Design of a Rectenna for Wireless Low-Power Transmission

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Chapter 1

Introduction

If you ask people what they think the future will bring with respect to electronics, they often come up with the buzz-word 'wireless'. Everything, they say, will become wireless. Well, that's a nice statement, and indeed, the advantages of wireless applications are obvious. No more struggling with annoying cables and the ease of portability make it a tempting picture of the future. An important indication of the 'wireless future', which we can see all around us already, is the widespread use of mobile phones. Other examples like wireless networking and wireless sensors confirm that there is a large market for wireless applications.

Still, there is one big problem with wireless applications. They need power. Most applications have some sort of battery to provide the needed power for the application to work. Every now and then, this battery has to be recharged or replaced. It would be very convenient if it would be possible to provide the power the application needs without the need of recharging the battery. A solution can be sought in the form of wireless power transfer. An appealing example of wireless power transfer is the use of solar cells. Obviously, these methods also have their drawbacks. What to do with solar cells when it is dark? Nevertheless, wireless power transfer is definitely an interesting subject to work on with a great potential for the future.

Other applications for wireless power transfer can be found in environments where physical connectors cannot be easily placed. A sensor that measures the air pressure in a tire is an example in this category. Applications in enclosed volumes, like fuel tanks, can also be interesting for the use of wireless power transfer.

An overview of the developments of wireless power transfer in history is presented in a paper of W. Brown [1]. Serious research on the subject is started in the late 1950's. In those days, the research mainly focused on the transfer of large amounts of power. Methods to collect energy from the sun through satellites with solar cells and antennas to beam it to the earth were investigated. Brown even described the use of wireless power transfer to fly a small helicopter.

In this thesis, research will be focused on wireless power transfer via electromagnetic radiation. The concept is that an energy source is available that radiates energy through an antenna. This energy is picked up by a receiving antenna and converted to usable power for the application. In general, the amount of power you are allowed to transmit with electromagnetic radiation in public areas, is limited. Therefore this concept is only suitable for low-power applications. For example, one might think of wireless sensors which only have to perform one measurement every hour.
Our work has focused on the modelling and realization of a practical model for wireless power transfer. The model receives the electromagnetic energy through an antenna. The received energy is converted to electric power for our application with a rectifying circuit. Because of the combination of antenna and rectifying circuit the system is often referred to as 'rectenna'. The challenge in our research is to create a rectenna which has limited dimensions and an efficient conversion of the received energy to useful power.

The building blocks of the rectenna can be divided in three main components. First of all, there is the need for an antenna. The antenna receives the radiated energy and is therefore essential in determining the amount of power that is available for the application. Secondly, the received radio-frequent (RF) electromagnetic signal has to be converted to a direct-current (DC) signal to be useful as power supply for the application. This conversion is performed with the use of a diode. In the third place, there is the need to suppress unwanted signals. This can be achieved via filters.

Since our rectenna is used for low-power applications, the dimensions of the rectenna are of importance. In general, wireless low-power applications have small dimensions. Often, this is one of the strong points of the application. Therefore a rectenna with limited dimensions is advantageous. Another important requirement of the rectenna is its conversion efficiency. The amount of received energy that is converted to usable power should be as high as possible. Especially for low-power environments, where the amount of received energy is limited, the importance of an efficient conversion is high. A third requirement for the rectenna is that it provides a stable output power. Low-power applications often contain integrated circuits which need a stable power supply to operate properly.

The practical model that we would like to design is used in the frequency range from 2.40-2.48 GHz. This is a frequency band where license-free operation is permitted. Wireless power transfer systems often use this frequency band, owing to the low losses in the atmosphere for electromagnetic radiation in this frequency band. Another advantage is the availability of cheap components for these frequencies. The maximum amount of radiated power that is allowed in this frequency band is 100 mW [2]. For other countries, other regulations may apply. In general, only a small portion of the transmitted energy is received by the rectenna. For our practical model, we assume a received power level of 1 mW. Although this choice is somewhat arbitrary, it lies within the range of expected power levels for our applications.

The goal is to model our rectenna via analytical expressions. The advantage of analytical expressions is that they often give a good insight in the physical behaviour of the system. Sub-circuits of our rectenna, e.g. the antenna, can also be simulated via numerical full-wave simulations. This can lead to more accurate results, at the cost of a significantly larger computation time. Therefore we will start with an analytical approach. Throughout the whole design process the analytical models will be verified with measurements.

The strategy is to analyze the different sub-circuits of the rectenna separately. The antenna is modelled in the second Chapter. The main component of the rectifying circuit, the diode, is analyzed in the third Chapter. Transmission lines, which are used to connect the several components, are considered in the fourth Chapter. For the implementation of the filters, the use of radial stubs is examined. This is presented in the fifth Chapter. In the sixth Chapter the several components are joined together to form a working rectenna. In this chapter the combination of the several models is analyzed and verified. Here, the analytical models are used to design a prototype rectenna.
Chapter 2

Microstrip Antennas

An analytical model of a microstrip patch antenna is presented. This model considers the radiation pattern and input impedance of this type of antennas. Full-wave simulations and measurements verify the derived model.

2.1 Introduction

The antenna is an essential part of the rectenna system. It receives the transmitted energy and therefore it is an important factor in determining the amount of power transmission that is possible. There is a wide range of available antenna types for our application. For this project we have chosen to use a microstrip patch antenna. The main reasons for this choice are its small dimensions and the ease of manufacturing. The desired operating frequency lies around 2.45 GHz. This frequency lies within an ISM-band (Industrial, Scientific and Medical band), where license-free operation is permitted.

Figure 2.1: microstrip patch antenna

Figure 2.1 shows the general layout of a microstrip patch antenna. The antenna consists of a ground plane, which is made of a good conductor. Usually copper is chosen for this purpose. On
top of the ground plane a dielectric medium is present and on top of that another conducting plane is positioned. The conducting plane on top of the antenna is called the patch. Usually the patch is etched on a printed circuit board (PCB). The width and length of the antenna typically lie around \( \lambda/2 \), where \( \lambda \) is the wavelength in the dielectric at the operating frequency. The relative permittivity of the dielectric is denoted by \( \varepsilon_r \). The height \( h \) of the antenna is much smaller than the wavelength and often corresponds to the thickness of the circuit board. Typical values of \( h \) lie around 1 mm.

The antenna is connected to a coaxial probe on the backside of the antenna. The signal is connected to the patch through the dielectric as shown in Figure 2.1. There are other means of connecting the antenna, e.g., by using a microstrip line or inductive/capacitive coupling [3]. For now, these methods are beyond our scope.

Within the antenna, the electric fields are perpendicular to the upper and lower conducting planes. Near the edges of the patch the fields will bend a little outwards (Figure 2.2). These so-called ‘fringe fields’ cause the microstrip antenna to radiate. For small heights of the antenna the effect of fringe fields is small, therefore it is ignored in the first setup of our analytical model in the next section.

\[ \begin{align*}
\mathbf{E} & = -j\omega \mu \mathbf{H}, \\
\nabla \times \mathbf{H} & = \mathbf{J} + j\omega \varepsilon \mathbf{E}.
\end{align*} \tag{2.1} \]

Figure 2.2: fringe fields antenna

\[ \begin{align*}
\mathbf{E} & = -j\omega \mu \mathbf{H}, \\
\nabla \times \mathbf{H} & = \mathbf{J} + j\omega \varepsilon \mathbf{E}.
\end{align*} \tag{2.1} \]

2.2 Cavity model

To calculate the electromagnetic fields in the microstrip antenna, the antenna is modelled as a cavity with a perfectly conducting ground and top plane [4]. The side walls are assumed to be perfect magnetic conductors. This model ignores the effect of fringe fields around the edges of the antenna. For heights \( h \) of the antenna much smaller than the wavelength this is a valid approach.

The fields inside the microstrip antenna are calculated by using Maxwell’s equations

\[ \begin{align*}
\nabla \times \mathbf{E} & = -j\omega \mu \mathbf{H}, \\
\nabla \times \mathbf{H} & = \mathbf{J} + j\omega \varepsilon \mathbf{E}.
\end{align*} \tag{2.1} \]

where \( \mathbf{E} \) represents the electric field vector, \( \mathbf{H} \) the magnetic field vector, \( \mathbf{J} \) the electric current source, \( \omega \) the radial operating frequency of the fields, \( \varepsilon \) the permittivity of the dielectric and \( \mu \) the permeability of the dielectric.

Since the height \( h \) of the microstrip antenna is small compared to the wavelength, the fields are assumed to be independent of \( z \). The resulting fields will be transverse magnetic (TM) with respect
to the $z$ direction. The related wave equation for the $z$-component is given by

$$\frac{\partial E_z}{\partial x^2} + \frac{\partial E_z}{\partial y^2} + k^2 E_z = j\omega \mu J_z, \quad (2.2)$$

with the propagation constant $k = \omega \sqrt{\mu \varepsilon} = \frac{2\pi}{\lambda}$. Here $\lambda$ represents the wavelength of the fields in the dielectric. The solution of the differential equation (2.2) describes the $z$-directed electric fields inside the cavity. To find this solution the boundary conditions have to be considered.

The cavity model implies the following boundary conditions

$$H_y = 0 \quad H_z = 0 \quad \text{at} \quad x = 0, a,$$

$$H_x = 0 \quad H_z = 0 \quad \text{at} \quad y = 0, b,$$

$$E_x = 0 \quad E_y = 0 \quad \text{at} \quad z = 0, h. \quad (2.3)$$

To find the solution of equation (2.2), the solution of the homogeneous wave equation is evaluated first. The homogenous wave equation equals

$$\frac{\partial E_z}{\partial x^2} + \frac{\partial E_z}{\partial y^2} + k^2 E_z = 0. \quad (2.4)$$

It describes the electric field in a source-free cavity. The solution of the homogeneous wave equation which satisfies the boundary conditions, Eq. (2.3), can be expressed as

$$E_z^{mn} = A_{mn} \psi_{mn}(x, y), \quad (2.5)$$

with $\psi_{mn}(x, y)$ the $z$-directed orthogonal electric field mode functions. They are the eigenfunctions of

$$\frac{\partial E_z}{\partial x^2} + \frac{\partial E_z}{\partial y^2} + k^2 E_z, \quad (2.6)$$

with the boundary condition $\vec{n} \times \vec{H} = 0$ on the magnetic sidewalls. Via separation of variables we obtain

$$\psi_{mn}(x, y) = \cos(k_m x) \cos(k_n y), \quad (2.7)$$

with

$$k_m = \frac{m\pi}{a},$$

$$k_n = \frac{n\pi}{b}. \quad (2.8)$$
The eigenvalues of Equation 2.6 result in the resonance condition

\[ k^2 = k_m^2 + k_n^2 = k_{mn}^2. \] (2.9)

Inside the cavity it is possible that multiple modes co-exist. The modes \( \psi_{mn}(x, y) \) are orthogonal under a \( L^2 \)-inner product. The total electric field can therefore be expressed as the sum of the orthogonal modes \( \psi_{mn}(x, y) \), i.e.,

\[ E_z = \sum_m \sum_n E_{mn}^z(x, y) \] (2.10)

To find the electric field when a current source is present, the homogeneous solution, Eq. (2.10), is substituted in the original wave equation (Eq. (2.2)), owing to completeness

\[ \sum_m \sum_n \left( \frac{\partial E_{mn}^z}{\partial x^2} + \frac{\partial E_{mn}^z}{\partial y^2} + k^2 E_{mn}^z \right) = j\omega \mu J_z. \] (2.11)

From Equation (2.4) and condition (2.9) we can derive the equality

\[ \frac{\partial E_{mn}^z}{\partial x^2} + \frac{\partial E_{mn}^z}{\partial y^2} = -k_{mn}^2 E_{mn}^z. \] (2.12)

Substituting this equality in Equation (2.11) gives the relation

\[ \sum_m \sum_n (-k_{mn}^2 E_{mn}^z + k^2 E_{mn}^z) = j\omega \mu J_z. \] (2.13)

From this relation, we want to find an expression for the amplitude coefficients \( A_{mn} \) of the electric field. Multiplying the left and right term by \( \psi_{mn} \) and integrating over the volume of the cavity gives

\[ \int_V \sum_m \sum_n (k_{mn}^2 - k^2) E_{mn}^z \psi_{m'n'} \, dV = -j\omega \mu \int_V J_z \psi_{m'n'} \, dV, \]

\[ \int_V \sum_m \sum_n (k_{mn}^2 - k^2) A_{mn} \psi_{mn} \psi_{m'n'} \, dV = -j\omega \mu \int_V J_z \psi_{m'n'} \, dV. \] (2.14)

This expression can be simplified when we use the orthogonality of the electric field functions. The
volume integral of the product of $\psi_{m_1n_1}$ and $\psi_{m_2n_2}$ equals

$$
\int_V \psi_{m_1n_1}\psi_{m_2n_2} \, dV = \int_{z=0}^{h} \int_{y=0}^{b} \int_{x=0}^{a} \psi_{m_1n_1}\psi_{m_2n_2} \, dx \, dy \, dz
$$

$$
= \begin{cases} 
0 & \text{for } m_1 \neq m_2 \lor n_1 \neq n_2 \\
\frac{abh}{\chi_{mn}^2} & \text{for } m_1 = m_2 \land n_1 = n_2 
\end{cases} 
$$

where

$$
\chi_{mn} = \begin{cases} 
1 & \text{for } m = 0 \land n = 0 \\
\sqrt{2} & \text{for } m = 0 \lor n = 0 \\
2 & \text{for } m \neq 0 \land n \neq 0 
\end{cases} 
$$

By using Eq. (2.15), the left hand side of Eq. (2.14) can be rewritten as

$$
\sum_{m} \sum_{n} (k_{mn}^2 - k^2) A_{mn} \frac{abh}{\chi_{mn}^2} = -j\omega\mu \int_V J_z \psi_{mn} \, dV. 
$$

This relation holds for each mode and therefore the amplitude coefficients are given by

$$
A_{mn} = j \frac{\chi_{mn}^2}{abh} \frac{\omega\mu}{k^2 - k_{mn}^2} \int_V J_z \psi_{mn}(x,y) \, dV. 
$$

With equation (2.10) the electric field

$$
E_z = j \frac{\eta}{abh} \sum_{m} \sum_{n} \frac{\chi_{mn}^2 k_{mn}}{k^2 - k_{mn}^2} \psi_{mn}(x,y) \int_V J_z \psi_{mn}(x,y) \, dV, 
$$

where $\eta = \sqrt{\frac{\mu}{\varepsilon}}$ represents the intrinsic impedance. Expression (2.19) is also presented in [5, p. 431-433] where a similar derivation is given.

