Determining Loss Models of Permanent Magnets and Investigation of the Losses in Permanent Magnets by Finite Element Analysis

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Internship Report:
Determining Loss Models of Permanent Magnets and Investigation of the Losses in Permanent Magnets by Finite Element Analysis

Bob van Ninhuijs

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Supervisors:
Dr. Dipl.-Ing. Christian Kral (Austrian Institute of Technology)
Prof. dr. Elena Lomonova (Eindhoven University of Technology)
## Contents

1 Introduction \hfill 3

2 Loss models \hfill 5
   2.1 Causes of losses in the rotor \hfill 5
   2.2 Solutions \hfill 5
   2.3 Analytical calculation \hfill 6

3 Implemented loss models \hfill 6
   3.1 Analytical calculation of commutation losses \hfill 6
      3.1.1 The analytical model \hfill 6
      3.1.2 Results \hfill 7
      3.1.3 Compromises \hfill 8
   3.2 Analytical calculation of PWM losses \hfill 10
      3.2.1 The analytical model \hfill 10
      3.2.2 Results \hfill 11
      3.2.3 Converting Bosch Rexroth machine properties \hfill 11

4 Loss calculation with FEM analysis \hfill 13
   4.1 The settings \hfill 13
   4.2 No excitation currents \hfill 15
   4.3 Sinusoidal excitated current \hfill 17
   4.4 PWM excited current \hfill 17
   4.5 Comparison \hfill 21

5 Conclusion and recommendation \hfill 25
   5.1 Conclusions \hfill 25
   5.2 Recommendation \hfill 26

A Materials and their physical properties \hfill 27
   A.1 Development of permanent magnets \hfill 27
   A.2 Skin effect \hfill 27
   A.3 Conclusion \hfill 29

B Functions, functions and more functions \hfill 30
   B.1 Functions for the commutation model \hfill 30
      B.1.1 The functions \hfill 30
      B.1.2 Reformulation for $C_{1_n}$ \hfill 31
   B.2 Functions for the PWM model \hfill 32
   B.3 Bessel and Kelvin functions \hfill 34

References \hfill 35
1 Introduction

The first magnetic material ever used is probably the “lodestone” which was the only magnetic material available in ancient times. Lodestone means literally “way stone” because it guided the marines on their way. The term “magnet” comes from the fact that lodestone was found in Magnesia, a district in Thessaly (one of the 13 regions of Greece). The first way to make artificial magnets was to “touch” or magnetize iron needles by lodestone. The first use of these needles were probably in the compass.

The evolution of creating permanent magnets can be divided in three milestones. The first was in the 19th century were very weak magnets with stability problems were used in devices because otherwise the devices would not function (e.g. a compass). Rather large sizes of the magnets were tolerated in these devices. The second milestone occurred in the 1940s were the permanent magnets could compete with the electromagnets, both functionally and economically. They were used in devices as loudspeakers and small d.c. machines. In the 1970s the most important milestone happened; the properties nearly improved tenfold. The rare-earth type became available, which allowed possibilities which weren’t possible with electromagnets. Because of the efficiency the cost were justifiable in spite of their very expensive raw materials and they had a high power and torque density which has major advantages over electromagnetism [1]. This is also the reason they still gain in popularity. In [2] two machines, one with ferrite magnets and the other with NdFeB magnets, with the same back EMF and output power were compared. The result of this study was that however the inductance of the ferrite is higher, so less iron losses, the machine with NdFeB magnets has a better overall efficiency. The reason for this is a great difference in the rotor structure, the rotor of the NdFeB is smaller than its opponent and therefore has less mechanical losses. However when the machine becomes bigger the iron losses also becomes bigger. At a given size the difference of the iron losses between the machines becomes bigger than the additional losses, so somewhere there is a turning point which makes the ferrite machine more efficient. This fact and that NdFeB is more expensive explains the reason that NdFeB is more used in small and medium sized machines.

The rare-earth magnets are, as everything in this world, not perfect. There are losses within these magnets that result in heat, which causes demagnetization when the magnets become too hot. This demagnetization is permanent and will cause change of properties of the machine. This is something that is not wanted because it could be possible that the machine then does not function anymore for the function it was designed for. Determining these losses in the permanent magnets is something that should already be taken into account when designing a machine. The machine should be designed in a way that the demagnetization does not occur. Also when the machine is used in a bigger system like a car the losses in the magnets should be monitored to see if they will not demagnetize. Also determining the losses in a bigger system makes it possible to determine the heat flow of this system or even determine a more accurate efficiency of the system. These are some of the many advantages when the losses in the magnets can be determined quickly.

These losses are mostly calculated using FEM analysis, but this analysis is very time consuming. A second problem with FEM is that is takes too much time to really check the machine for every situation possible or to optimize the machine. A third problem is that a FEM analysis can’t be used in a simulation of a bigger system like for that example again the car. Therefore an analytical model of the losses in a magnet is wanted. This analytical model should make it possible to get a quick estimate of the losses that arise in the magnets.

There has been a lot of work done in the field of determining an accurate analytical calculation. A lot of writers claim to have an improved model based on a previous model from there self or other writers. There have been models for brushless DC (BLDC) machines or brushless AC (BLAC) machines. Some models can be used for BLDC machines and BLAC machines and even account for two winding topologies.
are also papers which determine the losses of a BLAC machine with an fractional winding topologie. All of these models are only using commutation excited currents and even several papers even don’t mention which excitation they uses. Only one paper was found that claims to estimate the losses using an pulse-width modulation (PWM) excitation. These papers will be discussed in more detail in this report.

First the causes of the losses in the magnets are discusses, then there are some solution mentioned how to reduce the losses. This is done because to get a feeling about what causes the losses and how to reduce the effect of these causes. Then there are two models implemented were one is developed for commutation excitation and the other for PWM excitation. The results of these analytical models are further discussed. To verify these analytical models there is an FEM analysis done. But because the model is developed for PWM excitation the FEM analysis must also have a PWM excitation. Before this PWM excitation can be accomplished in the FEM program some additional data must be calculated using FEM. This additional data is the back EMF and the induced voltage when there flows a current. During these calculations also the losses are calculated to have a discussion about what the difference is between a PWM excitation and a sinusoidal excitation. Further this paper will be finalized with an conclusion and a recommendation.
2 Loss models

The goal of this project was to determine the loss models of permanent magnets (PM) with respect to permanent magnet synchronous machines (PMSM). These models would then be implemented in a PMSM model that is programmed in the Modelica programming language, to get a more accurate model of a PMSM. With this model it would be possible to get also a more accurate loss and heat flow calculations.

2.1 Causes of losses in the rotor

It already begins by looking at the machine when determining the back EMF, so no excitation currents just an open circuit. Because of the slot openings in the stator the flux density changes (the reluctance changes) when the rotor rotates. When the flux density changes there are eddy currents excited in the magnets and so there are losses. Knowing this it is no surprise that there are losses due to the stator and rotor shapes, which is analyzed in [3], [4] and [5]. The stator winding topology can also affect the stator losses [6]. The windings give rise to space harmonics so every winding topology has different harmonics and so different losses.

The excitation like in commutation and PWM control produce in the windings time harmonic losses due to imperfect currents. For the commutation control these are rectangular currents instead of sinusoidal currents, with the PWM control there is a ripple on the sinewave which causes these time harmonics. Rectangular currents also cause space harmonics due to the limited number of slots, it means basically that the spatial magnetomotive force (MMF) "jumps" at each slot. It should be noted that PWM has higher harmonics than the commutation excitation and so there is more skin effect, but these higher harmonics have less influence than the harmonics from the commutation control. This means that the losses in a commutation excited machine are bigger than those of a PWM excited machine. It should also be noted that commutation is probably more often used by a machine in DC mode and PWM is probably more used by a machine in AC mode. Therefore the losses in a DC mode machine are often larger than the losses in an AC mode machine.

2.2 Solutions

The two most known possibilities to reduce eddy currents is optimization and segmentation. By optimizing the stator slots and winding topologies you can reduce the space and time harmonics and so also the eddy currents. Unfortunately it has to be done with FEM analysis what takes again a lot of time. An other and more popular solution to reduce the eddy currents is to divide the magnets into segments. The idea is basically the same of that of a transformer, which is laminated to reduce the eddy currents. It is possible to divide the magnets in the circumferential and/or in the axial direction. When this is done the eddy currents will reduce significantly with respect to the number of segments. However there rises here a new problem; because the reduced eddy-currents in the magnets cause less shielding towards the rotor. The result is that the losses increase in the rotor. To avoid this problem we have to laminate the rotor [7]. A second effect of segmentation is that the losses in the segments can be significantly different. Such a distribution of eddy currents can cause large temperature difference and can result in partially demagnetization [8]. A third problem can be when the skin effect is not negligible anymore here segmentation can increase the losses in the magnets. However there is an optimum between skin effect and the number of segmentation to get the lowest losses [9]. In Appendix A is explained in more detail what the penetration depth is. Here some of the penetration depth of the most common magnetic material is discussed and compared with more known skin effect material like copper, aluminium and steel.
2.3 Analytical calculation

The reason to have an analytical calculation is because it gives a fast estimation of what the losses will be. The more accurate FEM analysis are very time consuming and therefore also very expensive. An example for wanting an estimation is for optimization, the setting for a FEM analysis starts often with some educated guesses and ends most of the times in the “trial and error” method. Also it is wanted for simulation of larger systems, like a machine in a car. Here the variables change often in a short time which can not be analysed in one FEM model, and also the FEM model would be to slow to get a fast result. With an analytical calculation it would be even possible to do realtime calculation of the losses using a DSP. With these calculation we can also calculate the temperature of the magnets and so monitor the machine. This is necessary because when the magnets are getting too hot they will demagnetize (NdFeB demagnetizes at about 120°C [10]).

