Depolarization by rain - some related thermal emission considerations

by

A. Mawira and J. Dijk
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T.H. Report 75-E-61
September 1975
I.S.B.N. 9.0.6.1440610
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Summary: This report deals with the electromagnetic properties of a medium containing axisymmetric raindrops. In section 2 are outlined the basic assumptions underlying the investigation. Section 3 concerns the propagation of a monochromatic plane wave through this medium: a corresponding differential equation is derived. Section 4 deals with the aspect of thermal emission closely connected to the problem. The Stokes spectral vector representation of the thermal emission field is presented. The differential equations derived in section 3, as well as some thermodynamic considerations are used to derive a transfer equation for this Stokes spectral vector. The solution of this equation then yields expressions for the generalized anisotropic effective temperature vector. Finally, in section 5, the relation between various quantities referring to monochromatic signals, such as the cross polarization parameters and the thermal emission magnitudes, are discussed.
Acknowledgement

The authors wish to thank Professor Dr. H. Bremmer for his valuable help in the preparation of this report.
Microwaves with wavelengths in the centimetre and millimetre ranges are strongly affected by the presence of rain. The individual raindrops absorb and scatter the incident wave, the absorption being dominant at frequencies below 15 GHz [1]. The combined effects of these mechanisms cause, among other things, the attenuation of the travelling wave. The non-spherical shape of raindrops with axial symmetry, combined with a non-random distribution of the orientations of their axes, generally leads to a depolarization of the propagating wave. In general, therefore, serious performance degradations, due to rain, may be expected for microwave radio systems. In particular, frequency reuse systems in which information is carried on orthogonal polarizations at the same frequency, may be hazardously affected by the depolarizing effect of rain.

The relation between rain and attenuation has been the subject of thorough theoretical and experimental investigations [1]-[7]. The application of frequency reuse systems to satellite-to-ground telecommunication has stimulated further relevant investigations on the phenomenon of rain depolarization [7]-[14]. In this report we shall deal with very general theoretical investigations concerning the propagation of electromagnetic waves through rain, considering also the associated thermal emission.

Finally, relations between thermal emission quantities and the parameters, connected with the depolarization phenomenon are discussed.
2. Physical model

The basic assumptions underlying this report are the following:

A. Composition of the rain medium

The rain medium should be composed of axisymmetric raindrops, i.e., each raindrop has an axis of rotational symmetry fixed by the unit vector $\vec{n}(\nu, \phi)$ along it (see Fig. 1). The raindrops may be of different types (for instance, spheres, oblate spheroids, prolate spheroids, etc.) and of different sizes, an effective radius $r$ being a measure of the latter. In the orthogonal $xyz$ system the $z$-axis is taken along the propagation direction of the incident wave, while the $y$-axis, perpendicular to it, is usually taken in a horizontal direction. The orientation of each raindrop is then fixed by the angles $\phi$ and $\theta$ (called the canting and incident angle respectively) which refer to this system according to Fig. 2. The composition of the rain medium is characterized by the distribution density $f_i(z, r, \theta, \phi)$ for raindrops of a certain type, labelled $i$, such that

$$dN_i(z) = N_i(z) f_i(z, r, \theta, \phi) \, drd\theta d\phi$$  \hspace{1cm} (2.1.1)

represents the number of raindrops of this type per $m^3$ that have their effective radii $r$, their canting angle $\phi$ and their angles of incidence $\theta$ situated in a special infinitesimal interval $drd\theta d\phi$.

B. Statistically independent single scattering

This assumption implies that the relevant electromagnetic properties of the raindrops are sufficiently described by the forward-scatter complex amplitude functions for each of them. For an axisymmetric raindrop these functions, viz. $S_{//}(i)(r, \theta, \omega)$ and $S_{\perp}(i)(r, \theta, \omega)$ for a special raindrop type labelled $i$, occur in the relation (see Fig. 3)

$$\begin{bmatrix} E_{//}^{\text{scat}} \\ E_{\perp}^{\text{scat}} \end{bmatrix} = \frac{k_o^2 \epsilon}{2\pi R} \begin{bmatrix} S_{//}(i) & 0 \\ 0 & S_{\perp}(i) \end{bmatrix} \begin{bmatrix} E_{//}^{\text{inc}} \\ E_{\perp}^{\text{inc}} \end{bmatrix}$$  \hspace{1cm} (2.1.2)

where $// \text{ and } \perp$ refer to the component of the electric field.
in the "plane of incidence" (containing the propagation direction and the symmetry axis), and the component perpendicular to it respectively; both these components are parallel to the $xy$ plane in view of the $TE$ character of the wave. The relation (2.1.2) determines the forward-scattered field at a distance $R$ versus the incident field $[1] ; \omega$ represents the angular frequency of the harmonic time dependence.

The diagonal character of the matrix in Eq. 2.1.2 is a consequence of the assumed symmetry property of the raindrops.

The relation between $x$ and $y$ components of the incident and the scattered field may now be obtained for arbitrary orientation of the raindrop by applying a suitable rotational transformation to 2.1.2 (Fig. 4). This results in

$$
\begin{bmatrix}
E_{\text{scat}}^x \\
E_{\text{scat}}^y
\end{bmatrix} = \frac{k^2_0 e^{-jkr}}{2\pi R} \begin{bmatrix}
\cos\phi & -\sin\phi \\
\sin\phi & \cos\phi
\end{bmatrix} \begin{bmatrix}
S_{/\parallel} (i) & 0 \\
0 & S_{/\perp} (i)
\end{bmatrix} \begin{bmatrix}
\cos\phi & \sin\phi \\
-\sin\phi & \cos\phi
\end{bmatrix} \begin{bmatrix}
E_{\text{inc}}^x \\
E_{\text{inc}}^y
\end{bmatrix},
$$

(2.1.3)

or, worked out, into the energy

$$
\begin{bmatrix}
E_{\text{scat}}^x \\
E_{\text{scat}}^y
\end{bmatrix} = \frac{k^2_0 e^{-jkr}}{2\pi R} \begin{bmatrix}
S_{/\parallel} (i) \cos^2 \phi + S_{/\perp} (i) \sin^2 \phi \\
(S_{/\parallel} (i) - S_{/\perp} (i)) \frac{\sin 2\phi}{2}
\end{bmatrix} \begin{bmatrix}
E_{\text{inc}}^x \\
E_{\text{inc}}^y
\end{bmatrix}.
$$

(2.1.4)

C. Forward scattering approximation

The single scattering albedo $\omega_0$ defined as the ratio of the scattered energy lost through both scattering and absorption (cf [2], [15]), is taken to be zero.

D. Local thermodynamic equilibrium of the rain medium.

The thermodynamic properties of an infinitesimal part of the medium are fixed by its temperature $T$ [16].
3. Propagation of a plane wave through the rain medium

According to our model the monochromatic TE wave travels through the medium in the $z$ direction. Its electric vector $\mathbf{E}(z,t)$ has the form

$$\mathbf{E}(z,t) = \{E_x(z) \mathbf{U}_x + E_y(z) \mathbf{U}_y\} e^{j\omega t},$$  

(3)

where $E_x$ and $E_y$ are complex functions of $z$, $\mathbf{U}_x$ and $\mathbf{U}_y$ being the unit vectors in the $x$ and $y$ directions respectively.

In the following section we shall derive a differential equation which governs the propagation of this electric field.

3.1. Derivation of the differential equation

Let us consider a space filled with raindrop particles. These particles may be labelled by the integer $l = 1, 2, 3, \ldots$.

The medium can be described by a properly chosen relative permittivity function $\varepsilon_r(x)$, when assuming the relative permeability $\mu_r$ to be unity. The Maxwell equations then lead to the following equation for the electric field strength $\mathbf{E}$:

$$\Delta \mathbf{E} + k_o^2 \varepsilon_r \mathbf{E} + \nabla (\frac{-\varepsilon_r}{\varepsilon_r^2}) \cdot \mathbf{E} = 0.$$  

(3.1.1)

By defining $n^2$ to be the operator

$$n^2 = \varepsilon_r + \mu \frac{-\varepsilon_r}{\varepsilon_r^2},$$  

(3.1.2)

we may express Eq. 3.1.1 as the Helmholtz equation

$$\Delta \mathbf{E} + k_o^2 \varepsilon_r \mathbf{E} = 0.$$  

(3.1.3)

In order to show the disturbing effect of the inhomogeneity of the medium this equation may also be represented by

$$\Delta \mathbf{E} + k_o^2 \varepsilon_r \mathbf{E} = -k_o^2 (n^2 - 1) \mathbf{E};$$  

(3.1.4)

as a matter of fact, $n^2 - 1$ only differs from zero inside the raindrops.

The solution to this equation is formally determined by the following integral equation:
$E(P) = E^0(P) + \frac{k_0^2}{4\pi} \int d\tau Q \frac{-jk_0 PQ}{PQ} \{ n^2 - 1 \} Q E(Q); \quad (3.1.5)$

$E^0$ is the free space solution that corresponds to the system of primary currents, $P$ is the point of observation, $Q$ the integration point and $PQ$ the distance from $P$ to $Q$.

The second term on the right hand side of Eq. 3.1.5 may be interpreted as the secondary field $E^S$, i.e.

$$E^S(P) = \frac{k_0^2}{4\pi} \int d\tau Q \frac{-jk_0 PQ}{PQ} \{ n^2 - 1 \} Q E(Q). \quad (3.1.6)$$

In other words $E$ is the sum

$$E = E^0 + E^S \quad (3.1.7)$$

of the primary field $E^0$ and the secondary field $E^S$.