If we assume that the microstrip antenna has a probe feed at position $x_0$ and $y_0$ with negligible diameter, the current source $J_z$ can be modelled as $J_z = I_0 \delta(x-x_0)\delta(y-y_0)$. Here $I_0$ can be chosen constant because of the limited height of the substrate. The resulting electric field in the microstrip antenna for this configuration is given by

$$
E_z = \sum_{m} \sum_{n} A_{mn} \psi_{mn}(x,y), 
$$
\[ A_{mn} = jI_0 \frac{\eta k \lambda_{mn}^2}{ab} \frac{k}{k^2 - k_{mn}^2} \psi_{mn}(x_0, y_0). \]  

(2.21)

### 2.3 Radiation pattern

The radiation pattern is calculated by modelling the microstrip antenna as four independent radiating apertures. The far field pattern can be calculated by introducing an equivalent magnetic current density \( \vec{M} = \vec{E} \times \vec{n} \) [6] on the walls of the antenna according to Love’s equivalence principle [5]. For the TM\(_{10}\) mode, the electric fields and the equivalent magnetic current densities are shown in Figure 2.3.

![Figure 2.3: equivalent magnetic currents for TM\(_{10}\) mode](image)

#### 2.3.1 Radiation pattern aperture

To calculate the radiation pattern of the antenna, the radiation pattern of one wall is analyzed first. As mentioned before, the radiation pattern of a wall is calculated by modelling the wall as an aperture with an equivalent magnetic current on an infinite ground plane. This allows us to use the electric vector potential [7, ch.5]

\[ \vec{F} = \frac{\varepsilon_0}{4\pi} \int_{S} \vec{M}_s \frac{e^{-jk_0|R|}}{|\vec{R}|} dS', \]  

(2.22)

where \( \varepsilon_0 \) represents the permittivity of free space and \( k_0 \) is the free-space wavenumber. The surface \( S \) represents the side wall and \( |\vec{R}| \) the distance between source and observation point as shown in Figure 2.4.

To calculate the electric vector potential in the far-field region, we are allowed to make some approximations [7, ch.3]. For the amplitude of the fields in the far-field region we can replace \(|\vec{R}|\) with \(|\vec{r}'|\), the distance from the origin to the observation point. The distance \(|\vec{R}|\) in the phase term \( e^{-jk_0|\vec{R}|} \) of the electric vector potential can be approximated with \(|\vec{R}| \approx |\vec{r}'| - |\vec{r}'| \cos(\xi)\). Where \( \vec{r}' \) is the distance between the origin and the source point and \( \xi \) the angle between \( \vec{r}' \) and \( \vec{r} \).

The equivalent magnetic surface current is \( \vec{M}_s = 2\vec{E} \times \vec{n} \). On the wall \( x = 0 \) of the antenna, the
electric field for the mode with indices $mn$ looks like (2.20)

$$\vec{E}_{\text{wall}} = A_{mn} \cos\left(\frac{n\pi}{b} y\right) \hat{a}_z,$$  \hspace{1cm} (2.23)

with $A_{mn}$ an amplitude factor depending on the mode indices $mn$ and radial frequency $\omega$ as in Eq. (2.18). The electric vector potential in the far-field region can be found by computing

$$\vec{F} = \frac{\mathbf{\epsilon}_0 \mathbf{\hat{a}}_y}{2\pi} \int_{z=0}^{h} \int_{y=0}^{b} A_{mn} \cos\left(\frac{n\pi}{b} y\right) \frac{e^{-j k_0 |r'| - |r| \cos(\xi)} \cos(\pi z'/l)}{r} \, dy' \, dz',$$  \hspace{1cm} (2.24)

where we express $|\mathbf{r}'|$ in Cartesian coordinates (Figure 2.4), i.e.,

$$|\mathbf{r}'| \cos(\xi) = r' \cdot \mathbf{\hat{a}}_r,$$

$$= y' \sin(\theta) \sin(\phi) + z' \cos(\theta).$$  \hspace{1cm} (2.25)

![Figure 2.4: spherical and Cartesian coordinates](image)

This results in

$$\vec{F} = \frac{\mathbf{\epsilon}_0 \mathbf{\hat{a}}_y}{2\pi r} A_{mn} e^{-jk_0 r} \int_{z=0}^{h} \int_{y=0}^{b} \cos\left(\frac{n\pi}{b} y\right) e^{-j k_0 [y' \sin(\theta) \sin(\phi) + z' \cos(\theta)]} \, dy' \, dz',$$  \hspace{1cm} (2.26)

$$= \frac{j \mathbf{\epsilon}_0 \mathbf{\hat{a}}_y}{\pi r} e^{-jk_0 r} A_{mn} Y' \left[\frac{(-1)^n e^{-j2Y} - 1}{4Y^2 - n^2 \pi^2} \sin(Z) e^{-jZ}\right],$$

where

$$Y = \frac{b}{2} k_0 \sin(\theta) \sin(\phi),$$

$$Z = \frac{h}{2} k_0 \cos(\theta).$$  \hspace{1cm} (2.27)
To find the radiated electric far field the relation, we use the relation

\[ \vec{E}_{rad} = \frac{1}{\varepsilon_0} \nabla \times \vec{F}. \]  

(2.28)

In the far-field region, the radial component of the fields is negligible compared to the \( \theta \)- and \( \phi \)-components. The relation \( \vec{a}_\theta = \sin(\theta)\sin(\phi)\vec{a}_r + \cos(\phi)\vec{a}_\phi + \cos(\theta)\sin(\phi)\vec{a}_\theta \) together with Equation (2.28) gives us an expression for the radiated electric field in the far-field region as

\[ \vec{E}_{rad} = -j\frac{k_0}{\varepsilon_0} F_y(\cos(\theta)\sin(\phi)\vec{a}_\phi + \cos(\phi)\vec{a}_\theta). \]  

(2.29)

When there are multiple modes excited in the antenna, the radiated field in the far-field region is simply the sum of the radiated fields due to the separate modes.

### 2.3.2 Radiation pattern of a microstrip antenna

Now the radiation pattern of a single wall has been determined, we can derive the radiation pattern of the resulting antenna by modelling it as an array of four apertures. Two opposite walls radiate identical fields, except for a possible phase shift of \( \pi \) radians depending on the mode of the antenna. Another phase shift occurs because of the spacing between the two side walls. The total radiated field for a single mode can be calculated by considering the antenna as two arrays of two aperture antennas.

The array factor for two identical antennas with phases \( \alpha_1, \alpha_2 \) and a vectorial spacing \( \vec{d} \) is given by [8]

\[ AF = 2 \cos \left( \frac{k_0}{2} \vec{d} \cdot \vec{r} + \frac{\alpha_1 - \alpha_2}{2} \right) e^{j\frac{\alpha_1 + \alpha_2}{2}}. \]  

(2.30)

Thus, the total radiated field is given by

\[ \vec{E}_{rad} = \vec{E}_{rad}^a AF_a + \vec{E}_{rad}^b AF_b, \]  

(2.31)

with

\[ \vec{E}_{rad}^a = k_0 \frac{bh}{\pi r} e^{-jk_0 r} A_{mn} \frac{Y[(-1)^n e^{-j2Y} - 1]}{4Y^2 - n^2\pi^2} \text{sinc}(Z)e^{-jZ}(\cos(\theta)\sin(\phi)\vec{a}_\phi + \cos(\phi)\vec{a}_\theta), \]

\[ \vec{E}_{rad}^b = k_0 \frac{ah}{\pi r} e^{-jk_0 r} A_{mn} \frac{X[(-1)^m e^{-j2X} - 1]}{4X^2 - m^2\pi^2} \text{sinc}(Z)e^{-jZ}(\cos(\theta)\cos(\phi)\vec{a}_\phi - \sin(\phi)\vec{a}_\theta), \]  

(2.32)

\[ AF_a = 2 \cos(X - \alpha_m) e^{j\alpha_m}, \]

\[ AF_b = 2 \cos(Y - \alpha_n) e^{j\alpha_n}, \]  

(2.33)
where
\[
X = \frac{a}{2} k_0 \sin(\theta) \cos(\phi),
\]
\[
Y = \frac{b}{2} k_0 \sin(\theta) \sin(\phi),
\]
\[
Z = \frac{h}{2} k_0 \cos(\theta),
\]
and
\[
\alpha_i = \begin{cases} 
\frac{\pi}{2} & \text{for } i \text{ even,} \\
0 & \text{for } i \text{ odd.} 
\end{cases}
\] (2.35)

Again, the radiated field for multiple modes is found by summing the radiated fields of the independent modes.

The total radiated power for the mode with indices \(mn\) can be found by integrating the square of the radiated field over a hemisphere, i.e.,
\[
P_{rad} = \int_{\theta=0}^{\frac{\pi}{2}} \int_{\phi=0}^{2\pi} \frac{1}{2\eta_0} |\vec{E}_{rad}|^2 r^2 \sin(\theta) \, d\phi \, d\theta,
\] (2.36)

with \(\eta_0 = \sqrt{\mu_0 / \varepsilon_0}\) the wave impedance for vacuum.

### 2.4 Input impedance

#### 2.4.1 Lossless input impedance

The input impedance of the antenna is defined as [5]
\[
Z_{in} = -\frac{1}{|I_0|^2} \int_V \vec{E} \cdot \vec{J}^* \, dV
\] (2.37)

Where \(I_0\) is the current that enters the port of the antenna. If we take the field distribution of equation (2.20) and the volume \(V\) equal to the inner space of the cavity, the input impedance will be imaginary because there are no losses in the cavity model. In this case the input impedance can be modelled as a sum of reactances \(X_{mn}\).

\[
Z_{in} = -j \sum_m \sum_n \frac{\eta X_{mn}^2}{ab} \frac{k}{k^2 - k_{mn}^2} \psi_{mn}^2(x_0, y_0),
\]
\[
= -j \sum_m \sum_n X_{mn}.
\] (2.38)
This is equivalent to a series of LC circuits as shown in Figure 2.5 for each mode \( mn \)

![Figure 2.5: LC circuit model](image)

with network parameters

\[
L = \sqrt{\mu \varepsilon} \frac{\eta \lambda_{mn}^2}{ck^2_{mn}} \psi_{mn}^2(x_0, y_0), \tag{2.39}
\]

\[
C = \sqrt{\mu \varepsilon} \frac{ab}{\eta \lambda_{mn}^2} \frac{1}{\psi_{mn}^2(x_0, y_0)}. \tag{2.39}
\]

### 2.4.2 Losses

In Equation (2.38) some important power losses are not considered. First of all, we have not accounted for radiation losses, which is obviously an important factor for an antenna. Further, there are some additional losses, because of the non-perfect isolation of the dielectric and the non-perfect conduction of the conducting planes.

To account for these losses an extra resistance term is added for each mode to the equivalent network as shown in Figure 2.6.

![Figure 2.6: RLC circuit model](image)

The input impedance of this circuit is given by

\[
Z_{in} = \frac{j\omega RL}{R - \omega^2 RLC + j\omega L},
\]

\[
= -\frac{j\omega \frac{1}{C}}{\omega^2 - \omega^2 (1 + \frac{1}{Q})}, \tag{2.40}
\]

where the resonance frequency equals \( \omega_r = \frac{1}{\sqrt{LC}} \) and the quality factor is given by \( Q = \frac{R}{\omega L} \).
Since we have no explicit expression for the losses \( R \) in the circuit model. The input impedance is calculated via the definition of the quality factor \( Q \) in terms of field concepts, i.e.,

\[
Q = \frac{\text{magnitude of reactive current density}}{\text{magnitude of dissipative current density}}
\]  

(2.41)

The total quality factor can be calculated from the quality factors that are related to the separate losses. For the radiation losses we find

\[
Q_{\text{rad}} = \frac{2\omega W_{E}}{P_{\text{rad}}}. 
\]  

(2.42)

Here \( P_{\text{rad}} \) is the total radiated power as defined in Equation (2.36) and the stored electric energy \( W_{E} \) is given by the volume integral over the squared electric field within the cavity, i.e.,

\[
W_{E} = \int_{V} \varepsilon |\vec{E}|^{2} dV, \\
= |A_{mn}|^{2} \frac{ab\varepsilon}{\chi_{mn}^{2}}. 
\]  

(2.43)

The quality factor for the losses in the conducting planes are given in [9] as

\[
Q_{c} = h\sqrt{\frac{\omega}{2 \mu_{0} \sigma}}, 
\]  

(2.44)

where \( \mu_{0} \) represents the permeability of vacuum and \( \sigma \) the conductivity of the patch and ground plane. Further, because of the losses in the dielectric we get the quality factor

\[
Q_{d} = \frac{1}{\tan \delta}, 
\]  

(2.45)

with \( \tan \delta \) the so-called loss tangent of the dielectric. \( \tan \delta = \frac{\sigma d}{\omega \varepsilon} \), where \( \sigma_{d} \) represents the conductivity of the dielectric.

The total \( Q \) can be easily found with the relation [5, pg. 76]

\[
\frac{1}{Q} = \frac{1}{Q_{\text{rad}}} + \frac{1}{Q_{c}} + \frac{1}{Q_{d}}. 
\]  

(2.46)

### 2.4.3 Input impedance model

The input impedance of the antenna can be modelled as a cascade of the RLC-circuits shown in Figure (2.6) for all modes. Still, this approach results in a slowly converging series which can take a long computation time. Therefore a few simplifications will be made.
The $TM_{00}$ mode has no inductance $L$ and a capacitance of $C = \frac{a b \epsilon}{h}$. The impedance of the $TM_{00}$ mode is therefore modelled as a parallel $RC$ circuit

$$Z_{00} = \frac{R}{1 + j \omega RC} = \frac{j \frac{1}{\omega C}}{1 - \frac{j}{Q}} ,$$

where $Q$ is calculated by Equation (2.46). In general, the $TM_{00}$ mode does not radiate much energy compared to the other modes, because no resonance occurs for this mode.

The non-radiating modes, usually referred to as higher order modes, have obviously no radiation losses. The inductive reactance of the higher order modes can be well approximated by [4]

$$X_L = \sqrt{\frac{\mu}{\varepsilon}} \tan(\omega \sqrt{\mu \varepsilon h}).$$

So we model the higher order modes as a parallel $RL$ circuit

$$Z_{RL} = \frac{j \omega RL}{R + j \omega L} = \frac{j \omega L}{1 + \frac{j}{Q}} ,$$

with $Q^{-1} = Q_e^{-1} + Q_d^{-1}$.

Finally, the resulting circuit model for the input impedance is given in Figure 2.7.
2.5 Example: \( TM_{10} \) mode

To clarify the results in the preceding sections, the electric field distribution, radiation pattern and input impedance is calculated for a microstrip antenna with a dominant \( TM_{10} \) mode.

