Depolarisation analysis
of the Olympus beacon signals

by

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Graduation Thesis

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Abstract

The analysis of depolarisation on micro-wave signals is of great importance because in case of microwave radio communications every frequency band is dually used, by using both orthogonal polarisation directions ("frequency reuse"). As soon as the propagation medium becomes anisotropic, due to the presence of raindrops or ice crystals, depolarisation will occur, which will result in crosstalk between the two transmitted signals. In order to predict the performance of micro-wave communication systems and to enlarge the insight in the depolarising influence of the atmosphere, the EUT examines the depolarisation of satellite beacon signals.

In the ground station of the EUT, the 12.5, 20 and 30 GHz beacon signals of the satellite Olympus are received with a 5.5 meter Cassegrain antenna. The first step in the signal analysis is the removal of the depolarisation, which is caused by the satellite and the ground station, by means of bias removal. We have two bias removal techniques at our disposal: one technique is called vector cancellation and can be performed on the single polarised 12.5 and 30 GHz signals, the other technique uses the specific characteristics of the dual-polarised 20 GHz beacon signal and is named matrix cancellation.

The relation between depolarisation and attenuation has been analyzed for a pure rain medium, a pure ice medium and a coincident rain and ice medium, in terms of XPD and CPA. In case of a pure rain medium, we have tested different theoretical rain depolarisation models by performing curvefits on scatterplots of corrected XPD versus measured CPA. Both "short-term" and "long-term" parameters have been derived, which characterize a general, abstracted XPD-CPA model. These parameter values have been compared with the theoretical parameter values of five well-known rain depolarisation models. The large standard deviations of the "short-term" parameter values show that these parameters can differ considerably from event to event. The "long-term" values match the values of the D/H/W-, Chu- and SIM models best. The relation between XPD and CPA for a pure ice medium is characterized by the high decay of XPD-values for low values of CPA. Threshold values of CPA have been determined above which no pure ice depolarisation was observed. These values are for the 12.5, 20 and 30 GHz frequency 1.0, 1.5 and 3.0 dB, respectively. We finally emphasize on the great importance of removing the event data of a coincident rain and ice medium from the pure rain event data, in order to test the rain depolarisation models.

For depolarisation analysis, the propagation medium can be characterized by three parameters: the mean canting angle, the differential attenuation and the differential phase shift. Both the mean raindrop and the mean ice crystal canting angle can vary a lot from event to event. Within an event, the course of the mean ice crystal canting angle is very whimsically and shows often large "jumps", probably due to lightning strokes, whereas the course of the mean raindrop canting angle is much smoother. The spread around the mean is much larger for the mean raindrop canting angle than for the mean ice crystal canting angle.

The differential attenuation $A$ and the differential phase shift $\beta$ have been calculated for a pure rain and a pure ice medium with different algorithms. After this, the calculated parameters have been compared with different theoretical derived relations. We recommend to calculate $A$, $\beta$ with $\Theta$, XPD- and DPH data, making as less assumptions as necessary.

Because a coincident rain and ice medium does generally not have the same axes of symmetry, as assumed for a single depolarising medium, we cannot calculate $\Theta$, $A$, $\beta$ for a double depolarising medium as we calculated these parameters for a single one. We have determined a discrimination method which uses the specific properties of a pure rain and a pure ice medium. By combining these properties we are yet able to select event periods in which a coincident rain and ice medium is present along the propagation path.
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1. Introduction

In July 1989 the European Space Agency (ESA) launched the experimental satellite Olympus, which is in the first place meant as a communication satellite and as such fitted for numerous telecommunication services. Besides, Olympus has also been equipped with transmitters which are especially suited for propagation research. Olympus transmits three beacon signals at frequencies of respectively 12.5, 20 and 30 GHz. The 12.5 and 30 GHz beacons are single (linearly) polarised, the 20 GHz beacon signal is dual polarised because it is switched with a frequency of 933 Hz between two orthogonal linear polarisations. By means of the three beacon signals it is possible to explore the influence of the atmosphere in the 30/20 GHz frequency band. This frequency band has been "reserved" for commercial satellite communications in the future and will be put into use if the, at the moment frequently used, 14/12 GHz band gets "full" and/or larger bandwidths are needed.

The research institutes that perform propagation experiments are nearly all united in the "Olympus Propagation Experiment" (OPEX) [1]. In this way the possibility exists to obtain a consistent collection of propagation data for three frequencies and for a great number of locations over Europe. The Eindhoven University of Technology (EUT) already participated in OPEX in an early stage. In the past years the department of telecommunications has designed and realised a measuring system which consists of a multi frequency beacon receiver, a multi frequency radiometer, a meteo station and a data acquisition system. With the beacon receiver it is possible to measure the amplitude and phase of the received satellite beacon signals in two orthogonal polarisation directions. By means of the radiometer the atmospheric noise emitted by gases and rain, which are responsible for micro wave attenuation, can be determined in a small frequency band close to the beacon frequency. The meteo station delivers the data which are relevant for propagation experiments such as rain intensity, temperature, pressure, wind velocity/direction and water vapour pressure. Presently the data analysis software is developed. This will be set up in such a way that existing prediction formulas and theoretical models can be verified and the processed measuring data can directly be used as input for the design of satellite communication systems in the 20/30 GHz band. For this, it is the intention to focus on the following three phenomena: attenuation, depolarisation and scintillation.

In this report we will emphasize on the depolarisation of the beacon signals due to rain and ice on the propagation path. The analysis of depolarisation on the micro wave signals is of great importance because in case of micro wave radio communications each frequency band is used twice, by using both orthogonal polarisation directions (frequency reuse). As soon as the propagation medium becomes anisotropic, due to the presence of raindrops or ice crystals, depolarisation will occur, which will result in crosstalk between the two transmitted signals. The 20 GHz beacon, which is switched between the two orthogonal polarisations, offers hereby the unique opportunity to characterize the medium fully. In chapter two the bias removal procedures will be discussed with which the depolarisation due to the satellite and ground station can be removed from the measuring data. We will particularly focus on one procedure which uses the specific characteristics of the dual polarised 20 GHz beacon signal, called matrix cancellation. In chapter three and four we will respectively analyze the influence of depolarisation due to rain and ice crystals on the beacon signals. Hereby we will investigate successively the relation between attenuation and depolarisation, calculate the canting angle of the hydrometeors and determine the differential attenuation and differential phase shift for the two symmetry axes of the propagation medium. Finally, in chapter five, we will analyze the
depolarisation due to a mixture of rain and ice and we will try to deduce a method to separate this double depolarising medium from a single depolarising rain or ice medium.
2 Crosspolar bias removal.

A problem in analyzing the depolarisation, measured by the ground station, is the fact that depolarisation is not only caused by the atmosphere, but also by the satellite and the ground station. In order to get insight in the depolarising influence of the propagation medium it is very desirable to remove these system influences. The removal of depolarisation caused by the system is often called crosspolar bias removal in propagation research.

2.1 Theory of crosspolar bias removal

When we intend to describe the received field in the ground station we can do this as follows.

\[ U = GTSE \] or

\[
U = \begin{bmatrix}
U_x & U_y \\
U_z & U_\eta
\end{bmatrix} = \begin{bmatrix}
G_{11} & G_{12} \\
G_{21} & G_{22}
\end{bmatrix} \begin{bmatrix}
t_{11} & t_{12} \\
t_{21} & t_{22}
\end{bmatrix} \begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix} \begin{bmatrix}
E_0 \\
0
\end{bmatrix}
\] (2.1)

with

\[ U = \] tension matrix, describing the received field in the ground station,
\[ G = \] ground station matrix, describing the attenuation and phase shift introduced by the ground station,
\[ T = \] transmission matrix, describing the propagation characteristics of the atmosphere,
\[ S = \] satellite depolarisation matrix, describing the attenuation and phase shift caused by the satellite,
\[ E = \] ideal transmitted electric field tensor, describing the two transmitted polarisations in the ideal case (i.e. no depolarisation by the satellite and equal powers in both polarisation directions.).

The crosspolar phasors measured in the ground station, which combine the depolarising influences of satellite, propagation medium and ground station are defined as:

\[
\delta_x^e = \frac{U_x}{U_\eta}
\] (2.2)

\[
\delta_y^e = \frac{U_y}{U_\eta}
\] (2.3)
The absolute values of these parameters are usually converted to a parameter called *crosspolar discrimination* (XPD). The crosspolar discrimination equals the ratio between the signal in the copolar channel and the signal in the crosspolar channel, expressed in dB’s. For the x- and y- polarisation we can respectively write:

\[
XPD_x = 20 \log \frac{|U_{xx}|}{|U_{yx}|} \quad (dB) \tag{2.4}
\]

\[
XPD_y = 20 \log \frac{|U_{yy}|}{|U_{yx}|} \quad (dB) \tag{2.5}
\]

We intend to remove above mentioned XPD’s from system influences by means of crosspolar bias removal, which will be implemented in the software.

If we turn back to formula (2.1) we should notice that the ratio between a cross- and copolar component is in practice always smaller than 1/10, so that we can neglect the products of the crosspolar components. The matrix U can now be approximated by:

\[
U = \begin{pmatrix}
G_{11}^1 t_{11}^1 S_{11} & G_{11}^1 t_{12}^1 S_{12} + G_{11}^1 t_{11}^1 S_{12} + G_{12}^1 t_{22}^1 S_{22} \\
G_{21}^1 t_{11}^1 S_{11} + G_{22}^1 t_{21}^1 S_{11} + G_{22}^1 t_{22}^1 S_{21} & G_{22}^1 t_{22}^1 S_{22}
\end{pmatrix}
\begin{pmatrix}
E_\varphi \\
0
\end{pmatrix}
\begin{pmatrix}
0 \\
E_\varphi
\end{pmatrix}
\]

By means of the approximated tension matrix U we find for \( \delta_y^* \):

\[
\delta_y^* = \frac{G_{12}^1 t_{22}^1 S_{22} + G_{11}^1 t_{12}^1 S_{12} + G_{11}^1 t_{11}^1 S_{12}}{G_{22}^1 t_{22}^1 S_{22}} \tag{2.7}
\]

or

\[
\delta_y^* = \frac{G_{11}^1 (t_{12} + t_{11} S_{12})}{G_{22}^1 (t_{22} + t_{22} S_{22})} \tag{2.8}
\]

We can interpret the five components in \( \delta_y^* \) as follows:

- \( G_{11}/G_{22} \) = ground station gain between x- and y- polarisation,
- \( t_{12}/t_{22} = \delta_y^* \) = crosspolar phasor of propagation path,
- \( t_{11}/t_{22} \) = quotient of copolar propagation parameters,
- \( S_{12}/S_{22} = \delta_y^* \) = crosspolar phasor of satellite,

---

1 The following formulas are valid for the y-polarisation. The equations for the x-polarisation can be found by interchanging the indices x, y and 1,2.
G_{12}/G_{22} = \delta_0^g = \text{crosspolar phasor of ground station.}

Consequently, we can now write \( \delta_0^g \) as:

\[
\delta_0^g = \frac{G_{11}}{G_{22}} \left( \delta_1^e + t_{12} \delta_2^e \right) + \delta_0^e
\]

The crosspolar phasor of the propagation path \( \delta_1^e \) is the component which we intend to measure, while we want to remove the other "system"-components.

Based on the above mentioned formulas we can deduce two crosspolar bias removal methods. One method, which can be used for the single polarised 12.5 and 30 GHz beacon signals, is called \textit{vector cancellation}. Vector cancellation has already been described and implemented in the software [2]. The other method, which uses the specific characteristics of the dual polarised 20 GHz beacon signal, is called \textit{matrix cancellation}. This method will be described in the next paragraphs both theoretically and in practice.

2.2 Matrix cancellation

2.2.1 Theory of matrix cancellation

In case of the 20 GHz beacon signal, all satellite- or all ground station influences can be eliminated by means of matrix cancellation. First the \textit{cancellation matrix} \( K \) is determined with aid of a measurement under "clear-sky" conditions (\( T \approx I \)).

\[
K = U_{\text{clear-sky}}^{-1} = (GSE)^{-1} = \frac{S^{-1}G^{-1}}{E_\phi}
\]

After this we can use matrix \( K \) to correct for in two ways:

Left cancellation:

\[
T_L = \begin{pmatrix} T_{\phi \phi} & T_{\phi \theta} \\ T_{\theta \phi} & T_{\theta \theta} \end{pmatrix} = KU = \frac{S^{-1}G^{-1}}{E_\phi} GTS \begin{pmatrix} E_\phi & 0 \\ 0 & E_\phi \end{pmatrix} = S^{-1}TS
\]

Right cancellation:

\[
T_R = \begin{pmatrix} T_{\phi \phi} & T_{\phi \theta} \\ T_{\theta \phi} & T_{\theta \theta} \end{pmatrix} = UK = GTS \begin{pmatrix} E_\phi & 0 \\ 0 & E_\phi \end{pmatrix} \frac{S^{-1}G^{-1}}{E_\phi} = GTG^{-1}
\]
T, and T can be considered as partially corrected descriptions of the transmission matrix T. (the error for left cancellation in the S'xTS matrix is ca. 0.5% in the copolar components and ca. 1.5% in the crosspolar components of the matrix T (assumptions: S/S = 0.99, S/S = 30 dB and φ(S - S) = ca. 1.5° (Olympus specifications)).

By means of left cancellation we correct for the influence of the ground station; with right cancellation for the influence of the satellite. The corrected crosspolar phasors are given by.

Left cancellation (correction of influence of ground station)

\[ \delta_{xL} = \frac{T_{xL}}{T_{xL}} = \frac{S_{11}}{S_{22}} \left( \delta'_{y} + (t_{11} - 1) \delta'_{x} \right) \]  

Right cancellation (correction of influence of satellite)

\[ \delta_{xR} = \frac{T_{xR}}{T_{xR}} = \frac{G_{11}}{G_{22}} \left( \delta'_{y} + \left( 1 - \frac{t_{11}}{t_{22}} \right) \delta'_{x} \right) \]  

The component \( t_{11}/t_{22} - 1 \) determines in case of left cancellation the residual influence of satellite and ground station depolarisation. Essentially there will be no error anymore in the XPD, if the assumption is correct that the satellite does not depolarise and the ground station depolarisation is completely corrected or vice versa. Because \( |t_{11}/t_{22} - 1| = |1 - t_{11}/t_{22}| \), the error \( \Delta XPD \) for right cancellation is as large as for left cancellation.

Table 2.1 gives \( \Delta XPD \) and \( \Delta \phi \) for three values of the satellite XPD and for a tilt angle \( \delta = -18.4° \) (EUT).

| error | \( |S_{12}/S_{22}| = -25 \text{ dB} \) | \( |S_{12}/S_{22}| = -30 \text{ dB} \) | \( |S_{12}/S_{22}| = -35 \text{ dB} \) |
|-------|-------------------------------|-------------------------------|-------------------------------|
| \( \Delta XPD \) | \( \pm 1.42 \text{ dB} \) | \( \pm 0.77 \text{ dB} \) | \( \pm 0.42 \text{ dB} \) |
| \( \Delta \phi \) | \( \pm 8.55° \) | \( \pm 4.83° \) | \( \pm 2.72° \) |

2 The errors \( \Delta XPD \) and \( \Delta \phi \) are the same as for vector cancellation [2]
The Olympus satellite XPD and the EUT ground station XPD are larger than 43 dB and 40 dB respectively [3]. Because the satellite unbalance component $S_{11}/S_{22}$ is smaller than the ground station unbalance component $G_{11}/G_{22}$, left cancellation should be preferred to right cancellation.

2.2.2 Matrix cancellation in practice

Matrix cancellation can in practice only be used for the analysis of the dual polarised 20 GHz beacon signal. We will need XPD-, DPH-, CPL- and VPHPH data of the EUT acquisition system for calculating the components of matrix $U$.

**Input**

$HCPL = \text{copolar level of the horizontal polarisation (dB)}$

$HXPD = \text{crosspolar discrimination of the horizontal polarisation (dB)}$

$HDPH = \text{phase shift between the cross- and copolar signal of the horizontal polarisation (°)}$

$VCPL = \text{copolar level of the vertical polarisation (dB)}$

$VXPD = \text{crosspolar discrimination of the vertical polarisation (dB)}$

$VDPH = \text{phase shift between the cross- and copolar signal of the vertical polarisation (°)}$

$VHPH = \text{phase shift between the copolar signals of the vertical and horizontal polarisation (°)}$

**Algorithm**

The matrix elements of the tension matrix $U$ can be represented as a vector $(I,Q)$ in a complex plane. The in-phase component $I$ is assumed to lie along the real axis, whereas the quadrature component $Q$ lies along the imaginary axis of the IQ-plane.

(Note: xx signal has been taken as reference of phase $\text{I} Q_{xx} = 0$)

\[
U = \begin{pmatrix} I_x & I_y + jQ_y \\ I_x + jQ_x & I_y + jQ_y \end{pmatrix}
\]  

(2.15)

with

\[
I_x = 10^{\frac{HCPL}{20}}
\]

(2.16a)

\[
I_x = 10^{\frac{HXPD}{20}} \cos(HPH)
\]

(2.16b)

\[
Q_x = 10^{\frac{VHPH}{20}} \sin(HDPH)
\]

(2.16c)
where $H_{\text{MAX}}$ and $V_{\text{MAX}}$ are the clear-sky copolar levels for horizontal and vertical polarisation, respectively.

After determination of the tension matrix $U$ under clear-sky conditions the cancellation matrix $K$ is calculated according to:

$$K = U_{\text{clear-sky}}^{-1} \quad (2.10)$$

where

$$U_{\text{clear-sky}}^{-1} = \frac{1}{\text{det}} \begin{pmatrix} I_{\gamma} + jQ_{\gamma} & -I_{\gamma} - jQ_{\gamma} \\ -I_{\gamma} - jQ_{\gamma} & I_{\gamma} \end{pmatrix} \quad (2.17)$$

with $\text{det}$ representing the determinant $U_{xx}U_{yy} - U_{xy}U_{yx}$ ($\approx U_{xx}U_{yy}$) of the matrix $U_{\text{clear-sky}}$.

