Multivariable H-infinity Control Design Toolbox

by

H.M. Falkus
A.A.H. Damen

EUT Report 94-E-282
ISBN 90-6144-282-6
April 1994
Multivariable H-infinity Control Design Toolbox

User Manual

by

H.M. Falkus
A.A.H. Damen

EUT Report 94-E-282
ISBN 90-6144-282-6

Eindhoven
April 1994
CIP-DATA KONINKLIJKE BIBLIOTHEEK, DEN HAAG

Falkus, H.M.

Multivariable H-infinity control design toolbox: user manual / by H.M. Falkus, A.A.H. Damen. - Eindhoven:
Eindhoven University of Technology, Faculty of Electrical Engineering. - Fig. - (EUT report, ISSN 0167 - 9708 ;
94-E-282)
With ref.
ISBN 90-6144-282-6
NUGI 832
Subject headings: robust control / multivariable control systems / control simulation software.
Abstract

Multivariable H-infinity Control Design Toolbox: User manual
H.M. Falkus and A.A.H. Damen

A MATLAB toolbox is presented for solving the multivariable $H_{\infty}$ control design problem. Algorithms are available (Robust control toolbox of MATLAB) which solve the problem, once the control design configuration including process model and weighting functions has been rewritten into a standard $H_{\infty}$ control problem. In this report a general package is described that facilitates the controller design for various control configurations, the standard $H_{\infty}$ control problem and the closed-loop system evaluation. Because no solution is known for translating design specifications such as desired behaviour, robustness, performance etc. directly into weighting functions in the frequency domain, the necessarily iterative design procedure has been implemented in a flexible, menu driven way.

Keywords: Robust control, Multivariable control systems, Control simulation software.

- Falkus, H.M. and A.A.H. Damen
  Multivariable H-infinity Control Design Toolbox: User manual
  Eindhoven: Faculty of Electrical Engineering,

- Address of the Authors:
  Measurement and Control Section, Faculty of Electrical Engineering,
  Eindhoven University of Technology,
  P.O. Box 513, 5600 MB Eindhoven, The Netherlands.
# Contents

1 Introduction ........................................................................................................ 1

2 General $H_{\infty}$ Control Design Framework .................................................. 5
   2.1 Structure Definition .................................................................................. 7
   2.2 Minimum Realization Generalized Plant .................................................. 8
   2.3 Controller Calculation ............................................................................ 10
   2.4 Evaluation Controller Design .................................................................. 12
   2.5 Installation and Requirements ................................................................. 12

3 Menu Description .................................................................................................. 13
   3.1 Options ..................................................................................................... 14
   3.2 Structure Initialization ............................................................................ 15
   3.3 Input Matrix Functions ........................................................................... 17
   3.4 Controller Design ................................................................................... 20
   3.5 System Evaluation .................................................................................. 24
   3.6 Disk Functions ......................................................................................... 27

4 Conclusions ........................................................................................................... 29

A Menu Overview .................................................................................................. 31

B Program Structure ............................................................................................. 33

C Function Description .......................................................................................... 35

D List of Variables .................................................................................................. 39

References ............................................................................................................. 43
Introduction

In the last few years, there has been much interest in the design of feedback controllers for linear systems that minimize the $H_\infty$ norm of a specified closed-loop transfer function. Since 1988, a state-space solution for general $H_\infty$ problems based on a "2-Riccati" approach, derived by Glover, K. and J.C. Doyle (1988), has been available for the representation of all stabilizing controllers that satisfy an $H_\infty$ norm bound:

$$\| \mathcal{F}(G,K) \|_\infty \leq \gamma$$

(1.1)

A more detailed explanation and a proof of its validity is outlined in Doyle, J.C. et. al. (1989). Standard program packages (Robust control toolbox of MATLAB) together with some numerical variations and extensions of the basic solution are available now and can be applied once the original problem has been translated into the standard control $H_\infty$ problem.

The generalized plant $G$ contains what usually is called the plant in a control problem and includes all weighting functions. The signal $w \in \mathbb{R}^{n_1}$ represents all external inputs, including disturbances, sensor noise and commands; the output $z \in \mathbb{R}^{p_1}$ is the error vector; $y \in \mathbb{R}^{p_2}$ is the observation vector; and $u \in \mathbb{R}^{m_2}$ is the control input. The generalized plant $G$ can be partitioned according to the dimensions of the signals:
which results in the following closed-loop transfer function from $w$ to $z$:

$$F(G,K) := G_{11} + G_{12} K (1 - G_{22} K)^{-1} G_{21}$$ \hfill (1.3)

The standard assumptions are:

- The triplet $(A,B_2,C_2)$ - namely the plant transfer $G_{22}$ - can be stabilized and detected, so that stabilizing controllers exist.
- $\text{rank}(D_{12}) = m_2$ and $\text{rank}(D_{21}) = p_2$ in order to ensure realizability of the controllers.
- No zeros on the imaginary axis, $p_1 \geq m_2$ and $m_1 \geq p_2$ ensures that the solution to the corresponding LQG problem is closed-loop asymptotically stable.

The main problem, however, is that every control problem has a different configuration because of different design constraints and control objectives. This implies that every new control problem has to be rewritten again into the standard $H_\infty$ control problem. In this report a MATLAB toolbox is presented which enables us (using computer routines) to transform every multivariable control problem into the standard $H_\infty$ control problem. After selecting the control setup, design constraints and objectives, the design configuration is defined in a fairly simple way. Because no general solution is known for translating design specifications such as desired behavior, robustness, performance etc. directly into weighting functions in the frequency domain, the $H_\infty$ control design is menu driven to ensure easy input.
of variables, controller calculation, and analysis of the results by computing both time and frequency responses. In this way the necessary iterative design procedure for optimizing the $H_{\infty}$ control design problem becomes much easier. All tools in this toolbox are implemented in MATLAB by means of standard .m-files.

In Section 2 the basic setup of the toolbox is presented using the process block diagram of a floating platform laboratory process as an example. The floating platform with rotating crane has been built on laboratory scale to evaluate identification and control theories. This particular process was chosen because it is an essentially linear MIMO system. It can be well described by three decoupled, second order SISO systems. The model errors are then mainly due to unmodeled waves, caused by the movement of the floats, which lead to linear transfers which are however difficult to model. The fact that $H_{\infty}$ control is said to be particularly suited for robust control in cases of unmodeled linear dynamics, makes this laboratory process an excellent example for testing the $H_{\infty}$ control synthesis procedure. On the platform, a crane has been mounted rotating a load and thereby tilting the platform. The control to be designed should prevent this tilting of the platform. A detailed description of the process together with the physical modeling, identification and control design can be found in Bouwels, J.P.H.M. (1991) and Damen, A.A.H. et. al. (1994).

A detailed menu description of the toolbox together with several design options is given in Section 3.
In this section the most important parts of the general framework will be explained. Fig. 2.1 depicts the \( H_\infty \) control design configuration for the floating platform. The solid part illustrates the basic control configuration, while the dashed part is added for the \( H_\infty \) control design. The main objectives in the design are disturbance attenuation (\( V_{d}n_{d} \) to prevent tilting of the platform due to the rotating crane) and robustness (model errors represented by \( V_{y}n_{y} \) due to unmodeled waves). In addition saturation of the actuators (\( W_{u}u_{p} \)) should be avoided.