The electric field in the cavity is given by Equation (2.20)

\[
E_z = A_{10} \psi_{10}(x, y) = A_{10} \cos \left( \frac{\pi}{a} x \right),
\]

with

\[
A_{10} = j I_0 \frac{\eta \chi_{mn}}{ab} \frac{k}{k^2 - k_{10}^2} \psi_{10}(x_0, y_0).
\]

For the wall at \( x = 0 \) we find the following electric vector potential in the far-field region from Equation (2.26)

\[
\vec{A}_x = \frac{\varepsilon_0 \beta \bar{h}}{2 \pi r} e^{-j k_{10} r} A_{10} \text{sinc}(Y) \text{sinc}(Z) e^{-j(Y+Z)} \mathbf{a}_y,
\]
where

\[
Y = \frac{b}{2} k_0 \sin(\theta) \sin(\phi),
\]

\[
Z = \frac{h}{2} k_0 \cos(\theta).
\]

and therefore the radiated electric field according to Equation (2.29) is given by

\[
\vec{E}_{rad} = -j \frac{k_0 b h}{2 \pi r} e^{-j k_0 r} A_{10} \text{sinc}(Y) \text{sinc}(Z) e^{-j(Y+Z)} (\cos(\theta) \sin(\phi) \alpha_\phi + \cos(\phi) \alpha_\theta). \tag{2.53}
\]

The wall at \( x = a \) has the same distribution as its opposite wall at \( x = 0 \), but with a phase difference of \( \pi \) radians. The electric fields at the side walls are in phase. The electric vector potential from sources at \( y = 0 \) is given by

\[
\vec{F}_b = j \frac{\varepsilon_0 a h}{\pi r} e^{-j k_0 r} A_{10} X \frac{-e^{-j2X} - 1}{4X^2 - \pi^2} \text{sinc}(Z) e^{-j(X+Z)} \alpha_z, \tag{2.54}
\]

where

\[
X = \frac{a}{2} k_0 \sin(\theta) \cos(\phi), \tag{2.55}
\]

and a radiated field in the far-field region

\[
\vec{E}_{rad}^b = \frac{k_0 a h}{\pi r} e^{-j k_0 r} A_{10} \frac{X(-e^{-j2X} - 1)}{4X^2 - \pi^2} \text{sinc}(Z) e^{-j k_0 (X+Z)} (\cos(\theta) \cos(\phi) \alpha_\phi - \sin(\phi) \alpha_\theta). \tag{2.56}
\]

The total radiation pattern of the microstrip antenna is calculated with (2.31) with appropriate antenna factors

\[
\vec{E}_{rad} = \vec{E}_{rad0} A_{Fa} + \vec{E}_{rad1} A_{Fb}
\]

\[
A_{Fa} = j 2 \sin(X)
\]

\[
A_{Fb} = 2 \cos(Y) \tag{2.57}
\]

Simulations and measurements have been performed for a rectangular patch antenna operating in \( TM_{10} \) mode at a frequency \( f \) around 2.45GHz. Its physical dimensions are \( a = 29.0 \) mm, \( b = 29.0 \) mm, \( h = 1.6 \) mm and probe position \( x_0 = 9.45 \) mm, \( y_0 = 14.6 \) mm. The conductance of the copper patch and ground plane equals \( \sigma = 5.8 \cdot 10^7 \Omega^{-1}m^{-1} \) and the dielectric constant \( \varepsilon_r = 4.28 \) with dielectric loss tangent \( \tan \delta = 1.6 \cdot 10^{-2} \).

The radiation pattern of the model is compared with a full-wave simulation of the antenna, made with Ansoft Ensemble student version 7. In the simulation, the ground plane is taken as an infinitely
Figures 2.8 and 2.9 show that the accuracy of the model is quite reasonable for $-30^\circ < \theta < 30^\circ$, though the results for larger angles are less precise compared to the full-wave simulation.

The reflection coefficients $\Gamma = \frac{\epsilon_0 - \epsilon}{\epsilon_0 + \epsilon}$ are plotted in Figure 2.10 to compare the analytical model with the full wave model. Here we see that there is an error in the resonance frequency of approximately 3%. The neglected fringe fields at the edges of the antenna are the main reason for this difference. To compensate for this effect, an improved model is proposed in the next section.
2.6 Fringe fields

The effect of fringe fields at the borders of the antenna is not accounted for in the derived model. To compensate for this effect a small extension of the physical width and length of the patch in the model can be applied. The resulting effective length can be used in the analytical model to get a more accurate result.

An empirical model for the effective length of a microstrip line has been proposed by Erik O. Hammerstad in 1975 [10]. This model is used to determine the effective length of an open end microstrip line with width $w$ and height $h$ (Figure 2.11). M. Kirschning et al. [11] have made some improvements to this model a few years later.

![Figure 2.10: reflection coefficient](image)

![Figure 2.11: effective length microstrip line](image)

For $TM_{0n}$ and $TM_{m0}$ modes, the field distribution in the antenna is equivalent to that of the microstrip line. Therefore we have tried to add an extension of $2\delta$ to the width or length of the antenna for these modes (Figure 2.12). For other (nonzero) modes the effect of fringe fields is
found to be less significant and no length extension is added for these modes. For $\delta$ we find [10]

$$\frac{\delta}{h} = 0.412 \frac{\varepsilon_r + 0.3}{\varepsilon_e - 0.258} \frac{\frac{w}{h} + 0.262}{\frac{w}{h} + 0.813},$$  \hspace{1cm} (2.59)

with

$$\varepsilon_e = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \sqrt{1 + \frac{12h}{w}}.$$  \hspace{1cm} (2.60)

![Figure 2.12: effective dimensions patch antenna](image)

The length extension formulas of Hammerstad [10] and Kirschning et al. [11] are based on the open end microstrip line. For the $TM_{0n}$ and $TM_{m0}$ modes, the antenna is seen as a microstrip line with two open ends. Therefore a length extension of $2\delta$ was initially proposed. However, measurements show that this extension is too much. A length extension of only $\delta$ gives more accurate results.

The use of effective dimensions in the analytical model gives some good results for the resonance frequency and input impedance. The Smith chart in Figure 2.13 shows the normalized reflection coefficient for $Z_0 = 50 \, \Omega$. The model is verified by a measurement, as indicated in the figure by crosses.

The reflection coefficient of the analytical model is calculated at the point where the probe is connected to the conducting plane. The reflection coefficient of the antenna is measured at the ground plane, where the physical connector is situated. To compensate for this difference, a transmission line model is used to model the phase shift at the ground plane (Figure 2.14). The reflection coefficient at the input port equals $\Gamma_A = \Gamma_L e^{-j2kh}$, with $k$ the propagation constant of the dielectric.

As can be seen from Figure 2.13, the analytical model performs quite well. The resonance frequency lies at 2.445 GHz according to the model, whereas the measurement resonates at 2.430 GHz, which corresponds to an error in resonance frequency of less than 1%.
2.7 Measurements

To verify the analytical model, a number of measurements have been performed for several patch antennas. The antennas are all etched on FR4 material with a metal layer made of copper which has a conductivity $\sigma = 5.8 \cdot 10^7 \, \text{\Omega}^{-1}\text{m}^{-1}$. The dielectric constant of the dielectric equals $\varepsilon_r = 4.28$ and the dielectric loss tangent is given by $\tan \delta = 1.6 \cdot 10^{-2}$. The values for $\varepsilon_r$ and $\tan \delta$ are based on previous measurement results of a microstrip line on this material.

Five antennas have been made with the probe position at different locations along the centerline in the $x$ direction. The dimensions of the first five antennas are $a = 29.00 \, \text{mm}$, $b = 29.00 \, \text{mm}$ and $h = 1.60 \, \text{mm}$. The probe positions are $x_0 = 3.0; 6.0; 8.0; 10.0; 12.0 \, \text{mm}$ with $y_0 = 14.5 \, \text{mm}$. Another five antennas are made with the probe position along the diagonal of the antenna. These antennas have smaller dimensions $a = 25.00 \, \text{mm}$, $b = 25.00 \, \text{mm}$ and $h = 1.60 \, \text{mm}$. The material constants are identical to the other case. Probe positions are $x_0 = y_0 = 3.0; 5.0; 7.0; 9.0; 11.0 \, \text{mm}$.

The antennas are measured at a frequency range from 2.0GHz to 3.6GHz. In this frequency range only the $TM_{10}$ and $TM_{01}$ modes are the dominant radiation modes. The reflection coefficient is measured at the end of the connector on the back side of the antenna. Therefore some extra phase difference has to be added to the antenna model. First of all, the phase shift of the current on the probe within the antenna is not modelled. As shown in the previous section, this phase shift equals $\Gamma_A = \Gamma L e^{-j2kh}$.

Another phase shift is caused by the connector. The reflection coefficient is measured at the end.
of the coaxial SMA connector on the back of the antenna. Therefore the model is extended with a small transmission line which models the phase shift due to the connector (Figure 2.15). The physical length of the connector is \( l = 6.9 \) mm. It has a teflon isolator with a relative permittivity of \( \varepsilon_r = 2.2 \). The phase shift of the reflection coefficient equals \( \Gamma = \Gamma_A e^{-j2kcl} \) with \( k_c \) the propagation constant in the connector.

![Figure 2.15: transmission line model with connector](image)

The transmission line model of Figure 2.15 still results in a phase error with the measured results. Unfortunately it is not clear what is causing this error. There is some suspicion that the calibration of the measurement equipment is not completely reliable. The reference plane for the impedance measurement seems to be shifted somewhat. For now, the length \( l \) of the connector in the transmission line model is adjusted such that the phase of the measurement agrees with the analytical model. Though \( l \) is adjusted, it is fixed for all cases of the measurements. Derived results are shown below.

### Measurement results probe position along center line

The Figures 2.16 through 2.20 show the measured reflection coefficient of the antennas with a varying probe position along the center line. The reflection coefficients are plotted in a Smith chart and are normalized for \( Z_0 = 50 \) Ω. The figures clearly show the effect of a change in probe position and also the accuracy of the analytical model, indicated by the dashed line.

![Figure 2.16: Smith chart \( x_0 = 3.0 \) mm, \( y_0 = 14.5 \) mm](image)

![Figure 2.17: Smith chart \( x_0 = 6.0 \) mm, \( y_0 = 14.5 \) mm](image)
Figure 2.18: Smith chart $x_0 = 8.0$ mm, $y_0 = 14.5$ mm

Figure 2.19: Smith chart $x_0 = 10.0$ mm, $y_0 = 14.5$ mm

Figure 2.20: Smith chart $x_0 = 12.0$ mm, $y_0 = 14.5$ mm

Measurement results probe position along diagonal

Another five antennas are measured with a varying probe position along the diagonal of the antenna. The reflection coefficient is plotted in a Smith chart with $Z_0 = 50\Omega$. The dimensions of the antenna are smaller than the dimensions of the previous case. For this measurement we have $a = b = 25.00$ mm. Again the model is found to be reasonably accurate. The difference between the modelled and measured reflection coefficient $||\Gamma_{\text{model}} - \Gamma_{\text{measurement}}||$ is less than 0.11 for all measured patch antennas.
Figure 2.21: Smith chart $x_0 = y_0 = 3.0$ mm

Figure 2.22: Smith chart $x_0 = y_0 = 5.0$ mm

Figure 2.23: Smith chart $x_0 = y_0 = 7.0$ mm

Figure 2.24: Smith chart $x_0 = y_0 = 9.0$ mm

Figure 2.25: Smith chart $x_0 = 10.3$ mm $y_0 = 11.0$ mm
Measurement 2-10 GHz

The antennas have also been measured for a wider frequency range from 2.0GHz to 10.0GHz. In this frequency range more radiating modes are exited. Radiating $TM$ modes from $TM_{10}$ up to $TM_{04}$ are recognized in the reflection measurement (Figure 2.26). This figure verifies the model for a wide frequency range. The measured antenna has dimensions $a = 29.00$ mm $b = 29.00$ mm $h = 1.6$ mm and probe position $x_0 = 8.0$ mm, $y_0 = 14.5$ mm. The model assumes radiating modes with indices $mn$ for all combinations of $0 < m < 4$ and $0 < n < 4$. The impedance of the higher-order modes is calculated by summing the impedances of the equivalent RLC circuits for all modes up to $m = 100$ and $n = 100$.

![Figure 2.26: Reflection coefficient wider frequency range](image)

2.8 Conclusions

In this chapter an analytical model for a probe-fed microstrip patch antenna has been derived. Starting from Maxwell's equations and boundary conditions, the field distribution within the antenna has been calculated. Here the concept of operation modes of the antenna has been introduced. Subsequently, the radiation behaviour of the antenna has been modelled using aperture antenna theory. This has been combined in a circuit model where additional copper and dielectric losses were also taken into account.

The analytical model was verified with measurements on microstrip antennas with two different dimensions and varying probe positions. The measurements agree reasonably well with the model. The resonance frequency as well as the reflection coefficient can be predicted accurately. At this point the model seems accurate enough to be used in the rectenna design.
Chapter 3

The Schottky diode

The rectifying circuit of the rectenna converts the received RF power to DC power. To achieve a high conversion efficiency, accurate models for the rectifying circuit are needed. An important part of the rectifying circuit is the Schottky diode. A nonlinear model of the Schottky diode is presented to accurately describe its behaviour for a wide range of received RF power levels.

3.1 Introduction

In previous chapter we have modelled the antenna of our rectenna system. In this chapter we focus on the rectifying circuit. The received RF power from the antenna has to be converted to DC power by a rectifying circuit. In this way, we are able to pass the received power to the application which we want to supply with electric energy. Obviously, we would like to convert as much of the RF power to DC power as possible. Therefore it is important to accurately model our rectifying circuit, such that we are able to achieve an efficient conversion.

A simple rectifying circuit is shown in Figure 3.1. Here we see that the sinusoidal generator voltage is rectified by the diode. The diode is a nonlinear device, because it has a nonlinear relation between the voltage over the diode and the current through it. The output voltage of the circuit will consist of the sum of a DC voltage and some sinusoidal harmonics. The capacitance $C_L$ is used to attenuate the harmonics such that a steady DC voltage is found at the output. The resistance $R_L$ is implemented to model the power dissipation of the application.

At a first glance, modelling the circuit of Figure 3.1 does not seem to be a problem. However, for

![Figure 3.1: rectifying circuit](image-url)

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our application it is essential that we can accurately describe the input impedance of the rectifying circuit at the operating frequency of the antenna. If we are able to model the input impedance of the rectifying circuit at this frequency, we can try to match the output impedance of the antenna with the input impedance of the rectifying circuit, such that there is minimal reflected power and thus, an efficient RF to DC conversion.

