MODELLING AND ROBUST
CONTROLLER DESIGN OF A PVC-
FILM PRODUCTION PROCESS

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carried out from February 1994 to October 1994
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Summary

Hoechst Holland N.V. in Weert is continuously searching for new ways to improve the quality of PVC-film by better controlling its production process. One of the current research programs is to realize a homogeneous filling of the first calender gap. To reach this goal a two step approach is used. The first step is stabilizing the level of material on top of the mixing rolls in an earlier stage of the process. Apart from this goal, stabilizing this level has two more reasons. The process is based on a rather critical temperature balance. To maintain this balance the amount of material has to remain as constant as possible and the transport lag has to be kept as short as possible. Secondly, to reduce the amount of wasting material the level should be kept at a certain low level to prevent the material from falling off the mixing rolls.

The control problem discussed concerns stabilizing the material level on top of the mixing rolls. As process input, the velocity of the screw in the filling section is used. The process output is the level of material, measured using a vision system. The main challenges of this control situation are the large dead time between input and output, the open-loop instability of the process, the large parameter uncertainty, the large measurement noise and the integrator like drift terms. Stabilizing the material level at a guaranteed low level yields a better product quality (less temperature variations) and less spill over.

For this process a sixth-order $H_\infty$ controller has been designed. Simulations and experiments at the plant have been carried out in order to study the properties and the effectiveness of the controller. The experiment with the robust controller indicate an improved dynamic behaviour. Also a comparison is made with respect to a Smith-predictor with PID-controller design.
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1 Introduction

This project has been carried out at Hoechst Holland N.V. in Weert in cooperation with Eindhoven University of Technology. The three main products produced at the plant in Weert are PVC-film/ sheet, Trespa boards and Depron polystyrene trays. The project is implemented at a calender installation of the PVC-film/ sheet production process.

Product quality is often the main issue in industrial processes. Complex processes are controlled satisfactory by trained operators. As the demands on product quality and production efficiency grow and the tasks of the operator become more complex, automatic control of (sub-)processes is required. Controlling the dynamic behaviour of a process requires investigation of those dynamics. A model of all relevant process dynamics and of the disturbances has to be obtained for optimal disturbance rejection.

The project includes process identification, process model estimation, disturbance modelling, $H_\infty$ controller calculation using a robust control toolbox, simulation of the process under control and measurements at the plant. The simulation results of the $H_\infty$ controller will be compared to a conventional Smith-predictor with PID-controller.
2 Process

2.1 Introduction

The PVC-sheet/ film production process consists roughly of the following parts (figure 1). The production starts at the raw material supply (a funnel) which is continuously feeding the transport section of the extruder. The material is transported into the extruder in which the raw material is being transformed into a kind of gel by heating and friction (gelatination). The temperature is a very important parameter in the gelatination-process ($|\Delta T/T|<3\%$). If the temperature is too low the gelatination-process stops, on the other hand if the temperature is too high the material burns in the extruder.

After leaving the extruder the material falls into a vibrating conveyer, through which it is transported towards the mixing rolls. A skin is formed around one of the two mixing rolls. The sheet is cut from the mixing roll by two knives and is transported over a conveyer belt towards the first calender gap. From here the sheet is stretched mainly in the longitudinal direction (up to ten times the original length). Finally the sheet is wound up.

The process of interest is the process from the raw material supply towards material level on top of the mixing rolls. The goal of the controller is to stabilize the material level on top of the mixing rolls at a user-defined level.

The input of this process is the angular velocity of the screw in the funnel (filling screw). The
amount of material delivered \( (\Phi_i, [m^3/s]) \) to the extruder is (approximately) linearly dependent on the angular velocity of the filling screw \( (u \ [rad/s]) \).

The output of the process is measured with a vision system, which is developed in an earlier stage of this project c.f. v.d. Pas (1992) [4]. The vision hardware in the computer grabs a two-dimensional image seen by a camera pointed at the scene. Each pixel of the discretized image is compared with an user-defined threshold. The number of samples above the threshold is supposed to be a quantitative, linearly dependent measure for the amount of material on top of the mixing rolls. The material on top of the mixing rolls is continuously in motion, due to the rotation of the mixing rolls and the input and output flow of material. Consequently, a large uncertainty (formulated as noise) is introduced in the measured output level.

Control goals are a small steady state tracking error, disturbance reduction (drift and oscillations), compensations of changes in input and output flow, smooth actuator behaviour, in order to guarantee the temperature balance, and insensitivity to measurement noise.

### 2.2 Modelling the process

#### 2.2.1 Introduction

The process is represented schematically as a transport lag and a column in figure 2. The input flow \( \Phi_i \) is controlled the filling screw \( u \). The material goes through the transport lag and is gathered in a column. The output is a continuous flow \( \Phi_o \ [m^3/s] \) taken from the column. The amount of material in the column \( y(t) \) written as function of the input flow and the output flow is (neglecting the pressure delivered by the material in the column):

![Figure 2. Process schematically represented as transport lag plus material gathering in a column.](image-url)
The input flow and output flow of this process are continuously disturbed by changes in the setpoints of the process parameters and fluctuations of the compound of the raw material. In table 1 the most important disturbances on input flow and output flow are mentioned.

In earlier stages of the project experiments have been done to determine the linearity, dead time and noise characteristics. The results are described below.

• **Linearity**

  Linearity tests have been performed on the open-loop process. The main problem during these experiments is the process output \( y \) drifting away continuously, due to the integration in the process. This makes it rather difficult to say something about the linearity of the process. The experiments indicate linearity of the process, although saturation effects in the actuator may occur. Apparent non-linearities could be caused by drifting away of \( y \).

• **Dead time**

  In earlier stages of the project only the results of estimations of the dead time are given without the calculations. The estimates vary from one minute up to nine minutes. Unfortunately, such variance makes these results useless and a new estimation has to be made. The available data sets from previous experiments will be used.

• **Noise**

  Experiments have been done to determine the noise characteristics of the process. The noise is introduced by the continuously moving material and the fact that a two-dimensional image is made, representing a mass (volume), which is in fact a projection of the mass (volume) on a sub-space (plane). The noise experiments have been done during a process stop, in which case no input or output flow was present. The results are somewhat deceiving because the input flow causes a rather large amount of extra noise. The noise measured in these experiments should be handled with care and should be taken as a minimum value.

### 2.2.2 Mathematical process model

A mathematical model, containing only the main properties of the process, will be derived. This model will be used to design a stabilizing controller for the process. The approximation of the relation between the input flow and \( u(t) \) is given by

\[
y(t) = y(0) + \int_0^t (\Phi(t - \tau) - \Phi_0(\tau)) d\tau
\]
Table 1. Disturbances of the input flow $\Phi_i$ and the output flow $\Phi_o$

<table>
<thead>
<tr>
<th>Disturbance of the input flow. A positive step on:</th>
<th>Effect on $y$:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. the angular velocity of the filling screw ($u$);</td>
<td>Ramp (+)</td>
</tr>
<tr>
<td>2. the angular velocity of the main screw ($\omega_m$);</td>
<td>Step (+)</td>
</tr>
<tr>
<td>3. the amount of material in the funnel i.e. the pressure in the funnel;</td>
<td>Ramp (+)</td>
</tr>
<tr>
<td>4. the amount of gelatination used;</td>
<td>Ramp (-)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Disturbance on output flow. A positive step on:</th>
<th>Effect on $y$:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. the angular velocity of the mixing rolls ($\omega_r$);</td>
<td>Ramp (-)</td>
</tr>
<tr>
<td>2. the distance between the two cutting knives ($d$);</td>
<td>Ramp (-)</td>
</tr>
<tr>
<td>3. the distance between the mixing rolls (hardly ever changed);</td>
<td>Ramp (-)</td>
</tr>
</tbody>
</table>

**Further:**

4. Manual removal of the skin around the mixing roll. This material is mostly used for obtaining a quick increase in the amount of material in the filling of the first calender gap;

5. Manual addition of material, which is removed from the conveyer belt to obtain a fast decrease in the amount of material in the filling of the first calender gap.

\[
\Phi_i(t) \left[ \frac{m^3}{s} \right] = C \left[ \frac{m^3}{\text{rad}} \right] u(t) \left[ \frac{\text{rad}}{s} \right] \tag{2}
\]

where $C$ is a constant.