There are several papers which determined analytical models to calculate the losses in the permanent magnets. In [11] there is an analytical model introduced that calculates the losses in the magnets and rotor core due to commutation excitation. In [12] the model of the previous paper is improved and should give a better result. The disadvantage of the previous two papers is that they only are usable for BLDC machines, were in [13] they found a way to determine a model that can be usable for both BLDC and BLAC. The reason that its suitable for BLDC and BLAC machines appears to be that its takes the time harmonics and spatial harmonics into account and associates it with overlapping and non overlapping. There are also analytical models for a BLAC machine with fractional number of slots per pole [14,15]. In [8] and [16] there are analytical models for segmented magnets determined. The main problem for the previous mentioned papers is that the analytical models are all for commutation excited machines determined. In several papers the way of excitation is not even mentioned, here we can only guess. What is known is that non of these papers use PWM excitation in there models so therefore it is guessed that they use commutation excitation. There is one paper found which determined the losses due to PWM switching, namely [17]. There is a model introduced which calculates the losses in the magnets, rotor core and stator core.

3 Implemented loss models

3.1 Analytical calculation of commutation losses

3.1.1 The analytical model

Before we start implementing the PWM model we first start with implementing the commutation model of the same author. This model, explained in [11], is easier than the PWM model and we want to build up experience on how we can program such a model. This analytical model divides the rotor into three parts, the magnets, the rotor section that is shielded by the magnets and the rotor section between the magnets (the interpolar section) which is marked with diagonal lines in figure 4.2.

The losses in the shielded section of the rotor is calculated with,

\[ P_{\text{core}} = 2p \frac{A_{1m}^2 LR' \sigma_1 \left( \frac{\pi}{p} - \psi' - \frac{1}{p} \sin (p\psi') \right)}{2\sigma_2} \cdot \sum_{n=1}^{\infty} \text{Real} \left\{ \frac{C_1^*}{\sigma_1^*} \frac{9}{2|C_n|^2 \frac{3\pi}{2} \left(36n^2 - 1\right)} \right\} \] (3.1)

the losses that arise in the magnet are calculated using,

\[ P_{\text{magnet}} = 2p \frac{A_{1m}^2 LR_{ad} \left( \frac{\pi}{p} - \psi' - \frac{1}{p} \sin (p\psi') \right)}{2\sigma_2} \cdot \sum_{n=1}^{\infty} \text{Real} \left\{ \frac{1}{C_2^*} \frac{9 \cdot |C_2|^2}{2\pi^2 \left(36n^2 - 1\right)} - P_{\text{core}} \right\} \] (3.2)
and the losses in the interpolar section of the rotor is calculated by,

\[
P'_{\text{core}} = \frac{2p^2}{p-1} \frac{L R' \left( \psi' + \frac{1}{2} \sin(p \psi') \right)}{2 \sigma} \cdot \sum_{n=1}^{\infty} \text{Real} \left\{ \frac{\tau_{1n}}{C_{1n}} \right\} \frac{9}{2\pi^2(36n^2-1)} \left| \frac{R^p_{1d} + R^p_{1q}}{2R_{1d}^p R_{1d}^q} + \frac{p \mu_1}{4R_{1d}^2 \tau_{1n} c_{1n}} \right|^2
\]

where \( p \) is the number of poles, \( L \) is the length of the motor, \( R' \) is the radius of the rotor without the magnet, \( R_{od} \) is the radius of the rotor without the magnet, \( \psi' \) is the half of the angle between the magnets, \( \mu_z \) is the permeability and \( \sigma_z \) stands for the conductivity. The subscript \( z \) stands for the material which is 1 for the rotor, 2 for the magnet and 3 for the interpolar section. The definition of \( R', R_{od} \) and \( \psi' \) can also be seen in figure 3.1. The functions \( A_{1n}, C_{1n}, C_{2n}, C \) and \( \tau_{1n} \) are presented in appendix B.1 to improve the readability of this report.

Unfortunately in the paper there was no information which machine properties were used to verify this model, therefore the properties of Bosch Rexroth MSK071E-0450 were used. The used data of this machine are presented in table 3.1. The Matlab program that is created has the same kind of figures like those presented in the paper, but it was not checked by FEM or measurements. It is also important to note that the Bosch Rexroth machine is an AC machine and the analytical model is determined for a DC machine. Therefore we have to make some compromises in the definition of the variables defined in table 3.1.

### 3.1.2 Results

Using this analytical method we can investigate what the losses in function of the harmonics, the mechanical speed and the current is. In figure 3.2 the losses per harmonic order can be seen. To make this figure we used a mechanical speed of 6000 rpm and a current of 28.3A (amplitude value), the machines nominal mechanical speed and the stator phase current. This figure is calculated till the 30th harmonic, because after the 30th harmonic an ill-conditioned situation will occur which means that the numbers become to big for the computer. Even when we apply the reformulation presented in appendix B.1.2 it will reach such a condition. However we can see that the losses of the 30th harmonic are very small with respect to the first harmonic which means that they any higher harmonics can be neglected.

How the losses increase with mechanical speed can be seen in figure 3.3. This figure was created with the nominal current and was calculated till the 30th harmonic. Here it can be seen that the losses increase like a logarithmic function when the speed increases.

The losses in function of the current are shown in figure 3.4, also here we don't go above the nominal
current. The results in figure 3.4 a) are calculated using the nominal speed and the harmonics are calculated until the 30th harmonic. In this figure it can be seen that the losses increase exponentially when we increase the current. It can also be seen that the losses in the magnets increase much more than those in the rotor core. This is because the magnets shield the rotor from eddy currents [7] and the interpolar section is very small (see also $\psi'$ in table 3.1).

In figure 3.4 b) is shown what the total losses are in function of the current for different mechanical speeds. It shows that the losses have the same function properties but the higher the speed the steeper the curve. Which is also logical because the more energy there flows the more losses there will be.

3.1.3 Compromises

The properties defined in table 3.1 were found in the datasheet of the Bosch Rexroth machine. The values for $q$ and $q_p$ are an approach because this machine uses a fractional winding topology, which means that the number of slots per pole is a fraction. The total number of slots are 36 and we have 8 poles, which means that we have 4.5 slots per pole and 1.5 slots per pole per phase because this machine works with 3 phases. To compromise this we choose to have 2 slots per pole per phase and because we have 3 phases this means 6
Figure 3.4: Losses in function of the current, a) total and regional losses, b) total losses for different speeds

slots per pole. This model is not developed for fractional windings so choosing 5 for the number of slots per pole will probably give a worse estimate.

The Bosch Rexroth machine has 4 times a 5 slot pitch winding of 10 turns and 2 times a 3 slot pitch winding of 11 turns which results in a number of coil turns per phase of \( \mathcal{N}_{ph} = 4 \cdot 10 + 2 \cdot 11 = 62 \text{ turns} \).

Further, \( \psi' \) is calculated by calculating the complete contour (using \( R_{od} \)) which we then divide by the number of poles to get the length of one pole pitch. In the datasheet it says that the magnet is 23.5mm, so we calculate the length of the interpolar section by subtracting the magnet width from the pole pitch length. By dividing the interpolar width by the contour and then multiplying this with \( 2\pi \text{rad} \) gives us the angle of the interpolar section. Now we only have to divide this by two and then we have \( \psi' \).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_0 )</td>
<td>permeability of free space</td>
<td>( 4\pi e^{-7} )</td>
<td>( \text{H/m} )</td>
</tr>
<tr>
<td>( \mu_1 )</td>
<td>permeability of the rotor</td>
<td>( 744 \cdot \mu_0 )</td>
<td>( \text{H/m} )</td>
</tr>
<tr>
<td>( \mu_2 )</td>
<td>permeability of the magnet</td>
<td>( 1.04457 \cdot \mu_0 )</td>
<td>( \text{H/m} )</td>
</tr>
<tr>
<td>( \mu_3 )</td>
<td>permeability of the interpolar section</td>
<td>( \mu_0 )</td>
<td>( \text{H/m} )</td>
</tr>
<tr>
<td>( \sigma_1 )</td>
<td>conductivity of the rotor</td>
<td>( 2270000 )</td>
<td>( \text{S/m} )</td>
</tr>
<tr>
<td>( \sigma_2 )</td>
<td>conductivity of the magnet</td>
<td>( 714000 )</td>
<td>( \text{S/m} )</td>
</tr>
<tr>
<td>( \sigma_3 )</td>
<td>conductivity of the interpolar section</td>
<td>( 0 )</td>
<td>( \text{S/m} )</td>
</tr>
<tr>
<td>( L_r )</td>
<td>Stack length</td>
<td>( 160e^{-3} )</td>
<td>( \text{m} )</td>
</tr>
<tr>
<td>( R' )</td>
<td>rotor radius without the magnet</td>
<td>( 31.2e^{-3} )</td>
<td>( \text{m} )</td>
</tr>
<tr>
<td>( R_{od} )</td>
<td>the outside radius of the rotor</td>
<td>( 33.8e^{-3} )</td>
<td>( \text{m} )</td>
</tr>
<tr>
<td>( \mathcal{N}_{ph} )</td>
<td>coil turns per phase</td>
<td>62</td>
<td>( \text{turns} )</td>
</tr>
<tr>
<td>( p )</td>
<td>number of poles</td>
<td>8</td>
<td>( \text{poles} )</td>
</tr>
<tr>
<td>( q )</td>
<td>slots per pole per phase</td>
<td>2</td>
<td>( \text{slots} )</td>
</tr>
<tr>
<td>( q_p )</td>
<td>Slots per pole</td>
<td>6</td>
<td>( \text{poles} )</td>
</tr>
<tr>
<td>( \psi' )</td>
<td>Interpolar angle, half of the angle between the magnets</td>
<td>0.0454</td>
<td>( \text{rad} )</td>
</tr>
</tbody>
</table>

Table 3.1: Bosch Rexroth MSK071E-0450 properties
3.2 Analytical calculation of PWM losses

3.2.1 The analytical model

In this section the PWM model introduced by [17] is implemented and the results are discussed. Also with this paper there was not enough information to create the exact result as presented in the paper. So the implemented program can not directly be compared with the results obtained in the paper. Further the model that is introduced is able to calculate the rotor loss, the magnet loss and the stator loss. In this report we only concentrate on the rotor and magnet losses because its our goal to determine the losses in the rotor and magnet. Further the model will be compared with a FEM analysis to determine the usability and correctness.