The ideal voltage-current characteristic of a diode is shown in Figure 3.2. In practice, the voltage-current characteristic is not ideal. The diode needs a certain voltage to operate the diode in forward bias. This voltage is called the forward bias voltage and is indicated by $V_{fb}$. For our application, we choose the Schottky diode, because of its low forward bias voltage. The Schottky diode consists of an n-type semiconductor and a metal which are brought into contact [12, Sec. 5.9]. At the junction, a depletion layer is formed where no free carriers (electrons) are present and positively charged donors are exposed. Between the metal and the semiconductor a contact potential is formed which is called the built-in potential. The Schottky diode has a fast switching time between forward and reverse bias, because of the relatively thin depletion layer. Figure 3.3 gives a schematic layout of a Schottky diode.

When an external voltage is applied to the Schottky diode, the width of the depletion region will be affected. When a positive voltage is applied, the width of the depletion region decreases. At the forward bias voltage, the depletion region has vanished and the resistance of the diode is mainly determined by the conductance of the semiconductor. When a negative voltage is applied, the depletion layer widens and only a very small current remains, which is due to thermal emission in the depletion region.

The voltage-current relation of the Schottky diode can be modelled well by the following equation.

$$i_d = I_s \left( e^{\alpha V_d - i_d R_s} - 1 \right) ,$$  \hspace{1cm} (3.1)
where \( v_d \) is the voltage over the Schottky diode, \( I_s \) the saturation current, \( R_s \) the resistance of the semiconductor and \( \alpha \) is given by

\[
\alpha = \frac{q}{nkT}.
\]  

(3.2)

Here \( q \) equals the charge of the electron, \( k \) Boltzmann's constant, \( n \) the diode ideality factor and \( T \) the temperature in Kelvin. The diode ideality factor can be found from the datasheet of the device. Typically, the value of \( n \) is close to unity.

### 3.2 Input impedance

#### 3.2.1 Nonlinear input impedance model

There are several theories concerning the input impedance of the diode in general and the Schottky diode in particular. Often, these theories use a linearized diode model. This is a valid approach when the voltage and current signals are relatively small. In these cases the nonlinear voltage-current characteristic of the diode can be linearized around a working point. However, in our case the input signals are large; typically the amplitude of the unloaded input voltage will exceed the forward bias voltage. Therefore we need a nonlinear model which describes the behaviour of the Schottky diode for large signals.

To calculate the input impedance of the Schottky diode, the nonlinear circuit model of Figure 3.4 is widely accepted [13], [14]. Here \( C_j \) represents the diodes junction capacitance, \( R_s \) the resistance of the substrate and \( D \) the nonlinear diode with voltage-current relation

\[
i_d = I_s(e^{\alpha v_d} - 1).
\]  

(3.3)

Here \( v_d \) is the voltage over the nonlinear diode D. Notice that equation 3.3 is different from equation 3.1. Eq. 3.1 describes the behavior of the physical Schottky diode, including substrate resistance, whereas Eq. 3.3 characterizes the exponential voltage-current behaviour of the nonlinear diode \( D \).

The junction capacitance depends on the width of the depletion layer and therefore it depends on the voltage over the diode. Consequently, the junction capacitance has a nonlinear behaviour. We will assume that the nonlinearity of the junction capacitance is not significant compared to the nonlinear behaviour of the voltage-current relation. The junction capacitance is therefore assumed to be constant in our first setup of the diode input impedance model.
Due to the packaging of the diode chip some parasitic capacitance and inductance is introduced. In general this can be modelled by adding a series inductance and a shunt capacitance to the diode model [15]. Figure 3.5 shows this package model.

The input impedance is defined as the ratio of incident voltage to incident current. To find this relation, the differential equations of the circuit of Figure 3.5 are employed, i.e.,

\[
\begin{align*}
    v_c &= v - L_p \frac{di}{dt}, \\
    i_{C_p} &= C_p \frac{dv_c}{dt}, \\
    i_{C_j} &= C_j \frac{dv_c - (i - i_{C_p}) R_s}{dt}, \\
    i_D &= I_s \left( e^{a[v_c - (i - i_{C_p}) R_s]} - 1 \right).
\end{align*}
\]

(3.4)

Using Kirchhoff’s current law, we find \( i = i_{C_j} + i_D + i_{C_p} \). Substitution of \( v_c \) and rearrangement of the terms results in

\[
\begin{align*}
    i &= I_s \left( e^{a[v+R_s C_p \frac{dv}{dt} - R_s i - L_p \frac{di}{dt} - R_s L_p C_p \frac{d^2i}{dt^2}]} - 1 \right) \\
    &+ (C_j + C_p) \frac{dv}{dt} + R_s C_j C_p \frac{dv}{dt^2} - R_s C_j \frac{di}{dt} - (C_j + C_p) L_p \frac{d^2i}{dt^2} - R_s L_p C_j C_p \frac{d^3i}{dt^3}.
\end{align*}
\]

(3.5)

We would like to determine the current through the diode for a given input voltage. The input voltage is assumed to be sinusoidal with amplitude \( a \) and an operating frequency \( \omega_0 \)

\[
v = a \cos(\omega_0 t).
\]

(3.6)

In general, the current in Equation (3.5) cannot be solved analytically for a sinusoidal input voltage. However, if we use our physical insight, a good approximation can be found. Because of the nonlinearity of the diode, the current through the diode will consist of a sinusoidal term at the operating frequency as well as a DC term and higher harmonic terms. The higher harmonic terms have a frequency which equals \( n \) times the frequency of the input voltage. Hence the current can
be written as

\[ i = i_0 + \sum_{n=1}^{\infty} (\kappa_n \cos(n\omega_0 t) + \zeta_n \sin(n\omega_0 t)) \quad (3.7) \]

If we are able to determine the DC coefficient \( i_0 \) and the coefficients \( \kappa_n, \zeta_n \) for all \( n = 1, 2, \ldots \), the exact solution is known. In practice it is only realizable to determine a finite number of coefficients. To determine these coefficients we use Galerkin’s theory [16], which is quite intuitive. Galerkin’s theory states that if we try to find a solution for a differential equation

\[ f(x, t, dt) = 0, \quad (3.8) \]

we can try a particular solution \( x = \psi \) and substitute it in the differential equation. Since we don’t have the exact solution, a nonzero term will remain. This term is called the error term and is defined by

\[ \epsilon(\psi, t) = f(\psi, t, dt). \quad (3.9) \]

A good approximation can be found by minimizing the square of this error term in time

\[ W_\epsilon(\psi) = \int_{t}^{t} \epsilon^2(\psi, t)dt. \quad (3.10) \]

Here, \( W_\epsilon \) is called the weight function for the error term.

In our case, the differential equation \( f(x, t, dt) \) can be found from Equation (3.5) and \( \psi \) equals the current with unknown coefficients. The coefficients of the input current which minimize \( W_\epsilon \) will give the best solution for our differential equation.

### 3.2.2 Input impedance model 0th harmonic

In the simplest case, only the DC component of the current is determined. When the amplitude of the harmonics is small enough this can be a valid approach. For this case a closed-form algebraic relation between input voltage and DC input current can be found [17]. In our case, however, the component of the current at the operating frequency is needed to determine the input impedance. Therefore we try an approximation for the current which consists of a DC term and a term at the operation frequency, i.e.,

\[ i = i_0 + \kappa_1 \cos(\omega_0 t) + \zeta_1 \sin(\omega_0 t). \quad (3.11) \]
Substituting Equation (3.11) in (3.5) results in an expression for the error term

\[ \varepsilon(t) = -i_0 + \tau \cos(\omega_0 t) + \mu \sin(\omega_0 t) + Is (e^{at} - 1), \]  

(3.12)

with

\[ v_d = -R_s i_0 + \phi \cos(\omega_0 t) + \rho \sin(\omega_0 t), \]  

(3.13)

and

\[ \tau = \omega_0^3 R_s L_p C_j C_p \kappa_1 + \omega_0^2 (L_p(C_j + C_p)\kappa_1 - R_s C_j C_p a) - \omega_0 R_s C_j \kappa_1 - \kappa_1, \]
\[ \mu = -\omega_0^3 R_s L_p C_j C_p \kappa_1 + \omega_0^2 (L_p C_p + L_p C_j) \kappa_1 + \omega_0 (R_s C_j \kappa_1 - (C_j + C_p)a) - \kappa_1, \]
\[ \phi = \omega_0^2 R_s L_p C_p \kappa_1 - \omega_0 L_p \kappa_1 + \alpha - R_s \kappa_1, \]
\[ \rho = \omega_0^2 R_s L_p C_p \kappa_1 - \omega_0 (L_p \kappa_1 - R_s C_p a) - R_s \kappa_1. \]  

(3.14)

Although the latter expressions are somewhat lengthy, it's mainly a matter of good bookkeeping.

The corresponding weight function of Eq. (3.10) for the error function of Equation (3.12) is given by

\[ W_e(i_0, \kappa_1, \kappa_1) = \frac{1}{T} \int_{t=0}^{T} [-i_0 + \tau \cos(\omega_0 t) + \mu \sin(\omega_0 t) + Is (e^{at} - 1)]^2 dt. \]  

(3.15)

Since our input voltage and input current are periodic functions, it is allowed to minimize our weight function over one period of time \( T = \frac{2\pi}{\omega_0}. \) Rewriting Eq. (3.15) results in

\[ W_e(i_0, \kappa_1, \kappa_1) = \frac{1}{T} \int_{t=0}^{T} i_0^2 + I_s^2 + 2i_0 I_s + \tau^2 \cos^2(\omega_0 t) + \mu^2 \sin^2(\omega_0 t) \]
\[ - 2\tau(i_0 + I_s) \cos(\omega_0 t) - 2\mu(i_0 + I_s) \sin(\omega_0 t) + 2\tau \mu \cos(\omega_0 t) \sin(\omega_0 t) \]
\[ - 2(i_0 I_s + I_s^2)e^{at} + 2\tau I_s \cos(\omega_0 t)e^{at} + 2\mu I_s \sin(\omega_0 t)e^{at} + I_s^2 e^{2at} dt. \]  

(3.16)

Here, the underlined expressions result in a zero integral contribution. Further, we have

\[ \int_{t=0}^{T} \cos^2(\omega_0 t) dt = \frac{1}{2} T, \]
\[ \int_{t=0}^{T} \sin^2(\omega_0 t) dt = \frac{1}{2} T, \]  

(3.17)
which leaves us with the problem of solving the integral for the exponential terms. Fortunately the tables of [18, Eq. 3.937] give two helpful relations

\[
\int_0^{2\pi} e^{p \cos(x) + q \sin(x)} \sin(a \cos(x) + b \sin(x) - mx) dx =
\]

\[
j\pi [(b - p)^2 + (a + q)^2]^{-\frac{m}{2}} \left\{ (A + jB)^{\frac{m}{2}} I_m(\sqrt{C - jD}) - (A - jB)^{\frac{m}{2}} I_m(\sqrt{C + jD}) \right\}, (3.18)
\]

and

\[
\int_0^{2\pi} e^{p \cos(x) + q \sin(x)} \cos(a \cos(x) + b \sin(x) - mx) dx =
\]

\[
\pi [(b - p)^2 + (a + q)^2]^{-\frac{m}{2}} \left\{ (A + jB)^{\frac{m}{2}} I_m(\sqrt{C - jD}) + (A - jB)^{\frac{m}{2}} I_m(\sqrt{C + jD}) \right\}, (3.19)
\]

with \( I_m \) the modified Bessel function of the first kind of order \( m \) and

\[
\begin{align*}
A &= p^2 - q^2 + a^2 - b^2, \\
B &= 2(pq + ab), \\
C &= p^2 + q^2 - a^2 - b^2, \\
D &= -2(ap + bq).
\end{align*}
\]

The above relations hold under the conditions that \( m \) is an integer and

\[
(b - p)^2 + (a + q)^2 > 0. \quad (3.21)
\]

Equations (3.18) and (3.19) can be used to determine the integrals with exponential terms in Equation (3.16). We will illustrate this for an exponential term from Eq. (3.16)

\[
\int_0^T 2\pi I_s \cos(\omega_0 t) e^{\alpha \nu t} dt = \int_0^T 2\pi I_s \cos(\omega_0 t) e^{\alpha[-\nu \cos(\omega_0 t) + \nu \sin(\omega_0 t)]} dt
\]

\[
= 2\pi I_s e^{-\alpha \nu i \omega_0} \frac{1}{\omega_0} \int_0^{2\pi} e^{\alpha \phi \cos(\omega_0 t) + \alpha \nu \sin(\omega_0 t)} \cos(\omega_0 t) d(\omega_0 t).
\]

This integral can be found from Eq. (3.19) with \( p = \alpha \phi, q = \alpha \nu, a = 0, b = 0, m = 1 \) and \( x = \omega_0 t \), which yields
\[
\int_0^T 2\tau I_s \cos(\omega_0 t)e^{\alpha \nu t} dt = 2\tau I_s e^{-\alpha R_s \omega_0} \frac{1}{\omega_0} \\
\pi \left[ ( \alpha \phi^2 + ( \alpha \rho )^2 \right]^{-\frac{1}{2}} \left\{ \left[ ( \alpha \phi^2 - ( \alpha \rho )^2 + j 2 \alpha^2 \phi \rho \right]^{\frac{1}{2}} I_1(\sqrt{\alpha \phi^2 + (\alpha \rho)^2}) \right. \\
\left. + \left[ ( \alpha \phi^2 - ( \alpha \rho )^2 - j 2 \alpha^2 \phi \rho \right]^{\frac{1}{2}} I_1(\sqrt{\alpha \phi^2 + (\alpha \rho)^2}) \right\}, \quad (3.23)
\]

and this can be simplified to

\[
\int_0^T 2\tau I_s \cos(\omega_0 t)e^{\alpha \nu t} dt = 2\tau I_s e^{-\alpha R_s \omega_0} T \frac{\phi}{\sqrt{\phi^2 + \rho^2}} I_1(\alpha \sqrt{\phi^2 + \rho^2}). \quad (3.24)
\]

In a similar way, the other integrals of the exponential terms in (3.16) can be calculated. This results in a closed-form algebraic expression for the weight function

\[
W_e = (i_0 + I_s)^2 + \frac{1}{2}(\tau^2 + \nu^2) - 2(i_0 I_s + I_s^2)e^{-\alpha R_s \omega_0} I_0(\alpha \sqrt{\phi^2 + \rho^2}) \\
+ 2I_s e^{-\alpha R_s \omega_0} \frac{\tau \phi + \nu \rho}{\sqrt{\phi^2 + \rho^2}} I_1(\alpha \sqrt{\phi^2 + \rho^2}) + I_s^2 e^{-2\alpha R_s \omega_0} I_0(2\alpha \sqrt{\phi^2 + \rho^2}). \quad (3.25)
\]

If we can minimize the weight function, the coefficients of the input current can be determined. The weight function is minimized by a Simplex method [19, sec. 10.4]. This is an N-dimensional minimization algorithm which needs a number of initial points, from which it searches for a local minimum. With this method, the input current coefficients are found for a given amplitude of the input voltage.

