Analytical and Numerical Modeling of Currents on Vivaldi Antennas for Radio Astronomy

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Analytical and Numerical Modeling of Currents on Vivaldi Antennas for Radio Astronomy

M.Sc. Thesis

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Terminology and Abbreviations

**ASTRON** A worldwide known Dutch institute, active in the field of radio astronomy.

**FARADAY** Focal-Plane Arrays for Radio Astronomy: Design, Access and Yield.

**FOV** Field of View.

**Gain** The ratio of the radiated intensity, in a given direction, to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically. The radiation intensity corresponding to the isotropically radiated power is equal to the power accepted (input) by the antenna divided by $4\pi$. (IEEE definition)

**MMIC** Monolithic Microwave Integrated Circuits.

**MoM** Methods of Moments; a computational method for solving electromagnetic field problems based on integral equations.

**Mutual Coupling Effect** In this thesis: the electromagnetic interaction between antennas.

**SKA** Square Kilometer Array.

**TSA** Tapered Slot Antenna.

**VSWR** Voltage Standing Wave Ratio.
Chapter 1

Introduction

1.1 The Square Kilometer Array (SKA) as a New Generation Radio Telescope

SKA is an international project for designing and developing a new generation Radio Telescope [1]. It should provide a wide Field of View (FOV) of the sky for astronomical observations and operate with a high sensitivity over the frequency range of 0.2-20 GHz. ASTRON is leading the research and development of the European concepts of SKA (EU SKA), which involves Aperture Phased Arrays, as the main concept, and Focal Plane Arrays as an intermediate concept. The results obtained during the first research phases demonstrated high potentials of MMIC array technology for SKA applications [5], [19].

In the framework of EU SKA, a sub-project has been defined called FARADAY [9], which concerns focal-plane arrays. The prime objective is to design a MMIC focal plane array feed for use on large reflecting radio telescopes in Europe (Westerbork) and Australia (Luneburg Lenses) in the 2-5 GHz band. The feed antenna that is going to be used is called the egg crate, which is a dense Vivaldi element array that can intercept two orthogonal polarizations.

1.2 Description of the Thesis Subject

The research presented in this thesis has been carried out as part of the SKA and FARADAY projects. To ASTRON’s best knowledge, Vivaldi-array technology is considered to be most suitable to satisfy the requirements both for aperture arrays and focal plane array antennas. Therefore, subject of this thesis is to develop a simple analytical model for finite linearly-polarized Vivaldi antennas and, subsequently, to verify the obtained results. The model needs to provide information about mutual coupling, which is an effect inherent in the structure used. Verification of the model is carried out numerically (MoM), as well as experimentally by comparing results with measurements. The pertaining model, either describing the isolated element or an embedded array element, provides insight into the radiation characteristics.
1.3 Literature Concerning the Thesis Subject

Keywords, such as analytical, Vivaldi, model, antenna, tapered, slot, etc., were used in search expressions in order to look for papers written after the first paper published on Vivaldi antennas in 1974 [8]. This led to the observation that literature concerning modeling of Vivaldi antennas is rare but available. The most important/useful papers for this thesis are described briefly in the next paragraphs.

The Tapered-Slot Antenna (TSA) element was introduced by Lewis et al. [8] in 1974. The element was applied in an 8x8 array application and characteristic antenna properties were measured. Subsequently, it was demonstrated that one element has broadband properties in terms of VSWR and gain. At that time, no fundamental attempts were made to describe the operating principle of the Vivaldi element.

As reported by Gibson in 1979 [4], the TSA can be considered as a new member of the class of aperiodic continuously scaled antenna structures and, as such, it has a theoretically unlimited instantaneous frequency bandwidth. Gibson called the element the Vivaldi element and stated that this antenna has significant gain and linear polarization in the principal planes. Also, it was mentioned that the element could be designed such that its gain does not depend on frequency. A traveling-wave mechanism was said to be responsible for the radiation, but was not analyzed analytically at that moment.

Nevertheless, a Vivaldi antenna model was developed in 1987, which could fairly well predict the radiation pattern of a single Vivaldi element by defining a stepped approximation to the tapered geometry using straight slot-line sections, each having a different slot size [6], [10]. Transmission-line techniques were applied successfully to determine the electric-field in each slot section and, afterwards, the entire slot. Far fields were calculated in the principal planes. However, the usage of infinitely wide slot-line sections, as well as the assumption of having no backward traveling waves due to reflection at the tip, limits the usefulness of this approach.

Later on, in 1990, a general method was presented to analyze broadband, symmetrical traveling wave antennas [20]. As an example, the gain as a function of frequency was investigated for V-shaped wire antennas. A global analytical current distribution model was presented that describes a single wave traveling forward and, due to reflection at the antenna end, one traveling backward. The amplitude was considered to be constant along the wire, which is not realistic since it is generally a leaky wave for a more general case. Several model parameters were determined using a numerical fitting procedure.

A more extended description of a traveling-wave model for dipole antennas can be found in [2] (1998). The author makes a summation of an infinite number of traveling waves to describe the current on a dipole antenna. An infinite number of waves needs to be taken into account because of multiple reflections along the structure, which implies that the currents will be reflected repeatedly from the antenna ends before the amplitude of the current has decreased significantly.

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In 2000 [18], the Vivaldi model from 1987 was improved by using annular linearly tapered sections instead of straight slot-line sections so as to determine the field distribution in the slot. Furthermore, the slot-line sections were considered to have a finite width for which field diffraction occurs at the edges of the slot. A three-edge field-diffraction method was developed to describe this phenomenon. However, the pertaining model does not account for a backward traveling wave as a consequence of reflection from the end of the Vivaldi tapering. Reflection would not occur when the antenna is long enough in terms of wavelength, therefore, a low-frequency limit for the model is introduced.

In addition to Vivaldi antenna models, a simplified current distribution model was presented in 2000 for Sierpinski fractal shaped antennas [17] based on traveling-wave currents. This model was developed after the behavior of a Sierpinski antenna was analyzed experimentally [12]. As a model, a multiperiodic traveling wave V model was used, which included many propagating modes only defined along the antenna edges. This model could reasonably well predict the radiation pattern at several frequencies, provided that the traveling-wave model parameters were chosen properly.

The concept of first analyzing the current distribution of a Vivaldi element and afterwards using traveling-wave techniques to model it, is a useful approach towards the development of an analytical model for Vivaldi (array) antennas. General theory on traveling-wave antennas has been described in the literature [21].

1.4 Description of the Thesis Contents

In chap. 2, a reference geometry for a single Vivaldi element is described and, subsequently, simulated with the aid of the MoM for a single frequency. The current distribution along the tapered edges of the element is extracted from the simulated data. Afterwards, an analytical traveling-wave model is proposed and fitted to the numerically obtained (MoM) current along the edges of the reference antenna. As a result, the traveling-wave parameters of the fitted model contain information about the physical characteristics of the element.

The verification of the traveling-wave model is performed in chap. 3, by comparing the E-plane power pattern of the traveling-wave model with the reference pattern, which is calculated by the MoM software. It will be demonstrated that, as a result, by only accounting for currents along the edges of the Vivaldi antenna, the shape of both patterns (model and reference) show a close resemblance.

To be able to compare radiation patterns in magnitude, and henceforth, to define a tolerance for the average pattern error, the edge currents need to be normalized. Two normalization procedures are described in chap. 4. First, an extraction method from the total surface current distribution is presented, which yields properly normalized currents and, subsequently, scaled patterns. Second, a normalization procedure is presented by fitting normalized pat-
terns for the reference antenna configurations. Finally, the frequency dependence of the traveling wave model parameters will be investigated and a region of validity will be determined by using the pattern errors as a tolerance.

A predictive model for the edge currents of an arbitrarily long, dense finite linear-array of Vivaldi elements is presented in chap. 6. The model uses scattering matrices to describe the transitions of currents from certain elements to neighbouring elements. In this manner, the model describes mutual coupling effects between elements. A procedure is presented to determine the scattering matrices for a 2x1 Vivaldi array. Furthermore, the matrices are used to predict the response and pattern of a 9x1 Vivaldi array. From these considerations it will become clear how accurate the model is.

Finally, conclusions are drawn and recommendations are presented.

Appendix E contains a separate document written at ASTRON. It presents simulated results\(^1\) for the reference Vivaldi model that has been used in this thesis.

\(^1\)All simulations were carried out by Zeland, a MoM software package used to solve integral equations in the spectral domain [22].
Chapter 2

A Simple Analytical Model for a Slot-Line Excited Vivaldi Element

The aim of this chapter is to demonstrate that the currents along the curved edges of a Vivaldi antenna element can successfully be approximated by traveling-wave currents. First, a reference geometry will be described and results obtained by the MoM will be presented. Second, a traveling-wave model will be fitted to the numerically determined currents. Finally, conclusions are drawn.

2.1 Traveling Wave Mechanism for a Single Vivaldi Element

2.1.1 Geometrical Parameters of a Vivaldi Antenna Element

A matched transition between a non-radiating slot line and free space can successfully be accomplished by using an exponentially tapered slot line (Vivaldi) that, as a result, transforms non-resonant traveling surface-waves into radiating leaky-waves [18]. An example of such a slot-line transition is shown in fig. 2.1. The relationship between the geometrical parameters and the antenna performance has experimentally been determined and described in [16].

The exponential taper of the infinitely thin metallic Vivaldi antenna, positioned in the xy-plane, is described by the points $P_1$ and $P_2$, at $(x_1, y_1)$ and $(x_2, y_2)$ respectively, and the opening rate $R$ as proposed in [16]. Between $P_1$ and $P_2$ the exponential taper is described by the following differential equation

$$\frac{dy}{dx} = Ry + C,$$  \hspace{1cm} (2.1)

in which the opening rate $R$ is expressed explicitly as a curvature factor and in which $C$ is an offset independent of $y$, which is chosen such that $P_1$ and $P_2$ are points on the line $y(x)$. The general solution of eq. (2.1) is

$$y = C_1 e^{Rx} + C_2,$$  \hspace{1cm} (2.2)
Figure 2.1: Geometrical parameters of a slot-line excited Vivaldi element. Excitation is accomplished by using a differential port connected to the slot line (extension for MMIC). A description of the available ports can be found in the user’s manual of the MoM software.

in which $P_1$ and $P_2$ have to be points on this line, therefore

$$C_1 = \frac{y_2 - y_1}{e^{Rz_2} - e^{Rz_1}} \quad (2.3)$$

and

$$C_2 = \frac{y_1 e^{Rz_2} - y_2 e^{Rz_1}}{e^{Rz_2} - e^{Rz_1}}. \quad (2.4)$$

Hence, a parameterization of the upper curved edge can be defined by

$$r(x) = x e_2 + [C_1 e^{Rz} + C_2] e_y, \quad x_1 \leq x \leq x_2. \quad (2.5)$$

The length of the curvature, denoted by $\ell(x)$, is simply expressed by

$$\ell(x) = \int_{\xi = x_1}^{x} |\dot{r}(\xi)| d\xi = \int_{\xi = x_1}^{x} \sqrt{1 + (C_1 R)^2 e^{2R\xi}} d\xi, \quad (2.6)$$

where $\dot{r}(\xi)$ is the derivative of $r(\xi)$. The integral in eq. 2.6 can be evaluated analytically by using the primitive function (app. D)

$$\int \sqrt{1 + (C_1 R)^2 e^{2R\xi}} d\xi = \frac{1}{R} \left[ \sqrt{1 + (C_1 R)^2 e^{2R\xi}} - \text{arctanh}(\sqrt{1 + (C_1 R)^2 e^{2R\xi}}) \right], \quad (2.7)$$

so that for $x_1 = 0$

$$\ell(x) = \frac{1}{R} \left[ \sqrt{1 + (C_1 R)^2 e^{2Rx}} - \text{arctanh}(\sqrt{1 + (C_1 R)^2 e^{2Rx}}) \right]. \quad (2.8)$$
CHAPTER 2. A SIMPLE ANALYTICAL MODEL FOR A SLOT-LINE EXCITED VIVALDI ELEMENT

2.1.2 A Vivaldi Reference Antenna Element Simulated by the MoM

To gain insight into the current distribution for a single Vivaldi element, a reference model has been defined and was simulated by the MoM at a frequency of 2.3 GHz using the following parameters: \( P_1 = (0, 0.35) \) and \( P_2 = (185, 46.25) \) in mm and \( R = 0.03 \text{ mm}^{-1} \). These dimensions were taken to be equal to the dimensions of the THEA element [19], except that our reference model does not have a flattened top and dielectric. The frequency was chosen in compliance with measurements that were carried out for the egg crate\(^1\) in Australia. The THEA element usually operates at low frequencies (0.2-1.5 GHz), however, our reference model is simulated at 2.3 GHz, also because of the absence of a dielectric substrate.

The Vivaldi antenna was excited by a slot line having a characteristic impedance of 100 \( \Omega \), which resulted in a \( S_{11} \) of less than -10 dB over a simulated frequency range of 2-5 GHz. The metal sheets were chosen to be perfect conductors and thin compared to the wavelength used. The magnitude of the average electric current density was determined and is illustrated in fig. 2.2.

![Figure 2.2: Magnitude of average electric current density on a single Vivaldi element.](image)

Clearly, the largest currents are concentrated along the edges, in particular the inner edges. A better knowledge of these edge currents should be obtained before presenting a simple analytical current distribution model. A .CDD-file was exported, containing complex values of the surface current density at discrete simulated mesh coordinates on the metal sheets (fig. 2.3). From these data it is possible to compute the phase distribution \( \psi(\ell, d) \) as well as the amplitude distribution density \( |J(\ell, d)| \) of the current at a certain distance \( d \) from the edge (\( |J(\ell)| \) would inevitably be singular) and as a function of the curvature length \( \ell \).

A Matlab program was written that searches for coordinates in the data file that are within the areas defined by the rectangles along the curvature as shown in fig. 2.3. The size of these rectangles can be altered by the user and should be chosen such, that it captures the currents which are near the edge. It should be mentioned that this size is strongly dependent on how the mesh was generated by the MoM software. Now, the phase \( \psi(\ell, d) \) and the magnitude

---

\(^1\)Name of the broadband dual-polarized Vivaldi array that is used for FARADAY.
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Figure 2.3: Obtaining the current density values, at a certain fixed distance \( d \) from the curved edge, from the current distribution data file.

\[ |J(\ell, d)| \] of the current can be determined in a way as described next.

Suppose that \( J(x_0, y_0) \) at a certain coordinate is determined along a curved edge, i.e.

\[ J(x_0, y_0) = |J_x|e^{j\varphi_x}e_x + |J_y|e^{j\varphi_y}e_y \quad [\text{Am}^{-1}]. \]  

(2.9)

In the time domain, this can be expressed as

\[
J(x_0, y_0, t) = \text{Re}\{|J_x|e^{j\varphi_x}e_x e^{-j\omega t}\}e_x + \text{Re}\{|J_y|e^{j\varphi_y}e_y e^{-j\omega t}\}e_y \\
= |J_x|\cos(\varphi_x - \omega t)e_x + |J_y|\cos(\varphi_y - \omega t)e_y, \tag{2.10}
\]

which can be interpreted as elliptical polarization\(^2\) for the current density at \((x_0, y_0)\). The method of extraction, to obtain \( \psi(\ell, d) \) and \( |J(\ell, d)| \), can now be explained with the aid of fig. 2.4.

Figure 2.4: Elliptical polarization of the current along the upper curved-edge.

The phase distribution \( \psi(\ell) \) is now defined as the angle of a current vector along the curvature at \( t = 0 \) and its corresponding maximum amplitude as shown in fig. 2.4. The phase is assumed

\(^2\)Formally, polarization is considered with reference to a vector perpendicular to the polarization ellipse (for TEM waves the propagation direction). In this case it will be the vector normal to the Vivaldi surface, i.e., \( e_z \).

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to be independent of $d$, since $d$ is small compared to the width of the element. $|\mathbf{J}(t, d)|$ is
simply defined as the maximum amplitude of the current density vector along the curvature.
The time for which $\mathbf{J}(x_0, y_0, t)$ reaches its maximum can be calculated according to
\[
\frac{\partial}{\partial t}|\mathbf{J}(x_0, y_0, t)|_{t=t_{\text{max}}} = 0, \tag{2.11}
\]
in which
\[
|\mathbf{J}(x_0, y_0, t)| = \sqrt{\frac{1}{2}|J_x|^2(1 + \cos(2\varphi_x - 2\omega t)) + \frac{1}{2}|J_y|^2(1 + \cos(2\varphi_y - 2\omega t))}. \tag{2.12}
\]
Substitution of (2.12) in (2.11), and evaluation of the derivative at $t = t_{\text{max}}$, yields
\[
|J_x|^2 \sin(2\varphi_x - 2\omega t_{\text{max}}) = -|J_y|^2 \sin(2\varphi_y - 2\omega t_{\text{max}}). \tag{2.13}
\]
Eq. (2.13) can be rewritten by making use of the relationship $\sin(a - b) = \sin(a)\cos(b) - \cos(a)\sin(b)$. This yields
\[
\tan(2\omega t_{\text{max}}) = \frac{|J_x|^2 \sin(2\varphi_x) + |J_y|^2 \sin(2\varphi_y)}{|J_x|^2 \cos(2\varphi_x) + |J_y|^2 \cos(2\varphi_y)}, \tag{2.14}
\]
from which $t_{\text{max}}$ can be solved, which is simply expressed as
\[
t_{\text{max}} = \frac{1}{2\omega} \left\{\arctan \left(\frac{|J_x|^2 \sin(2\varphi_x) + |J_y|^2 \sin(2\varphi_y)}{|J_x|^2 \cos(2\varphi_x) + |J_y|^2 \cos(2\varphi_y)}\right) + n\pi\right\} \quad n = 0, 1, 2 \ldots \tag{2.15}
\]
Since we have now determined two vectors, i.e. $\mathbf{J}_0 = \mathbf{J}(t = 0)$ and $\mathbf{J}_{\text{max}} = \mathbf{J}(t = t_{\text{max}})$,
for a certain position $(x_0, y_0)$, $\psi(x_0, y_0)$ can be calculated using
\[
\langle \mathbf{J}_0, \mathbf{J}_{\text{max}} \rangle = |\mathbf{J}_0||\mathbf{J}_{\text{max}}| \cos \psi. \tag{2.16}
\]
Hence, it is possible to define the amplitude distribution along the curvature as being $|\mathbf{J}_{\text{max}}|$ and the phase distribution as being $\psi$.