This model divides the rotor into three parts, the magnets, the rotor section that is shielded by the magnets and the rotor section between the magnets (the interpolar section). The losses for the shielded rotor sections is,

\[ P_{\text{core}} = pLR' \int_{\psi'}^{(\frac{2}{3})\pi-\psi'} \left\{ \sum_{k=1}^{\infty} \left[ \frac{\sigma_1 C'_1}{\sigma_2 C_1 \cdot 2\sqrt{2}} A_{k,m} \cdot \cos \left( k \left( \frac{p}{2} \psi - \frac{2\pi}{3} \right) \right) \right] \right\}^* d\psi \]

(3.4)

the losses in the magnets are,

\[ P_{\text{magnet}} = pL_{\text{rod}} \int_{\psi'}^{(\frac{2}{3})\pi-\psi'} \left\{ \sum_{k=1}^{\infty} \left[ \frac{1}{\sigma_2 C_2 \cdot 2\sqrt{2}} A_{k,m} \cdot \cos \left( k \left( \frac{p}{2} \psi - \frac{2\pi}{3} \right) \right) \right] \right\}^* d\psi - P_{\text{core}} \]

(3.5)

and the losses in the interpolar section of the rotor is calculated by,

\[ P_{\text{core}}' = pLR' \sum_{i=1}^{2} \int_{\psi_{i,1}}^{\psi_{i,2}} \left\{ \sum_{k=1}^{\infty} \left[ \frac{A_{k,m}}{2\sqrt{2}} \cdot \cos \left( k \left( \frac{p}{2} \psi - \frac{2\pi}{3} \right) \right) \right] \right\}^* \left\{ \left( \frac{R_{e}^p + R_{e}^{np}}{2R_{e}^p - R_{e}^{np}} \right) \right\} + \left\{ \left( \frac{R_{ad}^p - R_{ad}^{np}}{2R_{ad}^p - R_{ad}^{np}} \right) \right\} \}

(3.6)

where \( p \) is the number of poles, \( L \) is the length of the motor, \( R' \) is the radius of the rotor without the magnet, \( R_{\text{rod}} \) is the radius of the rotor without the magnet, \( \psi_{i,1}/\psi_{i,2} \) and \( \psi_{21}/\psi_{22} \) are \( 0/\psi' \) and \( \frac{2}{p} \pi - \psi'/\frac{2}{p} \pi \) respectively, \( \mu_z \) is the permeability and \( \sigma_z \) stands for the conductivity. The subscript \( z \) stands for the material which is 1 for the rotor, 2 for the magnet and 3 for the interpolar regions. The functions \( A_{1,m}, C'_1, C_2, \text{ and } \gamma_1 \) are placed in appendix B.2 to improve the readability of this report.

There was no information of the size of the machine in the paper, only the material properties were mentioned. So the properties of the Bosch Rexroth MSK071E-0450 are used, which are shown in table 3.2. Here the same compromises as mentioned in section 3.1.3 are applied because the analytical model is developed for and DC machine however the Bosch Rexroth is an AC machine. The figures can not be compared directly.
with the figures from the paper, but they are very similar and therefore it is assumed that the model is correctly implemented.

### 3.2.2 Results

This model makes it possible to see quickly what the losses are in function of the speed, current and how the losses act with different PWM frequencies. In figure 3.5 the losses versus the speed according to the implemented model are shown. Here the losses are calculated until the 25th harmonic with a current of 28.3A (amplitude value of the nominal current). The calculation can’t go higher than the 25th harmonic due to the ill-conditioned situation. The most notable in figure 3.5 is that the losses decrease after about 3500rpm and are almost zero at 6000rpm. Unfortunately the paper does not specify under which use of control this happens. Probably the losses are almost zero because the input voltage equals the back emf so there is almost no current. It seems also that the model is not correct after the nominal rotation speed because the figures in the paper all end when the figures reach the almost zero losses. However figure 3.5 is drawn a little further than the nominal speed to see what will happen after this point.

![Figure 3.5: The losses in function of the speed](image)

It is very interesting how the losses act in function of the current, which is shown in figure 3.6 a) and b). In figure 3.6 a) the losses in function of the current can be seen at 6000rpm and calculated till the 25th harmonic. In this figure it can be seen that the losses decreases when the current increases, which is something strange. This effect is not expected because higher current cause usually higher losses. In figure 3.6 b) the losses in function of the current for various speeds can be seen. This figure is very interesting because at lower speeds the losses keep almost constant but for the higher speeds the losses decreases for higher current. What further notable is, is that from figure 3.6 it follows that the losses for 6000rpm are always smaller than those of 5000rpm from 0A till 28.3A.

In figure 3.7 it can be seen that the losses decrease when the PWM frequency increases. The reason for this is that the higher the PWM frequency the better the approach of a sinusoidal excitation current. Consequently the time harmonic losses due to the excitation current become smaller.

### 3.2.3 Converting Bosch Rexroth machine properties

Before the back EMF constant can be used in the implemented model it has to be converted to match the back EMF constant properties that are used in the paper. The back EMF constant \((K_{\text{rms}} / \text{rpm})\) in the
Figure 3.6: The losses in function of the current, a) total and regional losses at 6000rpm, b) total losses for different speeds

Figure 3.7: The losses in function of the PWM frequency

datasheet of the Bosch Rexroth machine is $\frac{82.7V}{1000\text{rpm}}$ which refers to a RMS line to line value. The back EMF constant used in the model is an amplitude line to neutral value, so it has to be converted. Firstly it is converted to $V_{\text{rad}}$ unit by multiplying it with $\frac{60\pi}{2\pi \text{rad}}$, so $K_{e_{\text{rms}|l-l}} = 0.7897 \frac{V_s}{\text{rad}}$. The next step is to divide this number by $\sqrt{3}$ to get the line to neutral rms value, so $K_{e_{\text{rms}}|l-n} = 0.4559 \frac{V_s}{\text{rad}}$ and the last step is multiplying it by $\sqrt{2}$ to get the amplitude value $K_e = 0.645 \frac{V_s}{\text{rad}}$. Further it seems that the paper uses a back EMF constant per pole pair so the back EMF constant must also be multiplied by the number of polepairs. The paper does not mention how the back EMF constant must be defined, by using an amplitude, line to neutral rms value per polepair the results are the most realistic. It is also the only way to have zero losses at 6000rpm. How the direct voltage $V_{dc}$ is calculated will be explained in chapter 4, this value is namely determined with some FEM analysis results. This voltage is exact the same voltage as is used in figure 4.1.
<table>
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<td>permeability of free space</td>
<td>$4\pi e^{-7}$</td>
<td>H/m</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>permeability of the rotor</td>
<td>$744 \cdot \mu_0$</td>
<td>H/m</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>permeability of the magnet</td>
<td>$1.04457 \cdot \mu_0$</td>
<td>H/m</td>
</tr>
<tr>
<td>$\mu_3$</td>
<td>permeability of the interpolar section</td>
<td>$\mu_0$</td>
<td>H/m</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>conductivity of the rotor</td>
<td>$2270000$</td>
<td>S/m</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>conductivity of the magnet</td>
<td>$714000$</td>
<td>S/m</td>
</tr>
<tr>
<td>$\sigma_3$</td>
<td>conductivity of the interpolar section (air)</td>
<td>$0$</td>
<td>S/m</td>
</tr>
<tr>
<td>$f_{pwm}$</td>
<td>pulse width frequency</td>
<td>$20000$</td>
<td>Hz</td>
</tr>
<tr>
<td>$L$</td>
<td>Stack length</td>
<td>$160e^{-3}$</td>
<td>m</td>
</tr>
<tr>
<td>$R'$</td>
<td>rotor radius without the magnet</td>
<td>$31.2e^{-3}$</td>
<td>m</td>
</tr>
<tr>
<td>$R_{rod}$</td>
<td>the outside radius of the rotor</td>
<td>$33.8e^{-3}$</td>
<td>m</td>
</tr>
<tr>
<td>$N_{ph}$</td>
<td>coil turns per phase</td>
<td>62</td>
<td>turns</td>
</tr>
<tr>
<td>$p$</td>
<td>number of poles</td>
<td>8</td>
<td>poles</td>
</tr>
<tr>
<td>$q$</td>
<td>slots per pole per phase</td>
<td>2</td>
<td>slots</td>
</tr>
<tr>
<td>$q_p$</td>
<td>slots per pole</td>
<td>5</td>
<td>poles</td>
</tr>
<tr>
<td>$\psi'$</td>
<td>interpolar angle, half of the angle between the magnets</td>
<td>$0.0454$</td>
<td>rad</td>
</tr>
<tr>
<td>$I_{m}$</td>
<td>peak phase current (rms value)</td>
<td>20</td>
<td>A</td>
</tr>
<tr>
<td>$K_e$</td>
<td>back EMF constant (rms value)</td>
<td>$0.645 \cdot \frac{2}{p}$</td>
<td>V$_{rad}$/rad</td>
</tr>
<tr>
<td>$V_{dc}$</td>
<td>Battery voltage</td>
<td>872</td>
<td>V</td>
</tr>
<tr>
<td>$L_{in}$</td>
<td>equivalent phase inductance (L-N)</td>
<td>1.3</td>
<td>H</td>
</tr>
</tbody>
</table>

Table 3.2: Rexroth MSK071E-0450 properties for the PWM model

4 Loss calculation with FEM analysis

FEM analysis is a very nice and powerful tool to calculate the field in any magnetic structure. The software used to do these FEM calculation in this report is Ansoft’s Maxwell 13. In this report these calculations are used to determine the losses in the magnets of a BLAC machine. To save some time the calculation are done on an already modeled machine, this model is an approach of the Bosch Rexroth machine. This makes it possible to check the FEM analysis with the existing machine. Further it must be noted that the analysis is done on a 2D design, however eddy currents, which are the cause of the losses, are an 3D phenomenon.

4.1 The settings

First the constraints have to be determined because otherwise the results can not be compared with any other results. It is possible to calculate the losses with flux supporting current or flux weakening current but it is chosen to have the current is in phase with the back EMF. The losses due to the PWM switching is wanted so there must be a circuit made with Maxwell that simulates a PWM circuit. This circuit is made automatically by Maxwell which can be seen in figure 4.1, but the input voltage, the leakage inductance and the starting angle have to be calculated to achieve the constraint that the current is in phase with the back EMF. Unfortunately Maxwell does not calculate these values automatically. In the circuit shown in figure 4.1 the inductances $L_A$, $L_B$ and $L_C$ are the leakage inductances which must all be equal to provide a balanced circuit and are therefore represented as $L'_{\sigma}$ in this report. The resistors $R_A$, $R_B$ and $R_C$ are the winding resistences, which are all the same according to the datasheet and represented as $R_s$ in this report. The coils, $L_{PhaseA}$, $L_{PhaseB}$ and $L_{PhaseC}$ in the Maxwell circuit are linked with the coils in the FEM model of Maxwell. These coils have an inductance included which is reffered as $L'_{dm}$ in this report.