By means of the matrix $K$ we can now correct for the satellite (right matrix cancellation) or for the ground station (left matrix cancellation), which results in a (partly) corrected transmission matrix $T_{\text{corr}}$.

**Left cancellation** (correction of the influence of the ground station)

$$T_{\text{corr}} = KU_{\text{measured}} \quad (2.18)$$

**Right cancellation** (correction of the influence of the satellite)

$$T_{\text{corr}} = U_{\text{measured}}K \quad (2.19)$$

The corrected XPD and DPH of the $x$ polarisation can now be calculated with aid of the corrected transmission matrix $T_{\text{corr}}$.
We can find the formulas for the y polarisation by interchanging the x and y indices.

In contrast to vector cancellation [3] matrix cancellation corrects the system induced phase shift between the co- and crosspolar signals automatically. Subsequently we implemented the above algorithm in the software. As an illustration we will show the measured XPD for vertical and horizontal polarisation respectively, and the XPD's after matrix cancellation (left cancellation) for a rain event which took place on the 25th September 1991. Figure 2.1a and 2.1b show the measured VXPD and HXPD data, as received by the ground station. Figure 2.2a and 2.2b show the corrected VXPD and HXPD data after matrix cancellation. We can see that the XPD's, particularly outside the rain period, have been improved considerably.

\[ T_{\text{corr}} = \begin{pmatrix} I_{x_{\text{corr}}} + Q_{x_{\text{corr}}} & I_{y_{\text{corr}}} + jQ_{y_{\text{corr}}} \\ I_{y_{\text{corr}}} + jQ_{x_{\text{corr}}} & I_{y_{\text{corr}}} + jQ_{y_{\text{corr}}} \end{pmatrix} \]  

(2.20)

\[ \text{XPD}_{x_{\text{corr}}} = 10 \log \left( \frac{I_{x_{\text{corr}}}^2 + Q_{x_{\text{corr}}}^2}{I_{y_{\text{corr}}}^2 + Q_{y_{\text{corr}}}^2} \right) \text{ (dB)} \]  

(2.21)

\[ \text{DPH}_{x_{\text{corr}}} = \arctan \left( \frac{Q_{x_{\text{corr}}}}{I_{x_{\text{corr}}}} \right) - \arctan \left( \frac{Q_{x_{\text{corr}}}}{I_{x_{\text{corr}}}} \right) (0..360^\circ) \]  

(2.22)

Figure 2.1a: 20.0 GHz measured XPD, 25 september 1991. Vertical polarisation
Figure 2.1b: 20.0 GHz measured XPD, 25 September 1991. *Horizontal polarisation*

Figure 2.2a: 20.0 GHz XPD after matrix cancellation, 25 September 1991. *Vertical polarisation.*
2.3 Conclusions

For depolarisation analysis we can correct the depolarisation, which is not caused by the atmosphere, by means of two bias removal techniques. One technique, which can be used for the single polarised 12.5 and 30 GHz beacon signals is called vector cancellation and has already been implemented in the software [2]. The other technique which uses the specific characteristics of the dual polarised 20 GHz beacon signal is called matrix cancellation.

In case of matrix cancellation we assume that the system depolarisation is fully caused by either the satellite or by the ground station. If the assumption is correct we can fully correct either the depolarisation of the satellite (right matrix cancellation) or the depolarisation of the ground station (left matrix cancellation). If the assumption concerning the cause of the system depolarisation is not correct then there will remain a residual error in XPO after matrix cancellation. The maximum residual error $\Delta XPO$ is the same as for vector cancellation and only depends on the polarisation tilt angle and on either the satellite XPD (left matrix cancellation) or the ground station XPD (right matrix cancellation). Left matrix cancellation (correction of ground station) is preferred to right matrix cancellation.
3 Depolarisation due to rain

3.1 Introduction

Two independent information channels using the same frequency band can be transmitted over a single link by using orthogonal polarisations. This technique is used in satellite systems to effectively increase the available spectrum. Though the orthogonally polarised channels are completely isolated in theory, some degree of interference between them is inevitable. This is caused by less than theoretical performance of spacecraft and earth station antennas, and depolarising effects on the propagation path. The depolarisation at millimeter wave frequencies is mainly caused by hydrometeors, such as raindrops and ice crystals on the propagation path. A medium containing these hydrometeors is generally anisotropic and it has, projected onto the plane perpendicular to the propagation path (elevation angle = 0°), two orthogonal symmetry axes. For polarisations along the two symmetry axes the medium causes differential attenuation and phase shift. For waves, polarised in other than these two directions, this differential attenuation and phase shift causes depolarisation. As far as raindrops are concerned, the differential attenuation and phase shift occur because of the lack of spherical shape of the drops. The shape of the drops is generally oblate spheroidal, with the oblateness depending on the drop size. In the case of very large drops there will also be a concaveness at the lower side [4]. Furthermore, the symmetry axes of the drops are canted from the vertical, by an angle depending mainly on the vertical wind gradient [5].
3.2 Testing the theoretical rain depolarisation relations.

Depolarisation by hydrometeors is the major cause of crosstalk in dual-polarised satellite communication systems. The relation between depolarisation and attenuation is needed to predict the performance of these communication systems.

Both theoretically and empirically, it has been shown that there is a correlation between depolarisation and attenuation by rain. Owing to this a number of models were derived for this relation, in terms of XPD (crosspolarisation discrimination) and CPA (copolar attenuation). Because these models were based on different assumptions, and moreover were obtained in different parts of the world with different climates to match, it is not amazing that small differences between the models exist. In general, however, the models correspond well to each other. This is why the possibility exists to compare the measured data to a general rain depolarisation model.

3.2.1 Rain depolarisation models.

The different expressions for the relation between XPD and CPA have been derived from calculations, using theories of scattering by raindrops (Mie scattering: point matching technique), and using different models of raindrop size distribution and raindrop shape. On the results of these calculations, curve fitting has been performed to obtain an analytical expression for the relationship between XPD and CPA.

Of the different expressions, which have been proposed as models to predict XPD statistics from CPA statistics, five are presented here. These expressions all contain some specific parameters, which are:

- \( f \) = frequency (GHz)
- \( \delta \) = polarisation tilt angle (°) (= -18.4° for EUT ground station)
- \( \epsilon \) = elevation angle (°) (= 26.7° for EUT ground station)
- \( \theta \) = canting angle
- \( \theta_{av}, \sigma_{\theta_{av}} \) = average canting angle resp. standard deviation within an event
- \( \theta_{av}, \sigma_{\theta_{av}} \) = average canting angle resp. standard deviation from event to event

(In the models described below the assumption is made that \( \theta_{av} = 0 \). To use these models without making this assumption \( \delta \) should be replaced by \( \delta - \theta_{av} \).)

In the following formulas XPD and CPA are in dB's, the angles \( \theta, \sigma, \delta, \epsilon \) in degrees and \( f \) in GHz.

>1) CCIR model [6]

\[
XPD = C \log f + I(\delta, \sigma_{\theta_{av}}) - 40 \log \cos \epsilon + 0.0053 \sigma_{\theta_{av}}^2 - V \log \text{CPA}
\]  

(3.1)

with

\( C = 30 \)
\[ I(\delta, \sigma_{\text{em}}) = -10\log \frac{1}{2} \{1 - \cos 4\delta \exp(-0.0024\sigma_{\text{em}}^2)\} \]

V(12 GHz) = 20  
V(20 GHz) = 21  
V(30 GHz) = 23

Assumptions:
-Laws and Parsons drop size distribution  
-Pruppacher-Pitter drop shape model  
-rain temperature = 20° C  
Recommendation:  
\[ \sigma_{\text{em}} = 5^\circ \]

\(2\) Dissanayake/Haworth/Watson model (D/H/W-model) [7]

\[ XPD = S + C \log f + I(\delta, \sigma_{\text{em}}) - 40 \log \cos \phi + 0.0053\sigma_\theta^2 - V \log CPA \]  
(3.2)

with  
S = 8.16  
C = 21  
\[ I(\delta, \sigma_{\text{em}}) = -20 \log \sin 2\delta \]  
V = 20

Assumptions:
-Laws and Parsons drop size distribution  
-spherical drop shape  
Recommendation:  
\[ \sigma_\theta = 25^\circ \]

\(3\) Chu model for linear polarisation [8]

\[ XPD = S + C \log f + I(\delta, \sigma_{\text{em}}) - 40 \log \cos \phi + P \log CPA - V \log CPA \]  
(3.3)

with  
S = 11.5  
C = 20  
\[ I(\delta, \sigma_{\text{em}}) = -10\log \frac{1}{2} \{1 - \cos 4\delta \exp(-0.0024\sigma_{\text{em}}^2)\} \]
\[ P = +0.075\cos(\delta)\cos(2\delta) \text{ for vertical polarisation (}= 0.048 \text{ for EUT}) \]
\[ P = -0.075\cos(\delta)\cos(2\delta) \text{ for horizontal polarisation (}= -0.048 \text{ for EUT}) \]
\[ V = 20 \]

Recommendation:
\[ \sigma_{\text{max}} \leq 3' \]

>4) Simple Isolation model (SIM-model) [9]

\[ \text{XPD} = S + C\log f + I(\delta, \sigma_{\text{max}}) - 42\log \cos \epsilon + 0.0053\sigma_{\epsilon}^2 - 20\log F_0 - V\log CPA \quad (3.4) \]

with
- \( S = 9.5 \)
- \( C = 17.3 \)
- \( I(\delta, \sigma_{\text{max}}) = -10\log \frac{1}{2}(1 - \cos 4\delta \exp(-0.0024\sigma_{\text{max}}^2)) \)
- \( F_0 = \text{fraction of all rain drops which are non-spherical (flattened)} \)
- \( V = 19 \)

Assumptions:
- Laws and Parsons drop size distribution
- Scattering coefficients obtained by Uzunoglu, Evans and Holt (for spherical drop shape)

Recommendation:
- \( F_0 = 0.65 \) (65\% of all rain drops are non-spherical (flattened))
- \( \sigma_{\epsilon} = 12'', \sigma_{\text{max}} = 3'' \)

>5) Nowland/Olsen/Shkarofsky model (N/O/S-model) [10]

\[ \text{XPD} = S + C\log f + (V - 20)\log l + I(\delta, \sigma_{\text{max}}) - 40\log \cos \epsilon + 0.0053\sigma_{\epsilon}^2 - V\log CPA \quad (3.5) \]

with
- \( S = 4.1 \)
- \( C = 26 \)
- \( l = \text{effective path length through rain (km)} \)
- \( I(\delta, \sigma_{\text{max}}) = -20\log \sin 2\delta \)

\[ V = 12.8 f^{1.9} \text{ for } 10 < f \leq 20 \text{ GHz} \]
\[ V = 22.6 \text{ for } 20 < f \leq 40 \text{ GHz} \]

Assumptions:
- Marshall-Palmer drop size distribution (well correlated with the Laws and Parsons drop size distribution)
Pruppacher-Pitter drop shape model
- rain temperature = 20° C

The most important differences between the models are the coefficients C and V. In most models, these coefficients have been derived by curve fitting to the theoretical CPA- and f-dependence, calculated from the set of scattering coefficients used. Therefore, it is not surprising that differences occur, since different drop shapes have been used. Besides, because the drop shape depends on rain intensity, it would not be surprising either if different models would turn out to correspond better in different climatic regions of the world. The models are further equally composed and differ only in the neglection and approximation of different dependencies. Therefore, it seems useful not to test one of these models in the Olympus-project, but a general expression:

\[
XPD = U + V \log(CPA) \quad (\text{dB})
\]
\[
U = S + C \log(f) \quad (\text{dB})
\]

where CPA and XPD are in dB's and U and V depend on the elevation angle \( \epsilon \), the polarisation angle \( \delta \), the beacon frequency \( f \) and the canting angle \( \theta \).

We notice that the term \( P \cdot CPA \) in the Chu-model, which results in a different XPD for quasi-vertical and quasi-horizontal polarisation, is neglected in this abstracted version. (The difference in relation to the Chu-model is maximal 1.44 dB.)

Table 3.1 shows the theoretical values for U, V, S and C, calculated for the five XPD-CPA models. (We performed the following substitutions \( \epsilon = 26.7°, \delta = -18.4°, l = 5 \text{ km} \) and \( \sigma_\theta = 30° \) (SIM-model \( \sigma_\theta = 12° \)).

<table>
<thead>
<tr>
<th>frequency (GHz)</th>
<th>V (dB)</th>
<th>U (dB)</th>
<th>S (dB)</th>
<th>C (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12.5</td>
<td>20</td>
<td>30</td>
<td>all</td>
</tr>
<tr>
<td>CCIR model</td>
<td>-20</td>
<td>-21</td>
<td>-23</td>
<td>44.0</td>
</tr>
<tr>
<td>D/H/W model</td>
<td>-20</td>
<td>-20</td>
<td>-20</td>
<td>40.9</td>
</tr>
<tr>
<td>Chu model</td>
<td>-20</td>
<td>-20</td>
<td>-20</td>
<td>39.8</td>
</tr>
<tr>
<td>SIM model</td>
<td>-19</td>
<td>-19</td>
<td>-19</td>
<td>39.4</td>
</tr>
<tr>
<td>N/O/S model</td>
<td>-21</td>
<td>-23</td>
<td>-23</td>
<td>42.3</td>
</tr>
</tbody>
</table>
3.2.2 Testing the rain depolarisation models

To test the XPD-CPA models we have the crosspolar discrimination and the copolar signal level (XPD- resp. CPL-data) for every beacon frequency at our disposal. First the system influences are removed by means of a bias removal technique. After this the correspondence will be tested between the measured data and the semi-empirical relation for depolarisation and attenuation due to rain (formula 3.6) with aid of a curvefit.

3.2.2.1 Bias removal for the XPD-CPA analysis

Possible system influences are removed from the XPD-data by one of the two crosspolar bias removal methods (see chapter 2). Because the XPD-CPA models only describe the relation between the amplitude of XPD en CPA, also vector cancellation which does not correct for the system induced phase shift, can be used.

To obtain the copolar rain attenuation (CPA), the copolar signal level (CPL) should also be subjected to a kind of bias removal technique. For, the copolar attenuation is not caused by rain only, but also by the presence of oxygen, water vapour, clouds and sleet on the propagation path. Particularly the 20 GHz beacon signal suffers from strong attenuation due to the presence of water vapour. To consider rain attenuation only, other kinds of attenuation will have to be eliminated. In the ideal case radiometer data is available over a certain period of time, so that the 'clearsky'-attenuation caused by oxygen, water vapour and clouds can be determined. (see appendix A)

When no or incomplete radiometer data are available, the determination of the copolar attenuation for those events is highly simplified. During a period of dry, clear weather the clearsky copolar level (CPL\textsubscript{clearsky}) will be determined for every beacon signal.

To determine CPL\textsubscript{clearsky} we use the periods close before and after the event for calculating the mean CPL\textsubscript{clearsky pre event} and CPL\textsubscript{clearsky post event}:

\[
CPL_{\text{clearsky pre event}} = \frac{1}{N} \sum_{i=1}^{N} CPL_{\text{measured pre event}} 
\]

\[
CPL_{\text{clearsky post event}} = \frac{1}{M} \sum_{i=1}^{M} CPL_{\text{measured post event}} 
\]

where \(N\) and \(M\) are the numbers of samples in the period before respectively after the event.

Now there are two possible results:

- The CPL\textsubscript{clearsky pre event} and CPL\textsubscript{clearsky post event} are nearly the same. In this case CPL\textsubscript{clearsky} will be determined as follows:

\[
CPL_{\text{clearsky}} = \frac{CPL_{\text{clearsky pre event}} + CPL_{\text{clearsky post event}}}{2} 
\]

- The CPL\textsubscript{clearsky pre event} clearly differs from CPL\textsubscript{clearsky post event}. Now CPL\textsubscript{clearsky} will be calculated this way:
\[ \text{CPL}_{\text{clearsky}} = \text{CPL}_{\text{clearsky pre event}} + t \left( \frac{\text{CPL}_{\text{clearsky post event}} - \text{CPL}_{\text{clearsky pre event}}}{W} \right) \]  

(3.10)

where \( W \) is the number of samples in the event period and \( t \) (0 \(<\ t \leq W \)) indicates to which sample the calculated clearsky-sky value belongs to. The copolar attenuation (CPA), during an event, is now derived from the relation: \( \text{CPA} = \text{CPL}_{\text{clearsky}} - \text{CPL}_{\text{even}} \), where \( \text{CPL}_{\text{even}} \) is the copolar signal level during an event. This simplified procedure has of course some drawbacks. First, changing non-rain attenuation will not be corrected. Furthermore, possible day changes of the CPL can influence a measurement during a long event. However, the influences of month- or year-changes can satisfactorily be suppressed by determining the clearsky signal levels per each event analysis once again. Moreover, the 20 GHz water vapour attenuation can be distinguished by comparing the 20 GHz attenuation with 12.5 or 30 GHz attenuation, which will show much less water vapour attenuation.

A quite different aspect of bias removal is to ascertain that the data, which will be used for the analysis, will not be influenced by "out-of-lock" situations in the receiving ground station. In appendix B is shown how this can be prevented.