However, it is possible to simplify the previous expressions if the assumption of linear polarization is made along the curvature. In other words, the component of the surface current parallel to the edge will dominate over the component normal to the edge. Linear polarization of the surface currents close to the edges implies a high axial ratio ($AR$) of the elliptical polarized currents along the edges. The $AR$ can be calculated by
\[
AR = \frac{\text{Length Major Axis of the polarization ellipse}}{\text{Length Minor Axis of the polarization ellipse}}, \tag{2.17}
\]
which can be substituted by using expressions (2.15) and (2.12) for properly chosen $n$ values, so as to choose either the major or minor axis of the ellipse. The axial ratios for all discrete current distributions on both Vivaldi sheets were calculated and plotted in fig. 2.5.
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Figure 2.5: Axial ratios of the elliptically polarized surface currents on both Vivaldi sheets (MoM data).

As can be seen, the assumption of linear polarization along the edges can be made since the axial ratios of the discrete surface current distribution are high for currents close to the edges (AR ≈ 50). Furthermore, due to the boundary condition near the edges, the current has to be directed along the edges. Also, because it was observed that the magnitude of the currents is high along the edges, a line-current approximation is assumed close to the edges. This assumption simplifies the way to obtain phase and amplitude information along the edges, since eq. (2.10) reduces to

\[ J(x_0, y_0, t) \approx \cos(\phi - \omega t) \left\{ J_x e_x \pm J_y e_y \right\}, \]

(2.18)

in which the direction of \( J(x_0, y_0, t) \) is fully determined by \( |J_x| \) and \( |J_y| \). This direction is assumed to be parallel to the curved edge. After this simplification, \( \psi(\ell) \) as well as \( |J(\ell, d)| \) can be written according to

\[ |J(\ell, d)| = \sqrt{|J_x(\ell, d)|^2 + |J_y(\ell, d)|^2} = \sqrt{\mathbf{J} \cdot \mathbf{J}^*} \]

\[ \psi(\ell) = \varphi(\ell). \]

(2.19)

(2.20)

It is well known that the amplitude of the current density on a straight, infinitely long strip, has a square root dependence normal to the edges for the frequencies under consideration.
Therefore, every determined discrete amplitude $|J(\ell, d)|$ will be scaled to its corresponding edge by multiplication of a scaling factor, i.e.,

$$|\tilde{I}(\ell)| = |J(\ell, d)| \sqrt{d}, \quad 0 \leq \ell \leq L,$$

in which $|\tilde{I}(\ell)|$ is the amplitude of the line current along the curvature. Now, the proper shape of the current along the edge has been determined, however, the global strength is still unknown. Therefore, we define $|I(\ell)| = C_{\text{norm}} |\tilde{I}(\ell)|$ in which $|C_{\text{norm}}| = m^{1/2}$. In other words, $C_{\text{norm}}$ has to be determined in order to get globally scaled edge currents which provide a radiation pattern that is similar in terms of absolute values to the one obtained by the MoM software (reference pattern). As a first approach, only the shape of the current will be considered, i.e. $C_{\text{norm}} = 1$. Later on (chap. 4), $C_{\text{norm}}$ will be determined more precisely.

The simplified expressions (2.19)-(2.21) have been implemented with the aid of Matlab so as to obtain $|I(\ell)|$ and $\psi(\ell)$ along the curved edge. The results for the reference model are shown in fig. 2.6.

![Graph showing the determination of $|I(\ell)|$ and $\psi(\ell)$ along the upper curved edge.](image)

Figure 2.6: Determination of $|I(\ell)|$ and $\psi(\ell)$ along the upper curved edge.

It should be mentioned that the way in which the structure's mesh is determined, as well as the rectangular size to extract the edge currents, are still critical factors for the final result.

As can be seen, the current can be represented as a traveling-wave current along the curved edge. Therefore, the edge current might be regarded as a superposition of two waves traveling into opposite directions, forming a standing wave. Since the wave that travels into the negative $\ell$ direction is smaller than the wave traveling into the positive $\ell$ direction (due to radiation loss), an attenuating standing wave is observed and the phase decreases continuously. The amplitude attenuates gradually with increasing $\ell$, since the curvature is smooth. If the curvature is increased, attenuation of the current is increased because of radiation loss. It will be assumed that attenuation of the current is dependent on the curvature rate $R$. 

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The phase velocity of the total current equals approximately the speed of light. From these observations a simple current model along the curved edges can be assumed which will be explained in the next section.

2.1.3 A Simple Vivaldi Model Based on Traveling Wave Line Currents

Suppose that \( I(\ell) \) can be represented by an edge current, which travels into the positive \( \ell \) direction and a reflected edge current, which travels into the negative \( \ell \) direction. Reflection occurs at \( \ell = L \), and is described by a complex reflection coefficient \( \Gamma_L \). Both waves are assumed to be damped waves having an attenuation coefficient \( \alpha \) in \( \text{Np m}^{-1} \) and a single propagation coefficient \( \beta \) in \( \text{rad m}^{-1} \). The following expression holds for the wave traveling into the positive \( \ell \) direction

\[
I^+(\ell) = I_0^+ e^{-\alpha \ell} e^{-j\beta \ell}, \quad (2.22)
\]

in which \( I_0^+ \) is a complex amplitude, defined at \( \ell = 0 \). This wave propagates towards \( \ell = L \), where it is reflected. The complex amplitude for the reflected wave is then given by

\[
I^-_L = I_0^+ e^{-\alpha L} e^{-j\beta L} |\Gamma_L| e^{j\phi_L}. \quad (2.23)
\]

The reflected traveling wave, as a function of \( \ell \), is defined by

\[
I^-(\ell) = I^-_L e^{-\alpha(L-\ell)} e^{-j\beta(L-\ell)}, \quad (2.24)
\]

in which \( I^-_L \) is a complex amplitude, defined at \( \ell = L \). For any distance \( \ell \) we will detect both complex currents \( I^+(\ell) \) and \( I^-(\ell) \). The total current, which is expressed by \( I(\ell) \), is thus given by

\[
I(\ell) = I^+(\ell) + I^-(\ell) = I_0^+ e^{-\alpha \ell} e^{-j\beta \ell} + I_0^+ \Gamma_L e^{-\alpha (2L-\ell)} e^{-j\beta (2L-\ell)}. \quad (2.25)
\]

It is assumed that only a single reflection will take place at \( \ell = L \), since the magnitude of the reflected wave, when it returns at \( \ell = 0 \), is assumed to be negligibly small. This assumption is valid if the attenuation of the current along the curvature is large enough and the element is long enough in terms of wavelengths. The ratio between the “starting amplitude” \( |I_0^+| \) and the “returning amplitude” \( |I^-_L| \) is

\[
\frac{|I_0^+|}{|I^-_L|} = e^{-2\alpha(R)L} |\Gamma_L|, \quad (2.26)
\]

which becomes small when the criteria given above are satisfied. \( \alpha \) is assumed to be dependent on the curvature rate \( R \). Nevertheless, a global procedure to determine the region of validity of the model will be described in chap. 4.
2.1.4 Fitting the Traveling Wave Model on the Simulated Currents

In order to match the theoretical current model, denoted by $I(\ell)$ and derived in the last section, with the simulated current distribution, described by $I_s(\ell)$ and shown in fig. 2.6, it is convenient to introduce the error between both currents and to minimize this error in a least square sense. One defines this error $\Psi$ as

$$\Psi = \int_{\ell=0}^{L} w(\ell)|I_s(\ell) - I(\ell)|^2 d\ell, \quad (2.27)$$

in which $w(\ell)$ is a weighting function. $I(\ell)$ can be substituted using eq. (2.25). Or, alternatively, substitute $I(\ell) = Af(\ell, \alpha, \beta) +Bg(\ell, \alpha, \beta)$, in which $A$ and $B$ are complex constants and

$$f(\ell, \alpha, \beta) = e^{-\alpha \ell} e^{-j\beta \ell} \quad (2.28)$$

and

$$g(\ell, \alpha, \beta) = e^{-\alpha(2L-\ell)} e^{-j\beta(2L-\ell)} \quad (2.29)$$

After substitution of $I(\ell)$ one obtains

$$\Psi = \int_{\ell=0}^{L} w(\ell)|I_s(\ell) - Af(\ell, \alpha, \beta) -Bg(\ell, \alpha, \beta)|^2 d\ell, \quad (2.30)$$

which can be optimized linearly for $A$ and $B$ for each value of $\alpha$ and $\beta$, i.e. $\alpha = \alpha_0$ and $\beta = \beta_0$. Basically, $A$ and $B$ can be regarded as functions of $\alpha$ and $\beta$. If the minimum of $\Psi$ is reached for a certain $A = A_0$ and $B = B_0$, a small change of these coefficients, e.g. $A = A_0 + \Delta A$ and $B = B_0 + \Delta B$, will not influence the total error $\Psi$. Therefore, the linear terms of this Taylor expansion will vanish when a minimum is achieved, since $\frac{\partial \Psi}{\partial A} = 0$ and $\frac{\partial \Psi}{\partial B} = 0$. As a consequence, linear terms in $A$ and $B$ are chosen to be zero in the next derivations.

Substitution of $A = A_0 + \Delta A$, $B = B_0 + \Delta B$, $\alpha = \alpha_0$ and $\beta = \beta_0$ in eq. (2.30) and expanding the modulus squared into two product terms, one term containing the complex conjugate of the other term, yields

$$\Psi = \int_{\ell=0}^{L} w(\ell)(I_s(\ell) - A_0f(\ell) - \Delta Af(\ell) - B_0g(\ell) - \Delta Bg(\ell)) \cdot (I_s^*(\ell) - A_0^*f^*(\ell) - \Delta A^*f^*(\ell) - B_0^*g^*(\ell) - \Delta B^*g^*(\ell)) d\ell. \quad (2.31)$$

After multiplication we end up with the following terms

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The first term on the right-hand side of eq. (2.32) constitutes the optimal error achieved. The next terms represent the linearly dependent terms in $\Delta A$ and $\Delta B$, which should equal zero since the minimum error has to be taken. In enforcing this condition, it can be noticed that if $\int f(x)dx = 0$ also $\int f^*(x)dx = 0$, which implies that only an independent linear set of two equations remains. In matrix formalism the resulting system of equations reads

\[
\begin{pmatrix}
\int_0^L w(\ell)|f(\ell)|^2d\ell & \int_0^L w(\ell)g(\ell)f^*(\ell)d\ell \\
\int_0^L w(\ell)f(\ell)g^*(\ell)d\ell & \int_0^L w(\ell)|g(\ell)|^2d\ell
\end{pmatrix}
\begin{pmatrix}
A_0 \\
B_0
\end{pmatrix}
= \begin{pmatrix}
\int_0^L w(\ell)f^*(\ell)I_s(\ell)d\ell \\
\int_0^L w(\ell)g^*(\ell)I_s(\ell)d\ell
\end{pmatrix}.
\]

The matrix elements can be evaluated analytically, provided that $w(\ell)$ is of a simple form, while the terms on the right-hand side must be evaluated numerically, since $I_s$ is only available in discrete form. $w(\ell)$ will be taken to be equal to unity in order to obtain the matrix elements in an analytical form. The matrix elements, after evaluation, are thus expressed by

\[
\int_0^L |f(\ell)|^2d\ell = \int_0^L |e^{-\alpha_0\ell}e^{-j\beta_0\ell}|^2d\ell = \frac{1-e^{-\alpha_02L}}{2\alpha_0},
\]

\[
\int_0^L |g(\ell)|^2d\ell = \int_0^L |e^{-\alpha_0(2L-\ell)}e^{-j\beta_0(2L-\ell)}|^2d\ell = \frac{e^{\alpha_02L}-1}{2\alpha_0e^{\alpha_04L}},
\]

\[
\int_0^L g(\ell)f^*(\ell)d\ell = e^{-\alpha_02L}e^{-j\beta_02L}\int_0^Le^{j\beta_02\ell}d\ell = \frac{e^{-j\beta_02L}(e^{j\beta_02L}-1)}{2j\beta_0e^{\alpha_02L}},
\]

\[
\int_0^L f(\ell)g^*(\ell)d\ell = e^{-\alpha_02L}e^{j\beta_02L}\int_0^Le^{-j\beta_02\ell}d\ell = \frac{e^{j\beta_02L}(1-e^{-j\beta_02L})}{2j\beta_0e^{\alpha_02L}}.
\]

The terms on the right-hand side of the matrix equation are computed by means of the trapezoidal rule using a nonuniform interval width in $\ell$. Therefore, the right-hand side of the matrix equation is computed as

\[
\int_0^L f^*(\ell)I_s(\ell)d\ell \overset{\text{N}}{=} \frac{1}{2} \sum_{n=1}^{N-1} (\ell_{n+1} - \ell_n)\{f^*(\ell_n)I_s(\ell_n) + f^*(\ell_{n+1})I_s(\ell_{n+1})\}
\]

and

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\[
\int_0^L g^*(\ell) I_s(\ell) d\ell = \frac{1}{2} \sum_{n=1}^{N-1} (\ell_{n+1} - \ell_n) \{ g^*(\ell_n) I_s(\ell_n) + g^*(\ell_{n+1}) I_s(\ell_{n+1}) \},
\]

where \(N\) denotes the total number of data points between \(\ell_1 = 0\) and \(\ell_N = L\).

After one has determined all the integrals in eq. (2.33), one has to apply a matrix inversion in order to solve this matrix for \(A_0(\alpha)\) and \(B_0(\beta)\). After the determination of \(A_0\) and \(B_0\) for a particular \(\alpha\) and \(\beta\), \(\alpha\) and \(\beta\) can be altered to improve the fit the model. This procedure was carried out by a non-linear optimization routine within Matlab, called "fminsearch" (Nelder-Mead method), which looks for a better match of the model and data by optimization of the values \(\alpha\) and \(\beta\).

This optimization process has been applied to the reference model defined in sec. 2.1.2 and sec. 2.1.3. Initial values for \(\beta\) and \(\alpha\) were chosen for our optimizing process. \(\beta\) was initially assumed to be \(\omega / c_0 \approx 48 \text{ rad m}^{-1}\), where \(\omega = 2\pi \cdot 2.3 \times 10^9 \text{ rad sec}^{-1}\) and \(c_0 = 3 \times 10^8 \text{ m s}^{-1}\), the speed of light. In other words, we define the initial speed of the waves to be equal to the speed of light (no dielectric substrate). From fig. 2.6 it can be observed that the amplitude of the total current attenuates approximately with \(0.5/\lambda \approx 5 \text{ Np m}^{-1}\). Nevertheless, we define that \(\alpha = 1 \text{ Np m}^{-1}\) in order to observe the fitting process during several iterations.

After several iterations, the final matched current distribution, which is illustrated in fig. 2.7, was obtained for a fit till \(\ell = L\) and for a weighting function \(w(\ell) = 1\). The total number of iterations was limited by a tolerance, which was defined as the maximal amount of change of the model parameters \(\alpha\), \(\beta\), \(I_0^p\) and \(I_0^s\) after each iteration. This tolerance was set to be \(1 \times 10^{-4}\), which means that the model parameters that correspond to the minimum were at least accurate up to two digits. A quality factor, which indicates the error of the fit, was defined by

\[
Q = 100 \cdot \int_{\ell=0}^L \left| \frac{I(\ell) - I_s(\ell)}{I(\ell)} \right| d\ell,
\]

and is 4.4% (0.4 dB) for the fit as illustrated in fig. 2.7. \(Q\) is a number that denotes the global error. However, locally the fit can be less good and, as a consequence, could have an effect on the resulting pattern. Therefore, it is also necessary to consider the pattern error as will be done in the next chapter.

As can be seen, the analytical traveling-wave model resembles the simulated results more and more after each iteration. At a certain point the best fit has been reached; the optimization procedure has converged. As a result, the following optimized values were found for the currents along the upper and lower curvatures:

\[
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\]
After performing the optimization process for a prescribed amount of times in order to fit the electrical current model to the numerical data.

**Model parameters upper curvature**
- $\alpha = 5.3$ [Np m$^{-1}$]
- $\beta = 45$ [rad m$^{-1}$]
- $\Gamma_L = 0.75e^{j2.9}$ [-]
- $I_0^+ = 86e^{-j0.32}$ [mA]

**Model parameters lower curvature**
- $\alpha = 5.4$ [Np m$^{-1}$]
- $\beta = 46$ [rad m$^{-1}$]
- $\Gamma_L = 0.75e^{j3.0}$ [-]
- $I_0^+ = 87e^{j2.8}$ [mA]

Instead of a global fit, a local fit could be performed in order to prove consistency of the model. The results of the model parameters for a local fit, performed between $1/4L \leq \ell \leq 3/4L$, are

**Model parameters upper curvature**
- $\alpha = 5.1$ [Np m$^{-1}$]
- $\beta = 45$ [rad m$^{-1}$]
- $\Gamma_L = 0.78e^{j2.9}$ [-]
- $I_0^+ = 91e^{-j0.37}$ [mA]

Since the model parameters remain rather similar, consistency of the model is proved. However, a local fit between $0 \leq \ell \leq 1/2L$ or $1/2L \leq \ell \leq L$ showed large deviations for the model parameters (see fig. 2.8), which implies that the traveling-wave model should either be broken up into several parts along the curved edge (model refinement) or the weighting function has to be chosen different from 1 (different fitting strategy). The model needs refinements in order to describe the global current more accurate, e.g. the model could consist of three parts: a propagating part ($\alpha = 0$) for the slot line, a radiating part ($\alpha \neq 0$) for the curvature and a propagating part ($\alpha = 0$) in the tip. Nevertheless, the global fit will be used in the next chapters to determine the model parameters.
Figure 2.8: After performing the optimization process for $0 \leq \ell \leq 1/2L$ and $1/2L \leq \ell \leq L$. Only the fit for the amplitude is shown, since the phase was fitted properly.

