1401 bse

1.
STUDIEBIBLIOTHEEK
ELEKTROTECHNIEK
E - HOOGBOUW

EINDHOVEN UNIVERSITY OF TECHNOLOGY
EINDHOVEN THE NETHERLANDS
DEPARTMENT OF ELECTRICAL ENGINEERING

DIFFERENCE-MODES IN THE SCALAR FEED
BY
C.W.LAMBRECHTSE.

ANTENNA LABORATORY OF THE GROUP
THEORETICAL ELECTRICAL ENGINEERING

AUGUST 1970

ETA-15-1970

Contents.

- 0. Summary
- 1. Introduction
- 2. The radiation pattern of the $TM_{0\nu}$ -mode
 - 2.1 Introductory remarks
 - 2.2 $TM_{0\nu}$ -mode in corrugated conical waveguide
 - 2.3 Computation of the radiation pattern of the $TM_{0\nu}$ -mode
 - 2.4 Experimental radiation patterns of the $TM_{0\nu}$ -mode
- 3. The radiation pattern of the $HE_{2\nu}^{(1)}$ -mode
 - 3.1 The $HE_{2\nu}^{(1)}$ -mode in corrugated conical waveguide
 - 3.2 Calculation of the radiation pattern of the $HE_{2\nu}^{(1)}$ -mode
 - 3.3 Experimental radiation patterns of the $HE_{2\nu}^{(1)}$ -mode

Acknowledgements.

References

Appendix

Summary

Radiation of difference-modes in corrugated conical horn antennas is investigated. A theoretical explanation of the frequency independence property of the scalar feed is given.

Radiation patterns of the TM_{0v} -mode are computed. The agreement between theory and experiment is found to be very close. Furthermore is proven that the radiated power patterns of the second order hybrid-mode and of the TM_{0v} -mode are identical for small values of the flare angle of the cone.

1. Introduction

The scalar feed is a conical horn antenna with grooves perpendicular to the wall of the horn (Fig.1.1).

Fig.1.1 The scalar feed.

The propagation and radiation characteristics of corrugated cylinders and conical horns have been studied both theoretically and experimentally [1-16]. In the previously reported work, main attention is given to the dominant hybrid mode. The application of the scalar feed in autotrack systems made it necessary to investigate the difference modes as well.

Fig.2.1.1 Conical horn antenna with coordinate system.

Autotracking is frequently accomplished by using a TM_{01} -mode in a dual mode horn [17]. The performance of a system using the TM_{01} -mode is excellent when the polarization of the received signal is circular. However, with elliptically polarized signals, cross-coupling occurs between the two axes of the tracking antenna, and when the polarization is linear the system cannot detect essentially the angle error orthogonal to the plane of polarization.

Nakahashi e.a. [18] contrived a method to overcome the defect of the above mentioned system. They proposed the use of the circular TE_{21} -mode.

The radiation pattern of the TE_{21} -mode from circular horns, both in its radial and circumferential components, is a S-curve, which is the indispensable characteristic for the difference signal pattern of the simultaneous lobing system.

The present analysis shows that the spherical TM_{0v} -mode can exist in corrugated conical horn antennas.

For tracking of sources which radiate linear polarized waves, the spherical hybrid $HE_{2v}^{(1)}$ -mode is proposed.

2. The radiation pattern of the TM_{0v} -mode.

2.1 Introductory remarks.

When the field E', H' on the spherical-capped mouth S_A (Fig.2.1.1) of the scalar feed is known, then the intensity of the radiated field in the point P can be found from the following representation theorem [20]

$$\begin{aligned} \underline{E}(\underline{r}) &= \text{curl}_P \int_{S_A} \left[\underline{n}' \times \underline{E}(\underline{r}') \right] G(\underline{r}, \underline{r}') dS + \frac{1}{j\omega\epsilon_0} \text{curl}_P \text{curl}_P \int_{S_A} \left[\underline{n}' x \underline{H}(\underline{r}') \right] G(\underline{r}, \underline{r}') dS \\ \underline{H}(\underline{r}) &= \text{curl}_P \int_{S_A} \left[\underline{n}' \times \underline{H}(\underline{r}') \right] G(\underline{r}, \underline{r}') dS - \frac{1}{j\omega\mu_0} \text{curl}_P \text{curl}_P \int_{S_A} \left[\underline{n}' x \underline{E}(\underline{r}') \right] G(\underline{r}, \underline{r}') dS \end{aligned}$$

$$\text{with } G(\underline{r}, \underline{r}') = \frac{1}{4\pi} \frac{e^{-jk|\underline{r}-\underline{r}'|}}{|\underline{r}-\underline{r}'|} \quad \text{and } k = \omega(\epsilon_0 \mu_0)^{\frac{1}{2}} \quad (2.1.1)$$

In these expressions we have assumed that the outside of the horn antenna is perfectly conducting and no currents flow on the outside of the antenna.

Furthermore we assume that the field at the mouth of the horn is that which would exist at that cross section if the horn extended to infinity. Then solutions of the wave equation in conical waveguide can be substituted in (2.1.1).

2.2 TM_{0v}-mode in corrugated conical waveguide.

The components of the TM_{0v}-mode can be found by substituting the potential [21]

$$A_r(r', \theta', \phi') = A P_v(\cos \theta') H_v^{(2)}(kr') \quad (2.2.1)$$

in the following expressions

$$\begin{aligned} E_r &= \frac{1}{j\omega\epsilon_0} \left(\frac{\partial^2}{\partial r^2} + k^2 \right) A_r & H_r &= 0 \\ E_\theta &= \frac{1}{j\omega\epsilon_0} \frac{1}{r} \frac{\partial^2 A_r}{\partial r \partial \theta} & H_\theta &= \frac{1}{r \sin \theta} \frac{\partial A_r}{\partial \phi} \\ E_\phi &= \frac{1}{j\omega\epsilon_0} \frac{1}{r \sin \theta} \frac{\partial^2 A_r}{\partial r \partial \phi} & H_\phi &= -\frac{1}{r} \frac{\partial A_r}{\partial \theta} \end{aligned} \quad (2.2.2)$$

The results are

$$\begin{aligned} E_{r'} &= \frac{1}{j\omega\epsilon_0} \frac{v(v+1)}{r'^2} P_v(\cos \theta') H_v^{(2)}(kr') \\ E_{\theta'} &= \frac{1}{j\omega\epsilon_0} \frac{1}{r'} \frac{d P_v(\cos \theta')}{d \theta'} \frac{d H_v^{(2)}(kr')}{d r'} \\ E_{\phi'} &= 0 \\ H_{r'} &= 0 \\ H_{\theta'} &= 0 \\ H_{\phi'} &= -\frac{1}{r'} \frac{d P_v(\cos \theta')}{d \theta'} H_v^{(2)}(kr') \end{aligned} \quad (2.2.3)$$

It should be noted that primed coordinates are used for the description of the electromagnetic field in the horn. For the electromagnetic field in the grooves we use unprimed coordinates. Finally, the coordinate of a point outside the horn antenna will be unprimed again.

The characteristic equation for ν is derived from the condition that the longitudinal reactances of the waves in the cone and of the waves in the slots are matched on the surface $\theta' = \theta_0$.

From the expressions (2.2.3) the longitudinal reactance of the waves inside the cone is found,

$$X_{r,c} = jE_r / H_\phi = \frac{\nu(\nu+1)}{\omega \epsilon_0 r' \sin \theta' P'_\nu(\cos \theta')} \frac{P_\nu(\cos \theta')}{P'_\nu(\cos \theta')} \quad (2.2.4)$$

$$, P'_\nu(\cos \theta') \text{ denotes } \frac{d P_\nu(\cos \theta')}{d(\cos \theta')}$$

The derivation of the expression for the longitudinal reactance of the waves inside the slots is simplified when the slots perpendicular on the wall of the cone are approximated by spherical slots (Fig.2.2.1).