To calculate the unknown data we use a phaser diagram, which is shown in figure 4.2. The back EMF phaser of the first harmonic $V_{pl}$ can be determined by simulating the rotor to spin without excitation currents.
Then the phaser $V_{s1}$ of the input voltage’s first harmonic will be determined by using a perfect sinewave current and no leakage inductance. In Maxwell it is possible to inject the current in the FEM model without using any circuit. Knowing these two phasers the phaser $\omega L'_{d}I_{s}$ can be calculated using phthagoras. Note that $I_{s}$ is known because this is the injected current in the FEM model. The total inductance $L_{d}$ is experimental determined and has a value of $1.7mH$ and is defined as,

$$ L_{d} = L'_{\sigma} + L'_{dm} \quad (4.1) $$

From equation 4.1 $L'_{\sigma}$ can be determined, further $R_{s}$ is known from the datasheet, $V_{p1}$ is still known so $V_{s1}$ can be calculated. To complete the Maxwell circuit the input voltage $V_{dc}$ has to be calculated which is done by using,

$$ V_{dc} = V_{s1} \cdot \sqrt{3} \quad (4.2) $$

Note that $V_{s1}$ is a line to neutral voltage, and $V_{dc}$ is a line to line voltage, therefore it has to be multiplied with $\sqrt{3}$. 

Figure 4.2: Phaser diagram: current in phase with back EMF
4.2 No excitation currents

The back EMF can be determined by letting the rotor spin with no excitation currents in the windings. Because there is now an open circuit at the terminals of the BLAC machine, the back EMF can be measured at these terminals. This is also what is done with the simulation i.e. determining the induced voltage with no excitation current. In figure 4.3 the replacement circuit of the machine is drawn. Here it can be seen that by zero input current $I_s$ the voltages $V_{p1}$, $V'_{s1}$ and $V_{s1}$ are all equal. In this case the induced voltage $V'_{s1}$ that is calculated by Maxwell equals the back EMF voltage $V_{p1}$. It is important to note that the FEM model already includes the $L'_{dm}$ inductance in the windings, so the calculated induced voltage is $V'_{s1}$ and not $V_{p1}$. It is also important to note that the back EMF voltage is almost independent of the excitation current, but only dependent on the speed.

The back EMF consists of a sinusoidal waveform with several higher harmonics as can be seen in figure 4.4. It is also possible to see in this figure that the first harmonic of the back EMF is a sine wave with a period of 2.5ms. A more scientific way to determine this is by determining the phaser for e.g. 2.5ms and then calculate the angle of that phaser. If it is a sine wave the phaser should have an angle of -90° at 2.5ms (one period). The phasor is defined like,

$$V_{ph} = \frac{2}{3} \left( V_A + V_B \cdot e^{j\frac{2}{3}\pi} + V_C \cdot e^{j\frac{4}{3}\pi} \right)$$

where $V_A$, $V_B$ and $V_C$ represent the three phases. The angle that was calculated at 2.5ms using this method was -90.6°, this deviation of -0.6° is probably caused by the higher harmonics.

To determine the amplitude of the first harmonic the spectrum of the back EMF was calculated using the tools that Maxwell provides. The amplitude was then determined by calculating the average value of the three phases, which can be seen in table 4.1. In this table it is also possible to view the losses of the magnet and those of the rotor core with respect to the speed. This value of the losses is an average value because the losses are not constant as can be viewed in figure 4.5 a) and b).

In figure 4.6 the losses of the magnets and the rotor core are visualized. These losses are very likely due to the slot openings. When the rotor turns it sees these slot openings as a change in reluctance and this has a change in flux density as a result. This change in flux density then causes eddy currents to flow in the
Figure 4.4: The back EMF voltage waveforms

Figure 4.5: Losses in function of the time, a) magnet losses, b) rotor core losses
magnets, which increases when the speed increases as can be seen in figure 4.6. The values used in figure 4.6 are presented in table 4.2.

### 4.3 Sinusoidal excited current

Now since the back EMF is know the phase angle of the excitation current can be determined. As explained in chapter 4.2 the first harmonic of the back EMF is a sinewave. To satisfy the phaser diagram of figure 4.2 the excitation current which is injected must also be a sinewave. There are two simulation models used one with were the injected current had an amplitude of 16A and one with the nominal current (20A rms or 28.3A for the amplitude value). The reason to use two different currents is that this gives a small impression what will happen with the losses for different currents.

The induced voltage that is calculated by Maxwell is $V'_{s1}$, this value is determined for different speeds and currents as can be seen in table 4.3. The amplitude value of $V'_{s1}$ is determined by calculating the average voltage of the three phases first harmonic’s amplitude.

The losses that are calculated include the losses explained in chapter 4.2, because when the rotor turn we get for free the losses due to the change of the reluctance. In figure 4.7 a) the losses that occur in the magnets are shown and in figure 4.7 b) the losses that occur in the rotor core are shown. The values used in figure 4.7 are presented in table 4.4.

### 4.4 PWM excited current

Now it becomes interesting; what are the losses when a PWM excitation is used. Before these losses can be calculated the leakage inductance $L'_a$ and the input voltage $V_{dc}$ must be calculated. Well the back EMF and $V'_{s1}$ are know so the leakage inductance can be calculated using the phaser diagram shown in figure 4.2. Using this phaser diagram $L'_{dm}$ can be calculated by,

$$L'_{dm} = \frac{V'_{s1} \sin(\beta')}{\omega I_s}$$  \hspace{1cm} (4.4)
<table>
<thead>
<tr>
<th>speed [rpm]</th>
<th>Losses [W] for $I_s=16$A</th>
<th>speed [rpm]</th>
<th>Losses [W] for $I_s=16$A</th>
</tr>
</thead>
<tbody>
<tr>
<td>6500</td>
<td>45.16</td>
<td>6500</td>
<td>0.1375</td>
</tr>
<tr>
<td>6000</td>
<td>38.64</td>
<td>6000</td>
<td>0.1189</td>
</tr>
<tr>
<td>5500</td>
<td>32.58</td>
<td>5500</td>
<td>0.1015</td>
</tr>
<tr>
<td>5000</td>
<td>27.04</td>
<td>5000</td>
<td>0.0855</td>
</tr>
<tr>
<td>4500</td>
<td>21.99</td>
<td>4500</td>
<td>0.0708</td>
</tr>
<tr>
<td>4000</td>
<td>17.45</td>
<td>4000</td>
<td>0.0574</td>
</tr>
<tr>
<td>3500</td>
<td>13.43</td>
<td>3500</td>
<td>0.0454</td>
</tr>
<tr>
<td>3000</td>
<td>9.92</td>
<td>3000</td>
<td>0.0347</td>
</tr>
<tr>
<td>2500</td>
<td>6.94</td>
<td>2500</td>
<td>0.0256</td>
</tr>
<tr>
<td>2000</td>
<td>4.47</td>
<td>2000</td>
<td>0.0178</td>
</tr>
<tr>
<td>1500</td>
<td>2.54</td>
<td>1500</td>
<td>0.0113</td>
</tr>
<tr>
<td>1000</td>
<td>1.14</td>
<td>1000</td>
<td>0.0062</td>
</tr>
<tr>
<td>500</td>
<td>0.29</td>
<td>500</td>
<td>0.0026</td>
</tr>
</tbody>
</table>

Table 4.2: Values of the losses for the, a) magnets, b) rotor core

<table>
<thead>
<tr>
<th>speed [rpm]</th>
<th>$V'_{s1}$ [V] at $I_s=16$A</th>
<th>$V'_{s1}$ [V] at $I_s=28.3$A</th>
</tr>
</thead>
<tbody>
<tr>
<td>6000</td>
<td>424.9</td>
<td>430.2</td>
</tr>
<tr>
<td>4000</td>
<td>282.7</td>
<td>286.5</td>
</tr>
<tr>
<td>2000</td>
<td>141.4</td>
<td>142.9</td>
</tr>
</tbody>
</table>

Table 4.3: Induced voltage

Figure 4.7: Avarage losses versus the speed for 28.3A, a) Losses in the magnets, b) Rotor core losses
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6500</td>
<td>124.30</td>
<td>286.63</td>
<td>6500</td>
<td>0.602</td>
<td>1.351</td>
</tr>
<tr>
<td>6000</td>
<td>109.21</td>
<td>253.97</td>
<td>6000</td>
<td>0.541</td>
<td>1.221</td>
</tr>
<tr>
<td>5500</td>
<td>94.56</td>
<td>221.82</td>
<td>5500</td>
<td>0.475</td>
<td>1.084</td>
</tr>
<tr>
<td>5000</td>
<td>80.51</td>
<td>189.85</td>
<td>5000</td>
<td>0.415</td>
<td>0.953</td>
</tr>
<tr>
<td>4500</td>
<td>67.08</td>
<td>159.19</td>
<td>4500</td>
<td>0.357</td>
<td>0.82</td>
</tr>
<tr>
<td>4000</td>
<td>54.45</td>
<td>130</td>
<td>4000</td>
<td>0.302</td>
<td>0.7</td>
</tr>
<tr>
<td>3500</td>
<td>42.75</td>
<td>102.55</td>
<td>3500</td>
<td>0.25</td>
<td>0.577</td>
</tr>
<tr>
<td>3000</td>
<td>31.7</td>
<td>77.45</td>
<td>3000</td>
<td>0.213</td>
<td>0.469</td>
</tr>
<tr>
<td>2500</td>
<td>22.79</td>
<td>55.13</td>
<td>2500</td>
<td>0.156</td>
<td>0.361</td>
</tr>
<tr>
<td>2000</td>
<td>14.86</td>
<td>36.1</td>
<td>2000</td>
<td>0.119</td>
<td>0.267</td>
</tr>
<tr>
<td>1500</td>
<td>8.49</td>
<td>20.65</td>
<td>1500</td>
<td>0.0810</td>
<td>0.184</td>
</tr>
<tr>
<td>1000</td>
<td>3.62</td>
<td>9.31</td>
<td>1000</td>
<td>0.00392</td>
<td>0.109</td>
</tr>
<tr>
<td>500</td>
<td>0.96</td>
<td>2.35</td>
<td>500</td>
<td>0.0194</td>
<td>0.042</td>
</tr>
</tbody>
</table>