2.2 Conclusions

It has been demonstrated for a chosen reference Vivaldi antenna, that the surface current distribution close to the tapered edges can fairly well be approximated by traveling-wave line currents. A procedure to fit the traveling wave model to the extracted numerical surface currents has been proposed and has been implemented successfully. From a local fit it was observed that the model needs refinements in order to describe the global current more accurate, e.g. the model could consist of three parts: a propagating part ($\alpha = 0$) for the slot line, a radiating part ($\alpha \neq 0$) for the curvature and a propagating part ($\alpha = 0$) in the tip. Nevertheless, a global fit will be used for further investigations for the model proposed in eq. (2.25). This model only consists of two damped waves that propagate into opposite directions along an edge.

However, an issue remains still unanswered: will the shape of the radiation pattern, which is produced by two analytical line currents along the slot line tapering only (inner curved-edges), be similar to that of the radiation pattern obtained by the MoM (total surface-current distribution)? This question will be answered in the next chapter.
Chapter 3

Verification of the Traveling Wave Model by Comparing Power Patterns

As demonstrated in the previous chapter, the traveling-wave currents can be fitted fairly well to the numerically determined current distribution along the curved edges. Next, a comparison will be made between the reference radiation pattern (MoM solution) and the pattern which appears if only the currents along the curvatures are considered. The maximum of the patterns will be normalized to unity because the traveling-wave currents do not possess the proper amplitudes yet.

3.1 Numerical Computation of the E-Plane Power Pattern From Edge Currents

In order to compute the radiation pattern, the well known free-space far field expressions are used. In the frequency domain, the $H$-field is given as

$$H(r) = \frac{-j k_0 e^{-j k_0 r}}{4 \pi r} \hat{e}_r \times \iiint_V J_e(x') e^{j k_0 (\hat{e}_r \cdot r')} dV',$$  \hfill (3.1)

in which the wave number $k_0 = \beta_0 = w/\omega_0$ rad/m. $r$ is the distance in meters between the point of observation and the antenna (origin). $\hat{e}_r$ is a unit vector pointing in the direction of observation and $r'$ indicates the position of the electric current density $J_e$ inside the source volume $V$. The intrinsic impedance of free space is $Z_0 = \sqrt{\mu_0 / \varepsilon_0} \approx 120 \pi \Omega$.

At this moment, traveling-wave currents that flow only along the curvatures are considered. Therefore, we may restrict our integration area to these curved edges only. The current distribution, which is defined along the upper curvature and parameterized by $r_1(x)$, can be written as
CHAPTER 3. VERIFICATION OF THE TRAVELING WAVE MODEL BY
COMPARING POWER PATTERNS

\[ J_e(x) = \int_{x_1}^{x_2} |I_1(x)| e^{j\psi_1(x)} I_1(x) \delta(x - \ell_1(x)) |\ell'_1(x)| \, dx \]  
(3.2)

\[ = \int_{x_1}^{x_2} I_1(x) I_1(x) \delta(x - \ell_1(x)) \sqrt{1 + (C_1 R)^2 e^{2R_e x}} \, dx. \]  
(3.3)

in which \( I_1(x) \) is the upper curvature current that can be obtained from the analytical model as a function of the cartesian coordinate \( x \). \( \ell_1(x) \) is a tangential unit vector, indicating the direction of the current along the upper curvature and given by

\[ \ell_1(x) = \frac{1}{\sqrt{1 + (C_1 R)^2 e^{2R_e x}}} \left( C_1 R e^{R e x}, C_2 \right), \quad x_1 \leq x \leq x_2. \]  
(3.4)

Furthermore, if only the radiation pattern in the E-plane (xy-plane) as a function of \( r, \rho \) is considered, we may define

\[ \epsilon_r = \cos \varphi \epsilon_{x} + \sin \varphi \epsilon_{y}. \]  
(3.5)

The upper edge current \( I_1 \) from eq. (3.2), as well as the lower edge current \( I_2 \), can be substituted in eq. (3.1), so that the integral is written as

\[ V = \iiint_V J_e(x') e^{jk_0(r, x')} dV' \]
\[ = \int_{x_1}^{x_2} \left\{ I_1(x) \ell_1(x) \exp \left( jk_0 x \cos \varphi + (C_1 e^{R e x} + C_2) \sin \varphi \right) \right. \]
\[ + \left. I_2(x) \ell_2(x) \exp \left( jk_0 x \cos \varphi - (C_1 e^{R e x} + C_2) \sin \varphi \right) \right\} \sqrt{1 + (C_1 R)^2 e^{2R_e x}} \, dx, \]  
(3.6)

in which \( [k_0] = \text{rad mm}^{-1} \) and \( V \) is described in cartesian coordinates. Before evaluating this integral numerically for several constant \( \varphi \)-values, one should account for the transformation between \( \ell \) and \( x \), since \( I \) is defined as a function of \( \ell \). Therefore, the transformation (see also eq. (2.6))

\[ I_1(x = a) = I_1(\ell = \int_{x_1}^{a} \sqrt{1 + (C_1 R)^2 e^{2R_e \ell}} \, d\xi) \]  
(3.7)

has to be applied first, which can be evaluated analytically as seen in chap. 2. After computing \( V \) by means of the trapezoidal rule, eq. (3.1) can be evaluated further, i.e.

\[ H(r, \varphi) = \frac{-jk_0 e^{-jkr}}{4\pi r} (V_y \cos \varphi - V_x \sin \varphi) \epsilon_z \]  
[\text{A m}^{-1}]. \]  
(3.8)

For the electrical far-field \( E \), one can then simply write that

\[ E = Z_0 H_y \times \epsilon_r = Z_0 H_z (\cos \varphi \epsilon_y - \sin \varphi \epsilon_x) = Z_0 H_z \epsilon_{\varphi} \]  
[\text{V m}^{-1}]. \]  
(3.9)
3.2. POWER PATTERN OF MODEL COMPARED TO REFERENCE PATTERN

As can be seen, the edge currents give rise to a co-polar component in the E-plane only. The time-averaged Poynting vector is afterwards defined by

\[ S(r) = \frac{1}{2} \frac{1}{Z_0} |E(r)|^2 \varepsilon_r = \frac{1}{2} Z_0 |H_z|^2 \varepsilon_r \quad [\text{W m}^{-2}]. \tag{3.10} \]

Finally, the radiated E-plane power per steradian is

\[ P_r(\theta = 90^\circ, \varphi) = |\varepsilon|^2 S_r = \frac{Z_0 k_0^2}{32 \pi^2} (V_y \cos \varphi - V_x \sin \varphi)^2 \quad [\text{W Str}^{-1}]. \tag{3.11} \]

Before plotting the radiation diagram \( P_r(\varphi) \), a pattern normalization was introduced with respect to the maximum power, since the currents have not been normalized yet (chap. 4). Normalization to the maximum value is chosen because the MoM software is not able to export properly normalized patterns, this means that only the shape of the pattern can be compared. Later on (chap. 4) a method will be described to obtain scaled patterns from the MoM software. Results of the far-field computations were carried out by Matlab and are described in the next section.

3.2 Power Pattern of Model Compared to Reference Pattern

In this section power patterns are computed with the aid of the method presented in sec. 3.1. The magnitude of the radiation pattern, which is determined from the fitted model along the curved edges, is compared to the magnitude of the pattern that was determined by the original numerical data for the currents along the curved edges. Only the magnitude is compared because of limited possibilities of the MoM software. The errors between the fitted currents and the original edge currents can be observed in terms of patterns. Several of these patterns are shown in fig. 3.1 and fig. 3.2, under which a pattern determined by only the upper curved-edge currents, a pattern determined by only the lower curved-edge currents, a pattern determined by accounting for both edge currents ("2-edge pattern") and the total pattern produced by all the edge currents ("4-edge pattern"). The latter pattern has been compared with the total reference pattern, i.e. the pattern produced by the total surface current density.

As can be observed for the separate curved-edge patterns (fig. 3.1), main lobes are tilted due to the curvatures, which is expected according to theory on traveling-wave antennas [21]. The blue patterns match the red patterns rather well, which implies that a good fit is achieved to the numerical current distribution along the curved edges.

As can be observed for the 2-edge pattern (fig. 3.2, left), the backward radiation is minimal whereas it is nicely distributed in the forward direction due to the in-phase summation of the separate edge-patterns in the end-fire direction. Moreover, the patterns have still a similar shape, which implies that the far-field summation of both edge currents yield a pattern that is still good compared to the pattern produced by the numerical data along both edges. Also, the 2-edge pattern contributes less to the power in the far-field than the individual curved-edge contributions due to field cancellation.
CHAPTER 3. VERIFICATION OF THE TRAVELING WAVE MODEL BY COMPARING POWER PATTERNS

Figure 3.1: The power pattern produced by the fitted traveling-wave model, either along the upper or lower curved-edge, are plotted in red. The power pattern produced by the original numerical line data along either of the edges are plotted in blue. The patterns are all normalized to unity with respect to their maximum.

The reference radiation pattern for the total surface current distribution was computed by the MoM software and afterwards compared with the 2-edge pattern, which is shown on the left in fig. 3.2. Two edge currents were not sufficient to model the total reference pattern. However, by accounting for the currents along the straight edges as well (4-edge pattern), a rather high agreement could be obtained as shown in fig. 3.2 (right).

For the model parameters along the straight edges of the reference Vivaldi-element, the following values were found after fitting.

<table>
<thead>
<tr>
<th>Model parameters upper straight-edge</th>
<th>Model parameters lower straight-edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 1.8$ [Np m$^{-1}$]</td>
<td>$\alpha = 1.8$ [Np m$^{-1}$]</td>
</tr>
<tr>
<td>$\beta = 50$ [rad m$^{-1}$]</td>
<td>$\beta = 50$ [rad m$^{-1}$]</td>
</tr>
<tr>
<td>$\Gamma_L = 0.93e^{j2.9}$ [-]</td>
<td>$\Gamma_L = 0.93e^{j2.9}$ [-]</td>
</tr>
<tr>
<td>$I_0^+ = 16e^{j0.52}$ [mA]</td>
<td>$I_0^+ = 16e^{-j2.6}$ [mA].</td>
</tr>
</tbody>
</table>

As can be seen, the ratio between $|I_0^+|$ for the curved edge-current (previous chapter), and $|I_0^+|$ for the straight edge is approximately 86 mA/16 mA = 5.4. Although the currents along the straight edges are smaller, the contribution of power in the far-field is mainly dominated by the straight edges. This conclusion can be drawn when both patterns in fig. 3.2 are compared, since the radiated power in end-fire direction is only produced by the currents along the curved edges. Therefore, the currents along the straight edges are responsible for the main lobes in the total pattern. This effect is explicitly illustrated in fig. 3.3, where the contribution of the straight-edge currents and curved-edge currents can be observed independently. For this plot, the discrete current data were used and the ratio between the
3.2. POWER PATTERN OF MODEL COMPARED TO REFERENCE PATTERN

straight-edge current and curved-edge current was adjusted manually in order to achieve the best fit as is shown on the right-hand side of fig 3.3.

Figure 3.2: The power pattern produced by the fitted traveling-wave model, both along the upper or lower curved-edge, are plotted in red (left fig.). The power pattern produced by the original numerical data along either the edges are plotted in blue. The reference power pattern, obtained from the entire surface current density, is shown in blue (red). The pattern obtained by modeling all edge currents (4-edge pattern) is shown on the right.

Figure 3.3: Contribution of the currents along the straight edges and curved edges to the total pattern (right). The discrete line-current data was used and the ratio between the straight-edge current and the curved-edge current was adjusted manually in order to obtain a similar pattern as the reference pattern (blue) as is shown on the right.
One can easily verify that the straight-edge contribution can be reduced by changing the geometry of the element. As an experiment, a simulation was carried out for an element with slanted edges as is shown in fig 3.4. As a result, a radiation pattern is obtained which is more close to the pattern that is shown on the left-hand side of fig. 3.2. This implies that the power contribution, in case of straight edges, is higher than the power contribution in case of slanted edges.

![Figure 3.4: Minimization of the straight-edge far-field contribution by reducing the effect of the straight-edge currents.](image)

Another way to decrease the contribution of the straight-edge currents is to use a different excitation system, e.g. one could use a delta gap excitation in the slot as is used in app. E. The patterns obtained by using such an excitation system are uniformly distributed in forward direction. Nevertheless, the presented reference model will be used for further analysis.

It was also observed that, even for different frequencies, the straight-edge currents dominate over the curved-edge currents in terms of radiated power (app. C).

The ratio between the amplitude of the straight and curved edges was determined and will be applied as a fixed factor for further analysis, i.e. when the normalized current for the curved edge becomes known, the normalized straight-edge current will be known due to this ratio factor as well. A more convenient way to compare patterns will be described in chap. 4, where the model normalization will be discussed.

### 3.3 Conclusions

For the presented Vivaldi reference setup, the currents along the curved and straight edges should all be accounted for in an analytical model to obtain a similar shaped radiation pattern as produced by the entire surface current distribution. Apparently, currents along the straight edges should be avoided for this particular setup, since they dominate and contribute to a dip in the radiation pattern in end-fire direction and even increase backward radiation. Also, the currents along the straight edges travel with moderate speed, therefore, the straight
edges affect mainly the radiation that is directed between the broad-side and end-fire direction. It was also shown that the radiated power in the end-fire direction can only be caused by currents along edges which are curved.

In chap. 6 an array will be analyzed in which the elements are connected at the straight edges. It will be shown that the dip in end-fire direction gets less severe, due to the phenomenon that induced currents travel into opposite directions along the edges of the neighbouring elements. Another difference is that an infinite array does not have straight edges at all, therefore, every edge in the array contributes to the radiated power in end-fire direction.
Chapter 4

Normalizing Traveling Wave Currents for Vivaldi Antennas

This chapter proceeds with the description of two different normalization procedures for obtaining properly scaled edge currents in terms of amplitude. A scaling is needed in order to compute radiation patterns in absolute values, and henceforth, to be able to perform quantitative comparisons with respect to the reference pattern. Two normalization procedures will be described and the frequency dependency of the traveling-wave parameters will be investigated.

4.1 Normalizing Traveling Wave Currents by Transforming Surface Currents into Line Currents

One of the possibilities to determine normalized edge currents, is to make use of a simplified representation of the surface current distribution $J_s(r)$. First, separate simplified models are proposed for, respectively, the direction, phase and amplitude of $J_s(r)$. Afterwards, the normalization process will be described.

4.1.1 Determination of the Direction of the Surface Currents

Since only line currents are considered in our simplified approach, it is proposed to define the direction of the surface currents, denoted by $\mathcal{I}(x, y)$ along vertically curved lines as shown in fig. 4.1. As an example, the vertically curved line crossing an arbitrarily chosen pair $(x_0, y_0)$ is going to be determined in order to compute $\mathcal{I}(x_0, y_0)$. To achieve this, we only need to shift $y_1$ to $y_u$ to compute new values for $C_1$ and $C_2$, since $R$, $x_1$, $x_2$ and $y_2$ are kept constant. Unfortunately, $R$ cannot be made dependent on $y_u$ for constant $C_1$ and $C_2$, since $C_1$ and $C_2$ have to be altered as well in our definition in order to let the curved line cross $(x = 0, y = y_u)$.

So, $(x_0, y_0)$, $(x_1, y_u)$ and $(x_2, y_2)$ have to be coordinates on the curved line, which has earlier been defined by
4.1. NORMALIZING TRAVELING WAVE CURRENTS BY TRANSFORMING SURFACE CURRENTS INTO LINE CURRENTS

Figure 4.1: First step towards normalized edge currents; determination of $\tau(x, y)$.

\[ y = C_1 e^{R_x} + C_2. \]  
(4.1)

Substituting $(x_0, y_0)$, $(x_1, y_1)$ and $(x_2, y_2)$, yields

\[ y_0 = C_1(y_u)e^{R_{x_0}} + C_2(y_u), \]  
(4.2)

in which

\[ C_1(y_u) = \frac{y_2 - y_u}{e^{R_{x_2}} - e^{R_{x_1}}}, \]  
(4.3)

and

\[ C_2(y_u) = \frac{y_u e^{R_{x_2}} - y_2 e^{R_{x_1}}}{e^{R_{x_2}} - e^{R_{x_1}}}. \]  
(4.4)

After substituting eq. (4.4) and eq. (4.3) in eq. (4.2), and after some algebra, An expression for $y_u$ can be derived. This yields

\[ y_u = y_2 \left( \frac{e^{R_{x_0}} - e^{R_{x_1}}}{e^{R_{x_0}} - e^{R_{x_2}}} \right) - y_0 \left( \frac{e^{R_{x_2}} - e^{R_{x_1}}}{e^{R_{x_0}} - e^{R_{x_2}}} \right). \]  
(4.5)

The equation for the vertically curved dashed line, crossing $(x_0, y_0)$, is thus finally expressed by

\[ y = C_1(y_u)e^{R_x} + C_2(y_u). \]  
(4.6)

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Finally, the unit vector \( \tau(x_0, y_0) \), indicating the direction of the current at \((x_0, y_0)\), is defined by (see also chap. 1 for a parameterization of the curvature)

\[
\tau(x_0, y_0) = \frac{\partial_x(xe_x + [C_1(y_u) e^{Rx} + C_2(y_u)] e_y)}{\partial_x(xe_x + [C_1(y_u) e^{Rx} + C_2(y_u)] e_y)} \bigg|_{x=x_0, y=y_0} \tag{4.7}
\]

\[
= \frac{1}{\sqrt{1 + (C_1(y_u) R)^2 e^{2Rx_0}}} \sqrt{1 + (C_1(y_u) R)^2 e^{2Rx_0}} e_x + \frac{C_1(y_u) R e^{Rx_0}}{\sqrt{1 + (C_1(y_u) R)^2 e^{2Rx_0}}} e_y. \tag{4.8}
\]

In order to verify whether the main direction of the elliptical polarized surface currents is truly directed along these lines, electrical current data from the MoM package has been extracted and analyzed. The polarization direction \( \tau(x_n, y_n) \) of a discrete surface current \( J(x_n, y_n) \) is dominated by the direction of the major axis of the polarization ellipse, which is computed by

\[
\tau(x_n, y_n) = \text{Re}\{J_x(x_n, y_n)e^{-j\omega t_{max}}\} e_x + \text{Re}\{J_y(x_n, y_n)e^{j\phi} e^{-j\omega t_{max}}\} e_y, \tag{4.9}
\]

with index \( 1 \leq n \leq N \), where \( N \) is the total number of discrete current values and

\[
t_{\text{max}} = \frac{1}{2\omega} \arctan \left( \frac{|J_x|^2 \sin(2\varphi_x) + |J_y|^2 \sin(2\varphi_y)}{|J_x|^2 \cos(2\varphi_x) + |J_y|^2 \cos(2\varphi_y)} \right) + k\pi \tag{4.10}
\]

\( k = 0, 1, 2 \ldots \)

\( \tau \) is chosen such that \( \tau \) is pointing along the major axis of the corresponding ellipse. Alternatively, \( \tau \) can also be determined by making use of eq. (2.18), i.e.

\[
\tau(x_n, y_n) = \frac{|J_x(x_n, y_n)|}{\sqrt{|J_x(x_n, y_n)|^2 + |J_y(x_n, y_n)|^2}} e_x \pm \frac{|J_y(x_n, y_n)|}{\sqrt{|J_x(x_n, y_n)|^2 + |J_y(x_n, y_n)|^2}} e_y. \tag{4.11}
\]

\( \tau \) has been plotted onto the curved lines and is shown in fig. 4.2 for one Vivaldi sheet only.