Fig.2.2.1 Spherical slot and spherical coordinate system.

In [19] is proven that in the spherical slot only TM-modes can exist, provided the width of the slot is smaller than half a wavelength.

We now assume that in the slots the TM_{0n} -mode with the real value of the degree n is excited only.

The TM_{0n} -mode in the groove can be derived from the potential [21]

$$A_r = kr(a_n j_n(kr) + b_n y_n(kr))(c_n P_n(\cos\theta) + d_n Q_n(\cos\theta)) \quad (2.2.5)$$

by means of the expressions (2.2.2).

The characteristic equation for n is derived from the condition $E_\theta = 0$ on the boundaries I and II (Fig.2.2.1),

$$\begin{vmatrix} j_n(kr_1) + kr_1 j'_n(kr_1) & y_n(kr_1) + kr_1 y'_n(kr_1) \\ j_n(kr_2) + kr_2 j'_n(kr_2) & y_n(kr_2) + kr_2 y'_n(kr_2) \end{vmatrix} = 0 \quad (2.2.6)$$

Equation (2.2.6) has been solved in [19] where also was found, that $n \approx kr$. Making use of the boundary condition $E_r = 0$ for $\theta = \theta_2$ the following expression for the longitudinal reactance of the waves inside the grooves is derived

$$X_{r,s} = jE_r/H_\phi = \frac{n(n+1)}{\omega \epsilon_0 r \sin\theta} \frac{Q_n(\cos\theta_2)P_n(\cos\theta) - P_n(\cos\theta_2)Q_n(\cos\theta)}{Q_n(\cos\theta_2)P'_n(\cos\theta) - P_n(\cos\theta_2)Q'_n(\cos\theta)} \quad (2.2.7)$$

$\left[X_{r,s} \right]_{\theta=\theta_0}$ has been computed for two slots at a distance r_1 to the apex of the cone of 3.74 cm and 13.64 cm respectively

The values of the other parameters are: $\theta_0 = 60$ deg., $b = 2.6$ mm and $r_1 \theta_1 \approx d = 5.35$ mm. The results are plotted in Fig.2.2.2.

The computations are restricted to values of the propagation coefficient for which the longitudinal surface reactance is capacitive and no surface waves are supported

The characteristic equation for ν ,

$$\left[X_{r,c} - X_{r,s} \right]_{\theta=\theta_0} = 0 \quad (2.2.8)$$

has been solved for the two cases given above. The results are plotted in Fig.2.2.3.

For the special case $X_r = \infty$, which occurs if $d \approx \frac{1}{2}\lambda$, has been computed as a function of the flare angle θ_0 . The results are given in Fig.(2.2.4).

The conclusion is that a large change in the value of the longitudinal surface reactance, caused by a large change in frequency, gives rise to a small change of the degree ν , which implies a small change of the power distribution over the aperture and explains the frequency-independence property of the corrugated conical horn antenna.

Fig.2.2.2. Longitudinal surface reactance against propagation coefficient

— $r_1 = 13.64$ cm, --- $r_1 = 4.0$ cm
 $\theta_0 = 60$ deg., $r_1 \theta_1 \approx d = 5.35$ mm, $b = 2.6$ mm.

Fig.2.2.3. Solutions of the characteristic equation against the propagation coefficient

— $X_r = \infty$; — $r_1 = 13.64$ cm $X_r = X_r(k)$,
 --- $r_1 = 4.0$ cm $X = X(k)$.

Fig.2.2.4. Solutions of the characteristic equation, for $X_r = \infty$

— TM_{0v}-mode
 - - - HE_{2v}⁽¹⁾-mode

2.3 Computation of the radiation pattern of the $TM_{0\nu}$ -mode.

The far field approximation of (2.1.1) gives

$$E_{\theta}(r, \theta, \phi) = \frac{e^{-jkr}}{r} \frac{jk}{4\pi} \int_A \left[\left\{ E_{\phi}, \cos \theta' - Z_0 H_{\theta}, \cos \theta \right\} \sin(\phi - \phi') + \right. \\ \left. + \left\{ E_{\theta}, + Z_0 H_{\phi}, \cos \theta' \cos \theta \right\} \cos(\phi - \phi') + Z_0 H_{\phi}, \sin \theta \sin \theta' \right] x \quad (2.3.1) \\ \exp \left[+ jkr' \left\{ \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos(\phi - \phi') \right\} \right] (r')^2 \sin \theta' d\theta' d\phi'$$

$$E_{\phi}(r, \theta, \phi) = \frac{e^{-jkr}}{r} \frac{jk}{4\pi} \int_A \left[-\left\{ E_{\theta}, \cos \theta + Z_0 H_{\phi}, \cos \theta' \right\} \sin(\phi - \phi') + \right. \\ \left. + \left\{ -Z_0 H_{\theta}, + E_{\phi}, \cos \theta' \cos \theta \right\} \cos(\phi - \phi') + E_{\phi}, \sin \theta \sin \theta' \right] x \quad (2.3.2) \\ \exp \left[+ jkr' \left\{ \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos(\phi - \phi') \right\} \right] (r')^2 \sin \theta' d\theta' d\phi'$$

Substituting (2.2.3) in (2.3.1) and using the relation

$$e^{jkr' \sin \theta \sin \theta' \cos(\phi - \phi')} = J_0(kr' \sin \theta \sin \theta') + \sum_{n=1}^{\infty} 2j^n J_n(kr' \sin \theta \sin \theta') \cos n(\phi - \phi') \quad (2.3.3)$$

We obtain

$$E_{\theta} = -Z_0 H_{\nu}^{(2)}(kr') j \frac{ke^{-jkr}}{2r} r' F_{0\nu}(\theta, \theta_0, kr', \nu) \quad (2.3.4)$$

$$E_{\phi=0} \quad (2.3.5) \\ \text{where } F_{0\nu}(\theta, \theta_0, kr', \nu) = \int_0^{\theta_0} \sin \theta' \frac{dP_{\nu}(\cos \theta')}{d\theta'} \left[(1 + \cos \theta \cos \theta') \right]$$

$$x j J_2(kr' \sin \theta \sin \theta') + \sin \theta \sin \theta' J_0(kr' \sin \theta \sin \theta') \exp(jkr' \cos \theta \cos \theta') d\theta' \quad (2.3.6)$$

θ_m is the value of θ for which $|E_\theta|$ is maximal. From equation (2.3.4) follows the identity

$$\left| \frac{E_\theta(\theta, \theta_o, kr', \nu)}{E_\theta(\theta_m, \theta_o, kr', \nu)} \right| = \left| \frac{F(\theta, \theta_o, kr', \nu)}{F(\theta_m, \theta_o, kr', \nu)} \right|$$

Then the relative radiated power is

$$P_{0\nu}(\theta, \theta_o, kr', \nu) = 20^{10} \log \left| \frac{F(\theta, \theta_o, kr', \nu)}{F(\theta_m, \theta_o, kr', \nu)} \right|, \text{ dB} \quad (2.3.7)$$

The function $P_{0\nu}$ has been computed for several values of θ_o and kr' .

For ν the values ~~are~~ taken computed for the case $X_r = \infty$.

From these computations the beamwidth has been derived as a function of kr' for $\theta_o = 15, 30, 45, 60$ and 75 deg.. The results are plotted in Fig.2.3.1 to Fig.2.3.5 incl..

In Fig.2.3.6 the relative power of the aperture field,

$$P'_{0\nu}(\theta', \theta_o, \nu) = 20^{10} \log \left| \frac{E_{\theta'}(\theta', \theta_o, \nu)}{E_{\theta'}(\theta'_m, \theta_o, \nu)} \right| \quad (2.3.8)$$

, has been plotted for the case $\theta_o = 60$ deg., $X_r = \infty$.

A comparison with the radiation patterns of two horns with the same flare angle but with different lengths shows that for "long" horns the aperture field is a good approximation to the radiated field, provided $\theta < \theta_o$.