The output flow $\Phi_o$ is the volume of material produced per time unit. This flow depends on the wind up speed of the film $v_w$ [m/s], the width of the PVC-film $b$ [m], and the thickness of the film $d$ [m]

\[
\Phi_o = v_w b d \left[ \frac{m^3}{s} \right] \tag{3}
\]

Deviation of the input flow and the output flow are treated as disturbances on the output of the process. Now inserting (2) and (3) in (1) yields for $y(t)$
\[ y(t) = y(0) + \int_0^t (Cu(\tau - \tau_d) - v_w b_d) d\tau = y(0) + y_s + \Delta y(t) \]  

(4)

The constant level \( y_s \) is reached for \( \Phi_i = \Phi_o \) and \( u(t) = u_s \). Now \( u(t) \) is rewritten as

\[ u(t) = u_s + \Delta u(t) \]  

(5)

Combining (4) and (5) yields \( \Delta y(t) \)

\[ \Delta y(t) = C \int_0^t \Delta u(\tau - \tau_d) d\tau \]  

(6)

Taking the Laplace-transform

\[ \frac{\Delta Y(s)}{\Delta U(s)} = H_o(s) = \frac{Ce^{-s\tau_d}}{s} \]  

(7)

The process will be controlled by a digital controller. Therefore the process model is extended with a zero-order sample and hold. The sample time \( T_s = \tau / N \). This yields for \( y(k) \)

\[ y(k) = T_s C u(k - (N + 1)) + y(k - 1) \]  

(8)

### 2.2.3 Dead time estimation

The time necessary for the response of the output to a certain input, the dead time of the process, will be estimated using the open-loop step excitation experiment (v.d.Pas (1992) [4]). This experiment has been selected because of the large excitation level, making a clear distinction between the resulting process output \( y \) and the measurement noise \( m \), and because the process is disturbed minimally during this experiment.

The dead time \( \tau_d \) will be estimated using the cross correlation between the reference level \( r \) and the process output \( y \) (figure 3c). In the current situation no reference level exists, therefore the reference level \( r \) (figure 3b) is generated being the integral of the filling screw velocity \( u \) (figure 3a).

The result of the cross correlation between \( r \) and \( y \) is shown is figure 3d. The estimated dead time is \( \tau_d = 60 \) seconds.
2.2.4 Estimation of constant C

The estimation of the process constant C has been done using the experiment depicted in figure 4. In the first part of this experiment the velocity of the filling screw was too high, causing a rather sharp increase of the output level $y$. Next the velocity was decreased. This resulted in a slow decrease in the output level $y$. Somewhere in between a stabilizing velocity exists. Both parts are linearly approximated as depicted in figure 4 using $\tau_d$ derived in the previous

Figure 3. Dead time estimation.

Figure 4. Estimation of constant C.
paragraph.

The process constant $C$ is derived as follows:

$$C = \frac{1}{u_1-u_2} \left( \frac{Y_2-Y_1}{t_2-t_1} - \frac{Y_3-Y_2}{t_3-t_2} \right) = \frac{1}{0.653-0.557} \left( \frac{0.467-0.267}{625-0} - \frac{0.250-0.467}{2733-625} \right) = 4.4 \cdot 10^{-3} \left[ \frac{m^3}{rad} \right] \quad (9)$$
3 $H_\infty$ Robust controller design

3.1 Motivation

In an $H_\infty$ optimization procedure the known (nominal) part and the uncertain part of processes are modelled separately. No more information than upper bounds on errors is required to model the uncertain part of the process (unstructured uncertainty).

The advantages of using the $H_\infty$ robust controller design procedure are the guaranteed stability and performance of the nominal plant for modeled errors. Control goals such as disturbance reduction, tracking, sensor noise reduction and preventing the actuator from saturation are included during the design procedure. Not all control goals can be accomplished at the same time so a trade-off has to be made between them. In a $H_\infty$ controller design procedure these control goals are optimized by choosing appropriate weighting functions.

In this specific case, the $H_\infty$ design procedure yields profit because of the uncertainty in the process model, the large dead time, the large amount of sensor noise, the desired smooth actuator behaviour (temperature equilibrium) and the demand for disturbance reduction. Upper bounds can be derived for disturbances such as measurement noise and drift terms including process model uncertainty using previously done experiments. A smooth actuator behaviour can be obtained by using an appropriate weighting function.

3.2 Design procedure

3.2.1 Brief description of the $H_\infty$ controller design procedure

$H_\infty$ optimal control theory is a frequency domain approach which attempts to minimize the $H_\infty$-norm of a closed loop transfer function matrix. The $H_\infty$-norm of a transfer function matrix is the maximum over all frequencies of its largest singular value $\sigma$:

$$\|H\|_\infty = \max_{\omega} \sigma_i(\omega) \quad \text{with} \quad \sigma_1 > \sigma_i \quad \forall i, i \neq 1$$

(10)

For a SISO-system this turns out to be:
\[ \|H\|_\infty = \max_\omega |H(j\omega)| \]  

(11)

The standard unity gain feedback set up is depicted in figure 5. The symbols in this scheme are defined as follows:

- \( P(s) \): process
- \( K(s) \): controller
- \( \Delta(s) \): (process) uncertainty
- \( r \): reference signal
- \( e \): error signal
- \( d \): disturbance signal
- \( u \): actuator signal
- \( m \): measurement noise

Rewriting the feedback scheme in figure 5 into a general control scheme yields the system as depicted in figure 6. In this scheme vector \( w \) contains all external inputs, \( z \) contains all signals of interest, \( u \) contains the controller outputs and \( y \) contains the controller inputs:

\[
\begin{bmatrix}
  r \\
  d \\
  m
\end{bmatrix}
\begin{bmatrix}
  e \\
  y
\end{bmatrix}
\begin{bmatrix}
  u \\
  y
\end{bmatrix}
\]

(12)

Further, \( G \) is the generalized plant (including \( P \)) and \( K \) is the controller. In matrix notation the system is described by

\[
\begin{bmatrix}
  z \\
  y
\end{bmatrix} =
\begin{bmatrix}
  G_{11} & G_{12} \\
  G_{21} & G_{22}
\end{bmatrix}
\begin{bmatrix}
  w \\
  u
\end{bmatrix}
\]

(13)

\[ u = K_y \]

An admissible controller is a \( K(s) \) that stabilizes \( G \). Such an admissible \( K \) yields a closed loop system \( M \) described by

![Diagram](image.png)

Figure 5. Standard unity gain feedback set up.
\( z = Mw \)  
\[ M = G_{11} + G_{12}(I - G_{22}K)^{-1}G_{21} \]  

For stability robustness it is necessary that the closed loop \( M\Delta \) remains stable in the presence of the uncertainty bounded by \( \Delta \). To achieve this, the loop-gain \( \gamma \) should satisfy the condition

\[ \|M\Delta\|_\infty < \gamma < 1 \]  

(15)

So far it is only possible to state the stability robustness. To study the performance robustness, the problem is altered somewhat. Now the input and output vectors are also connected to the uncertainty \( \Delta \). By making a proper choice for the parameters for \( \Delta \) also performance robustness for a class of input and output signal can be achieved. To make this more explicit \( \Delta \) is written as \( V\Delta W \) with \( \|\Delta\|_\infty \leq 1 \), which is always possible as \( V \) and \( W \) are minimum-phase transfer functions (no poles or zeros in the right half plane). For this situation should hold

\[ \|WMV\|_\infty < \gamma \]  

or after pre-multiplication with \( W^T \) and post-multiplication with \( V^T \)

\[ \|M\|_\infty < \gamma \|W^{-1}V^{-1}\|_\infty = \frac{\gamma}{\|WV\|_\infty} \]  

(17)

The interpretation of (17) is as follows. Assume, for example, that for a process a maximum low frequency tracking error, from input \( r \) to error \( e \), of 0.1% (-60 dB) is allowed. For frequencies \( \omega \geq \omega_0 \) no specific demands are made. In this case the product, of the error filter and the reference filter, \( W, V, \) should have a low pass characteristic with a gain of at least 60 dB for \( \omega < \omega_0 \). In this case the calculation yields a controller \( K \) with \( \|M\|_{\infty} < 10^{-1} \) for \( \omega < \omega_0 \). If the demands are too high, no stabilizing controller \( K \) satisfying the conditions exists, so the filters have to be adjusted by weakening the demands.
3.2.2 Using MHC

The robust controller will be designed using the Multiple Input Multiple Output $H_\infty$ Control (MHC) software in Matlab written by ir.H.Falkus [6]. MHC is a general package that facilitates the controller design for various control configurations, the standard $H_\infty$ control problem and the closed-loop system evaluation. MHC calculates a controller minimizing infinity norm of the closed-loop response i.e. minimizing $\gamma$ in (17).