The transfers of Fig. 2.1 are described as follows:

\[
\begin{align*}
P : & \text{ Platform Dynamics} \\
R : & \text{ Disturbance Dynamics} \\
V_{d} : & \text{ Shaping Crane Disturbance} \\
V_{y} : & \text{ Shaping Model Disturbance} \\
W_{y} : & \text{ Weighting Process Output} \\
W_{u} : & \text{ Weighting Control Input} \\
C_{fb} : & \text{ Feedback Controller} \\
C_{ff} : & \text{ Feedforward Controller}
\end{align*}
\]

where we can define the following standard signals:

\[
z = \begin{bmatrix} y \\ u \end{bmatrix}, \quad y = \begin{bmatrix} y_{m} \\ d \end{bmatrix}, \quad w = \begin{bmatrix} n_{d} \\ n_{y} \end{bmatrix}, \quad u = u_{p}
\]  

\[ (2.1) \]
After deriving the various relations between the inputs and outputs, it follows that, in terms of the standard $H_\infty$ problem, the generalized plant $G$ is defined by:

$$
\begin{align*}
\begin{pmatrix}
\bar{y} \\
\bar{u} \\
y_m \\
d
\end{pmatrix} &=
\begin{pmatrix}
W_y V_v & W_y R V_d & W_y P \\
0 & 0 & W_u \\
V_v & R V_d & P \\
0 & V_d & 0
\end{pmatrix}
\begin{pmatrix}
n_v \\
n_d \\
u_p
\end{pmatrix} \\
&= \begin{pmatrix}
G_{11} & G_{12} \\
G_{21} & G_{22}
\end{pmatrix}
\begin{pmatrix}
w \\
u
\end{pmatrix} \\
(2.2)
\end{align*}
$$

The control acting on the outputs $y$ is represented by:

$$
\begin{align*}
u &= u_p = (C_{fb} \mid C_{ff}) \begin{pmatrix} y_m \\ d \end{pmatrix} = K y \\
&= \begin{pmatrix}
C_{fb} \\
C_{ff}
\end{pmatrix} \begin{pmatrix}
y_m \\
d
\end{pmatrix} = K y \\
(2.3)
\end{align*}
$$

This yields the closed-loop system $\mathcal{F} := \mathcal{F}(G,K)$ mapping $w \rightarrow z$ which has $H_\infty$ norm:
2.1 Structure Definition

\[
\mathcal{F}_{\infty} = \begin{bmatrix} \mathcal{F}_{11} & \mathcal{F}_{12} \\ \mathcal{F}_{21} & \mathcal{F}_{22} \end{bmatrix}
\]

\[
= \begin{bmatrix} W_y(I-PC_{fb})^{-1}V_v & W_y(I-PC_{fb})^{-1}(R+PC_{ff})V_d \\ W_uC_{fb}(I-PC_{fb})^{-1}V_v & W_u(I-PC_{fb})^{-1}(C_{fb}R+PC_{ff})V_d \end{bmatrix}_{\infty}
\]

(2.4)

Here the various transfer functions are named as follows:

- \( \mathcal{F}_{11} \): Sensitivity
- \( \mathcal{F}_{12} \): Disturbance Attenuation
- \( \mathcal{F}_{21} \): Control Sensitivity
- \( \mathcal{F}_{22} \): Saturation

Until now we have only described the standard approach to define the \( \mathcal{H}_\infty \) control problem for the floating platform to achieve disturbance attenuation and avoiding saturation of the control input. This results in the closed-loop system \( \mathcal{F}(G,K) \) of Eq. 2.4. This configuration however is specific to the floating platform and can be significantly different for other control problems. To avoid this procedure of building (and implementing) different configurations every time when new control problems occur, we will generalize this setup.

2.1 Structure Definition

Basicly, the design configuration (Fig. 2.1) can be built up of four major blocks (Fig. 2.2):

1.2) Process models \( P_1 \) & \( P_2 \)
3) Shaping filters \( V \) for the input signals \( w \)
4) Weighting filters \( W \) for the output signals \( z \).

The extra process block \( P_2 \) is sometimes necessary if there exists already a known feedback. Further, these four blocks are somehow connected. Rearranging of Fig. 2.1 into the blocks \( P_1, P_2, V \) and \( W \) is carried out in Fig. 2.3 where \( IM_1 \) and \( IM_2 \) define the structure of the control design configuration. \( IM_1 \) and \( IM_2 \) reflect the interconnection structure of the various blocks. These are constant matrices with entries \( \pm 1 \) and \( 0 \), each entry corresponding to a specific adding, subtracting or no connection of signals. The matrices \( I_1 \) to \( I_4 \) define the feed-through of signals which are necessary to build the state-space representation of the generalized plant \( G \).

We emphasize that this structure is general. That is, every control configuration of the form shown in Fig. 2.1 can be represented by a configuration of Fig. 2.2 resulting in Fig. 2.3.
2.2 Minimal Realization Generalized Plant

The control design configuration of the floating platform can now be described in a simple way by defining the dimensions of $V$ ($V_d$ & $V_v$), $P_1$ ($R$ & $P$), $P_2$ ($\emptyset$), $W$ ($W_y$ & $W_u$) and $I_1$ to $I_4$ together with the structure of the configuration using the interconnection blocks $IM_1$ and $IM_2$. This is effectively all input which is needed for the toolbox to convert it into a standard $H_\infty$ control problem.
Therefore we use the approach outlined in Munro, N. et. al. (1971). For every row in a transfer function matrix, the smallest common denominator is determined and the numerators are updated if necessary. The new MISO transfer functions can be transformed into an observer canonical state-space representation which is minimal. Combining the state matrices of the MISO systems for every row in a block-diagonal form and adding the input, output and feed-through matrices correctly results in an overall state-space representation which is observable but not necessarily controllable. In Dooren, P.M. v. (1981) it is proven that if the controllability matrix of \((A,B)\) has rank \(r \leq n\), where \(n\) is the size of \(A\), then there exists a similarity transformation \(T\) such that:

\[
\bar{A} = TAT^T, \quad \bar{B} = TB, \quad \bar{C} = CT^T, \quad \bar{D} = D
\]

and the transformed system has a staircase form with the uncontrollable modes (being the eigenvalues of \(A_{uc}\), if any, in the upper left-hand corner.

\[
\bar{A} = \begin{pmatrix} A_{uc} & 0 \\ A_{21} & A_c \end{pmatrix}, \quad \bar{B} = \begin{pmatrix} 0 \\ B_c \end{pmatrix}, \quad \bar{C} = \begin{pmatrix} C_{uc} & C_c \end{pmatrix}
\] (2.5)

where \((A_c,B_c)\) is controllable, \((A_{uc},B_{uc}=0)\) is uncontrollable and \(C_c(sI-A_c)^{-1}B_c = C(sI-A)^{-1}B\).

If the process \(P_1, P_2\) and the design blocks \(V\) and \(W\) are given in transfer function matrices, this approach can be used to derive a minimum state-space representation of every block.