![Figure 4.2: Main directions of elliptical polarized currents (MoM) in comparison with theoretically proposed polarization lines.](image)

The surface currents are, on average and for this particular case, reasonably well directed along the theoretically proposed curved lines. However, near the edges deviations are observed, since the major axes of the ellipses are not directed along the edge directions. One of the reasons for these deviations could be attributed to the way the integral equation is
4.1. NORMALIZING TRAVELING WAVE CURRENTS BY TRANSFORMING SURFACE CURRENTS INTO LINE CURRENTS

solved by the MoM software close to the edges. For instance, when Dirac functions are used as test functions (point matching), solutions for the currents near the edges are not as accurate as for other well known test functions. Nevertheless, it would not affect the accuracy of the normalization much due to the averaging effect of this error. For further analysis it is assumed that the currents are directed along these theoretically proposed curved lines only.

4.1.2 Constant Phase Lines of the Surface Currents

A simplification of the direction of the surface currents has been made. Next, a simplified representation of the phase distribution, denoted by $\psi(x, y)$, of the surface current distribution will be considered. According to eq. (2.18), we simply define that for linear polarized currents on the Vivaldi sheet

$$\psi(x, y) \approx \text{arg}\{J_x(x, y)\} \approx \text{arg}\{J_y(x, y)\} \pm \frac{n\pi}{m} = \arctan\left(\frac{\text{Im}\{J_x(x, y)\}}{\text{Re}\{J_x(x, y)\}}\right).$$ \hspace{1cm} (4.12)

The surface currents are only available at fixed discrete mesh points, therefore, an interpolation routine was implemented to compute the phase for arbitrary $(x, y)$ on the Vivaldi sheet [app. A]. The phase distribution along the previously determined curvatures has been computed by using the numerical data and is shown in fig. 4.3.

![Figure 4.3: Phase distribution of the surface currents, which has numerically been determined along longitudinal curvatures.](image)

Subsequently, lines of equal phase could be determined by using an algorithm, which searches for points of constant phase on the Vivaldi element. However, determination of the phase near the edge is, because of the absence of data and inaccuracy, impossible. Therefore, extrapolation was needed using polynomial functions, which were fitted through the equi-phase points as shown in fig. 4.4.

Unfortunately, it was impossible to determine equi-phase lines in the tip of the Vivaldi element. As can be observed, the equi-phase lines are nonlinearly shaped curvatures due to different phase velocities along the longitudinal curvatures and different corresponding reflection coefficients at the upper corner. Also, phase centers are visible on the straight
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edge. The last phase center was detected in the tip of the element, therefore, the upper part of the equi-phase lines close to the tip should reach this tip. Because of a lack of phase information in the tip, polynomial functions could not be fitted properly in these areas. Moreover, when edges come together, the amplitudes of the currents get very small and the MoM software has problems to mesh the tip and to solve the fields accurately. All effects together cause a lot of problems in the tip, which should be analyzed more deeply in the future. One possibility to circumvent this problem would be to use an analytical model for the tip of the element.

4.1.3 Simplification of the Amplitude Distribution of the Surface Currents

The amplitude of the surface current density will be defined by taking the amplitude of the component of the surface current density along the previously (sec. 4.1.1) proposed curved lines, i.e. \(|J_r(x, y)|\). Next, \(J_r\) will be described as a function of \(\ell_1\) and \(\ell_{c_1}\) as also illustrated in fig. 4.5. \(\ell_{c_1}\) is the distance along an equi-phase line, starting from the edge at a certain \(\ell_1\). So, \(|J_r(\ell_1, \ell_{c_1})| = |\langle J_x(\ell_1, \ell_{c_1}), \overline{z}(\ell_1, \ell_{c_1}) \rangle|\).

As can be noticed, the amplitude of the current increases rapidly for small values of \(\ell_{c_1}\) (near the edge). As illustrated in fig. 4.5, a square-root dependence of the amplitude is observed close to the edge, which is according to our expectations. However, no amplitudes could be determined for very small values of \(\ell_{c_1}\), because the mesh size is finite. Therefore, a square-root extrapolation was used near the edges to predict the behavior of the amplitude in these areas. The accuracy of this method is not known and hard to verify as an intermediate step. Errors made in this assumption will become clear in the final normalized pattern.

4.1.4 Normalization of the Edge Currents for a Single Vivaldi Element

The surface current distribution has now been simplified in terms of line currents on the element. From this simplification it is possible to obtain scaled line currents along the edges. As an example, the normalized edge current \(I_1(\ell_1)\) along the upper curved edge is going

Figure 4.4: Equi-phase lines for the phase distribution of the surface currents, which are crossing the longitudinal curvatures.
4.1. NORMALIZING TRAVELING WAVE CURRENTS BY TRANSFORMING SURFACE CURRENTS INTO LINE CURRENTS

Figure 4.5: The amplitude distribution \(|J_r(\ell_1 = \text{Constant}, \ell_c_1)|\) of the surface-current density along a certain equi-phase line as a function of an equi-phase curvature length.

to be determined. Consider for this purpose fig. 4.6, where \(a(\ell_1)\) is the distance from the edge along an equi-phase line for which a square-root extrapolation is needed. This distance is dependent on different positions \(\ell_1\) along the curved edge. The distance along an equi-phase line, for a certain \(\ell_1\), is denoted by \(\ell_c_1\). \(b(\ell_1)\) is the distance from the curvature and has been positioned in the center of one Vivaldi sheet to divide the Vivaldi arm into two segments.

Now, the normalized amplitude of the upper curvature current \(|I_1(\ell_1)|\) is calculated by

\[
|I_1(\ell_1)| = \int_0^{b(\ell_1)} |J_r(\ell_1, \ell_c_1)| \, d\ell_c_1 = \int_0^{b(\ell_1)} |J_r(\ell_1, \ell_c_1)| \, d\ell_c_1
\]

in which the integral on the left-hand side of eq. (4.13) can be calculated analytically, since \(|J_r|\) is assumed to be a square-root function for \(0 \leq \ell_c_1 \leq a(\ell_1)\) with a singularity at \(\ell_c_1 = 0\), i.e.,

\[
|J_r(\ell_1, \ell_c_1)| = \frac{\sqrt{a(\ell_1)}}{\ell_c_1}, \quad 0 \leq \ell_c_1 \leq a(\ell_1).
\]

Substituting eq. (4.14) in eq. (4.13) yields

\[
|I_1(\ell_1)| = \int_0^{a(\ell_1)} \frac{\sqrt{a(\ell_1)}}{\ell_c_1} |J_r(\ell_1, \ell_c_1 = a)| \, d\ell_c_1 + \int_{a(\ell_1)}^{b(\ell_1)} |J_r(\ell_1, \ell_c_1)| \, d\ell_c_1
\]

\[
= 2a(\ell_1)|J_r(\ell_1, \ell_c_1 = a)| + \int_{a(\ell_1)}^{b(\ell_1)} |J_r(\ell_1, \ell_c_1)| \, d\ell_c_1,
\]

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Figure 4.6: Normalization process in order to obtain proper scaled edge currents.

in which the right-hand side of eq. (4.15) has to be computed numerically.

The normalized phase distribution of the upper curvature current, i.e. \( \arg\{I_1(\ell_1)\} \), is simply taken to be equal to the corresponding phases that were determined for the several equi-phase lines along \( \ell_1 \). As a result, the normalized complex current \( I_1 \) can be determined for all \( \ell_1 \).

Finally, the traveling-wave model can be fitted to the normalized edge current \( I_1 \). A fit for the normalized normalized upper-curvature current is shown in fig. 4.7.

![Normalized model parameters upper curvature](image)

<table>
<thead>
<tr>
<th>Normalized model parameters upper curvature</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha ) = 4.9 \hspace{1cm} [\text{Np m}^{-1}]</td>
</tr>
<tr>
<td>( \beta ) = 48 \hspace{1cm} [\text{rad m}^{-1}]</td>
</tr>
<tr>
<td>( \Gamma_L ) = 0.89e^{-j2.5} \hspace{1cm} [-]</td>
</tr>
<tr>
<td>( I_0^+ ) = 17e^{-j0.16} \hspace{1cm} [\text{mA}]</td>
</tr>
</tbody>
</table>

Although \( |I_0^+| \) now constitutes the properly scaled amplitude of the upper edge current, the remaining normalized model parameters keep approximately the the same values as determined before, since the shape of the normalized edge current has not been changed significantly compared to the non-normalized current. This has graphically been illustrated in fig. 4.8, where the non-normalized current model as well as the normalized current model.
have been drawn in order to make comparisons. For this purpose, a proper normalization for the non-normalized current was chosen.

Deviations between both currents occur in the tip of the element, because the normalized current could not be determined well in this area. As a consequence, the position of the maximum of the amplitude in the tip is shifted with respect to the non-normalized current, causing $\beta$ to be slightly different ($\approx 6\%$).

Nevertheless, the shape of the non-normalized current will be used because of the accuracy of this distribution in the tip and the ease of the procedure to determine this current. Also, the properly scaled $|I_0^+|$ can be found easily by only considering a single equi-phase line for integration. Subsequently, the normalized straight edge current is now known as well, because the ratio between $|I_0^+|$ for the curved edge and $|I_0^+|$ for the straight edge is known.
Normalized parameters upper curved-edge & Normalized parameters upper straight-edge \\
$\alpha = 5.3$ [Np m$^{-1}$] & $\alpha = 1.8$ [Np m$^{-1}$] \\
$\beta = 45$ [rad m$^{-1}$] & $\beta = 50$ [rad m$^{-1}$] \\
$\Gamma_L = 0.75e^{j2.9}$ & $\Gamma_L = 0.93e^{j2.9}$ \\
$I_0^+ = 13e^{-j0.32}$ [mA] & $I_0^+ = 4.0e^{j0.52}$ [mA]

The E-plane power pattern, produced by the normalized edge currents, is illustrated in fig. 4.9. The average pattern error, denoted by $\Upsilon$, was chosen to be

$$\Upsilon = \frac{1}{2\pi} \int_0^{2\pi} |P_m(\phi) - P_r(\phi)| d\phi,$$

(4.16)

in which $P_m$ is the E-plane power pattern in dB of the model and $P_r$ is the E-plane power pattern of the reference in dB. The error is thus defined as the average difference between the plots that are illustrated in fig. 4.9.

![Comparison between the reference power-pattern (MoM software) and the power pattern produced by the traveling-wave edge-currents. The average error between both patterns is 2.1 dB](image)

Figure 4.9: Comparison between the reference power-pattern (MoM software) and the power pattern produced by the traveling-wave edge-currents. The average error between both patterns is 2.1 dB

As can be observed, the average pattern error between the reference pattern and the model is, on average, 2.1 dB. We will define that an amplitude error of 3 dB is satisfactory, which is a requirement for focal plane array applications. However, interpretation of the results should be carried out with care, since a relative pattern error as well as an absolute pattern error would yield different results both in power and dB (4 different possibilities). For example, the difference in absolute power between the patterns can be very small in the nulls, whereas the relative difference can be still significant. Also, if the patterns are computed in dB, while afterwards the average absolute difference between the patterns is computed, a form of a relative error will be obtained. Therefore, the error in the nulls can be large (as in fig. 4.9).
4.2 Normalizing Traveling-Wave Currents by Fitting Patterns

4.2.1 Conceptual Description

An alternative manner to normalize edge currents is to fit the total E-plane radiation-pattern, which is a superposition of several patterns obtained from several non-normalized edge currents, to the normalized E-plane reference pattern obtained from the total surface current density (MoM). The procedure to accomplish this normalization will be described next in more detail.

First, we consider the E-plane field-pattern that is produced by a line current $I(x)$ along a single arbitrarily curved edge as shown in fig. 4.10.

Figure 4.10: Determination of the complex E-plane field-pattern $E_z(r, \theta)$, produced by a line current $I(x)$ along an arbitrarily curved edge.

In a similar way as described in chap. 3, we can define for this particular setup

$$E(r, \theta, \varphi = 0) = Z_0 H_y(r, \theta) e^\theta, \quad (4.17)$$

in which

$$H_y(r, \theta) = \frac{-j k_0 e^{-j k r}}{4 \pi r} (V_z \cos \theta + V_z \sin \theta) \quad (4.18)$$

and

$$V = \int \int_V I_z'(r') e^{j k_0(s_x, s_y, r')} dV' = \int_{x_1}^{x_2} I(x) I_z(x) e^{j k_0(s_x, s_y)} \sqrt{1 + \frac{(C_1 R)^2 e^{2 R x}}{1 + (C_1 R)^2 e^{2 R x}}} \, dx. \quad (4.19)$$

In case of a curved edge, which can either be curved up or down, we use

$$\left\{ \begin{array}{c} I(x) = x e_x + \left[ z_{offset} \pm (C_1 e^{R x} + C_2) \right] e_x \\ I'(x) = \frac{1}{\sqrt{1 + (C_1 R)^2 e^{2 R x}}} e_x \pm \frac{C_1 R e^{R x}}{\sqrt{1 + (C_1 R)^2 e^{2 R x}}} e_y \end{array} \right. ,$$

while, in case of a straight edge, we use

$$\left\{ \begin{array}{c} I(x) = x e_x + z_{offset} e_x \\ I'(x) = e_x \end{array} \right. .$$
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$z_{\text{offset}} \in \mathbb{R}$, $x$ is in mm and $k_0$ in rad mm$^{-1}$.

Next, we consider the case of the complex E-plane field-pattern that is produced by $N$ normalized edge currents along $N$ arbitrarily curved edges. Normalization implies that edge current $n$ is given by $C_n I_n(x)$, in which $C_n$ is the normalization constant for the edge current under consideration and is initially taken to be real-valued. A real quantity is taken because only the amplitude will be scaled, the phase is taken equal to the original phase values that were determined close to the edges. Hence, the total pattern $E$ is a superposition of individual edge patterns, i.e.

$$E(r, \theta) = \sum_{n=1}^{N} C_n E_n(r, \theta) \quad \text{[V m$^{-1}$]}, \quad (4.20)$$

Finally, the total power pattern for $N$ normalized edge currents is given by

$$P_r(\theta) = |r^2 S_r| = \frac{r^2}{2Z_0} |E(r, \theta)|^2 = \frac{r^2}{2Z_0} \left| \sum_{n=1}^{N} C_n E_n(r, \theta) \right|^2 \quad \text{[W Sr$^{-1}$]} \quad (4.21)$$

$C_n$ can be chosen such that $P_r$ resembles the normalized reference pattern $P_{\text{ref}}$ best. This process can be done automatically by fitting $P_r$ to $P_{\text{ref}}$. However, in order to obtain $P_{\text{ref}}$, the pattern obtained from the MoM package, called $P_{\text{zel}}$, had to be normalized to W Sr$^{-1}$. This procedure was needed because the MoM software is only able to export E-plane field patterns that are normalized to unity at their maximum. Therefore, the following transformation was used

$$P_{\text{ref}}(\theta) = (4\pi)^{-1} 10^{\frac{D}{10}} P_{\text{rad}} P_{\text{zel}}(\theta), \quad (4.22)$$

in which $D$ is the directivity in dB, $P_{\text{rad}}$ is the total radiated power in W and $P_{\text{zel}}$ is the linear field-pattern that is obtained from software. Finally, $P_r(\theta)$ was fitted to the normalized reference pattern $P_{\text{ref}}(\theta)$. The fit can be achieved by minimizing $\Psi$, which constitutes the error between $P_r$ and $P_{\text{ref}}$. For this particular case we choose,

$$\Psi(C_1, \ldots, C_N) = \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} |P_r(\theta) - P_{\text{ref}}(\theta)|^2 d\theta$$

$$= \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \left| \frac{r^2}{2Z_0} \sum_{n=1}^{N} C_n E_n(r, \theta) - P_{\text{ref}}(\theta) \right|^2 d\theta. \quad (4.23)$$

Eq. (4.23) can be written as a linear set of equations by using a method similar to the one described in chap. 2. Hence, a minimum for $\Psi$ was determined by using a linear optimization algorithm (matrix inversion), to find optimal normalization constants $C_1, \ldots, C_N$. 

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4.2. NORMALIZING TRAVELING-WAVE CURRENTS BY FITTING PATTERNS

4.2.2 Normalization Results

The pattern normalization procedure has been applied for the earlier defined reference Vivaldi element and the results will be presented next.

First, the constants $C_1$ and $C_2$ were determined, for the numerical current distribution along the curved and straight edge of one Vivaldi sheet, by using the pattern normalization procedure. $C_3$ and $C_4$, the normalization coefficients for the lower sheet-currents, will be equal to $C_1$ and $C_2$ respectively, because of symmetry. The result for the pattern after normalizing the numerical edge-currents is shown in fig. 4.11.

![Figure 4.11](image)

Figure 4.11: The pattern (black) fitted to the reference pattern (blue) in order to determine the normalization coefficients for the currents. The average pattern error is 2.9 dB for $0^\circ \leq \varphi \leq 360^\circ$.

From this procedure, traveling-wave models can be fit to the normalized currents. Hence, the pattern of the traveling-wave model can be calculated and be compared to the reference pattern. Fig. 4.12 illustrates the result for the pattern of the traveling-wave model in comparison with the reference pattern.