The radiation pattern for is shown in Fig.2.3.7 and Fig.2.3.8

In Fig.2.3.7 and Fig.2.3.8 the computed radiation patterns of horns with a length r' of 13.64 and 4.0 cm respectively are shown for the frequencies 14.0, 19.1 and 23.9 GHz. At 14.0 GHz the depth of the slots is equal to a quarter of a wavelength. From these patterns can be concluded that the deviations in the beamwidth of the "long" horn due to the change of frequency are very small, but for the "short" horn the frequency-dependence may be of practical interest.

Fig.2.3.1. Beamwidth against kr' for $\theta_0 = 15$ deg.; dots indicate experimental results obtained with antenna 1 at a frequency of 10 GHz; diameter waveguide is 28 mm

Fig.2.3.2. Beamwidth against kr' for $\theta_0 = 30^\circ$; dots indicate experimental results obtained with the antennas 3,4 and 5 at a frequency of 14.5 GHz; diameter waveguide is 18 mm.

Fig.2.3.3. Beamwidth against kr' for $\theta_0 = 45^\circ$; dots indicate experimental results obtained with the antennas 6,7,8,9 and 10 at a frequency of 14.5 GHz; diameter waveguide is 18 mm.

Fig.2.3.4. Beamwidth against kr' for $\theta_0 = 60$ deg.; dots indicate experimental results obtained with the antennas 12 and 13 at a frequency of 14.5 GHz.; diameter waveguide is 18 mm.

Fig.2.3.5. Beamwidth against kr' for $\theta_0 = 75$ deg.; dots indicate experimental results obtained with the antennas 14 and 15 at a frequency of 14.5 GHz.; diameter waveguide is 18 mm.

Fig.2.3.6. Computed power patterns for $\theta_0 = 60$ deg.;

- aperture field
- - - far field , $kr' = 12.5$
- far field , $kr' = 40.0$.

Fig.2.3.7. Computed radiation patterns for $\theta_0 = 60$ deg., $r' = 13.64$ cm.

————	frequency 14.0 GHz , $d/\lambda = 0.25$
-----	" 19.1 GHz , " = 0.34
.....	" 23.9 GHz " = 0.43

Fig.2.3.8. Computed radiation patterns for $\theta_0 = 60$ deg, $r' = 4.0$ cm.

—	frequency 14.0 GHz ;	$d/\lambda = 0.25$
- - -	" 19.1 GHz	" = 0.34
.....	" 23.9 GHz.	" = 0.43

2.4 Experimental radiation patterns of the TM_{0v} -mode.

The experimental verification of the theoretical curves of Fig.2.3.1 to Fig.2.3.5 incl. has been done by measuring the radiation patterns of the antennas listed, with their relevant dimensions, in table I.

All the slots are of the same depth and this was a quarter of a wavelength at the frequency of 14.0 GHz. It should be noted that the antenna with $\theta_0 = 15^\circ$ is an exception to this rule.

Table I

antenna	θ_0	r' [cm]	d [cm]	b [cm]	t [cm]
1	15°	17.60	0.9	0.20	0.20
2	30°	3.50	0.535	0.26	0.04
3	30°	4.20	0.535	0.26	0.04
4	30°	6.60	0.535	0.26	0.04
5	30°	9.00	0.535	0.26	0.04
6	45°	2.78	0.535	0.26	0.04
7	45°	3.71	0.535	0.26	0.04
8	45°	6.17	0.535	0.26	0.04
9	45°	8.66	0.535	0.26	0.04
10	45°	13.57	0.535	0.26	0.04
11	60°	2.80	0.535	0.26	0.04
12	60°	4.00	0.535	0.26	0.04
13	60°	13.64	0.535	0.26	0.04
14	75°	5.08	0.535	0.26	0.04
15	75°	10.23	0.535	0.26	0.04
16	90°	10.64	0.535	0.26	0.04

The radiation patterns of these antennas has been measured at the frequency of 14.5 GHz. The results are presented in the Fig.2.4.1 to Fig.2.4.14 incl. The calculated patterns are also given. For ν the value was taken computed for the case $X_r = \infty$.

The conclusion is that the experimental results are in good agreement with the theoretical predictions, especially for $\theta_o < 75$ deg..

3. The radiation pattern of the $HE_{2\nu}^{(1)}$ -mode.

3.1 The $HE_{2\nu}^{(1)}$ -mode in corrugated conical waveguide.

The $HE_{2\nu}^{(1)}$ -mode can be understood as the sum of a TE-mode and a TM-mode. The components of this hybrid mode can be found by substituting the potentials [21]

$$A_r = A_1 P_\nu^2(\cos\theta') \cos 2\phi' H_\nu^{(2)}(kr') \quad (3.1.1)$$

and

$$F_r = A_2 P_\nu^2(\cos\theta') \sin 2\phi' H_\nu^{(2)}(kr') \quad (3.1.2)$$

in expressions (2.2.2) and in the following expressions

$$\begin{aligned} E_r &= 0 & H_r &= \frac{1}{j\omega\mu_0} \left(\frac{\partial^2}{\partial r^2} + k^2 \right) F_r \\ E_\theta &= \frac{-1}{r \sin\theta} \frac{\partial F_r}{\partial \phi} & H_\theta &= \frac{1}{j\omega\mu_0} \frac{1}{r} \frac{\partial^2 F_r}{\partial r \partial \theta} \\ E_\phi &= \frac{1}{r} \frac{\partial F_r}{\partial \theta} & H_\phi &= \frac{1}{j\omega\mu_0} \frac{1}{r \sin\theta} \frac{\partial^2 F_r}{\partial r \partial \phi} \end{aligned} \quad (3.1.3)$$

The results are

$$E_{r'} = A_1 \frac{1}{j\omega\epsilon_0} \frac{\nu(\nu+1)}{r'^2} P_\nu^2(\cos\theta') \cos 2\phi' \hat{H}_\nu^{(2)}(kr') \quad (3.1.4)$$

$$E_{\theta'} = \frac{A_1 Z_0 \hat{H}_\nu^{(2)}(kr')}{r'} \left[\frac{dP_\nu^2(\cos\theta')}{d\theta'} \frac{1}{jk \hat{H}_\nu^{(2)}(kr')} \frac{d\hat{H}_\nu^{(2)}(kr')}{dr'} - \frac{2A_2}{A_1 Z_0} \frac{1}{\sin\theta'} P_\nu^2(\cos\theta') \cos 2\phi' \right] \quad (3.1.5)$$

$$E_{\phi'} = \frac{A_1 Z_0 \hat{H}_\nu^{(2)}(kr')}{r'} \left[-\frac{2}{\sin\theta'} P_\nu^2(\cos\theta') \frac{1}{jk \hat{H}_\nu^{(2)}(kr')} \frac{d\hat{H}_\nu^{(2)}(kr')}{dr'} + \frac{A_2}{A_1 Z_0} \frac{dP_\nu^2(\cos\theta')}{d\theta'} \right] \sin 2\phi' \quad (3.1.6)$$

$$Z_o H_{r'} = \frac{1}{j\omega\mu_o} Z_o A_2 \frac{\nu(\nu+1)}{r'^2} P_\nu^2(\cos\theta') \sin\phi' \hat{H}_\nu^{(2)}(kr') \quad (3.1.7)$$

$$Z_o H_{\theta'} = \frac{A_1 Z_o \hat{H}_\nu^{(2)}(kr')}{r'} \left[\frac{A_2}{A_1 Z_o} \frac{dP_\nu^2(\cos\theta')}{d\theta'} \frac{1}{j k \hat{H}_\nu^{(2)}(kr')} \frac{d\hat{H}_\nu^{(2)}(kr')}{dr'} \right. \quad (3.1.8)$$

$$\left. - \frac{2}{\sin\theta'} P_\nu^2(\cos\theta') \sin\phi' \right] \sin\phi'$$

$$Z_o H_{\phi'} = \frac{A_1 Z_o \hat{H}_\nu^{(2)}(kr')}{r'} \left[- \frac{dP_\nu^2(\cos\theta')}{d\theta'} + \frac{2A_2}{A_1 Z_o} \frac{1}{\sin\theta'} P_\nu^2(\cos\theta') \times \right. \quad (3.1.9)$$

$$\left. \frac{1}{j k \hat{H}_\nu^{(2)}(kr')} \frac{d\hat{H}_\nu^{(2)}(kr')}{dr'} \right] \cos\phi'$$