Once the input current is known for a given input voltage, we can calculate the complex input impedance at the operation frequency. The input voltage and input current at the operating frequency are given by

\[
u_{\omega_0} = a \cos(\omega_0 t), \quad i_{\omega_0} = \kappa_1 \cos(\omega_0 t) + \zeta_1 \sin(\omega_0 t), \quad (3.26)
\]

The corresponding complex input impedance is easily calculated as

\[
Z_{\text{in}} = \frac{V}{I} = \frac{a}{\kappa_1 - j\zeta_1}, \quad (3.27)
\]

with \( V \) and \( I \) the complex voltage and complex current, respectively.
3.2.3 Input impedance model for higher harmonics

Under the assumption that the amplitude of the higher harmonics is small compared to the amplitude of the zeroth harmonic, the model presented in Section 3.2.2 can be used to determine the input impedance. When it is unsure whether it is a valid assumption to ignore the effect of the higher harmonics, an extended model can be used. This section describes an approach to determine the input impedance that takes into account the effect of higher harmonics.

Suppose we want to include the effect of $N$ harmonics. The corresponding input current is given by

$$i = i_0 + \sum_{n=1}^{N+1} (\kappa_n \cos(n\omega_0 t) + \zeta_n \sin(n\omega_0 t)). \quad (3.28)$$

Again the same approach as given in Section 3.2.1 can be used. The error term is found from Eq. (3.5) and equals

$$\epsilon = I_s \left( e^{v-iR_s-iL_s \frac{di}{dt}} - 1 \right) + (C_j + C_p) \frac{dv}{dt} - R_s C_j \frac{di}{dt} - (L_p C_j + L_p C_p) \frac{d^2i}{dt^2} - i. \quad (3.29)$$

Unlike in the preceding section, it is not possible to obtain a closed-form expression for the weight function of Eq. (3.10). We can, however, compute the weight function by solving the integral numerically for given coefficients. Again, the searched coefficients for the input current can be found via a multi-dimensional minimization algorithm [19, Sec. 10.4] that operates on the weight function.

Because of the need to calculate the integral for the weight function numerically, the computation time of this method is larger than the computation time of the model presented in preceding section. Still, it is not always sufficiently accurate to ignore higher harmonics. Therefore this method can be very convenient to accurately model the diode's input impedance.

3.3 Measurements

To validate our nonlinear models, a number of measurements have been performed. The input impedance of a single diode is measured for a frequency range from 100 MHz to 4.1GHz. For the measurements we have used a HSMS-2852 Schottky diode from Agilent Technologies [20]. The input impedance is measured for several input powers using a HP8530a network analyzer with an S-parameter test set. The diode is mounted directly on an SMA connector as shown in Figure 3.6.

The input impedance of the diode is measured for several levels of incident power. The incident power level can be controlled by the network analyzer, but some additional losses have to be considered. The reason for this is that the S-parameter test set introduces some losses too, as well as the cable from the network analyzer to the S-parameter test set. These losses have been measured for a number of operating frequencies. Figure 3.7 shows the measured power loss for our measurement equipment for a frequency range from 45 MHz to 10 GHz. The measured losses in the cable are represented by the circled points. The dashed line indicates the interpolated values.
from these measurements. The measured losses in the S-parameter test set are given by the crosses. The dotted line represents the approximated loss curve for the S-parameter test set. The total power loss is given by the solid line.

The measurements are shown in Figures 3.8-3.17. The incident power level as indicated in the caption of the figures represents the output power of the network analyzer. The losses of Figure 3.7 should be taken into account to determine the input power of the diode. The measurements are given by the solid line. The input impedance according to the model is given by the dashed line. As we can see, the model predicts the measurements quite accurately for incident power levels up to 4 dBm. For higher power levels, some inaccuracy is introduced, although the global behaviour of the diode is predicted well.

The model that is used to compare the measurement results considers an input current with frequency components up to the first harmonic. The coefficients for the input current are calculated following the approach as described in Section 3.2.3. Here, we need a numerical integration algorithm to calculate the weight function. For this need we have used a ten-point Gauss-Legendre integration algorithm [19, Sec. 4.5].

For some power levels, we see significant noise on the measured signals. The main reason for this effect is the calibration of the measurement equipment. The calibration of the S-parameter test set needs a number of calibrated loads. In our calibration we used loads which were not perfectly accurate. A measurement has been performed with better calibration tools on a different network analyzer. The more accurate calibration reduced the noise significantly.
Figure 3.8: $\text{Re}\{Z_{in}\}, P_{inc}=-10\text{dBm}$

Figure 3.9: $\text{Im}\{Z_{in}\}, P_{inc}=-10\text{dBm}$

Figure 3.10: $\text{Re}\{Z_{in}\}, P_{inc}=0\text{dBm}$

Figure 3.11: $\text{Im}\{Z_{in}\}, P_{inc}=0\text{dBm}$

Figure 3.12: $\text{Re}\{Z_{in}\}, P_{inc}=4\text{dBm}$

Figure 3.13: $\text{Im}\{Z_{in}\}, P_{inc}=4\text{dBm}$
Figure 3.14: $\text{Re}\{Z_{in}\}, P_{\text{inc}}=8\text{dBm}$

Figure 3.15: $\text{Im}\{Z_{in}\}, P_{\text{inc}}=8\text{dBm}$

Figure 3.16: $\text{Re}\{Z_{in}\}, P_{\text{inc}}=12\text{dBm}$

Figure 3.17: $\text{Im}\{Z_{in}\}, P_{\text{inc}}=12\text{dBm}$
3.4 Model generalization

The diode model presented in previous sections seems to perform quite well. However, in a practical setup, the diode is placed in an extensive electric circuit. The model in the previous section assumed a voltage source at the input of the diode and a short circuit at its output. To be able to model the behaviour of the diode in a more extensive circuit the model has to be generalized somewhat.

Let us assume that we have an arbitrary, but linear, electric circuit at the input and output of the diode (Figure 3.18). Moreover, let us assume that the sources in our electric circuit operate at a single frequency \( \omega_0 \). The rigorous approach would be to formulate the differential equation for this circuit in the time domain in the same manner as proposed in Section 3.2. The drawback of this approach is that the system of nonlinear differential equations that describes the circuit can become rather large. A more tempting solution is to solve the linear parts of our circuit in the frequency domain and only the nonlinear part in the time domain. This decreases the complexity of the system of nonlinear differential equations. Since the diode generates a DC component and harmonics of the operating frequency, the linear network has to be evaluated for all these frequencies.

A convenient way of evaluating the linear circuit is to apply Thévenin's theorem [21]. Thévenin's theorem states that we can characterize the behaviour of a linear electric circuit at its outputs by an equivalent network consisting of a voltage source \( V_{th} \) and an output impedance \( Z_{th} \) (Figure 3.19). The Thévenin voltage \( V_{th} \) is given by the unloaded circuit output voltage of the linear circuit and \( Z_{th} \) is defined as the ratio of the unloaded output voltage and the short-circuit current.

We use Thévenin’s theorem to acquire an equivalent network which describes the behaviour of all linear components of the circuit at the operating frequency. In our analysis only the voltage-current characteristic of the diode is assumed nonlinear. Therefore this is the only part which is not characterized by the equivalent Thévenin circuit.

To calculate the effect of the linear circuit on the harmonics generated by the diode, the input impedance of the linear circuit seen from the nonlinear diode has to be evaluated as well. In the
frequency domain the diode can be modelled as a current source for the harmonics and a load impedance at the operating frequency.

Define the current \( i(t) \) in the linear circuit as

\[
i = i_0 + \sum_{n=1}^{\infty} \left( \kappa_n \cos(n\omega_0 t) + \zeta_n \sin(n\omega_0 t) \right),
\]

then the voltage over the nonlinear diode can be expressed as

\[
v_i(t) = \text{Re} \{ V_{th} \} \cos(\omega_0 t) - \text{Im} \{ V_{th} \} \sin(\omega_0 t) - i_0 R_0 - \sum_{n=1}^{\infty} \left[ \text{Re} \{ I_n Z_n \} \cos(n\omega_0 t) - \text{Im} \{ I_n Z_n \} \sin(n\omega_0 t) \right],
\]

where \( I_n = \kappa_n - j\zeta_n \) is the complex current of frequency \( n\omega_0 \) through the nonlinear diode and \( Z_n \) the input impedance for the frequency \( n\omega_0 \), seen from the nonlinear diode. The DC voltage drop is accounted for by the term \( i_0 R_0 \), with \( i_0 \) the direct current and \( R_0 \) the DC load.

From the voltage-current characteristic of the nonlinear diode we also know that the current through the diode is given by

\[
i_D(t) = I_s \left( e^{\alpha v_i(t)} - 1 \right).
\]

Minimization of the error between \( i_D(t) \) and the current in the linear circuit \( i(t) \) gives the searched coefficients \( (\kappa_n, \zeta_n) \) for the current. Once these coefficients are known, the circuit can be characterized completely. The error can be minimized in an identical way as presented in Section 3.2.

### 3.4.1 Example: Schottky diode on a transmission line

To demonstrate the use and validity of the model, a relatively simple example is given. Suppose we have a measurement setup as shown in Figure 3.20. The measurements consists of a Schottky diode mounted on two transmission lines of lengths \( l_{cd} \) and \( l_{dl} \) with a propagation constant \( \gamma \) and a characteristic impedance \( Z_0 \). The RF generator is characterized by \( V_g \) and \( Z_g \) and is short-circuited for DC. The transmission line is loaded with an RF load \( R_{RF} \) and a DC load \( R_0 \).

We would like to determine the equivalent Thévenin circuit for this setup at the operating frequency \( \omega_0 \). We also need to determine the input impedance of the circuit for the harmonic terms. Incorporating the package model for the Schottky diode (Figure 3.5) gives the electric circuit of Figure 3.21.

To derive the equivalent circuit we first determine the unloaded output voltage at the pins of the
ideal diode

\[ V_{th} = \frac{1}{1 + j\omega R_s C_j} \frac{Z_d}{j\omega L_p + Z_d + Z_c} V_b, \]  

(3.33)

where

\[ V_b = V_a \cosh(\gamma l_{cd}) - Z_0 I_a \sinh(\gamma l_{cd}), \]

\[ V_a = \frac{Z_a}{Z_a + Z_g} V_g, \]

\[ I_a = \frac{V_g}{Z_a + Z_g}, \]

(3.34)
and

\[
Z_a = \frac{Z_0}{Z_0 + Z_b \tanh(\gamma_{\text{cd}})}
\]

\[
Z_b = j\omega L_p + Z_d + Z_c,
\]

\[
Z_c = \frac{Z_{\text{RF}}}{Z_0 + Z_{\text{RF}} \tanh(\gamma_{\text{dl}})}
\]

\[
Z_d = \frac{\left( \frac{1}{j\omega C_p} + R_s \right) \frac{1}{j\omega C_p}}{-j\omega C_j + R_s + \frac{1}{j\omega C_p}}.
\]

(3.35)

The short-circuit current is given by

\[
I_{\text{sc}} = \frac{1}{1 + j\omega R_s C_p} \frac{V'_b}{-j\omega L_p + \frac{R_s}{1 + j\omega R_s C_p} + Z_c}
\]

(3.36)

where

\[
V'_b = V'_a \cosh(\gamma_{\text{cd}}) - Z_0 I'_a \sinh(\gamma_{\text{cd}}),
\]

\[
V'_a = \frac{Z'_a}{Z'_a + Z_g} V_g,
\]

\[
I'_a = \frac{V_g}{Z'_a + Z_g},
\]

(3.37)

and

\[
Z'_a = \frac{Z'_b + Z_0 \tanh(\gamma_{\text{cd}})}{Z_0 + Z'_b \tanh(\gamma_{\text{cd}})},
\]

\[
Z'_b = j\omega L_p + Z'_d + Z_c,
\]

(3.38)

\[
Z'_d = \frac{R_s}{1 + j\omega R_s C_p}.
\]

The output resistance of the equivalent network now equals

\[
Z_{\text{th}} = \frac{V_{\text{th}}}{I_{\text{sc}}}.\]

(3.39)

The input impedance of the \(n\)th harmonic term \((\omega = n\omega_0)\), as seen from the diode into the linear circuit is given by

\[
Z_n = \frac{R_s + Z_t}{1 + jn\omega_0 C_j (R_s + Z_t)},
\]

(3.40)
where

\[
Z_t = \frac{Z_c + Z_e + jn\omega_0 L_p}{1 + jn\omega_0 C_p(Z_c + Z_e + jn\omega_0 L_p)} ,
\]

\[
Z_e = \frac{Z_0 Z_g + Z_0 \tanh(\gamma l_{cd})}{Z_0 + Z_g \tanh(\gamma l_{cd})} .
\]

(3.41)

For \( \omega = 0 \) the input impedance is equal to \( R_0 = R_s + R_{DC} \).

Now we have determined the equivalent Thévenin circuit and the input impedance for the harmonics, the behaviour of the circuit can be simulated. The circuit is also measured to verify the modelling. For the circuit we use FR4 material with a dielectric permittivity \( \varepsilon_r = 4.28 \), a loss tangent \( \tan \delta = 0.02 \) and a height \( h = 1.6 \text{ mm} \). The transmission line lengths are \( l_{cd} = 9.00 \text{ mm} \) and \( l_{dl} = 8.95 \text{ mm} \). The width of the lines is \( w = 0.54 \text{ mm} \). The generator impedance \( Z_g = 50 \, \Omega \), the RF load is \( R_{RF} = 50 \, \Omega \) and the DC load is \( R_{DC} = 1 \, \text{k}\Omega \). The same Schottky diode as used in Section 3.3 is employed.

For the simulation we have taken 3 harmonic terms into account. Figures 3.22 and 3.23 show the simulated and measured input impedance of the circuit seen at the beginning of the transmission line. Figure 3.24 shows the DC output voltage. From these figures we conclude that the model matches the physical behaviour of the circuit. The difference between the impedance \( |Z| \) of the model and the measurement is less than 22\( \Omega \) for the frequency range of interest (2-3GHz).

The behaviour of the nonlinear Schottky diode can be modelled with a relative simple approach. The assumption made in Section 3.2.1 that the nonlinear junction capacitance can be approximated as a constant capacitance is proven to be accurate enough. The accuracy of the model improves when more harmonic terms are incorporated. Still, for a limited number of harmonic terms, the model already gives an accurate prediction of the practical behaviour of the diode.