The dual approach for realizing a minimum state-space representation can also be used. In that case a controllable but not necessarily observable state-space representation can be derived and all unobservable states have to be removed. So if the observability matrix of \((A,C)\) has rank \(r \leq n\), there exists again a similarity transformation such that the transformed system has a staircase form with the unobservable modes, if any, in the upper left-hand corner.

\[
\bar{A} = \begin{pmatrix} A_{uo} & A_{12} \\ 0 & A_o \end{pmatrix}, \quad \bar{B} = \begin{pmatrix} B_{uo} \\ B_o \end{pmatrix}, \quad \bar{C} = \begin{pmatrix} 0 & C_o \end{pmatrix}
\] (2.6)

Because the blocks \(V, P_1, P_2\) and \(W\) are now available as minimum state-space realizations, straightforward matrix computations for connecting state-space systems in series or parallel can be used to build the generalized plant:

1) Build \(1_1, V\) and \(1_2\) parallel (System 1).
2) Build \(P_1\) and \(1_3\) parallel (System 2).
3) Build \(P_2, W\) and \(1_4\) parallel (System 3).
4) Connect system 1 in series with \(IM_1\) (System 4).
5) Connect system 2 in series with IM₂ (System 5).
6) Connect system 4 in series with system 5 (System 6).
7) Connect system 6 in series with system 3 (System 7).
8) Partition system 7 according to the defined inputs/outputs.
9) Close the loop around P₂ and I₁.

The state-space system of the generalized plant might not be a minimum realization because of common modes in the various blocks. Removing again all uncontrollable (2.5) and unobservable (2.6) modes will yield a minimum state-space realization of the generalized plant. This approach has been selected because obtaining the same minimal realization after building the generalized plant using the non-minimal state-space realizations of the various blocks and applying Eq. 2.5 and 2.6 only once, might not be achievable due to numerical problems (e.g. round-off errors).

The constructed minimal state-space realization of the augmented plant might be badly conditioned depending on the design filters and process behaviour. This can result in numerical problems when calculating the $H_\infty$ controller. Balancing of the augmented plant is therefore often desired to improve numerical reliability. The balancing approach described in Weiland, S. (1993) is used in order to handle unstable as well as stable systems.

### 2.3 Controller Calculation

Because a minimum state-space representation is available, the standard solution method based on solving two Algebraic Riccati equations and implemented in the Robust Control Toolbox of MATLAB, Chiang, R.C. et. al. (1988), can be applied. The methods available:

1) Safonov/Limebeer/Chiang loop-shifting formulae; Safonov, M.G. et. al. (1989).
2) Glover/Doyle all-solution formulae; Glover, K. et. al. (1988).
3) Limebeer/Kasenally all-solution formulae; Limebeer, D.J.N. et. al. (1988).

are only different in circumventing some of the numerical problems which generally arise when a design approaches its $H_\infty$ performance limits.

The solutions to the Riccati equations can be solved either by an eigenvalue or Schur decomposition. The eigenvalue approach is the fastest but for design filters close to the $H_\infty$ performance limits the Schur approach is numerically more reliable.

These routines calculate a controller, if one exists, only for a fixed value of $\gamma$. That is, a controller is computed achieving $\| F \|_{\infty} \leq \gamma$. However, we are interested in $\gamma_{\text{opt}}$ for which a stabilizing controller still exists. Therefore the basic routine has been extended as follows with an iterative search procedure:
- A start value $\gamma_o$ and a step size $\alpha$ ($\alpha > 1$) are defined.
- An interval $[\gamma_{\text{min}}, \gamma_{\text{max}}]$ is computed which contains the optimal solution.
  1) If a solution exists for $\gamma_o$ ($\gamma_{\text{opt}} \leq \gamma_o$) define $\gamma_{\text{max}} = \gamma_o$. The lower bound of the interval can be found by decreasing $\gamma$ ($\gamma_{k+1} = \gamma_k / \alpha^k$) until no solution exists defining $\gamma_{\text{min}} = \gamma_{k+1}$.
  2) If no solution exists for $\gamma_o$ ($\gamma_{\text{opt}} > \gamma_o$) define $\gamma_{\text{min}} = \gamma_o$. The upper bound of the interval can be found by increasing $\gamma$ ($\gamma_{k+1} = \alpha^k \gamma_k$) until a solution exists defining $\gamma_{\text{max}} = \gamma_{k+1}$.
- Bisection search is used to find $\gamma_{\text{opt}}$ within a certain tolerance margin for which a stabilizing solution exists.
  1) Define $\gamma_k = (\gamma_{\text{max}} + \gamma_{\text{min}})/2$.
  2) If a solution exists for $\gamma_k$ adjust the upper bound $\gamma_{\text{max}} = \gamma_k$. If no solution exists for $\gamma_k$ adjust the lower bound $\gamma_{\text{min}} = \gamma_k$.
  3) Repeat 1 & 2 until $(\gamma_{\text{max}} - \gamma_{\text{min}})/\gamma_{\text{min}} \leq \text{tol}$.

When starting the controller design, no information is available about $\gamma_{\text{opt}}$ which depends of course on the design filters and the process. Because the final goal is to achieve $\gamma \uparrow 1$, it is recommended to start with $\gamma_o = 1$ and $\alpha = 2$ to reduce the number of iterations. This approach has the advantage that it is reasonably fast (7 to 15 iterations depending on the tolerance margin) and that independent of the start value $\gamma_o$ a sub-optimal solution is found. The variable tolerance margin has been introduced to speed up the design (fewer iterations) and because of the fact that if this margin becomes too small the Riccati equations cannot be solved properly anymore. Using the method proposed in Bruinsma, N.A. (1990), the $H_{\infty}$ norm of the closed-loop system can be used to check the solution $\gamma_{\text{opt}}$ of the search procedure a posteriori. Since the standard solution is only available for the continuous-time case, it should be mentioned here that the discrete-time case is solved via bilinear transformation. In Stoorvogel, A.A. et. al. (1993) and Iglesias, P.A. et. al. (1993), it is shown that designing a discrete-time controller via a bilinear transformation to the continuous-time domain might introduce an implicit and undesirable additional weighting function. A simple free stable contraction map is added to eliminate this additional weighting.

In general, the resulting $H_{\infty}$-controllers are of high order because the order is equal to the order of the generalized plant (process & all design filters). To reduce the order of the controller, the following reduction techniques can be applied to the resulting controller:

1) Minimal state-space realization (reduces within a predefined tolerance margin).
2) Optimal Hankel reduction.
3) Schur reduction.
4) Relative Schur reduction.
For the reduction methods $2$ to $4$ an additional option can be selected to reduce the controller with variable order and fixed error bound or fixed order and variable error bound. A detailed description and more references for these reduction techniques can be found in Chiang, R.C. et. al. (1988).

### 2.4 Evaluation Controller Design

After calculating the $H_\infty$ controller, the closed-loop system is derived (without shaping and weighting functions) in order to evaluate the controller design. For this purpose time as well as frequency responses can be calculated. Time simulations can be performed to check the closed-loop behaviour with respect to design objectives and constraints in the time domain like disturbance attenuation, reference tracking and/or input saturation. Frequency response analysis can be used to verify sensitivity and complimentary sensitivity functions. Whenever the design functions $V$ and $W$ are defined as diagonal blocks, which is recommended to keep the design as simple as possible, the closed-loop behaviour from every input to every output can be compared with the corresponding inverse weighting functions (scaled with the $H_\infty$-norm $\gamma$). This can simplify the iterative controller design because the Bode plots indicate which function in which frequency range is the limiting factor and where and how the design can be improved.

If the controller design is not satisfactory, the menu driven structure of the toolbox ensures that the design filters can be changed fast and a new controller can be calculated easily in order to optimize the controller design. At every stage of the design procedure, the controller configuration as well as the actual results can be saved to ensure continuation if necessary. The toolbox is built up in such a way that the input required from the user is minimized and that correct data transfer between the various functions is guaranteed.