Table 4.4: Values of the losses for the, a) magnets, b) rotor core

<table>
<thead>
<tr>
<th>current [A]</th>
<th>$L'_d [mH]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_s=28.3$A</td>
<td>0.57</td>
</tr>
<tr>
<td>$I_s=16$A</td>
<td>0.45</td>
</tr>
<tr>
<td>$I_s=0$A</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Table 4.5: Leakage inductance

Further the total inductance $L_d$ is known and as follows from equation 4.1,

\[ L'_d = L_d - L'_{dm} \]  \hspace{1cm} (4.5)

and the result from this is represented in table 4.5. Then the phaser of the first harmonic of the input voltage is calculated by,

\[ V_{s1} = V_{p1} + j\omega L_d I_s + R_s I_s \]

from which the amplitude and angle $\beta$ can determined. The amplitude is determine by taking the absolute value of the phaser, this can be done using phytagoras were the real and the imaginary part are the rectangle sides. The results of these calculations are placed in tabel 4.6 and drawn in figure 4.8 a). In figure 4.8 a) it can also be seen that the voltage increases linear in function of the speed. However there is some nonlinearity expected at lower speeds because the effect of the resistance of the windings is then noticeable.

The angle $\beta$, which is the angle between $V_{p1}$ and $V_{s1}$, can be determined by subtracting the angles of both phasers. The angle is at higher speeds constant and drops a little bit at lower speeds because here the resistance of the windings is not neglectable anymore. In figure 4.8 b) this effect can be seen, and in tabel 4.7

<table>
<thead>
<tr>
<th>speed [rpm]</th>
<th>$V_{s1}$ [V] at $I_s=16$A</th>
<th>$V_{s1}$ [V] at $I_s=28.3$A</th>
</tr>
</thead>
<tbody>
<tr>
<td>6000</td>
<td>429.2</td>
<td>442.7</td>
</tr>
<tr>
<td>4000</td>
<td>286.8</td>
<td>296.3</td>
</tr>
<tr>
<td>2000</td>
<td>144.6</td>
<td>150.4</td>
</tr>
</tbody>
</table>

Table 4.6: Input voltage $V_{s1}$
Table 4.7: The angle $\beta$

<table>
<thead>
<tr>
<th>Speed [rpm]</th>
<th>$\beta$ [degrees] at $I_s = 16A$</th>
<th>$\beta$ [degrees] at $I_s = 28.3A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6000</td>
<td>9.18</td>
<td>15.9</td>
</tr>
<tr>
<td>4000</td>
<td>9.16</td>
<td>15.8</td>
</tr>
<tr>
<td>2000</td>
<td>9.08</td>
<td>15.6</td>
</tr>
</tbody>
</table>

The only thing that rests is the direct voltage $V_{dc}$ which is the source as shown in figure 4.1. The amplitude of this voltage is also the amplitude of the line to line voltage of the machine. Further the amplitude of the line to neutral voltage is known which is $V_{s1}$ and from here it follows that,

$$V_{dc} = V_{s1} \cdot \sqrt{3}$$

which gives a value of 743V when a current of 16A is used and 787V when the nominal current is used. This is a constant value and is determined for the nominal speed (6000rpm). However after some simulation it appeared that using these results the current was not 16A but 12.5A and it was not a perfect sine wave, it was lagging with about 1.34 degrees for the 6000rpm. It seemed also that the first harmonic of the line to neutral voltage was not what it supposed to be. This was solved by multiplying the direct voltage $V_{dc}$ and the angle $\beta$ by a correction factor of 1.137. Apparently this correction factor is a constant and is used for each simulation to get better results. Why this correction factor is needed is unknown, one of the reason can be that the modulation index can not be 1 in Maxwell’s circuit editor. Also this constant is used to determine the $V_{dc}$ used in chapter 3.2. Further it is important to note that all the values presented in this chapter are without the correction factor otherwise the phaser diagram shown in figure 4.2 would not fit.

The simulations are done with three different currents namely: 0A, 16A and 28.3A, so it can be seen what happens with the losses for different currents. Because it is very time consuming to get all the variables good the excitation currents are not exactly the numbers as mentioned before. The real values of the currents are stated in table 4.8

The losses calculated with these settings are displayed in figure 4.9; in figure 4.9a) the losses in the magnets can be viewed and in figure 4.9b) the losses in the rotor core can be viewed. It is also possible to
<table>
<thead>
<tr>
<th>speed [rpm]</th>
<th>$I_s = 0$ [A]</th>
<th>$I_s = 16$ [A]</th>
<th>$I_s = 28.3$ [A]</th>
</tr>
</thead>
<tbody>
<tr>
<td>6000</td>
<td>0.07</td>
<td>16.5</td>
<td>29</td>
</tr>
<tr>
<td>4000</td>
<td>0.9</td>
<td>15.8</td>
<td>29.4</td>
</tr>
<tr>
<td>2000</td>
<td>0.6</td>
<td>16.2</td>
<td>28.6</td>
</tr>
</tbody>
</table>

Table 4.8: Real excitation currents

![Figure 4.9: Losses in function of the speed for a) the magnets b) the rotor core](image)

see that the losses are almost a linear function. It is also notable in figure 4.9 b) that the distance between the blue line ($I_s = 0$ A) and the red line ($I_s = 16$ A) is much smaller than the distance between the red line and the black line ($I_s = 28.3$ A). In figure 4.10 the losses in function of the current can be seen. In figure 4.10 a) the losses in the magnets are shown and in b) the losses in the rotor core are shown. Here it is obvious that it is a quadratic function, which means that the losses increase quadratic in function of the current. The values used in figures 4.9 and 4.10 are presented in table 4.9.

One attempt to explain the increase of losses is to investigate the current ripple. To investigate this the spectrum of the current is evaluated at the PWM frequency, which is in this case 20kHz. At 20kHz there are two sidebands, the average of these sidebands, further referred as the current ripple, is taken and placed in the table 4.10. In this table it can be seen that the current ripple increases when the speed increases, and in figure 4.9 it can also be seen that the losses increase when the speed increases. This means that there could be a relation between the rise of the current ripple and the speed. However it can also be seen that the current ripple not increases when the input current $I_s$ increases. Unlike the results of figure 4.10 it can be seen that the losses increase quadratically in function of the current, which means this can not be explained because of the increase of the current ripple.

4.5 Comparison

Now lets compare the results achieve in the previous section of this chapter with the results achieved from the PWM excitation. The results will not be compared with the results achieved from the analytical model because they are too different. The losses due to the PWM excitation in function of the speed can be viewed in figure 4.11. In figure 4.11 a) the losses in the magnets are shown and in b) the losses in the rotor core are shown. Further in figure 4.11 and figure 4.12 the blue dashed line are the losses when there is no excitation
Figure 4.10: Losses in function of the current for, a) the magnets, b) the rotor core

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6000</td>
<td>77</td>
<td>145</td>
<td>337.3</td>
</tr>
<tr>
<td>4000</td>
<td>41</td>
<td>70.5</td>
<td>162.7</td>
</tr>
<tr>
<td>2000</td>
<td>15.9</td>
<td>23</td>
<td>49.4</td>
</tr>
</tbody>
</table>

Table 4.9: Values of the losses for the, a) magnets, b) rotor core

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6000</td>
<td>0.64</td>
<td>0.8</td>
<td>1.78</td>
</tr>
<tr>
<td>4000</td>
<td>0.35</td>
<td>0.44</td>
<td>1.10</td>
</tr>
<tr>
<td>2000</td>
<td>0.13</td>
<td>0.2</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Table 4.10: Average of the PWM related sidebands RMS current components
Figure 4.11: Losses in function of the speed for a) the magnets b) the rotor core

Figure 4.12: Losses in function of the current for a) the magnets b) the rotor core

current, calculated during the determining of the back EMF. The red and black dashed lines are losses using the sinusoidal excitation currents 16A and 28.3A respectively. The solid lines are the losses from the PWM excited currents. It is notable that the losses due to the PWM excitation are all a little bit bigger because of the ripple on the current. This ripple on the current can be viewed in figure 4.13. In figure 4.14 a) the ripple of the current by zero current configuration of the PWM excitation can be viewed. In figure 4.14 b) only one phase is shown and here it is more clearly to see that the average current is almost zero. Because of this ripple the losses in the zero current configuration are bigger than those of the back EMF. It can be seen in figure 4.13 that there is a direct current present, this is because as mentioned before the current that flows is a little bit different from the design current because some inaccuracy in the calculation of the variables.
Figure 4.13: PWM excited current at 28.3A by 6000rpm

Figure 4.14: Current at zero current configuration by 6000rpm is shown for a) three phases b) one phase
5 Conclusion and recommendation

5.1 Conclusions

Well well, this was quite some work, but the results are therefore also quite a bit nicer. There were two analytical models implemented, which were taken from a technical paper. The implemented programs were verified by comparing the results with those of the paper. Unfortunately only the shape of the figures could be compared because there was no data of the machine that was used in the paper. The data of a Bosch Rexroth MSK071E-0450 machine was used in this model because this machine was available in the lab. The results of the analytical model with PWM excitation could not be explained, therefore the assignment changed in determining the losses in the magnets with FEM analysis. The FEM model that was taken was from a Bosch Rexroth machine which is an 8 surfaced mounted pole brushless AC machine. However the analytical models were developed for a DC machine this model was taken because it was already made and would save quite some time. To simulate the FEM model with PWM excited currents it was necessary to determine the back EMF and the induced voltages by sinusoidal currents of 16A and its nominal current. There were two different currents taken to see what will happen with the losses for different currents. During the calculation of the back EMF the losses in the magnets and rotor core were also calculated. These losses are very likely due to the change of reluctance in the stator because of the slot openings. The losses due this change in reluctance increase quadratically in function of the speed, both for the losses in the magnets as for the rotor core losses. After this the induced voltage with sinusoidal currents was calculated for the currents 16A and the nominal current, also here the losses were calculated. These losses seem to start as quadratic function but end as an linear function. The next step was determining the losses in a PWM excited machine, were in the previous FEM analysis there was chosen to do the FEM analysis for 500rpm till 6000rpm with a step of 500rpm, is here chosen to analyze only 2000, 4000, 6000rpm. The reason for this is that the time step for a PWM excited signal, which was 20kHz, must be very small, otherwise the analysis would not be correct. The losses for the PWM excitation appeared to be linear in function of the speed and quadratic in function of the current. The losses of the PWM excitation were compared with the losses of the back EMF and the induced voltages of the sinusoidal currents. It appeared here that the losses caused by the PWM excitation had just increased by an almost constant fraction compared to sinusoidal excitation. Which means that the function is from the same kind.