![Figure 4.12](image)

Figure 4.12: The pattern of the traveling-wave model (green) compared to the reference pattern (blue). The average pattern error is 2.4 dB for $0^\circ \leq \varphi \leq 360^\circ$. 
With an average error of 2.9 dB, the E-plane pattern of the normalized currents resembles the reference pattern rather well. Moreover, after fitting the traveling-wave model onto the normalized currents, the pattern error becomes 2.4 dB. However, the magnitude of the main lobes are less well modeled for the fitted traveling-wave model. In contrast to the magnitude, the positions of the lobes are well modeled, which implies that it is justified to choose $C_n$ real-valued. The pattern error is comparable to the pattern error obtained for the former normalization procedure that has been described (c.f. fig. 4.9). Both normalization procedures thus yield similar results. Nevertheless, normalization obtained by surface current integration (first procedure presented) is a more accepted method because all modeling is done with currents. However, pattern fitting is a more simple way to obtain normalized currents.

4.3 Frequency Dependence of the Model Parameters

By applying either of the two presented normalization procedures for the reference Vivaldi element, normalized edge currents can be determined. Next, a fit of the traveling-wave models can be performed to these edge currents for several Vivaldi geometries and/or frequencies. Hence, the range of validity of the traveling wave model can be investigated and determined by considering the pattern error. As a result, a model is obtained with which it is possible to synthesize Vivaldi geometries for a certain design easily.

As an example, the frequency dependence of the traveling-wave model will be investigated. As a constraint, the error between the E-plane power pattern of the model and the reference will be limited in order to define a range of validity. The traveling-wave model parameters as a function of frequency, for the normalized currents along one curved edge only (upper one), are shown in fig. 4.13.

As can be observed, the attenuation coefficient $\alpha$ remains approximately constant for frequencies above 2 GHz. This is expected since the attenuation of the current along the edge is assumed to be only dependent on the curvature rate $R$ and not on frequency. $\beta$ is linearly dependent on frequency, which implies that the phase velocity of the traveling wave is constant for every frequency and approximately equal to $c_0$, the speed of light. $\Gamma_L$ is -1 for very low frequencies, so that total reflection of the current occurs at the tip of the element, whereas less is reflected from the tip for higher frequencies.

The E-plane power patterns were computed for different frequencies and compared to the corresponding reference power patterns (app. C). The error between both patterns is shown in fig. 4.13 as well as the total radiated power.

The element starts to radiate a constant power of 10 mW for frequencies above 2 GHz. For low frequencies the element is not radiating yet and the traveling-wave model parameters, e.g. $\alpha$, yield non-physical values (negative for $f < 0.5$ GHz), whereas for higher frequencies the pattern error between model and reference starts to increase and might cross a certain
4.3. FREQUENCY DEPENDENCE OF THE MODEL PARAMETERS

Figure 4.13: Determined model parameters $\alpha(f)$, $\beta(f)$ and $\Gamma_L(f)$ as a function of frequency.

Figure 4.14: Error between the power patterns of the traveling-wave models and the corresponding reference patterns as a function of frequency. The total radiated power as a function of frequency is also shown.
tolerance. Both phenomena limit the range of validity of the model in frequency domain.

Another interesting physical parameter is the passive antenna input impedance. The passive input impedance has been calculated with the aid of the MoM and is shown in fig. 4.15.

Figure 4.15: Left: Simulated input impedance as a function of frequency. Right: $|S_{11}|$ as a function of frequency for a 100 $\Omega$ terminating resistance.

Anomalies (resonances) are observed for $Z_{11}$ at fixed frequency intervals. The real part of the input impedance oscillates towards 100 $\Omega$ and the imaginary part oscillates close to 0 $\Omega$ for increasing frequency. Therefore, $|S_{11}|$ remains below -7 dB for frequencies higher than 2 GHz for a terminating resistance of $100 \Omega$. In other words, the element starts to radiate for frequencies above 2 GHz. A similar tendency of the input impedance is observed in [3]. Moreover, it is known that the lower cut-off frequency for Vivaldi elements is determined by the distance between the Vivaldi tips, which is said to be 0.5 $\lambda$ [4]. For our reference model the tip spacing is 0.6 $\lambda$ at 2 GHz, which is in good agreement.

By using the traveling-wave model, one is able to predict the positions of the anomalies. Anomalies occur at frequencies for which the amplitude of the current at the input ($\ell = 0$) is minimal, say zero. For completeness, we repeat the traveling wave model, which is defined by

$$I(\ell) = I^+(\ell) + I^-(\ell) = I_0^+ e^{-\alpha \ell} e^{-j\beta \ell} + I_0^+ \Gamma L e^{-\alpha (2L-\ell)} e^{-j\beta (2L-\ell)}. \quad (4.24)$$

The amplitude of the current at $\ell = 0$ is

$$|I(\ell = 0)| = |I_0| = \left| I_0^+ \right| \left| 1 + \Gamma L e^{-2\alpha L} e^{-2j\beta L} \right|. \quad (4.25)$$

Since $|I_0|$ has to be zero, $|I_0|^2$ has to be zero as well, so

$$|I_0|^2 = I_0 I_0^* = |I_0|^2 + 2 \text{Re}\left\{ |I_0|^2 \Gamma L e^{-2\alpha L} e^{-2j\beta L} \right\} + |I_0|^2 |\Gamma L|^2 e^{-4\alpha L} = 0. \quad (4.26)$$
4.4. CONCLUSIONS

Only low frequencies will be considered, since the anomalies are most important for the lower frequencies (see fig. 4.15). Consequently, the reflection coefficient $\Gamma_L$ will be set to $-1$ (total tip-reflection), which was already clear from the parameter investigation. Also, $\beta = \omega/c_0$. Hence, eq. (4.26) reduces to

$$|I_0|^2 \approx |I_0^+|^2 \left[ 1 - 2e^{-2\alpha L} \cos \left( \frac{2\omega L}{c_0} \right) + e^{-4\alpha L} \right], \quad (4.27)$$

which is zero if $I_0^+ = 0$, which is a trivial solution, or $\left[ 1 - 2e^{-2\alpha L} \cos \left( \frac{2\omega L}{c_0} \right) + e^{-4\alpha L} \right] = 0$. From the parameter investigation it was observed that $\alpha$ is small for low frequencies, therefore, the following function will be solved

$$[1 - 2\cos \left( \frac{2\omega L}{c_0} \right) + 1] = 0$$
$$\omega_n = \frac{\pi c_0}{L} n, \quad n = 0, 1, 2, \ldots \quad (4.28)$$

Anomalies are rather well predicted at the following frequencies: 0 GHz, 0.75 GHz, 1.5 GHz, 2.25 GHz, 3 GHz, etc.

In conclusion, the traveling-wave model provides a physical description of the properties of an antenna that lead to a better understanding of the radiating mechanism of a tapered slot element even for different frequencies. Different geometries can be analyzed as well, but this is outside the scope of this thesis.

4.4 Conclusions

Two procedures to normalize traveling wave edge-currents for a Vivaldi element have been presented and successfully implemented. The first one is based on a normalization obtained by simplifying the surface current distribution and the second one on a normalization obtained by fitting patterns. As an validation of both procedures, a frequency dependence investigation of the model was carried out.

The following conclusions can be drawn:

- Normalization, obtained by transforming surface currents into scaled edge currents, showed a high degree of agreement between the absolute pattern values of the reference pattern and the pattern of the traveling wave model, i.e. 1.9 dB. However, the normalization process is rather complicated.

- The procedure to obtain normalized currents by fitting patterns is less complicated. However, pattern comparisons are made as a final verification. Therefore, it is preferred to extract all information from the surface-current distribution in order to build a normalized traveling-wave model.

- Both normalization procedures yield similar normalized edge currents. Therefore, after fitting the traveling-wave model, the pattern error between model and reference is for both cases typically in the order of 2 to 3 dB.
• The traveling wave-model constitutes physical properties that lead to a better understanding of the radiating mechanism of a tapered-slot element. It has been proven that the reference model is valid for a frequency range of 2-5 GHz. The model allows us to predict the positions of anomalies rather accurately.

It should be mentioned that the pattern of the traveling-wave model can also be fitted to the reference model by adjusting the traveling-wave model parameters. However, the results of this procedure have not been described in this thesis.
Chapter 5

Model Applicability

This chapter is meant to illustrate two important applicabilities of an edge-current model for a Vivaldi element. It will be shown that some physical aspects can be analyzed rather easily by an edge-current model to provide insight into the operating principle of the element.

5.1 Effective Radiating Area of a Vivaldi Element

The region of the Vivaldi element, which ties together the transmission line and the tapered-slot line, is a special transition area (fig. 5.1). This important transition determines the propagating and radiating phenomena in the antenna operation and, as a consequence, the effective radiating area of the element. In practice, it is rather difficult to define the radiating effective area of the Vivaldi element, due to the problem to specify this boundary exactly. Therefore, analytical models take on special significance for the prediction of this transition zone while satisfying certain criteria. For instance, as it was demonstrated for a discrete-stepped slot-line approach in [6], the boundary of the "radiation-propagation area" was determined by the comparison between characteristic impedance and terminal load impedance variations (they should be equal at the boundary).

The edge-current model proposed allows us to consider the problem from another point of view, assuming the radiating power as a measure for the radiating area of a Vivaldi element and the edge currents as sources of the field. Then, the analysis of the operating mechanism is reduced to the study of the contribution from the corresponding parts of the edges (lower and upper part) into the radiated field. It will be shown that the upper part of the Vivaldi element is the part that is responsible for radiation. This part determines the real part of the passive antenna input-impedance, since this corresponds to the radiating resistance. The position of the transition zone can be determined in a way as described next.

Suppose the E-plane power pattern is calculated by accounting for the currents along the edges up to a certain position \( x = X \), i.e., the amplitude of the edge currents for the right part \( (x > X) \) will be taken zero. Now, the E-plane power that is radiated only by the left part of the element, \( x \leq X \), can be regarded as a part of the maximum E-plane power, i.e.
for \( x = x_{\text{max}} \). The ratio will be denoted by \( P_{\%}(x) \) and is defined by (c.f. eq. (3.11) and eq. (3.6))

\[
P_{\%}(x = X) = \frac{\int_{\varphi=0}^{2\pi} \epsilon_z \cdot \left( \epsilon_r \times \iint_{x \leq X} J_e(t') e^{jko(\epsilon_r,t')} dA' \right) d\varphi}{\int_{\varphi=0}^{2\pi} \epsilon_z \cdot \left( \epsilon_r \times \iint_{x \leq x_{\text{max}}} J_e(t') e^{jko(\epsilon_r,t')} dA' \right) d\varphi} \times 100\%.
\]

in which \( J_e \) is a surface-current density that will be reduced to edge currents only. \( P_{\%}(x) \) as a function of \( x/x_{\text{max}} \) is shown in fig. 5.2 for the reference model. It needs to be mentioned that this function is also dependent on the geometry and frequency.

The multi-extremum behavior of \( P(x) \) shows that the radiated power is accumulated differently with a linear increasing height of the element. The majority of the power, is subscribed
5.2. PHASE CENTER DETERMINATION

Figure 5.3: *Relative accumulated radiated E-plane power as a function of the normalized element height $x/x_{max}$, for both the curved edges and straight edges.*

to the right part of the Vivaldi element, which can be considered as the radiating area of the antenna. From the figure it turns out that approximately 70% of the total radiated power is only due to the last 20% of the element.

The result is of great importance for the model development as well, since the radiating part of the element is most critical i.e., small phase and amplitude errors of the edge currents in this area of the element may result in erroneous radiation patterns.

In addition, the contribution of the radiated power for the curved and straight edges were investigated separately and shown in fig. 5.3 with respect to the total E-plane power. The result confirms conclusions drawn earlier about the importance of the straight edges for the Vivaldi element model. Not only the shape of the radiation pattern is affected due to an increment of the side radiation level, but also the contribution to the total power. As one can see, in the propagation area of the element, which is the area where the curved edges hardly contribute to the total radiated power, the straight edges produce a significant fraction of the power (about 20%, according to fig. 5.2).

5.2 Phase Center Determination

According to the IEEE definition is the phase center the location of a point associated with an antenna such that, if it is taken as the center of a sphere whose radius extends into the far-field, the phase of a given field component over the surface of the radiation sphere is essentially constant, at least over that portion of the surface where the radiation is significant. It is noted, that not every antenna has a unique phase center. In fact, an antenna can have multiple phase centers from which spherical waves originate.
The position of the global phase center for some antennas is obvious, for example at the feed point of a dipole, whereas it is not for broadband antennas. The phase center is of great importance for measurements, since an unknown position can lead to measurement errors and even calibration errors [11].

To explain the procedure that has been used for determining the position of the phase center in the E-plane, we start by looking at fig. 5.4.

![Diagram](image.png)

Figure 5.4: Procedure for determining the position of the phase center in the E-plane.

\[ \Delta \] represents the distance between two neighbouring points on a rectangular grid. Consider \( H_z^k(\varphi_n) \), in which \( H_z \) is the complex magnetic far-field component produced by currents along the Vivaldi edges and determined with respect to the \( k \)th origin \( O^k \). \( \varphi_n \) indicates a discrete azimuthal coordinate in the far field. Now, a value will be assigned to \( O^k \) that indicates the flatness of the phase in the far field with respect to this particular origin. A measure of flatness could be the standard deviation of the discrete set \( \{ \arg H_z^k(\varphi_1), \ldots, \arg H_z^k(\varphi_N) \} \) for a particular origin \( O^k \). The standard deviation is given by [15]

\[
\sigma^k = \sqrt{\frac{\sum_{n=1}^{N} (\arg H_z^k(\varphi_n) - \mu^k)^2}{N}} \tag{5.2}
\]

where \( N \) is the total number of azimuthal steps taken.

A 2D plot for \( \sigma^k \) has been made for the reference element operating at 2.3 GHz. The result is shown in fig. 5.5, where the straight edges and curved edges are considered separately.

Apparently, the phase center is located in the the radiating part of the element. This observation is expected since, in general, the phase center indicates the center of the radiating...
5.3 Conclusions

The applicability of a line-current model has been demonstrated for two important practical examples, i.e. the radiating effective area and the phase center definition.

The numerical results obtained with the model allowed us to explore peculiarities of the source, which for a dipole is in the center. By reducing the straight edge-currents, the phase center tends to move to the right. By increasing the curved-edge currents the opposite occurs. The more the phase center moves to the right, the more it radiates into end-fire direction, whereas it radiates more into broad-side direction for a low position of the phase center.

Figure 5.5: Interpolated value of $\sigma^k$ in the $xy$-plane. The position for which $\sigma^k$ is low (blue) corresponds to the phase center of the element. The phase center moves along the axis whenever either the currents along the curved edges or straight edges are omitted.
antenna operating mechanism, in particular:

- To show the presence of a propagating and radiating area; to specify the size of the radiating effective area and the location of the transition zone.
- To detect the contribution of the straight edges into the total power ($\approx 20\%$);
- To define the position of the phase center of the antenna, which shifts down into the propagating part of the element when the straight-edge currents are significant.

The effects observed during the analysis of these characteristics can be also used as a means of determining a tolerance constraint for the edge-current model.
Chapter 6

A Feasibility Study for a Linear Vivaldi Array Model

The applicability of the traveling-wave model will be analyzed for a dense array of Vivaldi elements by means of a feasibility study. A scattering model will be presented, which describes the transitions of the currents from the excited element to the neighbouring elements. If these transitions are known, a prediction of the edge currents can be made for long arrays and the power pattern for such arrays can be predicted.

6.1 Scattering Matrix Formalism

A physical way to describe the transitions of the traveling waves to neighbouring elements can be achieved by using the so-called scattering-matrix formalism [7]. Consider fig. 6.1.

Figure 6.1: Left- and right-propagating waves at $\ell = 0$, $\ell = L$ and $\ell = 2L$ and corresponding amplitudes.

The matrix $S(0; L)$ expresses the outgoing waves in terms of the incident waves in the following way

$$
\begin{pmatrix}
  b \\
  c
\end{pmatrix} = S(0; L) \begin{pmatrix}
  a \\
  d
\end{pmatrix} = \begin{pmatrix}
  S_{11}^A & S_{12}^A \\
  S_{21}^A & S_{22}^A
\end{pmatrix} \begin{pmatrix}
  a \\
  d
\end{pmatrix}
$$

(6.1)

if we call the interval $0 < \ell < L$ section A. Similarly, we have for section B.
\[
\begin{pmatrix}
  d \\
  e
\end{pmatrix} = S(L; 2L) \begin{pmatrix}
  c \\
  f
\end{pmatrix} = \begin{pmatrix}
  S_{11}^B & S_{12}^B \\
  S_{21}^B & S_{22}^B
\end{pmatrix} \begin{pmatrix}
  c \\
  f
\end{pmatrix}.
\]

(6.2)

For the entire section, we need to express \( b \) and \( e \) in terms of \( a \) and \( f \). To this end, we need to eliminate the constants \( c \) and \( d \). This is possible because the entire system is still a linear reacting system.

For the amplitude of the outgoing wave at \( \ell = 0 \), we find

\[
b = S_{11}^A a + S_{12}^A S_{12}^B a + S_{12}^B f,
\]

(6.3)

while the amplitude of the outgoing wave at \( \ell = 2L \) is given by

\[
e = S_{21}^B S_{11}^A a + S_{22}^A S_{12}^B f + S_{22}^B f.
\]

(6.4)

This means that we can now write

\[
S(0; 2L) = \begin{pmatrix}
  S_{11}^A + \frac{S_{12}^A S_{12}^B}{1 - S_{22}^A S_{22}^B} & \frac{S_{12}^A S_{12}^B}{1 - S_{22}^A S_{22}^B} \\
  S_{12}^B S_{22}^A S_{22}^B & S_{22}^B + \frac{S_{22}^A S_{22}^B}{1 - S_{22}^A S_{22}^B}
\end{pmatrix},
\]

(6.5)

which is known as the Redheffer star product \[14\]

\[
S(0; 2L) = S(L; 2L) \ast S(0; L),
\]

(6.6)

so that we have

\[
\begin{pmatrix}
  b \\
  e
\end{pmatrix} = S(0; 2L) \begin{pmatrix}
  a \\
  f
\end{pmatrix}.
\]

(6.7)

Now, several scattering matrices are going to be determined for a 2-element Vivaldi array. Later on, these matrices will be used to predict the transitions of the currents to neighbouring elements for very long arrays.