### 2.5 Installation and Requirements

All names of the .m-files in the toolbox start with "rnhc" (App. B) and can be copied (e.g. copy rnhc*.m) to the working directory of MATLAB. If the files are copied to a directory different than the workspace of MATLAB, this directory has to be added to the matlabpath. The routines in this toolbox make use of standard MATLAB functions and the following MATLAB toolboxes:

- Signal processing toolbox
- Control system toolbox
- Robust control toolbox
Before starting the $H_\infty$ control design, the specific control problem, including design filters (Fig. 2.1) must be transformed into the standard configuration defined for this toolbox (Fig. 2.2) resulting in the required input information (Fig. 2.3). The Multivariable $H_\infty$ Control design toolbox (MHC) is menu driven to ensure easy input of variables, controller calculation and analysis of the results. All menus of the toolbox will be described briefly and the controller design for a floating platform will be used as an example. Any of the menu options can be selected by typing the correct number and pressing ENTER. The previous menu will appear again by pressing just ENTER. Every menu is provided with a help screen (menu option 0) describing briefly the several menu options.

**Startup**: **MHC**

To start the controller design procedure, execute **MHC** from inside MATLAB. The main menu, which is depicted in Fig. 3.1 will appear on the screen.

---

**Fig. 3.1**: Main menu.
Fig. 3.2 depicts the help screen of the main menu.

<table>
<thead>
<tr>
<th>Menu Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Help Main Menu H-infinity Control Design</td>
</tr>
<tr>
<td>This menu structured H-infinity control design package can be used to ensure easy definition of the design configuration, input of variables, controller calculation and analysis of the results by computing time and frequency responses.</td>
</tr>
<tr>
<td>Structure Initialisation : Defines the design configuration including process and weighting filters as a standard problem. The structure is fixed by defining the dimensions and two interconnection matrices.</td>
</tr>
<tr>
<td>Input Matrix Functions V, P and W : Definition of process and weighting filters via SISO transfer functions or MIMO state-space matrices.</td>
</tr>
<tr>
<td>Controller Design : H-infinity control design parametrizing all stabilizing controllers such that a specified closed-loop transfer function has H-infinity norm less than a given scalar. This involves the solution to two algebraic Riccati equations, each with the same order as the system, and further gives feasible controllers also with this order.</td>
</tr>
</tbody>
</table>

Press any key to continue

Fig. 3.2a : Help main menu.

<table>
<thead>
<tr>
<th>Menu Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Help Main Menu H-infinity Control Design</td>
</tr>
<tr>
<td>System Evaluation : Analysis of closed-loop system by computing time and frequency responses.</td>
</tr>
<tr>
<td>Options : Definition of general options for design package.</td>
</tr>
<tr>
<td>Disk Functions : Menu to load and save variables. WARNING all variables are initialized in a standard form when starting up the design package.</td>
</tr>
</tbody>
</table>

Press any key to continue

Fig. 3.2b : Help main menu (cont.).

### 3.1 Options

Before starting the actual controller design, several options must be defined in the options menu. The default menu is shown in Fig. 3.3.

<table>
<thead>
<tr>
<th>Options</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Mode : Continuous</td>
</tr>
<tr>
<td>2) Tolerance margin for minimization procedure : 1e-010</td>
</tr>
<tr>
<td>3) Selected input signal :</td>
</tr>
<tr>
<td>4) Generating MKTA files : No</td>
</tr>
<tr>
<td>5) Lower frequency bound : 2 rad</td>
</tr>
<tr>
<td>6) Upper frequency bound : 2 rad</td>
</tr>
<tr>
<td>7) Number of frequency points : 50</td>
</tr>
<tr>
<td>8) End of time interval : 1 sec</td>
</tr>
<tr>
<td>0) Help</td>
</tr>
</tbody>
</table>

Select menu option or press ENTER to Exit :

Fig. 3.3 : Options menu.

Change for the floating platform design example the following options into :

1) **Mode :**

**Discrete**
3.2 Structure Initialization

The structure initialization menu is depicted in Fig. 3.5. The general structure as well as IM1 & IM2 can be changed.
The generalized plant is described by defining the 11 signal dimensions of the blocks within the basic structure (Fig. 3.6). For the floating platform the transformation of the control configuration into this structure is shown in Section 2.1. The first step in the design is the definition of the dimensions.

**Dimensions:**  \[ \{0 0 2 1 2 2 2 3 1 2\} \]

The dimensions of the interconnection matrices are fixed now and can be defined row by row. After selecting a row number, this row can be defined as an array in MATLAB notation. Fig. 3.7 depicts the screen for IM1.

**Interconnection matrices:**

\[
IM1 = \begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}, \quad IM2 = \begin{bmatrix}
1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
\end{bmatrix}
\]
3.3 Input Matrix Functions

Fig. 3.8 shows the help screen of the structure initialization menu.

Help Structure Initialization Menu

H-infinity standard problem:

\[ \begin{array}{cccc}
\ldots & n_1 & \ldots & n_2 \\
\ldots & -+ & \ldots & -+ \\
w & n_5 & \ldots & n_6 \\
\ldots & -+ & \ldots & -+ \\
u & n_9 & \ldots & n_{10} \\
\ldots & -+ & \ldots & -+ \\
\end{array} \]

Press any key to continue

Fig. 3.8a: Help structure initialization menu.

Help Structure Initialization Menu

Before designing an H-infinity controller, the control setup has to be defined together with the design constraints and objectives. (e.g. tracking, disturbance rejection, input saturation etc.) This design configuration must be transformed into the H-infinity standard problem with an augmented plant that contains what usually is called the plant in a control problem plus all weighting functions.

The augmented plant consists of four basic blocks: 1) \( \Phi \), 2) \( \Gamma \) - Process models, 3) \( \mathcal{V} \) - Shaping the disturbance vector \( w(t) \) and 4) \( \mathcal{W} \) - Weighting the error vector \( z(t) \). Further these four blocks are somehow connected through the interconnection matrices \( IM_1 \) & \( IM_2 \) containing only the elements ±1 and 0 which corresponds with adding, subtracting or no connection.

Every control configuration can now be described in a simple way by defining the dimensions \( n_1 \ldots n_{11} \) together with the structure of the configuration using the interconnection matrices.

Press any key to continue

Fig. 3.8b: Help structure initialization menu (cont.).

3.3 Input Matrix Functions

After initializing the structure, the blocks \( \Phi \), \( \Gamma \), \( \mathcal{V} \) and \( \mathcal{W} \) must be defined. This menu is depicted in Fig. 3.9. The blocks can be defined either as SISO transfer functions per entry or as MIMO state-space matrices. An additional viewing option has been included to verify the magnitude plots.

Fig. 3.9: Input matrix functions \( \Phi, \Gamma, \mathcal{V} \) or \( \mathcal{W} \)
Fig. 3.10 shows the help screen for the input matrix functions menu.

Help Input Matrix Functions P1, P2, V or W
As described in the Structure Initialization menu, the augmented plant of the standard problem consists of three basic blocks:
1) P1 - The process model (part 1).
2) P2 - The process model (part 2).
3) V - Shaping of the disturbance vector w(t).
4) W - Weighing of the error vector z(t).
These four blocks can be entered into the design package either as SISO transfer functions or as MIMO state-space matrices. For the SISO case the user must define the filters as numerator and denominator polynomials for every entry of the matrix function. For the MIMO case the A,B,C or D matrices must be defined. The number of inputs and outputs of these blocks depends of course on the dimensions entered in the Structure Initialization menu.

Press any key to continue

Fig. 3.10a: Help input matrix functions P1, P2, V or W.