### 3.2.2.2 Performing a curvefit

After the bias removal the CPA-data is plotted against the XPD-data. With aid of a curvefit the agreement between the measured and the semi-empirical relation between rain depolarisation and -attenuation will be tested. For performing an appropriate curve we will use the "least squares" fitting method. In 3.2.1 the following abstracted rain depolarisation model has been proposed:

\[
\begin{align*}
\text{XPD} &= U + V \log(\text{CPA}) \\
U &= S + C \log(f)
\end{align*}
\]  

(3.6)

We notice that according to this model there is a linear relation between XPD and \( \log(\text{CPA}) \). Supposing a number of points \((\log(\text{CPA})_1, \text{XPD}_1), ..., (\log(\text{CPA})_N, \text{XPD}_N)\) which coincide with pairs of data results we intend to find a curve \( \text{XPD} = U + V \log(\text{CPA}) \) which fits these points "as good as possible" ("as good as possible" means that the sum of the quadratures of the vertical deviation is minimal). The so found curve is called regression curve. A regression curve gives per event per beacon frequency the \( U \) - and \( V \)-values. Comparing the values of \( U \) for the three beacon frequencies gives us the possibility to estimate the \( S \) - and \( C \)-values. In practice we do not fit directly to the points \((\log(\text{CPA})_1, \text{XPD}_1)\). Because normally there will be much more points having low CPA-values than large ones, the CPA-range is divided in a number of bins, each having a binsize of 0.1 dB, after which a mean XPD-value is calculated per bin. The curvefit is subsequently performed on basis of the points \((\log(\text{CPA}_{\text{bin}(1)}), \text{XPD}_{\text{mean}(1)}), ..., (\log(\text{CPA}_{\text{bin}(N)}), \text{XPD}_{\text{mean}(N)})\), where \( \text{CPA}_{\text{bin}(i)} \) is the value of the \( i \)-th bin and \( N \) the total number of bins.

### 3.2.3 Testing the rain depolarisation models with Olympus data
V, curvefits have been performed on all analyzed events separately. After this, the mean values and the standard deviation have been calculated for U and V. "Long-term" regression values have been obtained by performing curvefits on matrices, in which all available event data has been collected. Both the "short-term"- as the "long-term"-values for U and V have been compared with the semi-empirical values from table 3.1. Hereafter the values of S and C have been calculated for the three frequencies.

3.2.3.1 Short-term values of U and V.

The short-term values for U and V have been obtained by performing a curvefit on every event separately. Only the, according to the algorithm described in appendix B, as reliable marked data will be shown. Also, the calculated regression curve and the semi-empirical curve according to the Chu-model have been shown, as done by Hogers [2]. Figures 3.1a, 3.1b, 3.1c and 3.1d are scatterplots of the 12.5, 20 (V/H) and 30 GHz measured copolar attenuation versus the corrected XPD for an event that took place on 25 sept. '91 (pure rain event)1.

We notice that in figure 3.1d the CPA-axis only runs till 15 dB, because data beyond this value can not be trusted because of "out of lock" situations in the receiving ground station (see appendix B).

---

1 To ascertain that only a pure rain medium was present along the propagation path, we first subjected the event data to two algorithms which will be described in the next chapters. One algorithm removes pure ice data (by leaving out the data where severe depolarisation was observed in the absence of significant copolar attenuation), whereas the other removes coincident rain and ice (see § 5.3.1)
The regression curves match the semi-empirical curves according to the Chu-model pretty well. Analysis of other events, however, has shown that the "short-term" values for U and V can significantly differ from event to event. In table 3.2 this is confirmed by the large standard deviations of the "short-term" values. However, the agreement between the mean "short-term" values from table 3.2 and the different semi-empirical values is not bad.
The "short-term" values for S and C have been determined on basis of "short-term" values for U for f = 12.5 GHz, 20 GHz (vertical polarisation) and 30 GHz. In practice this results in fitting a system of three equations:

\[
\begin{align*}
U_{12.5 \text{ GHz}} &= S + \text{Clog}(12.5) \\
U_{20 \text{ GHz}} &= S + \text{Clog}(20) \\
U_{30 \text{ GHz}} &= S + \text{Clog}(30)
\end{align*}
\]  

(3.7)

We first remark that the different models give rather diverge values of S and C, as we can see in table 3.2. In this context do the experimental determined "short-term" values of S and C not differ startlingly from the semi-empirical values. Nevertheless, we notice that the standard deviations of the "short-term" values are very large.

3.2.3.2 Long-term values for U and V.

To obtain the "long-term" values for U and V all available pure rain event data, whereby the peak attenuation was considerable (CPA > 10 dB for f= 20 GHz, see appendix E), have been collected in a matrix for the three beacon signals. Such a matrix can be compared to a scatterplot. The horizontal CPA-axis always runs from 0 till 30 dB and has been divided in 150 bins (binsize = 0.2 dB). The vertical XPD-axis

---

2 To ascertain that only a pure rain medium was present along the propagation path, we first subjected the event data to two algorithms which will be described in the next chapters. One algorithm removes pure ice data (by leaving out the data where severe depolarisation was observed in the absence of significant copolar attenuation), whereas the other removes coincident rain and ice (see § 5.3.1)

23
runs from 15 till 65 dB and has been divided in 100 bins (binsize = 0.5 dB). The binsize has been suggested by the assumed precision of measurement of the parameters belonging to it and the maximum size of the matrix (64 kB). If we add a new event to the matrix, for every XPD-CPA combination the belonging matrix element is determined after which the value of that element is raised by one. The "long-term" values of U and V are subsequently determined by performing a curvefit on all data that are collected in the matrix. (The way of performing a curvefit does not differ from the way of performing a curvefit on a single event, as described in 3.2.2). The 12.5, 20 (both vertical and horizontal) and 30 GHz matrices are graphically shown in the figures 3.2a, 3.2b, 3.2c and 3.2d respectively.

Figure 3.2a: Scatterplot of 55.04 hours corrected 12.5 GHz XPD versus 12.5 GHz CPA. (- - = theory Chu, --- = curvefit)
Figure 3.2b: Scatterplot of 55.04 hours corrected 20 GHz XPD versus 20 GHz CPA. *Vertical polarisation* (- = theory Chu, -- = curvefit)

Figure 3.2c: Scatterplot of 55.04 hours corrected 20 GHz XPD versus 20 GHz CPA. *Horizontal polarisation* (- = theory Chu, -- = curvefit)
Figure 3.2d: Scatterplot of 55.04 hours corrected 30 GHz XPD versus 30 GHz CPA. 
(- - = theory Chu, — = curvefit)

The "long-term" values for U and V, which have been derived by performing curvefits on these matrices, match the semi-empirical values of the D/H/W-, Chu- and SIM rain depolarisation models best. The (absolute) values of U and V according to the CCIR- and N/O/S models are clearly too high for the events that occurred in Eindhoven. This means that the CCIR- and N/O/S models predict too high XPD-values for low CPA-values and a too steep decay of XPD for rising CPA. As in case of the "short-term" values of S and C, the "long-term" ones need to be presented with serious reservedness. Nevertheless, the "long-term" values of S and C do not differ very much from the values of the D/H/W-, Chu- and SIM models. We finally remark that, in contrast to other models, the Chu model contains an extra term P0CPA. This term results in different XPD's for quasi-vertical and quasi-horizontal polarisations. This may explain the small difference in the XPD-CPA relation found in figure 3.2b and 3.2c. One should therefore be careful neglecting this term.
3.2.4 Conclusions

Both theoretically and empirically, it has been shown that there is a strong correlation between depolarisation and attenuation by rain. Owing to this a number of models have been derived for this relation in terms of XPD and CPA. Based on these models one general, abstracted rain depolarisation model has been determined, which is described by four parameters U, V, S and C. By means of performing curvefits on the measuring data, whereby the attenuation was considerable (CPA > 10 dB for f = 20 GHz), we have determined experimental "short-term" and long-term" values of U, V, S and C.

First we have calculated the mean values and standard deviations from the "short-term" values for U and V. The large standard deviations of the "short-term" values of U and V show that these parameters can differ considerably from event to event. The "long-term" values for U and V match the semi-empirical values of
the D/H/W-, Chu- and SIM rain depolarisation models best. The (absolute) values of U and V according to the CCIR- and N/O/S- models are obviously too high for the events which have been observed in Eindhoven. This means that the CCIR- and N/O/S models predict too high XPD-values for low CPA-values and a too steep decay of XPD for rising CPA.

Furthermore, we have also calculated the mean values and the standard deviations of S and C, based on the "short-term" values for U for the 12.5, 20 (vertical polarisation) and 30 GHz frequencies. Once more we notice that very large standard deviations were found. The "long-term" values of S and C do not differ much from the values of the D/H/W-, Chu- and SIM models. Finally we remark that the term P*CPA in the Chu-model, which is not present in the other rain depolarisation models can not simply be neglected.
3.3 Calculation of canting angle, differential attenuation and phase shift.

For depolarisation analysis the rain medium can be characterized by the differential attenuation (A), the differential phase shift (β) and the canting angle (θ). The difference in attenuation and phase shift for the two polarisations along the symmetry axes of the propagation medium are defined as the differential attenuation A and the differential phase shift β. The canting angle θ describes the orientation of the raindrops with respect to the local horizontal.

In this chapter we will first enunciate how the canting angle of a rain medium can be calculated. After this a number of algorithms are deduced for calculating the differential attenuation and phase shift, based on the calculated canting angle.

3.3.1 Calculation of canting angle

The calculation of a canting angle is mostly based on a model that introduces eigendirections in the propagation medium. In such models two (or more) principal planes are defined, which are characterized by their own propagation constants [1]. These propagation constants describe the attenuation A and phase shift β for waves with polarisations lying in the concerning plane. The directions of the principal planes are called eigendirections. For modelling the propagation medium the eigendirections of the medium are defined by the shape and orientation of the hydrometeors, in our case raindrops, taking into account the assumption that all raindrops are equally shaped and orientated. The shape of a raindrop is generally equalled to a spheroid, top and bottom of which are flattened. The mean angle between the long symmetry axis of the raindrop and the local horizontal is defined as the raindrop canting angle θ (figure 3.3).

Fig 3.3: The canting angle of a raindrop.
We assume that the rain medium has two orthogonal eigendirections, coinciding with the long and short symmetry axes of the raindrops (x and y). Each polarisation of an electric field, dual-polarised in those directions, will be received as a signal which has been attenuated and phase shifted by the medium, however, the polarisation state of the two waves $E_m$ and $E_n$ are unchanged after passing through the raindrops and can be represented by:

$$
\begin{pmatrix}
E_{rx} \\
E_{ry}
\end{pmatrix} = T_0 \begin{pmatrix}
E_{cx} \\
E_{cy}
\end{pmatrix} = \begin{pmatrix}
T_{11} & 0 \\
0 & T_{22}
\end{pmatrix} \begin{pmatrix}
E_{cx} \\
E_{cy}
\end{pmatrix}
$$

(3.8)

where matrix $T_0$ describes the influences of a propagation medium on em-waves, which are polarised along one (or two) eigendirections of the medium and $E_{rx}$ and $E_{ry}$ are attenuated waves after passing through raindrops. The transmission coefficients $T_{11}$ and $T_{22}$ are defined as:

$$
T_{11} = A_x \exp(i\beta_x) \\
T_{22} = A_y \exp(i\beta_y)
$$

(3.9)

where,

$A_x$, $A_y$ = attenuation coefficients and \\
$\beta_x$, $\beta_y$ = phase shift coefficients in the eigendirections of the medium.

Here we choose x as the principal long axis of the raindrop. This means that the attenuation and phase shift will be the largest for axis x.

Waves with a polarisation vector, which lies along an arbitrary axis, will on the other hand generally be depolarised. Such waves $E'_m$ and $E'_n$ can be obtained from $E_m$ and $E_n$ by a coordinate transformation:

$$
\begin{pmatrix}
E'_m \\
E'_n
\end{pmatrix} = T \begin{pmatrix}
E_m \\
E_n
\end{pmatrix} = \begin{pmatrix}
\cos\phi_0 & -\sin\phi_0 \\
\sin\phi_0 & \cos\phi_0
\end{pmatrix} \begin{pmatrix}
T_{11} & 0 \\
0 & T_{22}
\end{pmatrix} \begin{pmatrix}
\cos\phi_0 & \sin\phi_0 \\
-\sin\phi_0 & \cos\phi_0
\end{pmatrix} \begin{pmatrix}
E_m \\
E_n
\end{pmatrix}
$$

(3.10)

with

$$
T = \begin{pmatrix}
t_{11} & t_{12} \\
t_{21} & t_{22}
\end{pmatrix}
$$

$T$ is the transmission matrix, describing the influences of a propagation medium on em-waves, who are arbitrarily polarised.

$\phi_0$ is the angle between the long symmetry axis of the raindrop and the x-polarisation (rotation angle (°))
This results in the components of the transmission matrix $T$ becoming.

$$t_{11} = T_{11} \cos^2 \phi_0 + T_{22} \sin^2 \phi_0$$

$$t_{12} = t_{21} = \frac{1}{2} (T_{11} - T_{22}) \sin 2\phi_0$$

$$t_{22} = T_{22} \cos^2 \phi_0 + T_{11} \sin^2 \phi_0$$

(3.11)

We notice that the model results in $t_{12} = t_{21}$. In practice however, it will inevitably turn out that $t_{12}$ will not exactly equal $t_{21}$. In that case the above system will be overdimensioned. (Still, to use the proposed formulas one could use an average $(t_{12} + t_{21})/2$).

With the above three equations we are yet able to calculate the rotation angle $\phi_0$.

$$\phi_0 = \frac{1}{2} \arctan \frac{2t_{12}}{t_{11} - t_{22}}$$

or

$$\phi_0 = \frac{1}{2} \arctan \frac{2}{D_z - D_y}$$

(3.12)

with

$$D_z = \frac{t_{11}}{t_{21}}$$

$$D_y = \frac{t_{22}}{t_{12}}$$

where $D_z$ and $D_y$ are the "depolarisation ratios". They are related to measured quantities as:

$$HXPD = 20 \log |D_x|$$
$$VXPD = 20 \log |D_y|$$
$$HDPH = \arg D_x$$
$$VDPH = \arg D_y$$

with,

(V)(H)XPD = XPD, measured for a signal, transmitted in y resp. x direction;
(V)(H)DPH = co/crosspolar relative phase, measured for the same signal.

In the model of two principal planes, it can be proved that

$$\text{Im } D_x = \text{Im } D_y [11]$$
and thus the result of (3.12) is real. In the data processing, the result can forced to fit this model replacing (3.12) by either

\[ \phi_0 = \frac{1}{2} \arctan \frac{2}{D_x - D_y} \]

or

\[ \phi_0 = \frac{1}{2} \arctan \frac{2}{\text{Re}(D_x - D_y)} \] (3.13)

A problem, associated with all of the above proposed formulae, is the following. The crosspolar phases VDPH and HDPH are needed for these calculations, and the result may be very sensitive to errors in VDPH and HDPH [11]. This means that if the crosspolar phase is either not or not accurately enough measured, the canting angle can not be calculated. Since the canting angle is usually the first of a set of modelling parameters which are to be calculated, the other parameters can not be determined either. In order to avoid this difficulty, it is desirable to find a formula for the canting angle which does not make use of the crosspolar phase but e.g. of the relative phase between the two copolar signals.

For a medium with two principal planes, it can be shown that the two received crosspolar signals are equal. Consequently it follows that

\[ R = \frac{D_x}{D_y} \] (3.14)

where R is the "copolar signal ratio".

R is related to measured quantities as

\[ B_1 \text{VHCPL} = 20 \log |R| \] and  
\[ B_1 \text{VHPH} = \text{arg } R. \]

with,

\[ B_1 \text{VHCPL} = \text{relative copolar signal level between two polarisations.} \]
\[ B_1 \text{VHPH} = \text{relative copolar signal phase between two polarisations.} \]

Using this, eq. (3.12) can be written as:

\[ \phi_0 = \frac{1}{2} \arctan \frac{2}{D_y(R - 1)} \] (3.15)
Assuming that the result should be real, it can be forced to be real by taking

\[
\phi_0 = \frac{1}{2} \arctan \frac{2}{|P_R|} = \frac{1}{2} \arctan \frac{2}{\sqrt{|P_R|^2 + 1 - 2|P_R| \cos \phi_r}}
\]  \hspace{1cm} (3.16)

with \( \phi_r = \arg R \).

For this formula, which will be used for the EUT event analysis, \( \phi_r \) is not needed.

### 3.3.1.1 Calculation of \( \Theta \) for 20 GHz beacon signal

In case of the dual-polarised 20 GHz beacon signal we are able to obtain a transmission matrix \( T_r \) or \( T_b \) by means of right or left matrix cancellation (see chapter 2). To calculate these matrices we also use the copolar signal levels. These signals are influenced by the presence of gases, water vapour, clouds and possibly hydrometeors (rain, snow, ice, etc.) on the propagation path. If we intend to describe only the influence of hydrometeors, the copolar signal levels should be corrected for attenuation which is caused by gases, water vapour and clouds. Appendix A describes how, by means of radiometer data, the "clear-sky" attenuation can be calculated.

Consequently, to calculate \( \Theta \), we use the following procedure

> **input**
> B1VXPD \( \text{dB, 20 GHz crosspolarisation discrimination, vertical polarisation} \)
> B1VHCPL \( \text{dB, 20 GHz relative copolar signal level between two polarisations} \)
> B1VHHPH \( ^\circ, 20 \text{ GHz relative copolar signal phase between two polarisations} \)

> **algorithm**
> First the input signals are converted to usable quantities for the calculations.

\[
|P_R| = 10^{B1VXPD/20}
\]

\[
|R| = 10^{B1VHCPL/20}
\]

if \( B1VHHPH > 180^\circ \) then \( B1VHHPH = B1VHHPH - 360^\circ \)

\[
\phi_r = \pi \times \frac{B1VHHPH}{180^\circ}
\]

(\( \phi_r \) has a value which lies between \(-\pi\) and \(+\pi\) in contrast to the value of the \( B1VHHPH \) signal who lies between \( 0^\circ \) and \( 360^\circ \).)
Further we will insert these parameters in formula (3.16) after which we will calculate $\Theta$, according to:

$$\Theta = \delta + \phi_0$$

(3.17)

with $\delta = \text{polarisation tilt angle (-18.4 }^\circ \text{ for EUT)}$. 

It would be interesting to determine the sensitivity to data variations of formula (3.16). This is because it is important to know in which amount e.g. a small error in the measured phase of $B1VPH$ would result in a deviation of $\phi_0$. In appendix C this has been investigated.