At the start there was no idea what the losses in the magnets were and how they act in a brushless machine. After a literature study there was an analytical model implemented to determine a quick estimate of the losses. However after the implementation of the analytical models some of the results from the implemented analytical PWM model could not be explained. These results could also not be compared to any other results to verify if there were correct because there weren’t any other results. Therefore a FEM analysis was done to determine these losses. Now it is known that the losses of the PWM excited currents are not completely different of those of the sinusoidal excitation, they only differ with some additional losses. This makes it possible to only calculate the sinusoidal losses, which saves time, to get an estimate of the losses. Also these results can help with the verification of any other implementation of an analytical model in the future.

Unfortunately the main goal was not accomplished, because the analytical PWM model does not look like the losses calculated with the FEM analysis. This can mean two things, namely the analytical model is incorrect or incorrectly programmed, however the figure are very look alike of those in the paper. Or this is because the used machine in the FEM calculation is an AC machine and the analytical model is developed for a DC machine. A second drawback is that there is a 2D FEM model used and eddy currents are a 3D
phenomena. This means that it is probably the case that there is an error in the results which is unknown.

5.2 Recommendation

The implemented analytical model that calculates the losses in the magnets and in the rotor core was not suitable for an AC machine. This does not mean that the analytical model is incorrect because it is not fully clear under which conditions the losses of the brushless machine have been determined in [17]. From here the same analysis should be done with a similar machine but in DC mode to compare the results with those of the AC machine. Also these results should be compared with the analytical model to see if they will match.

Further when the FEM model of the BLDC machine has been made it could also be used to check the analytical model for commutation excitation. There is in this report nothing done to verify this model with any FEM analysis, it is only verified by comparing it with the results achieved in [11]. Also in this paper there was no information about the used machine, so the results are only checked by the figures which are of the same kind.

Doing the same analysis with a 3D FEM model would give probably a more accurate solution for the losses. These results then can be used to compare them with the 2D results from were the error can be determined. If the error is small enough a 2D analysis can be used instead because this is less time consuming.

One last possibility would be when the above is done is to expand the model and check what segmentation does with the losses. It appears to be a popular ability to reduce the eddy current losses significantly. Here is some research already done and there also exist papers which claim to developed models to calculate the losses in the segments analytically [8, 16]. But as everything in this world, it has its drawbacks which also have to be examined.

Therefore a possible next step would be checking what the losses would be if this machine operates as a DC machine. This means however that the FEM model that was created must be converted from AC mode to DC mode and how much works this takes is unknown. It should be possible to make a 3D model from this machine and do the calculation again to check if there is an error and how big it is. If the error is small enough it is known for the future that a 2D analysis is enough and a 3D analysis is not needed which can save quite some time. Further it is possible to check the losses with the real machine, which is available in the lab. However the model is not exactly the same as the machine, which is reported in the internship report of Jeroen Waarma. A further next step could be to extent the analysis for segmentation of permanent magnets. This will reduce the eddy current in the magnets drastically but unfortunately it also has its downfall.
A Materials and their physical properties

A.1 Development of permanent magnets

Permanent magnets can be found in a variety of applications, from consumer to industrial applications. Because the materials for the permanent magnets are there in quite some variations of sizes and strengths they can be used in mostly every machine, from low cost to high end machines. The permanent magnets became more and more popular since the 1970s when the “rare earth” type were discovered. Since the 1970s, these rare earth magnets were improved dramatically every year till the 1990s when they reached maturity [18]. One disadvantage of the most rare earth magnets are that they contain metal and therefore have eddy current losses due to e.g. space and time harmonics in an electrical machine. These eddy current losses result in heating up the magnet and when a permanent magnet gets too hot they demagnetize (NdFeB demagnetizes at about 120 °C) [10]. A second disadvantage of the metal in the magnets is that they have a penetration depth. Every magnet has a different penetration depth because they have different physical properties. These physical properties will be mentioned and compared in this short report.

A.2 Skin effect

The phenomenon skin effect is defined as having a non uniformly distributed current density in a conductor when high frequency signals are used. The higher the frequency, the higher the current density at the surface and the lower the current density in the middle of a conductor. That this happens with the more common materials like copper, aluminium and steel is well known. Less known is that it also happens within the permanent magnets in an electrical machine. To get a good feeling about when this happens we will investigate this.

The feeling we want to get is specially for permanent magnet synchronous machines (PMSM), focusing on the permanent magnets. So we will mention some design rules to know when the skin effect is something to take into account. In PMSM the magnet size is roughly about ten times the air gap length, were the air gap length is approximately 0.2mm for small machines, for medium size machine it is approximately 0.4mm and for large machines it is approximately 0.5mm or bigger [19]. When we apply this rule we get a magnet that can vary from 1.27mm to 8.89mm.

To see what the penetration depth of a material is at a given frequency we can calculate it using,

\[
\delta = \sqrt{\frac{2 \cdot \rho}{\mu \cdot \omega}}
\]  

(A.2.1)

where \( \rho \) is the resistivity, \( \mu \) is the permeability and \( \omega \) is the angular speed. Note that in this report we used the fact that \( \omega \) can also be written as \( 2 \pi \cdot f \).

In table A.2.1 the penetration depth is calculated for a frequency of 50Hz and 10kHz. The material properties of the magnets were found in [1, 19]. Copper, aluminium and steel are also in the table so we can compare them with the magnets. These metals are well known in combination with the penetration depth and now we can see how the magnets behave with respect to these metals. Here we can see that for 50Hz the penetration depth is at minimum 18.3mm (Alnico) so the biggest magnet (8.89mm) is smaller then the skin depth which means that there is no significant skin effect. For higher frequencies like 10kHz the penetration depth is much smaller and there will be skin effect in some of the materials in the large machines. For example by NdFeB the penetration depth is 6mm for a frequency of 10kHz, using this in a small machine, the magnet size is some were between 1.27mm and 2.54mm, we see that there is no significant skin effect present for this frequency. But using Alnico we see that the penetration depth about 1.3mm at 10kHz. If
### Table A.2.1: penetration depth for different materials

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho [\mu\Omega m]$</th>
<th>$\rho_{relative} [H/m]$</th>
<th>$\delta_{f=50Hz}[mm]$</th>
<th>$\delta_{f=10kHz}[mm]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper</td>
<td>17.2</td>
<td>0.99</td>
<td>9.3</td>
<td>0.66</td>
</tr>
<tr>
<td>Aluminium</td>
<td>28.3</td>
<td>1</td>
<td>12</td>
<td>0.85</td>
</tr>
<tr>
<td>Steel</td>
<td>470</td>
<td>100</td>
<td>4.9</td>
<td>0.35</td>
</tr>
<tr>
<td>Alnico</td>
<td>470</td>
<td>1.9 to 7</td>
<td>18.4</td>
<td>1.3</td>
</tr>
<tr>
<td>SmCo</td>
<td>860</td>
<td>1.02</td>
<td>65.4</td>
<td>4.6</td>
</tr>
<tr>
<td>NdFe30</td>
<td>1400</td>
<td>1.04</td>
<td>82.4</td>
<td>5.8</td>
</tr>
<tr>
<td>NdFeB</td>
<td>1500</td>
<td>1.05</td>
<td>85.1</td>
<td>6</td>
</tr>
<tr>
<td>Ferrite</td>
<td>$10^{21}$</td>
<td>1.05</td>
<td>7293</td>
<td>515.7</td>
</tr>
</tbody>
</table>

To get a better overview when the problems really starts we draw the figures A.2.1a) and A.2.1b), this figure is drawn on a linear scale from 50 to 5000Hz. Note that steel has the smallest penetration depth but not the lowest resistivity, this is because of the high relative permeability of steel [Fig. A.2.1 a)]. For the magnets [Fig. A.2.1 b)] we see that the skin effect is negligible for rather small magnets.

When we look at a further range of the frequency, for example for the harmonics by a commutation exited or PWM exited machine. We see in figure A.2.2 a) and b) that the penetration depth is getting really small. Some were here there is a trade off because at a given frequency the amplitude of the harmonics becomes so small that its losses become negligible against the total losses.

Note that none of the linear scaled figures include Ferrite, this is because its penetration dept is so large that it can be neglected in almost every machine e.g. at 10kHz this value is still 0.515m. When the magnets are this large the machine must be really big. In figure A.2.3 are all the penetration depths of the materials mentioned in this report can be seen in one figure. It shows the results of the previous figures in a log scale, here it is also easy to see which of the materials are the most sensitive to the skin effect. Note that NdFeB and NdFe30 are almost the same material and so have almost the same penetration depths.

One popular solution to minimize the eddy currents, and so the skin effect losses, is to segment the
Figure A.2.2: Penetration depth of the materials (linear scale), a) metals, b) magnets

Figure A.2.3: Penetration depth (log scale)

magnets i.e. divide one magnet into more magnets like done with lamination of the stator. But then the rotor has less shielding what results in more eddy currents in the rotor, to solve this problem the rotor has to be laminated to get an optimal solution [20].

A.3 Conclusion

In this report the penetration depths of the most used magnet types are calculated. Also these results were compared to more known penetration depths i.e. copper, aluminium and steel. We can see that the skin effect is different for each material.