Help Input Matrix Functions P1, P2, V or W
When entering the filters as SISO transfer functions, the corresponding state-space representation is derived automatically and vice versa. A consequence of this representation of the blocks P1, P2, V and W in transfer function matrices and state-space matrices is that all the process and design blocks must be proper. In addition the magnitude of the designed filters can be plotted.

REMARK: To simplify the H-infinity control design it is recommended to define the shaping and weighting blocks, V and W respectively, as square functions (equal number of inputs and outputs) with elements only on the diagonal.

Press any key to continue

Fig. 3.10b: Help input matrix functions P1, P2, V or W (cont.).

When defining a block as SISO transfer function, the correct element of the matrix must be selected first.

Every transfer function is defined by its numerator and denominator. Both polynomials can be entered as arrays in MATLAB notation. The polynomials must be defined in powers of 'z' or 's' for the discrete or continuous time respectively.

--- Define Discrete Transfer Matrix P1 ---
Enter input number (1 -- 1) or press ENTER to Exit:
Enter output number (1 -- 1):

Fig. 3.11: Selecting element of transfer matrix P1.

--- Define Discrete Transfer Function of P1(11) ---
Please define the following polynomials in MATLAB notation.
Example: \[ z^3 + 2z^2 - 3z + 1 \] \[ \rightarrow [1 2 -3 1] \]
Press ENTER if a polynomial should not be changed !!!!!

Old numerator: [0]
New numerator:
Old denominator: [1]
New denominator:

Fig. 3.12: Defining SISO transfer function.
3.3 Input Matrix Functions

The menu to define the state-space matrices A, B, C & D is depicted in Fig. 3.13. These matrices can be entered exactly the same way as the interconnection matrices IM1 & IM2 (Fig. 3.7).

The help screen to define state-space systems is shown in Fig. 3.14.

Before describing the actual $H_\infty$ control design, the following block information for the floating platform should be entered using the input menus for transfer functions and state-space matrices (Fig. 3.11, 3.12 & 3.13). More detailed information about the modeling, identification and $H_\infty$ filter design can be found in Bouwels, J.P.H.M (1991) and Damen, A.A.H. et. al. (1994).

\[
A_{p1} = \begin{bmatrix} 1 & 0.1 & 0 \\ 0 & 1 & 0.1 \\ 0 & -3.794215 & 0.4044 \end{bmatrix}, \quad B_{p1} = \begin{bmatrix} -0.0017 \\ 0.0311 \\ -0.1213 \end{bmatrix}, \quad C_{p1} = \begin{bmatrix} 1 \\ 0.1 \\ 0 \end{bmatrix}, \quad D_{p1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad A_{p2} = B_{p2} = C_{p2} = \begin{bmatrix} \end{bmatrix}, \quad D_{p2} = \begin{bmatrix} 0 \end{bmatrix}
\]

\[
V_d = -0.3104 (z^2 - 0.2z) \\
\frac{1}{z^2 - 1.9944z + 0.9950}
\]

\[
V_r = \frac{(z + 0.4172) (z - 0.5727 - 0.57251) (z - 0.5727 + 0.57251)}{60 (z - 1) (z - 0.7022 - 0.53921) (z - 0.7022 + 0.53921)}
\]

Design filters:

\[
W_r = \frac{0.1 (z - 0.8 - 0.011) (z - 0.8 + 0.011)}{(z - 0.999 - 0.0011) (z - 0.999 + 0.0011)}
\]

\[
W_u = 40 (z - 0.995 - 0.00991) (z - 0.995 + 0.00991) \\
(z + 0.9)^2
\]
The numerator and denominator polynomials can be defined in several ways in the transfer function entry of Fig. 3.12. For example:

**Numerator** $V_o'$:

$$-0.3104 \times [1 -0.2 0]$$

**Denominator** $V_p$:

$$60 \times \text{poly}([1; 0.7022 +0.5392 \times i; 0.7022 -0.5392 \times i])$$

Note that MATLAB commands can be used as well to define the transfer functions.

### 3.4 Controller Design

Once the complete generalized plant has been defined, the controller design becomes fairly simple.

The help screen for the controller design menu is depicted in Fig. 3.16.

**Help Controller Design Menu**

In this menu the actual controller design is performed. All stabilizing controllers such that a specified closed-loop transfer function has $H$-infinity norm less than a given scalar. This characterization involves the solution to two algebraic Riccati equations, each with the same order as the system, and further gives feasible controllers also with this order.

$H$-infinity Controller Options: In this menu some controller relevant settings are defined, like type of $H$-infinity solution, type of Riccati equation solution and a tolerance margin indicating the accuracy of the closed-loop $H$-infinity norm with respect to the optimal gamma. In addition an option for balancing of the augmented plant can be selected. This can improve the numerical stability of the controller design. Also a reduction technique can be selected to reduce the order of the $H$-infinity controller.

Press any key to continue

**Fig. 3.16a**: Help controller design.
3.4 Controller Design

The options described in Section 2.3 can be defined in the controller options menu shown in Fig. 3.17.

Help Controller Design Menu

Calculate H-infinity controller: After checking the conditions for the existence of a stabilizing controller satisfying an H-infinity norm of the closed-loop system, like whether the system is stabilizable and detectable, rank conditions to ensure that the controllers are proper, and no zeros on the imaginary axis of some transfers, a start value and step size for gamma must be defined. After deriving an interval containing the optimal solution, the nearly optimal gamma will be derived using bisection search until the required accuracy has been obtained.

Controller Reduction: Because the order of the controllers will be the same as the order of the system, controller reduction is often required to obtain lower order controllers.

Load Original Controller: The controller reduction step can result in less accurate closed-loop performance, which makes it necessary to use the original high order controllers.

Press any key to continue

Fig. 3.16b: Help controller design (Cont.).

The options described in Section 2.3 can be defined in the controller options menu shown in Fig. 3.17.

Fig. 3.17: Controller options menu

Change for the floating platform design example the following options into:

3) Tolerance margin optimizing gamma: 0.001
4) Balancing augmented plant: Yes

The help screen for the controller options menu is depicted in Fig. 3.18.

Help Controller Options Menu

Type of H-infinity solution: Several routines are available which solve the H-infinity control problem in different ways according to Safonov/Limebeer/Chiang loop-shifting formulae, Glover/Doyle all-solution formulae or Limebeer/Kasenally all-solution formulae.

Type of Riccati Equation Approach: The calculation of the H-infinity involves the solution to two algebraic Riccati equations. These two equations can be solved either by eigenvalue or Schur decomposition. The eigenvalue decomposition is faster but the Schur decomposition is numerically robust for badly conditioned design problems.

Tolerance Margin Optimizing Gamma: The H-infinity controller will be designed in such a way that the H-infinity norm of the closed-loop system will be less than gamma. Because the optimal gamma can only be approximated, a tolerance margin must be defined indicating when the iterative design procedure optimizing gamma can be stopped.

Press any key to continue

Fig. 3.18a: Help controller options.
The following options are available to solve the $H_\infty$ control design problem based on the "2-Riccati" equation approach.

For controller reduction the options depicted in Fig. 3.20 are available.

After selecting the control design options, the actual controller calculation is started (option 2 of the controller design menu). In this case some information will scroll over the screen describing the minimal realization of the generalized plant, bilinear transformation, balancing and ...
checking the conditions to solve the $H_{\infty}$ control problem:
- $G_{22} : (A, B_2)$ stabilizable and $(A, C_2)$ detectable.
- $D_{21}$ full column rank.
- $G_{12} : (A, B_2, C_1, D_{12})$ full column rank.
- $G_{21} : (A, B_1, C_2, D_{21})$ full row rank.