From the sensitivity analysis it followed, that small variations of $\phi$, and $|R|$, when the VXPD-value is high, cause large variations in $\phi_\nu$. We therefore, recommended to exclude $\phi_\nu$ data, for which VXPD exceeds 40 dB.

3.3.2 Calculation of differential attenuation and phase shift.

In § 3.3.1. it has been explained how the orientation of a raindrop (given by the canting angle $\Theta$) and also the shape of it, introduces eigendirections in the propagation medium, which each have their own specific attenuation and phase shift.

A number of algorithms are deduced to calculate the differential attenuation and phase shift based on a known canting angle [3]. This chapter will again summarize these algorithms and will provide them with new views. There has been made a distinction between the dual-polarised 20 GHz beacon signal and the single-polarised 12.5 and 30 GHz beacon signals.

3.3.2.1 Calculation of $\alpha$ and $\beta$ for dual-polarised signals.

In case of the 20 GHz beacon signal, for which both the horizontal and the vertical polarisation are available, the rotation angle $\phi_0$ can simply be computed. This means that the differential attenuation and phase shift too can be determined without further assumptions. If $\phi_0$ is not available, we will be forced to make the assumption $\Theta = 0^\circ (\phi_0 = -\delta)$, which means that the eigendirections of the medium are horizontal and vertical. Furthermore we will discriminate between the situations in which phase-information is available and in which it is not.

i) Calculation of 20 GHz $\alpha, \beta$ with phase-information.

We assume that the data is cleared with help of matrix cancellation. In case of having the complete set of 20 GHz signals at our disposal (all XPD amplitudes and phases), we use the following algorithm.

---

1 The algorithms are illustrated for the 20 GHz, vertical polarisation state. For using the algorithms in case of the 20 GHz, horizontal polarisation state VXPD, VDPH and $\phi_\nu$ should be replaced by HXPD, HDPH and $\phi_h$ respectively.
\[ \phi_r = \text{calculated rotation angle (°) (a), or } \phi_0 = -\delta \text{ (b).} \]

B1VXPD (dB, 20 GHz crosspolarisation discrimination, vertical polarisation)

B1VDPH (°, 20 GHz differential phase shift between the cross- and copolar signal, vert. polarisation)

Algorithm 1

The XPD is converted to:

\[ |D_\perp| = 10^{B1VXP/20} \]

if B1VDPH > 180° then B1VDPH = B1VDPH - 360°

\[ \phi_r = \pi \times \frac{\text{B1VDPH}}{180°} \]

(\( \phi_r \) has a value which lies between \( -\pi \) and \( +\pi \) in contrast to the value of the B1VHPH signal who lies between 0° and 360°.)

The differential attenuation \( A \) and the differential phase shift \( \beta \) are next computed with.

\[
|D_\perp| = \sqrt{D_\perp^2 \sin^2 \phi_0 + 2D_\perp \sin 2\phi_0 \cos 2\phi \cos \phi_\perp + \cos^2 2\phi_0}
\]

\[
\beta = \arctan \left( \frac{-2D_\perp \sin 2\phi \sin \phi_\perp}{|D_\perp|^2 - 1} \right)
\]

\[
A = -20 \log \sqrt{D_\perp^2 + 2\cos 2\phi_0 + 1 + 2D_\perp \sin 2\phi_0 \cos \phi_\perp \cos \phi_\perp}
\]

l) Calculation of 20 GHz \( A, \beta \) without phase-information.

When no phase-information is available (no \( \phi_\perp \) ) \( A \) and \( \beta \) can only be computed when we make one of the following assumptions:

\( XPD \) is caused by a pure differential phase shift \( (A=1) \)

From tables by Chu [12] it may be concluded that a rain medium can be assumed causing purely differential phase shift for frequencies below 15 GHz.

\[ \phi_\perp = \text{calculated rotation angle (°) (a), or } \phi_0 = -\delta \text{ (b).} \]

B1VXPD (dB, 20 GHz crosspolarisation discrimination, vertical polarisation)
The XPD is converted to:

\[ |p_d| = 10^{B_{1VXPD20}} \]

The differential phase shift \( \beta \) is next computed with:

\[ |p_d| = \sqrt{P_x \sin^2 \phi_0 - \cos^2 \phi_0} \]

\[ \beta = \arctan \frac{2|p_d|}{|p_d|^2 - 1} \]  

\( \ast \) **XPD is caused by a pure differential attenuation** \( (\beta = 0^\circ) \).

From tables by Chu [12] it may be concluded that a rain medium can be assumed causing purely differential attenuation for frequencies above 15 GHz.

**input**

\( \phi_a = \) calculated rotation angle \( (') (a), \) or \( \phi_0 = -\delta (b). \)

B1VXPD \( (\text{dB, 20 GHz crosspolarisation discrimination, vertical polarisation}) \)

**algorithm 3**

The XPD is converted to:

\[ |p_d| = 10^{B_{1VXPD20}} \]

The differential attenuation \( A \) is next computed with:

\[ |p_d| = |p_d| \sin 2\phi_0 - \cos 2\phi_0 \]  

\[ A = -20\log \frac{|p_d| - 1}{|p_d| + 1} \]  

\[ \text{3.3.2.2 Calculation of } A, \beta \text{ for single-polarised signals.} \]

In case of single-polarised 12.5 and 30 GHz beacon signals only the vertical polarisation is available. On the result of this the needed rotation angle can not simply be computed on basis of the 12.5 and 30 GHz signals. We notice, however, that the 12.5 and 30 GHz signals propagate through the same medium as the 20 GHz
signals. Consequently, if 20 GHz data is available for a certain event, then the so derived rotation angle, can serve for the computation of $A$, $\beta$ for 12.5 and 30 GHz beacon signals. If either no or incomplete 20 GHz data is available for a certain event then we will assume that $\Theta = 0^\circ$ ($\phi_0 = -\delta$). We will again make a distinction between the situations in which phase-information is available and in which it is not.

i) Computation of 12.5 and 30 GHz $A$, $\beta$ with phase-information.

We assume that the data is cleared with help of vector cancellation. In case of having the complete set of 12.5 & 30 GHz signals at our disposal (all XPD amplitudes and phases), we use the following algorithm.

>input
$\phi_0 =$ with 20 GHz data calculated rotation angle(*) (a),or $\phi_0 = -\delta$ (b)

$B(0)(2)VXPD$ (dB, 12.5/30 GHz crosspolarisation discrimination)
$B(0)(2)VDPH$ ($^\circ$, 12.5/30 GHz differential phase shift between the cross- and copolar signal)

>algorithm 1

The calculations are again preceded by some conversions:

$$|P| = 10^{B(0)(2)\text{VXPD}20}$$

if $B(0)(2)VDPH > 180^\circ$ then $B(0)(2)VDPH = B(0)(2)VDPH - 360^\circ$

$\phi_\gamma = \pi \times \frac{B(0)(2)VDPH}{180^\circ}$

The differential phase shift $\beta$ and the differential attenuation $A$ are next computed according to the formulas 3.19 and 3.20 (§ 3.3.2.1).

ii) Computation of 12.5 and 30 GHz $A$, $\beta$ without phase-information.

When no phase-information is available (no $\phi_\gamma$) $A$ and $\beta$ can only then be computed if we make one of the following assumptions:

- **XPD is caused by a pure differential phase shift ($A = 1$)**
  A rain medium can be assumed causing purely differential phase shift for frequencies below 15 GHz.

>input
$\phi_0 =$ with 20 GHz data calculated rotation angle(*) (a),or $\phi_0 = -\delta$ (b)

$B(0)(2)VXPD$ (dB, 12.5/30 GHz crosspolarisation discrimination, vertical polarisation)
The XPD is converted to:

\[ |P_x| = 10^{0.1 \cdot \text{XPD}_{20}} \]

The differential phase shift \( \beta \) is next computed according to formula 3.22 (§ 3.3.2.1).

\( \text{XPD is caused by a pure differential attenuation } (\beta = 0^\circ) \)

A rain medium can be assumed causing purely differential attenuation for frequencies above 15 GHz.

\( \phi_0 = \text{with } 20 \text{ GHz data calculated rotation angle}(^\circ) \) (a), or \( \phi_0 = -\delta \) (b)

\( B(0)(2) \text{VXP} \) (dB, 12.5/30 GHz crosspolarisation discrimination, vertical polarisation)

The XPD is converted to:

\[ |P_x| = 10^{0.1 \cdot \text{XPD}_{20}} \]

The differential attenuation \( A \) is next computed according to formula 3.24 (§ 3.3.2.1).

We can see that the computation of \( A \) and \( \beta \) for the 12.5 & 30 GHz single-polarised signals passes off the same as for the dual-polarised 20 GHz beacon signal.

3.3.3 Theoretical \( A, \beta - \) relations.

We would like to check the computed \( A \) and \( \beta \) with theoretical and empirical results. Fukuchi et al. [13],[14] derived the following theoretical relations for \( A, \beta \) caused by rain:

\[ A = p A_\omega \mu_I \rho'_0 \cos2\phi_0 \cos^2\varepsilon \exp\left(-0.00061\sigma^2_4\right) = k_f A_\omega^* \quad \text{(3.25)} \]

\[ \beta = \mu A_\omega \mu_I \rho'_0 \cos2\phi_0 \cos^2\varepsilon \exp\left(-2\sigma^2_4\right) = k_f A_\omega^* \quad \text{(3.26)} \]

with

\( \phi_0 = \text{rotation angle } (= 18.4^\circ \text{ for EUT}) \)

\( \varepsilon = \text{elevation angle } (= 26.7^\circ \text{ for EUT}) \)

\( \sigma_4 = \text{effective standard deviation of canting angle } (\text{recommended } 0^\circ) \)

\( l_{\text{eff}} = \text{effective path length through rain } (= 5.1 \text{ km for EUT}) \)

\( A_\omega = \text{mean copolar attenuation } (A_x + A_y)/2 \)

\( \mu, \nu, p, q = \text{parameters which are functions of the frequency and the raindrop size distribution.} \)
Table 3.3 gives the values for $\mu$, $v$, $p$, $q$, $k_1$ and $k_2$ which have been calculated for EUT [3].

<table>
<thead>
<tr>
<th></th>
<th>$p$</th>
<th>$q$</th>
<th>$\mu$</th>
<th>$v$</th>
<th>$k_1$</th>
<th>$k_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B0</td>
<td>0.157</td>
<td>1.27</td>
<td>0.042</td>
<td>1.0</td>
<td>0.065</td>
<td>1.54</td>
</tr>
<tr>
<td>B1</td>
<td>0.116</td>
<td>1.27</td>
<td>0.022</td>
<td>1.0</td>
<td>0.048</td>
<td>0.80</td>
</tr>
<tr>
<td>B2</td>
<td>0.089</td>
<td>1.27</td>
<td>0.0096</td>
<td>0.85</td>
<td>0.037</td>
<td>0.45</td>
</tr>
</tbody>
</table>

3.3.4 Calculation of $\Theta$, $A$ and $\beta$ with Olympus data

By means of 20 GHz Olympus data, the canting angle $\Theta$ has been derived for a number of events. Hereafter both with the 20 GHz and the 12.5 and 30 GHz data we have calculated the differential attenuation $A$ and phase shift $\beta$. Algorithms 1, 2 and 3 have been used for all calculations with the according assumptions. (see § 3.3.2). For the 12.5 and 30 GHz data, we used vector cancellation and corrected the system induced phase shift for calculating the correct values of $A$ and $\beta$.

3.3.4.1 Calculation of $\Theta$ with Olympus data.

We will consider the results of the calculation of $\Theta$, according to the procedure deduced in § 3.3.1, for an event which took place on 25 september '91 (rain event). In figure 3.4a the canting angle $\Theta$, has been plotted during 3½ hours of time. Under that (figure 3.4b), the 20 GHz crosspolar discrimination (vertical polarisation) after matrix cancellation is shown for the same period.
Fig 3.4a: Canting angle $\theta$ of a rain medium, 25 september 1991.

Figure 3.4b: 20 GHz crosspolar discrimination (vertical polarisation) after matrix cancellation, 25 september 1991.

We should notice that much of the data can not be trusted (see appendix C). In fact we are only interested in calculating the canting angle during the rain period (from approximately 11.45 till 12.05). During this period we see a mean canting angle of ca. -2°, with deviations as large as $\pm 5^\circ$ (figure 3.5). Other events showed that the mean canting angle can vary between ca. $+20^\circ$ and $-20^\circ$ with much spread in it.
Fig 3.5: Canting angle $\Theta$ during the rain period, 25 september 1991.

3.3.4.2 Calculation of $A, \beta$ with Olympus data.

To obtain the differential attenuation and phase shift we use the same event as for calculating the canting angle. In figures 3.6a and 3.7a the differential phase shift $\beta$ has been set out versus respectively 20 GHz vertical copolar attenuation (CPA) and 20 GHz horizontal CPA. Figures 3.6b and 3.7b show scatterplots of the differential attenuation $A$ versus respectively 20 GHz vertical CPA and 20 GHz horizontal CPA. For all these computations we used both the canting angle $\Theta$ and the XPD and DPH data. (algorithm 1a). The broken line in these figures shows the predicted behaviour of $A$ and $\beta$ as given by Fukuchi (§ 3.3.3).

Figure 3.6a: 20 GHz calculated differential phase shift versus 20 GHz vertical copolar attenuation, 25 september 1991. (- - = theory Fukuchi)
Figure 3.6b: 20 GHz calculated differential attenuation versus 20 GHz vertical copolar attenuation, 25 September 1991. (- - = theory Fukuchi)

Figure 3.7a: 20 GHz calculated differential phase shift versus 20 GHz horizontal copolar attenuation, 25 September 1991. (- - = theory Fukuchi)
We can see that the computed values of $A$ match the theoretical values reasonably well. The computed values of $A$, nevertheless, lie under the predicted curves.

In figures 3.8a,b and 3.9a,b we calculated the differential attenuation and phase shift for respectively the 12.5 and 30 GHz beacon signals. Again we used both the canting angle $\Theta$ (computed with 20 GHz data) and the XPD and DPH data (algorithm 1a). We remark that in the ideal case (i.e., no measuring errors) the canting angle, calculated according to formula (3.16), should be the same as if we had calculated this parameter with data of any other frequency. However, we notice that our calculations will be influenced by measuring errors. From appendix C, we know that the sensitivity of formula (3.16) to data variations strongly differs for among others different values of XPD (and thus different frequencies). One can minimize this sensitivity by smoothing the noisy course of the canting angle. We have done so, by taking the mean canting angle ($2^\circ$) during the event period and using this value as input for our calculations.
Figure 3.8a: 12.5 GHz calculated differential phase shift versus 12.5 GHz copolar attenuation, 25 September 1991. (--- = theory Fukuchi)

Figure 3.8b: 12.5 GHz calculated differential attenuation versus 12.5 GHz copolar attenuation, 25 September 1991. (--- = theory Fukuchi)
Figure 3.9a: 30 GHz calculated differential phase shift versus 30 GHz copolar attenuation, 25 September 1991. (- - = theory Fukuchi)

Figure 3.9b: 30 GHz calculated differential attenuation versus 30 GHz copolar attenuation, 25 September 1991. (- - = theory Fukuchi)

We can see that the computed values of the differential phase shift $\phi$ match the theoretical values well. The computed values of the differential attenuation $A$ lie under the predicted curves, as we also saw in case of the 20 GHz data.
In figures 3.10 till 3.13 we calculated the differential phase shift and differential attenuation with the assumption that the canting angle equals zero ($\Theta = 0^\circ$) (algorithm 1b).

Figure 3.10a: 20 GHz calculated differential phase shift versus 20 GHz vertical copolar attenuation, 25 september 1991. Assumption: $\Theta = 0^\circ$.

Figure 3.10b: 20 GHz calculated differential attenuation versus 20 GHz vertical copolar attenuation, 25 september 1991. Assumption: $\Theta = 0^\circ$.
Figure 3.11a: 20 GHz calculated differential phase shift versus 20 GHz horizontal copolar attenuation, 25 September 1991. Assumption: $\Theta=0^\circ$.
(- - = theory Fukuchi)

Figure 3.11b: 20 GHz calculated differential attenuation versus 20 GHz horizontal copolar attenuation, 25 September 1991. Assumption: $\Theta=0^\circ$.
(- - = theory Fukuchi)
Figure 3.12a: 12.5 GHz calculated differential phase shift versus 12.5 GHz copolar attenuation, 25 september 1991. Assumption: $\Theta = 0^\circ$.

(--- = theory Fukuchi)

Figure 3.12b: 12.5 GHz calculated differential attenuation versus 12.5 GHz copolar attenuation, 25 september 1991. Assumption: $\Theta = 0^\circ$.

(--- = theory Fukuchi)
Figure 3.13a: 30 GHz calculated differential phase shift versus 30 GHz copolar attenuation, 25 september 1991. Assumption: $\Theta = 0^\circ$.

We can see that the values of $A$ and $\beta$, computed with the assumption $\Theta = 0^\circ$, do not differ much from the values of $A$ and $\beta$ as calculated without this assumption. This accounts for all frequencies. Nevertheless, particularly in the figures 3.10a and 3.11a, we notice that the spread around the theoretical curve for high CPA values is more severe now.

The 12.5 GHz differential phase shift (shown in fig 3.14a) is computed with algorithm 2, which is based on the assumption that depolarisation is caused by a pure differential phase shift. The 20 GHz (V/H) and 30 GHz differential attenuation (fig 3.14b,c,d) are computed with aid of algorithm 3, which is based on the
assumption that depolarisation is caused by a pure differential attenuation.
Also these approximations match the predicted behaviour of A and β as calculated without these assumptions well. In figure 3.14c, however, the calculated values for A at \( f = 20 \) GHz (horizontal), assuming \( \beta = 0^\circ \), lie twice as high as predicted.