The model is generalized in Section 3.4. The equivalent Thévenin circuit is incorporated to analyze the linear parts of the circuit in the frequency domain. This results in a more convenient analysis
Figure 3.24: DC voltage

of the linear and nonlinear circuit. The validation of the generalized model with measurements gives a good confidence in the applicability of the model in practical circuits.
Chapter 4

Microstrip transmission line

4.1 Introduction

The several components of our rectenna are interconnected through microstrip transmission lines. To incorporate the effects of the transmission lines an accurate description of their behaviour is needed. A microstrip transmission line consists of a conducting line of thickness $t$ and width $w$ above a conducting ground plane. The line and the ground plane are separated by a dielectric layer of height $h$ with a relative permittivity $\varepsilon_r$ (Figure 4.1).

4.2 Microstrip modelling

At the edges of the line, the electric field will fringe a little outwards. To include this effect, the microstrip transmission line is modelled by an effective strip width $w_e$ and an effective relative permittivity $\varepsilon_e$. The characteristic impedance $Z_0$ and propagation constant $k = \beta - j\alpha$ can be determined from the convenient empirical relations of E. Hammerstand and O. Jensen [22].

The effective width is given by

$$w_e = w + \frac{h\Delta w_1}{2}\left(1 + \frac{1}{\cosh(\sqrt{\varepsilon_r} - 1)}\right),$$

(4.1)

Figure 4.1: microstrip transmission line
where
\[
\Delta u_1 = \frac{t}{\pi} \ln \left( 1 + \frac{4e}{t \coth^2 (\sqrt{6.517}u)} \right),
\] (4.2)

and \( u = \frac{w}{h} \) the normalized strip width. For the characteristic impedance we get
\[
Z_0 = \frac{Z_{01}(w_e)}{\sqrt{\varepsilon_{e1}}},
\] (4.3)

with \( Z_{01} \) the characteristic impedance of a microstrip line in free space and \( \varepsilon_{e1} \) the effective relative permittivity in free space, i.e.,
\[
Z_{01}(w) = 60.0 \ln \left( \frac{f}{w} + \sqrt{1 + \left( \frac{4}{w} \right)^2} \right),
\] (4.4)
\[
\varepsilon_{e1} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left( 1 + \frac{10}{u} \right)^{-\alpha b},
\]

where we use
\[
f = 6 + (2\pi - 6) \exp \left( - \left( \frac{30.666}{w} \right)^{0.7528} \right),
\]
\[
a = 1 + \frac{1}{49} \ln \left( \frac{u^4 + \left( \frac{w}{52} \right)^2}{u^4 + 0.432} \right) + \frac{1}{18.7} \ln \left( 1 + \left( \frac{w}{18.1} \right)^3 \right),
\] (4.5)
\[
b = 0.564 \left( \frac{\varepsilon_r - 0.9}{\varepsilon_r + 3} \right)^{0.053}.
\]

For the effective relative permittivity we find
\[
\varepsilon_e = \varepsilon_{e1} \left( \frac{Z_{01}(w_{e1})}{Z_{01}(w_e)} \right)^2,
\] (4.6)

where \( w_{e1} \) denotes the effective width in free space. The wavenumber for the microstrip transmission line can be calculated from the effective relative permittivity as
\[
\beta = \omega \sqrt{\mu_0 \varepsilon_0 \varepsilon_e}.
\] (4.7)

The attenuation coefficient \( \alpha \) along the microstrip line due to conduction and dielectric losses can
be found from [8] and is given in Neper per meter (Np/m) as

\[
\alpha = \alpha_c + \alpha_d,
\]

\[
\alpha_c = \sqrt{\frac{\omega \mu_0}{2\sigma}} \frac{1}{w Z_0},
\]

\[
\alpha_d = \beta \frac{\varepsilon_e - 1}{\varepsilon_r - 1} \frac{\varepsilon_r \tan(\delta)}{2},
\]

where \(\alpha_c\) is the attenuation due to the Ohmic losses, \(\alpha_d\) is the attenuation due to the dielectric losses, \(\sigma\) is the surface conductivity of the conducting line and ground plane and \(\tan(\delta)\) is the loss tangent of the dielectric.

### 4.3 Measurement

To verify the model a transmission line was etched on FR4 material. The substrate has a dielectric constant \(\varepsilon_r = 4.28\), a loss tangent \(\tan(\delta) = 0.02\) and a height \(h = 1.6\) mm. The first transmission line was designed to have a characteristic impedance \(Z_0\) of about 50 \(\Omega\). This results in a transmission line width of \(w = 2.8\) mm. The transmission line has a length of 21 mm and is measured in a frequency range from 100 MHz to 4.1 GHz. Figures 4.2 and 4.3 give the results.

We observe that for higher frequencies the measurements look somewhat noisy. This is probably caused by the calibration of the measurement equipment. The measurement is performed with a self-made calibration set. Therefore the measurements for frequencies from 4 GHz and higher are less accurate. Up to 3 GHz, we see that the model is in good agreement with the measurements.

![Figure 4.2: Re\(\{Z_{in}\}\) transmission line](image1)

![Figure 4.3: Im\(\{Z_{in}\}\) transmission line](image2)

### 4.4 Conclusions

In this chapter a model is presented that describes the behaviour of the microstrip transmission line. This model introduces an effective relative permittivity and effective physical dimensions of
the microstrip transmission line to incorporate the effect of the fringe fields at the edges of the line. The model is verified with measurements. It is shown that the model is sufficiently accurate for frequencies up to 3 GHz.
Chapter 5

Radial stub

5.1 Introduction

As mentioned in Chapter 1, we would like to provide our application with a steady DC power-supply and convert as much RF power of the signal to DC power. Therefore we would like to suppress the sinusoidal terms of our output voltage and output current. This can be realized by a low-pass filter. The initial setup of our rectifying circuit in Figure 3.1 already contained a low pass filter in the form of the capacitor load. In theory, this is a good solution. In practice, however, there is the problem of finding an adequate capacitor. Of-the-shelf capacitors for frequencies up to 4 GHz are not available. Therefore other solutions have to be found.

To suppress a sinusoidal voltage term at radial frequency $\omega_0$, a shunt quarter-lambda stub can be used. Schematically, the layout and the input impedance of a lossless quarter-lambda stub is given in Figure 5.1. At the frequency where the length of the quarter-wave stub equals a quarter of the wavelength ($\lambda$), the input impedance of the stub is identical to zero. Therefore we would like to design the stub such that this frequency is identical to the operating frequency $\omega_0$.

![Figure 5.1: quarter-lambda stub](image)

The problem with practical quarter-lambda stubs is that the length of the stub cannot be determined exactly. This makes it difficult to predict the resonance frequency. To solve this problem, a shunt-connected radial stub can be used. The general layout of a radial stub is given in Figure 5.2.
The radial stub consists of a circular strip with an annular section of $\phi$ radians, an inner radius $r_i$ and an outer radius $r_o$. Because of this layout, the insertion point $p$ is well defined. Another advantage of the radial stub is its limited dimensions compared to quarter-wave length straight stub. Moreover, the radial stub has a low-impedance point over a wider frequency range compared to the quarter-wave length straight stub [25].

5.2 Radial stub modelling

5.2.1 Model introduction

An analytical model for the input impedance of the radial stub is given by several authors, e.g. [23], [24], [25] and [26]. The work of J.P. Vinding [23] is the basis for most discussions on the input impedance of the radial stub. It gives a relatively simple analytical expression for the input impedance. However, this expression does not seem to be really accurate. The model neglects the effects of fringe fields at the edges of the stub. Therefore the prediction of the resonance frequency of the radial stub is not in agreement with the measured values in most cases. Vinding's model can therefore only be used as a first estimate. H.A. Atwater [24] tried to improve Vinding's model by introducing an effective dielectric permittivity $\varepsilon_{\text{eff}}$. The effective permittivity is determined in a rule-of-thumb sort of manner, which still does not seem to give really accurate results. Giannini et al. [25] derived a more rigorous model. This model calculates the input impedance from an electromagnetic field expansion in terms of resonant modes. It includes the effect of fringing fields and also incorporates losses. Sorrentino [26] proposed a simplification to the model of Giannini et al. while maintaining its accuracy.

Although the model of Giannini et al. seems to perform quite well, it is rather difficult to implement the presented work. Unfortunately the papers concerning radial stubs written by Giannini et al. contain several printing errors and not all introduced parameters are explained univocally. To overcome these difficulties we will try to present an extensive model which includes all derivations and at least gives a clear overview of the model used in our simulations.

Giannini's model uses an electromagnetic field expansion in terms of resonant modes. The radial stub is equivalent to a circular sector microstrip ring (Figure 5.3). Neglecting fringe fields and additional losses, a resonant mode of the $z$-directed electric field inside the radial stub can be expressed as [27, Ch. 5]

$$E_z = E_0 \left[ J_m(k_{mn}r)Y'_n(k_{mn}r_i) - J'_m(k_{mn}r_i)Y_m(k_{mn}r) \right] \cos(m\phi), \quad (5.1)$$
where $J_m$ and $Y_m$ are the Bessel functions of the first and second kind, and of order $m$, respectively. The resonant modes are transverse magnetic (TM) to the $z$ direction. A specific mode is denoted $TM_{mn}$ where $m$ determines the field distribution in the annular direction and $n$ represents the $n^{th}$ solution of the characteristic equation

$$J'_m(k_{mn}r_o)Y'_m(k_{mn}r_i) - J'_m(k_{mn}r_i)Y'_m(k_{mn}r_o) = 0. \tag{5.2}$$

The characteristic equation follows from the boundary conditions. The solutions $k_{mn}$ of this equation determine the resonance frequencies of the radial stub.

### 5.2.2 Model proposed by Giannini et al.

The assumption that the inner radius is small compared to the outer radius leads to the conclusion that the lowest resonant modes of the radial stub are of the type $TM_{0n}$. The field distribution in the annular direction is constant for these modes. The model of Giannini et al. uses this assumption to derive an expression for the input impedance of a radial stub.

To account for the fringe fields at the edges of the radial stub, effective dimensions and are introduced in combination with an effective permittivity for the static mode and each resonant mode $TM_{0n}$. The effective dimension are shown in Figure 5.4 and are calculated from
\[ r^* = \frac{2p + w_e - w}{2 \cos(\frac{\phi}{2})}, \]
\[ \Delta = \frac{f(\phi r^*) - \phi r^*}{2}, \]
\[ w_g = (2p + w_e - w) \tan(\frac{\phi}{2}), \]
\[ \frac{w_{ge}}{w} = w_g + \frac{2\Delta}{\cos(\frac{\phi}{2})}, \]
\[ r_{ie} = r^* + \frac{\Delta}{\tan(\frac{\phi}{2})} + \Delta \tan(\frac{\phi}{2}), \]
\[ r_{oe} = r_o \left[ 1 + \frac{2h}{\pi \varepsilon_o r_o} \left( \ln \left( \frac{\pi r_o}{2h} \right) + 1.7726 \right) \right]^\frac{1}{2}. \]

Here \( f(\phi r^*) \) represents the effective width of a microstrip line of width \( \phi r^* \) as given by Hammerstad (Section 4.2). The effective outer radius is found from W.C. Chew and J.A. Kong [28] and is different from Giannini's [29] by the term \( \varepsilon_r \).

The expression for the input impedance of a radial stub with dimension as shown in Figure 5.4 is given by

\[
Z_{in} = \frac{j \varepsilon_0}{k^2 \varepsilon_{do}} - j k_g P_0^2 + \sum_{n=1}^{\infty} \frac{k_g P_0^2}{k_{0n}^2 (1 + \frac{j}{Q_{t0n}}) - k^2 \varepsilon_{do}}. \tag{5.4}
\]

The input impedance is normalized with respect to the characteristic impedance of the radial stub, which is defined as

\[
Z_0^* = \frac{120\pi h}{r_{ie} \alpha \sqrt{\varepsilon_{eff}}}, \tag{5.5}
\]

here \( \varepsilon_{eff} \) is the effective permittivity of a microstrip line of width \( w_g \) and \( h \) is the height of the dielectric. Further we have

- \( k_g \): wavenumber of the feeding line,
- \( k \): free space wavenumber,
- \( k_{0n} \): eigenvalue of the \( TM_{0n} \) mode,
- \( P_0 \): coupling coefficient quasi-TEM mode and static mode of stub,
- \( P_{0n} \): coupling coefficient quasi-TEM mode and \( TM_{0n} \) mode,
- \( \varepsilon_{do} \): effective permittivity of the static mode,
- \( \varepsilon_{d0n} \): dynamic effective permittivity of the \( TM_{0n} \) mode,
- \( Q_{t0} \): quality factor static mode,
- \( Q_{t0n} \): global quality factor.

The wavenumber of the feeding line, \( k_g \), can be found from the equations of Hammerstadt presented in Section 4.2. The free-space wavenumber equals \( k = \omega \sqrt{\mu_0 \varepsilon_0} \). The eigenvalue of the \( TM_{0n} \) mode can be found from Equation (5.2). \( P_{0n} \) represents the coupling coefficient between the quasi-TEM...
wave on the feeding line and the $TM_{0n}$ modi on the radial stub. The coupling coefficients are given by

$$P_0 = \sqrt{\frac{2u_{ge}}{\alpha(r_{oe}^2 - r_{ie}^2)}},$$

$$P_{0n} = \sqrt{u_{ge}} \left[ A_{0n} J_0(k_{0n}r_{ie}) + B_{0n} Y_0(k_{0n}r_{ie}) \right].$$

where

$$A_{0n} = \sqrt{\frac{2}{\phi}} \left\{ r_{oe}^2 \left[ J_0(k_{0n}r_{oe}) + K_n Y_0(k_{0n}r_{oe}) \right]^2 - r_{ie}^2 \left[ J_0(k_{0n}r_{ie}) + K_n Y_0(k_{0n}r_{ie}) \right]^2 \right\}^{-\frac{1}{2}},$$

$$B_{0n} = K_n A_{0n},$$

$$K_n = -\frac{J_1(k_{0n}r_{oe})}{Y_1(k_{0n}r_{oe})}.$$  

Dynamic dielectric constant

The dynamic permittivities of the static and $TM_{0n}$ modes can be found from the work of J. Vrba [30]. The work of Vrba is based on previous work from N. Knoppik [31]. Knoppik presented a method to calculate the dynamic permittivity of a ring resonator. Vrba has generalized this work to a circular sector microstrip ring. However, the formulas of Knoppik presented in Knoppik's and Vrba's papers do not seem to give the results shown in the figures of those papers. The formula derived by Vrba, based on Knoppik's formula, gives the desired results. Suspicion arises the presented formula from Knoppik contains some typographic errors. Therefore the generalized model of Vrba is presented here.