Only if a condition is violated an error message will appear.

For $\gamma$ minimization a new start value and step size must be defined:

Start Value : 1
Step size : 1.1

The tolerance margin defined in the controller options determines the number of $\gamma$ iterations (Fig. 3.24). For an optimal design, $\gamma$ should be slightly smaller than 1. The $H_{\infty}$ norm of the closed-loop system can be used to verify $\gamma$ and the difference should be of the same order as the tolerance margin.

Whenever the Riccati equations are not solved properly (large residuals or other numerical problems), the closed-loop system might not be stable although the $\gamma$ iteration has been
terminated successfully. Redefining the design filters and reducing the constraints and objectives can often help to overcome this problem. Because in general $H_\infty$ controllers are of high order (order generalized plant), some controller reduction options (option 3 of the controller design menu) have been included to realize lower order controllers. If this reduction results in an unstable closed-loop system, the original high-order controller can be loaded again without new calculations (option 4 of the controller design menu).

### 3.5 System Evaluation

Fig. 3.25 shows the menu to analyze the closed-loop behavior by computing time and frequency responses.

Help screen of the system evaluation menu.

**Fig. 3.26a** : Help system evaluation.

**Fig. 3.26b** : Help system evaluation (Cont.).
3.5 System Evaluation

The single transfers in the closed-loop system are shown by selecting the corresponding input and output.

Fig. 3.27: Plot closed-loop transfer.

Only the magnitude plots for the closed-loop evaluation will be shown here.

Fig. 3.28: Closed-loop transfer 1 → 1.

Fig. 3.29: Closed-loop transfer 2 → 1.

Fig. 3.30: Closed-loop transfer 1 → 2.

Fig. 3.31: Closed-loop transfer 2 → 2.

Before we can evaluate the time simulations, an input signal must be defined in the options menu (Fig. 3.3). The variable name of the input signal matrix can be entered. If the variable
name exists in the workspace (for example generated before starting up MHC) and the number of columns correspond to the defined input dimensions, the time simulations can be performed. However, if the variable name does not exist in the workspace, the input matrix must be defined first. Also, the variable name of the output signal matrix must be defined. For the floating platform example a disturbance signal can be generated corresponding with 3 rotations of the crane (rotation frequency 0.04 Hz) and a load of 1 kg. After 10 seconds (100 samples, sample time = 0.1 sec) the crane starts rotating and zeros have been added to create a time simulation of 100 seconds.

Name input : dis
Input signal : 
\[
\begin{bmatrix}
\text{zeros}(100,1);-9.81*\sin(2*\pi*0.04*(0:0.1:75))' \\
\text{zeros}(149,1)\text{ zeros}(1000,1)
\end{bmatrix}
\]
Name output : dis_out

![Discrete Time Simulation of Closed-loop System](image)

**Fig. 3.32** : Simulation output signal.  

![Discrete Time Simulation of Closed-loop System](image)

**Fig. 3.33** : Simulation control signal.

The corresponding time simulations are depicted in Fig. 3.32 & 3.33. The relative bad disturbance rejection in Fig. 3.32 after 10 and 85 sec. are caused by starting and stopping the rotation of the crane. Note that compared to the designs described in Bouwels, J.P.H.M. (1991) and Damen, A.A.H. et al. (1994) the scaling of the filters has been adjusted such that $H_\infty$-norm $\gamma$ becomes smaller than 1.
3.6 Disk Functions

All information can be saved and loaded using the disk options menu (Fig. 3.34). Information should be stored regularly during the design because MATLAB errors due to numerical problems can terminate MHC.

The help screen for the disk options menu is depicted in Fig. 3.35.

The most important features of the $H_\infty$ control design toolbox have been described together with the menus which will appear on the computer screen. The exact screen input has not been described because the control design is rather straightforward and the required user input is fairly simple. The example of the floating platform should be sufficient to guide the user through all menus of the design procedure. It is not the intention to show with this example a complete $H_\infty$ control design procedure for all shaping and weighting filters. This is described in more detail in Bouwels, J.P.H.M. (1991).
Conclusions

Any control configuration can be rewritten in the presented basic structure which is automatically transformed into a standard $H_\infty$ control problem. The menu driven structure of the toolbox makes the necessarily iterative design procedure fast, due to easy input of variables and simple analysis of the results by calculating time and frequency responses. The $H_\infty$ control design of a laboratory process has been used to show the user how to define the basic control structure. An extensive description of all menus and help-facilities should guide the user through the design and explain all options.
Menu Overview

The menus of the multivariable $H_\infty$ control design (MHC) toolbox are presented in one scheme to provide an overview of the most important functions. This overview can be used as quick reference guide by the user during the $H_\infty$ control design.
Program Structure

The global program structure including all mhc-functions is depicted in Fig. B.1. Note that the standard MATLAB functions (including the toolboxes) which are used in the multivariable $H_\infty$ control design toolbox are not mentioned in the overview. The required toolboxes are described in Section 2.5.
Fig. B.1: Global program structure.
A brief description of all functions (in alphabetical order) presented in the overview of Appendix A will be given.