![Figure 3.14a: 12.5 GHz calculated differential phase shift versus 12.5 GHz copolar attenuation, 25 September 1991. Assumptions: A=1, θ=0°.](image)

(--- = theory Fukuchi)

![Figure 3.14b: 20 GHz calculated differential attenuation versus 20 GHz vertical copolar attenuation, 25 September 1991. Assumptions: \( \beta = 0^\circ \), θ=0°.](image)

(--- = theory Fukuchi)

50
Figure 3.14c: 20 GHz calculated differential attenuation versus 20 GHz horizontal copolar attenuation, 25 September 1991. Assumptions: $A = 1, \Theta = 0^\circ$.

(- - = theory Fukuchi)

Figure 3.14d: 30 GHz calculated differential attenuation versus 30 GHz copolar attenuation, 25 September 1991. Assumptions: $\beta = 0^\circ, \Theta = 0^\circ$.

(- - = theory Fukuchi)
3.3.5 Conclusions

Different algorithms have been deduced for calculating the canting angle $\Theta$, the differential attenuation $A$ and differential phase shift $\beta$ for a rain medium. The calculations for the single-polarised 12.5 and 30 GHz beacon signals are similar to the calculations for the 20 GHz dual-polarised signal. If vector cancellation is used as bias removal technique, the system induced offset in the crosspolar phase will have to be corrected for.

Essentially we use the canting angle $\Theta$ of the raindrops, the XPD and DPH data for our calculations. If no data is available for the canting angle $\Theta$, we assume $\Theta = 0^\circ$, which means that the eigendirections of the medium are horizontal and vertical. If no phase-information is available, $A$ and $\beta$ can be approximated by assuming that depolarisation is caused either by a pure differential phase shift ($f < 15 \text{ GHz}$) or by a pure differential attenuation ($f > 15 \text{ GHz}$).

A difficulty in calculating the canting angle is the huge sensitivity to data variations when the VXPD value is high. Fortunately these VXPD-value sinks considerably during an intense period of rain, so that the canting angle in this time can be calculated reliably. We found that the mean canting angle can vary a lot from event to event (between approximately $-20^\circ$ and $+20^\circ$) and also within an event large variations around the mean were found (ca. $5^\circ$). Striking was moreover the noisy course of the mean canting angle, which caused a lot of spread around the mean.

The relation between the calculated $A$ and $\beta$ and the copolar attenuation, calculated for a rain event (25 September 1991), matches the theoretical relations of Fukuchi reasonably well for all used algorithms and for all three the frequencies. Nevertheless, we essentially want to make as few assumptions as possible for calculating $A$ and $\beta$, so we recommend to use $\Theta$, XPD and DPH data for our calculations (algorithm 1a).
4 Depolarisation due to ice crystals

4.1 Introduction

Until the middle 1970’s it was assumed that depolarisation along the propagation path was only caused by raindrops, following in average the laws presented in the previous chapter. This would imply that depolarisation can only occur under conditions of severe copolar attenuation. However, on satellite-earth paths in '75-'76 during the European phase of ATS-6 ("Application Technology Satellite") [15,16] significant depolarisation was observed in the absence of significant copolar attenuation. From among others, comparison with radar reflections alongside the satellite link, it was found that depolarisation was due to the presence of clouds of high-altitude ice crystals along the radio path. Various experiments, SIRIO, OTS, COMSTAR, CTS, ETS, etc., followed and the collected data were analyzed. Still, in contrast with rain depolarisation, much less theory is known about depolarisation due to ice crystals.

Of the ice particles, it are the needle-like and plate-like particles which have enough eccentricity to cause significant depolarisation with low attenuation, when present in considerable quantity ($\approx 10^4$ particles/m$^3$). Ice pellets and hail are only slightly eccentric, have relatively little orientation and are generally not present in sufficient large volumes, to cause significant depolarisation.

Some theories state that the ice particles, which can be approximated by ice needles and plates, are aligned by the electric fields in thunderstorms ("lightning"). The latter conclusion was obtained from electric field measurements together with XPD-measurements [17].

According to chapter 3, we will present the experimental XPD-CPA relation and calculate the parameters $\Theta$, $\Delta$ and $\beta$ characterizing the ice medium. We will, when possible, compare our measurements with models or other experiments.
4.2 Relation between XPD and CPA

Since the main characteristic of ice depolarisation is the low value of copolar attenuation relative to that due to rain, we will investigate the relation between copolar attenuation (CPA) and crosspolar discrimination (XPD). Although, since ice depolarisation was first observed, some models have been derived, based on the presence of ice needles and plates, none of them can actually be used to test with Olympus data. The main difference, in relation to the expressions for rain depolarisation is that in the icy case, the attenuation dependent term has been replaced by the ice content. CCIR has proposed to use the CCIR XPD-CPA relation for rain depolarisation [11] and to subtract 5 dB from the XPD-value to derive a simple empirical relation for ice depolarisation. We will investigate the XPD-CPA relation and test the CCIR law for ice, both for "short-term"- and "long-term" measurements.

4.2.1 Analysis of XPD-CPA relation with Olympus data.

For analyzing the XPD-CPA relation, we have the crosspolar discrimination and the copolar signal level of the 12.5, 20 (vertical and horizontal) and the 30 GHz beacon signals at our disposal. The data are first subjected to one of the bias removal techniques, as mentioned in chapter 3. The XPD-data are cleared from system influences by a crosspolar bias removal technique (for the 20 GHz beacon signal we use matrix cancellation, in case of the 12.5 and 30 GHz signals vector cancellation is performed) whereas the CPL-data are cleared from attenuation which is not caused by rain (oxygen, water vapour, clouds etc.) by means of linear trend removal. After the bias removal techniques the CPA-data are plotted versus the XPD-data.

4.2.1.1 "Short-term" measurements.

The "short-term" XPD-CPA scatterplots have been obtained by selecting the XPD and CPA data during an event where pure ice-depolarisation appeared. Only the, according to the algorithm described in appendix B, as reliable marked data will be presented. Also the semi-empirical curve as proposed by CCIR will be shown. Figures 4.1a, 4.1b, 4.1c and 4.1d are scatterplots of respectively 12.5, 20 (vertical and horizontal) and 30 GHz measured corrected XPD versus the CPA for a period in which ice-depolarisation arose (5 July 1990, 18.22.00 -18.41.00).

---

1 To ascertain that only a pure ice medium is present along the propagation path, we only use event data for analyzing purposes where depolarisation is observed in the absence of significant copolar attenuation.
Figure 4.1a: 12.5 GHz XPD after bias removal versus 12.5 GHz copolar attenuation, 5 July 1990.
(- - = theory CCIR)

Figure 4.1b: 20 GHz XPD after bias removal versus 20 GHz copolar attenuation, 5 July 1990.
Vertical polarisation (- - = theory CCIR)
Figure 4.1c: 20 GHz XPD after bias removal versus 20 GHz copolar attenuation, 5 July 1990. 

*Horizontal polarisation* (- - = theory CCIR)

Figure 4.1d: 30 GHz XPD after bias removal versus 30 GHz copolar attenuation, 5 July 1990. 
(- - = theory CCIR)

The figures show that crosspolar discrimination decreases much faster than may be expected from the CCIR curve for ice. We find XPD-values as low as approximately 25, 20 and 17 dB, concerning respectively 12.5, 20 (both vertical and horizontal) and 30 GHz frequencies, for negligible values of CPA. Golé and Mon [17] also showed the inadequacy of the CCIR law for ice. Although no linear relation can be determined between
XPD and CPA, there seems to be a threshold value of CPA above which no pure ice-induced XPD exists. In case of this event the CPA-threshold may be approximately 0.9 dB for 12.5 GHz, 1.5 dB for 20 GHz and 2.9 dB for 30 GHz. This values will be tested for with "long-term" measurements, in order to get more reliable threshold values.

4.2.1.2 "Long-term" measurements

To analyze the XPD-CPA relation in case of an ice medium more accurately, all available event data per beacon frequency, considering ice depolarization, have been collected in a matrix. The horizontal CPA-axis always runs from 0 till 30 dB and has been divided in 150 bins (binsize = 0.2 dB). The vertical XPD-axis runs from 15 till 65 dB and has been divided in 100 bins (binsize = 0.5 dB). The 12.5, 20 (resp. vertical and horizontal) and 30 GHz matrices are graphically shown in figures 4.2a, 4.2b, 4.2c, and 4.2d. The broken curves in the figures indicate the theoretical XPD-CPA law as suggested by the CCIR.

In all matrices we can see extreme low XPD-values for negligible values of CPA. As suggested in § 4.2.1.1 we can now determine the \( \text{CPA}_{\text{th}} \) values more reliably. The \( \text{CPA}_{\text{th}} \) values for the 12.5, 20 and 30 GHz frequencies are approximately resp. 1.0, 1.5 and 3.0 dB. Above these values there will, most certainly, occur no pure ice depolarisation.

Figure 4.2a: Scatterplot of 1.29 hours corrected 12.5 GHz XPD versus 12.5 GHz CPA. (--- = theory CCIR)
Figure 4.2b: Scatterplot of 1.29 hours corrected 20 GHz XPD versus 20 GHz CPA. *Vertical polarisation* (- - = theory CCIR)

Figure 4.2c: Scatterplot of 1.29 hours corrected 20 GHz XPD versus 20 GHz CPA. *Horizontal polarisation* (- - = theory CCIR)
4.2.2 Conclusions

The main characteristic of ice depolarisation is the low value of the copolar attenuation relative to that due to rain. Although some models have been derived since ice depolarisation was first observed, none of them can actually be tested with Olympus data. CCIR has proposed to use the CCIR law for rain depolarisation and to subtract 5 dB from the XPD-value. We have investigated the relation between copolar attenuation (CPA) and crosspolar discrimination (XPD) for the three beacon signals, both for "short-term" measurements and "long-term" ones. In both cases we noticed that the values of XPD decreased much faster than might be expected from the CCIR law and that no linear relation could be determined between XPD and CPA. However, we could determine threshold values of CPA for the three beacon frequencies above which no pure ice depolarisation was observed. The CPA_{\text{threshold}} values for the 12.5, 20 and 30 GHz frequencies are approximately resp. 1.0, 1.5 and 3.0 dB. Above these values there will, most certainly, occur no pure ice depolarisation.

Figure 4.2d: Scatterplot of 1.29 hours corrected 30 GHz XPD versus 30 GHz CPA. \textit{Horizontal polarisation} \((- - = \text{theory CCIR})\)
4.3 Calculation of canting angle, differential attenuation and differential phase shift

To calculate the parameters which characterize an ice medium, the medium is first modelled. Like, in case of a rain medium, two eigendirections are introduced in the propagation medium, which in the icy case coincide with the two symmetry axes of the ice particles (approximated by ice-needles and ice-plates). The angle between the long symmetry axis of the ice particle (again we assume that all ice particles are equally aligned) and the local horizontal is called the ice crystal canting angle. If the eigendirections of the medium do not coincide with the polarisation state of the transmitted wave a differential phase shift will be introduced to the field components along these eigendirections. Because the modelling of an ice medium essentially passes off the same as for a rain medium we can use the formulas as presented in chapter 3.

4.3.1 Calculation of canting angle $\Theta$

The determination of the ice medium canting angle $\Theta$ is basically equivalent to the determination of the rain medium canting angle (see 3.3.1). The phenomenon of ice crystal canting may be explained in terms of electrostatic fields during thunderstorms ("lightning") tending to align a particle so that the longest axis is along the field. We will consider the results of the calculations of $\Theta$, according to the procedure deduced in 3.3.1, assuming that both the XPD-data and the CPA-data have been cleared with help of a bias removal procedure. The procedure of calculating $\Theta$ is once more summarized below.

**Input**
- $B_{1VXPD}$ (dB, 20 GHz crosspolarisation discrimination, vertical polarisation)
- $B_{1VHCPL}$ (dB, 20 GHz relative copolar signal level between two polarisations)
- $B_{1VHPH}$ (°, 20 GHz relative copolar signal phase between two polarisations)

**Algorithm**
First the input signals are converted to usable quantities for the calculations.

$$ |P| = 10^{B_{1VXPD}/20} $$

$$ \phi = 10^{B_{1VHCPL}/20} $$

if $B_{1VHPH} > 180^\circ$ then $B_{1VHPH} \equiv B_{1VHPH} - 360^\circ$

$$ \phi = \pi \cdot B_{1VHPH}/180^\circ $$

($\phi$, has a value which lies between $-\pi$ and $+\pi$ in contrast to the value of the $B_{1VHPH}$ signal which lies between $0^\circ$ and $360^\circ$.)

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Further we will insert these parameters in formula:

\[ \phi_0 = \frac{1}{2} \arctan \frac{2}{D_p \sqrt{1 + 2R \cos \phi_e}} \]  

(3.16)

after which we will calculate \( \Theta \), according to:

\[ \Theta = \delta + \phi_0 \]  

(3.17)

with \( \delta = \) polarisation tilt angle (-18.4° for EUT).

4.3.2 Calculation of differential attenuation \( A \) and differential phase shift \( \beta \)

The orientation of an ice particle (given by the canting angle \( \Theta \)) and also the shape of it, introduces eigendirections in the propagation medium, which each have their own specific attenuation and phase shift. In 3.3.2 a number of procedures have been deduced to calculate the differential attenuation and differential phase shift which will as far as an ice medium is concerned be summarized below. The procedures concern the 12.5, 20 (vertical and horizontal) and 30 GHz frequencies and there will be discriminated between the situations in which phase-information is available and in which it is not.

i) Calculation of \( A \) and \( \beta \) with phase-information.

We assume that the data is cleared with help of matrix cancellation (20 GHz) or vector cancellation (12.5 and 30 GHz). In case of having the complete set of signals at our disposal (all XPD amplitudes and phases), we use the following algorithm.

>input

- \( \phi_0 \) = calculated rotation angle (°) (a), or \( \phi_0 = -\delta \) (b).
- XPD (dB, crosspolarisation discrimination)
- DPH (°, differential phase shift between the cross- and copolar signal)

>algorithm 1

The XPD is converted to:

\[ |D_p| = 10^{XPD/20} \]
if DPH > 180° then DPH = DPH - 360°

φ_y = π * DPH/180°

(φ_y has a value which lies between -π and +π in contrast to the value of the DPH signal which lies between 0° and 360°.)

The differential attenuation A and differential phase shift β are next computed with:

\[ |P_d| = \sqrt{|D_r|^2 \sin^2 φ_y + 2|D_r| \sin 2φ_y \cos 2φ_y \cos φ_y + \cos 2φ_y} \]  \hspace{1cm} (3.18)

\[ A = -20 \log \left( \frac{|D_r|^2 + 2 \cos 2φ_y + 1 + 2|D_r| \sin 2φ_y \cos φ_y}{|D_r|^2 - 2 \cos 2φ_y + 1 - 2|D_r| \sin 2φ_y \cos φ_y} \right) \]  \hspace{1cm} (3.19)

\[ β = \arctan \left( \frac{-2|D_r| \sin 2φ_y \sin φ_y}{|P_d|^2 - 1} \right) \]  \hspace{1cm} (3.20)

ii) Calculation of β without phase-information.

When no phase-information is available (no φ_y) only β can be computed (considering an ice medium) by making the following assumption:

**XPD is caused by a pure differential phase shift (A=1)**

From tables by among others Chu [11] it may be concluded that an ice medium can be assumed causing purely differential phase shift.

**Input**

φ_0 = calculated rotation angle (°) (a), or φ_0 = -δ (b).

XPD (dB, crosspolarisation discrimination)

**Algorithm 2**

The XPD is converted to:

\[ |P_d| = 10^{XPD/20} \]

The differential phase shift β is next computed with:

\[ |P_d| = \sqrt{|P_d|^2 \sin^2 φ_0 - \cos^2 2φ_0} \]  \hspace{1cm} (3.21)

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\[ \beta = \arctan \frac{2D_x}{|D_y|^2 - 1} \]  

(3.22)

4.3.3 Theoretical relation between XPD and \( \beta \)

Now we have computed \( A \) and \( \beta \), we would like to check these parameters with theoretical and/or empirical results. In contrast to rain depolarisation there are no linear relations between CPA and \( A, \beta \) in the icy case. Tsolakis and Stutzman [18] calculated the result of single scattering on ice needles and plates, approximated respectively by large narrow prolate and flat oblate spheroids. They derived an expression to represent the XPD of a homogeneous (i.e. containing identical and equally aligned particles) and purely phase shifting (i.e. not attenuating) medium, as a function of the differential phase shift along the principal axes of the medium. (These principal axes are in this case also the symmetry axes of the ice particles.) The result is:

\[
XPD = -20 \log \left| \frac{\beta \pi \sin 2(\theta_m - \delta)}{360} \right| \quad (4.1)
\]

with

- \( \beta \) = differential phase shift (\(^\circ\)),
- \( \theta_m \) = canting angle of the medium (\(^\circ\)),
- \( \delta \) = polarisation tilt angle (\(^\circ\)) (= -18.4\(^\circ\) for EUT ground station)

for

\( \beta \ll 1 \) (rad) and differential attenuation \( A = 1 \).

The XPD, as seen in (4.1), is proportional to the differential phase shift \( \beta \). XPD is also proportional to \( \sin 2(\theta_m - \delta) \), indicating (as expected) no depolarisation when the electric field is aligned with the medium axes \( (\theta_m = \delta) \), and maximum depolarisation occurs for \( \theta_m - \delta = 45^\circ \).

To test this expression we rewrite formula 4.1 in a more general form:

\[
XPD = A + B \log \beta \quad (4.2)
\]

where \( A \) depends on \( \theta_m \) and \( \delta \), \( B \) has a constant value.