One method of solving Eq. 3.1.5 is to proceed with the following Neumann-Liouville expansion (cf[17]),

$$E(P) = \sum_{m=0}^{\infty} (m)E(P), \quad (3.1.8)$$

where $(o)E = E(o)$, while the next terms can be deduced with the aid of the following recurrence relation

$$(m)E(P) = \frac{k_0^2}{4\pi} \int d\tau Q \frac{-jk_0 PQ}{PQ} \{ n^2 - 1 \} Q (m-1)E(Q). (m+1) \quad (3.1.9)$$

The term $(m)E$ may be interpreted as the contribution associated with successive $m$ scatterings. This includes the scattering processes inside each drop as well.

We shall now derive an alternative representation of $E$. To do so, we split the volume integration in $E^S$ into particle contributions $E^S_z$ in which the integration only extends over the volume $V_z$ of an individual raindrop labelled $z$. Hence

$$E^S(P) = \sum_{i=1}^{\infty} E^S_z(P) \quad (3.1.10)$$

where
By introducing a proper linear operator $T_l$, we may represent Eq. 3.1.11 by the following concise relation:

$$\tilde{E}^S_l (\nu) = T_l \{ \tilde{E} \} \quad (\nu = 1, 2, 3, \ldots)$$  \hspace{1cm} (3.1.12)

Eq. 3.1.12 constitutes an integral equation, since $\tilde{E}^S$ contains the term $\tilde{E}^S_l$ through Eqs. 3.1.7 and 3.1.10. The new equation 3.1.12 can be solved by an expression analogous to the Neumann-Liouville expression of Eq. 3.1.6, so as to have

$$\tilde{E}^S_l = \sum_{\nu=1}^{\infty} \tilde{E}^S_{l, \nu} \quad (l = 1, 2, 3, \ldots)$$  \hspace{1cm} (3.1.13)

Substituting this expression in Eq. 3.1.12 while applying the Eqs. 3.1.7 and 3.1.10, we obtain the equation

$$\sum_{\nu=1}^{\infty} \tilde{E}^S_{l, \nu} = T_l \{ \tilde{E}^O + \sum_{m=1}^{\infty} \sum_{\nu=1}^{\infty} \tilde{E}^S_{m, \nu} \} \quad (l = 1, 2, 3, \ldots)$$  \hspace{1cm} (3.1.14)

A solution of this equation is obtained, by solving the set of equations

$$\tilde{E}^S_{l, \nu} = T_l \{ \tilde{E}^O + \sum_{m=1}^{\infty} \sum_{\nu=1}^{\infty} \tilde{E}^S_{m, \nu} \} \quad (l = 1, 2, 3, \ldots)$$  \hspace{1cm} (3.1.15)

for all $l$, and also the set

$$\tilde{E}^S_{l, \nu} = T_l \{ \sum_{m \neq l}^{\infty} \tilde{E}^S_{m, \nu-1} + \tilde{E}^S_{l, \nu} \} \quad (l = 1, 2, 3, \ldots)$$  \hspace{1cm} (3.1.16)

for all $l$ and $\nu$. In fact, a summation of Eq. 3.1.16 over $\nu = 2, 3, 4, \ldots$, while applying Eq. 3.1.15 leads after elementary reductions to Eq. 3.1.12.

Comparing Eqs. 3.1.15 with 3.1.12, we see that $\tilde{E}^S_{l, \nu}$ is the secondary field that would be obtained if all drops other than $l$ were removed, provided of course that the primary currents are not changed. $\tilde{E}^S_{l, \nu}$ may be given an analogous interpretation, with the difference that the primary field is then given by the form $\sum \tilde{E}^S_{m, \nu-1}$ instead of $\tilde{E}^O$. Thus, finally $\tilde{E}^S_{l, \nu}$ represents the contribution resulting from $\nu$ successive raindrop scatterings, the last of these taking place in the $l$th raindrop (the number of scatterings inside the raindrops are not counted, as was done in the first representation).
Our numerical applications only need to take into account the first term of Eq. 3.1.13, so as to obtain the following approximation of $\bar{E}$

$$\bar{E} = \bar{E}^0 + \sum_{l=1}^{\infty} \bar{E}^S_{l,1}$$

(3.1.17)

This approximation is known as the Born approximation. Eq. 3.1.15 shows that $\bar{E}^S_{l,1}$ depends linearly on $\bar{E}^0$, i.e.

$$\bar{E}^S_{l,1} = S_{l}(\bar{E}^0),$$

(3.1.18)

$S_{l} = \{1-T_{l}\}^{-1} T_{l}$ being a linear operator that solves Eq. 3.1.15. In the case of a plane wave $\bar{E}^0$, while the distance from the observation point towards the drop is large (admitting asymptotic approximations) we find the following explicit form of Eq. 3.1.18

$$\bar{E}^S_{l,1}(P) = k_o^2 \bar{S}_{l}(P_{Q_{l}}) \left[ \frac{e^{-j k_o P_{Q_{l}}}}{P_{Q_{l}}} \right] \bar{E}^0(Q_{l})$$

(3.1.19)

$$\bar{S}_{l}$$ is the scattering matrix [33] for the particle $l$, $Q_{l}$ is the position of the (properly defined) centre of the particle, and $P_{Q_{l}}$ is the unit vector along the direction from $Q_{l}$ to $P$.

The secondary field at $P$ can here be approximated as follows

$$\bar{E}^S(P) = \frac{k_o^2}{2\pi} \sum_{l=1}^{\infty} \left[ \frac{e^{-j k_o P_{Q_{l}}}}{P_{Q_{l}}} \bar{S}_{l}(P_{Q_{l}}) \right] \bar{E}^0(Q_{l})$$

(3.1.20)

The summation over discrete elements may be replaced by a smooth integration, provided that a proper raindrop density $N$ can be defined; hence:

$$\bar{E}^S(P) = \frac{k_o^2}{2\pi} \int dQ \frac{e^{-j k_o P_{Q}}}{P_{Q}} \ N(Q) \ \bar{S}(Q) \ \bar{E}^0(Q)$$

(3.1.21)

We shall now investigate the above Born approximation for the "plane-wave problem". In this problem we have the primary plane wave

$$\bar{E}^0(z) = \bar{E}^0(o) e^{-j k_o z},$$

(3.1.22)

arriving from the half space $z < z_o$ and entering at $z = z_o$ into the scattering half space $z > z_o$ containing the raindrop particles. Eq.3.1.21 has now the specific form
\[ E^B(x, y, z) = \frac{k^2}{2\pi} \iiint d\xi d\eta d\zeta \left\{ -j \frac{1}{\sqrt{(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2}} \right\} \]

(3.1.23)

The variables \( \xi \) and \( \eta \) may be eliminated by the following procedure. This derivation is according to one given by Bremmer (cf[17][18]). First we introduce the operator relation

\[ N(\xi, \eta, \zeta) \ S(\xi, \eta, \zeta) = e^{(\xi-x) \frac{\partial}{\partial x} + (\eta-y) \frac{\partial}{\partial y} \left\{ N(x, y, \zeta) \ S(x, y, \zeta) \right\}} \]

(3.1.24)

and introducing the new variables \( r \) and \( \phi \) according to

\[ \xi - x = r \cos \phi, \]
\[ \eta - y = r \sin \phi. \]

Integration of the \( \phi \) variables then delivers the relation

\[ E^B(x, y, z) = \frac{k^2}{2\pi} \int_0^\infty dr \int_0^\infty r \left\{ -j k_0 \sqrt{r^2 + (z-\zeta)^2} + \zeta \right\} e^{-j k_0 \sqrt{r^2 + (z-\zeta)^2} \ S(x, y, z)} \]

The integration in the \( r \) variables can be reduced to the well-known Sommerfeld integral (cf[34]) and results in

\[ E^B(x, y, z) = -j k_0 \int d\zeta e^{-j k_0 \sqrt{k_0^2 + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_0^2 \zeta}} \]

(3.1.25)

If the effects of back scattering are negligible, then the integration in Eq. 3.1.25 can be restricted to regions \( \zeta < z \). This corresponds to what is called the forward scatter approximation.

For high frequencies and a not very high degree of inhomogeneity, i.e.
we may neglect the effect of the operator $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. This approximation is known as the geometrical-optical approximation. Eq. 3.1.25 is therefore reduced to the following simple form:

$$\tilde{E}(x,y,z) = -j\kappa_0 e^{-j\kappa_0 z} \int_0^z d\zeta N(x,y,\zeta) \tilde{S}(x,y,\zeta) \tilde{E}^0(o),$$

(3.1.27)

where $z$ is taken to be larger than $z_o$. The geometrical-optical nature of the above approximation is illustrated by the independence of Eq. 3.1.27 of the medium properties outside of the ray trajectory.