In MHC the design configuration can be build up of major blocks (figure 7): Process models $P_1$ and $P_2$, shaping filters $V$ for the input signals $w$, and weighting filters $W$ for the output signals $z$. The extra process block $P_2$ is sometimes necessary if there exists already a known feedback. The matrices $IM_1$ and $IM_2$ reflect the interconnection structure of the various blocks. These are constant matrices with entries $\pm 1$ and 0, each entry corresponding to a specific adding, subtracting or no connection of signals. The matrices $I_1$ to $I_4$ define the feed-through of signals which are necessary to build the state-space representation of the generalized $G$.

The structure used for the controller design is illustrated in figure 8. The encadred part is the real system (process plus controller(s)). The other part has been added for controller design purposes. As described before, the process model consists of a dead time plus an integrator. $u$ is the process input, and $y$ is the process output. $r$ is the reference level. $v$ combines the drift term in the output and the uncertainty of the process model. $m$ represents the measurement or sensor noise. The error $e$ is defined as the difference between the $N$ samples delayed reference level $r$ (input) and the output level $y$. Note that the error signal includes the drift term but does not include the measurement noise. The measurement noise is introduced by the sensor (i.e. the way the output is registered) and therefore it is not a process error.

In the scheme two controllers are added: a feed-forward controller $C_{ff}$ for the reference level and
a feedback controller $C_f$ for the output. To be able to calculate the feed-forward $C_f$ and the feedback controller $C_f$ fulfilling the control goals it is necessary to connect weighting functions to the inputs and the outputs of interest as described in the previous chapter.

The control structure of figure 8 is rewritten into the general MHC structure (figure 9).

3.2.3 Filter design

The filters are a translation of the control goals defined by the a-priori information about the inputs (filters $V$) and the demands made with respect to the outputs (filters $W$). This information mainly covers frequency characteristics, like low or high frequent behaviour and the number of integrations. The gain, the cut-off frequency and the order of these filters are optimized during the design procedure.

The a-priori information about the inputs and the outputs are briefly described in table 2. The incorporation of this information in the filters will follow afterwards.
Modelling and robust controller design of a PVC-film production process

Figure 9. Representation of the complete process model in MHC.

Table 2. A-priori information.

- **Reference input** $r$  
The reference level $r$ is changed stepwise.

- **Disturbance input** $v$  
The disturbances $v$ influencing the process are of an integrative nature. Examples are a change in the velocity of the mixing rolls and change in distance between the cutting knives.

- **Measurement noise** $m$  
The measurement noise $m$ is assumed to be white noise.

- **Actuator** $u$  
To maintain the temperature balance in the extruder, the actuator should be driven as smoothly as possible. This means that high frequencies should be removed from the actuator.

- **Error** $e$  
Error $e$ is only an artificial output of the system. The performance of the closed-loop system is mainly determined by this output. The demands with respect to $e$ are as follows: Both the tracking of the reference level and the disturbance reduction should have a steady-state error that is zero or tends to zero. No specific demands are made with respect to high frequency behaviour.

Filter structure design

As mentioned above, the filters are designed using the a-priori information about the inputs and the demands made with respect to the outputs. The way the a-priori information is incorporated...
in the filters is described below.

The filters will be determined using the steady-state error $e_s$ of the closed-loop transfer. According to (15) the closed-loop system, including the controller and the weighting filters, is bounded in $\infty$-norm by $\gamma$. For the closed-loop transfer function $H_{re}(z)$ from $r$ to $e$ this at least means

$$|H_{re}(z)| < \frac{\gamma}{|V_r(z)W_e(z)|} \tag{18}$$

Changes in the reference level $r$ are made stepwise. The steady-state tracking error to this input should be as small as possible. For the absolute value $e_s$ of the steady-state error of the step-response of $H_{re}(z)$ follows:

$$e_s = \lim_{z \to 1} \left| \frac{z-1}{z} H_{re}(z) \frac{z}{z-1} \right| = \lim_{z \to 1} |H_{re}(z)| < \lim_{z \to 1} \frac{\gamma}{|V_r(z)W_e(z)|} \tag{19}$$

To obtain a small finite steady-state error with no interest for high frequencies the product of the filters $V_r$ and $W_e$ should have a low-pass characteristic with a high gain. For example, to achieve a finite steady-state error less than 1% (0.01), the gain of $|V_rW_e|$ should be at least 100 for low frequencies. If a zero steady-state error is wanted then the product $V_rW_e$ should at least contain one integration. In this case a small finite steady-state error is chosen.

The disturbances acting on the process often have an integrative nature ($U(z)$ is a ramp). This type of disturbances should have (at least) a finite steady-state error. Excitation of the transfer $H_{re}(z)$ with a ramp yields an absolute steady-state error:

$$e_s = \lim_{z \to 1} \left| \frac{z-1}{z} H_{re}(z) \frac{T_z}{(z-1)^2} \right| = \lim_{z \to 1} \left| \frac{T_z}{z-1} H_{re}(z) \right| < \lim_{z \to 1} \frac{T_z}{z-1} \frac{1}{V_rW_e} \tag{20}$$

For a finite steady-state error $e_s$ this means that the product of $V_r$ and $W_e$ should contain one integration, cancelling the integration in the denominator.

By choosing the weighting filters in this way MHC is forced to find, if possible, a controller that satisfies the specifications. If no stabilizing controller yielding an $\infty$-norm $\gamma<1$ can be found, the specifications have to be altered (weakened).

The way the a-priori information is incorporated in the filters is described in table 3.

### 3.2.4 Controller calculation
Table 3. Incorporation of a-priori information in weighting filters.

- **Reference input** \( r \),
- **Output** \( y \)  
  To decrease the overshoot in case of a step at input \( r \) \( W_y \) has been assigned a constant \( c \) times \( V_r^{-1} \). The maximum amplification (overshoot) from \( r \) to \( y \) is now the maximum of the inverse of the product of \( V_r \) and \( W_y \): \( \max(V_r^{-1} W_y) = V_r^{-1} \).

- **Disturbance input** \( v \)  
  The integration in \( V_v \) expresses the integrative nature of the disturbances. Together with the filters \( W_u \) and \( W_e \) it determines the closed-loop transfer from \( v \) to \( u \) respectively \( e \).

- **Measurement noise** \( m \)  
  The measurement noise is expected have an equal power over the whole frequency range and is therefore a constant.

- **Actuator** \( u \)  
  The second order high-pass filter puts a heavy weight on high frequencies and rejects those frequencies from the actuator. As a result of the integration in the process model, all closed-loop transfers concerning \( u \) will have at least a +20 dB slope for low frequencies.

The algorithm used to solve the \( H_\infty \) problem is the Glover-Doyle algorithm; a proof of its validity is outlined by Doyle et al. (1988) [1]. Although the process model is rather simple the calculations will yield a controller of a relatively high order. This is due to the fact all perturbations and demands are added to this model as weighting functions. The order of the controller will be the sum of the order of the process model plus the order of all weighting functions: \( \text{O(Controller)} = \text{O(P)} + \text{O}(V_r) + \text{O}(V_v) + \text{O}(V_u) + \text{O}(W_e) + \text{O}(W_u) + \text{O}(W_y) = 5 + 1 + 0 + 1 + 2 + 1 = 11 \) with \( \text{O(P)} = z^4 + \text{integration} = 5 \).

In several steps the eleventh-order controller is found. Without too much performance reduction the controller can be reduced to sixth-order. To guarantee the robustness over a wide range of modelling errors and process disturbances, this controller is rather conservative.

The controller is determined iteratively. In every step the closed-loop transfers and the closed-loop time simulations, with the calculated controller embodied, from all inputs to all outputs are judged. Based on engineering insight the filters were adapted and have achieved their final structure. The controller fulfills the control goals but is not unique because the selection of the filters is rather subjective.
3.2.5 Properties of the controller

In table 4 the steady-state errors are given for various input signals of the closed-loop response from $r$, $v$ and $m$. From this table it can be seen that the steady-state error of the step response at input $r$ is almost zero. Further it is clear that step disturbances are cancelled completely and ramp disturbances are suppressed considerably. Measurement noise, that is impulse-like, is cancelled entirely.

Table 4. Steady-state error for various input signals.