In the expressions 3.1.4 to 3.1.9 incl. the unknown quantities are $\frac{A_1}{A_2}$ and ν , and can be found after applying the boundary conditions $E_{\phi'}=0$ and $Z_o H_{\phi'}=0$ for $\theta'=\theta_o$ [19]. From these conditions we find that

$$\frac{A_2}{A_1 Z_o} = \pm 1 \quad (3.1.10)$$

So two modes with the same ϕ' -dependence, but different θ' -dependence can exist in the corrugated conical horn. The mode for which $A_2 = Z_o A_1$ is called the $HE_{2\nu}^{(1)}$ -mode, while the other is the $HE_{2\nu}^{(2)}$ -mode.

Substitution of (3.1.10) in the equation $Z_o H_{\phi'}=0$ yields the characteristic equations

$$\left[\frac{dP_\nu^2(\cos\theta')}{d\theta'} \pm \frac{2}{\sin\theta'} P_\nu^2(\cos\theta') \xi_\nu(kr') \right]_{\theta'=\theta_o} = 0 \quad (3.1.11)$$

The function $\xi_\nu(kr') = \frac{1}{jk} \frac{d\hat{H}_\nu^{(2)}(kr')}{dr'} \frac{1}{\hat{H}_\nu^{(2)}(kr')}$ and is a complex function

Using the asymptotic expansion of $\hat{H}_\nu^{(2)}(kr')$ we see that $\lim_{kr' \rightarrow \infty} \xi_\nu(kr') = -1$

For this case we have calculated the mode patterns which are presented in the Fig.3.1.1 and Fig.3.1.2. The calculations are given in the appendix. For the case that the hybrid mode is excited by the circular TE_{21} -mode the excitation of the $HE_{2\nu}^{(1)}$ -mode is to expect as can be concluded from

Fig.3.1.1. $HE_{2v}^{(1)}$ -mode pattern for $\theta_0 = 60^\circ$.

Fig.3.1.2. $HE_{2v}^{(2)}$ -mode pattern for $\theta_0 = 60 \text{ deg}$.

a comparison of the hybrid mode patterns with the TE_{21} -mode pattern. The characteristic equation for the $HE_{2\nu}^{(1)}$ -mode has been solved for the lowest value ν . The results are plotted in Fig.2.2.4.

Substitution of $A_1 Z_0 = A_2$ in (3.1.5) to (3.1.9) gives

$$E_{\theta'} = - \frac{A_1 Z_0 \hat{H}_{\nu}^{(2)}(kr')}{r'} f_{2\nu}^{(1)}(\theta') \cos 2\phi' \quad (3.1.12)$$

$$E_{\phi'} = \frac{A_1 Z_0 \hat{H}_{\nu}^{(2)}(kr')}{r'} f_{2\nu}^{(1)}(\theta') \sin 2\phi' \quad (3.1.13)$$

$$Z_0 H_{\theta'} = - \frac{A_1 Z_0 \hat{H}_{\nu}^{(2)}(kr')}{r'} f_{2\nu}^{(1)}(\theta') \sin 2\phi' \quad (3.1.14)$$

$$Z_0 H_{\phi'} = - \frac{A_1 Z_0 \hat{H}_{\nu}^{(2)}(kr')}{r'} f_{2\nu}^{(1)}(\theta') \cos 2\phi' \quad (3.1.15)$$

$$\text{with } f_{2\nu}^{(1)}(\theta') = \frac{dP_{\nu}^{(1)}(\cos\theta')}{d\theta'} + \frac{2}{\sin\theta'} P_{\nu}^{(1)}(\cos\theta') \quad (3.1.16)$$

3.2 Calculation of the radiation pattern of the $HE_{2\nu}^{(1)}$ -mode.

Substituting (3.1.12) to (3.1.15) incl. in (2.3.1) and (2.3.2) and using the relation (2.3.3) we obtain

$$E_{\theta} = + \frac{k}{4} \frac{e^{-jkr}}{r} A_1 Z_0 \frac{\hat{H}_{\nu}^{(2)}(kr')}{r'} (r')^2 \cos 2\phi F_{2\nu}^{(\theta, \theta_0, kr')} \quad (3.2.1)$$

and

$$E_{\phi} = - \frac{k}{4} \frac{e^{-jkr}}{r} A_1 Z_0 \frac{\hat{H}_{\nu}^{(2)}(kr')}{r'} (r')^2 \sin 2\phi F_{2\nu}^{(\theta, \theta_0, kr')} \quad (3.2.2)$$

with

$$\begin{aligned} F_{2\nu}^{(\theta, \theta_0, kr')} = & \int_0^{\theta_0} \left\{ \cos \theta + \cos \theta' \right\} \left\{ J_1(kr' \sin \theta \sin \theta') + J_3(kr' \sin \theta \sin \theta') \right\} + \\ & + \left\{ 1 + \cos \theta \cos \theta' \right\} \left\{ J_1(kr' \sin \theta \sin \theta') - J_3(kr' \sin \theta \sin \theta') \right\} + \\ & + 2j \sin \theta \sin \theta' J_2(kr' \sin \theta \sin \theta') \right] f_{2\nu}^{(1)}(\theta') \exp[jkr' \cos \theta \cos \theta'] \sin \theta' d\theta' \end{aligned} \quad (3.2.3)$$

From the equation (3.2.1) and (3.2.2) we derive that $|E_{\theta}|^2 + |E_{\phi}|^2$ is independent of ϕ , and that

$$\left| \frac{E_{\theta}(\theta, \theta_0, kr')}{E_{\theta}(\theta_m, \theta_0, kr')} \right| = \left| \frac{E_{\phi}(\theta, \theta_0, kr')}{E_{\phi}(\theta_m, \theta_0, kr')} \right| = \left| \frac{F_{2\nu}(\theta, \theta_0, kr')}{F_{2\nu}(\theta_m, \theta_0, kr')} \right| \quad (3.2.4)$$

where θ_m is the value of θ for which $|F_{2\nu}|$ is maximal.

The function $P_{2\nu}^{(1)} = 20^{10} \log \left| \frac{F_{2\nu}(\theta, \theta_0, kr')}{F_{2\nu}(\theta_m, \theta_0, kr')} \right|$ has been calculated

for $\theta_0 = 40$ deg., $kr' = 20$. The numerical results are given in table II.

Also in table II the computed pattern of the $TM_{0\nu}$ -mode for the same values of the parameters θ_0 and kr' is given.

Table II

θ THE- Γ A	$P_{2v}^{(1)}$, dB	P_{0v} , dB
+2	-12.701	-12.696
+4	-6.956	-6.951
+6	-3.891	-3.887
+8	-2.024	-2.021
+10	-.885	-.883
+12	-.253	-.252
+14	-.034	-.035
+16	-.280	-.281
+20	-.673	-.672
+22	-1.158	-1.155
+24	-1.696	-1.691
+26	-2.271	-2.264
+28	-2.886	-2.877
+30	-3.560	-3.550
+32	-4.316	-4.306
+34	-5.174	-5.164
+36	-6.144	-6.136
+38	-7.226	-7.220
+40	-8.405	-8.399
+42	-9.653	-9.647
+44	-10.931	-10.923
+46	-12.192	-12.180
+48	-13.373	-13.375
+50	-14.518	-14.484
+52	-15.533	-15.504
+54	-16.487	-16.455
+56	-17.402	-17.370
+58	-18.310	-18.280
+60	-19.237	-19.212

A comparison of the two patterns shows a very close agreement. This can be explained by the agreement existing between the relative power distributions over the aperture of both modes, provided the flare angle is "small".