The total field is now given by

$$\tilde{E}(x,y,z) = e^{-j\kappa_0 z} \{1-jk_0 \int_0^z d\zeta N(x,y,\zeta) \tilde{S}(x,y,\zeta) \} \tilde{E}^0(o).$$

(3.1.28)

By differentiating this equation to $z$, we then obtain the following differential equation:

$$\frac{\partial E}{\partial z}(x,y,z) = -j\kappa_0 \{1+\hat{N}(x,y,z) \tilde{S}(x,y,z)\}$$

(3.1.29)

where $\hat{N}$ is the matrix operator given by

$$\hat{N}(x,y,z) = jk_0 \{1+\hat{N}(x,y,z) \tilde{S}(x,y,z)\}$$

(3.1.30)

The 2 by 2 transmission matrix $\Gamma$ has the following representation with respect to the bases $\overline{U}_x, \overline{U}_y$

$$\Gamma = \begin{bmatrix} \Gamma_{xx} & \Gamma_{xy} \\ \Gamma_{yx} & \Gamma_{yy} \end{bmatrix},$$

(3.1.31)

The elements of which may be obtained from the elements on the right hand side of Eq.3.1.28. Here we have a simple form of density times the scattering matrix, in fact we have drops of different types and sizes with different orientation, so that a weighted summation must be carried out. This then leads to the following forms for the elements of matrix $\Gamma$:

$$\Gamma_{xy}(x,\omega) = jk_0 \left\{1+ \sum \int_0^z d\zeta \int_0^z \int_0^\infty f(z,r_2,\theta_2,\phi) \left[ S_{\parallel}(\theta_2,\phi) \right] \cos^2 \phi + \left[ S_{\perp}(\theta_2,\phi) \right] \sin^2 \phi \right\}$$

(3.1.32)
In general, $\Gamma_{xy}$ differs from zero. However, $\Pi$ has a diagonal representation with respect to its own equations as bases; the eigenvalues then constitute the diagonal elements. The corresponding normalized eigenvectors $\vec{M}_A$ and $\vec{M}_B$ and their eigenvalues $\Gamma_A$, $\Gamma_B$, can be derived by solving the equations

$$\vec{M}_A = \Gamma_A \vec{M}_A, \quad \vec{M}_A^* \vec{M}_A = 1,$$

$$\vec{M}_B = \Gamma_B \vec{M}_B, \quad \vec{M}_B^* \vec{M}_B = 1.$$  

(3.1.33)

The solution with respect to the bases $\vec{U}_x$ and $\vec{U}_y$ [see section A.1] is represented by

$$\begin{bmatrix} \vec{M}_A \\ \vec{M}_B \end{bmatrix} = D^{-\frac{1}{2}} \begin{bmatrix} \cos \phi_o & \sin \phi_o \\ -\sin \phi_o & \cos \phi_o \end{bmatrix} \begin{bmatrix} \vec{U}_x \\ \vec{U}_y \end{bmatrix},$$

(3.1.34)

$$\Gamma_A(z,\omega) = jk_0 \left\{ \sum_i N_i(z) \int \int \int f_i(z,r,\theta,\phi) \left[ S_{ii}(r,\theta,\omega) - S_{ii}(r,\theta,\omega) \right] \sin \theta \frac{d\theta d\phi}{2} \right\} \cdot$$

$$+ \sum_i N_i(z) \int \int \int f_i(z,r,\theta,\phi) \left[ S_{ii}(r,\theta,\omega) - S_{ii}(r,\theta,\omega) \right] \sin \theta \frac{d\theta d\phi}{2}$$

(3.1.35)

where the "effective average complex canting angle" $\phi_o$ is given by

$$\tan[2\phi_o] = \frac{\sum_i N_i(z) \int \int \int f_i(z,r,\theta,\phi) \left[ S_{ii}(r,\theta,\omega) - S_{ii}(r,\theta,\omega) \right]}{\sum_i N_i(z) \int \int \int f_i(z,r,\theta,\phi) \left[ S_{ii}(r,\theta,\omega) - S_{ii}(r,\theta,\omega) \right] \cdot \sin(\phi) \frac{d\theta d\phi}{2} \cdot \cos(2\phi) \frac{d\theta d\phi}{2}}.$$  

(3.1.36)
Eq. 3.1.36 shows that, if $\phi_0$ is independent of $z$, the medium admits two non-interfering channels; in fact, the elliptically polarized electric field components $E_A \overline{M}_A$ and $E_B \overline{M}_B$, directed along the complex directions $\overline{M}_A$ and $\overline{M}_B$, then propagate through the medium without interfering with each other. This is verified by solving Eq. 3.1.27 in a representation referring to $\overline{M}_A$ and $\overline{M}_B$. We obtain:

\begin{align}
E_A(z) &= e^{-\int_{z_0}^{z} \Gamma_A(z',\omega) \, dz'} E_A(z_0), \\
E_B(z) &= e^{-\int_{z_0}^{z} \Gamma_B(z',\omega) \, dz'} E_B(z_0).
\end{align}

(3.1.37) (3.1.38)

These expressions relate the field strength incident at $z_0$ to the more remote field strength at $z$, and shows the non-interference of $E_A$ and $E_B$ mentioned. Further, the two elliptical polarizations reduce to two orthogonal linear polarizations in the case of a real $\phi_0$.

As an illustrative example we shall further consider the special case of a medium composed of raindrops of a single type only, for which, moreover, the relation

\begin{equation}
S_{ij}^{(i)}(r,\theta,\omega) = S_{ji}^{(i)}(r,\theta,\omega) = \Delta S^{(i)}(r,\omega) \sin^2 \theta,
\end{equation}

(3.1.39)

holds, in which

\begin{equation}
S_{ij}^{(i)}(r,\omega) = S_{ji}^{(i)}(r,\omega) = S^{(i)}(r,\omega).
\end{equation}

(3.1.40)

This situation corresponds to dipole approximations of the raindrop scattering mechanisms (cf[15]). In this approximation each raindrop is characterized by three planes of symmetry with the property that an electric field vector parallel to any of the three associated principal axes induces dipole moments proportional to this field vector. The scattered field can then be derived from the values of these dipole moments (cf. section A.2). Eq.3.1.36 now becomes:

\begin{equation}
\tan(2\phi_0) = \frac{\iint f(r,\theta,\phi) \Delta S(r,\omega) \sin(2\phi) \sin^2 \theta \, dr \, d\theta \, d\phi}{\iint f(r,\theta,\phi) \Delta S(r,\omega) \cos(2\phi) \sin^2 \theta \, dr \, d\theta \, d\phi},
\end{equation}

(3.1.41)

Moreover, if the distribution of the orientations of the raindrops
happens to be independent of their sizes, i.e. if \( f(z, r, \theta, \phi) = g(z, r)h(z, \theta, \phi) \) then \( \tan(2\phi_o) \) will become real according to the relation

\[
\tan(2\phi_o) = \frac{\iint h(z, \theta, \phi) \sin(2\phi) \sin^2 \theta d\phi d\theta}{\iint h(z, \theta, \phi) \cos(2\phi) \sin^2 \theta d\phi d\theta}
\]

The two eigenvectors \( \vec{M}_A \) and \( \vec{M}_B \) then represent two orthogonal linear polarizations; in the case of a homogenous medium these polarizations do not interfere throughout. From this special example it may be inferred that the possible dependence of the raindrop orientations on their sizes, or possibly on differences between their shapes, may lead to a complex value of \( \phi_o \).
3.2. Solution of the differential equation; the cross-polarization parameter

In general the field strength incident at $z_0$ and that observed in the medium at $z$ are connected by a linear transform which can be defined by the relation

$$\bar{E}[z] = \bar{F}[z, z_0; \omega] \bar{E}[z_0]$$  \hspace{1cm} (3.2.1)

the $2\times 2$ evolution matrix operator $\bar{F}$ is to be determined by solving Eq. 3.1.29. To do so we first represent $\bar{F}$ as follows:

$$\bar{F} = \Gamma_0 - \eta \delta \Gamma_0 \bar{W}, \hspace{1cm} (3.2.2)$$

where $\Gamma_0$ and $\delta \Gamma_0$ are defined by the expressions

$$\Gamma_0 = \frac{1}{2} \{ \Gamma_A + \Gamma_B \}, \hspace{0.5cm} \delta \Gamma_0 = \Gamma_B - \Gamma_A \hspace{1cm} (3.2.3)$$

$\eta$ is a dummy variable equalling unity; while $\bar{W}$ is given by the following representation referring to the $xy$ coordinate system (see section A3)

$$\bar{W} = \begin{bmatrix} \cos(2\phi_0) & \sin(2\phi_0) \\ \sin(2\phi_0) & -\cos(2\phi_0) \end{bmatrix} \hspace{1cm} (3.2.4)$$

Next, $\bar{E}(z)$ is expanded in a power series of $\eta$, according to

$$\bar{E}(z) = \sum_{n=0}^{\infty} \eta^n \bar{E}^{(n)}(z) \hspace{1cm} (3.2.5)$$

Substituting this equation, together with Eq. 3.2.2, in Eq. 3.1.29, and then assembling the terms containing special powers of $\eta$, we may obtain the following set of equations

$$\frac{d\bar{E}^{(0)}(z)}{dz} = -\Gamma_0 \bar{E}^{(0)}(z) \hspace{1cm} (3.2.6)$$

$$\frac{d\bar{E}^{(n)}(z)}{dz} = -\Gamma_0 \bar{E}^{(n)}(z) + \frac{1}{2} \delta \Gamma_0 \bar{W} \bar{E}^{(n-1)}(z), \hspace{0.5cm} (n \geq 1). \hspace{1cm} (3.2.7)$$

We introduce the boundary conditions

$$\bar{E}(z_0) = \bar{E}^{(0)}(z_0) \hspace{1cm} (3.2.8)$$
\[ E^{(n)}(z_o) = 0 \quad , \quad (n \geq 1) \quad . \quad (3.2.9) \]

The solution of Eq. 3.2.6 is obtained at once

\[ E^{(0)}(z) = e^{-\int_{z_o}^{z} \Gamma_o(z')dz'} E(z_o) \quad , \quad (3.2.10) \]

while the higher-order terms \( E^{(n)} \) are given by

\[ E^{(n)}(z) = \frac{1}{n} \int_{z_o}^{z} \Gamma_o(z')dz' \]

\[ \times \frac{1}{n-1} \int_{z_o}^{z} \Gamma_o(z')dz' \quad , \quad (n \geq 1) \quad (3.2.11) \]

where the factors \( \frac{1}{n} \) are to be determined from the recurrence relation

\[ \frac{1}{n} \int_{z_o}^{z} \Gamma_o(z')dz' \times \frac{1}{n-1} \int_{z_o}^{z} \Gamma_o(z')dz' \quad \]

provided that we define \( \mathbb{1} \) to be the unit operator. This last equation can easily be verified by substituting Eq. 3.2.11 in Eq. 3.2.8 while remembering Eq. 3.2.9.