The theoretical values of \( A \) and \( B \), which have been calculated for EUT, are respectively 45.6 and 20.0 (assuming: \( \theta_m = 0^\circ \)). By plotting the calculated XPD-data versus the \( \beta \)-data and performing a curvefit (which is of the same form as formula 4.2, and uses the "least squares" fitting method) we can compare the theoretical values of \( A \) and \( \beta \) with the measured ones.
4.3.4 Calculation of $\theta$, $A$ and $\beta$ with Olympus data

By means of 20 GHz Olympus data, the canting angle $\theta$ has been derived for a number of events. Hereafter both with the 20 GHz and the 12.5 and 30 GHz data we have calculated the differential attenuation and differential phase shift $\beta$. Algorithms 1 and 2 have been used for all calculations with the according assumptions. In case of 20 GHz data, we performed matrix cancellation as bias removal procedure to correct the XPD-data. For the 12.5 and 30 GHz data, we used vector cancellation and removed the system induced phase shift for calculating the correct value of $\beta$.

4.3.4.1 Calculation of $\theta$ with Olympus data.

We will consider the results of the calculation of $\theta$, according to the procedure deduced in § 4.3.2, for an event which took place on 5 July 1990. Though the 5 July 1990-event is not the only one that shows ice depolarisation, it is indeed the only event where depolarisation due to ice appears that clearly separated from rain. In figure 4.4a the canting angle $\theta$, has been plotted during $3\frac{1}{2}$ hours of time. Under that in figure 4.4b, the 20 GHz crosspolar discrimination, after matrix cancellation (XPDCor, shown below) together with cleared 20 GHz copolar signal level (BIVCPL) have been shown for the same period (verticalpolarisation). The values on the left axis-scale concern the CPL-data, the values on the right axis the XPD-data. The period where depolarisation is due to ice crystals can be deduced from figure 4.4b. Ice depolarisation is namely characterized by the low value of copolar attenuation; this is in the period from approximately 18.22.00 till 18.41.00, for which period the canting angle is once more shown in figure 4.5 (above). During the period where ice crystals are present along the propagation path the canting angle varies a lot (from approximately $8^\circ$ till $+15^\circ$) with jumps as large as $15^\circ$. These variations and jumps are much larger as we saw for the rain canting angle. Lightning strokes have been found to cause sudden changes in crosspolar signal level of as much as 10 dB, as reported by Haworth, McEwan and Watson [19]. This effect can be explained by the fact that lightning strokes, being a discharge mechanism of charged thunderclouds, cause a sudden change in the electric field strength, resulting in a sudden change in the alignment of the ice crystals. In figure 4.5 we can see that a large jump in the canting angle data coincides with a sudden change in the XPD-data (figure 4.5, under) and thus is likely to be caused by a lightning stroke.

Spread seems to be less severe for the ice medium canting angle than for the rain medium canting angle, which may be explained by the fact that the ice particles are better aligned than the raindrops.
Fig. 4.4a: Canting angle $\Theta$ of an ice medium, 5 July 1990. (time: 17.00.00 - 20.30.00)

Fig. 4.4b: 20 GHz crosspolar discrimination (vertical polarisation) after matrix cancellation together with cleared 20 GHz copolar signal level, 5 July 1990 (time: 17.00.00 - 20.30.00).
4.3.4.2 Calculation of $A$ and $\beta$ with Olympus data.

To obtain the differential attenuation $A$ and differential phase shift $\beta$ we use the same event as for calculating the canting angle. After calculating $\beta$ we plotted this parameter versus XPD during the period where ice depolarisation was present (from approximately 18:22:00 till 18:41:00) and performed a curvefit. The theoretical curve, which we plotted in all figures has been calculated according to the theoretical relation derived by Tsolakis and Stutzman, assuming that the differential attenuation $A$ is one and the canting angle $\Theta$ is zero.

In figures 4.6a, 4.6b, 4.6c and 4.6d we also made these assumptions (algorithm 2b) for calculating $\beta$. In figures 4.6a and 4.6b the differential phase shift $\beta$ has been set out versus 20 GHz (vertical respectively horizontal) crosspolar discrimination XPD. Figures 4.6c and 4.6d show scatterplots of the differential phase shift $\beta$ versus respectively 12.5 GHz and 30 GHz XPD. The solid line in these figures shows the performed curvefit, the broken line is the theoretical relation.

Fig. 4.5: Canting angle $\Theta$ (above) and 20 GHz crosspolar discrimination XPD (vertical polarisation) of an ice medium, 5 July 1990. (time: 18.22.00 - 18.41.00)
Figure 4.6a: 20 GHz calculated differential phase shift versus 20 GHz *vertical* crosspolar discrimination, 5 July 1990.

Assumptions: \( A = 1, \Theta = 0^\circ \). (- = theory, - = curvefit)

Figure 4.6b: 20 GHz calculated differential phase shift versus 20 GHz *horizontal* crosspolar discrimination, 5 July 1990.

Assumptions: \( A = 1, \Theta = 0^\circ \). (- = theory, - = curvefit)
Figure 4.6c: 12.5 GHz calculated differential phase shift versus 12.5 GHz crosspolar discrimination, 5 July 1990.
Assumptions: $A=1$, $\Theta=0^\circ$. (- - = theory, - = curvefit)

Figure 4.6d: 30 GHz calculated differential phase shift versus 30 GHz crosspolar discrimination, 5 July 1990.
Assumptions: $A=1$, $\Theta=0^\circ$. (- - = theory, - = curvefit)

We can see that the regression curves match the theoretical relation well, for all frequencies.

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In figures 4.7a till 4.7d, we omitted the assumption that $\Theta = 0^\circ$ and calculated the differential phase shift according to algorithm 2a. The solid line in these figures shows the performed curvefit, the broken lines are the theoretical relations for maximum, mean and minimum values of the ice crystal canting angle ($12^\circ$, $2^\circ$, $-8^\circ$ respectively). We will call the performed curvefit a regression curve.

Figure 4.7a: 20 GHz calculated differential phase shift versus 20 GHz vertical crosspolar discrimination, 5 July 1990.
Assumption: $A=1$. (--- theory (max., mean, min.), - = curvefit)

Figure 4.7b: 20 GHz calculated differential phase shift versus 20 GHz horizontal crosspolar discrimination, 5 July 1990.
Assumption: $A=1$. (--- theory (max., mean, min.), - = curvefit)
Figure 4.7c: 12.5 GHz calculated differential phase shift versus 12.5 GHz crosspolar discrimination, 5 July 1990.
Assumption: $A = 1$. (--- = theory (max., mean, min.), - = curvefit)

Figure 4.7d: 30 GHz calculated differential phase shift versus 30 GHz crosspolar discrimination, 5 July 1990.
Assumption: $A = 1$. (--- = theory (max., mean, min.), - = curvefit)

The regression curves still match the theoretical relation well. However, especially for higher values of $\beta$, we can see some spread on both sides of the regression curve.

Tsolakis and Stutzman derived an expression to represent the XPD of a homogeneous and purely phase
shifting (i.e. not attenuating) medium as a function of the differential phase shift. We will now check if the assumption of no differential attenuation is justified (A≠1). In figures 4.8a till 4.8h, we calculated first A and next β according to algorithm 1b.

Figure 4.8a: 20 GHz calculated differential attenuation versus 20 GHz vertical crosspolar discrimination, 5 July 1990.
Assumption: θ=0°. ( – = theory, - = curvefit)

Figure 4.8b: 20 GHz calculated differential phase shift versus 20 GHz vertical crosspolar discrimination, 5 July 1990.
Assumption: θ=0°. ( – = theory, - = curvefit)
Figure 4.8c: 20 GHz calculated differential attenuation versus 20 GHz horizontal crosspolar discrimination, 5 July 1990.
Assumption: $\Theta = 0^\circ$. (- = theory, - = curvefit)

Figure 4.8d: 20 GHz calculated differential phase shift versus 20 GHz horizontal crosspolar discrimination, 5 July 1990.
Assumption: $\Theta = 0^\circ$. (- = theory, - = curvefit)
Figure 4.8e: 12.5 GHz calculated differential attenuation versus 12.5 GHz crosspolar discrimination, 5 July 1990.
Assumption: $\theta = 0^\circ$. (--- = theory, - = curvefit)

Figure 4.8f: 12.5 GHz calculated differential phase shift versus 12.5 GHz crosspolar discrimination, 5 July 1990.
Assumption: $\theta = 0^\circ$. (--- = theory, - = curvefit)
Figure 4.8a: 30 GHz calculated differential attenuation versus 30 GHz crosspolar discrimination, 5 July 1990. Assumption: \( \Theta = 0° \). (- = theory, - = curvefit)

Figure 4.8b: 30 GHz calculated differential phase shift versus 30 GHz crosspolar discrimination, 5 July 1990. Assumption: \( \Theta = 0° \). (- - = theory, - = curvefit)

We can see that the differential attenuation \( A \) differs from 1 for low values of XPD however the deviations are minor (maximal 1.04 for \( f=30 \) GHz). The assumption that \( A \) equals 1, made by Tsolakis and Stutzman, can consequently be justified for all frequencies. For the relation between XPD and \( \beta \) we see no striking difference with algorithm 2b, as expected.

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In figures 4.9a till 4.9h we calculated $A$ and next $\beta$ according to algorithm 1a, making no assumptions.

Figure 4.9a: 20 GHz calculated differential attenuation versus 20 GHz vertical crosspolar discrimination, 5 July 1990.
(- = theory (max., mean, min.), - = curvefit)

Figure 4.9b: 20 GHz calculated differential phase shift versus 20 GHz vertical crosspolar discrimination, 5 July 1990.
(- = theory (max., mean, min.), - = curvefit)
Figure 4.9c: 20 GHz calculated differential attenuation versus 20 GHz horizontal crosspolar discrimination, 5 July 1990.

(- - = theory, - = curvefit)

Figure 4.9d: 20 GHz calculated differential phase shift versus 20 GHz horizontal crosspolar discrimination, 5 July 1990.

(- - = theory (max., mean, min.), - = curvefit)
Figure 4.9e: 12.5 GHz calculated differential attenuation versus 12.5 GHz crosspolar discrimination, 5 July 1990.

(- - = theory, - = curvefit)

Figure 4.9f: 12.5 GHz calculated differential phase shift versus 12.5 GHz crosspolar discrimination, 5 July 1990.

(- - = theory (max., mean, min.), - = curvefit)
Figure 4.9a: 30 GHz calculated differential attenuation versus 30 GHz crosspolar discrimination, 5 July 1990. 
(- = theory, - = curvefit)

Figure 4.9b: 30 GHz calculated differential phase shift versus 30 GHz crosspolar discrimination, 5 July 1990. 
(- = theory (max., mean, min.), - = curvefit)

Still the differential attenuation \( A \) differs only a little from 1 (maximal 1.07 for \( f = 30 \) GHz). Regarding the relation between XPD and \( \beta \) we can now see that the regression curves differ slightly from the theoretical relation of Tsolakis and Stutzman. Like in algorithm 2a some spread for higher values of \( \beta \) is present.
4.3.5 Conclusions

Because the theory of modelling an ice medium is basically the same as for a rain medium we are able to use the procedures deduced in chapter 3 to calculate the parameters characterizing ice crystal depolarisation. The canting angle of an ice medium can vary a lot from event to event and also within an event large deviations were found. Striking was moreover that jumps as large as 15° were found, probably due to lightning strokes.

Furthermore we calculated the differential attenuation and the differential phase shift and plotted the latter versus XPD and performed a curvefit in order to test the theoretical relation derived by Tsolakis and Stutzman. Essentially we only use the XPD data for our calculations (algorithm 2b), according to the assumptions made for deriving the theoretical relation between β and XPD (differential attenuation A = 1 and canting angle Θ = 0°). After this, we omitted the suggested assumptions calculating β according to algorithm 2a (A = 1, Θ = 0°), algorithm 1b (A ≠ 1, Θ = 0°) and algorithm 1a (A ≠ 1, Θ ≠ 0°).

We saw that the assumption that A = 1 is justified for all frequencies, however the canting angle can not simply be equalled to zero.
5 Depolarisation due to a mixture of rain and ice

5.1 Introduction

In previous chapters we have calculated the characteristics of the depolarising mechanisms of both a rain and an ice medium. For this, the assumption has been made that the depolarising medium has two axes of symmetry. We saw that in contrast to a rain medium, an ice medium does not cause significant attenuation. As a result, the depolarisation is mainly caused by differential phase shift between the two axes of symmetry of the medium and not by differential attenuation. Another consequence is that the depolarisation does not coincide with significant copolar attenuation, as is the case of rain depolarisation. This property makes it possible to distinguish this effect from depolarisation due to rain, if they occur temporally separated from each other. If on the other hand, the two media are present on the propagation path at the same time they both will cause an amount of depolarisation, which can not easily be separated in rain and ice depolarisation any more. Since generally the two media do not have the same axes of symmetry, the assumption made above is no longer valid and the characteristics have to be calculated in an other way. For analysis of a depolarising medium it is therefore essential to know when we are dealing with a single depolarising medium or a double depolarising one.

In this chapter we will try to analyze the effects of a double depolarising medium; first we will show the XPD-CPA relation both for "short-term" and "long-term" measurements, after which we will describe a method to distinguish a double depolarising medium (mixture of rain and ice) from a single depolarising rain or ice medium.
5.2 Relation between XPD and CPA

In chapter three we noticed that there is a strong correlation between the crosspolar discrimination (XPD) and the copolar attenuation (CPA) for a rain medium. We derived a general rain depolarisation model, which was successful to describe this relation. In chapter four we found that an ice medium strongly differs from this model, due to the fact that ice crystals do not cause significant attenuation. Threshold values of CPA were determined for the three beacon signals above which no ice depolarisation was observed. However, by analyzing the relation between XPD and CPA we sometimes perceived "lobes" in the XPD-CPA scatterplot which could neither be adjusted to pure rain depolarisation nor to pure ice depolarisation. In the following paragraphs we will show the results of our examinations of such events and we will try to analyze them.

5.2.1 Analysis of XPD-CPA relation with Olympus data

As already depicted in previous chapters we have the crosspolar discrimination and the copolar level (XPD- respectively CPL-data) for every beacon signal at our disposal to analyze the XPD-CPA relation. First the measured XPD is subjected to a crosspolar bias removal technique to remove the system influences (for the 20 GHZ beacon signal we used matrix cancellation, in case of 125 and 30 GHz signals vector cancellation was performed), whereas the measured CPL-data is cleared from non-hydrometeor attenuation. Before plotting the data, we execute an algorithm which ensures that the data are not determined during situations that the receivers were "out-of-lock" and thus not appropriate for analyzing purposes. We will show the XPD-CPA relation both for "short-term" measurements and "long-term" ones.

5.2.1.1 "Short-term" measurements.

In figures 5.1a, 5.1b, 5.1c and 5.1d, scatterplots are shown of respectively 12.5, 20 (vertical and horizontal polarisation) and 30 GHz corrected XPD-data versus CPA-data for an event which took place on 8 June 1992 and lasted from 15:57:00 till 17:00:00. The vertical XPD-axis always runs from 15 till 65 dB, whereas the horizontal CPA-axis reaches from 0 till 25 dB. The broken curve indicates the theoretical rain depolarisation relation derived in chapter three. We notice that in case of the 30 GHz beacon signal many of the data are not valid because of "out-of-lock" situations of the receiver, so that only few scatterpoints are indicated for high CPA-values.

In chapter four we determined threshold values of CPA above which no pure ice depolarisation was found. These values are respectively for the 12.5, 20 and 30 GHz beacon signals, 1.0, 1.5 and 3.0. The lobes which lie below these threshold values can be adjusted to a pure ice depolarising medium. However, we also see lobes which clearly lie above this threshold values and seriously differ from the theoretical XPD- CPA curve for rain (XPD values ≤ 15 dB !, for f = 30 GHz ). These lobes are most probably caused by a mixture of rain and ice on the propagation path.

In order to determine where these lobes generally occur in the scatterplots we will analyze the XPD-CPA relations for "long-term" measurements in the next paragraph.
Figure 5.1a: scatterplot of 12.5 GHz XPD after bias removal (vertical axis) versus 12.5 GHz copolar attenuation, 8 June 1992. (- - = theory Chu)

Figure 5.1b: scatterplot of 20.0 GHz XPD after bias removal (vertical axis) versus 20.0 GHz copolar attenuation, 8 June 1992. Vertical polarisation (- - = theory Chu)
Figure 5.1c: scatterplot of 20.0 GHz XPD after bias removal (vertical axis) versus 20.0 GHz copolar attenuation, 8 June 1992. *Horizontal polarisation* (- - = theory Chu)

Figure 5.1d: scatterplot of 30.0 GHz XPD after bias removal (vertical axis) versus 30.0 GHz copolar attenuation, 8 June 1992. (- - = theory Chu)
5.2.1.1 "Long-term" measurements.

We have 0.57 hours of event data at our disposal to analyze the XPD-CPA relation for a coincident rain and ice medium. This coincident rain and ice event data have been collected in three matrices, considering the 12.5, 20 and 30 GHz frequencies. The horizontal CPA-axis reaches from 0 till 30 dB and has been divided in 150 bins (binsize = 0.2 dB). The vertical XPD-axis has a range from 15 till 65 dB and has been divided in 100 bins (binsize = 0.5 dB). The 12.5, 20 (resp. vertical and horizontal) and 30 GHz matrices are graphically shown in the figures 5.2a, 5.2b, 5.2c and 5.2d. The broken curves in the figures indicate the theoretical XPD-CPA relation according to the Chu-model [8].

In all matrices we can see that most data points are gathered in a relative small area. This area, which contains extremely low values of XPD, has a horizontal CPA range from the threshold values of CPA for a pure ice medium (CPA\(_{\text{threshold}}\)) till approximately 2.5, 4 and 9 dB for resp. 12.5, 20 and 30 GHz frequencies. We also can see some data points for higher values of CPA that coincide with very high values of XPD. These points can neither be adjusted to pure ice depolarisation nor to pure rain depolarisation and will therefore be adjusted to coincident rain and ice on the propagation path.

To analyze the XPD-CPA relation for a coincident rain and ice medium, we first subjected the event data to an algorithm which will be described in next paragraph (see § 5.3). By doing so, we removed the pure ice event data and the pure rain event data.