<table>
<thead>
<tr>
<th>Function name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>mhc.m</td>
<td>Initialization script file showing the main menu.</td>
</tr>
<tr>
<td>mhc_are.m</td>
<td>Computes the algebraic Riccati equation for the $H_\infty$ control problem</td>
</tr>
<tr>
<td>mhc_c2o.m</td>
<td>State-space transformation from controller canonical form to observer canonical form.</td>
</tr>
<tr>
<td>mhc_ccl.m</td>
<td>Calculates the closed-loop system consisting of the process blocks $P_1$ &amp; $P_2$ and the $H_\infty$ controller.</td>
</tr>
<tr>
<td>mhc_cm.m</td>
<td>Function to change rows in a matrix.</td>
</tr>
<tr>
<td>Function name</td>
<td>Description</td>
</tr>
<tr>
<td>---------------</td>
<td>-------------</td>
</tr>
<tr>
<td>mhc_crm.m</td>
<td>Computing controller reduction according to several methods.</td>
</tr>
<tr>
<td>mhc_csc.m</td>
<td>Checks the conditions to solve the $H_\infty$ control problem.</td>
</tr>
<tr>
<td>mhc_disk.m</td>
<td>Disk options menu.</td>
</tr>
<tr>
<td>mhc_djml.m</td>
<td>$H_\infty$ all solution formulae derived by Limebeer and Kasenally.</td>
</tr>
<tr>
<td>mhc_dtf.m</td>
<td>Define transfer function.</td>
</tr>
<tr>
<td>mhc_h1.m</td>
<td>Help screen for structure initialization menu.</td>
</tr>
<tr>
<td>mhc_h2.m</td>
<td>Help screen for input matrix functions menu.</td>
</tr>
<tr>
<td>mhc_h21.m</td>
<td>Help screen for define state-space matrices menu.</td>
</tr>
<tr>
<td>mhc_h3.m</td>
<td>Help screen for controller design menu.</td>
</tr>
<tr>
<td>mhc_h31.m</td>
<td>Help screen for controller options menu.</td>
</tr>
<tr>
<td>mhc_h4.m</td>
<td>Help screen for system evaluation menu.</td>
</tr>
<tr>
<td>mhc_h5.m</td>
<td>Help screen for options menu.</td>
</tr>
<tr>
<td>mhc_h6.m</td>
<td>Help screen for disk options menu.</td>
</tr>
<tr>
<td>mhc_hcb.m</td>
<td>$H_\infty$ controller basic function which prepares the variables for the general MIMO configuration and minimizes $\gamma$ to calculate the optimal controller.</td>
</tr>
<tr>
<td>mhc_hcm.m</td>
<td>Script file to generate $H_\infty$ control menu.</td>
</tr>
<tr>
<td>mhc_hco.m</td>
<td>Shows $H_\infty$ controller options menu.</td>
</tr>
<tr>
<td>mhc_hin.m</td>
<td>Routine to calculate $H_\infty$-norm of a state-space system which is the maximum over all frequencies of the maximum singular value.</td>
</tr>
<tr>
<td>Function name</td>
<td>Description</td>
</tr>
<tr>
<td>----------------</td>
<td>----------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>mhc_hm.m</td>
<td>Help screen for main menu.</td>
</tr>
<tr>
<td>mhc_im.m</td>
<td>Function to build interconnection matrices.</td>
</tr>
<tr>
<td>mhc_imf.m</td>
<td>Script file to generate input matrix functions menu.</td>
</tr>
<tr>
<td>mhc_kgjd.m</td>
<td>$H_\infty$ all solution formulae derived by Glover and Doyle.</td>
</tr>
<tr>
<td>mhc_map.m</td>
<td>Function to construct minimal realization of the augmented plant for the basic structure.</td>
</tr>
<tr>
<td>mhc_meta.m</td>
<td>This function file generates a meta file using a filename defined by the user and writes the current graph to for late processing.</td>
</tr>
<tr>
<td>mhc_mss.m</td>
<td>Routine to calculate minimal state-space realization.</td>
</tr>
<tr>
<td>mhc_opt.m</td>
<td>Script file to generate options menu.</td>
</tr>
<tr>
<td>mhc_pcl.m</td>
<td>Function to plot closed-loop transfer function. If the design filters $V$ and $W$ are diagonal matrices the inverse design function is also plotted.</td>
</tr>
<tr>
<td>mhc_pzc.m</td>
<td>Function to check pole-zero cancellations in SISO transfer functions.</td>
</tr>
<tr>
<td>mhc_rbal.m</td>
<td>Returns the LQG or Riccati balanced state-space representation of stable and unstable systems.</td>
</tr>
<tr>
<td>mhc_rtf.m</td>
<td>Function to replace an element in a transfer function matrix.</td>
</tr>
<tr>
<td>mhc_sbp.m</td>
<td>Routine to show Bode plot of a SISO transfer function.</td>
</tr>
<tr>
<td>mhc_sem.m</td>
<td>Script file to generate system evaluation menu.</td>
</tr>
<tr>
<td>mhc_sim.m</td>
<td>Function to calculate and show time simulation.</td>
</tr>
<tr>
<td>mhc_slrc.m</td>
<td>$H_\infty$ loop-shifting formulae derived by Safonov, Limebeer and Chiang.</td>
</tr>
<tr>
<td>Function name</td>
<td>Description</td>
</tr>
<tr>
<td>------------------</td>
<td>----------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>mhc_ssr.m</td>
<td>Routine to show and define state-space representation of a system.</td>
</tr>
<tr>
<td>mhc_stin.m</td>
<td>Structure initialization function for $H_\infty$ control design.</td>
</tr>
<tr>
<td>mhc_stm.m</td>
<td>Script file to generate structure menu to define augmented plant.</td>
</tr>
<tr>
<td>mhc_tfss.m</td>
<td>MIMO transfer function matrix to state-space conversion.</td>
</tr>
</tbody>
</table>
In this appendix a list of variables in alphabetical order with a short description is given which are used as input/output arguments of the MHC functions described in Appendix C.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ac, Bc, Cc, Dc</td>
<td>State-space matrices of controller in continuous time domain.</td>
</tr>
<tr>
<td>Acl, Bcl, Ccl, Dcl</td>
<td>State-space matrices of closed-loop system without design functions.</td>
</tr>
<tr>
<td>Acon, Bcon, Ccon, Dcon</td>
<td>State-space matrices of final controller (discrete/continuous time, high/low order depending on design options).</td>
</tr>
<tr>
<td>Acor, Bcor, Ccor, Dcor</td>
<td>State-space matrices of original controller in continuous time domain (backup of Ac, Bc, Cc, Dc if controller reduction fails).</td>
</tr>
<tr>
<td>Ap1, Bp1, Cp1, Dp1</td>
<td>State-space representation of process block P1.</td>
</tr>
<tr>
<td>Variables</td>
<td>Description</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>Av, Bv, Cv, Dv</td>
<td>State-space representation of design block V.</td>
</tr>
<tr>
<td>Aw, Bw, Cw, Dw</td>
<td>State-space representation of design block W.</td>
</tr>
<tr>
<td>IM1, IM2</td>
<td>Interconnection matrices for basic MHC structure</td>
</tr>
<tr>
<td>alpha</td>
<td>Step size</td>
</tr>
<tr>
<td>dim</td>
<td>Dimension array for basic MHC structure.</td>
</tr>
</tbody>
</table>

**flag**

Information array about current status:

1) Mode; 1 = Continuous, 2 = Discrete
2) Configuration structure; 1 = Known, 0 = Unknown
3) Process block P1
4) Process block P2; 0 = Unknown, 1 = Transfer function
5) Shaping block V; 2 = State-space matrices
6) Weighing block W
7) Generating META files; 1 = Yes, 0 = No
8) Valid controller design; 1 = Yes, 0 = No
9) Valid controller reduction; 1 = Yes, 0 = No
10) $H_\infty$ type approach:
   1 = Safonov/Limebeer/Chiang loop-shifting formulae
   2 = Glover/Doyle all-solution formulae
   3 = Limebeer/Kasenally all-solution formulae
11) Type of Riccati solution approach; 1 = Eigen, 2 = Schur
12) Balancing augmented plant; 1 = Yes, 0 = No
13) Controller reduction method:
   1 = Minimal realization
   2 = Optimal Hankel method,
   3 = Schur reduction method
   4 = Relative Schur reduction
14) Type of controller reduction for method 2, 3 & 4:
   1 = Variable order & Fixed error bound
   2 = Fixed order & Variable error bound
<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
</tr>
</thead>
</table>
| freq      | Array defining frequency information:  
            1) Lower bound  
            2) Upper bound  
            3) Number of frequency points |
| gamtol    | Tolerance margin for $\gamma$ minimization |
| num_P1, den_P1 | Numerator/denominator transfer matrix of process block P1 |
| num_P2, den_P2 | Numerator/denominator transfer matrix of process block P2 |
| num_V, den_V  | Numerator/denominator transfer matrix of design block V |
| num_W, den_W  | Numerator/denominator transfer matrix of design block W |
| time      | Time information: End of time interval (continuous mode) or Sample time (discrete mode) |
| tol       | Tolerance margin for minimal state-space realization |
References

Bouwels, J.P.H.M.
Ontwikkeling en beproeving van verschillende regelaars (LQG, $H_\infty$) voor de horizontale afregeling van een drijvend platform met een draaiende kraan als verstoring.
Measurement and Control Section, Faculty of Electrical Engineering, Eindhoven University of Technology (The Netherlands), 1991.
M. Sc. graduation report.

Bruinsma, N.A. and M. Steinbuch
A fast algorithm to compute the $H_\infty$-norm of a transfer function matrix.