We finally arrive at the following expression:

\[ E(z) = \frac{1}{n} \int_{z_o}^{z} \Gamma_o(z')dz' \]

where

\[ \frac{1}{n} \int_{z_o}^{z} \Gamma_o(z')dz' \]

\[ \times \frac{1}{n-1} \int_{z_o}^{z} \Gamma_o(z')dz' \quad \]

The effect of rain on some radio systems may be analyzed by using Eqs. 3.2.13 and 3.2.14. For systems applying orthogonal polarized channels, the so-called cross polarization parameters are often used as measures of performance. These parameters indicate the degree of depolarization, and are defined by

\[ X_{PLX}(z_o, z) = \left| \frac{F_{yy}(z, z_o)}{F_{yy}(z, z_o)} \right|^2 \quad (3.2.15) \]

for transmitting \( x \) polarization, and by

\[ X_{PLY}(z_o, z) = \left| \frac{F_{yy}(z, z_o)}{F_{yy}(z, z_o)} \right|^2 \quad (3.2.16) \]
for transmitting y polarization.

In general the elements of $F$ depend on many terms of the series in Eq. 3.2.14, and therefore are inconvenient. However, if the rainpath $[z, z']$ causes only weak depolarization effects, i.e. if

$$\left| (z-z') \delta \Gamma_0 \right| \ll 1$$

(3.2.17)

the series in Eq. 3.2.14 will converge rapidly. In this case only the matrices $F_0$ and $F_1$ are needed for approximate analysis. The first order approximation of the cross-polarization parameters are then given by

$$XPL \approx XPLY \approx \frac{1}{2} \left| \int_{z_0}^{z} \delta \Gamma_0(z') \sin \{2\phi_0(z') \} \, dz' \right|^2$$

(3.2.18)

The integrand here occurring can be represented as follows

$$\delta \Gamma_0 \sin(2\phi_0) = \{\delta \Gamma_R + j \delta \Gamma_I\}\{\sin(2\phi_R)\cosh(2\phi_I) + j \cos(2\phi_R)\sinh(2\phi_I)\},$$

(3.2.19)

where

$$\delta \Gamma_R = \text{Re}\{\delta \Gamma_0\}, \quad \delta \Gamma_I = \text{Im}\{\delta \Gamma_0\}.$$  

(3.2.20)

This formula shows that the value of $XPL$ may increase if the effective average canting angle $\phi_0$ is complex instead of real. Fig. 5 represents $|\sin(2\phi_0)/\sin(2\phi_R)|^2$ and $|\sin(2\phi_0)|^2$ as functions of $\phi_R$ and $\phi_I$. These quantities show the increase in $XPL$ due to the imaginary part of $\phi_0$ when the rain medium is homogeneous. However, in the case of rain containing only oblate spheroidal raindrops, and, with a canting angle mechanism only caused by the laminar flow of air above the ground surface (cf[19]), the numerical values of $\phi_0$ prove to be negligible (section A4). Further, Eq. 3.2.8 implies some kind of statistical averaging over the rainpath $[z, z']$ and may cause, for instance, a decrease in the $XPL$ value with respect to that of the homogeneous case.

These considerations suggest that in order to obtain some statistical knowledge of $XPL$ from Eq. 3.2.8, it is necessary to know statistical data concerning the variation of $\phi_0$ and $\delta \Gamma_0$ along the rainpath. Since these quantities are determined by the distribution of the orientations, sizes and shapes of the
3.3. Representation of $\mathbf{F}$ in the case of orthogonal circular polarizations

The transition of the basis $\mathbf{U}_x, \mathbf{U}_y$, associated with linearly polarized field components to a new basis $\mathbf{U}_\uparrow, \mathbf{U}_\downarrow$, i.e.

$$
\begin{bmatrix}
\mathbf{U}^\uparrow_r \\
\mathbf{U}^\downarrow_r
\end{bmatrix} = [[D]]
\begin{bmatrix}
\mathbf{U}^\uparrow_x \\
\mathbf{U}^\downarrow_x
\end{bmatrix}
$$

connected with two orthogonal circularly polarized field components is given by the coordinate transformation matrix

$$[[D]] = \begin{bmatrix}
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{bmatrix}.$$  \hspace{1cm} (3.3.1)

The representation of $\mathbf{F}$ in the $\mathbf{U}_x, \mathbf{U}_y$ system

$$[[F_{[xy]}]] = \begin{bmatrix}
F_{xx} & F_{xy} \\
F_{yx} & F_{yy}
\end{bmatrix}$$

is associated with its representation $[[F_{[p,z]}]]$ in the $\mathbf{U}_\uparrow, \mathbf{U}_\downarrow$ system by the relation

$$[[F_{[p,z]}]] = [[D]]^{-T} [[F_{[xy]}]] [[D]]^T,$$  \hspace{1cm} (3.3.3)

which leads to the explicit form

$$[[F_{[p,z]}]] = \frac{1}{2} \begin{bmatrix}
(F_{xx} + F_{yy}) + j(F_{yx} - F_{xy}) & (F_{xx} - F_{yy}) + j(F_{yx} + F_{xy}) \\
(F_{xx} - F_{yy}) - j(F_{yx} + F_{xy}) & (F_{xx} + F_{yy}) - j(F_{yx} - F_{xy})
\end{bmatrix}.$$  \hspace{1cm} (3.3.5)
In the case of condition (3.2.17), i.e. if the rainpath causes only weak depolarization effects, we may approximate the above representation by:

\[
[[F[R,z]]] = e^{\mathcal{J}_0(z')} \begin{bmatrix} 1 \\ \mathcal{J}_2(1) \end{bmatrix}
\]

\[
\mathcal{J}_0(z') e^{j2\phi(z')} d\mathbf{z}'
\]

\[
\frac{1}{\mathcal{J}_0(z')} e^{j2\phi(z')} d\mathbf{z}' 1
\]

(3.3.6)

The cross-polarization parameters analogous to (3.2.15) and (3.2.18) are now approximated by

\[
X_{PCR} = \frac{1}{\mathcal{J}_0(z')} \left| \int_{\mathcal{J}_0(z')}^{z} \delta_{\mathcal{J}_0(z')} e^{j2\phi(z')} d\mathbf{z}' \right|^2
\]

(3.3.7)

for transmitting right circular polarization, and by

\[
X_{PCL} = \frac{1}{\mathcal{J}_0(z')} \left| \int_{\mathcal{J}_0(z')}^{z} \delta_{\mathcal{J}_0(z')} e^{j2\phi(z')} d\mathbf{z}' \right|^2
\]

(3.3.8)

for transmitting left circular polarization.
4. Thermal emission

4.1. The transfer equation

Any absorbing medium emits noise-like electromagnetic energy, known as thermal emission. The power spectrum of this emission is related to that of a black body (cf. [23]), if the medium is in a state of local thermodynamic equilibrium.

Let us consider a real plane TE wave propagating in the $z$ direction. Its electric vector can be given by

$$\mathbf{E}(r)(z, t) = \mathbf{E}_x(r)(z, t) \hat{\mathbf{u}}_x + \mathbf{E}_y(r)(z, t) \hat{\mathbf{u}}_y$$  \hspace{1cm} (4.1.1)

where $\mathbf{E}_x(r)(z, t)$ and $\mathbf{E}_y(r)(z, t)$ are real functions of $z$ and $t$. We next assume that the associated real functions [20] $\mathbf{E}_x^{(i)}(z, t)$ and $\mathbf{E}_y^{(i)}(z, t)$, defined by the following Hilbert reciprocity relations,

$$\mathbf{E}_x^{(i)}(z, t) = \frac{P}{\pi} \int_{-\infty}^{\infty} \frac{\mathbf{E}_x^{(r)}(z, t')}{t-t'} \, dt'$$  \hspace{1cm} (4.1.2)

$$\mathbf{E}_y^{(i)}(z, t) = \frac{P}{\pi} \int_{-\infty}^{\infty} \frac{\mathbf{E}_y^{(r)}(z, t')}{t-t'} \, dt'$$  \hspace{1cm} (4.1.3)

do exist ($P = \text{Cauchy principal value}$). We may then construct the analytic functions (cf. [20], [21])

$$\mathbf{E}_x^{(r)}(z, t) = \mathbf{E}_x^{(r)}(z, t) + j\mathbf{E}_x^{(i)}(z, t)$$  \hspace{1cm} (4.1.4)

$$\mathbf{E}_y^{(r)}(z, t) = \mathbf{E}_y^{(r)}(z, t) + j\mathbf{E}_y^{(i)}(z, t)$$

with the associated analytic electric vector $\mathbf{E}$

$$\bar{\mathbf{E}}(z, t) = \mathbf{E}_x^{(r)}(z, t) \hat{\mathbf{u}}_x + \mathbf{E}_y^{(r)}(z, t) \hat{\mathbf{u}}_y$$  \hspace{1cm} (4.1.5)