<table>
<thead>
<tr>
<th>Input</th>
<th>Impulse</th>
<th>Step</th>
<th>Ramp</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r \rightarrow e$</td>
<td>0</td>
<td>$2.0 \cdot 10^{-3}$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$v \rightarrow e$</td>
<td>0</td>
<td>0</td>
<td>1.5</td>
</tr>
<tr>
<td>$m \rightarrow e$</td>
<td>0</td>
<td>1</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

Figure 10a shows the closed-loop frequency response from $r$ to $e$ and figure 10b shows the closed-loop frequency response from $m$ to $u$. The dashed line is the bound as expressed in (18). The solid line is the actual closed-loop response. The plots show the shaping of the closed-loop response by the weighting filters: Low frequent tracking is demanded and no high frequent actuator behaviour is allowed.

The closed-loop responses of all other input-output relation are given in appendix 2. In
appendix 3 the plots of the feedforward controller and feedback controller can be found.
4 Practical implementation

4.1 Software

4.1.1 Controller implementation

The implementation of the calculated controller in a PC consists of adjusting the previously written measurement software and adding new software for control purposes. The controller matrix $S$ has the following structure (between brackets the dimensions are given)

$$
S = \begin{bmatrix}
A(n \times n) & B(n \times 2) \\
\cdot & \cdot \\
C(1 \times n) & D(1 \times 2)
\end{bmatrix}
$$

and is read in from a MATLAB-file named $SMATRIX.MAT$. The system matrix $A$ of the controller is of variable order $n$, which is automatically detected by the program. The input matrix $B$ is a $n$ by 2 matrix. The first input is the reference $r$ level and the second input is the measured level $y$. The output matrix $C$ is a 1 by $n$ matrix mapping the states of the controller on the output $u$. The input-output matrix $D$ is, consequently, an 1 by 2 matrix.

The programs always starts in the 'measure'-mode. The user can switch to the 'control'-mode any time he likes. Switching from the 'measure'-mode to 'control'-mode, the current filtered output level $y$ is taken as reference level so no control action is needed in the first instance. The assumption is made that the operator has tuned the process parameters in a way that is close to the (unstable) equilibrium. In this situation (almost) no adjustments have to be made to the current angular velocity of the filling screw $u$. This $u$ is taken as $u_s$ (5); the start value of the process input.

4.1.2 Visualisation

In the previous version of the program only figures were displayed. Due to the rather high noise level and the slow behaviour of the process, it is rather difficult to detect trends in, for example, the output level. The control experiment require knowledge about the history of the process input and output to judge their behaviour.

To overcome these difficulties, the measured data is directly displayed in a graph on the screen. Actually two graphs are made: one containing the output level $y$ and the reference level $r$ and one containing $u$, $I_n$, $w_m$, $I_m$ or $w_r$, selectable by the user. The graphs can be scaled independently and contain information about the preceding ±15 minutes.
All measured data is stored in MATLAB compatible files for analyses purposes afterwards.

4.1.3 Automatic logging file

To be sure that all necessary information about the process condition during the experiment is stored the program asks for these conditions and stores it with the measured data. The logging file contains information of the kind of sheet produced, the width, etc.. Further the sample rate and the date and time of the experiment are stored. The logging data is stored in an ASCII-file. For an example see appendix 1.

4.2 Hardware

4.2.1 Adjustment for the actuator

Before starting the experiments some adaptations have to be made to generate a proper reference signal to control the angular velocity of the filling screw. In the current situation it is not (yet) possible to connect the digital-to-analogue converter directly to the velocity controller of the filling screw. The main reason therefore is that a failure of the computer may not lead to an uncontrollable situation or a filling screw standstill. The solution for this problem could be the usage of a PLC (programmable logic control). Until a PLC or similar solution is available a temporary solution is made.

This temporary solution is an imitation of the manual control situation. The operator uses a three function switch: slower (lowering the angular velocity), 0 (holding the angular velocity), faster (raising the angular velocity). The time the switch is set in the position slower or the position faster quantifies the decrease or the increase of the angular velocity of the filling screw. In other terms, the slower or faster signal is passed through an integrator. The output of the integrator is directly passed through to the control unit for the angular velocity of the filling screw.

In the automatic control situation the switch is replaced by two relays (figure 11): one selecting slower and one selecting faster; if neither of the two is selected, the current angular velocity is held. These relays are driven by the digital output of the DA-converter in the computer. At time $k$, the difference between the calculated $u(k)$ and $u(k-1)$ is converted to the pulling time of the relays, necessary to make the desired change of the filling screw velocity. The pulling time is discretised to a whole number of clock tick interrupts; the clock tick interrupt of the computer operates at a frequency of $18.2 \text{ Hz}$ ($1 \text{ tick is } \pm55 \text{ ms}$).

The main disadvantage of using this scheme for controlling the angular velocity of the filling
screw is adding integrator to the process. After a period of time (always) a difference will grow between the real angular velocity and the calculated angular velocity. This difference is compensated as described in the following paragraph.

### 4.2.2 Compensator for the switch time difference

The addition of the relais (figure 11) as time switch will lead to a difference between the calculated and the measured velocity of the filling screw, if no feedback is used.

Another fact, that showed up during the experiments, is that the velocity of the filling screw only can be set at discrete velocity's with a step size of approximately \(0.010 - 0.011 \text{ [rad/s]}\).

The switch time compensator is build up as follows. In the time between two control actions, the computer compares the set-point and the measured velocity every two seconds. If the difference is greater or equal to \(0.010\) then the half of the time necessary to compensate this difference the switch is set. Or in pseudo language:

\[
\text{Diff} = \text{Measured Velocity} - \text{Calculated Velocity} \\
\text{if } (\text{abs(Diff)} < 0.010) \text{ then} \\
\quad \text{Switch Time} = 0 \\
\text{else} \\
\quad \text{Switch Time} = \text{Tau} \times \text{Diff}/2 \\
\quad // \text{Remark: Up: Switch Time } > 0; \text{ Down: Switch Time } < 0 \\
\text{Do_Switch(Switch Time)}
\]

The choice to hold the switch only half the time necessary is made empirical. The actuator often responds with a small delay. So the previous adaption could not have taken effect yet. Setting the switch time to one yielded a very restless actuator signal. Setting the switch time back to

![Figure 11. Connection to process input for automatic control.](image)
one third did not improve the results.

4.2.3 Actuator saturation compensation

Although the saturation is unexpected it is better to prepare the controller for saturation of the actuator than leaving it on its own. Different types of saturation compensators will be compared in simulations. The compensator yielding the best results will be selected.

Three different schemes will be compared with the normal not saturated situation (table 5).

| Table 5. Different types of saturation compensators |
|------------------------------------------|------------------|
| Switch Time Compensator                  | Saturation Compensator |
| Normal                                   | Yes              |
| Type 1                                   | No               |
| Type 2                                   | Yes              |
| Type 3                                   | Yes              |

The saturation compensator type 1 is the so called 'anti-wind up'-compensator. The process input $u$ recovers from saturation as soon as the calculated velocity starts decreasing. Type 2 is the normal behaviour without any type of compensation. The controller takes its normal actions without knowing that the process input $u$ has saturated.

The saturation compensator of scheme number 3 functions as follows. If the process input $u$ saturates the difference between the measured velocity and the setpoint velocity is held. As soon as the process input $u$ recovers from saturation the missed part will be send afterward. In this case the integral over the non-saturated velocity and the saturated velocity is constant.

The outputs of interest $u$ and $y$ (or $e$) will be compared for two types of inputs:
1) A step at the reference input $r$;
2) A ramp at the disturbance input $d$.

These are the most likely situations in which a saturation of the actuator may occur. In a specific case, for example when the process is running close to the maximum production capacity, the excitation levels resulting from those inputs can be large enough to drive the process input in saturation.
The saturation level of the process input $u$ has been chosen at approximately 75% of the maximum excitation of the non-saturated response of the process input. If this level is taken much lower an uncontrollable situation would occur, in which even the best compensator does not lead to satisfying results. If this situation occurs during normal production, it means that something outside the process of interest is seriously wrong. It should not be expected that the controller can create material itself.

The results of the simulations are depicted in figure 12 and (22). The line types used in these figures agree with the types as displayed in table 6.

The simulations show a great difference between the different types of compensators. In this case, type 2 is superior to both type 1 and 3. This type of compensation shows the best behaviour for both the actuator and the output. Compensation type 3 leads to a large overshoot and a large step in the velocity, while a smooth behaviour of the actuator is required.