6.2 A 2-element Vivaldi Array, Used to Predict Edge Currents in Long Arrays

A dense 2-element Vivaldi array will be used as a basis configuration in order to predict the edge currents and, hence, the power pattern of an arbitrary long single-polarized array of Vivaldi elements. The basis array configuration that is going to be used is shown in fig. 6.2, which is an extension of the reference geometry proposed earlier.

Apparently, a strong mutual coupling effect between currents of both elements exists, since traveling wave currents in the element next to the excited element are induced, which in turn
6.2. A 2-ELEMENT VIVALDI ARRAY, USED TO PREDICT EDGE CURRENTS IN LONG ARRAYS

enlarges the effective antenna surface for current transport.

The coupling between antenna ports is expressed by the S-parameter matrix for the slot-line input ports 1 and 2. The matrix was determined by the MoM software for terminating slot-line impedances of 100 Ω and reads

\[
S = \begin{pmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{pmatrix} = \begin{pmatrix}
0.32e^{j \frac{\pi}{8}} & 0.16e^{j \frac{3\pi}{8}} \\
0.16e^{j \frac{3\pi}{8}} & 0.32e^{j \frac{3\pi}{8}}
\end{pmatrix}.
\]

As can be seen, the coupling between both antenna ports, which is given by $|S_{12}|$ and $|S_{21}|$, is low compared to $|S_{11}|$ and $|S_{22}|$. In other words, radiation will mainly be produced by the excited elements and less by the neighbouring elements. Therefore, it will be important to model the currents close to a point of excitation more accurate than its corresponding induced currents at a significant distance in the array.

The "pattern normalization procedure" (see chap. 4) has been applied for the 2x1 Vivaldi array, which resulted in normalized edge currents that produce a pattern as is shown in fig. 6.3.

Although the average pattern error is only 1.4 dB, the positions and depths of some nulls are not well predicted. However, a change of the overall phase of the current along the most upper curved-edge (fig. 6.2) by 23 degrees, yields a difference as illustrated in fig. 6.4. As a consequence, certain nulls can be modeled fairly well now, due to the field cancellation that occurs as a consequence of cancellations or contributions from individual currents.

The accuracy of the edge currents for a 2x1 array should already be rather high in order to match the reference pattern well, especially in the tip of the element, as was concluded...
Figure 6.3: Comparison between the reference pattern and the pattern produced by the normalized edge currents. The average error between both patterns is 1.4 dB.

Figure 6.4: Effect on a phase change for one edge current by 23 degrees on the E-plane radiation pattern. The average error between both patterns is 1.9 dB.

from fig. 5.2 as well. A small change of the overall phase of one of the edge currents by 10% yields already rather large differences for positions and depths of nulls in the resulting pattern. Edge currents in the tip of the elements are difficult to determine by using the methods described in this thesis. Therefore, a recommendation is made to analyze the tip of the element in more detail by means of analytical techniques, or in general, to model the currents for several intervals along the edge separately and to satisfy Kirchhoff at the boundaries of the intervals.

6.3 Scattering Matrices for a Very Long Linear Vivaldi Array

Despite the pattern deviations presented in the last section, it is assumed that the previously described 2x1 Vivaldi array can be used to predict the edge currents, and hence the power pattern, for a very long linear array of Vivaldi elements. Very long means long enough to neglect radiation from straight edges due to truncation effects. Consider the lower element being excited for the 2x1 array at port 2 as shown in fig. 6.5.
6.3. SCATTERING MATRICES FOR A VERY LONG LINEAR VIVALDI ARRAY

Figure 6.5: Normalized currents along $l_1$ and $l_2$ for excitation at port 2. Equi-phase lines on the Vivaldi sheets are shown as well (c.f. fig. 4.4).

If $\ell_k$ denotes the length along the $k$th edge, with $k = 1, 2, \ldots$, then $I^+_k$ denotes the complex amplitude of the current at $\ell_k = 0$. This amplitude is multiplied by a complex propagation factor, so that the amplitude of the current that reaches $\ell_k = L$ is given by $I^+_k$. $L$ is the total length of a curved edge. $I^-_k$ is the amplitude of the current at $\ell_k = L$ that starts to travel back. Finally, $I^-_0$ is the amplitude that has returned to $\ell_k = 0$.

Next, the edge current along $l_1$ is again approximated by means of two traveling waves, i.e. (c.f. eq. (2.25))

$$I_1(l_1) = I^+_1(l_1) + I^-_1(l_1) = I^+_0 e^{-\alpha_c l_1} e^{-j\beta_c l_1} + I^-_1 e^{-\alpha_c (L-l_1)} e^{-j\beta_c (L-l_1)}.$$  \hspace{1cm} (6.9)

$I^+_0$ and $I^-_1$ become known after $I_1$ has been fitted to the normalized currents. This process will be explained in more detail in sec. 6.5. Subsequently, $I^+_L$ and $I^-_0$ can be derived and are given by

$$I^+_L = I^+_0 e^{-\alpha_c L} e^{-j\beta_c L}$$  \hspace{1cm} (6.10)

and

$$I^-_0 = I^-_1 e^{-\alpha_c L} e^{-j\beta_c L}.$$  \hspace{1cm} (6.11)

$\alpha_c$ and $\beta_c$ will be chosen such that they can be applied for both the currents $I_1$ and $I_2$, since $\alpha_c$ and $\beta_c$ are assumed to be dependent on frequency and geometry of the elements only. Eq. (6.10) and eq. (6.11) can be written in terms of a scattering matrix called $S_{\text{prop}}$, i.e.
Similarly, $I^+_0$, $I^-_0$, $I^+_L$ and $I^-_L$ can be determined using a fitting procedure for fixed chosen $\alpha_c$ and $\beta_c$.

Next, the relationship between the amplitudes of the currents at the tip will be described by a scattering formalism in a way as shown below:

$$\begin{pmatrix} I^-_1 \\ I^-_2 \end{pmatrix} = S_{\text{tip}} \begin{pmatrix} I^+_1 \\ I^+_2 \end{pmatrix}.$$ \hfill (6.13)

Excitation at port 1, instead of port 2, yields similar currents along $I_1$ and $I_2$, except that $I_1$ and $I_2$ are interchanged. Because of this symmetry, we conclude that matrix $S_{\text{tip}}$ has to be symmetrical, i.e. $S_{\text{tip}} = (S_{\text{tip}})^T$, and the diagonal has to contain equal values. In that case we write

$$\begin{pmatrix} I^-_1 \\ I^-_2 \end{pmatrix} = \begin{pmatrix} \Gamma_1 & \tau_1 \\ \tau_1 & \Gamma_1 \end{pmatrix} \begin{pmatrix} I^+_1 \\ I^+_2 \end{pmatrix},$$ \hfill (6.14)

so that

$$\begin{pmatrix} \Gamma_1 \\ \tau_1 \end{pmatrix} = \begin{pmatrix} I^+_1 & I^+_2 \\ I^+_2 & I^+_1 \end{pmatrix}^{-1} \begin{pmatrix} I^-_1 \\ I^-_2 \end{pmatrix} = \frac{1}{(I^+_1)^2 - (I^+_2)^2} \begin{pmatrix} (I^-_1 I^+_2 - I^-_2 I^+_1) & (I^-_1 I^+_2 - I^-_2 I^+_1) \\ (I^-_2 I^+_1 - I^-_1 I^+_2) & (I^-_2 I^+_1 - I^-_1 I^+_2) \end{pmatrix} \begin{pmatrix} I^-_1 \\ I^-_2 \end{pmatrix}$$ \hfill (6.15)

and thus

$$S_{\text{tip}} = \frac{1}{(I^+_1)^2 - (I^+_2)^2} \begin{pmatrix} (I^-_1 I^+_2 - I^-_2 I^+_1) & (I^-_1 I^+_2 - I^-_2 I^+_1) \\ (I^-_2 I^+_1 - I^-_1 I^+_2) & (I^-_2 I^+_1 - I^-_1 I^+_2) \end{pmatrix}.$$ \hfill (6.16)

If Kirchhoff's theorem has to be enforced at the tip, then eq. (6.14) can be reduced by substituting $\Gamma_1 = \Gamma$ and $\tau_1 = 1 - \Gamma$. However, it was observed that by only choosing $\Gamma$ at the tip, the currents along $\ell_1$ and $\ell_2$ could not be modeled properly due to a lack of model parameters. Therefore, an oscillating electrical charge has to be permitted at the tip and eq. (6.14) cannot be reduced further.

In a similar way, the gap at port 1 is modeled using a scattering formalism. It concerns the complex amplitudes $I^+_0$ and $I^-_0$ at $I_2 = 0$, as well as $I^+_3$ and $I^-_3$ at $I_3 = 0$. Symmetry is assumed again for $S_{\text{gap}}$, since this matrix will be applied for both ports 1 and 2. Therefore,

$$S_{\text{gap}} = \frac{1}{(I^-_0)^2 - (I^+_0)^2} \begin{pmatrix} (I^-_0 I^+_2 - I^-_2 I^+_0) & (I^-_0 I^+_2 - I^-_2 I^+_0) \\ (I^-_0 I^+_3 - I^-_3 I^+_0) & (I^-_0 I^+_3 - I^-_3 I^+_0) \end{pmatrix}.$$ \hfill (6.17)
In future, $S_{\text{gap}}$ could be expanded to a larger matrix, which describes the transition of the currents to the slot line explicitly. Currently, only the interaction between the curved edges are considered in the proposed gap model. Nevertheless, $S_{\text{gap}}$ indirectly includes information about the matching with the slot line, but should be extracted from this matrix in future.

The scattering of the currents at the gap, tip and along the edge are modeled now, however, the point of excitation has to be considered separately. It will be assumed that $I_{10}^+$, i.e. the “starting amplitude”, will be similar for both the very long array case as well as the 2x1 basic-array configuration.

Now, a prediction of the currents can be made in case of a very long array by means of the method that is described in the next section.

### 6.4 A Predictive Model for a Very Long Linear Vivaldi Array

The scattering matrices $S_{\text{gap}}$, $S_{\text{prop}}$ and $S_{\text{tip}}$, which were determined for a 2-element array in the last section, will be used in order to predict the edge currents for a very long linear Vivaldi array. The procedure to do so will be explained with the aid of fig. 6.6.

![Figure 6.6: A predictive model for a very long linear polarized array of Vivaldi elements; a traveling wave scattering model.](image)

It is assumed that the previously determined scattering matrices for a short array may be applied for every Vivaldi element in a long array, with the exception of the edge elements. Suppose that the array is excited at port 1 i.e., $I_{10}^+$ is known and equal to the 2x1 Vivaldi-array case. By using the scattering matrices, the remaining amplitudes can then be calculated as described next.

Because of symmetry, only the left part of the array will be considered, since the right part of the array comprises currents having a 180° phase shift with respect to the left part. The following relation holds between the amplitudes of the edge currents at element 1 and element $n$:

---

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in which the chain of scattering matrices is formed by Redheffer star products. From eq. (6.18) it is obvious that if we assume that the current at the outer tip of element $n$ is totally reflected, the amplitude of the current is zero at the tip and $I_{(2n-1)_{L}}^{-} = -I_{n-1}_{L}^{+}$. Hence,

$$I_{10}^{-} = I_{10}^{+} \left( S_{11}^{\text{tot}} - \frac{S_{12}^{\text{tot}} S_{11}^{\text{tot}}}{1 + S_{22}^{\text{tot}}} \right). \quad (6.19)$$

In conclusion, by assuming that the current is completely reflected at element $n$, which can be the last element in a very long array, or an element that is considered to be the last one, $I_{1}$ is known since both $I_{10}^{+}$ and $I_{10}^{-}$ are known now (c.f. eq. (6.9)). Henceforth, the remaining currents $I_{2} \ldots I_{(2n-1)}$ can be determined in a manner which is straightforward by using the scattering matrices. For instance,

$$I_{2L}^{+} = \frac{I_{1L}^{-} - I_{1L}^{+} S_{11}^{\text{tip}}}{S_{12}^{\text{tip}}} \quad (6.20)$$

and

$$I_{2L}^{-} = I_{2L}^{+} S_{22}^{\text{tip}} + I_{1L}^{+} S_{21}^{\text{tip}}. \quad (6.21)$$

### 6.5 Results for the Determination of the Scattering Matrices

In order to determine $S_{\text{prop}}$, $S_{\text{tip}}$ and $S_{\text{gap}}$ for a 2x1 Vivaldi array, the current models $I_{1}$, $I_{2}$ and $I_{3}$ were fitted to the normalized currents along $I_{1}$, $I_{2}$ and $I_{3}$ respectively. The fitting procedure yields normalized complex amplitudes that can be substituted in eq. (6.12), (6.16) and (6.17).

The normalization of the currents was obtained as described in the first section. The elements were positioned along the $z$-axis and the traveling-wave models were fit to the normalized edge currents in a way as described next.

First, the complex amplitudes $I_{10}^{+}$ and $I_{10}^{-}$ were found by fitting $I_{1}$ in a least-square sense for an initial estimate of $\alpha_{c}$ and $\beta_{c}$ (c.f. sec. 2.7). Also, $I_{2}$ was fitted similarly for the same $\alpha_{c}$ and $\beta_{c}$. Afterwards, a nonlinear optimization routine yielded better values for $\alpha_{c}$ and $\beta_{c}$ for both $I_{1}$ and $I_{2}$. In other words, both $I_{1}$ and $I_{2}$ are optimized as a function of $\alpha_{c}$ and $\beta_{c}$. The final result, after several iterations, has been depicted in fig. 6.7.
6.5. RESULTS FOR THE DETERMINATION OF THE SCATTERING MATRICES

Figure 6.7: The final fitted currents along $l_1$ and $l_2$ after several iterations.

It seems that the phase fitting along $l_2$ is performed wrong, however, the difference between the fitted phase and the real data is exactly $2\pi$. The phase jump occurs, because the phase of the model is already increasing before it makes its last jump. The jump occurs at a position for which the magnitude of the current is low, therefore, the phase values could be wrong at the jump but have no consequences.

As a result, it is found that

$$\alpha_c = 5.8 \text{ Npm}^{-1} \quad \text{and} \quad \beta_c = 45 \text{ radm}^{-1}.$$ 

<table>
<thead>
<tr>
<th></th>
<th>$I_1$</th>
<th>$I_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{10}^+$</td>
<td>$29e^{-j0.31}$ [mA]</td>
<td>$3.5e^{-j0.17}$ [mA]</td>
</tr>
<tr>
<td>$I_{10}^-$</td>
<td>$1.6e^{j3.1}$ [mA]</td>
<td>$1.0e^{-j2.8}$ [mA]</td>
</tr>
<tr>
<td>$I_{1L}^+$</td>
<td>$9.3e^{-j3.0}$ [mA]</td>
<td>$1.1e^{-j2.9}$ [mA]</td>
</tr>
<tr>
<td>$I_{1L}^-$</td>
<td>$5.0e^{-j0.48}$ [mA]</td>
<td>$3.3e^{-j0.086}$ [mA]</td>
</tr>
</tbody>
</table>

Subsequently, the scattering matrices $S^{\text{tip}}$ and $S^{\text{prop}}$ can be computed according to eq. (6.12) and eq. (6.16), which gives

$$S^{\text{prop}} = \begin{pmatrix} 0 & 0.32e^{-j2.7} \\ 0.32e^{-j2.7} & 0 \end{pmatrix}$$

and

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CHAPTER 6. A FEASIBILITY STUDY FOR A LINEAR VIVALDI ARRAY

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\[ S_{\text{tip}} = \begin{pmatrix} 0.51e^{2.5} & 0.29e^{j3.0} \\ 0.29e^{j3.0} & 0.51e^{2.5} \end{pmatrix} \]

Next, the current \( I_3 \) has to be fitted along \( l_3 \), in order to obtain \( I_{30}^+ \) and \( I_{32}^- \). This can be done in one single turn, because the optimal \( \alpha_e \) and \( \beta_e \) have already been determined. The fitted current \( I_3 \) is illustrated in fig. 6.8.

\[
\begin{array}{c|c}
I_3 & \\
\hline
I_{30}^+ & 1.4e^{2.0} [\text{mA}] \\
I_{30}^- & 1.1e^{-j1.1} [\text{mA}] \\
I_{32}^- & 0.5e^{-j0.77} [\text{mA}] \\
I_{32}^- & 3.5e^{j1.6} [\text{mA}] \\
\end{array}
\]

Figure 6.8: Fitted current along \( l_3 \).

Finally, \( S_{\text{gap}} \) becomes

\[ S_{\text{gap}} = \begin{pmatrix} 2.2e^{2.7} & 1.1e^{j0.82} \\ 1.1e^{j0.82} & 2.2e^{2.7} \end{pmatrix} \]

As can be noticed, \(|S_{\text{gap}}^{11}| > 1\) and \(|S_{\text{gap}}^{22}| > 1\). The reason for this remarkable effect is that, apparently, the part of the wave that travels towards the tip along \( l_2 \), is larger than its counterpart, which travels towards the gap. This phenomenon is attributed to coupling between the currents along \( l_1 \) and \( l_2 \). In order to model this, the traveling-wave models along the edges should be extended. The extension could consist of coupling terms that are added to the differential equations for both the traveling waves \( I_1 \) and \( I_2 \). Hence, a coupled system of differential equations has to be solved. However, coupling between edge currents will not be investigated in this thesis. In a similar way it can be explained that \(|S_{\text{gap}}^{11}| > 1\) and \(|S_{\text{gap}}^{22}| > 1\).

6.6 Results for the Predictive Vivaldi-Array Model

As a next step, the response of a 9x1 dense Vivaldi array will be predicted by using the previously derived results. The array is excited at port 5, which is obvious when the current

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6.6. RESULTS FOR THE PREDICTIVE VIVALDI-ARRAY MODEL

distribution in fig 6.9 is considered.

Figure 6.9: A 9x1 single polarized Vivaldi array.

As a verification, the scattering matrices were determined for the 9x1 Vivaldi array and showed similarities with the matrices for a 2x1 reference array and also for a 3x1 array. Therefore, an element of a 2x1 array configuration can already be regarded as a representative infinite array element. The tip matrices for the 9x1 and 3x1 Vivaldi-array are respectively given by

\[
\mathbf{S}_{\text{tip}}^{9\times1} = \begin{pmatrix}
0.34e^{j2.3} & 0.42e^{-j2.9} \\
0.42e^{-j2.9} & 0.34e^{j2.5}
\end{pmatrix}, \quad (6.25)
\]

and

\[
\mathbf{S}_{\text{tip}}^{3\times1} = \begin{pmatrix}
0.29e^{j2.5} & 0.41e^{-j3.0} \\
0.41e^{-j3.0} & 0.29e^{j2.5}
\end{pmatrix}. \quad (6.26)
\]

The currents along the edges were predicted by using the scattering matrices in a manner as presented in sec. 6.4. The final result is illustrated in fig. 6.10.