The dynamic dielectric constant of a microstrip ring resonator with resonant mode $TM_{0n}$ is given by

$$\varepsilon_{d0} = \frac{C_{d0}(\varepsilon_r) + C_{df}(\varepsilon_r) + 2C_{ds}(\varepsilon_r)}{\varepsilon_r},$$

$$\varepsilon_{d0n} = \frac{C_{d0n}(\varepsilon_r) + C_{df}(\varepsilon_r) + C_{ds}(\varepsilon_r)}{\varepsilon_r},$$

where $C_{d0n}$ equals the dynamic capacitance of the ring sector, $C_{df}$ represents the capacitance of the fringe field at the inner and outer radius of the ring sector and $C_{ds}$ represents the capacitance at
the side walls of the sector. These capacities are calculated as follows

\[
C_{d0} = \frac{\varepsilon_0 \varepsilon_r \phi (r_o^2 - r_i^2)}{2h},
\]

\[
C_{d0n} = \frac{\varepsilon_0 \varepsilon_r \phi}{2h} \left\{ \frac{r_o^2 - r_i^2}{r_o^2 + r_i^2} \left[ \frac{J_0(k_0 r_o)}{J_0(k_0 r_i)} + \frac{K_n(Y_0(k_0 r_o))}{K_n(Y_0(k_0 r_i))} \right]^2 \right\},
\]

\[
C_{df} = \frac{(r_o + r_i) \phi}{2} \left[ \frac{1}{Z_\phi v_\phi} - \frac{\varepsilon_0 \varepsilon_r (r_o - r_i)}{h} \right],
\]

\[
C_{ds} = \frac{r_o - r_i}{2} \left[ \frac{1}{Z_r v_r} - \frac{\varepsilon_0 \varepsilon_r (r_o + r_i) \phi}{2h} \right].
\]

(5.11)

where \(Z_\phi\) and \(Z_r\) is the wave impedance in the annular direction and radial direction, respectively. \(v_\phi\) and \(v_r\) are the phase velocities of the ring sector in the annular and radial direction. The impedance and phase velocity in the annular direction can be found from the wave impedance and phase velocity of a microstrip of width \(w = r_o - r_i\) from Hammerstad (Section 4.2). The wave impedance and phase velocity in the radial direction depend on the radius \(r\). In the papers presented by Giannini et al. is not explained how these parameters are determined. Therefore we propose to calculate the average wave impedance and phase velocity by integrating the wave impedance and phase velocity of a microstrip of width \(w = \phi r\) in the radial direction

\[
\left\langle \frac{1}{Z_r v_r} \right\rangle = \frac{1}{\phi (r_o - r_i)} \int_{r_i}^{r_o} \frac{1}{Z_r (\phi r') v_r (\phi r')} dr'.
\]

(5.12)

The capacitances of a radial stub with outer radius \(r_o = 10\ mm\) made on FR4 material \((\varepsilon_r = 4.28)\) with a height \(h = 1.6\ mm\) and a feeding line of width \(w = 0.54\ mm\) are shown in Figure 5.5 for a \(TM_{03}\) mode. We observe that the edge capacitances \(C_{df}\) and \(C_{ds}\) have an important effect on the total capacitance. The accompanying dynamic dielectric constant is shown in Figure 5.6 for the static mode and \(TM_{0n}\) modi up to \(n = 3\). The dynamic permittivities for the \(TM_{0n}\) modes have very close values, especially for small radii \(r_i\).

![Figure 5.5: capacitances radial stub](image-url)
Figure 5.6: dynamic dielectric constant

**Quality factor**

To account for losses in the stub, quality factors are introduced. The global quality factors are given by [25]

\[
Q_{t0} = \frac{1}{\tan \delta}, \\
Q_{t0n} = \left( \frac{1}{Q_{t0}} + \frac{1}{Q_{c0n}} + \frac{1}{Q_{r0n}} \right)^{-1},
\]

(5.13)

with \( \tan \delta \) the loss tangent of the substrate, \( Q_{c0n} \) the losses in the conductor and \( Q_{r0n} \) the losses due to radiation

\[
Q_{c0n} = \frac{\omega_0 \mu_0 h}{2R_s}, \\
Q_{r0n} = \left( \frac{\omega_0 \mu_0 \varepsilon_0 I_\theta}{\varepsilon_{d0n} K} \right)^{-1},
\]

(5.14)

where

\[
R_s = \sqrt{\frac{\omega_0 \mu_0}{2\sigma}}, \\
I_\theta = \int_0^\pi J_1^2(Kr_{oe} \sin \theta) \sin \theta d\theta,
\]

(5.15)

\[
K = 1 - \frac{\tau_{1e}^2 [J_0(k_0 r_{1e})Y_1(k_0 r_{1e}) - J_1(k_0 r_{1e})Y_0(k_0 r_{1e})]^2}{\tau_{oe}^2 [J_0(k_0 r_{oe})Y_1(k_0 r_{oe}) - J_1(k_0 r_{oe})Y_0(k_0 r_{oe})]^2}.
\]
5.3 Measurements

The model of Giannini et al. is verified by a measurement of a radial stub. Figure 5.7 shows the sample with the measured radial stub. The layout of the stub is given in Figure 5.8. The outer radius of the stub $r_o = 6.6$ mm, the inner radius $r_i = 0.5$ mm, the length $l_{es} = 11.4$ mm, $l_{sl} = 9.6$ mm and the load $R_{RF}$ equals 50 $\Omega$. The width of the transmission line $w = 2.8$ mm. This corresponds with a characteristic impedance of 50 $\Omega$ for the transmission lines. The stub is etched on FR4 material with a relative permittivity of $\varepsilon_r = 4.28$, a loss tangent $\tan \delta = 0.02$ and a height $h = 1.6$ mm. The input impedance is calculated from basic transmission line equations and the equations from Hammerstad (Section 4.2) for the characteristic impedance and propagation constant of the transmission line.

The measured and modelled input impedance is shown in Figures 5.9 and 5.10. We see that the measured resonance frequency is in agreement with the model. The quality factor, which determines the sharpness of the resonance peak, has a smaller value in the measurements. This means that there are more losses in the measurement setup than accounted for in the model. Still, the difference is not alarming; the model is a satisfactory reproduction of the input impedance of the radial stub. Figure 5.10 shows the scattering coefficient $|S_{21}|$. The figure shows that the $|S_{21}|$ has quite some discrepancy between measurement and model. Apparently, the model is not yet completely accurate.

5.4 Conclusions

In this chapter we have presented an analytical model that characterizes the impedance of the radial stub. This model is based on the model presented by Giannini et al. [25]. It uses an electromagnetic field expansion in terms of resonant modes of the radial stub to derive an expression for the input impedance. Effective parameters are introduced to account for fringe fields at the edges of the stub.
Measurements indicate that the model is not completely accurate. Still, it gives an impression of the behaviour of the radial stub. Additional experiments can give a better understanding of the model's accuracy.
Chapter 6

Rectenna

Now that we have modelled the building blocks of our rectenna, a smart combination should do the trick. The goal is to achieve a high conversion efficiency while keeping the physical dimensions of our rectenna small. Measurements will indicate the range of validity of our analytical models.

6.1 Introduction

Now that we have acquired an accurate modelling of the key components of our rectenna system, we can combine these models to a rectenna. The interconnection of these models should be given careful attention, not only mathematically, but also practically. We have to keep in mind that modelled layouts also have to be realized physically.

Schematically, a general rectenna system is depicted in Figure 6.1. Here, we see that the RF energy that is received by the antenna is sent to the rectifier through an impedance matching network. The task of the impedance matching network is to adapt the output impedance of the antenna to the input impedance of the rectifier. In this way the efficiency of the rectenna can be increased, since reflection between antenna and rectifier is minimized.

As suggested by H. Visser [32], an even better option is to design the antenna and rectifier system such that their input and output impedance already match. In this way, an impedance matching network is superfluous, and precious space and power loss is avoided. Another improvement is the use of a low-pass filter after the rectifier circuit. This setup is shown in Figure 6.2. The low-pass filter blocks the RF power and therefore less unwanted RF energy is consumed in the load. An additional advantage of the low-pass filter is that it reduces the influence of changing loads on the RF behaviour of the rectenna system.

![Figure 6.1: schematic rectenna system](image-url)
6.2 Model interconnection

Initially, we are interested in the validation of our interconnected analytical models. When these models agree with measurements of a complete rectenna system, attention can be given to the optimization of the conversion efficiency. A detailed layout of the first setup of a total rectenna system is shown in Figure 6.3.

The patch antenna is connected to the rectifier through an interconnect. This is a coaxial transmission line with a teflon dielectric and a characteristic impedance of 50 $\Omega$. The interconnect is assumed lossless. The purpose of the inductor is to provide a short circuit at DC whereas it has a high impedance at the operating frequency. The Schottky diode and the radial stub are described comprehensively in preceding chapters. The microstrip transmission lines on the PCB also have a characteristic impedance of 50 $\Omega$. The output of the radial stub is connected to a network analyzer. The network analyzer has an internal RF load of 50 $\Omega$ and a low-pass filter with an external connection which is used to provide a DC load for the rectenna circuit.

The equivalent electric circuit of the rectenna layout shown in Figure 6.3 is given in Figure 6.4. The grey blocks in the figure represent transmission lines. The figure shows that the power collected by the antenna is modelled as a voltage source with an output impedance equal to the input impedance of the antenna. This is a valid model to determine what happens at the output of the antenna [33]. The physical inductor is modelled as a parallel circuit of a resistance $R_{\text{ind}}$, inductor $L_{\text{ind}}$ and a capacitor $C_{\text{ind}}$. The parasitic capacitance is caused by the internal capacitance between the turns of the coil. The resistance models the losses in the coil.

The electric circuit of Figure 6.4 can be simplified to the one shown in Figure 6.5. Here, the linear components before and after the Schottky diode are lumped together in a generator impedance and a load impedance. From this circuit, the current through the diode can be calculated by the methods described in Chapter 3.
6.3 Verification

We would like to match the output impedance of the antenna to the input impedance of the rectifier. To verify whether our models predict the proper impedances, they are measured and simulated separately.

Antenna

First, the input impedance of the antenna is measured. As mentioned before, the input impedance of the antenna is equivalent to its output impedance. For the measurement we have used a microstrip patch antenna with width and length dimensions $a = b = 29.0$ mm and a height $h = 1.6$ mm. The dielectric has a relative permittivity $\varepsilon_r = 4.28$ and a loss tangent $\tan \delta = 0.02$. The probe is located at $x_0 = 3.0$ mm and $y_0 = 14.5$ mm. For the simulation, the higher order modes up to $m = n = 10$ have been taken into account. Figures 6.6 and 6.7 show that the simulated results agree satisfactorily with the measurements. The measured and modelled resonance frequencies have a discrepancy less than 0.8%.

Schottky diode and radial stub

The combination of the Schottky diode and the radial stub is also simulated with a DC load of $R_{DC} = 470 \, \Omega$ and an incident power of 0 dBm. The diode is modelled including the effects of the first harmonic. The results for the input impedance and the DC output voltage are presented in Figures 6.8 - 6.10

From Figures 6.8 and 6.9 we see that the input impedance of Schottky diode and radial stub is modelled well. For higher frequencies the imaginary part of the input impedance has a slight offset, but this error is relatively small. The DC output voltage has a remarkable agreement with the measurements. This result gives a good confidence in the accuracy of the model.
Inductor

To provide a short circuit at DC for the antenna, the inductor shown in Figure 6.3 was introduced. The inductor is modelled as a parallel RLC circuit. To determine the values for resistance, inductance and capacitance the impedance of the inductor was measured. Figures 6.11 and 6.12 show the results, where the inductor is modelled with $R_{\text{ind}} = 5 \, \text{k}\Omega$, $L_{\text{ind}} = 40 \, \text{nH}$ and $C_{\text{ind}} = 0.4 \, \text{pF}$.

Rectenna

The performance of the rectenna was measured by the setup shown in Figure 6.13. The output of the network analyzer is connected to an amplifier which has a gain of approximately 20 dB. The amplified signal is transmitted by a small horn antenna. Part of the transmitted signal is received by the patch antenna and rectified by the rectifying circuit. The resulting DC signal is measured by an oscilloscope. To determine the amount of power that was received by the patch antenna, the transmission coefficient $S_{21}$ of the setup was measured without the rectifying circuit. The received power equals $P_r = P_t |S_{21}|^2$, with $P_t$ the transmitted power. Figure 6.14 shows the received power of the patch antenna when the network analyzer is connected.
From the measurement of \( S_{21} \), the equivalent Thévenin circuit for the patch antenna can be determined (Figure 6.15). The network analyzer has an input impedance of \( Z_l = 50 \ \Omega \). The output impedance is equal to the antenna impedance. The voltage \( V_g \) can be found from

\[
P_r = \frac{1}{2} \text{Re}\{VI^*\},
\]
\[
= \frac{1}{2} \left| \frac{V_g}{Z_{\text{ant}} + Z_l} \right|^2 \text{Re}\{Z_l\}.
\]  

(6.1)

Now that we have determined the equivalent Thévenin model for the transmit-receive circuit, the antenna can be connected to the rectifying circuit. The DC voltage is measured and simulated with the incident power found from the previous measurement. The results are shown in Figure 6.16. An accurate agreement between the model and the measurement is found. This indicates that the applied models are appropriate.
Figure 6.13: measurement setup rectenna

Figure 6.14: received power antenna

Figure 6.15: equivalent Thévenin model patch antenna
6.4 Improvement of design

At this stage we are quite pleased. The several building blocks of our rectenna have been modelled and verified with measurements. With this knowledge, we can try to design an improved prototype of a rectenna. The rectenna should operate at 2.45 GHz and should be as efficient as possible for an input power of approximately 1 mW. The efficiency $\eta$ of the rectenna is defined as the ratio of DC power and received power, i.e.,

$$\eta = \frac{P_{DC}}{P_r}. \quad (6.2)$$

The rectenna that was put together in previous section had an efficiency of 26%. Improvement of the efficiency is discussed below.

6.4.1 General considerations

To acquire an efficient rectenna, there are two main concerns. First of all, the losses in the rectenna have to be minimized. All losses involve dissipation of power and consequentially, a reduced efficiency. Secondly, it is important to achieve a high conversion efficiency, i.e. the portion of received RF power that is converted to DC power should be as high as possible.

Losses

To minimize the losses in the rectenna, several mechanisms of dissipation have to be considered. There are a number of mechanisms that dissipate power. The important ones we distinguish are

- dissipation in material,
- dissipation in diode,
• radiation of reflected power,
• dissipation of RF power in load.

The dissipation in the material is due to the finite conductivity of the metal and the non-ideal isolation of the dielectric. For frequencies up to 2.5GHz the losses in good-conducting metals like copper are small. The losses in the dielectric, however, can be significant at these frequencies for less appropriate materials like FR4.

The main dissipation in the diode is due to the resistance of the substrate material. A Schottky diode with a low substrate resistance is a favorable choice.