Instead of the bases $\hat{\mathbf{u}}_x$, $\hat{\mathbf{u}}_y$, we can take, also in the $xy$ plane, a pair of two independent complex phasors $\bar{\mathbf{u}}_p$, $\bar{\mathbf{u}}_q$. The analytic electric vector is then given by

$$\bar{\mathbf{E}}(z, t) = \mathbf{E}_p^{(r)}(z, t) \bar{\mathbf{u}}_p + \mathbf{E}_q^{(r)}(z, t) \bar{\mathbf{u}}_q$$.
while the analytic functions \( f'_{x}, f'_{q} \) are associated with the analytic functions \( f_{x}, f_{y} \) by the transformation
\[
\begin{bmatrix}
E_{x} \\
E_{y}
\end{bmatrix} = \begin{bmatrix}
(\tilde{u}_{x}, \tilde{u}_{p}) & (\tilde{u}_{x}, \tilde{u}_{q}) \\
(\tilde{u}_{y}, \tilde{u}_{p}) & (\tilde{u}_{y}, \tilde{u}_{q})
\end{bmatrix} \begin{bmatrix}
E_{p} \\
E_{q}
\end{bmatrix}.
\]
(4.1.6)

Since the thermal-emission field is of a stochastic nature, it is convenient to describe it by a set of suitably defined correlation parameters. The Stokes correlation parameters are thus defined as a four-dimensional vector
\[
\mathcal{C} \equiv \{ C_{p}, C_{q}, C(p), C(q) \},
\]
(4.1.7)
the components of which are:
\[
\begin{align*}
C_{p}(z, \tau) &= \alpha \langle E_{p}(z, t+\tau) E_{p}^{*}(z, t) \rangle, \\
C_{q}(z, \tau) &= \alpha \langle E_{q}(z, t+\tau) E_{q}^{*}(z, t) \rangle, \\
C(p)(z, \tau) &= \alpha \{ \langle E_{p}(z, t+\tau) E_{q}^{*}(z, t) \rangle + \langle E_{q}(z, t) E_{p}^{*}(z, t+\tau) \rangle \}, \\
C(q)(z, \tau) &= \alpha \{ \langle E_{p}(z, t+\tau) E_{q}^{*}(z, t) \rangle - \langle E_{q}(z, t) E_{p}^{*}(z, t+\tau) \rangle \}.
\end{align*}
\]
(4.1.8)

\( \alpha \) is a constant to be so chosen that \( C_{x}(z, \varnothing) \) and \( C_{y}(z, \varnothing) \) may be interpreted as the power-flux density per unit solid angle, at \( z \), flowing in the \( z \) direction. The normalization of power per steradians instead of the usual one per surface is relevant to the property that the thermal-emission field is considered being composed of plane waves propagating in all directions. The ensemble averages here occurring will be assumed to be identical with the time averages for a special realization of the ensemble, i.e. the field is ergodic [22]. We then have all the averages
\[
\langle h(z, t, \tau) \rangle = \lim_{T' \to \infty} \frac{1}{2T'} \int_{-T'}^{T'} h(z, t, \tau) \, dt
\]
(4.1.9)
The Stokes spectral vector \( \mathcal{I} \) (cf[22]) is next introduced as the Fourier transform of the above Stokes correlation vector, so as to have
This definition implies real values of the components of \( \tilde{I} \).

The propagation of an electromagnetic field, characterized by its Stokes spectral vector \( \tilde{I} \), is governed by the extinction and emission mechanisms of the medium. In fact, it will be shown below that a radiative transfer equation of the form

\[
\frac{\partial}{\partial z} \tilde{I}(z, \omega) = -\kappa(z, \omega) \{ \tilde{I}(z, \omega) - \tilde{S}(z, \omega) \}
\]  

(4.1.11)

can be derived by considering the variation of the Stokes spectral vector along an infinitesimal distance \( dz \).

In this equation \( \kappa \) is the 4 by 4 extinction coefficient matrix. The first term of the right hand side of Eq. 4.1.11 represents the variation of \( I \) due to the absorption and scatter losses in the medium, while \( \tilde{S} \) is the source function vector representing the thermal emission.

We shall first verify Eq. 4.1.11 without considering the effect of thermal emission, i.e. the contribution depends on \( \tilde{S} \). In the case of a monochromatic wave the equation in question is readily derived from Eq. 3.1.28 and the definition of the Stokes vector. When dealing with stochastic signals we define the truncated signals as

\[
E_{T'P}^{(r)}(z,t) = \begin{cases} \\
E_p^{(r)}(z,t) & \text{when } |t| < T', \\
o & \text{when } |t| > T', 
\end{cases}
\]  

(4.1.12)

\[
E_{T'Q}^{(r)}(z,t) = \begin{cases} \\
E_q^{(r)}(z,t) & \text{when } |t| < T', \\
o & \text{when } |t| > T', 
\end{cases}
\]

(4.1.13)

\( E_{T'P}^{(i)} \) and \( E_{T'Q}^{(i)} \) are defined here by relations analogous to Eq. 4.1.2, as the associated functions connected to \( E_{T'P}^{(r)} \) and \( E_{T'Q}^{(r)} \) respectively. The resulting analytic functions

\[
E_{T'P}^{(i)} = E_{T'P}^{(r)} + j E_{T'P}^{(i)} , \\
E_{T'Q}^{(i)} = E_{T'Q}^{(r)} + j E_{T'Q}^{(i)} ,
\]

will have Fourier spectra according to
\[ E_{T',p}(z,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega t} \tilde{E}_{T',p}(z,\omega) \, d\omega, \]

\[ E_{T',q}(z,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega t} \tilde{E}_{T',q}(z,\omega) \, d\omega, \]

since \( E_{T',p} \) and \( E_{T',q} \) may be assumed to be square integrable.

We next define the "truncated" Stokes spectral parameters

\[ I_{T',p}(z,\omega) = |\tilde{E}_{T',p}(z,\omega)|^2 \tau'^{-1} \]

\[ I_{T',q}(z,\omega) = |\tilde{E}_{T',q}(z,\omega)|^2 \tau'^{-1} \]

\[ I_{T',p}^{(p)}(z,\omega) = 2\alpha \Re(\tilde{E}_{T',p}(z,\omega) \tilde{E}_{T',q}^{*}(z,\omega)) \tau'^{-1} \]

\[ I_{T',q}^{(q)}(z,\omega) = -2\alpha \Im(\tilde{E}_{T',p}(z,\omega) \tilde{E}_{T',q}^{*}(z,\omega)) \tau'^{-1} \]

In view of the linearity and time independence of the medium, the functions \( \tilde{E}_{T',p}(\omega) \) and \( \tilde{E}_{T',q}(z,\omega) \) may be assumed to satisfy Eq. 3.1.29. Differentiation of Eq. 4.1.15 then yields the following relation (section A.5)

\[ \frac{\partial}{\partial z} \tilde{I}_{T',p}(z,\omega) = \kappa(z,\omega) \tilde{I}_{T',p}(z,\omega) \]

If \( p \) and \( q \) refer to the component of \( \tilde{F} \) with respect to the normalized eigenvectors \( \tilde{M}_A \) and \( \tilde{M}_B \) (at frequency \( \omega \)) \( \kappa \) is given explicitly by

\[ \kappa = \begin{bmatrix} \kappa_{AA} & 0 & 0 & 0 \\ 0 & \kappa_{BB} & 0 & 0 \\ 0 & 0 & \kappa_{AB} & \kappa_{BB} \\ 0 & 0 & \kappa_{BA} & \kappa_{BB} \end{bmatrix} \]

where the elements are given by

\[ \kappa_{AA} = 2 \Re(\Gamma_{A}) \]  
\[ \kappa_{BB} = 2 \Re(\Gamma_{B}) \]
\[ \kappa_{AB} = \kappa_{BA} = \Re(\Gamma_{A} \Gamma_{B}^{*}) \]

\[ \kappa_{BB} = \Im(\Gamma_{A} \Gamma_{B}^{*}) \]
Now, under suitable assumptions concerning the stochastic nature of the signals (stationarity, ergodicity, etc.), we have the relation

\[ \lim_{T' \to \infty} \text{Est}\{ \tilde{I}_{T'}(z, \omega) \} = \tilde{I}(z, \omega) \]  \hspace{1cm} (4.1.19)

where \( \text{Est}\{ \tilde{I}_{T'}(z, \omega) \} \) denotes ensemble averaging\[20\]. Therefore, from Eq. 4.1.16, we have

\[ \frac{\partial}{\partial z} \tilde{I}(z, \omega) = -\bar{\kappa}(z, \omega) \tilde{I}(z, \omega) \]  \hspace{1cm} (4.1.20)