Apparently, edge currents close to the point of excitation can be predicted fairly well. However, the amplitude of the predicted edge currents close to the outer elements deviate severely with respect to the reference model. Also, the phase distributions of the currents start to deviate when the outer elements are considered. Nevertheless, the amplitudes of these currents are small and might not contribute much to the total pattern.

We are interested in the total E-plane radiation pattern, which is produced by the predicted edge currents, in comparison with the reference pattern for a 9x1 Vivaldi array (MoM). The predicted pattern as well as the reference pattern are shown in fig. 6.11.
Figure 6.10: Predicted edge currents for a 9x1 array configuration for center element excitation (blue) compared to the normalized edge currents (green). Currents are shown for the center element ($I_1$), for one Vivaldi element left from the center ($I_2$ and $I_4$) and for one edge next to the this element ($I_4$).
6.6. RESULTS FOR THE PREDICTIVE VIVALDI-ARRAY MODEL

Figure 6.11: Left: reference power pattern of a 9x1 Vivaldi array (red). The power pattern produced by the predicted currents for the center element only (magenta). The power pattern of two neighbouring elements next to center element (blue). Resultant power pattern for all edge currents (green). Right: Error between the reference pattern and the total pattern produced by the predicted model (green), which is 0.8 dB for forward direction.

It can be observed that the position of nulls and maxima are not well predicted. However, this could be expected since the positions of nulls and maxima for the 2x1 Vivaldi array, could not be approximated well either. As a result of accumulated errors, the pattern error for $0^\circ \leq \theta \leq 180^\circ$ is 0.8 dB. It is shown in fig 6.12 that the original sampled data of the currents along the edges, which was used to build the model, already produces a pattern which has wrong positions of nulls and maxima. Therefore, the model can never predict a radiation pattern that is better than the (black) pattern as is shown in fig 6.12.

Figure 6.12: Patterns, which are produced by the original currents along the edges, compared to the reference pattern (red). Pattern of all edges (black). Pattern of the center element (blue) Pattern of three elements surrounding the point of excitation (green).
From the results presented we can conclude that the original edge currents should be
determined more accurately before fitting the model to these currents (data-extraction refine­
ment). Also, when one wants to predict the reference pattern as good as the green pattern
in fig. 6.12, currents should be predicted properly along $\ell_1$, $\ell_2$ as well as along $\ell_3$. A way to
achieve this is to determine the matrices for a group of edges, so that the currents will be
predicted properly along several more edges (model reduction by using less information).

To prove that the matrices can be determined more accurate, so as to predict more edge
currents properly, we consider the matrices at the tip for a 9x1 Vivaldi array. As can be seen
below, the matrices that are determined at three tips from the center are different. Especially
close to the point of excitation (matrix 1), which means that the procedure to determine the
matrices has to be improved in order to predict the edge currents properly along several
edges.

\[
S_{\text{tip}1} = \begin{pmatrix}
0.34 e^{j2.5} & 0.42 e^{-j2.9} \\
0.42 e^{-j2.9} & 0.34 e^{j2.5}
\end{pmatrix},
\]

(6.27)

\[
S_{\text{tip}2} = \begin{pmatrix}
1.9 e^{-j1.9} & 1.6 e^{-j1.8} \\
1.6 e^{-j1.8} & 1.9 e^{-j1.9}
\end{pmatrix},
\]

(6.28)

\[
S_{\text{tip}3} = \begin{pmatrix}
2.9 e^{j0.15} & 3.0 e^{-j2.7} \\
3.0 e^{-j2.7} & 2.9 e^{j0.15}
\end{pmatrix}.
\]

(6.29)

6.7 Conclusions

A procedure has been described to extract scattering matrices from a 2x1 reference Vivaldi
array in order to predict the response for a very long array of Vivaldi elements. It was
shown that the currents along the edges of the 2x1 array have to be determined accurately,
especially at the radiating part of the element, in order to approximate the reference pattern
well. Although some deviations in the power pattern existed, a scattering-matrix model was
built to describe edge currents along all the edges of a long Vivaldi array. A peculiarity of
the matrix model is that it allows an oscillating charge on the tips of the elements.

As a result:

- The pattern produced by the traveling-wave currents resembles the reference pattern
  reasonably well, apart from the prediction of the nulls and maxima. The main reason
  for this deviation is that the currents are not extracted accurate enough from the
  MoM data. This should be performed more accurately before fitting the models and
  extracting the scattering-matrices from this fit.

- Strong coupling between edge currents exists, which should be modeled in future by
  introducing coupling terms in the traveling-wave model.

- The edge currents for a 9x1 array were predicted well for elements close to the point
  of excitation but less accurate for the remaining elements. The way in which the matrices
were determined can be improved so as to predict more edge currents properly. In general, it is recommended to look for more accurate techniques to determine edge currents in the tip of the Vivaldi element, which can be achieved either analytically and/or numerically.
Chapter 7

Conclusions/recommendations

7.1 Conclusions

It has been demonstrated that the E-plane power pattern of a slot line based Vivaldi element can be approximated well by the pattern produced by a traveling wave model, which constitutes currents that travel along the edges of the element only.

The traveling-wave model only accounts for a current that propagates towards the tip and a reflected current that propagates back. This seems to be sufficient to model the currents and to predict the magnitude of patterns at least 3 dB accurate for a frequency range of 2-5 GHz, which satisfies the requirements for focal plane array applications.

The reference model showed that the outer (straight) edges for a single element needed to be modeled as well, in order to successfully predict a dip for the pattern into the end-fire direction. The contributions of the straight-edge currents and curved-edge currents to the power pattern were analyzed separately. Although the magnitude of the straight-edge currents is much smaller than the curved-edge currents, the straight edge currents contribute more to the total radiated power. Several ways to decrease this contribution were proposed.

Two normalization procedures were described and implemented to determine the amplitude of the traveling-wave edge currents. As a result, the absolute radiated power could be predicted well. First, a method was presented that transforms surface currents into normalized edge currents by integration. Second, a normalization was presented by fitting non-normalized patterns, obtained from non-normalized edge currents, to the normalized reference pattern. Both methods yielded similar results.

The traveling-wave model parameters were analyzed for different frequencies. From this study it was concluded that reflection at the tip gets less for higher frequencies, the velocity of the waves is frequency independent and equal to the speed of light and the attenuation coefficient of the currents is more or less constant for a broad spectrum. Moreover, the passive input impedance was simulated with the aid of the MoM and anomalies were observed
at fixed frequency intervals. The positions of the anomalies could be well predicted by the traveling-wave model for low frequencies.

The applicability of the model has been demonstrated for two important practical parameters, i.e. the radiating effective area and the phase center definition. The size of the radiating effective area was determined as well as the location of the transition zone between the radiating and propagating parts of the element, which is 80% of the total element height. The phase center was analyzed as a function of the straight and curved-edge currents. The phase center moves into the Vivaldi slot when the magnitude of the currents along the straight edges are increased with respect to the currents along the curved edges and vice versa.

A feasibility study has been performed for a Vivaldi array model. A scattering-matrix model has been suggested and implemented in order to predict the response for very long linear Vivaldi arrays. The model allows us to describe the transition of the currents from one edge to its neighbouring edges (mutual coupling). The capabilities and limitations of the scattering-matrix model were shown for a 9x1 Vivaldi array and recommendations were developed.

In general, it has been proven that currents on a Vivaldi element can be well represented by traveling waves along the edges. In such a case, simple network theory can be used and integration with electronic models can be made, which is highly desirable for large aperture arrays. Capabilities of the traveling-wave model presented are shown. This thesis also describes when the model fails to work and what can be done to extend the model in order to describe a larger class of elements more accurately. Eventually, the model can be used to perform antenna synthesis efficiently and to obtain results quickly in a simple and fundamental way.

7.2 Recommendations

The coupling between the several edge currents is significant for the slot-line excitation and should be modeled. In general, it should be studied more extensively how edge currents are coupled. Hence, the predictive model can be adapted and currents can be predicted more accurately.

Apart from the coupling between the edge currents, the currents itself can be modeled more accurate along the edges to achieve a better fit. For instance, the traveling wave model can be extended such, that it includes the description of a continuous reflection coefficient along the curvature, dependent on the curvature rate. This (transmission-line) model could be fit for the propagating part of the Vivaldi element. The radiating part could be treated similarly, however, by considering the radiating part of a Vivaldi element as a canonical structure, analytical expressions for the currents can be used. It is recommended to look for more accurate techniques to determine edge currents in the tip of the Vivaldi element, the radiating part, since small deviations in the tip lead to large changes for the resulting pattern.
The currents obtained from numerical solutions (MoM) could be obtained in an alternative way. The present currents show some oscillations in the amplitude distribution. By controlling the mesh manually or by using an alternative solver, more smooth and accurate current distributions might be obtained, which improves the accuracy of the model parameters after fitting.

The coupling between the antennas is considered to take place in the tip of the elements, since the propagating parts (lower parts) of the elements are assumed to be independent of each other and can be modeled by transmission-line theory. The radiating part (the actual antenna) needs to be analyzed more deeply in terms of coupling.
Bibliography


Appendix A

Numerical Determination of $J(r_0)$ by Linear Interpolation

Consider the vector field as illustrated in fig. A.1. The aim is to determine $J(r_0)$ with the aid of linear interpolation.

First, three mesh points are searched for that are closest to the vector at position $r_0$. In this case $r_1$, $r_2$ and $r_3$ are most close. Next, a test should be performed to verify whether $r_0$ is in the triangle formed by the surrounding mesh points. This test can be performed by first writing

$$r_0 = r_1 + p(r_2 - r_1) + q(r_3 - r_1), \quad (A.1)$$

in which $r_n$ is the vector pointing to position $n$. If $r_0$ is within the triangle, then it has to be possible to form $r_0$ by linear combination of $(r_2 - r_1)$ and $(r_3 - r_1)$. However, not every combination is allowed, since

$$0 < p < 1, \quad 0 < q < 1 - p, \quad (A.2)$$
which describes two sides of the triangle. The surface is eventually described by

\[ p + q \leq 1. \]  

(A.3)

If these conditions are satisfied, then \( \mathbf{r}_0 \) is within the triangle. \( p \) and \( q \) can easily be obtained from eq. (A.1), which yields

\[ p = \frac{(r_{0z} - r_{1z})(r_{3y} - r_{1y}) - (r_{0y} - r_{1y})(r_{3z} - r_{1z})}{(r_{2z} - r_{1z})(r_{3y} - r_{1y}) - (r_{2y} - r_{1y})(r_{3z} - r_{1z})} \]  

(A.4)

and

\[ q = \frac{(r_{0y} - r_{1y})(r_{2z} - r_{1z}) - (r_{0z} - r_{1z})(r_{2y} - r_{1y})}{(r_{2z} - r_{1z})(r_{3y} - r_{1y}) - (r_{2y} - r_{1y})(r_{3z} - r_{1z})}. \]  

(A.5)

Finally, a linear approximation of \( J(\mathbf{r}_0) \) can be obtained by computing

\[ J(\mathbf{r}_0) = J(\mathbf{r}_1) + p[J(\mathbf{r}_2) - J(\mathbf{r}_1)] + q[J(\mathbf{r}_3) - J(\mathbf{r}_1)], \]  

(A.6)

or by defining a plane through the three coordinates and applying interpolation by a matrix inversion, which is the same but a less efficient approach.
Appendix B

Determined Edge Currents for Two 3x1 Vivaldi Array Configurations

On the following two pages edge currents are illustrated that were obtained for two different array configuration, which are shown in sec. 4.2.2, in case of center element excitation. The two array configurations are called:

- The slot line model; a 3x1 Vivaldi array model based on slot lines. The simulation was carried out at 2.3 GHz, for a slot line excitation with terminating impedances of 100 Ω.

- The cavity model; a 3x1 Vivaldi array model based on a tapered slot line and a circular cavity. The simulation was carried out at 1.12 GHz, for delta gap excitation with terminating impedances of 100 Ω.
APPENDIX B. DETERMINED EDGE CURRENTS FOR TWO 3X1 VIVALDI ARRAY CONFIGURATIONS

Figure B.1: Amplitude and phase distributions of non-normalized currents along the edges. A polynomial function has been fitted through the obtained currents in order to calculate the patterns more easily.
Figure B.2: Amplitude and phase distributions of non-normalized currents along the edges. A polynomial function has been fitted through the obtained currents in order to calculate the patterns more easily.
Appendix C

Slot Line Vivaldi Element Patterns For Several Frequencies

The traveling wave models were fitted along the edges of a single tapered slot line antenna after normalizing the edge currents. Power patterns can be calculated from these model currents and, hence, be compared with the reference pattern for several frequencies as shown in fig. C.1 and C.2.

As can be concluded, the tapered slot line element starts to radiate with a small intensity
Figure C.2: E-plane power pattern for model (red) and reference (blue). Upper left: 3 GHz, Upper right: 4 GHz, Lower center: 5 GHz.

into broadside direction, while the patterns becomes more directive for end-fire direction at higher frequencies. The dip in forward direction is caused by the currents along the straight edges, which is less severe in case of the cavity model. Nevertheless, also the cavity model show patterns that become more narrow in the E-plane for increasing frequencies.
Appendix D

Prove of Equation (2.7)

We want to prove the following primitive function

\[ \int \sqrt{1 + (C_1 R)^2 e^{2 R \xi}} d\xi = \frac{1}{R} \left[ \sqrt{1 + (C_1 R)^2 e^{2 R \xi}} - \text{arctanh} \left( \sqrt{1 + (C_1 R)^2 e^{2 R \xi}} \right) \right]. \]

This primitive function is used to calculate the length along the curved edge of a Vivaldi antenna. The prove is straight-forward, since

\[ \frac{\partial}{\partial \xi} \left( \sqrt{1 + (C_1 R)^2 e^{2 R \xi}} \right) = \frac{1}{2} \left( 1 + (C_1 R)^2 e^{2 R \xi} \right)^{-\frac{1}{2}} \frac{(C_1 R)^2 e^{2 R \xi}}{1 - (1 + (C_1 R)^2 e^{2 R \xi})}. \]

and

\[ \frac{\partial}{\partial \xi} \left( \text{arctanh} \left( \sqrt{1 + (C_1 R)^2 e^{2 R \xi}} \right) \right) = \frac{1}{1 - (1 + (C_1 R)^2 e^{2 R \xi})} \frac{(C_1 R)^2 e^{2 R \xi}}{\sqrt{1 + (C_1 R)^2 e^{2 R \xi}}}. \]

Combination of these derivatives yields

\[ \frac{1}{R} \frac{\partial}{\partial \xi} \left( \sqrt{1 + (C_1 R)^2 e^{2 R \xi}} - \text{arctanh} \left( \sqrt{1 + (C_1 R)^2 e^{2 R \xi}} \right) \right) = \frac{(C_1 R)^2 e^{2 R \xi}}{1 - (1 + (C_1 R)^2 e^{2 R \xi})} = \sqrt{1 + (C_1 R)^2 e^{2 R \xi}}. \]

Therefore,

\[ \int \sqrt{1 + (C_1 R)^2 e^{2 R \xi}} d\xi = \frac{1}{R} \left[ \sqrt{1 + (C_1 R)^2 e^{2 R \xi}} - \text{arctanh} \left( \sqrt{1 + (C_1 R)^2 e^{2 R \xi}} \right) \right]. \]
APPENDIX E. USING ZELAND TO SIMULATE FINITE-VIVALDI-ELEMENT-ARRAYS

Using Zeland to Simulate Finite-Vivaldi-Element-Arrays

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1 Introduction

Research activities, in a framework of Large Aperture Array concepts, involve study of a number of composite electromagnetic field problems: fields in Far- and Near-regions, polarization issues, mechanism of excitation, noise and mutual coupling effects between the elements, scattering etc.

The progress in solving them in many respects is determined by availability of the necessary mathematical-physical model (developed and approved techniques) as well as computing facilities.

A present market of software/hardware provides us with a number of the professional packages, which can be used for the research, and computes powerful enough to analyze complicated antenna configurations in a wide frequency band.

This document demonstrates the most important features that Zeland comprises by performing calculations on Vivaldi antenna configurations.

2 What is Zeland?  

Zeland is a Method of Moment (MoM) package that tackles electromagnetic field problems in frequency domain by solving an Electric Field Integral Equation.

Zeland is able to display and export radiation patterns, calculate coupling parameters between different excitation ports in array configurations, display and export current distributions and analyze a feed system using a spice-like circuit simulator. It even has possibilities to perform simple antenna optimizations.

This document shows the complexity of the antenna configurations that can be analyzed if Zeland is running on hardware that is currently available.

Simulations were carried out on a 2Gb RAM, 2GHz CPU machine, using Zeland 7.01.

3 Getting Started

3.1 Geometry Setup and Inputs for Feed Source

The geometry of an antenna element can be drawn using MGRID. The Vivaldi structure is built from polygons, as shown for the element in Figure 1 (FARADAY geometry). The element consists of a tapered slot-line, which has been generated by MATLAB and subsequently been imported by MGRID. A circular cavity and a metal connection in the slot, which is needed to create a place for excitation. A feed structure including a radial stub has been positioned just above the Vivaldi element and excites the slot.

The main limitations of MGRID are:
- Only infinity large substrate layers in the XY-plane can be defined.
- The excitation sources that can be used are:
  1) A vertical localized source, which is defined in the z-direction and can be used as a delta gap source inside the slot of the Vivaldi element. Horizontal localized sources are included in later versions of Zeland.
  2) A port from which waves are excited that is connected to the feeding strip line as shown in (Figure 1).
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The next step is to spatially discretize the antenna element by using the automatic triangular mesh generator. A large matrix needs to be inverted if the mesh is chosen small relative to the total antenna surface. However, a very sparse mesh in terms of wavelength or geometry will, however, yield inaccurate results.

3.2 Reference Frames

It is known that polarized radiation can be fully described by the projection of the electric field vector $E$, in a 2-dimensional orthogonal basis.