To prevent radiation of reflected RF power, there are two important measures we can take. To prevent reflection at the operating frequency, the antenna has to be matched to the rectifier circuit. In this way the power transfer from antenna to rectifier circuit is optimized and no reflection occurs. To avoid that the higher harmonics radiate from the antenna, we can use a band-stop filter for these frequencies between the antenna and the diode, which generates the harmonics.

The RF power dissipation in the DC load can be minimized by a low-pass filter. This filter blocks the RF power and thus prevents RF power dissipation in the load. Instead of a low-pass filter, we can also use a band-stop filter which blocks the operating frequency and its harmonics. We should point out here, that although this filter is placed between the antenna and the rectifier, its primary use is not to match the antenna with the rectifying circuit. Therefore this setup is substantially different from the setup presented in Figure 6.1.

Conversion efficiency

An important parameter for the conversion efficiency of the diode is the amplitude of the input voltage. If the amplitude of the input signal is large compared to the built-in voltage of the diode, the efficiency is high. In this case the diode is in forward bias for a long time during the conversion cycle. This results in a higher DC output voltage.

The junction capacitance of the diode also plays a role in the conversion efficiency. This capacitance has to be charged and discharged every conversion cycle. This causes a lag in the output voltage. As a result, the period of time that the output voltage and output current are both in forward direction becomes shorter and the DC output voltage decreases. Although the effect of the junction capacitance on the conversion efficiency is not that influential at our frequencies, a small junction capacitance is preferred.

6.5 Prototype design

Given the considerations in the previous section, we end up with the design setup as shown in Figure 6.17. This setup is used in the design of a prototype. For the antenna of our prototype we use the patch antenna. The band-stop filters can be composed of radial stubs. The rectifier consists of a single Schottky diode and the load is assumed purely resistive.

Let us focus on the patch antenna first. The antenna should resonate at the operating frequency of \( f_0 = 2.45 \, \text{GHz} \). From this specification, the length of the antenna is fixed. In our prototype
the width of the antenna is chosen identical to the length of the antenna. If the probe position is chosen along the centerline in the length direction, the $TM_{10}$ mode in the antenna will be excited as the lowest resonance frequency. The output impedance of the antenna depends on the probe position. If the probe is located near the edge of the patch, the output impedance is high. An antenna with a high output impedance gives a larger output voltage, which is advantageous for the conversion efficiency, at the cost of a smaller output current.

The band-stop filters are designed by employing radial stubs. To prevent the harmonics that are generated by the diode from radiating, they are reflected by the band-stop filter. In our prototype the filter is designed to stop only the first harmonic term. The radial stubs are designed to resonate at $f_1 = 4.9 \text{ GHz}$. To improve the filter a butterfly stub-layout is chosen (Figure 6.18). The band-stop filter between the diode and the load consists of a radial stub which resonates at the operating frequency and a radial stub to block the first harmonic.

For the rectifier we have chosen the same HSMS-2852 Schottky diode as the one used throughout this thesis [20]. This diode is not entirely suitable for our application. The substrate resistance $R_s = 25 \text{ n}$ is relatively high for our low-power rectenna. Y. Suh and K. Chang [34] have reported the use of a diode with $R_s = 4 \text{ n}$. Still, the HSMS-2852 is the only one available to us, so it is incorporated in our design.

The load of the rectenna models the application that uses the power from the rectenna. We assume the load to be purely resistive. It is noted that there is an optimal load which maximizes the rectenna efficiency. In our prototype design, the load was chosen to be fixed as $R_L = 1 \text{ k}$.

Given these considerations, we end up with an initial layout as shown in Figure 6.18. The practical realization is shown in Figure 6.19. The rectenna consists of two parts: the antenna and the rectifier circuit with filters. Both parts have been etched on FR4 material and have been stacked. The probe of the patch antenna is guided through the dielectric to the rectifier circuit. To provide a short-circuit for the antenna at DC, the inductor of Figure 6.3 is implemented as a shorted transmission line of length $l_{\text{ind}}$.

The lengths of the transmission lines have to be determined. The lengths have to be chosen such that the antenna is matched with the circuit for the operating frequency and such that power dissipation of the generated harmonics is minimized. To minimize the power dissipation of the harmonics, the input impedance as seen by the diode for these frequencies should be high. A high input impedance results in a low current for the harmonics and consequently a lower dissipation. To acquire a high impedance, the radial stubs that block the first harmonic term are placed at a distance of $\frac{\lambda_1}{4}$ from the diode. Here $\lambda_1$ is the wavelength of the first harmonic of the transmission line. The width of the transmission lines should be chosen small, such that the characteristic impedance of the line is high. In this case the input impedance from the radial stubs that are connected to the diode becomes even higher.

The lengths $l_{dso}$ and $l_{\text{ind}}$ are used to match the circuit to the antenna’s output impedance. Figure 6.20 shows the input impedance of the circuit as a function of the length $l_{\text{ind}}$ for $l_{dso} = 0 \text{ mm}$. The
Figure 6.18: prototype rectenna layout

Figure 6.19: prototype rectenna
Figure 6.20: circuit matching

The figure shows the dependence of the input impedance on the length of the shorted transmission line. We also observe that the sensitivity of the input impedance for the length of the line is quite high. A length variation of one millimeter already has a considerable impact on the input impedance.

6.5.1 Prototype layout

The dimensions of the prototype are given by $l_{nd} = 2.11$ mm, $l_{pd} = l_{soe} = 9.03$ mm and $l_{dso} = 0.0$ mm. The width of the transmission lines is chosen identical to the width of the diode pins and equals $w_{ms} = 0.54$ mm. The length and width of the antenna are both 29.0 mm. The probe of the antenna is placed at 4.0 mm from the edge. The dielectric is made of FR4 material with an effective permittivity of $\varepsilon_r = 4.28$, a loss tangent $\tan \delta = 0.02$ and a height $h = 1.6$ mm. The outer radius of the large radial stub is $r_o = 7.55$ mm, the outer radius of the small stubs is $r_o = 4.25$ mm. The inner radius of the stubs is $r_i = 0.1$ mm and the angle of all stubs equals $\phi = \frac{\pi}{2}$ radians.

The radii of the stubs should be chosen such that the stubs resonate at the operating frequency and its harmonic. Unfortunately the outer radii have been determined with a preliminary model, different from the one given in Chapter 5, which is not sufficiently accurate. As a result, the radii were chosen too small. Nevertheless we will use these dimensions in our further analysis, although it will result in a poorer performance for our rectenna.

The incident power of the antenna under matched load conditions is shown in Figure 6.21. The DC output voltage of the rectenna is shown in Figure 6.22. The modelled and measured input impedance of the rectifying circuit of the prototype are shown in Figures 6.23 and 6.24 for the frequency range from 100 MHz to 4.1 GHz. The DC output voltage of the rectifying circuit is shown in Figure 6.25, where the rectifying circuit is connected to a generator with an output impedance of 50 $\Omega$ and a power level of 0 dBm.
Figure 6.21: incident power rectenna

Figure 6.22: DC output voltage rectenna

Figure 6.23: Re\{Z_{in}\} rectifying circuit

Figure 6.24: Im\{Z_{in}\} rectifying circuit

Figure 6.25: DC output voltage rectifying circuit
6.5.2 Discussion

The modelled and measured DC output voltage have a reasonable resemblance, although there is a slight frequency shift between the measured and modelled curves. The height of the output voltage is predicted well. The measured efficiency of the rectenna lies around $\eta = 40\%$.

What we observe is, that the models of the rectifying circuit do not agree well with the measurements. Still, we can see the trend of the modelled impedance curve in the measurements. An important reason for this discrepancy is the tolerance introduced during the fabrication process. To measure the impedance of the rectifying circuit, a connector is placed at the ground plane. A hole with a diameter of 1.0 mm was drilled to connect the probe of the connector to the transmission line of the circuit. This line is only 0.54 mm wide. As a result, the inner radii of the butterfly stubs and transmission line lengths are not accurately defined. The effect of the larger probe diameter is not accounted for in the model.

Another problem is the accuracy of the transmission line model for small line widths. The transmission lines are relatively thin. The width is 0.54 mm whereas the height of the substrate is 1.6 mm. A measurement of such a transmission line with a length $l = 20$ mm indicates that there is a difference between model and measurement. Figures 6.26 and 6.27 show the input impedance. The model we use is verified with a simulation performed in Ansoft Serenade v8.0. This is a circuit simulator. It gives results which are very similar to our model. Hence, suspicion arises that the measurements are not completely representative. It is possible that the transition from connector to the small microstrip transmission gives undesired effects.

The modelling of the radial stubs is also a point of uncertainty. As the model seemed to perform reasonably well for the stub in Chapter 3, the model for the newly designed stubs apparently introduces some inaccuracy. This is shown in Figures 6.28 - 6.30. Here, the input impedance and scattering parameter $|S_{21}|$ for a radial stub with outer radius $r_o = 7.55$ mm is plotted. The radial stub is connected to a transmission line of width $w_{ms} = 0.54$ mm. The circuit is analyzed in four different ways, which are listed below:
The model of Giannini et al. was presented in Chapter 3. Ansoft Serenade v8.0 is a circuit simulator. It uses models based on the models by Vinding and Giannini. Ansoft Ensemble is a 2.5D full-wave simulator. The full-wave simulations have been performed by H. Visser. From other experimental results, the accuracy of the full-wave simulations are found to be good. In general, full-wave simulated and measured resonance frequencies have an error of less than 5%. Regarding the input impedance, we see from Figures 6.28 and 6.29, that the model of Giannini et al. lies closest to the measured values. The results from Ansoft Serenade seems to lie closer to the full-wave simulations. At present, it is not clear whether the discrepancy is due to the full-wave model or the measurements. The transmission line measurement indicated that the connector might introduce some inaccuracy. Still, there is uncertainty whether the correct model lies closer to the measurement or closer to the full-wave simulation.

The scattering parameter $|S_{21}|$ is measured as well. The difference between the measurement of the input impedance and the scattering parameter $|S_{21}|$ should be pointed out. For the measurement of the input impedance, the position of the radial stub on the microstrip transmission line is an important parameter. For the measurement of the scattering parameter $|S_{21}|$ only the total transmission line length is relevant.

The measurement of the scattering parameter $|S_{21}|$ is more convincing (Fig. 6.30). Two measurements of two radial stubs with identical circuit layouts are measured on two different network analyzers. They come up with similar results for the $|S_{21}|$. The full-wave simulation also lies close to the measurements. The model of Giannini et al. does not seem to behave according to the measurement and the full-wave simulation. The resonance frequency is quite inaccurate. It indicates that more work on the radial stub model has to be done.
6.5.3 Efficiency

As mentioned before, the efficiency of the prototype is about 40%. At first sight, this value does not seem to be very high compared to reported results in [35] and [34] who achieve an efficiency of 82% and 84%, respectively. Still, the main difference between our setup and theirs, is the incident power level. Reported conversion efficiencies are achieved at incident power levels of 30mW and higher, whereas our setup is optimized for an incident power level of 1mW. For these low incident power levels a lower conversion efficiency is realized. As the incident power level decreases, it is harder to open the diode in forward bias. This results in a lower conversion efficiency.

Nevertheless, there is room for improvement of the efficiency of our rectenna. First of all the radial stubs should be redesigned such that they actually resonate at the chosen operating frequency and its harmonic. Preliminary measurements with enlarged radial stubs indicate a conversion efficiency of 49%. As pointed out in Section 6.4.1, the Schottky diode has a rather high substrate resistance. This causes unwanted losses. We also note that the loss tangent of the chosen dielectric causes additional losses which can be avoided with a better choice of dielectric material.

6.6 Conclusions

As seen in Section 6.3, the rectenna system can be modelled accurately with an interconnection of the derived models for the sub-circuits. This validates the separate analysis of the sub-circuits. A good agreement between measurement and model is found in the prediction of the DC output power of the rectenna.

Although the measured and modelled DC output voltage of our prototype rectenna gave a good resemblance, the design also gives rise to new questions. The input impedance of the rectifying circuit was not modelled well. This could be caused by the tolerance introduced by the fabrication process, but chances are that the used models for transmission lines and radial stubs are not entirely adequate. The prototype designed in the pertaining section clearly indicates that there is still room for more research.
The achieved conversion efficiency also leaves room for improvement. Conversion efficiency ratios as reported by McSpadden et al. [35] and Suh and Chang [34] are not realizable for our incident power level, but with a suitable choice for the dielectric, Schottky diode, and filters efficiencies of more than 50% should be achievable.
Chapter 7

Conclusions & Recommendations

The design and modelling of a rectenna was investigated. The rectenna was designed for small dimensions and a low incident power level. An actual rectenna was built, modelled and measured. The modelling was performed by using analytical models, and it was shown that these models can produce accurate results when used appropriately.

As is seen throughout this thesis the importance of verification with measurements of the derived models is significant. Not only the verification of the model is important, also the practical aspects concerning measurement give a lot of insight. For the layout of transmission lines and other structures at microwave frequencies every millimeter has significant effects on its behaviour. This requires that the model and the measurement use accurate and identical dimensions to be able to compare them in a sensible manner. In this light, also the used the materials should be well-defined. In our case, the used dielectric was not well-defined. The FR4 material is known to have some variation in relative permittivity, not only between different batches, but also locally on a single batch. Therefore a recommendation is to use a microwave laminate as dielectric which is characterized properly for microwave frequencies and has more suitable properties.

An important conclusion can be drawn from Chapter 3 about the Schottky diode. In this chapter it was shown that with a relatively simple approach the nonlinearity of the diode can be modelled accurately. This analysis was the basis of the design of the rectenna. It allowed us to translate the nonlinear circuit to a set of linear circuits for the harmonic frequencies and the DC frequency. This approach can be easily adopted for use with different antennas and filters. Validity of the models was demonstrated for a wide frequency range from 100 MHz to 4.1 GHz.

A remarkable observation regards the difficulty of the implementation of the radial stub model as presented by Giannini et al. Although they claim accurate results, the exact method and derivations of their formulas is not completely presented in their papers. At times, figures and formulas are erroneous and parameters are introduced without univocally introducing them.

The DC output voltage of the prototype rectenna lies around 0.6 V for an incident power of 1 mW. For a lot applications this is too low. Most low-power integrated circuits need at least 1 V to operate. The output voltage of the rectenna can be doubled if we use two diodes in the rectifying circuit [32]. This is an important recommendation for further research. The modelling of such a circuit leads to a rectenna with higher output voltages.
Also, more attention should be given to the efficiency of the rectenna circuit. The efficiency of the rectenna can be increased with a better choice for the diode, dielectric and filters. The load impedance can be optimized for efficiency. In this thesis an initial design was proposed, which can be used as a start for further improvement.
Bibliography


