad \text{s.t.} \quad \forall i \in J \quad i \notin \{m + 1\}} } \prod_{i \in J} x_i$$

$$= \sum_{j=0}^{m+1} (-1)^j \sum_{\substack{J \subseteq \{1, \ldots, m + 1\} \quad \text{s.t.} \quad \forall i \in J \quad i \notin \{m + 1\}} } \prod_{i \in J} x_i$$

where in the third line we used that the only subset $J \subseteq \{1, \ldots, m + 1\}$ satisfying $|J| = m + 1$ is $\{1, \ldots, m + 1\}$ itself. In the last line we combined the terms again, switching back to calling our index sets $I$ instead of $J$. We conclude that the statement is also true for $n = m + 1$, so that by the principle of induction it is true for any $n \geq 1$.

For the proof of the theorem, we will also need the following result.

**Lemma 2.** Under the assumptions of Theorem 1, for $I \subseteq \{1, \ldots, n\}$ with $|I| = m$ we have that

$$E \left[ \prod_{i \in I} X_i \right] = E \left[ \prod_{i=1}^{m} X_i \right]$$
Proof  This follows straight from the definition of the expected value, where we heavily rely on
the assumption that the $X_i$ are identically distributed.

Proof of theorem 1  By lemma 1 with $x_i = X_i$, we get that

$$\prod_{i=1}^{n} (1 - X_i) = \sum_{j=0}^{n} (-1)^j \sum_{\|I\|=j} \prod_{i \in I} X_i$$

We now take the expected value on both sides of the above equality. Because the function $E$ is
linear, we then see that

$$E\left[\prod_{i=1}^{n} (1 - X_i)\right] = \sum_{j=0}^{n} (-1)^j \sum_{\|I\|=j} E\left[\prod_{i \in I} X_i\right] = \sum_{j=0}^{n} (-1)^j \sum_{\|I\|=j} E\left[\prod_{i=1}^{j} X_i\right]$$

where the last equality follows from Lemma 2. Now we only need to count the amount of
subsets $I \subseteq \{1,\ldots,n\}$ satisfying $|I| = j$, as the inner summand on the right hand side is
independent of $I$. It is clear that the amount of such $I$ is equal to $\binom{n}{j}$. Using this we obtain

$$E\left[\prod_{i=1}^{n} (1 - X_i)\right] = \sum_{j=0}^{n} (-1)^j \binom{n}{j} E\left[\prod_{i=1}^{j} X_i\right]$$

as desired.
C Effect pooling & correlation

Table A.1: Effect of pooling & correlation with power utility, $w_0 = 0, \gamma = 0.6, p = 0.05$

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</table>

Figure A.1: Effect of pooling & correlation with power utility, $w_0 = 0, \gamma = 0.6, p = 0.05$

Table A.2: Effect of pooling & correlation with exponential utility, $w_0 = 0, \alpha = 0.02, p = 0.05$

<table>
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Figure A.2: Effect of pooling & correlation with exponential utility, $w_0 = 0, \alpha = 0.02, p = 0.05$
Table A.3: Effect of pooling & correlation with power utility, \( w_0 = 0, \gamma = 0.9, \rho = 0.05 \)

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Figure A.3: Effect of pooling & correlation with power utility, \( w_0 = 0, \gamma = 0.9, \rho = 0.05 \)

Table A.4: Effect of pooling & correlation with power utility, \( w_0 = 0, \gamma = 0.3, \rho = 0.05 \)

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Figure A.4: Effect of pooling & correlation with power utility, \( w_0 = 0, \gamma = 0.3, \rho = 0.05 \)
Table A.5: Effect of pooling & correlation with power utility, \( w_0 = 50, \gamma = 0.6, p = 0.05 \)

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Figure A.5: Effect of pooling & correlation with power utility, \( w_0 = 50, \gamma = 0.6, p = 0.05 \)

Table A.6: Effect of pooling & correlation with power utility, \( w_0 = 0, \gamma = 0.6, p = 0.1 \)

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Figure A.6: Effect of pooling & correlation with power utility, \( w_0 = 0, \gamma = 0.6, p = 0.1 \)
Pricing decision

This example shows the value of combining the financial decision on doing an IPR deal with the operational aspects of the pricing decision. For this example I will assume a demand \((D)\) curve that is linearly dependent on the price \((P)\) of the product. I assume that the demand is always between 0 and \(M\): \(D \in [0, M]\). The demand curve is simple:

\[
D = M - P.
\]

And therefore also \(P \in [0, M]\). Further I define the variable cost \(c\) and the profit \(\pi\). The object is to maximize the profit:

\[
\pi = (P - c)D
\]

\[
\pi = (P - c)(M - P)
\]

\[
\pi = -P^2 + (c + M)P - cM
\]

To find the optimal pricing \(P_{opt}\), I take the derivative and set it equal to zero:

\[
-2P_{opt} + c + M = 0
\]

\[
P_{opt} = \frac{c + M}{2}
\]

With this pricing, the maximum profit is:

\[
\pi_{max} = -P_{opt}^2 + (c + M)P_{opt} - cM = -\frac{(c + M)^2}{4} + \frac{(c + M)^2}{2} - cM = \frac{c^2}{4} + \frac{M^2}{4} - \frac{cM}{2}
\]

Now, an IPR deal is made, which decreases the financing cost of the supplier. Since the financing cost decreases, the variable costs are also decreasing. I define the new variable costs as:

\[
c' = \lambda c,
\]

where \(\lambda \in [0,1]\).

Under these new variable costs, the optimal pricing decreases to \(P'_{opt} = \frac{c' + M}{2}\) and the maximum profit increases to \(\pi'_{max} = \frac{c'^2}{4} + \frac{M^2}{4} - \frac{c'M}{2}\). Now, the question is whether value is lost when the pricing is not updated to the new parameter settings. Therefore I will compare the
maximal profit under the new pricing with the profit under the old pricing. The difference
between the two is the gain of making this financial and operational decision in conjunction.

\[
\text{Gain} = \pi'(P_{opt}') - \pi'(P_{opt}) = \left( \frac{c' + M}{2} - c' \right) \left( M - \frac{c' + M}{2} \right) - \left( \frac{c + M}{2} - c' \right) \left( M - \frac{c + M}{2} \right) = \\
\frac{c'M}{2} + \frac{M^2}{4} - \frac{(c' + M)^2}{4} - c'M + \frac{c'^2}{2} + \frac{c'M}{2} - \left[ \frac{cM}{2} + \frac{M^2}{4} - \frac{(c + M)(c' + M)}{4} - \frac{c'M}{2} + \frac{c'c}{2} + \frac{c'M}{2} \right] = \\
\frac{M^2}{4} + \frac{c'^2}{2} - \frac{c'M}{2} - \left[ \frac{cM}{2} + \frac{M^2}{4} - \frac{c^2}{4} - \frac{cM}{2} - \frac{c'M}{2} + \frac{c'c}{2} \right] = \\
\frac{c'^2}{4} + \frac{c^2}{4} - \frac{c'c}{2} = \\
\frac{(1 + \lambda^2)\lambda^2}{4} - \frac{\lambda c^2}{4} = \\
\frac{c^2 \left( \lambda^2 - 2\lambda + 1 \right)}{4}
\]

Since \( \lambda \in [0,1] \), it holds that \( \lambda^2 - 2\lambda + 1 \geq 0 \), so there is a gain when the operational pricing
decision is adapted to the financing decision.