In the case of Zeland:

- The center of the reference frame of the fields to be calculated coincides with the center of the coordinate basis of the prescribed feed source.
- The feed sources in Zeland are attached to certain coordinate systems a priori. As a result, orientation of the antenna geometry coordinate system, with respect to the source frame, is fixed and cannot be changed. In case of a vertical localized source (the most accurate model for the excitation of the vivaldi-element), a vertical location of the elements is required in its coordinate basis (See Figure 2, the element is placed in the XOZ-plane).

4 List of Examples

The list of examples of antenna characteristics presented here are for the Vivaldi array geometry corresponding to the THEA prototype and partly for focal plane array (FARADAY).

4.1 A Single Vivaldi Element

4.1.1 Current Distribution Analysis

The current distribution on the surface of the element was obtained for a THEA configuration that was simulated at 1.12GHz. The current distribution, which is shown in Figure 2, was processed by CURVIEW.
The magnitude of the average current density distribution of a THEA element (without a flattened top), operating at 1.12GHz.

In this case, the vertical localized source was chosen to be a current source with an internal impedance of infinity, which is a common way in literature to excite and to model a Vivaldi element. Nevertheless, an arbitrary impedance and/or a voltage source can be chosen at the point of excitation instead.

4.1.2 Radiation Patterns

Element patterns for the co-polarization as well as the cross polarization were calculated and can be viewed e.g. in 3D form, as shown in Figure 3.

![Figure 3: Linear normalized E-field element patterns for the co-, cross-polarization and total pattern, obtained for a single THEA element.](image)

4.1.3 Near-Field Patterns

Near field calculations can also be performed and, subsequently, be exported. As an example, the THEA element, which is illustrated in Figure 2, has been used for near field analysis. Several near field grid planes were positioned in front of the antenna as shown in Figure 4. The magnitude of the total electrical field was computed for the several grid planes, which together form a near field volume.

As a result, the amplitude of the total electrical field decreases for increasing distance between the Vivaldi antenna and the grid plane under consideration. Moreover, the amplitude of the electrical field distribution gets more and more focused for increasing distance, i.e. two separate field distributions, which are generated by currents along both curved arms of the Vivaldi element, coincide.
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Figure 4: Magnitude of total electrical field in near field planes, positioned at several different distances in front of the Vivaldi antenna. A circular shaped amplitude distribution of the field pattern is observed if the distance from the Vivaldi tips is larger than 0.3, and grid planes together form a total volume in which the near field is calculated. Grid planes are positioned at 0.0, 0.1, 0.2, 0.3, and 0.4 from the Vivaldi tips respectively.

From Figure 5 it can be concluded that, in the two orthogonal planes phi=0 (E-plane) and theta =90 (H-plane), the magnitude of the cross-polar component (Ey) is very small, which can be expected for a linear polarized element.

Figure 5: Magnitude of Ey in a near field grid plane in front of the Vivaldi antenna. As can be seen, Ey=0 in the E and H-plane, as expected for a linear polarized element.

Limitations of the pattern viewer (CURVIEW), are:
- Only normalized far field patterns can be exported for post processing. However, pattern parameters such as total radiated power can be calculated and, henceforth, be used to renormalize the patterns.
- Near fields cannot be calculated for z=0.

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4.2 Vivaldi-Element Array (THEA Prototype)

Emphasis has been made on investigation of the array, rather than individual elements, since mutual coupling contribution into the total radiation is significantly large.

4.2.1 Single Elements in the Environment of the Array

An example of a linear polarized dense phased array antenna is the THEA tile, which consists of 64 Vivaldi elements (8x8) on top of a finite ground plane, as illustrated in Figure 6.

The simulation of a complete THEA tile\(^4\) took approximately 2 hours. The matrix solver had to solve a linear system of 9000 unknowns, which is rather acceptable for a mesh size of 6 steps per wavelength.

![Figure 6: The THEA tile, an 8x8 single polarized Vivaldi array. Magnitude of average current distribution is shown for center element excitation (left) as well as for corner element excitation (right).](image)

As can be seen, due to mutual coupling effects between the elements, other elements are also excited. Because of this coupling effect, together with finite array effects, element patterns for the center elements differ much from element patterns of corner elements, as is illustrated in Figure 7.

Mutual coupling is an important issue in dense array technology. Therefore, it is useful to know which coupling parameters can be investigated by using the Zeland software.

\(^4\) A complete THEA tile comprises a ground plane. Nevertheless, a simulation that was performed without accounting for a ground plane, showed only a significant declination in the element patterns for the backward radiation.
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Figure 7: Upper left: co-polar radiation pattern in E-plane for center excitation. Upper right: co-polar radiation pattern in H-plane for center excitation. Lower left: co-polar radiation pattern in E-plane for corner excitation. Lower right: co-polar radiation pattern in H-plane for corner excitation.

4.2.2 How to Control Coupling Effects

4.2.2.1 Influence of the Terminating Impedances on the Coupling Mechanism

The scattering matrix \( S \) describes the coupling effects between the array elements in the transmitting situation. The impedance matrix \( Z \) describes the mutual and self-impedances of the antennas and remains unchanged for a fixed frequency and geometry. The impedance matrix \( Z_0 \) describes the terminating impedance at each antenna port and is, in general, a diagonal matrix. The following relation holds between \( S \) and \( Z_0 \):

\[
S = (z + I)'(z - I),
\]

in which

\[
I \quad \text{is the unity matrix and } \quad z = \begin{pmatrix}
\frac{1}{Z_0} & 0 & 0 \\
0 & \frac{1}{Z_0} & 0 \\
0 & 0 & \frac{1}{Z_0}
\end{pmatrix} \quad \text{and} \quad Z = \begin{pmatrix}
\frac{1}{Z_0} & 0 & 0 \\
0 & \frac{1}{Z_0} & 0 \\
0 & 0 & \frac{1}{Z_0}
\end{pmatrix}
\]
In conclusion, if the full impedance matrix $Z$ is known and we terminate the array with an impedance matrix $Z_2$, the scattering matrix $S$ can be calculated by using the equations shown above. In other words, the scattering matrix $S$ is a function of the terminating impedance $Z_2$, so we can control the coupling of the array elements by choosing a proper terminating impedance matrix $Z_2$.

Figure 8: Coupling mechanism in the THEA array with 2 center element excited for three different terminating impedances: Left figure, every element is short circuited ($Z_2 = 0$), so that coupling is strongest in the E-plane. Center figure, $Z_2 = 100$ Ohm, which gives coupling in both E and H-plane. Right figure: $Z_2 = 51$ Ohm (open), which gives significant coupling in the H-plane. The coupling in the E-plane seems to be constant, whereas the H-plane coupling can be influenced strongly.

4.2.2.2 Results of the Simulations
Figure 8 demonstrates the coupling effects occurring in a dense array antenna when 2 center elements are active, while the rest is passive.

As one can observe, the current distribution of 2 elements fed in the dense array is dependent on the terminating impedances of the neighbouring elements. As a consequence, the radiation patterns and the total received amount of power for the array are functions of the terminating impedances and can thus be controlled by $Z_2$. Therefore, by using the mutual coupling mechanism (termination), a compromise should be found between the required shapes of the array pattern in transmitting situation, or the maximum total received amount of power and minimal contribution of noise emitted by the loads in receiving situation.

4.2.2.3 Effective Area of the Array as a Function of Coupling Effects
In case the effective area of the antenna is considered, it can be noticed that the effective antenna area is a function of the terminating impedances as well, since the terminating impedances control the total amount of power received. Since effective area is an important issue in radio astronomy, we would like to mention an important relation between effective area of an infinite array and the element coupling, although this might not belong to the scope of this document.

The well-known equation for the effective area for a unit cell of an infinite antenna array, is defined by:
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\[ A_d = A \cdot \left( 1 - \left| \Gamma_r(\theta, \phi) \right|^2 \right) \cdot \cos \theta, \]

in which

- \( A \) is the surface of one unit cell of the array,
- \( \theta \) is the angle of observation with respect to broadside,
- \( \Gamma_r \) is the active reflection coefficient.

In this equation, the active reflection coefficient \( \Gamma_r \) is dependent on the termination and thus the effective area can be controlled by the termination.

- The full S-matrix for a certain termination can be exported with Zeland (MODUA), as well as the impedance matrix \( Z \) and the admittance matrix \( Y \).

4.2.3 Analyzes of Scanning Capabilities

4.2.3.1 Transmitting situation

Scanning capabilities of phased arrays are useful issues to examine. As an example, we let the main beam of the THEA tile, which is depicted in Figure 9, point towards two different directions, e.g. \((\theta = 90, \phi = 0)\) and \((\theta = 45, \phi = 135)\). It should be noted that the reference spherical coordinate system for the radiation field is not given according to the standard basis of the spherical coordinates. Here, E- and H-planes are defined by planes of the reference frame as \((\theta = \vartheta, \phi = 0)\) and \((\theta = 90, \phi = \varphi)\), correspondingly (See also 3.2). The results shown are for 1.12GHz.

Example 1

In order to point at the direction \((\theta = 90, \phi = 0)\), we excited every element with an equal phase and amplitude. As a result, the patterns that have been depicted in Figure 9 were obtained. As expected, the uniform current distribution gives maximal gain (the 3dB beam width is 12° in E plane) and rather low level of side illumination (the first side lobe is about of 112dB).
Figure 9: (left): The uniform current distribution of the 8x8 Vivaldi array excited by using current sources with internal impedances of infinity. (right): patterns of co-polarization component of the radiation field in two orthogonal planes. Since the ground plane is infinitely thin, the backward radiation might be unreliable i.e. currents were found on the backside of the ground plane as well.

Example 2

This example shows the importance of studying the behavior of the radiation field in a wide field of view and the relation between co- and cross-polarization components during scanning.

In order to be able to point the main beam towards \( (\theta = 45, \varphi = 315) \), we are making use of well-known array theory as a first approach. We assume that only the array factor is important enough to be considered and each element pattern is supposed to be isotropic and normalized to 1. These assumptions are allowed since we are only interested in how to define the location of the maximum of the total pattern into a certain direction. Thus

\[
F(\theta, \varphi) = F_a(\theta, \varphi) * F_{array}(\theta, \varphi),
\]

in which

\[
F_a(\theta, \varphi) \text{ is the single element pattern (equals 1 for all elements)}
\]

and

\[
F_{array}(\theta, \varphi) \text{ is the array factor.}
\]
Next, in order to point the maximum of the array pattern towards \((\theta = 45, \phi = 315)\), we use the following well-known excitation model for the \(M \times N\) antenna array with element spacing \(d^*\):

\[
F_{\text{array}}(\theta, \phi) = \sum_{n=1}^{M} \sum_{m=1}^{N} e^{i \left( \phi_n \sin \theta + \phi_m \cos \theta \right)}
\]

which is the array factor produced by \(M \times N\) isotropic radiators, each having a certain phase shift. From this formula it is well known that if we want to point the main beam towards a certain direction \((\theta_0, \phi_0)\), we need to use the following phase settings for element \(mn\):

\[
\text{Phase}_{mn} = \frac{180}{\pi} \sin \left( \frac{M+1}{2} \phi_n \sin \theta_n + \frac{N+1}{2} \cos \theta_n \right)
\]

For \(\phi = 2\pi \cdot 1.12 \times 10^6\) [radians], \(d = 0.127\) [m], \(c_0 = 3\times10^8\) [m/s], \(\phi_0 = 315\) [''], \(\theta_0 = 45\) [''], \(M = 8\) and \(N = 8\), we determined the following phase settings for the excitation coefficients of the antenna array:

\[
\begin{array}{cccccccc}
-124 & 151 & 65.6 & -19.8 & -105 & 170 & 84.2 & -1.14 \\
-3.03 & -88.4 & -174 & 101 & 15.6 & -69.8 & -155 & 120 \\
118 & 32.3 & -53.0 & -138 & 136 & 50.9 & -34.4 & -120 \\
-122 & 153 & 67.7 & -17.7 & -103 & 172 & 86.3 & 0.949 \\
-0.949 & -86.3 & -172 & 103 & 17.7 & -67.7 & -153 & 122 \\
120 & 34.4 & -59.9 & -136 & 138 & 53.0 & -32.3 & -118 \\
-120 & 155 & 69.8 & -15.6 & -101 & 174 & 88.4 & 3.03 \\
1.14 & -84.2 & -170 & 105 & 19.8 & -65.6 & -151 & 124 \\
\end{array}
\]
Figure 10: Graphical interpolative representation of the phase settings for the excitation coefficients of the 8x8 antenna array at the scan situation of $\theta = 45^\circ$ and $\phi = 315^\circ$. The figure represents the phase settings for the ports without phase jumps, i.e., the phases are not limited between $\pm 180$ and 180 degrees. The phase settings obtained were applied to the 64 current sources, each having an internal impedance of $1 \times 10^{-6}$ [W] and a uniform amplitude distribution. The resulting co- and cross-polar patterns for the cut planes $\theta = 45^\circ$ and $\phi = 315^\circ$ are shown in Figure 11.
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Figure 11: Radiation patterns for co- and cross-polar components in two cut planes, i.e. $\theta_n = 45^\circ$ and $\phi_n = 315^\circ$

This example demonstrates the worst case situation in scanning performance of arrays when the level of cross- and co-polar component are equally large. It means that linear polarization is lost for this particular scan angle.

4.2.3.3 Receiving Situation

Zeland has possibilities to excite antennas by incident plane waves and to monitor the voltages and currents on the ports.

Suppose a H-polarized plane wave is incident from $\theta_n = 315^\circ$ and $\phi_n = 45^\circ$. i.e. we now consider the reciprocal receiving situation instead of transmitting situation as was described in example 2. The plane wave arriving from the given direction reaches first the corner element with index $m=1, n=8$ (see Figure 12 (left)). The incident electromagnetic field induces surface currents along the elements that, in turn, create voltages on the open ports of the elements. It is possible to monitor the open port voltages using the CURVIEW application within Zeland and to plot the amplitude of the complex received voltage as well as the phase of the complex voltage for every antenna port. The receiving complex voltages for every antenna port have been graphically illustrated in Figure 12 (right).
Figure 12: Response of the THEA array on an incoming plane wave incident from the direction \( \phi_0 = 315° \) and \( \theta_0 = 45° \). Left: Current distribution along the aperture of the array. Right: Complex voltages received at the antenna ports. The figure represents the phase settings for the ports without phase jumps, i.e., the phases are not limited between \( \pm 180° \) and \( 180° \) degrees.

Figure 12 illustrates that, when the direction of observation is off from the axial direction, the amplitude of the current distribution along the array aperture is non-uniform. It can be explained by laws of Ray Optics: certain elements are illuminated by direct rays of the plane wave, while the others can be partly located in shadow and only tops or edges of the elements are in a region of light. As a consequence of these effects, the amplitude of the voltage distribution on the ports has a non-linear tapering character. It means that the former assumption of a uniform amplitude distribution for the currents in the reciprocal transmitting situation was not completely right and should be corrected for the excitations in the transmitting situation; in order to point the main beam into the same scanning direction as in example 2. By varying the amplitudes of the excitations in transmitting situation, the observation direction of the main beam remains the same only if the element patterns are equal.

In this particular example, the ratio between the maximum and the minimum received amplitude of the voltage is approximately 2. Please note that the element with indexes \( n=1 \) and \( m=8 \) gets the largest port voltage, while the element in the opposite diagonal corner gets the smallest amplitude.

As one can see from Figure 12 and Figure 10, the phase distributions of the port voltages in a receiving situation is the conjugate case for the phase distribution of the port voltages in the transmitting situation.
4.3 Dual Polarized Vivaldi-Element Array

An example of a current distribution for a dual polarized array is shown in Figure 13.

Unfortunately, coupling parameters between dual polarized elements cannot be investigated with the current version of Zeland, since the excitation systems (the vertical localized sources) are only defined in the z-direction.

![Figure 13: A 5x3 dual polarized Vivaldi element array. Center element is excited with a voltage source having an internal impedance of 50 Ω. The other elements are short circuited by connecting the Vivaldi tapering with a metal strip at the antenna ports.](image)

Elements in the XY-plane cannot be excited using the vertical localized sources, and henceforth, not be terminated using an excitation port. Therefore we need to leave the port open or need to make it a short.
Comparison of Measured & Simulated Patterns

This section shows the results for the radiation patterns obtained by numerical simulations and those obtained by NFST measurements for the THEA tile.

The patterns are presented for two particular cases, i.e. 2 and 4 active center-elements. The current distribution for the numerical model (2 active elements) is illustrated in Figure 14 together with the response of the current on the ground plane. Figure 15 shows the theoretical and measured patterns of the radiation field of the array in E- and H-planes.

Figure 14: The current distribution for the THEA tile with 2 active center elements (numerical model).

Figure 15: Radiation patterns obtained by Zeland simulations and NFST measurements for the THEA tile (2- and 4-center elements are active respectively.)
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A comparative analysis of the results presented here shows a rather high degree of agreement between the simulations and measurements. It allows us to observe the following features of the dense Vivaldi-element arrays:

- The main beam width of the radiation pattern in H-plane is broader than those in E-plane for both cases of 2-active elements: (HPBWH,HPBW_E) = 10° Measurements 20° Simulations
  4-active elements: (HPBWH,HPBW_E) = 4° Measurements 56° Simulations
- Using 4 active elements in the array leads to narrowing of the E-plane pattern in about 2 times with respect to the E-plane pattern for a 2-element array, while the H-plane pattern remains the same.
- Both measurements and simulations give good prediction of first nulls.
6 Conclusions/recommendations

6.1 Conclusions

This report demonstrates the potential of Zeland to simulate finite single- or dual-polarized Vivaldi element array configurations.

Zeland is capable to investigate the main radiation and impedance characteristics of Vivaldi antennas such as:

- Current distributions
- 3D far and near field patterns
- Coupling effects between array elements
- Scanning behavior, both in transmit and receive situations.

As an example of a Vivaldi element array the THEA tile has been considered. Results are presented for both simulations and measurements.

From the examples it is clear that it is highly desirable to use Zeland for further numerical analysis, in order to investigate focal plane arrays and phased array antennas.

6.2 Recommendations

It is in general recommended to verify results obtained with results that have been presented in literature. Other in-house software, such as Schauberts TDIE code, can be useful for comparison with Zeland simulations as well.