In deriving the term representing the thermal emission in Eq. 4.1.11 we shall assume that the medium is stationary, i.e. no temperature fluctuations should occur in time. We also assume that the thermal emission of the medium contributes linearly to the change of \( \tilde{I} \)

\[ \frac{\partial}{\partial z} \tilde{I}(z, \omega) = -\bar{\kappa}(z, \omega) \tilde{I}(z, \omega) + \tilde{J}(z, \omega) \]  \hspace{1cm} (4.1.21)

where \( \tilde{J}(z, \omega) \) represents this latter contribution. In order to evaluate \( \tilde{J}(z, \omega) \) at say \( z = z_i \), we consider the case of a fictitious homogeneous medium with a constant extinction matrix \( \bar{\kappa}_i \):

\[ \bar{\kappa}_i = \bar{\kappa}(z_i, \omega) \]  \hspace{1cm} (4.1.22)

enclosed by black-body radiator of temperature \( T \) equal to that of the inhomogeneous medium at \( z = z_i \). The homogeneous medium mentioned will also have the temperature \( T \) throughout when the state of thermodynamic equilibrium between it and the black-body radiator is reached. According to thermodynamics and the black-body law of radiation (cf.[23]), the intensity of the thermal emission flowing in the \( z \) direction is given by

\[ \tilde{I}(\omega) = \bar{S}(T, \omega) \]  \hspace{1cm} (4.1.23)

where the source-function vector \( \bar{S} \) is fixed by the following representation referring to the bases \( \bar{U}_x, \bar{U}_y \)

\[ \bar{S} = \{ \kappa B_T(T), \kappa B_y(T), 0, 0 \} \]  \hspace{1cm} (4.1.24)
the black-body radiator, is given by the following expression \cite{23}:

\[ B(v, T) = \frac{2 \pi \nu^2 e^{-\frac{\nu}{k_B T}} \left\{ \exp \left( \frac{\nu}{k_B T} \right) - 1 \right\} }{\nu^3} \]  

(4.1.25)

where \( \nu = \omega / 2\pi \) is the frequency, \( \hbar \) Planck's constant, \( k_B \) Boltzmann's constant, and \( c \) the velocity of light. When \( \hbar \nu < kBT \), we can apply the so-called Rayleigh-Jeans approximation for \( B(v, T) \), viz.

\[ B(v, T) \approx 2 \pi \frac{k_B T \nu^2}{\hbar^3} \]  

(4.1.26)

Next, substituting Eqs. 4.1.23 and 4.1.24 in Eq. 4.1.21, and applying the property

\[ \frac{\partial}{\partial z} \tilde{J}(z, \omega) = \frac{\partial}{\partial z} \tilde{S}(T, \omega) = 0 \]  

(4.1.27)

we obtain the relation

\[ 0 = \kappa \tilde{S}(T, \omega) - \tilde{J}(T, \omega) \]  

(4.1.28)

or

\[ \tilde{J}(T, \omega) = \kappa \tilde{S}(T, \omega) \]  

(4.1.29)

where \( \kappa \) indicates the value of \( \kappa \) at \( z = z_i \). The latter equation is a generalization of Kirchhoff's law for the radiation from an absorbing body in anisotropic media.

If we now assume that, in the case of a stationary inhomogeneous medium, \( \tilde{J}(z, \omega) \) is to be determined completely by the local values of \( \kappa \) and \( T \), the relation

\[ \tilde{J}(z, \omega) = \kappa(z, \omega) \tilde{S}(T(z), \omega) \]  

(4.1.30)

will be satisfied in such a medium.
4.2. Solution of the transfer equation; partial polarization of emission

The general solution of the radiative transfer equation Eq. 4.1.11 can formally be derived by a method analogous to that described in section 3.2 for solving Eq. 3.21 without an emission term. We find

$$\bar{T}(z,z_0;\omega) = \bar{H}(z,z_0;\omega) \bar{T}(z_0;\omega) + \int_{z_0}^{z} dz' \bar{H}(z,z';\omega) \bar{\kappa}(z';\omega) \bar{S}(z';\omega); \quad (4.2.1)$$

where the 4 by 4 evolution-matrix operator $\bar{H}$ given explicitly below, is the analogon of the 2 by 2 evolution-matrix operator $\bar{F}$ described in section 3.2.

The validity of Eq. 4.2.1 is readily verified by substituting it in the transfer equation.

As in section 3.2, $\bar{H}$ may be arrived at by representing $\bar{\kappa}$ as follows:

$$\bar{\kappa} = \kappa_0 + \text{Re} \{ \bar{\sigma}_c \cos(2\phi_0) + \bar{\sigma}_s \sin(2\phi_0) \}; \quad (4.2.2)$$

where $\kappa_0$ is the average extinction coefficient given by

$$\kappa_0 = 2Re\Gamma_0, \quad (4.2.3)$$

while the 4 by 4 matrix operators $\bar{\sigma}_c$ and $\bar{\sigma}_s$ are defined by their representations

$$\bar{\sigma}_c = \delta \Gamma_0 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{bmatrix}, \quad \bar{\sigma}_s = \delta \Gamma_0 \begin{bmatrix} 0 & 0 & \frac{1}{2} & \frac{i}{2} \\ 0 & 0 & \frac{1}{2} & -\frac{i}{2} \\ 1 & 1 & 0 & 0 \\ -i & i & 0 & 0 \end{bmatrix}, \quad (4.2.4)$$

that refer to the $xy$ coordinate system (cf. section A.5).

An evaluation similar to that worked out in section 3.2 leads to the representation

$$\bar{H}(z,z_0;\omega) = \left\{ \sum_{n=0}^{\infty} \bar{H}_n(z,z_0;\omega) \right\} e^{-\beta(z,z_0;\omega)}; \quad (4.2.5)$$

here $\bar{H}_0 = 1$, while the other factors $\bar{H}_n$ result from the recurrence relation
\[ \bar{H}_i(z,z_o;\omega) = -\Re \int_{z_o}^{z} \frac{dz'}{z'} \left[ \sigma_s(z') \cos \left[ 2\phi_0(z') \right] + \sigma_o(z') \sin \left[ 2\phi_0(z') \right] \right] \bar{H}(z',z_o;\omega) \]

(4.2.6)

\[ \beta(z,z_o;\omega) = \int_{z_o}^{z} dz' \kappa(z',\omega) \]

(4.2.7)

\( \beta \) being defined by

When dealing with thermal emission, it is convenient to express the intensities in terms of temperatures instead of powers. The "apparent temperature" vector \( \bar{T}_e \) is thus defined by the relation

\[ \bar{T}_e = \frac{\hbar}{k_B} \ln \left( \frac{T}{T_0} \right) \]

Likewise, the "source temperature" vector \( \bar{T} \) is given by

\[ \bar{T} = \frac{\hbar}{k_B} \ln \left( \frac{T}{T_0} \right) \]

At microwave frequencies, \( B_\nu \) may be replaced by its Rayleigh-Jeans approximation. This leads to the following expression for \( \bar{T} \) in the \( xy \) system

\[ \bar{T}(z) = \{ T(z), T(z), 0, 0 \} \]

(4.2.10)

\( T(z) \) is the temperature of the medium at a special \( z \) level.

Eq. 4.2.1 can then be represented by

\[ \bar{T}_e(z;w) = \bar{H}(z,z_0;\omega) \bar{T}_e(z_0;\omega) + \int_{z_0}^{z} dz' \bar{H}(z,z';\omega) \kappa(z';\omega) \bar{T}(z') \]

(4.2.11)

The solution of this latter equation is as follows if the temperature \( T \) is independent of \( z \) throughout the rainpath \( [z,z_0] \)

\[ \bar{T}_e(z;w) = \bar{H}(z,z_0;\omega) \bar{T}_e(z_0;\omega) + \{ 1 - \bar{H}(z,z_0;\omega) \} \bar{T} \]

(4.2.12)
Usually, the "apparent temperature" \( \tilde{T}_c(z_o, \omega) \) incident at \( z_o \) is the totally unpolarized sky-emission temperature so that \( \tilde{T}_c(z_o, \omega) \) will have the same representation as Eq. 4.2.10 with a special temperature \( T_{inc} \) instead of \( T \).

A first-order approximation of \( \bar{H} \) will suffice when \( \delta \Gamma_R \) and \( \delta \Gamma_I \) are sufficiently small. We may then arrive at the expressions

\[
\begin{align*}
T_{ex}(x) &= 2e^{\frac{-B(z, z_o, \omega)}{(T-T_{inc})}} \Re \int dz' \delta \Gamma_0(z') \cos[2\Phi_0(z')] \\
T_{ex}(y) &= 0
\end{align*}
\]

Eqs. 4.2.13-15 show that the thermal emission in the atmosphere due to rain may be slightly polarized. From Eq. 4.2.15 we may conclude that the polarized part of the thermal emission is linear.

Fig 6+ shows \( \Delta T_e = T_{ex} - T_{ex} \) as functions of different rain-pathlengths \( L = z - z_o \), and different rain intensities \( p \), for a homogeneous rain at frequencies 11, 18.1, and 30 GHz. It has been assumed there that the raindrops are of the oblate type, and that they are all oriented according to \( \phi = 25^\circ \) and \( \theta = 90^\circ \); the values used for the extinction coefficient are those given by [14], where Laws and Parsons' raindrop-size distributions have been assumed. These figures show the above mentioned polarization to be small, \( |\Delta T_e| \) being always smaller than 13 K in the range considered.

4.3. Partial polarization of emission due to scattering.

The partial polarization of thermal emission, as derived in the previous section, is uniquely due to the anisotropy of the extinction property of the medium. In this section we shall show that scattering in directions differing from the forward direction may also cause some polarization of thermal emission.