It is always a trade off when to take the skin effect into account, to get an accurate loss calculation. Of course in large machines and/or high speed machines it is almost certain that it has to be taken into account, but for smaller machines it is neglectable. For machines in between it is the best to look at what the impact (amplitude) of the harmonics are. Then calculate the penetration depth for these harmonics and see if the penetration depth is getting small with respect to the size of the magnet.
B Functions, functions and more functions

B.1 Functions for the commutation model

B.1.1 The functions

In this appendix the functions needed to complete equation 3.1 are placed. The definition of the variables for equation B.1.1 through B.1.3 are placed in table B.1.1. The definition of the variables for equation B.1.4 through B.1.9 are placed in table B.1.2.

\[ A_{1m} = \frac{\pi}{2} F_{1m} \frac{2}{\tau_p} \]  

(B.1.1)

where

\[ F_{1m} = \frac{N_{ph}}{p} I_m \frac{4}{\pi} \sum_{k=1}^{q} \sin \left( \frac{(q_p - 2k + 1)\pi}{2q_p} \right) \]  

(B.1.2)

and

\[ \tau_p = \frac{2\pi R_{od}}{p} \]  

(B.1.3)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_{ph} )</td>
<td>coil turns per phase</td>
</tr>
<tr>
<td>( p )</td>
<td>number of poles</td>
</tr>
<tr>
<td>( I_m )</td>
<td>magnitude of the stator phase current</td>
</tr>
<tr>
<td>( q )</td>
<td>slots per pole per phase</td>
</tr>
<tr>
<td>( q_p )</td>
<td>slots per pole</td>
</tr>
<tr>
<td>( \tau_p )</td>
<td>pole pitch length</td>
</tr>
<tr>
<td>( R_{od} )</td>
<td>the outside radius (rotor+magnet)</td>
</tr>
</tbody>
</table>

Table B.1.1: Definition of the variables for equations B.1.1 through B.1.3

The equations B.1.4, B.1.5 and B.1.7 are using the modified Bessel functions \( I_\nu(x) \) and \( K_\nu(x) \) which we can not calculate directly because \( x \) must be a real non-negative number which is not the case as we see in equation B.1.8. One way to solve this problem is explained in appendix B.3.

\[ C_n = \frac{1}{\tau_{2\pi}} \left[ I'_{1\frac{\nu}{2}} (\tau_{2n} R_{od}) K'_{1\frac{\nu}{2}} (\tau_{2n} R') - I'_{1\frac{\nu}{2}} (\tau_{2n} R') K'_{1\frac{\nu}{2}} (\tau_{2n} R_{od}) \right] \]

\[ \frac{\sigma_1 C_1}{\tau_{2\pi} n} \left[ -I'_{1\frac{\nu}{2}} (\tau_{2n} R_{od}) K_{1\frac{\nu}{2}} (\tau_{2n} R') + I_{1\frac{\nu}{2}} (\tau_{2n} R') K_{1\frac{\nu}{2}} (\tau_{2n} R_{od}) \right] \]  

\[ + \left[ I_{1\frac{\nu}{2}} (\tau_{2n} R_{od}) K'_{1\frac{\nu}{2}} (\tau_{2n} R') - I'_{1\frac{\nu}{2}} (\tau_{2n} R') K'_{1\frac{\nu}{2}} (\tau_{2n} R_{od}) \right] \]  

(B.1.4)

and

\[ C_{2n} = \frac{I_{1\frac{\nu}{2}} (\tau_{2n} R_{od}) K_{1\frac{\nu}{2}} (\tau_{2n} R') - I_{1\frac{\nu}{2}} (\tau_{2n} R') K_{1\frac{\nu}{2}} (\tau_{2n} R_{od})}{N} \]

\[ \frac{\sigma_1 C_1}{\tau_{2\pi} n} \left[ -I_{1\frac{\nu}{2}} (\tau_{2n} R_{od}) K_{1\frac{\nu}{2}} (\tau_{2n} R') + I_{1\frac{\nu}{2}} (\tau_{2n} R') K_{1\frac{\nu}{2}} (\tau_{2n} R_{od}) \right] \]  

\[ + \left[ I_{1\frac{\nu}{2}} (\tau_{2n} R_{od}) K'_{1\frac{\nu}{2}} (\tau_{2n} R') - I'_{1\frac{\nu}{2}} (\tau_{2n} R') K'_{1\frac{\nu}{2}} (\tau_{2n} R_{od}) \right] \]  

(B.1.5)
where
\[
N = \frac{1}{\tau_{2_n}} \left[ I'_{\frac{\nu}{2}} (\tau_{2_n} R_{od}) K'_{\frac{\nu}{2}} (\tau_{2_n} R') - I'_{\frac{\nu}{2}} (\tau_{2_n} R') K'_{\frac{\nu}{2}} (\tau_{2_n} R_{od}) \right]
+ \frac{\sigma_1 C_{1_n}}{\sigma_2 \tau_{1_n}} \left[ -I'_{\frac{\nu}{2}} (\tau_{2_n} R_{od}) K_{\frac{\nu}{2}} (\tau_{2_n} R') + I'_{\frac{\nu}{2}} (\tau_{2_n} R') K_{\frac{\nu}{2}} (\tau_{2_n} R_{od}) \right]
\]

(B.1.6)

and
\[
C_{1_n} = \frac{I'_{\frac{\nu}{2}} (\tau_{1_n} R')}{I'_{\frac{\nu}{2}} (\tau_{1_n} R)}
\]

(B.1.7)

which we have to reformulate in case of ill-conditioned situations which is shown in section B.1.2, further
\[
\tau_{z_n} = \sqrt{j\mu_z \sigma_z \omega_{z_n}}
\]

(B.1.8)

and
\[
\omega_n = 6n\omega
\]

(B.1.9)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R' )</td>
<td>radius of the rotor without the magnet (rotor-magnet)</td>
</tr>
<tr>
<td>( R_{od} )</td>
<td>the outside radius (rotor+magnet)</td>
</tr>
<tr>
<td>( \mu_z )</td>
<td>permeability of material ( z ), where ( z ) is 1 (rotor core), 2 (magnet) or 3 (interpol regions)</td>
</tr>
<tr>
<td>( \sigma_z )</td>
<td>conductivity of material ( z ), where ( z ) is 1 (rotor core), 2 (magnet) or 3 (interpol regions)</td>
</tr>
<tr>
<td>( \omega )</td>
<td>the motor's synchronous speed (electrical)</td>
</tr>
<tr>
<td>( n )</td>
<td>the ( n )th-order harmonic component of a variable</td>
</tr>
<tr>
<td>( I_{\nu}(x) )</td>
<td>modified Bessel function (see also appendix B.3)</td>
</tr>
<tr>
<td>( I'_{\nu}(x) )</td>
<td>first derivative of the modified Bessel function (see also appendix B.3)</td>
</tr>
<tr>
<td>( K_{\nu}(x) )</td>
<td>modified Bessel function (see also appendix B.3)</td>
</tr>
<tr>
<td>( K'_{\nu}(x) )</td>
<td>first derivative of the modified Bessel function (see also appendix B.3)</td>
</tr>
</tbody>
</table>

Table B.1.2: Definition of the variables for equations B.1.4 through B.1.9

### B.1.2 Reformulation for \( C_{1_n} \)

For very small skin depth which results in a very large \( \tau_{k_n} \) (equation B.1.8), \( I_{\nu}(x) \) and \( I'_{\nu}(x) \) will to very large numbers which can be out of range of a computer. In this case we get an ill-conditioned situation which we can reduce by reformulating \( C_{1_n} \).

For \( \nu \geq 2 \) we get
\[
C_{1_n} = \frac{-1}{2(n-1)^{\frac{3}{2}}} \cdot \frac{x}{\text{ber}_{n-2}(x) + j\text{bei}_{n-2}(x)} + \frac{1}{2(n-1)^{\frac{3}{2}}} \cdot \frac{x}{\text{ber}_{n-1}(x) + j\text{bei}_{n-1}(x)}
\]

(B.1.10)

and for \( \nu = 0 \)
\[
C_{1_n} = \frac{1}{\sqrt{j}} \cdot \frac{\text{ber}_0(x) + j\text{bei}_0(x)}{\text{ber}_0(x) + j\text{bei}_0(x)} = \frac{1}{\sqrt{j}} \cdot \phi(x)
\]

(B.1.11)

and for \( \nu = 1 \)
\[
C_{1_n} = \frac{1}{\sqrt{j}} \cdot \left[ \frac{1}{\sqrt{j}} \cdot \frac{\text{ber}_0(x) + j\text{bei}_0(x)}{\text{ber}_1(x) + j\text{bei}_1(x)} - \frac{1}{x} \right] = \frac{1}{\sqrt{j}} \cdot \left[ -\frac{1}{\sqrt{j}} \cdot \phi(x) - \frac{1}{x} \right]
\]

(B.1.12)

where
\[
\frac{\text{ber}_0(x) + j\text{bei}_0(x)}{\text{ber}_1(x) + j\text{bei}_1(x)} = \phi(x)
\]

\[
\frac{1}{\sqrt{j}} \cdot \left[ -\frac{1}{\sqrt{j}} \cdot \phi(x) - \frac{1}{x} \right]
\]
\( \phi (x) \) is given in [21] on page 385.

### B.2 Functions for the PWM model

The functions to complete equation 3.4, 3.5 and 3.6 from section 3.2.1 are placed in this appendix section. The definition of the variables for equation B.2.1 through B.2.8 are placed in table B.2.1. The definition of the variables for equation B.2.9 through B.2.13 are placed in table B.2.2.

\[
A_k = \frac{\pi}{2} F_{km} \frac{2}{\tau_{pk}} \quad (B.2.1)
\]

where

\[
F_{km} = \frac{4N_{ph}}{p\pi} \Delta I \sum_{j=1}^{\eta} \sin \left( \frac{k\pi}{2} \right) \cdot \frac{\sin \left( \frac{(k(q_r-2j+1)\pi)}{2q_r} \right)}{k} \quad (B.2.2)
\]

\[
\tau_{pk} = \frac{2\pi R_{ad}}{pk} \quad (B.2.3)
\]

\[
\Delta I = \frac{1}{2} \left( \frac{p \omega_m k_i (I_{ms} - I_m)}{f_{pwm}} \right) \cdot d_i \quad (B.2.4)
\]

\[
I_{ms} = \frac{2}{p\omega_m k_i L_{ln}} \sqrt{\left( \frac{V}{2} k_v \right)^2 - \left( \frac{p}{2} k_v \omega_m \right)^2} \quad (B.2.5)
\]

and

\[
d = \frac{V_p}{k_v \frac{V}{2}} \quad (B.2.6)
\]

where

\[
V_p = \frac{p}{2} \sqrt{(k_v \omega_m)^2 + (\omega_m L_{ln} I_m k_i)^2} \quad (B.2.7)
\]

For a squarewave current waveform it holds that

\[
k_i = \frac{I_p}{I_m} = \frac{4}{\pi} \sin \left( \frac{\alpha}{2} \right) \quad (B.2.8)
\]

and for a sinusoidal waveform current it holds that

\[
k_i = 1
\]

Further it is important to notice that the voltage drop due to the winding phase resistance is neglected. The reason to neglect the phase resistance is that it’s amount is insignificant in large machines.