This can be shown easily by developing $f_2^{(1)} v$ in terms of $P_v^{(1)}(\cos \theta)$ and making use of the recurrence relations [21]

$$\sin \theta \frac{d P_v^{\mu}(\cos \theta)}{d \theta} = v \cos \theta P_v^{\mu}(\cos \theta) - (v+\mu) P_{v-1}^{\mu}(\cos \theta) \quad (3.2.5)$$

and

$$P_v^{\mu+1}(\cos \theta) = (\cos \theta - 1)^{-\frac{1}{2}} \{ (v-\mu) \cos \theta P_v^{\mu}(\cos \theta) - (v+\mu) P_{v-1}^{\mu}(\cos \theta) \} \quad (3.2.6)$$

Then follows $f_{2v}^{(1)} = \frac{d P_v^2(\cos\theta')}{d\theta} + \frac{2}{\sin\theta'} P_v^2(\cos\theta') =$

$$\begin{aligned} \frac{P_v(\cos\theta')}{\sin\theta'} \{ & \frac{v(2+v\cos\theta')(v-1)\cos^2\theta}{\cos^2\theta-1} - \frac{v(2+v\cos\theta')(v+1)(v-1)}{(\cos^2\theta-1)(v-1)} \\ & + \frac{v(v+2)(v-2)(v-1)\cos\theta' + v^2(v+2)(v-2)\cos\theta' - v^2(v+2)(2v-3)\cos\theta'}{(\cos^2\theta-1)(v-1)} \} \\ + \frac{P_{v-1}(\cos\theta')}{\sin\theta'} \{ & \frac{-v(v+2)(2v-3)(2v-1)\cos^2\theta' + (2+v\cos\theta')(v+1)(v-1)(2v-1)\cos\theta'}{(\cos^2\theta-1)(v-1)} \\ & + \frac{(v+2)(v-2)(v-1)(2v-1)\cos^2\theta' + v(v+2)(v-2)(2v-1)\cos^2\theta'}{(\cos^2\theta-1)(v-1)} \\ & + \frac{v(v+2)(v-1) - v(2+v\cos\theta')(v-1)\cos\theta - (2+v\cos\theta')(v+1)(v-1)\cos\theta'}{\cos^2\theta'-1} \\ & - \frac{(v+2)(v-2)(v-1)\cos^2\theta'}{\cos^2\theta'-1} \} \end{aligned} \quad (3.2.7)$$

If $v \gg 1$ and $\cos\theta' \approx 1$ then (3.2.7) can be further simplified to

$$f_{2v}^{(1)} \approx \frac{v^3}{\sin\theta'} \{ P_v(\cos\theta') - P_{v-1}(\cos\theta') \} = v^2 \frac{d P_v(\cos\theta')}{d\theta'} \quad (3.2.8)$$

The difference between the values of the degree of the TM_{0v} -mode and of the $HE_{2v}^{(1)}$ -mode has been found to be very small especially for small values of the flare angle (See Fig.2.2.4)

The relative power distribution of the $HE_{2v}^{(1)}$ -mode over the aperture

$$P_{2v}^{(1)} = \frac{\left| \frac{f_{2v}^{(1)}(\theta', \theta_o, kr')}{f_{2v}^{(1)}(\theta'_m, \theta_o, kr')} \right|^2}{\left| \frac{\frac{d P_v(\cos\theta')}{d\theta'}}{\frac{d P_v(\cos\theta')}{d\theta'} \bigg|_{\theta'=\theta'_m}} \right|^2} \quad (3.2.9)$$

and is equivalent to the relative power distribution of the TM_{0v} -mode, calculated in section 2.3.

The conclusion is that the computed beamwidth curves for the TM_{0v} -mode are a good approximation for the beamwidth curves of the $HE_{2v}^{(1)}$ -mode especially for "small" values of the flare angle of the horn.

3.3 Experimental radiation patterns of the $HE_{2v}^{(1)}$ -mode.

In Fig.(3.3.1) the measured radiation pattern of the $HE_{2v}^{(1)}$ -mode of antenna 10 is given. The cut-off frequency for the TE_{21} -mode in the circular waveguide is 16.203 GHz. The measurement is done at a frequency of 16.5 GHz.

A comparison with the TM_{0v} -mode pattern given in Fig.(2.4.9) shows that both patterns are identical as was predicted by theory.

Fig 3 3 1 HF (1) -mode radiation pattern. $\theta = 45$ deg. frequency 16.5 GHz.

Acknowledgements

The author is particular indebted to Drs.M.E.J.Jeuken, who encouraged and provided many of the ideas for the development of the hybrid-mode technology presented in this report.

The solutions of the frequency dependent characteristic equation (2.2.8) are given by J.F.Mayer.

I.Ongers wrote the computer program for the drawings of the mode-patterns.

The author wish to thank Prof.Dr.Ir.A.A.Th.M.van Trier for giving him the opportunity to carry out the research described in this report.

References

- 1 Kay, A.F.: "The scalar feed", TRG report, contract AF 19(604)-8057, March 1964.
- 2 Minnett, H.C. and Thomas, B.MACA.: "A method of synthesizing radiation patterns with axial symmetry", IEEE Trans., 1966, AP-14.
- 3 Jeuken, M.E.J. and Kikkert, J.S., "A broadband aperture antenna with a narrow beam", Alta Frequenza, 38, 1969, Numero Speciale pp. 270-276.
- 4 Clarricoats, P.J.B.: "Analysis of spherical hybrid modes in a corrugated conical horn", Electron. Lett., 1969, 5, pp. 189-190.
- 5 Clarricoats, P.J.B. and Saha, P.K.: "Theoretical analysis of cylindrical hybrid modes in a corrugated horn", ibid., 1969, 5, pp 187-189.
- 6 Clarricoats, P.J.B. and Saha, P.K.: "Radiation from wide-flare-angle scalar horns", ibid., 1969, 5, pp 376-378.
- 7 Jeuken, M.E.J.: "Experimental radiation pattern of the corrugated conical horn antenna with small flare angle", ibid., 1969, 5, pp 484-485.
- 8 Jeuken, M.E.J. and Lambrechtse, C.W.: "Small corrugated conical-horn antenna with wide flare angle", ibid., 1969, 5, pp 489-490.
- 9 Clarricoats, P.J.B.: "Similarities in the electromagnetic behaviour of optical waveguides and corrugated feeds", ibid., 1970, 6, pp 178-180.
- 10 Booker, D.D., and McInnes, P.A.: "Computer-predicted performance of corrugated conical feeds using experimental primary-radiation patterns", ibid., 1970, 6, pp 18-20.

- 11 Thomas, B.MACA.: "Bandwidth properties of corrugated conical horns",
ibid., 1969, 5, pp 561-563.
- 12 Narasimhan, M.S., and Rao, B.V.: "Hybrid modes in corrugated conical
horns", ibid., 1970, 6, pp 32-34.
- 13 Baldwin, R., and McInnes, P.A.: "Surface-Wave radiation from a corrugated
horn", ibid., 1970, 6, pp 259-260.
- 14 Clarricoats, P.J.B., and Saha, P.K.: "Attenuation in corrugated circular
waveguide", ibid., 1970, 6, pp 370-372.
- 15 Thomas, B.MACA.: "Prime-focus one- and two-hybrid mode feeds", ibid.,
1970, 6, pp 460-461.
- 16 Narasimhan, M.S., and Rao, B.V.: "Diffraction by wide-flare-angle
corrugated conical horns", ibid., 1970, 6, pp 469-471.
- 17 Cook, J.S., and Lowell, R.: "The autotrack system", BSTJ, 42, No. 4, pt. 2,
pp 1283-1307, July 1963.
- 18 Nakahashi, N., Kakinuma, Y., and Shirai, T.: "A multimode Autotrack
system employing the circular TE_{11} and TE_{21} modes", Journal of the radio
research laboratories, 14, No. 73, pp 129-152, May 1967.
- 19 Jansen, J.K.M., Jeuken, M.E.J., and Lambrechtse, C.W.: "The scalar feed",
T.H. Report 70-E-12, November 1969.
- 20 Flügge, S.: "Handbuch der Physik", Bd 25, S 238-240 Springer-Verlag,
Berlin, Göttingen, Heidelberg, 1961.
- 21 Harrington, R.F.: "Time-harmonic electromagnetic fields", Chapter 6,
Mc Graw-Hill 1961.
- 22 Smythe, W.R.: "Static and dynamic electricity", Chapter 1,
Mc Graw-Hill 1950.