<table>
<thead>
<tr>
<th></th>
<th>Step $r$</th>
<th>Ramp $d$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$u/y$</td>
<td>$u/y$</td>
</tr>
<tr>
<td>Type 1</td>
<td>+/-</td>
<td>+/-</td>
</tr>
<tr>
<td>Type 2</td>
<td>+/+</td>
<td>+/+</td>
</tr>
<tr>
<td>Type 3</td>
<td>+/-</td>
<td>-/-</td>
</tr>
</tbody>
</table>

Figure 12. Saturated step-response from input $r$. 
Comparison of these types learns that the actuator saturation is compensated best by type 2.
5 Measurements

Two real control experiments have been selected to illustrate the controller behaviour: one experiment during a smooth running process and one during a permanently perturbed running process. For illustrative reasons the results of the experiments are filtered with an anti-causal fourth order Butterworth filter with cut-off frequency $1/20T_s$ and are depicted in figure 14 and 15. In figure 14b and 15b the dashed line is the set-point level and solid line is the measured level. The figure shows that the level is stabilized within a 0.04 error margin for the calm running process. In figure 14b also a typical manually controlled output is drawn. This level fluctuates over the full range.

In figure 14a and 15a the dashed line is the driving current of the main screw $I_m$ and the solid line is the velocity of the filling screw $u$. The driving current of the main screw $I_m$ reflects the torque delivered by the main screw. Increasing $\omega_i$ by turning $u$ up should result in an increment of $I_m$. During the first experiment this correlation is clearly visible. Due to variations in the filling degree in the funnel and variations in the mixture of material in the funnel, this correlation has disappeared partly in the second experiment. In this case, the output level is stabilized within an 0.07 error margin except for one major disturbance. The cause of this disturbance was an almost empty funnel. During the disturbance the funnel is only filled with granulate causing a sharp decrease in $C$. After refilling the output level increases rather far above its set-point but stabilizes afterwards. Simulations also showed this behaviour because of the change in $C$ twice within a short time.

![Figure 14. Calm running process under control (filtered signals).](image-url)
Figure 15. Perturbed process under control (filtered signals).
6 PID-controller design with Smith-predictor

In this section a comparison will be made between the robust controller design for this plant and a classical PID-controller design combined with a Smith-predictor. The Smith-predictor was proposed in the 1950s to improve the closed-loop performance for systems with time-delay.

The process dealt with is apparently a simple process: First order process plus dead time. For this type of process a classical PID-controller plus Smith-predictor should offer sufficient degrees of freedom to control the process satisfactory. In figure 16 the control scheme is depicted. In this scheme $P_n$ is the real process, $P_m$ is the process model and $P_m^*$ is the process model without dead time. A second order Butterworth low pass filter is added to smoothen the actuator signal.

The controller is tuned for the nominal situation and yields comparable results as with the robust controller. However, the robust controller performs considerably better in case of parameter variations and process disturbances; resulting in a larger stability margin and a better sensor noise reduction.

Figure 16. Control structure with Smith-predictor and filter.
7 Conclusion

A simple process model, which is based on a-priori knowledge of the process and some simple measurements, offers sufficient information to design a stabilizing $H_\infty$-controller. The process is stabilized for a wide range of modelling errors and process disturbances.

Owing to this control configuration, the variance of the level reduces considerably which allows a lower reference level. This, in turn, yields a better product quality (less temperature drop due to a smaller stay) while running empty is still prevented.

By controlling the level of PVC on top of the mixing rolls a better conditioning of the production process is achieved without influencing the temperature balance in the extruder too much. Stabilizing this part of the process also means that the following part of the process becomes settled.
Literature


## Variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma )</td>
<td>( H_\gamma )-norm</td>
</tr>
<tr>
<td>( \Delta(s), \Delta(z) )</td>
<td>Uncertainty in ( s ) and ( z ) domain</td>
</tr>
<tr>
<td>( \tau_d )</td>
<td>[s] Dead time</td>
</tr>
<tr>
<td>( \Phi_i )</td>
<td>([m^2/s]) Input flow</td>
</tr>
<tr>
<td>( \Phi_o )</td>
<td>([m^2/s]) Output flow</td>
</tr>
<tr>
<td>( \omega_m )</td>
<td>([rad/s]) Velocity main screw</td>
</tr>
<tr>
<td>( \omega_r )</td>
<td>([m/s]) Velocity mixing rolls</td>
</tr>
<tr>
<td>( b )</td>
<td>([m]) Width film</td>
</tr>
<tr>
<td>( C )</td>
<td>Integration constant</td>
</tr>
<tr>
<td>( C_{fb} )</td>
<td>Feedback controller</td>
</tr>
<tr>
<td>( C_{ff} )</td>
<td>Feedforward controller</td>
</tr>
<tr>
<td>( d )</td>
<td>([m]) Distance between the cutting knives</td>
</tr>
<tr>
<td>( d )</td>
<td>Disturbance input</td>
</tr>
<tr>
<td>( e )</td>
<td>Error: ( r - y )</td>
</tr>
<tr>
<td>( G(s), G(z) )</td>
<td>Generalized process in ( s ) and ( z ) domain</td>
</tr>
<tr>
<td>( H(s), H(z) )</td>
<td>General transfer function in ( s ) and ( z ) domain</td>
</tr>
<tr>
<td>( I_f )</td>
<td>([A]) Driving current filling screw</td>
</tr>
<tr>
<td>( I_m )</td>
<td>([A]) Driving current main screw</td>
</tr>
<tr>
<td>( K )</td>
<td>([m^2/rad]) Gain filling screw</td>
</tr>
<tr>
<td>( K(s), K(z) )</td>
<td>Controller in ( s ) and ( z ) domain</td>
</tr>
<tr>
<td>( m )</td>
<td>Measurement noise input</td>
</tr>
<tr>
<td>( M(s), M(z) )</td>
<td>Closed-loop ( G - K )</td>
</tr>
<tr>
<td>( N )</td>
<td>Dead time in number of samples</td>
</tr>
<tr>
<td>( P(s), P(z) )</td>
<td>Process in ( s ) and ( z ) domain</td>
</tr>
<tr>
<td>( r )</td>
<td>Reference input</td>
</tr>
<tr>
<td>( T_s )</td>
<td>[s] Sample time</td>
</tr>
<tr>
<td>( U(s), U(z) )</td>
<td>Laplace- and Z-transform of ( u )</td>
</tr>
<tr>
<td>( u )</td>
<td>([rad/s]) Actuator. Filling screw velocity</td>
</tr>
<tr>
<td>( u_s )</td>
<td>([rad/s]) Stabilizing filling screw velocity</td>
</tr>
<tr>
<td>( V_r, V_d, V_m )</td>
<td>Input weighting filters for resp. ( r, d, m ).</td>
</tr>
<tr>
<td>( v_w )</td>
<td>([m/s]) Wind up speed film</td>
</tr>
<tr>
<td>( W_r, W_d, W_y )</td>
<td>Output weighting filters for resp. ( u, e, y )</td>
</tr>
<tr>
<td>( y )</td>
<td>Output level</td>
</tr>
<tr>
<td>( Y(s), Y(z) )</td>
<td>Laplace- and Z-transform of process output ( y )</td>
</tr>
</tbody>
</table>
Appendix 1 - Logfile

Meting soort : aaaaaaaaaaaaaaaaaaaaaaaaaaaaa
Datum (start) : dd-mm-yyyy
Tijd (start) : hh:mm:ss
ADC data : adcnnn.mat
Vision data : visnnn.mat
Logfile : lognnn.txt

Folie gegevens:
Charge nummer : Mnnn.nnn
Folie soort : aa.nn
Folie breedte : nnnn mm
Folie dikte : nn um
Vel dikte : nn um

Data gegevens:
Sample freq. : nn.nn Hz
Beeld data :
Venster : xw x yw : nnn x nnn = nnnnn
xs : nnn yx : nnn
Belichting : background or foreground
ADC data : 

Aantal samples : nnnnn
Datum (eind) : dd-mm-yyyy
Tijd (eind) : hh:mm:ss

----------- Einde logfile nr nn -----------
Appendix 2 - Closed-loop plots

Figure 17. Closed-loop r-u (solid) and 1/V,W_e (dashed).

Figure 18. Closed-loop r-e (solid) and 1/V,W_e (dashed).
Figure 19. Closed-loop r-y (solid) and 1/V_r W_r (dashed).

Figure 20. Closed-loop d-u (solid) and 1/V_d W_u (dashed).
Figure 21. Closed-loop $d-e$ (solid) and $1/V_W$ (dashed).

Figure 22. Closed-loop $d-y$ (solid) and $1/V_W$ (dashed).
Figure 23. Closed-loop $m-u$ (solid) and $1/V_m W_u$ (dashed).