The equations B.2.9, B.2.11 and B.2.12 are using the modified Bessel functions \( I_n(x) \) and \( K_n(x) \) which we can not calculate directly because \( x \) must be a real non-negative number which is not the case as we see in equation B.2.13. One way to solve this problem is explained in appendix B.3.

\[
C_2 = \frac{\left[ I_{k_{\frac{p}{2}}} (\tau_2 R_{ad}) K_{k_{\frac{p}{2}}} (\tau_2 R') - I'_{k_{\frac{p}{2}}} (\tau_2 R') K_{k_{\frac{p}{2}}} (\tau_2 R_{ad}) \right]}{N} \\
+ \frac{\sigma_1 C_{1,\tau_2}}{\sigma_2 \tau_2} \frac{\left[ -I_{k_{\frac{p}{2}}} (\tau_2 R_{ad}) K_{k_{\frac{p}{2}}} (\tau_2 R') + I'_{k_{\frac{p}{2}}} (\tau_2 R') K_{k_{\frac{p}{2}}} (\tau_2 R_{ad}) \right]}{N} \quad (B.2.9)
\]
Table B.2.1: Definition of the variables for equations B.2.1 through B.2.8

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{km}$</td>
<td>Space harmonic amplitude of the stator resultant MMF</td>
</tr>
<tr>
<td>$A_{km}$</td>
<td>current density</td>
</tr>
<tr>
<td>$N_{ph}$</td>
<td>coil turns per phase</td>
</tr>
<tr>
<td>$p$</td>
<td>number of poles</td>
</tr>
<tr>
<td>$q$</td>
<td>slots per pole per phase</td>
</tr>
<tr>
<td>$q_p$</td>
<td>slots per pole</td>
</tr>
<tr>
<td>$R_{od}$</td>
<td>the outside radius (rotor+magnet)</td>
</tr>
<tr>
<td>$k$</td>
<td>space harmonic order</td>
</tr>
<tr>
<td>$\Delta I$</td>
<td>current ripples due to the PWM switching</td>
</tr>
<tr>
<td>$\omega_m$</td>
<td>motor speed (mechanical speed)</td>
</tr>
<tr>
<td>$I_{ms}$</td>
<td>motor’s square wave peak phase current under the full battery voltage or saturated PWM</td>
</tr>
<tr>
<td>$I_m$</td>
<td>Peak phase current</td>
</tr>
<tr>
<td>$f_{pwm}$</td>
<td>PWM frequency</td>
</tr>
<tr>
<td>$d$</td>
<td>duty cycle</td>
</tr>
<tr>
<td>$L_{in}$</td>
<td>equivalent phase inductance (L-N)</td>
</tr>
<tr>
<td>$V_s$</td>
<td>Battery voltage</td>
</tr>
<tr>
<td>$k_e$</td>
<td>Back EMF constant</td>
</tr>
<tr>
<td>$V_p$</td>
<td>effective phase voltage</td>
</tr>
<tr>
<td>$I_p$</td>
<td>amplitude of the fundamental component</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>conduction angle</td>
</tr>
</tbody>
</table>

where

$$\tau_z = \sqrt{j2\pi f_{pwm} \mu_z \sigma_z} \quad (B.2.13)$$

Note that the functions in section B.1 are almost the same of those of this section (section B.2). It is possible to program the functions in such a way that they can be used for both the commutation model and PWM model, this to avoid redundant code. In the implemented program the commutated model was programmed first, to make this program then work with the functions of this section would introduce quite some work and possibly introduce some debugging. The implemented models were just a prototype to compare
<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R'$</td>
<td>radius of the rotor without the magnet (rotor-magnet)</td>
</tr>
<tr>
<td>$R_{od}$</td>
<td>the outside radius (rotor+magnet)</td>
</tr>
<tr>
<td>$\mu_z$</td>
<td>permeability of material $z$, were $z$ is 1 (rotor core), 2 (magnet) or 3 (interpolar regions)</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>conductivity of material $z$, were $z$ is 1 (rotor core), 2 (magnet) or 3 (interpolar regions)</td>
</tr>
<tr>
<td>$\omega_m$</td>
<td>motor speed</td>
</tr>
<tr>
<td>$n$</td>
<td>the $n$'th-order harmonic component of a variable</td>
</tr>
<tr>
<td>$I_\nu(x)$</td>
<td>modified Bessel function (see also appendix B.3)</td>
</tr>
<tr>
<td>$I'_\nu(x)$</td>
<td>first derivative of the modified Bessel function (see also appendix B.3)</td>
</tr>
<tr>
<td>$K_\nu(x)$</td>
<td>modified Bessel function (see also appendix B.3)</td>
</tr>
<tr>
<td>$K'_\nu(x)$</td>
<td>first derivative of the modified Bessel function (see also appendix B.3)</td>
</tr>
</tbody>
</table>

Table B.2.2: Definition of the variables for equations B.2.9 through B.2.13

the results with the FEM analysis results and therefore there was no effort put into avoiding redundant code.

### B.3 Bessel and Kelvin functions

To determine the value of $C_{1n}$ (equation B.2.12) and $C_n$ (equation B.1.4) we need the modified Bessel functions $I_\nu(x)$ and $K_\nu(x)$, were $\nu$ is real, $x$ is real and non-negative and $n$ is a positive integer or zero. The value for $x$ we want to calculate the Bessel function is $\tau_{zn} \cdot r = \sqrt{j \mu_z \sigma_z \omega_n \cdot r}$ which is a pure imaginary number and thus not allowed. However if we use Kelvin functions we can solve this problem using the following equations

\[
I_\nu(z) = j^{-\nu} [\text{ber}_\nu(x) + j\text{bei}_\nu(x)]
\]

\[
K_\nu(z) = j^{\nu} [\text{ker}_\nu(x) + j\text{kei}_\nu(x)]
\]

\[
I'_\nu(z) = \frac{1}{\sqrt{j}} j^{-\nu} [\text{ber}'_\nu(x) + j\text{bei}'_\nu(x)]
\]

\[
K'_\nu(z) = \frac{1}{\sqrt{j}} j^{\nu} [\text{ker}'_\nu(x) + j\text{kei}'_\nu(x)]
\]

where $x = |z|$ and $z = \tau_{zn} \cdot r = \sqrt{j \mu_z \sigma_z \omega_n \cdot r}$ is. What we do is basically first calculating the values of the Kelvin function for the absolute value of $x$ en than using equations B.3.1 through B.3.4 to calculate the bessel function value for the imaginary input.

The Kelvin functions can be calculated on two different ways. the first one is expressing the Kelvin functions in Bessel functions as done in equation B.3.5 and B.3.6.

\[
\text{ber}_\nu(x) + j\text{bei}_\nu(x) = J_\nu \left( xe^{-\frac{3\pi}{4}} \right) = e^{\frac{3\nu\pi}{2}j} I_\nu \left( xe^{-\frac{3\pi}{4}} \right) \] (B.3.5)

\[
\text{ker}_\nu(x) + j\text{kei}_\nu(x) = e^{-\frac{3\nu\pi}{2}j} K_\nu \left( xe^{\frac{3\pi}{4}} \right) \] (B.3.6)

But the Kelvin functions can also be calculated using these recurrence relations:

\[
f_{\nu+1} + f_{\nu-1} = -\frac{\nu \sqrt{2}}{x} (f_{\nu} - g_{\nu}) \] (B.3.7)

\[
f'_\nu = \frac{1}{2\sqrt{2}} (f_{\nu+1} + g_{\nu+1} - f_{\nu-1} - g_{\nu-1}) \] (B.3.8)
\[ f'_v - \frac{\nu}{x} f_v = \frac{1}{\sqrt{2}} (f_{\nu+1} + g_{\nu+1}) \quad (B.3.9) \]

\[ f'_v + \frac{\nu}{x} f_v = -\frac{1}{\sqrt{2}} (f_{\nu-1} + g_{\nu-1}) \quad (B.3.10) \]

where there are four combinations are possible

\[
\begin{align*}
\begin{cases}
    f_v = \text{ber}_\nu(x) \\
g_v = \text{bei}_\nu(x)
\end{cases} & \quad \begin{cases}
    f_v = \text{bei}_\nu(x) \\
g_v = -\text{ber}_\nu(x)
\end{cases} \\
\begin{cases}
    f_v = \text{ker}_\nu(x) \\
g_v = \text{kei}_\nu(x)
\end{cases} & \quad \begin{cases}
    f_v = \text{kei}_\nu(x) \\
g_v = -\text{ker}_\nu(x)
\end{cases}
\end{align*}
\]

and the first value can be calculated using

\[ \sqrt{2} \cdot \text{ber}'(x) = \text{ber}_1(x) + \text{bei}_1(x) \]

\[ \sqrt{2} \cdot \text{bei}'(x) = -\text{ber}_1(x) + \text{bei}_1(x) \]

and

\[ \sqrt{2} \cdot \text{ker}'(x) = \text{ker}_1(x) + \text{kei}_1(x) \]

\[ \sqrt{2} \cdot \text{kei}'(x) = -\text{ker}_1(x) + \text{kei}_1(x) \]

The initial values of the Kelvin functions can be calculated using an polynomial approximation which can be found in [21] on pages 383-385.

**References**