Let us consider a rain-medium for which the following transfer equation holds

( and Table 1 )
\[(\bar{\rho}.\nabla)\bar{I}(\bar{r};\bar{\rho}) = -\kappa(\bar{r};\bar{\rho})\{\bar{I}(\bar{r};\bar{\rho}) - (1-\sigma_0)\bar{S}(\bar{r})\} - \frac{\sigma_0}{4\pi} \int d\Omega_{\bar{\rho}'} \bar{P}(\bar{r};\bar{\rho},\bar{\rho}') \bar{I}(\bar{r};\bar{\rho}')\}

(4.3.1)

where

- \bar{\rho} = unit vector fixing a special direction
- \bar{r} = space coordinates
- \bar{I}(\bar{r};\bar{\rho}) = the Stokes spectral vector at \bar{r}, representing the power flow in the direction \bar{\rho}
- \kappa(\bar{r};\bar{\rho}) = the by extinction matrix operator at \bar{r}, referring to waves travelling in the \bar{\rho} direction
- \sigma_0 = albedo for single scattering, representing the ratio of the scattered energy to the energy lost through both scattering and absorption (for an infinitesimal volume element of the medium), see also [15].
- \bar{P}(\bar{r};\bar{\rho},\bar{\rho}') = the phase matrix (cf. [15]), giving the fraction of the power flow in \bar{\rho}' direction that is scattered in the \bar{\rho} direction.
- d\Omega_{\bar{\rho}'} = an element of solid angle in the \bar{\rho}' direction.

Equation 4.3.1 is a generalization of the classical transfer equation for a scattering and absorbing medium [2],[15]. The generalization being apparent from the matrix form of the extinction coefficient.

In section 4 it is assumed that \sigma_0 vanishes. In this case Eq.4.3.1 reduces to the form of the transfer equation 4.1.10. As a matter of fact, \sigma_0 does not vanish, but leads to the third term in the right hand side of Eq.4.3.1. This term represents the contribution due to scattering to the total variation of \bar{I} in the \bar{\rho} direction. In operator notation, we could write this term concisely as:

\[\sigma_0 \bar{R}(\bar{r},\bar{\rho}) \equiv \sigma_0 \int d\Omega_{\bar{\rho}'} \bar{P}(\bar{r};\bar{\rho},\bar{\rho}') \bar{I}(\bar{r};\bar{\rho}')\]

(4.3.2)

For our cases of interest \sigma_0 has a small value. The anisotropy of the medium is slight, i.e.

\[\bar{\kappa} = \kappa_0 + \eta \delta \kappa\]

(4.3.3)

where the absolute values of the elements of \delta \kappa are small compared to the
average extinction coefficient $\kappa_0$. It is now useful to apply a Born series representation of $\mathcal{I}$ to solve Eq.4.3.1, viz.

$$\mathcal{I}(\mathbf{r};\mathbf{p}) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \eta \sigma_0 m \mathcal{F}(\mathbf{r};\mathbf{p};n,m)$$  \hspace{1cm} (4.3.4)

Substituting this form in Eq.4.3.1, and grouping the contributions proportional to equal powers of $\sigma_0$ and $\eta$, the following systems of equations result:

$$ (\hat{\rho}, \text{grad}) \mathcal{I}(0,0) = -\kappa_0 [\mathcal{I}(0,0) - \mathcal{S}] $$
$$ (\mathbf{p}, \text{grad}) \mathcal{I}(0,n,0) = -\kappa_0 [\mathcal{I}(n,0) - \kappa_0^{-1} \delta \kappa \mathcal{I}(n-1,0)], \quad (n \geq 1) \quad (4.3.5) $$

$$ (\hat{\rho}, \text{grad}) \mathcal{I}(0,1) = -\kappa_0 [\mathcal{I}(0,1) + \mathcal{S} - \mathcal{R}(\mathcal{I}(0,0))], $$
$$ (\mathbf{p}, \text{grad}) \mathcal{I}(0,m) = -\kappa_0 [\mathcal{I}(0,m) - \mathcal{R}(\mathcal{I}(0,m-1))] \quad , \quad (m \geq 2) \quad (4.3.6) $$

and for $n$ and $m \geq 1$:

$$ (\hat{\rho}, \text{grad}) \mathcal{I}(n,m) = -\kappa_0 [\mathcal{I}(n,m) - \mathcal{R}(\mathcal{I}(n,m-1)) - \kappa_0^{-1} \delta \kappa (\mathcal{I}(n-1,m) - \mathcal{R}(\mathcal{I}(n-1,m-1)))] \quad (4.3.7) $$

where the notation $\mathcal{I}(n,m) \equiv \mathcal{I}(\mathbf{r};\mathbf{p};n,m)$ has been used.

System Eq.4.3.5 corresponds with the transfer equation introduced in section 4; in fact, $\mathcal{I}(0,0) + \mathcal{I}(1,0)$ represents the first order approximation mentioned in section 4.2.

To complete the above group of equations, we must specify the boundary conditions. A natural choice is:

$$ \mathcal{I}(\mathbf{r}_{\text{boundary}};\mathbf{p};0,0) = \mathcal{I}(\mathbf{r}_{\text{boundary}};\mathbf{p}) \quad (4.3.8) $$

while

$$ \mathcal{I}(\mathbf{r}_{\text{boundary}};\mathbf{p};n,m) = 0 \quad , \quad (4.3.9) $$

for all $(n,m) \neq (0,0)$.

Since $\sigma_0$ and $\kappa_0^{-1} \delta \kappa$ may be expected to be small, we still have the following approximation:

$$ \mathcal{I} = \mathcal{I}(0,0) + \mathcal{I}(1,0) + \sigma_0 \mathcal{F}(0,1) \quad . \quad (4.3.10) $$
The first two terms on the right hand side of this equation have already been evaluated in section 4.2. \( \tilde{I}(0,1) \) is a solution of the following set of equations:

\[ (\hat{p}, \text{grad}) \tilde{I}(0,1) = -\kappa_0 [\tilde{I}(0,1) - \tilde{S}'] \]  
\[ \tilde{I}(\tilde{r}_{\text{boundary}}, \hat{p}; 0, 0) = 0 \]

where

\[ \tilde{S'} = \frac{1}{4\pi} \int d\tilde{r} \tilde{I}(0,0) - \tilde{S} \]

We shall now calculate \( \tilde{I}(0,1) \) for the following situation, illustrated by Fig.7:

1. the rain medium extends indefinitely in the horizontal direction and is enclosed below and above by the non-scattering ground and by clouds, respectively; the \( z \) direction is assumed parallel to the \( \xi \eta \) plane;
2. the rain medium is homogeneous, i.e. \( \kappa_0 \) is independent of \( \tilde{r} \);
3. the temperature \( T \) of the rain medium is constant and equal to the ground temperature;
4. the ground is considered to be a black-body radiator;
5. the clouds are assumed to be non-reflecting, the emission there being the totally unpolarized sky emission;
6. the phase function is a Rayleigh phase function [15], and given by the following representation referring to a local spherical coordinate system:

\[ \hat{p} \equiv \begin{bmatrix} 2\sin^2 \varphi \sin^2 \varphi' + \cos^2 \varphi \cos^2 \varphi' & \cos^2 \varphi' \\ \cos^2 \varphi' & 1 \end{bmatrix} \]  

\[ (\varphi, \varphi'), (\varphi', \varphi') \] indicate the \( \hat{p} \) and \( \hat{p}' \) directions respectively; the orthogonal linear polarizations have been taken along the meridian and longitudinal directions respectively.

The solution of the first equation of system 4.3.5, corresponding to a known
situation at the boundary points, where \( r = \tilde{r}_o \), reads

\[
\tilde{I}(\tilde{r}_o; \tilde{r}, 0; 0, 0) = \tilde{I}(\tilde{r}_o; \tilde{r}, 0; 0, 0) \rho_o \rho_o ^* (\tilde{r} - \tilde{r}_o) + [1 - e^{-\kappa_o \rho_o ^* (\tilde{r} - \tilde{r}_o)}] \tilde{S}
\]

Substitution of this formula in Eq. 4.3.11 yields the equation

\[
(\rho, \text{grad})\tilde{I}(\tilde{r}_o; \tilde{r}, 0; 0, 1) = -\kappa_o \left[ \tilde{I}(\tilde{r}_o; \tilde{r}, 0; 0, 1) - \frac{1}{4\pi} \int d\Omega_{\tilde{r}_o} \tilde{P}(\tilde{r}, \tilde{r}_o, \tilde{r}_o') \left\{ \tilde{I}(\tilde{r}_o; \tilde{r}_o', \tilde{r}_o) - \tilde{S} \right\} e^{-\kappa_o \tilde{r}_o' (\tilde{r} - \tilde{r}_o)} \right]
\]

where the relation

\[
\int d\Omega_{\tilde{r}_o} \tilde{P}(\tilde{r}, \tilde{r}_o, \tilde{r}_o') \tilde{S} = \{ \int d\Omega_{\tilde{r}_o} \tilde{P}(\tilde{r}, \tilde{r}_o, \tilde{r}_o') \} \tilde{S} = 4\pi \tilde{S}
\]

has been applied ( \( \tilde{S} \) is independent of the direction \( \tilde{r}_o' \)).