Figure 24. Closed-loop $m-e$ (solid) and $1/V_m W_e$ (dashed).
Figure 25. Closed-loop $m-y$ (solid) and $1/V_mW_y$ (dashed).
Appendix 3 - Controller plots

Figure 26. Feedforward controller $C_f$

Figure 27. Feedback controller $C_f$
Summary

The control problem discussed concerns stabilizing the material level on top of mixing rolls at the Hoechst PVC-film production process. As process input, the velocity of the screw in the filling section is used. The process output is the level of material, measured using a vision system. The main challenges of this control situation are the large dead time between input and output, the open-loop instability of the process, the large parameter uncertainty, the large measurement noise and the integrator-like drift terms. Stabilizing the material level at a guaranteed low level yields a better product quality (fewer temperature variations) and less spill over.

For this process a sixth-order robust $H\infty$ controller has been designed. Simulations and experiments at the plant have been carried out in order to study the properties and the effectiveness of the controller. Experiments with the robust controller indicate an improved dynamic behaviour. Also a comparison is made with respect to a Smith-predictor with PID-controller design.
1 Introduction

This paper describes a project carried out at Hoechst Holland N.V. in Weert in cooperation with Eindhoven University of Technology. The three main products produced at the plant in Weert are PVC-film/sheet, Trespa boards and Depron polystyrene trays. The project is implemented at a calender installation of the PVC-film/sheet production process.

Product quality is often the main issue in industrial processes. Complex processes are controlled satisfactorily by trained operators. As the demands on product quality and production efficiency grow and the tasks of the operator become more complex, automatic control of (sub-)processes is required. Controlling the dynamic behaviour of a process requires investigation of those dynamics. A model of all relevant process dynamics and of the disturbances has to be obtained for optimal disturbance rejection.

The project includes process identification, process model estimation, disturbance modelling, $H_\infty$ controller calculation, simulation of the process under control and measurements at the plant. The simulation results of the $H_\infty$ controller will be compared with a conventional PID-controller.
2 Process

2.1 Introduction

The PVC-sheet film production process consists roughly of the following parts (figure 1). The production starts at the raw material supply (a funnel) which is continuously feeding the transport section of the extruder. The material is transported into the extruder in which the raw material is being transformed into a kind of gel by heating and friction (gelatination). The temperature is an important parameter in the gelatination-process (\( |\Delta T/T| < 3\% \)). If the temperature is too low the gelatination-process stops, on the other hand if the temperature is too high the material burns in the extruder.

After leaving the extruder, the material falls into a vibrating conveyor, through which it is transported towards the mixing rolls. A skin is formed, around one of the two mixing rolls. The sheet is cut from the mixing roll by two knives and is transported over a conveyor belt towards the first calender gap. From here the sheet is stretched mainly in the longitudinal direction (up to ten times the original length). Finally the sheet is wound up.

The process of interest is the process from the raw material supply towards material level on top of the mixing rolls. The process input is the angular velocity of the screw in the funnel (filling screw). The amount of material delivered (\( \Phi \ [m^3/s] \)) to the extruder is (approximately) linearly dependent on the angular velocity of the filling screw (\( u \ [rad/s] \)).

The output of the process is measured with a vision system c.f. [4]. The vision hardware in the computer grabs a two-dimensional image seen by a camera pointed at the scene. Each pixel of the discretized image is compared with a user-defined threshold. The number of samples above the threshold is supposed to be a quantitative, linearly dependent measure for the amount of material on top of the mixing rolls. The material on top of the mixing rolls is continuously in motion, due to the rotation of the mixing rolls and the input and output flow of material. Consequently, a large uncertainty (formulated as noise) is introduced in the measured output level.

![Figure 1. Schematic overview of the PVC-film production process.](image-url)
Control goals are a small steady state tracking error, disturbance reduction (drift and oscillations), compensations of changes in input and output flow, smooth actuator behaviour, in order to guarantee the temperature balance, and insensitivity to measurement noise.

2.2 Modelling the process

2.2.1 Introduction

The input flow \( \Phi_i \) is controlled by the filling screw \( u \). The material goes through the transport lag \( (\tau_d [s]) \) and is gathered at the mixing rolls. The output is a continuous flow \( \Phi_o [m^3/s] \) taken by the sheet leaving the mixing rolls. The amount of material on top of the mixing rolls \( y(t) [m^3] \) as function of the input flow and the output flow is:

\[
y(t) = y(0) + \int_{0}^{t} [\Phi_i(\tau - \tau_d) - \Phi_o(\tau)] d\tau
\]

The input flow and output flow of this process are continuously disturbed by changes in set-points of process parameters and fluctuations of the compound of the raw material. In table 1 the most important disturbances on input flow and output flow are mentioned.

Linearity

Linearity tests have been performed on the open-loop process. The main problem during these experiments is the process output \( y \) drifting away continuously, due to the integration in the process. This makes it rather difficult to say something about the linearity of the process. The experiments indicate linearity of the process, although saturation effects in the actuator may occur. Apparent non-linearities could be caused by drifting away of \( y \).

2.2.2 Mathematical process model

A mathematical model, containing only the main properties of the process, will be derived. This model will be used to design a stabilizing controller for the process. The approximation of the relation between the input flow and \( u(t) \) is given by

\[
\Phi_i(t) = C u(t) \left[ \frac{m^3}{s} \right] = \left[ \frac{m^3}{rad} \right] \left[ \frac{rad}{s} \right]
\]

where \( C \) is a constant.

The output flow \( \Phi_o \) depends on the wind up speed of the film \( \nu_w [m/s] \), the width of the PVC-film \( b [m] \), and the thickness of the film \( d [m] \)

\[
\Phi_o = \nu_w b d \left[ \frac{m^3}{s} \right] = \left[ \frac{m}{s} \right] \left[ m \right] \left[ m \right]
\]

Deviation of the input flow and the output flow are treated as disturbances on the output of the process. Now inserting (2) and (3) in (1) yields for \( y(t) \):
Table 1. Disturbances of the input flow \( \Phi_i \) and the output flow \( \Phi_o \)

<table>
<thead>
<tr>
<th>Disturbance of the input flow. A positive step on:</th>
<th>Effect on ( y ):</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. the angular velocity of the filling screw (( u ));</td>
<td>Ramp (+)</td>
</tr>
<tr>
<td>2. the angular velocity of the main screw (( \omega_m ));</td>
<td>Step (+)</td>
</tr>
<tr>
<td>3. the amount of material in the funnel i.e. the pressure in the funnel;</td>
<td>Ramp (+)</td>
</tr>
<tr>
<td>4. the amount of gelatination used;</td>
<td>Ramp (-)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Disturbance on output flow. A positive step on:</th>
<th>Effect on ( y ):</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. the angular velocity of the mixing rolls (( \omega_r ));</td>
<td>Ramp (-)</td>
</tr>
<tr>
<td>2. the distance between the two cutting knives (( d ));</td>
<td>Ramp (-)</td>
</tr>
<tr>
<td>3. the distance between the mixing rolls (hardly ever changed);</td>
<td>Step (-)</td>
</tr>
<tr>
<td>Further:</td>
<td></td>
</tr>
<tr>
<td>4. Manual removal of the skin around the mixing roll. This material is mostly used for obtaining a quick increase in the amount of material in the filling of the first calender gap;</td>
<td>Step (+)</td>
</tr>
<tr>
<td>5. Manual addition of material, which is removed from the conveyer belt to obtain a fast decrease in the amount of material in the filling of the first calender gap.</td>
<td></td>
</tr>
</tbody>
</table>

\[
y(t) = y(0) + \int_0^t (Cu(\tau - \tau_d) - \nu_w b d) d\tau = y(0) + y_s + \Delta y(t) \tag{4}
\]

The constant level \( y_s \) is reached for \( \Phi_i = \Phi_o \) and \( u(t) = u_s \). Now \( u(t) \) is rewritten as

\[
u(t) = u_s + \Delta u(t) \tag{5}
\]

Combining (4) and (5) yields \( \Delta y(t) \)

\[
\Delta y(t) = Cu \int_0^t (\tau - \tau_d) d\tau \tag{6}
\]

Taking the Laplace-transform

\[
\frac{\Delta Y(s)}{\Delta U(s)} = H_p(s) = \frac{Ce^{-\tau_d}}{s} \tag{7}
\]

The process will be controlled by a digital controller. Therefore the process model is extended with a zero-order sample and hold. The sample time \( T_s = \tau/N \). This yields for \( y(k) \)

\[
y(k) = T_s Cu(k - (N + 1)) + y(k-1) \tag{8}
\]
2.2.3 Dead time estimation

The time necessary for the response of the output to a certain input, the dead time of the process, will be estimated using the open-loop step excitation experiment (v.d.Pas [4]). This experiment has been selected because of the large excitation level, making a clear distinction between the resulting process output $y$ and the measurement noise $m$, and because the process is disturbed minimally during this experiment.