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1 To analyze the XPD-CPA relation for a coincident rain and ice medium, we first subjected the event data to an algorithm which will be described in next paragraph (see § 5.3). By doing so, we removed the pure ice event data and the pure rain event data.
Figure 5.2b: Scatterplot of 00.57 hours of corrected 20 GHz XPD versus 20 GHz CPA.  
*Vertical polarisation* (- - = theory Chu)

Figure 5.2c: Scatterplot of 00.57 hours of corrected 20 GHz XPD versus 20 GHz CPA.  
*Horizontal polarisation* (- - = theory Chu)
5.2.2 Conclusions

Rain and ice depolarisation, which occur temporally separated from each other on the propagation path, can be distinguished by among others the different XPD-CPA relations for both mechanisms. If on the other hand, the two media are present on the propagation path at the same time, the XPD-CPA relation during this time will differ from those of rain and ice. We investigated this XPD-CPA relation for coincident rain and ice both for "short-term" and "long-term" measurements. It appeared that most event data are gathered in a relative small area. This area, which contains extremely low values of XPD, has a horizontal CPA range from the threshold values of CPA for a pure ice medium (CPA_{threshold}) till approximately 2.5, 4 and 9 dB for resp. 12.5, 20 and 30 GHz frequencies.
5.3 Discrimination between a double depolarising medium and a single depolarising one

If we intend to calculate the parameters which characterize the depolarisation mechanisms on the propagation path, we first have to model the medium. A single depolarising medium can be modelled by assuming that the hydrometeors (raindrops or ice crystals) have two axes of symmetry. The differential attenuation and/or differential phase shift between these two eigendirections are the cause of depolarisation. In case of a double depolarising medium, we generally can not assume the media to have the same axes of symmetry, so that the above mentioned model does not suffice any longer. Before developing a new model, we first should solve the problem of distinguishing a double depolarising medium from a single depolarising one, in order to know for which periods of time the models should be applied for.

5.3.1 Decision criteria.

In § 5.2 we found some different lobes in the XPD-CPA scatterplots, which could not be attributed to a single depolarising rain or ice medium. Nevertheless it will be very hard to determine the exact size of the lobes from these figures and attribute them to coincident rain and ice. Moreover, the XPD-CPA relationship is an average and very many measurements will differ slightly from the ideal curve, without any necessary contribution of ice.

Better criteria can be found using characteristic conditions for a single depolarising medium. From previous chapters we know that the differential phase shift ($\beta$) between the two eigendirections of the medium is both responsible for rain and ice depolarisation, but that this parameter shows a different relation with respectively the copolar attenuation (CPA) of a rain and the CPA of an ice medium. Fukuchi [13][14] derived a relation (see § 3.3.3), which was successful to describe the $\beta$-CPA relation for a rain medium. Moreover, we determined different threshold values of CPA ($\text{CPA}_{\text{threshold}}$), for an ice medium, above which no ice depolarisation was observed. Consequently, above these threshold values there will exist no differential phase shift. If we combine these properties of the single media we may conclude that serious deviations from the $\beta$-CPA relation for $\text{CPA} > \text{CPA}_{\text{threshold}}$ will be caused by a double depolarising medium.

Paraboni [20] presented criteria, distinguishing three types of media:
- homogeneous with principal planes (characteristic for a single depolarising medium);
- homogeneous without principal planes;
- inhomogeneous (characteristic for a double depolarising medium);

He showed that a characteristic condition for the existence of principal planes is:

\[ \text{Im } D_x = \text{Im } D_y, \]  \hspace{1cm} (5.1)

where $D_x$ and $D_y$ are the "depolarisation ratios". They are related to measured quantities as:

\[ \text{HXPD} = 20 \log |D_x|; \]
\[ \text{VXPD} = 20 \log |D_y|; \]
HDPH = \text{arg}(D_y) (\phi_y);
VDPH = \text{arg}(D_x) (\phi_x);

with

(V)(H)XPD = XPD, measured for a signal, transmitted in y respectively x direction;
(V)(H)DPH = \text{co/crosspolar relative phase}, measured for the same signal.
The characteristic condition for the two principal planes can be translated in the "imaginary difference" I:

\[ I = \text{Im } D_x - \text{Im } D_y = |D_x| \sin \phi_x - |D_y| \sin \phi_y \] (5.2)

Equation (5.1) is equivalent to "I=0". If I exceeds a certain threshold level, the medium should be treated as a double medium. Empirical investigation will have to make clear whether such a distinction can be made at all, and what the threshold value should be in that case. A deviation of I from zero will result in a complex canting angle \( \eta \) (see eq. 3.12). Therefore the imaginary part of \( \eta \) could also be used as a criterion. However, since this shows no great advantages, this will not be used.

Furthermore, Paraboni showed that a characteristic condition for homogeneity is:

\[ R = \frac{D_x}{D_y} \] (5.3)

where R is the "copolar signal ratio".
R is related to measured quantities as:
B1VHCPL = 20\log |R|;
B1VHPH = \text{arg}(R) (\phi_y);

with
B1VHCPL = relative copolar signal level between two polarisations (20 GHz);
B1VHPH = relative copolar signal phase between two polarisations (20 GHz);

The characteristic condition for homogeneity can be written as follows, using the "antidiagonal ratio" S:

\[ S = \frac{D_x}{(R \cdot D_y)} \] (5.4)

Equation (5.3) is equivalent to "S=1". If S exceeds a certain threshold level (in absolute value or in phase), the medium should be treated as a double medium. As in the previous case, the value of this threshold level should be empirically determined. One problem associated with this criterion is that S will probably differ very little from unity. Paraboni already showed this, concluding that the depolarising behaviour of an inhomogeneous medium differs only slightly from that of a homogeneous medium without principal planes (for which S =1).

In following paragraphs we will test the decision criteria. We will illustrate our examinations by means of the 8 June 1992 event for the 20 GHz beacon frequency (both vertical and horizontal polarisation).
5.3.1.1 Testing the decision criteria with Olympus data.

Now we have presented the possibilities to separate a single depolarising medium from a double depolarising one, we will in this paragraph describe the results of our examinations to test the decision criteria with Olympus data.

First of all we will examine the relation between the differential phase shift $\beta$ and the copolar attenuation CPA. We will also show the relationship between the differential attenuation $A$ and CPA, to check the special property of ice particles, namely that ice particles do not cause significant differential attenuation. We will calculate $A$ and $\beta$ assuming that the hydrometeors have two axes of symmetry\(^1\). The theory of modelling such a medium has already been fully described in previous chapters, so that we will confine ourselves to show only the results of our calculations. Furthermore $A$ and $\beta$ will be determined according to the recommendations made in chapters three and four, namely that $A$ and $\beta$ could best be calculated using XPD, DPH and canting angle data.

In figures 5.2a and 5.2b the differential phase shift has been plotted versus 20 GHz (respectively vertical and horizontal) copolar attenuation. Figures 5.3a and 5.3b show scatterplots of the differential attenuation $A$ versus 20 GHz (respectively vertical and horizontal) CPA. The broken curves in these figures indicate the predicted behaviour of $A$ and $\beta$ as given by Fukuchi for a rain medium.

![Figure 5.2a: Scatterplot of 20.0 GHz calculated differential phase shift (vertical axis) versus 20.0 GHz vertical copolar attenuation, 8 June 1992. (--- = theory Fukuchi)](image)

\(^1\) NOTE: not to much attention should be paid to the exact values of $A$ and $\beta$ during the time that two depolarising mechanisms occur, because for the whole event period we made the assumption that the hydrometeors have the same axes of symmetry. If two depolarising media are present along the propagation path, this will generally not be the case, so that the values of $A$ and $\beta$ may not have very much physical meaning.
Figure 5.2b: Scatterplot of 20.0 GHz calculated differential phase shift (vertical axis) versus 20.0 GHz horizontal copolar attenuation, 8 June 1992. (--- = theory Fukuchi)

Figure 5.3a: Scatterplot of 20.0 GHz calculated differential attenuation (vertical axis) versus 20.0 GHz vertical copolar attenuation, 8 June 1992. (--- = theory Fukuchi)
In figures 5.2a and 5.2b we can distinguish between two areas on the CPA-range: one area above approximately 10 dB where the $\beta$-CPA relation matches the theoretical relation reasonably well and can therefore be adjusted to a pure rain medium, and an other area below 10 dB where we see some different lobes in the scatterplots, which are too large to be adjusted to a pure rain medium. One lobe lies entirely under the CPA$_{\text{threshold}}$ value of 1.5 dB and therefore this will be attributed to a pure ice depolarising medium. The other lobes lie above this CPA$_{\text{threshold}}$ and must be caused by the occurrence of a mixture of raindrops and ice crystals on the propagation path. In figures 5.3a and 5.3b, we notice that ice particles on the propagation path cause negligible differential attenuation, as expected.

Further, we calculated the "imaginary difference" I, which is shown in figure 5.4 for the event period. To determine I, we first cleared the XPD and DPH data, with aid of matrix cancellation, in order to remove the "system influences". Before plotting I, we performed an algorithm which ensures that the data were not determined during situations that the receivers were "out-of-lock". The time periods where the data are not valid have been left out in figure 5.4. Nevertheless, from figure 5.4 it is not possible to determine a threshold level for I, therefore we also calculated I for other events but never could clearly discriminate between a single depolarising medium and a double depolarising one. In fact this is not as amazing as it may seem, we should namely notice that our calculations are influenced by measurement errors. The sensitivity of $D_x$ and $D_y$ (and thus also I) to measurement errors in $\phi_x$ and $\phi_y$ grows dramatically when the XPD values become higher. Therefore, it is not fair to compare values of I for XPD values reaching from 15 till 40 dB!

We also calculated the "antidiagonal ratio" S. However, both in absolute value and in phase, S deviates too little from unity to see in a figure any deviations. So, we can not distinguish between a homogeneous and inhomogeneous medium, as expected. Calculations for other events confirmed this.

In the following paragraph we will develop the decision criterion considering the $\beta$-CPA relation further on, in order to use it as a way to separate a double depolarising medium from a single depolarising one in the time domain.
5.3.2 A discrimination method.

Now we know which lobes in the $\beta$-CPA scatterplots are an indication for a double depolarising medium, we intend to find out during which period of time in the event a mixture of rain and ice was present on the propagation path. A convenient property of the $\beta$-CPA relationship, as derived by Fukuchi, is the fact that the slope of the theoretical curve in the $\beta$-CPA scatterplot is constant (at least in case of 20 GHz frequency!), so that the ratio between $\beta$ and CPA plotted versus time will equal a constant value in the theoretical case. This constant value has already been determined in § 3.3.3 and equals 0.8. From measurements we do know that the experimental, with Olympus data determined relation matches the theoretical relation very well, in case of a pure rain medium and that the spread around the mean is small (see chapter 3). It is therefore possible to use this relation to distinguish between a single and a double depolarising medium. If we do so, and leave out the CPA-values where CPA lies under the CPA$_{\text{threshold}}$ value of 1.5 dB we should find serious deviations from the constant 0.8 for the periods in which also ice-particles are present. The extent of the deviations (or lobes) are also a criterion for the ratio between ice- and rain content in the mixture; a large deviation from the constant 0.8 means that ice-particles are dominant on the propagation path, whereas small deviations must be caused by rain dominated periods.

In figure 5.5a and 5.5b we plotted the ratio between $\beta$ and CPA for the event period (15:57:00-17:00:00). The time periods where the data was invalid and/or CPA did not exceed the CPA$_{\text{threshold}}$ value of 1.5 dB have been left out in the figures. For a great period of time we see that $\beta$/CPA equals the theoretical value of

---

2 To exclude measurement errors or small inhomogeneities in a rain medium we will only consider the $\beta$/CPA values above 1.6 (twice the theoretical value). However, we remark that this value is an arbitrary one.
0.8 well; during this period of time rain will be the only depolarising medium. Furthermore we can distinguish more or less four lobes, which differ seriously from the constant value. During the period of time where lobes 2, 3 and 4 occur, which peaks reach values of approximately $10^\text{dB}$, ice must be the dominant hydrometeor, whereas during the time lobe 1 ice-particles will play a less dominant role.

Figure 5.5a: 20.0 GHz calculated $\beta$/VCPA versus time, 8 June 1992 (time periods where the data are invalid and/or CPA > CPA$_\text{threshold}$ of an ice medium (= 1.5 dB) have been left out.).

Figure 5.5b: 20.0 GHz calculated $\beta$/HCPA versus time, 8 June 1992 (time periods where the data are invalid and/or CPA > CPA$_\text{threshold}$ of an ice medium (= 1.5 dB) have been left out.).

However, by doing so, we can only distinguish the time periods where $\beta$/CPA exceeds 0.8 clearly. These are in fact the lobes which lie above the theoretical curve. The lobes which lie below the theoretical curve can
be better distinguished by plotting the ratio between CPA and $\beta$. In figure 5.6a and 5.6b we plotted the ratio between CPA and $\beta$ for the event period (15:57:00-17:00:00). The time periods where the data was invalid and/or CPA did not exceed the $\text{CPA}_{\text{threshold}}$ value of 1.5 dB have been left out in the figures. For a great period of time we see that $\beta/\text{CPA}$ equals the theoretical value of 1.25 ($0.8^{-1}$) well; during this period of time rain will be the only depolarising medium. Furthermore we can distinguish one lobe, which lies significantly above the constant value. During this time period coincident rain and ice will also be presents along the propagation path.

Figure 5.6a: 20.0 GHz calculated $\frac{\text{VCPA}}{\beta}$ versus time, 8 June 1992 (time periods where the data are invalid and/or CPA $> \text{CPA}_{\text{threshold}}$ of an ice medium ($= 1.5$ dB) have been left out.).

Figure 5.6b: 20.0 GHz calculated $\frac{\text{HCPA}}{\beta}$ versus time, 8 June 1992 (time periods where the data are invalid and/or CPA $> \text{CPA}_{\text{threshold}}$ of an ice medium ($= 1.5$ dB) have been left out.).
By taking out the time periods where the different lobes occur, we can separate the time-periods where a double depolarising medium was present from the time-periods where a single depolarising medium occurred.

In figure 5.7 we plotted the XPD and CPL-data (vertical polarisation) for the same event period.

Figure 5.7: 20.0 GHz crosspolar discrimination (vertical polarisation) after bias removal (below) together with 20.0 GHz copolar signal level, 8 June 1992.

5.3.3 Conclusions

The parameters that characterize a coincident rain and ice medium can not be calculated in the way they are calculated for a single depolarising medium. This is due to the fact that generally the two media do not have the same axes of symmetry, which assumption is made for modelling a single depolarising medium. For analysis of a depolarising medium it is therefore essential to know when we are dealing with a single depolarising medium or a double depolarising one. Several decision criteria have been presented to tell the two media from each other. One criterion, which uses the specific properties of both a pure rain medium and a pure ice medium appeared to be the most appropriate one. By plotting $CPA/\beta$ and $\beta/CPA$ versus time for CPA values above which no pure ice depolarisation occurs ($CPA_{threshold}$) we can distinguish a coincident rain and ice medium by the serious deviations from the theoretical constant values that are valid for a pure rain medium. Because, in practice some spread around the theoretical constant value is present for a pure rain medium, we suggest to take out those peaks which values exceed a certain threshold level.
6 Conclusions and recommendation

Before we examined the influence of depolarisation due to hydrometeors, we first removed the depolarising influences that are not caused by the atmosphere. We used two bias removal techniques to remove these system influences. One technique, which was used for the single polarised 12.5 and 30 GHz Olympus beacon signals is called vector cancellation and was already implemented in the software. The other technique that uses the specific characteristics of the dual polarised 20 GHz beacon signal is called matrix cancellation. In the case of matrix cancellation we assumed that the system depolarisation is fully caused by either the depolarisation of the satellite or the depolarisation of the ground station. If the assumption is correct we can fully correct either the depolarisation of the satellite (right matrix cancellation) or the depolarisation of the ground station (left matrix cancellation). If the assumption concerning the cause of the system depolarisation is not correct then there remains a residual error in XPD after matrix cancellation. The maximum residual error \( \Delta \text{XPD} \) is demonstrated to be the same as for vector cancellation and only depends on the polarisation tilt angle and on either the satellite XPD (left matrix cancellation) or the ground station XPD (right matrix cancellation). Left matrix cancellation (correction of ground station) is preferred to right matrix cancellation.

The relation between depolarisation and attenuation differs strongly for a pure rain medium, a pure ice medium and a coincident rain and ice medium, respectively. The relation between depolarisation and attenuation for a pure rain medium has both theoretically and empirically been described in different rain depolarisation models in terms of XPD and CPA. Based on these different models, one general abstracted rain depolarisation model has been determined which is characterized by four parameters \( U, V, S \) and \( C \). By means of performing curvefits on the measuring data we have determined "short-term" and "long-term" values of \( U, V, S \) and \( C \) for the three beacon signals. The "short-term" parameter values have been determined by performing curvefits on the measuring data of one single event, whereas the "long-term" parameter values have been obtained by performing one curvefit on all collected event data (55.04 hours of event data, whereby CPA\( \geq 10 \) dB for \( f = 20 \) GHz). After this we have compared these experimental determined values with the theoretical derived values of the different rain depolarisation models. The large standard deviations of the "short-term" parameter values show that the parameters can rather differ from event to event. Especially the \( S \) and \( C \) "short-term" values show a huge standard deviation. The "long-term" parameter values match the values of the D/H/W-, Chu- and SIM models best.

The relation between XPD and CPA for a pure ice medium is characterized by the huge decay of XPD values for relative low values of CPA. We noticed that above certain values of CPA (CPA\( _{threshold} \)) no pure ice depolarisation was observed at all. These threshold values of CPA have been determined for the three beacon signals. The CPA\( _{threshold} \) values for the 12.5, 20 and 30 GHz frequencies are 1.0, 1.5 and 3.0 dB respectively, based on 1.29 hours of data.