APPENDIX

A.1 Calculation of the $HE_{2v}^{(1)}$ - and $HE_{2v}^{(2)}$ -conical waveguide mode-patterns.

An electric **field** is visualized by drawing the "lines of force" [22]
A line of force is a directed curve in an electric field such that the forward drawn tangent at any point has the direction of the electric intensity there.

It follows that if $\underline{ds}$ is an element of this curve,

$$\underline{ds} = \lambda \underline{E} \quad (A.1)$$

where λ is a scalar factor. Writing out the components in polar coordinates, we have the differential equation of the lines of force

$$\frac{d\rho}{\rho d\phi} = \frac{E_\rho}{E_\phi} \quad (A.2)$$

The lines of force on the spherical aperture are projected on a plane perpendicular on the axis of the cone (Fig.A.1)

Fig.A.1. Coordinate system used for the calculation of the lines of force.

The normalized radius of the aperture is 1 . Then

$$E_\rho = E_\theta \cos\theta = E_\theta \sqrt{1 - (\rho \sin\theta_0)^2} \quad (A.3)$$

Substitution of the expressions for the aperture fields (3.1) for the case $\xi_v(kr') = -1$ in (A.2) gives

$$\frac{d\rho}{\rho \sqrt{1 - (\rho \sin\theta_0)^2}} = \pm \frac{\cos 2\phi}{\sin 2\phi} d\phi \quad (A.4)$$

Integration of equation (A.4) gives
for the $HE_{2v}^{(1)}$ -mode

$$t = \frac{\sqrt{|\sin 2\phi|} - C}{\sqrt{|\sin 2\phi|} + C}, \quad \rho = \frac{1}{\sin \theta_0} \frac{1-t^2}{1+t^2} \quad (A.5)$$

and for the $HE_{2v}^{(2)}$ -mode

$$t = \frac{1-C\sqrt{|\sin 2\phi|}}{1+C\sqrt{|\sin 2\phi|}}, \quad \rho = \frac{1}{\sin \theta_0} \frac{1-t^2}{1+t^2} \quad (A.6)$$

This are the equations giving the lines of force shown in the
Figs. 3.1.1 and 3.1.2.

✓

```

1 BEGIN COMMENT PROGRAMMA 06064494 VAN I. C. ONGERS , BEREKENEN EN PLOTTEN VAN VELDLIJNEN VAN EEN HOORNANTENNE T.B.V. C. (A MORECHTSE)
2 REAL PI, THETA0, C, PHI, T, R;
3 INTEGER TELLER, J, I;
4 REAL ARRAY XCIRK, YCIRK, XX, YY, Y[-400 : 400];
5 LIBRARY PLOTTEN, PLOTPICTURE, PLOTAXIS, PLOTAXIS 2, MARKED CURVE, SCALE;
6 BOOLEAN GRAFIEK, NUMERIEK;

7 PI := 3.14159265355; THETA0 := READ; TELLER := 0; THETA0 := THETA0 * PI / 180; GRAFIEK := 0 = READ; NUMERIEK := 0 = READ;
8 FOR C := READ WHILE C # -10 DO
9   BEGIN FOR I := 0 STEP 1 UNTIL 360 DO
10     BEGIN PHI := I * PI / 180; T := (1 - SQRT(ABS(SIN(2 * PHI))) * C) / (1 + C * SQRT(ABS(SIN(2 * PHI)))));
11       R := (1 - T * T) / (1 + T * T) / SIN(THETA0);
12       IF R > 1 THEN R := 1; XCIRK[I] := COS(PHI); YCIRK[I] := SIN(PHI);
13       X[I] := R * COS(PHI); Y[I] := R * SIN(PHI);
14     END;
15   IF NUMERIEK THEN BEGIN NLCR; PRINTTEXT($ VELDLIJNEN BEREKENING $); NLCR; PRINTTEXT($ C = $); FIXT(3,3,C); NLCR; END;
16   TELLER := TELLER + 1;
17   IF GRAFIEK THEN
18     BEGIN
19       IF TELLER = 1 THEN
20         BEGIN PLOTPICTURE(XCIRK[I], YCIRK[I], 1, 360, 10, 0, 0033, -1, 1, 100, 2000, $ $,
21           -1, 1, 100, 2000, $ $);
22       END;
23       PLOTPICTURE(X[I], Y[I], 1, 360, 10, 0, 2077, -1, 1, 100, 2000, $ $, -1, 1, 100, 2000, $ $);
24     END;
25   IF NUMERIEK THEN
26     BEGIN
27       PRINTTEXT($ X Y $) NLCR;
28       FOR I := 0 STEP 1 UNTIL 60 DO
29         BEGIN FIXT(1, 4, X[I]); FIXT(1, 4, Y[I]); SPACE(4);
30           FIXT(1, 4, X[60 + I]); FIXT(1, 4, Y[60 + I]); SPACE(4);
31           FIXT(1, 4, X[120 + I]); FIXT(1, 4, Y[120 + I]); SPACE(4);
32           FIXT(1, 4, X[180 + I]); FIXT(1, 4, Y[180 + I]); SPACE(4);
33           FIXT(1, 4, X[240 + I]); FIXT(1, 4, Y[240 + I]); SPACE(4);
34           FIXT(1, 4, X[300 + I]); FIXT(1, 4, Y[300 + I]); SPACE(4); NLCR;
35         END;
36       END;
37     END; ENDPLODOC;
38     TELLER := 0;
39   FOR C := READ WHILE C # -10 DO
40     BEGIN FOR I := 0 STEP 1 UNTIL 360 DO
41       BEGIN PHI := I * PI / 180; XCIRK[I] := COS(PHI); YCIRK[I] := SIN(PHI);
42       END;
43       I := 0;
44     NEXT I;
45     PHI := (I + 90) * PI / 360; T := (SQRT(ABS(SIN(2 * PHI))) - C) / (SQRT(ABS(SIN(2 * PHI))) + C);
46     R := (1 - T * T) / (1 + T * T) / SIN(THETA0);
47     IF R < 1 - I # 90 THEN BEGIN XX[I] := R * COS(PHI); YY[I] := R * SIN(PHI); I := I + 1; GO TO NEXT;
48     END;
49     FOR J := 0 STEP -1 UNTIL -I + 1 DO
50       BEGIN X[J] := YY[-J]; Y[J] := XX[-J]; END;
51     FOR J := 1 STEP 1 UNTIL I DO
52       BEGIN X[J] := X[J - 1]; Y[J] := Y[J - 1]; END;
53     FOR J := I + 1 STEP 1 UNTIL 2 * I - 1 DO
54       BEGIN X[J] := XX[J - 1]; Y[J] := YY[J - 1]; END;
55     IF NUMERIEK THEN BEGIN NLCR; PRINTTEXT($ VELDLIJNEN BEREKENING $); NLCR; PRINTTEXT($ C = $); FIXT(3,3,C); NLCR; END;
56     TELLER := TELLER + 1;
57     IF GRAFIEK THEN
58       BEGIN
59         IF TELLER = 1 THEN