Chiang, R.C. and M.G. Safonov
Robust control toolbox : User's guide.

Damen, A.A.H. and H.M. Falkus, J.P.H.M. Bouwels
Modeling and control of a floating platform.
References

Doyle J. and B. Francis, A. Tannenbaum
Feedback control theory.

Doyle J.C. and K. Glover, P.P. Khargonekar, B.A. Francis
State-space solutions to standard $H_2$ and $H_\infty$ control problems.

Falkus, H.M. and A.A.H. Damen, J. Bouwels
General MIMO $H_\infty$ control design framework.

Francis, B.A.
A course in $H_\infty$ control theory.
Lecture notes in control and information sciences, Vol. 88.

Glover, K. and J.C. Doyle
State-space formulae for all stabilizing controllers that satisfy an $H_\infty$-norm bound and relations to risk sensitivity.

Grace, A. and A.J. Laub, J.N. Little, C. Thompson
Control system toolbox : User's guide.

Iglesias, P.A. and D. Mustafa.
State-space solution of the discrete-time minimum entropy control problem via separation.

Kailath, T.
Linear systems.
Klompstra M. and T. van den Boom, A. Damen
A comparison of classical and modern controller design: A case study.
Eindhoven: Faculty of Electrical Engineering,
Eindhoven University of Technology (The Netherlands), 1990.
EUT Report 90-E-244.

Limebeer, D.J.N. and E.M. Kasenally, I. Jaimoukha, M.G. Safonov
All solutions to the four block general distance problem.
In: Proc. 27th IEEE Conf. on Decision and Control, Austin, Texas, December 7-9, 1992.

Maciejowski, J.M.
Multivariable feedback design.

McFarlane, D.C. and K. Glover
Robust controller design using normalized coprime plant descriptions.

Morari, M. and E. Zafiriou
Robust process control.

Munro, N. and C. Eng, R.S. McLeod
Minimal realization of transfer function matrices using the system matrix.

Safonov, M.G. and D.J.N. Limebeer, R.Y. Chiang
Simplifying the $H_{\infty}$ theory via loop-shifting, matrix-pencil and descriptor concepts.

Stoorvogel, A.A. and J.H.A. Ludlage
The discrete time minimum entropy $H_{\infty}$ control problem.
Faculty of Mathematics and Computing Science,
Eindhoven University of Technology (The Netherlands), 1993.
Internal report.
Dooren, P.M. van
The generalized eigenstructure problem in linear system theory.

Weiland, S.
A behavioral approach to balanced representations of dynamical systems.
Eindhoven : Faculty of Electrical Engineering,
Eindhoven University of Technology (The Netherlands), 1993.
EUT Report 93-E-277

Weinman, W.
Uncertain models and robust control.
(256) Backx, A.C.P.M. and A.A.H. Damen
IDENTIFICATION FOR THE CONTROL OF MIMO INDUSTRIAL PROCESSES.

(257) Maagt, P.J.J. de and H.G. ter Morsche, J.L.M. van den Broek
A SPATIAL RECONSTRUCTION TECHNIQUE APPLICABLE TO MICROWAVE RADIOMETRY

(258) Vleenhouwers, J.M.
DERIVATION OF A MODEL OF THE EXCITER OF A BRUSHLESS SYNCHRONOUS MACHINE.

(259) Orlov, V.B.
DEFECT MOTION AS THE ORIGIN OF THE 1/F CONDUCTANCE NOISE IN SOLIDS.

(260) Rooijackers, J.E.
ALGORITHMS FOR SPEECH CODING SYSTEMS BASED ON LINEAR PREDICTION.

(261) Boom, T.J.J. van den and A.A.H. Damen, Martin Klimstra
IDENTIFICATION FOR ROBUST CONTROL USING AN H-infinity NORM.

(262) Groten, M. and W. van Etten
LASER LINewidth MEASUREMENT IN THE PRESENCE OF RIN AND USING THE RECIRCULATING SELF HETERODYNE METHOD.

(263) Spoelers, A.B.
RIGOROUS ANALYSIS OF THICK MICROSTRIP ANTENNAS AND WIRE ANTENNAS EMBEDDED IN A SUBSTRATE.

(264) Freriks, L.W. and P.J.M. Cluitmans, M.J. van Gilis
THE ADAPTIVE RESONANCE THEORY NETWORK: (Clustering-) behaviour in relation with brainstem auditory evoked potential patterns.

MANUFACTURING AND CHARACTERIZATION OF GaAs/AlGaAs MULTIPLE QUANTUMWELL RIDGE WAVEGUIDE LASERS

(266) Cluitmans, L.J.M.
USING GENETIC ALGORITHMS FOR SCHEDULING DATA FLOW GRAPHS

(267) Jozwik, L. and A.P.H. van Dijk
A METHOD FOR GENERAL SIMULTANEOUS FULL DECOMPOSITION OF SEQUENTIAL MACHINES: Algorithms and implementation.

(268) Boom, H. van den and W. van Etten, W.H.C. de Krom, P. van Bennekum, F. Haliksen, L. Wijerssen, P. de Leijer
AN OPTICAL ASK AND FSK PHASE DIVERSITY TRANSMISSION SYSTEM.
(159) Putten, P.H.A. van der
MULTIDISCIPLINARY SPECIFICEREN EN OMTWERPEN VAN MICROELEKTRONICA IN PRODUKTEN (in Dutch).

(170) Bloks, R.H.J.
PROGAIL. A language for the definition of protocol grammars.

(171) Bloks, R.H.J.
CODE GENERATION FOR THE ATTRIBUTE EVALUATOR OF THE PROTOCOL ENGINE GRAMMAR PROCESSOR UNIT.

(172) Van Kepping and E.M. van Veldhuzen
FLUE GAS CLEANING BY PULSE CORONA STREAMER.

(173) Smolders, A.J.
FINITE STACKED MICROSTRIP ARRAYS WITH THICK SUBSTRATES.

(174) Bolien, M.H.J. and M.A. van Ruten
ON INSULAR POWER SYSTEMS: Drawing up an inventory of phenomena and research possibilities.

(175) Geurzen, A.J. van
ELECTROMAGNETIC COMPATIBILITY: Part 5. installation and mitigation guidelines. section 3.
cabling and wiring.

(176) Bolien, M.H.J.
LITERATURE SEARCH FOR RELIABILITY DATA OF COMPONENTS IN ELECTRIC DISTRIBUTION NETWORKS.

(177) Welland, Siep
A BEHAVIORAL APPROACH TO BALANCED REPRESENTATIONS OF DYNAMICAL SYSTEMS.

(178) Gorskirov, Yu. A. and V.I. Vladimirov
LINE REVERSAL GAS FLOW TEMPERATURE MEASUREMENTS. Evaluations of the optical arrangements for the instrument.

(179) Creveldtma, Y.T.M. and W.R. Rutgers, E.M. van Veldhuzen
IN-SITU INVESTIGATION OF PULSED CORONA DISCHARGE.

(180) Li, H.Q. and R.P.P. Smiers
GAP-LENGTH DEPENDENT PHENOMENA OF HIGH-FREQUENCY VACUUM ARCS

(181) Li, Channing and Jochen A.G. Jess
ON THE DEVELOPMENT OF A FAST AND ACCURATE BRIDGING FAULT SIMULATOR.

(182) Falkus, H.M. and A.H. Damen
MULTIVARIABLE H-INFINITY CONTROL DESIGN TOOLBOX User manual