By analyzing the relation between XPD and CPA we sometimes perceived "lobes" in the XPD-CPA scatterplots which could neither be caused by a pure rain medium nor by a pure ice medium and have therefore been adjusted to coincident rain and ice. It appeared that most data points were concentrated in a relative small area. This area, which contains extreme low values of XPD, has for our event selection a CPA range from the threshold values of CPA for a pure ice medium (CPA\( _{threshold} \)) till approximately 2.5, 4 and 9 dB for the 12.5, 20 and 30 GHz frequencies, respectively. These values have been obtained from a
collection of 0.57 hours of event data.

For depolarisation analysis, the medium can be characterized by three parameters: the mean canting angle, the differential attenuation and the differential phase shift. Both the mean raindrop canting angle and the mean ice crystal canting angle can vary a lot from event to event (from approximately -20° till +20°). The course of the mean canting angle within a single event is in case of a pure ice medium much more whimsically than for a pure rain event. Moreover, the course of the mean ice crystal canting angle is sometimes characterized by huge jumps (circa 15°), probably due to "lightning" strokes. The course of the raindrop canting angle is much noisier (resulting in a lot of spread around the mean) than for the ice crystal canting angle. This might be explained by the fact that ice crystals are better aligned than raindrops. The differential attenuation $A$ and the differential phase shift $\beta$ have both for a pure rain and a pure ice medium been calculated with different algorithms. After this, the calculated parameters have been compared with theoretical/semi-empirical derived relations. Although, small differences have been observed between the parameters that are calculated according to the different algorithms, none of them can actually be preferred above the other. We recommend to make as less assumptions as possible for calculating $A$ and $\beta$ using $\Theta$, XPD- and DPH data.

The parameters $\Theta$, $A$ and $\beta$ that characterize a coincident rain and ice medium can not be calculated in the way they are calculated for a single depolarising pure rain medium or pure ice medium. This is due to the fact that generally the two media do not have the same axes of symmetry, an assumption that is made for modelling a single depolarising medium. For depolarising analysis it is consequently essential to know when we are dealing with a single depolarising medium or a double one. We have determined a discrimination method which uses the specific properties of both a pure rain and a pure ice medium. By plotting $\text{CPA}/\beta$ and $\beta/\text{CPA}$ versus time for CPA-values above which no pure ice depolarisation occurs ($\text{CPA}_{\text{threshold}}$) we can distinguish a coincident rain and ice medium by the serious deviations from the theoretical constant values that account for a pure rain medium (at least in case of the 20 GHz frequency!). Because, in practice there is some spread around the theoretical constant value in case of a pure rain medium, we recommend to take out only those peaks, of which the values exceed a certain threshold value.

Finally we recommend to develop an algorithm to separate rain from ice in a coincident rain and ice medium, now we know the time periods where a coincident rain and ice medium is present along the propagation path.
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Appendix A

Copolar signal correction by means of radiometer data.

For propagation research it is often desirable that only pure rain attenuation is taken into account for analysis. In that case it is necessary to determine the signal attenuation by gases, water vapour and clouds separately during the clearsky periods. The course of these attenuation during a day is called a template. The actual rain attenuation can be determined after this by subtracting the measured signal from the template. The attenuation owing to gases, water vapour and clouds can be determined with aid of a radiometer [3]. The attenuation at radiometer frequency can be calculated with:

$$|T_{\text{clearsky}}| = 10 \log \left[ \frac{T_m - T_c}{T_m - T_g} \right] \quad (\text{dB}) \quad \text{(A.1)}$$

with

$$|T_{\text{clearsky}}| = \text{clearsky attenuation (at radiometer frequency)} \geq 0 \text{ dB},$$

$$T_m = \text{effective medium temperature (270 K)},$$

$$T_c = \text{cosmic-noise temperature (4K)},$$

$$T_s = \text{sky-noise temperature (K)}.$$

$T_s$ is given by:

$$T_g = \frac{T_s}{h} + \left(1 - \frac{1}{h}\right) T_g \quad (K) \quad \text{(A.2)}$$

with

$T_a = \text{antenna noise temperature (K)}$

($T_a$ is registered by the EUT acquisition system per beacon frequency, channel names: $RnTANT; n = 0,1,2$),

$T_g = \text{ground noise temperature (K)}$ and defined by, $T_g = A_g + B_g T_e$, $A_g$ and $B_g$ have constant values (see table A.1) and $T_e$ is the temperature of environment,$$
$$

($T_e$ is registered by the EUT acquisition system, channel name: $MITAMB$),

$h = \text{antenna integration factor (see table A.2)}.$
Table A.1: Values of the Ag and Bg for 12.5, 20 and 30 GHz (derived by EUT).

<table>
<thead>
<tr>
<th>frequency (GHz)</th>
<th>12.5</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>season</td>
<td>summer</td>
<td>winter</td>
<td>summer</td>
</tr>
<tr>
<td>Ag</td>
<td>2.0</td>
<td>3.7</td>
<td>5.6</td>
</tr>
<tr>
<td>Bg</td>
<td>0.48</td>
<td>0.45</td>
<td>0.53</td>
</tr>
</tbody>
</table>

Table A.2: Values of the antenna integration factor h.

<table>
<thead>
<tr>
<th>frequency (GHz)</th>
<th>12.5</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td>0.91</td>
<td>0.93</td>
<td>0.92</td>
</tr>
</tbody>
</table>
Appendix B

Reliability of Olympus data

B.1 EUT receivers in "out-of-lock" situations

For data processing it is necessary to know when the data is unreliable and thus cannot be used for analyzing purposes because of "out-of-lock" situations of the receivers. The data acquisition system of the EUT ground station does not know if the receivers function properly during a period of measurement and consequently also samples and digitalizes the signal levels during periods in which one or more receivers are "out-of-lock".

For all three beacon signals, two receivers per beacon signals are installed in the EUT ground station, which receive respectively the copolar and crosspolar signals.

The 12.5 and 30 GHz receivers are analogue and equipped with phase locked loops (PLL) which take care of the local oscillator signals, with which the received signals are modulated, having the right frequency all the time. For a good functioning of the PLL it is important that the received signal level is large enough with respect to the system noise level. If the signal level becomes too small regarding the system noise level, the PLL and thus the receiver will get "out-of-lock". The signal levels for which the receivers get "out-of-lock", differ per receiver. We will call these signal levels "out-of-lock-levels" and note them in dB's below clear-sky level. The crosspolar-receivers have in contrast to the copolar receivers a signal level, by which they are formally still in lock but already show unstable behaviour in practice. The "out-of-lock- levels" have been shown in table B.1 per receiver.

<table>
<thead>
<tr>
<th>receiver</th>
<th>out_of_lock level (level 1)</th>
<th>stability level (level 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.5 GHz copolar receiver</td>
<td>30 dB</td>
<td>-</td>
</tr>
<tr>
<td>12.5 GHz crosspolar receiver</td>
<td>55 dB</td>
<td>45 dB</td>
</tr>
<tr>
<td>20 GHz V copolar receiver</td>
<td>-</td>
<td>30 dB</td>
</tr>
<tr>
<td>20 GHz V crosspolar receiver</td>
<td>-</td>
<td>45 dB</td>
</tr>
<tr>
<td>20 GHz H copolar receiver</td>
<td>-</td>
<td>30 dB</td>
</tr>
<tr>
<td>20 GHz H crosspolar receiver</td>
<td>-</td>
<td>45 dB</td>
</tr>
<tr>
<td>30 GHz copolar receiver</td>
<td>30 dB</td>
<td>-</td>
</tr>
<tr>
<td>30 GHz crosspolar receiver</td>
<td>42 dB</td>
<td>41 dB</td>
</tr>
</tbody>
</table>
The 20 GHz receiver has digitally been realised. The dynamic range of this receiver is limited by the number of bits of the A/D converter and by the precision (in dB's) at the lower side of the range. (So, formally it is not right to speak from "out-of-lock" situations). In table 3.1 is shown how many dB's the 20 GHz signals are allowed to get under the copolar clearsky level before the lower boundaries of the dynamic range will be attained.

B.2 An algorithm to determine the reliability of the data

Now we know at which levels the receivers get "out of lock" respectively become unstable, an important perception can be made for the crosspolar receivers. Let us take the 20 GHz (horizontal) receiver as an example. The 20 GHz crosspolar receiver becomes unstable at a crosspolar level \( X_{PL} = 30 - 45 = -15 \) dB, or

\[
X_{PL_{\text{unstable}}} = CPL - XPD \geq -15 \text{ dB} \quad (20 \text{ GHz}) \tag{B.1}
\]

Together with the relation which applies for the copolar attenuation (CPA)

\[
CPA = CPL_{\text{clear-sky}} - CPL = 30 - CPL \text{ (dB)} \tag{B.2}
\]

it follows

\[
X_{PD_{\text{unstable}}}(CPA) \leq 45 - CPA \text{ (dB)} \tag{B.3}
\]

The XPD-CPA plane is, because of the above inequality, divided in two parts. The points lying under the line \( 45 - CPA \) (dB) can be marked as reliable, the points lying above this line as unreliable. In practice there is of course not such a sharp crossing between reliable and unreliable data; the line \( XPD = 45 - CPA \) can be considered as a noise threshold under which the receiving system does not function properly. Both theoretically and empirically has been shown that there is a relation between XPD and CPA caused by rain:

\[
X_{PD_{\text{db}}} = U - V \log(CPA_{\text{db}}) \tag{B.4}
\]

where \( U \) and \( V \) are functions of a large number of parameters (frequency, elevation, etc.)

Figures B.1, B.2 and B.3 show the relation between XPD and CPA for the 3 Olympus beacon frequencies, according to five well known rain depolarisation models. We notice that, particularly for the 30 GHz beacon signal, the measured values will be close to the noise threshold. For a reliable analysis of the measured data we will therefore continuously have to take the possibility into account of invalid data.
Figure B1: theoretical relation between XPD and CPA with regard to the noise threshold (12.5 GHz).

Figure B2: theoretical relation between XPD and CPA with regard to the noise threshold (20 GHz).
We can do this by subjecting the measured data to an algorithm, which determines whether the data is reliable or not. In the EUT data acquisition system not the co- and crosspolar signal levels are sampled (CPL and XPL) but the copolar level and the crosspolar discrimination (CPL and XPD). However with the relation \( XPD = CPL - XPL \) (dB) the crosspolar signal level can easily be calculated. Following algorithm creates a file which shows per sample whether the data is reliable or not.

```plaintext
• initialize \( \text{wait for lock} = 0, \text{CPL in lock}, \text{XPL in lock} \)
• do for all samples {
  • read CPL and convert to dB's
  • read XPD and convert to dB's
  • \( \text{XPL} = \text{CPL} - \text{XPD} \)
  • if frequency = 20 GHz then \( \text{wait for lock} = 0 \)
    else if (\( \text{wait for lock} <> 0 \)) then \( \text{wait for lock} = \text{wait for lock} - 1 \)
    else if (\( \text{CPL} < \text{clearsky co level} - 1 \)) then \( \text{wait for lock} = 1 \)
  • if (\( \text{CPL} < \text{clearsky co level} - 1 \)) then \( \text{CPL out lock} \) else \( \text{CPL in lock} \)
  • if (\( \text{XPL} < \text{clearsky cross level 1} \) or \( \text{wait for lock} <> 0 \)) then \( \text{XPL out lock} \) else \( \text{XPL in lock} \)
  • if (\( \text{XPL} < \text{clearsky cross level 2} \)) then \( \text{data unstable} \) else \( \text{data stable} \)
  • if (\( \text{CPL out lock} \) or \( \text{XPL out lock} \) or \( \text{data unstable} \)) then
    write to reliability file: \( \text{data unreliable} \)
    write to reliability file: \( \text{data reliable} \)
} 
```

Figure B3: theoretical relation between XPD and CPA with regard to the noise threshold (12.5 GHz).
Appendix C

Analysis of the sensitivity of $\phi_0$ to data variations

Before examining the sensitivity of $\phi_0$ to data variations it is worth taking the expected values of $\phi_0$ and $R$ into account. In papers describing rain depolarisation models, mentioned in § 3.2, it is stated that $\bar{\Theta}$ and $\sigma_\Theta$ (the time average of $\Theta_m$) are $\theta^o$ and $5^o$. However, from measurements we know that the canting angle $\Theta$ can vary considerably ($\sigma_\Theta \approx 15^o$). We, consequently assume $\phi_0$ to be around $\delta = 18.4^o$ and choose $3^o$ and $33^o$ as most extreme values. Furthermore, if the depolarisation is not severe, $D_x$ and $D_y$ are large and close together and consequently $R$ is close to unity ($R \approx 1$).

We take these assumptions into account for examining the sensitivity to data variations in $\phi_0$, B1VHCPL and B1HXPD, where $\phi_0$ is defined according to formula (3.16) in § 3.3.1:

$$\phi_0 = \frac{1}{2} \arctan \frac{2}{\sqrt{\mathcal{A}^2 + \mathcal{B}^2}}$$

C.1 The sensitivity of $\phi_0$ to data variations in $\phi_r$

We shall first investigate the sensitivity of $\phi_r$ to data variations in $\phi_0$. Table C1 shows these calculations, where we made the assumptions that $\phi_0 = 18.4^o (\theta = 0^o)$ and B1VHCPL = 0 dB ($|R| = 1$)

<table>
<thead>
<tr>
<th>VXPD = 20 dB</th>
<th>VXPD = 30 dB</th>
<th>VXPD = 40 dB</th>
</tr>
</thead>
<tbody>
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<td>10.3</td>
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It seems reasonable to exclude data, by which VXPD exceeds 40 dB, since a 1° error in $\phi_r$ can result in deviations as large as 15° (chosen as most extreme value).

C.2 The sensitivity of $\phi_0$ to data variations in VXPD

Now we have investigated the influence of the phase of $R$ on $\phi_0$ it is also interesting to know the influence of data variations of the amplitude on $R$ (registered by the EUT-data acquisition as B1VHCPL). Table C2 shows the calculations where we assumed $\phi_0 = 18.4° (\Theta = 0°)$ and $\phi_r = 0°$.

<table>
<thead>
<tr>
<th>VXPD = 20 dB</th>
<th>VXPD = 30 dB</th>
<th>VXPD = 40 dB</th>
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<tr>
<td>1.88</td>
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<td>0.47</td>
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</tbody>
</table>

This seems reasonable, since a $\approx 0.2$ dB (maximal) error in B1VHCPL (VXPD = 40 dB) causes $\phi_0$ to vary in the limited range of 15°.
C.3 The sensitivity of $\phi_0$ to data variations in VXPD

<table>
<thead>
<tr>
<th>VXPD</th>
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<td>65.6</td>
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<td>48.4</td>
<td>18.6</td>
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<tr>
<td>47.5</td>
<td>20.1</td>
</tr>
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</table>

Also huge errors (1 dB) in VXPD cause a negligible deviation in $\phi_0$.

C.4 Conclusions

The influence of $\phi_r$ on $\phi_0$ seems to be the hugest. A 1° error in $\phi_r$ (VXPD = 40 dB) can result in a 15° deviation of $\phi_0$ (chosen as most extreme value of standard deviation). We therefore recommend to exclude data, where VXPD exceeds 40 dB, from calculations of $\phi_0$. 
Appendix D

Time registration of event files

Figure D.1a: 12.5 GHz copolar signal level (above) together with 12.5 GHz crosspolar discrimination after bias removal, 25 September 1991 (time: 10:00:00 - 13:30:00). *Vertical polarisation*

Figure D.1b: 20 GHz copolar signal level (above) together with 20 GHz crosspolar discrimination after bias removal, 25 September 1991 (time: 10:00:00 - 13:30:00). *Vertical polarisation*
Figure D.1c: 20 GHz copolar signal level (above) together with 20 GHz crosspolar discrimination after bias removal, 25 September 1991 (time: 10:00:00 - 13:30:00). Horizontal polarisation

Figure D.1d: 30 GHz copolar signal level (above) together with 30 GHz crosspolar discrimination after bias removal, 25 September 1991 (time: 10:00:00 - 13:30:00). Vertical polarisation
Figure D.2a: 12.5 GHz copolar signal level (above) together with 12.5 GHz crosspolar discrimination after bias removal, 5 July 1990 (time: 17:00:00 - 20:30:00). Vertical polarisation

Figure D.2b: 20 GHz copolar signal level (above) together with 20 GHz crosspolar discrimination after bias removal, 5 July 1990 (time: 17:00:00 - 20:30:00). Vertical polarisation
Figure D.2c: 20 GHz copolar signal level (above) together with 20 GHz crosspolar discrimination after bias removal, 5 July 1990 (time: 17:00:00 - 20:30:00). *Horizontal polarisation*

Figure D.2d: 30 GHz copolar signal level (above) together with 30 GHz crosspolar discrimination after bias removal, 5 July 1990 (time: 17:00:00 - 20:30:00). *Vertical polarisation*
Figure D.3a: 12.5 GHz copolar signal level (above) together with 12.5 GHz crosspolar discrimination after bias removal, 8 June 1992 (time: 14:00:00 - 17:00:00). Vertical polarisation

Figure D.3b: 20 GHz copolar signal level (above) together with 20 GHz crosspolar discrimination after bias removal, 8 June 1992 (time: 14:00:00 - 17:00:00). Vertical polarisation
Figure D.3c 20 GHz copolar signal level (above) together with 20 GHz crosspolar discrimination after bias removal, 8 June 1992 (time: 14:00:00 - 17:00:00). *Horizontal polarisation*

Figure D.3d: 30 GHz copolar signal level (above) together with 30 GHz crosspolar discrimination after bias removal, 8 June 1992 (time: 14:00:00 - 17:00:00). *Vertical polarisation*
### Appendix E

Analysed events for testing XPD-CPA relation

<table>
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<tr>
<th>Parameters</th>
<th>V (dB)</th>
<th>U (dB)</th>
<th>S (dB)</th>
<th>C (dB)</th>
<th>1 time</th>
<th>2 time</th>
<th>3 time</th>
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<td>20 (H)</td>
<td>30</td>
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1. pure rain
2. canting angle
3. pure ice
4. coincident rain and ice