```

190870 - 123 - 6064494 LAMBRECHTSE

2

```
59      BEGIN PLOTPICTURE( XCIRK(J), YCIRK(J), J, 360, 10, 0, 0033, -1, 1, 100, 2000, + +,  
60      -1, 1, 100, 2000, + +);  
61      END;  
62      PLOTPICTURE(X(J), Y(J), J, 2 * I - 1, 10, 0, 2077, -1, 1, 100, 2000, + +, -1, 1, 100, 2000, + +);  
63      PLOTPICTURE(- X(J), Y(J), J, 2 * I - 1, 10, 0, 2077, -1, 1, 100, 2000, + +, -1, 1, 100, 2000, + +);  
64      PLOTPICTURE(- X(J), - Y(J), J, 2 * I - 1, 10, 0, 2077, -1, 1, 100, 2000, + +, -1, 1, 100, 2000, + +);  
65      PLOTPICTURE( X(J), - Y(J), J, 2 * I - 1, 10, 0, 2077, -1, 1, 100, 2000, + +, -1, 1, 100, 2000, + +);  
66      END;  
67      IF NUMERIEK IMEN  
68      BEGIN  
69          PRINTTEXT(+ X Y +); NLCR;  
70          EQB J := 1 STEP 1 UNTIL 2 * I - 1 DO  
71              BEGIN FIST(1, 4, X(J)); FIST(1, 4, Y(J)); NLCR;  
72              END  
73          END;  
74      END; ENVELODOG;  
75      END
```

```

lalgol06064164 lambrechtse,0,1
begin comment program 06064164      c.w.lambrechtse

      radiation pattern of the conical horn T HE(1)-2n-mode;

integer i;

real procedure gamma(x);value x;real x;
begin if x>1 then gamma:=gamma(x-1)*(x-1) else gamma:= 1/x+ (-0.577191652+x*(0.988205891+x*(-0.897056937+x*(
0.918206857
      +x*(-0.756704078+x*(0.482199394+x*(-0.193527818+x*0.035868343))))));
end gamma;

real procedure F(a,b,c,z,i,re);value a,b,c,z,re;real a,b,c,z,re;integer i;
      begin      real FG,t1,t2;FG:=t1:=1;i:=1;
herhaal:t2:=t1*(a+i-1)*(b+i-1)*z/((c+i-1)*1);FG:=FG+t2;t1:=t2;i:=i+1;
      if abs(t2/FG)>re then goto herhaal;
F:=FG      endF;

real procedure P(theta,m,n,re);value theta,re,n,m;real theta,re,m,n;
      begin real pi,f1,f2;pi:=3.14159265359;
      if theta<pi/4 then begin P:=(sin(theta)/km)*F((n+m+1)/2,(m-n)/2,m+1,sin(theta)*sin(theta),1,re)*gamma(
n+m+1)/
      (((-2)/km)*gamma(m+1)*gamma(n-m+1));
      end
      else begin f1:=F((1-n-m)/2,(n-m+2)/2,1.5,cos(theta)*cos(theta),1,re);      f2:=F((-
n-m)/2,(n-m+1)/2,0.5,cos(theta)
      xcos(theta),1,re);
      pi*(n+m-1)/2)*gamma((n+m-1)/2)
      P:=(((2)/km)*sqrt(pi)/(sin(theta)/km))*(-f2*((n+m-1)/2)*sin(
/(pi*gamma((n-m+2)/2))
      +f1*cos(theta)*(n+m)*sin(pi*(n+m)/2)*gamma((n+m)/2)/(pi*gamma((n-m+1)/
2)))
      end
endP;

```

```

real procedure dP(theta,m,n,re);value theta,re,m,n;real theta,re,m,n;
    begin dP:=(n+1)xcos(theta)XP(theta,m,n,re)+(n-m+1)XP(theta,m,n+1,re)
    end dP;

```

```

real procedure J2(x);value x;real x;
    begin if x=0 then J2:=0 else J2:=2XJ1(x)/x -J0(x)
    end J2;

```

```

real procedure J3(x);value x;real x;
    begin if x=0 then J3:=0 else J3:=4XJ2(x)/x -J1(x)
    end J3;

```

```

real procedure J0(x); value x; real x;
begin comment A.J.M. Hitchcock, Polynomial approximations to Bessel functions
    of order zero and one and to related functions (1957), 86-88.;
    real t; x := abs(x);
    if x < 4 then begin t := (x/4) ^ 2;
        comment The maximum absolute error in the polynomial approximation is 10-10;
        J0 := if x < 2-6 then 1
            else 1 + t x (-3.99999 98721 + t x (3.99999 73021 + t x (-1.77775 60599
                + t x (.44435 84263 + t x (-.07092 53492 + t x (.00767 71853
                + t x (-.00050 1415))))))
    end
    else begin real P0, Q0, p14; t := (4/x) ^ 2; p14 := .78539 81633 97448;
        comment The maximum absolute error in the polynomial approximations
        for P0 and Q0 are 12-10 and 25-10, respectively;
        P0 := if x > 0.5-6 then .79788 45608 02865
            else 2 x (.39894 22793 + t x (-.00175 30620 + t x (.00017 34300 + t x
                + t x (.0001 73565 + t x (-.00000 37043)))));
        Q0 := if x > 0.5-6 then -.79788 45608 02865/(8 x x)
            else 8/x x (-.01246 69441 + t x (.00045 64324 + t x (-.00008 69791
                + t x (.00003 42468 + t x (-.00001 42078 + t x (.00000 32312)))));
        J0 := (P0 x cos(x - p14) - Q0 x sin(x - p14))/sqrt(x)
    end
end J0(x);

```

```

real procedure J1(x); value x; real x;
begin comment A.J.M. Hitchcock, Polynomial approximations to Bessel functions
      of order zero and one and to related functions
      mtac(1957), 86-88.;
      integer signx; real t; signx := sign(x); x := abs(x);
      if x < 4 then begin t := (x/4) ^ 2;
        comment The maximum absolute error in the polynomial approximation is 310-10;
        J1 := if x < 210-6 then signx × x/2
              else signx × x × .25 × (1.99999 99998 + t × (-3.99999 99710 + t × (2.66
666 60544
              + t × (-.88888 39649 + t × (.17775 82922 + t × (-.02366 16773 + t × (.00220 69
155
              + t × (-.00012 89769)))))))))
      end
      else begin real P1, Q1, p134; t := (4/x) ^ 2; p134 := 2.35619 44901 92345;
        comment The maximum absolute error in the polynomial approximations
          for P1 and Q1 are 1510-10 and 2510-10, respectively;
        P1 := if x > 0.5106 then .79788 45608 02865
              else 2 × (.39894 22819 + t × (.00292 18256 + t × (-.00022 32030 + t
× (.00005 80759
              + t × (-.00002 00920 + t × (.00000 42414))))));
        Q1 := if x > 0.5106 then .79788 45608 02865 × 3/(8 × x)
              else 8/x × (.03740 08364 + t × (-.00063 90400 + t × (.00010 64741
              + t × (-.00003 98708 + t × (.00001 62200 + t × (-.00000 36504))))));
        J1 := signx × (P1 × cos(x - p134) - Q1 × sin(x - p134))/sqrt(x)
      end
end J1(x);