The dead time $T_d$ will be estimated using the cross correlation between the reference level $r$ and the process output $y$ (figure 2c). In the current situation no reference level exists, therefore the reference level $r$ (figure 2b) is generated being the integral of the filling screw velocity $u$ (figure 2a).

The result of the cross correlation between $r$ and $y$ is shown is figure 2d. The estimated dead time is $T_d=60$ seconds.

2.2.4 Estimation of constant C

The estimation of the process constant $C$ has been done using the experiment depicted in figure 3. In the first part of this experiment the velocity of the filling screw was too high, causing a rather sharp increase of the output level $y$. Next the velocity was decreased. This resulted in a slow decrease in the output level $y$. Somewhere in between a stabilizing velocity exists. Both parts are linearly approximated as depicted in figure 3 using $T_d$ derived in the previous paragraph.

![Figure 2. Dead time estimation.](image_url)
The process constant $C$ is derived as follows:

$$ C = \frac{1}{u_1 - u_2} \left( \frac{y_2 - y_1}{t_2 - t_1} - \frac{y_3 - y_2}{t_3 - t_2} \right) = \frac{1}{0.653 - 0.557} \left( \frac{0.467 - 0.267}{625 - 0} - \frac{0.250 - 0.467}{2733 - 625} \right) = 4.4 \cdot 10^{-3} \text{[m}^3\text{rad]} \quad (9) $$
3 $H_\infty$ Robust controller design

3.1 Motivation

In an $H_\infty$ optimization procedure the known (nominal) part and the uncertain part of processes are modelled separately. No more information than upper bounds on errors is required to model the uncertain part of the process (unstructured uncertainty).

The advantages of using the $H_\infty$ robust controller design procedure are the guaranteed stability and performance of the nominal plant for modelled errors. Control goals such as disturbance reduction, tracking, sensor noise reduction and preventing the actuator from saturation are included during the design procedure. Not all control goals can be accomplished at the same time so a trade-off has to be made between them. In a $H_\infty$ controller design procedure these control goals are optimised by choosing appropriate weighting functions.

In this specific case, the $H_\infty$ design procedure yields profit because of the uncertainty in the process model, the large dead time, the large amount of sensor noise, the desired smooth actuator behaviour (temperature equilibrium) and the demand for disturbance reduction. Upper bounds can be derived for disturbances such as measurement noise and drift terms including process model uncertainty using previously done experiments. A smooth actuator behaviour can be obtained by using an appropriate weighting function.

3.2 Design procedure

3.2.1 Brief description of the $H_\infty$ controller design procedure

$H_\infty$ optimal control theory is a frequency domain approach which attempts to minimize the $H_\infty$-norm of a closed loop transfer function matrix. The $H_\infty$-norm of a transfer function matrix is the maximum over all frequencies of its largest singular value $\sigma_i$:

$$\|H\|_\infty = \max_\omega \sigma_i(\omega) \quad \text{with} \quad \sigma_i > \sigma_i \quad \forall i, i \neq 1$$

For a SISO-system this turns out to be:

$$\|H\|_\infty = \max_\omega |H(j\omega)|$$

The standard unity gain feedback set up is depicted in figure 4 [2]. The symbols in this scheme are defined as follows:

- $P(s)$: process
- $r$: reference signal
- $y$: process output
- $K(s)$: controller
- $d$: disturbance signal
- $e$: error signal
- $\Delta(s)$: (process) uncertainty
- $m$: measurement noise
- $u$: actuator signal

Rewriting the feedback scheme in figure 4 into a general control scheme yields the system as depicted in figure 5. In this scheme vector $w$ contains all external inputs, $z$ contains all signals of interest, $y$ contains the controller outputs and $x$ contains the controller inputs:
Further, \( G \) is the generalized plant (including \( P \)) and \( K \) is the controller. In matrix notation the system is described by

\[
\begin{pmatrix}
G_{11} & G_{12} \\
G_{21} & G_{22}
\end{pmatrix}
\begin{pmatrix}
w \\
u
\end{pmatrix}
\]

\( u = Ky \)  

An admissible controller is a \( K(s) \) that stabilizes \( G \). Such an admissible \( K \) yields a closed loop system \( M \) described by

\[
\begin{pmatrix}
z \\
y
\end{pmatrix}
= \begin{pmatrix}
G_{11} & G_{12} \\
G_{21} & G_{22}
\end{pmatrix}
\begin{pmatrix}
w \\
u
\end{pmatrix}
\]

\[ M = G_{11} + G_{12}(I - G_{22}K)^{-1}G_{21} \]  

For stability robustness it is necessary that the closed loop \( M\Delta \) remains stable in the presence of the
uncertainty bounded by $\Delta$. To achieve this, the loop-gain $\gamma$ should satisfy the condition
\[ |M\Delta|_\infty = \gamma < 1 \] (15)

So far it is only possible to state the stability robustness. To study the performance robustness, the problem is altered somewhat. Now the input and output vectors are also connected to the uncertainty $\Delta$. By making a proper choice for the parameters for $\Delta$ also performance robustness for a class of input and output signal can be achieved. To make this more explicit $\Delta$ is written as $V\Delta W$ with $\|\Delta\|_\infty \leq 1$, which is always possible as $V$ and $W$ are minimum-phase transfer functions (no poles or zeros in the right half plane). For this situation should hold
\[ |WMV|_\infty < \gamma \] (16)

or after pre-multiplication with $W^{-1}$ and post-multiplication with $V^{-1}$
\[ |M|_\infty < \gamma \frac{|W^{-1}V^{-1}|_\infty}{|WV|_\infty} = \frac{\gamma}{|WV|_\infty} \] (17)

The interpretation of (17) is as follows. Assume, for example, that for a process a maximum low frequency tracking error, from input $r$ to error $e$, of 0.1% (-60 dB) is allowed. For frequencies $\omega \geq \omega_0$ no specific demands are made. In this case the product of the error filter and the reference filter, $W_r V_r$, should have a low pass characteristic with a gain of at least 60 dB for $\omega < \omega_0$. In this case the calculation yields a controller $K$ with $|M|_\infty < 10^3$ for $\omega < \omega_0$. If the demands are too high, no stabilizing controller $K$ satisfying the conditions exists, so the filters have to be adjusted by weakening the demands.

### 3.2.2 Using MHC

The robust controller will be designed using the Multiple Input Multiple Output $H_\infty$ Control (MHC) software in Matlab [5]. MHC is a general package that facilitates the controller design for various control configurations, the standard $H_\infty$ control problem and the closed-loop system evaluation. MHC calculates a controller by minimizing the $H_\infty$-norm $\gamma$ of the closed-loop response (17).

The structure used for the controller design is illustrated in figure 6. The encadred part is the real system (process plus controller(s)). The other part has been added for controller design purposes.

Figure 6. Addition of weighting filters to the process.
Note that the error signal $e$ includes the drift term but does not include the measurement noise. The measurement noise is introduced by the sensor (i.e. the way the output is registered) and therefore it is not a process error.

The scheme includes two controllers, to be calculated, a feed-forward controller $C_{ff}$ for the reference level and a feedback controller $C_J$ for the output. To calculate the feed-forward $C_{ff}$ and the feedback controller $C_J$, it is necessary to connect weighting functions to the inputs and the outputs of interest.

### 3.2.3 Filter design

The filters are a translation of the control goals defined by the a-priori information about the inputs (filter $V$) and the demands made with respect to the outputs (filters $W$). This information mainly covers frequency characteristics, like low or high frequent behaviour and the number of integrations. The gain, the cut-off frequency and the order of these filters are optimized during the design procedure.

The a-priori information about the inputs and the outputs are briefly described in table 2. The incorporation of this information in the filters will follow afterwards.

<table>
<thead>
<tr>
<th>Table 2. A-priori information.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reference input $r$</strong></td>
</tr>
<tr>
<td><strong>Disturbance input $v$</strong></td>
</tr>
<tr>
<td><strong>Measurement noise $m$</strong></td>
</tr>
<tr>
<td><strong>Actuator $u$</strong></td>
</tr>
<tr>
<td><strong>Error $e$</strong></td>
</tr>
<tr>
<td><strong>Output $y$</strong></td>
</tr>
</tbody>
</table>
Filter structure design

As mentioned above, the filters are designed using the a-priori information about the inputs and the demands made with respect to the outputs. The way the a-priori information is incorporated in the filters is described below.