```



```

begin real a,b,c,nmul,p,u,r,thetmul,thet,theta,n,pi,sk,dk,ek,k,thetanul,thetanulgr; integer dia,sit, it,its;
array d,e[1:4],IEI,IER,IHR,IHI[0:180];
complex hank,q,s,j,t;
complex array IH,IE[0:180]; library INTEGRAL,ARG,PLOTPICTURE,SCALE,MARKED CURVE,REGULA FALSI,PLOTAXIS,PLOTAXIS 2,
PLOTTEN;
begin:a:=0;
j:=com(0,1);pi:=3.14159265359;dia:=READ;thetanulgr:=READ;thetanul:=thetanulgr*pi/180;n:=READ;r:=READ;sk:=READ;dk:=R
EAD;ek:=READ;
e[1]:=READ;e[2]:=READ;e[3]:=READ;e[4]:=READ;thetmul:=READ;sit:=READ;
for k:=sk step dk until ek do begin a:=0;
for it:=sit step sit until thetnul do begin thet:=it*pi/180; IER[it]:=INTEGRAL(theta,re
((dp(theta,2,n,e[2]*xn-2)+2*x(theta,2,n,e[2]*xn-2))x(1x(1+cos(theta)xcos(thet))x(J1((p:=k*xr*xsin(theta)xsin(thet))
)-J3(p))
+(cos(thet)+cos(theta))x(J1(p)+J3(p))+2*xj*xsin(theta)xsin(thet)xJ2(p))x(cos((u:=k*xr*cos(theta)x
cos(thet)))+j*xsin(u)))

,a,thetanul,d,e,false,false);IEI[it]:=INTEGRAL(theta,im
((dp(theta,2,n,e[2]*xn-2)+2*x(theta,2,n,e[2]*xn-2))x(1x(1+cos(theta)xcos(thet))x(J1((p:=k*xr*xsin(theta)xsin(thet)))-
J3(p))
+(cos(thet)+cos(theta))x(J1(p)+J3(p))+2*xj*xsin(theta)xsin(thet)xJ2(p))x(cos((u:=k*xr*cos(theta)x
cos(thet)))+j*xsin(u)))

,a,thetanul,d,e,false,false);IE[it/sit]:=com(IEI[it],IER[it]);
end;CARRIAGE(4);PRINTTEXT('corrugated horn HE(1)-2n-mode: THETANUL,n,r en k zijn respectievelijk:');
FIXT(2,2,thetanulgr);FIXT(2,8,n);FIXT(2,3,r);FIXT(3,2,k);CARRIAGE(2);
for it:=2 step 1 until thetnul/DO BEGIN if mod(IE[it])<mod(IE[it-1]) then go to AAend;AA:its:=it-1;PRINTTEXT(
'theta mod(I/IO) ARG(I/IO) 20ln(I/IO)');b:=-10;
for it:=1 step 1 until thetnul/sit do begin NLGR;FIXT(3,0,it*xsit);SPACE(10);FIXT(1,5,(p:=mod(IE[it]/IE[its])));SPA
CE(10);
FIXT(1,3,ARG(IE[it]/IE[its]));SPACE(10);FIXT(2,3,(a:=20xln(p)/ln(10)));SPACE(10);
for c:=0,3,10,15,20,25,30,35,40 do begin if (a<-c/b>c)V(a>-c/b<-c) then FIXT(2,1,sitx(it-1-(b+c)/(a-b)))end;b:=a;
end;if dia=0 then begin
PLOTPICTURE(1xsit,if(p:=20xln(mod(IE[i]/IE[its]))/ln(10))>0 then 0 else (if p<-40 then -40 else p),1,thetmul/sit
,2,-1,0066,0,thetmul,12,
2540xthetmul/180,
'theta (deg.)',-40,0,2,2500,'
relative power in dB');
PLOTTTEXT(12,-38,90,30,0,true,2,'HE(1)-2n-mode ,calculated ');
PLOTTTEXT(18,-38,90,30,0,true,0,'kr=');FIXPLOT(0,0,90,30,0,false,0,3,2,kxr);
PLOTTTEXT(0,0,90,30,0,false,0,'thetanul=');FIXPLOT(0,0,90,30,0,false,0,2,0,thetanulgr);
PLOTTTEXT(0,0,90,30,0,false,0,'deg. ');ENDPLODOC
end end ;p:=read;if p#0 then goto begin; end end progend

```

```

begin real a,b,c,nmul,p,u,r,thetmul,thet,theta,n,pi,sk,dk,ek,k,thetmulgr,thetmulgr; integer dia,sit, it,its; array
  d,e[1:4],IEI,IER,IHR,IHI[0:180];
complex hank,q,s,j,t;
complex array IH,IE[0:180]; library INTEGRAL,ARG,PLOTPICTURE,SCALE,MARKED CURVE,REGULA FALSI,PLOTAXIS,PLOTAXIS
2,PLOTTEN;
begin:a:=0;
j:=com(0,1);pi:=3.14159265359;dia:=READ;thetmulgr:=READ;thetmul:=thetmulgr*pi/180;n:=READ;r:=READ;sk:=READ;
dk:=READ;ek:=READ;
e[1]:=READ;e[2]:=READ;e[3]:=READ;e[4]:=READ;thetmul:=READ;sit:=READ;
for k:=sk step dk until ek do begin a:=0;
  for it:=sit step sit until thetmul do begin thet:=it*pi/180; IER[it]:=INTEGRAL(theta,re
    (dP(theta,0,n,e[2]*xn-2)*(j*(1+cos(theta)*cos(thet)))*J1((p:=k*xr*sin(theta)*sin(thet)))+sin(theta)*sin(thet)*J0(
    p))*(cos((u:=k*xr*cos(theta)*
    cos(thet)))+j*sin(u)))

,a,thetmul,d,e,false,false);IEI[it]:=INTEGRAL(theta,im
  (dP(theta,0,n,e[2]*xn-2)*(j*(1+cos(theta)*cos(thet)))*J1((p:=k*xr*sin(theta)*sin(thet)))+sin(theta)*sin(thet)*J0(
  p))*(cos((u:=k*xr*cos(theta)*
  cos(thet)))+j*sin(u)))

,a,thetmul,d,e,false,false);IE[it/sit]:=com(IER[it],IEI[it]);
end;CARRIAGE(4);PRINTTEXT(⟨corrugated horn tm-01-mode: THETANUL,n,r en k zijn respectievelijk:⟩);
FIXT(2,2,thetmulgr);FIXT(2,8,n);FIXT(2,3,r);FIXT(3,2,k);CARRIAGE(2);
for it:=2 step 1 until thetmul/sit do begin if mod(IE[it])<mod(IE[it-1]) then go to AAend;AA:its:=it-1;PRINTTEXT(
  ⟨theta mod(I/I0) ARG(I/I0) 20ln(I/I0)⟩);b:=-10;
for it:=1 step 1 until thetmul/sit do begin NLGR;FIXT(3,0,it*sit);SPACE(10);FIXT(1,5,(p:=mod(IE[it]/IE[its])));SP
  ACE(10);
FIXT(1,3,ARG(IE[it]/IE[its]));SPACE(10);FIXT(2,3,(a:=20*ln(p)/ln(10)));SPACE(10);
for c:=0,3,10,15,20,25,30,35,40 do begin if (a<-c/b>c)∨(a>c/b<-c) then FIXT(2,1,sit*(it-1-(b+c)/(a-b)))end;b:=a
;end;if dia=0 then begin
  PLOTPICTURE(1*sit,if(p:=20*ln(mod(IE[1]/IE[its]))/ln(10))>0 then 0 else (if p<-40 then -40 else p),1,thetmul/sit,2,
  -1,0066,0,thetmul,12,
  2540*thetmul/180,
  ⟨ theta (deg.)⟩,-40,0,2,2500,⟨ relative power in dB⟩);
  PLOTTTEXT(36,-38,90,30,0,true,2,⟨TM-0n-mode , n=⟩);FIXPLOT(0,0,90,30,0,false,0,2,4,n);
  PLOTTTEXT(48,-38,90,30,0,true,0,⟨kr=⟩);FIXPLOT(0,0,90,30,0,false,0,3,2,kxr);
  PLOTTTEXT(0,0,90,30,0,false,0,⟨ thetmul=⟩);FIXPLOT(0,0,90,30,0,false,0,2,0,thetmulgr);
  PLOTTTEXT(0,0,90,30,0,false,0,⟨deg.⟩);ENDPLOC
end
end ;p:=read;if p≠0 then goto begin;
end
end progend

```