The filters will be determined using the steady-state error $e_s$ of the closed-loop transfer. According to (15) the closed-loop system, including the controller and the weighting filters, is bounded in $\infty$-norm by 1. For the closed-loop transfer function $H_\infty(z)$ from $r$ to $e$ this at least means

$$ |H_\infty(z)| < \frac{\gamma}{|V_r(z)W_e(z)|} $$  \hspace{1cm} (18)

Changes in the reference level $r$ are made stepwise. The steady-state tracking error to this input should be as small as possible. For the absolute value $e_s$ of the steady-state error of the step-response of $H_\infty(z)$ follows:

$$ e_s = \lim_{z \to 1} \left| \frac{z-1}{z} H_\infty(z) \frac{z}{z-1} \right| = \lim_{z \to 1} |H_\infty(z)| < \lim_{z \to 1} \frac{\gamma}{|V_r(z)W_e(z)|} $$  \hspace{1cm} (19)

To obtain a small finite steady-state error with no interest for high frequencies the product of the filters $V_r$ and $W_e$ should have a low-pass characteristic with a high gain. For example, to achieve a finite steady-state error less than 1% ($0.01$), the gain of $|V_rW_e|$ should be at least $100$ for low frequencies. If a zero steady-state error is wanted then the product $V_rW_e$ should at least contain one integration. In this case a small finite steady-state error is chosen.

The disturbances acting on the process often have an integrative nature ($U(z)$ is a ramp). This type of disturbances should have (at least) a finite steady-state error. Excitation of the transfer $H_\infty(z)$ with a ramp yields an absolute steady-state error:

$$ e_s = \lim_{z \to 1} \left| \frac{z-1}{z} H_\infty(z) \frac{T_s}{(z-1)^2} \right| = \lim_{z \to 1} \frac{T_s}{z-1} \frac{H_\infty(z)}{z-1} < \lim_{z \to 1} \frac{T_s}{z-1} \frac{1}{V_rW_e} $$  \hspace{1cm} (20)

For a finite steady-state error $e_s$ this means that the product of $V_r$ and $W_e$ should contain one integration, cancelling the integration in the denominator.

By choosing the weighting filters in this way MHC is forced to find, if possible, a controller that satisfies the specifications. If no stabilizing controller yielding an $\infty$-norm smaller than one can be found, the specifications have to be altered (weakened).

The way the a-priori information is incorporated in the filters is described in table 3.

3.2.4 Controller calculation

The algorithm used to solve the $H_\infty$ problem is the Glover-Doyle algorithm; a proof of its validity is outlined by Doyle [1]. Although the input output relation of the process is rather simple, the complete process model including the weighting filters, representing the disturbances and the control goals, yield a model of eleventh-order. As result an eleventh-order controller is found. Without too much performance reduction the controller can be reduced to sixth-order. To
Table 3. Incorporation of a-priori information in weighting filters.

- **Reference input** $r$.
- **Output** $y$  
  To decrease the overshoot in case of a step at input $r$ $W_r$ has been assigned a constant $c$ times $V_r^{-1}$. The maximum amplification (overshoot) from $r$ to $y$ is now the maximum of the inverse of the product of $V_r$ and $W_r$: $\max(\gamma/V_r W_r) = \gamma/c$.

- **Disturbance input** $v$.  
  The integration in $V_v$ expresses the integrative nature of the disturbances. Together with the filters $W_v$ and $W_e$ it determines the closed-loop transfer from $v$ to $u$ respectively $e$.

- **Measurement noise** $m$.  
  The measurement noise is expected to be white over the frequency range of interest and is therefore a constant.

- **Actuator** $u$.  
  The second order high-pass filter puts a heavy weight on high frequencies and rejects those frequencies from the actuator. As a result of the integration in the process model, all closed-loop transfers concerning $u$ will have at least a $+20$ dB slope for low frequencies.

- **Error** $e$.  
  The error is only of interest in the low frequency range. In the filter $W_e$ this expressed as a low-pass filter.

guarantee the robustness over a wide range of modelling errors and process disturbances, this controller is rather conservative.

The controller is determined iteratively. In every step the closed-loop transfers and the closed-loop time simulations, with the calculated controller embodied, from all inputs to all outputs are judged. Based on engineering insight the filters were adapted and have achieved their final structure. The controller fulfils the control goals but is not unique because the selection of the filters is rather subjective.

### 3.2.5 Properties of the controller

In table 4 the steady-state errors are given for various input signals of the closed-loop response from $r$, $v$ and $m$. From this table it can be seen that the steady-state error of the step response at input $r$ is almost zero. Further it is clear that step disturbances are cancelled completely and ramp disturbances are suppressed considerably. Measurement noise, that is impulse-like, is cancelled entirely.
Table 4. Steady-state error for various input signals.

<table>
<thead>
<tr>
<th></th>
<th>Impulse</th>
<th>Step</th>
<th>Ramp</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r \rightarrow e$</td>
<td>0</td>
<td>$2.0 \cdot 10^3$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$v \rightarrow e$</td>
<td>0</td>
<td>0</td>
<td>1.5</td>
</tr>
<tr>
<td>$m \rightarrow e$</td>
<td>0</td>
<td>1</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

Figure 7a shows the closed-loop frequency response from $r$ to $e$ and figure 7b shows the closed-loop frequency response from $m$ to $u$. The dashed line is the bound as expressed in (18). The solid line is the actual closed-loop response. The plots show the shaping of the closed-loop response by the weighting filters: Low frequent tracking is demanded and no high frequent actuator behaviour is allowed.

Figure 7. Closed-loop transfer functions.
4 Measurements

Two real control experiments have been selected to illustrate the controller behaviour: one experiment during a smooth running process and one during a permanently perturbed running process. For illustrative reasons the results of the experiments are filtered with an anti-causal fourth order Butterworth filter with cut-off frequency $1/20T$, and are depicted in figure 8 and 9. In figure 8b and 9b the dashed line is the set-point level and solid line is the measured level. The figure shows that the level is stabilized within a 0.04 error margin for the calm running process. In figure 8b also a typical manually controlled output is drawn. This level fluctuates over the full range.

In figure 8a and 9a the dashed line is the driving current of the main screw $I_m$ and the solid line is the velocity of the filling screw $u$. The driving current of the main screw $I_m$ reflects the torque delivered by the main screw. Increasing $\phi$ by turning $u$ up should result in an increment of $I_m$. During the first experiment this correlation is clearly visible. Due to variations in the filling degree in the funnel and variations in the mixture of material in the funnel, this correlation has disappeared partly in the second experiment. In this case, the output level is stabilized within an 0.07 error margin except for one major disturbance. The cause of this disturbance was an almost empty funnel. During the disturbance the funnel is only filled with granulate causing a sharp decrease in $C$. After refilling the output level increases rather far above its set-point but stabilizes afterwards. Simulations also showed this behaviour because of the change in $C$ twice within a short time.

![Figure 8. Calm running process under control (filtered signals).](image-url)
Figure 9. Perturbed process under control (filtered signals).
5 PID-controller design with Smith-predictor

In this section a comparison will be made between the robust controller design for this plant and a classical PID-controller design combined with a Smith-predictor. The Smith-predictor was proposed in the 1950s to improve the closed-loop performance for systems with time-delay.

The process dealt with is apparently a simple process: First order process plus dead time. For this type of process a classical PID-controller plus Smith-predictor should offer sufficient degrees of freedom to control the process satisfactory. In figure 10 the control scheme is depicted. In this scheme $P_n$ is the real process, $P_m$ is the process model and $P_m^*$ is the process model without dead time. A second order Butterworth low pass filter is added to smoothen the actuator signal.

The controller is tuned for the nominal situation and yields comparable results as with the robust controller. However, the robust controller performs considerably better in case of parameter variations and process disturbances; Resulting in a larger stability margin and a better sensor noise reduction.

![Figure 10. Control structure with Smith-predictor and filter.](image-url)
6 Conclusion

A simple process model, which is based on a-priori knowledge of the process and some simple measurements, offers sufficient information to design a stabilizing $H_i$-controller. The process is stabilized for a wide range of modelling errors and process disturbances.

Owing to this control configuration, the variance of the level reduces considerably which allows a lower reference level. This, in turn, yields a better product quality (less temperature drop due to a smaller stay) while running empty is still prevented.

By controlling the level of PVC on top of the mixing rolls a better conditioning of the production process is achieved without influencing the temperature balance in the extruder too much. Stabilizing this part of the process also means that the following part of the process becomes settled.
References