In Eq. 4.3.15 we may so choose the boundary points \( \tilde{r}_o \) that the integral on the right hand side can be split into two terms:

\[
\int d\Omega_{\tilde{r}_o} \tilde{P}(\tilde{r}, \tilde{r}_o, \tilde{r}_o') \tilde{S} = \int d\Omega_{\tilde{r}_o} \tilde{P}(\tilde{r}, \tilde{r}_o, \tilde{r}_o') \tilde{S}_{\text{ground}} + \int d\Omega_{\tilde{r}_o} \tilde{P}(\tilde{r}, \tilde{r}_o, \tilde{r}_o') \tilde{S}_{\text{cloud}}
\]

the first contribution refers to all boundary points on the ground, and vanishes in view of the assumed black-body character of the ground, i.e. \( \tilde{I}(\tilde{r}_o) = \tilde{S} \), while the second refers to the boundary points on the cloud ceiling. This latter contribution may be approximated by

\[
-\left\{ \int d\Omega_{\tilde{r}_o} \tilde{P}(\tilde{r}, \tilde{r}_o, \tilde{r}_o') e^{-\kappa_o \tilde{r}_o' (\tilde{r} - \tilde{r}_o)} \right\} \tilde{S}_{\text{cloud}}
\]

since the intensity of the emission (sky) due to the clouds is usually much smaller than \( \tilde{S} \).

Now, by taking

\[
\tilde{r}_o = \tilde{u}_z
\]

and

\[
\tilde{I}(\tilde{r}_o; \tilde{r}_o, 0; 0, 1)_{\text{cloud}} = 0
\]

(4.3.21)
and introducing
\[
\mu = \cos \nu, \quad \mu' = \cos \nu',
\]
we obtain the following expression for the solution of Eq. 4.3.16:
\[
\begin{align*}
\left[ I_x(0,1) \right] & = \frac{3}{2} \int_0^L \int_0^1 \kappa_0 \left[ 2(1-\mu^2)(1-\mu'^2)+\mu^2\mu'^2 \right] \mu' \left[ \kappa_0^0 \right]_0^L \left( L \cdot \nu' \right) + \alpha' \mu' / \mu' \\
\left[ I_y(0,1) \right] \bigg|_{x=L} & = \mu'^2 \left[ \begin{array}{c}
1 \\
0
\end{array} \right]
\end{align*}
\]

The difference between intensities in \( x \) and \( y \) polarizations, \( \Delta I = I_x - I_y \), is given by the expression
\[
\Delta I[I_1,0] = \Delta I[I_1,0] + \sigma_0 \Delta I[I_1,0]
\]

(4.3.24)

where
\[
\Delta I[I_1,0] = I_x[I_1,0] - I_y[I_1,0]
\]

(4.3.25)

represents the contribution to emission polarization due to the anisotropy and in our special case is given by the following explicit form:
\[
\Delta I[I_1,0] = \cos(2\phi_0) \delta \kappa_0 L \beta B_\nu(T) e^{-\kappa_0 L}
\]

(4.3.26)

while
\[
\sigma_0 \Delta I[I_1,0] = \sigma_0 \left( I_x[I_1,0] - I_y[I_1,0] \right)
\]

(4.3.27)

represents the contribution due to scattering; from Eq. 4.3.23 we have
\[
\sigma_0 \Delta I[I_0,1] = 3/16 \sigma_0 \left( 1-\mu^2 \right) \gamma B_\nu(T)
\]

(4.3.28)

where
\[
\gamma = e^{-\kappa_0 L} \int_0^L \int_0^1 \left( 1-\beta^2 \right) e^{-\kappa_0 L} \left( 1-\mu' / \mu' \right)
\]

(4.3.29)

\( \text{assuming} \ \phi_0 \ \text{real} \)
A simple calculation shows that
\[
\gamma = E_2(\eta) - E_4(\eta) \tag{4.3.30}
\]
where
\[
\eta = \mu \kappa_0 L \tag{4.3.31}
\]
and where the exponential integral \( E_n(\eta) \) is defined by[24]:
\[
E_n(\eta) = \int_1^{\infty} dt \, e^{-\eta t} \, t^{-n} \tag{4.3.32}
\]

Fig. 8 shows \( \gamma \) as a function of \( \Pi \).

The quotient \( T \) of \( \omega \Delta I[0,1] \) and \( \Delta I[1,0] \) is given by the expression
\[
T = 3/6 \, \omega \, q^{-1} (1-\mu^2) \cos^{-1}(2\phi_0) \, \beta_o^{-1} e^{\beta_0} \tag{4.3.33}
\]
where \( q \) represents the factor
\[
q = \delta \kappa_0 / \kappa_0 \tag{4.3.34}
\]
while \( \beta_o \), which is given by \( \beta_o = \kappa_0 L \), represents the total damping along our rain path.

For rain intensities and path lengths that cause \( \Delta I[1,0] \) to become maximum, \( \beta_o \) is equal to one. In these cases \( T \) is given by the expression:
\[
T = 3/6 \, \omega \, q^{-1} (1-\mu^2) \cos^{-1}(2\phi_0) \, e^\gamma \tag{4.3.35}
\]

For \( \mu=\frac{1}{2} \) and \( \cos(2\phi_0)=0.7 \), we obtain the following values of \( |T| \), when using the values of \( \omega \) and \( q \) as given by[2] and[14] respectively; the rain intensity is assumed to be 25 mm/h.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>10%</th>
<th>24%</th>
<th>34%</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 GHz</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18 GHz</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30 GHz</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.4. The sun as a source of unpolarized emission

Eq. 4.2.12 gives the apparent temperature \( \overline{T}_e(z, \omega) \) when \( \overline{H}, \overline{T} \) and \( \overline{T}_e(z_o, \omega) \) are known. For a fixed observation point, these quantities still depend on the direction of \( \overline{U}_z \) chosen, so that we may appropriately write Eq. 4.2.12 in the following form:

\[
\overline{T}_e(\Omega) = \overline{H}(\Omega) \overline{T}_{inc}(\Omega) + \{1 - \overline{H}(\Omega)\} \overline{T}(\Omega) 
\]

(4.4.1)

Here \( \Omega \) represents the solid angle indicating a special \( \overline{U}_z \) direction, see Fig. 9, while \( \overline{T}_e(\Omega), \overline{H}(\Omega), \overline{T}_{inc}(\Omega) \) and \( \overline{T}(\Omega) \) are defined to be equal to \( \overline{T}_e(z, \omega), \overline{H}(z, z_o; \omega), \overline{T}_e(z_o, \omega) \) and \( \overline{T} \) respectively along the rain path in that special direction.

If \( \Omega \) points to the empty sky, then \( \overline{T}_{inc} \) will be the totally unpolarized sky-emission temperature which is usually much lower than the atmospheric temperature \( T \). If \( \Omega \) points to the surface of the sun, \( \overline{T}_{inc} \) will be equal to the emission at the surface of the sun \( \overline{T}_{sun} \), i.e.

\[
\overline{T}(\Omega) = \overline{T}_{sun}(\Omega)
\]

(4.4.2)

Since the intensity of sun emission is much greater than that of the atmosphere, \( \Delta T_e \) and \( m(x) \) should be much higher when observed in the sun direction than otherwise.

The emission from the sun (solar radio emission) is known to be made up of three distinct components, originating from the quiet sun, from the bright regions, and from such transient phenomena as flares[25]:

(i) the quiet sun component \( B \) is the residual radiation that occurs in the absence of localized sources on the sun, and is due to the thermal emission in the solar atmosphere. This component is totally unpolarized[25];

(ii) the slowly varying component \( S \) is the component originating from the bright regions, on the solar surface, and is also due to thermal emission; this component is partially polarized, the polarized part being right or left circular[25]. However, since
the sources of polarized radiation are confined to small areas, and it seems that right and left circular polarizations occur in the same amount, we may, for our practical purposes, consider the polarization of this $S$ component to be random (totally unpolarized). Fig. 10, taken from [26], shows the intensity distribution at the surface of the sun on April 19, 1958; (iii) the radio bursts [26] generally associated with solar flares. At centimetre wavelengths the bursts are partially polarized. However, since bursts at these wavelengths last no longer than a few tenths of minutes, it will not be taken into further consideration.

Fig. 11, taken from [26], shows the intensity of the above components for some different wavelengths.

These considerations show that an emission of the sun may be considered to be totally unpolarized, i.e. in the $xy$ coordinate system

$$\overline{T}_{sun} = \{1, 1, 0, 0\} T_{sun}$$

(4.4.3)

where $T_{sun}$ is the sum of the B and S intensities.

The temperature difference $T_e = T_{ex} - T_{ey}$ and the component $T_e(x)$ is now given by the expressions:

$$\Delta T_e = -2e^{-\beta}(T_{sun} - T) \Re \int dz'd\Gamma' \cos(2\Phi)$$

(4.4.4)

$$T_e(x) = -2e^{-\beta}(T_{sun} - T) \Re \int dz'd\Gamma' \sin(2\Phi)$$

(4.4.5)

analogously to Eqs. 4.2.13 and 4.2.14. Since, as can be seen from Fig. 11, $T_{sun}$ is of the order of thousands of degrees Kelvin, the value of $\Delta T_e$ and $T_e(x)$ should be much larger when observed in the sun direction than otherwise. Further, because the sun subtends a small angle at the earth's surface (ca 0.6° [25]), the values of $\Delta T_e$ and $T_e(x)$ in the direction of the sun will be much less influenced by contributions from scattering effects than otherwise (cf. section 4.3).

It seems, therefore, that the sun might be a suitable radio source to measure the polarizing properties of the rain medium. An obvious disadvantage of this method is that a mechanism must be used to make the observing antenna follow the sun as this moves through the sky.
5. Determination of some parameters

The above considerations suggest the possibility of using thermal-emission measurements for the evaluation of the \( XPL \) parameter. Also of interest is the number of rain-parameters, such as \( \phi_0, \Gamma_0, \delta \Gamma_0 \), etc. that may be determined from such measurements. However, when determining the rain-parameters, experiments using radio signals are usually preferred. Another possibility is to use both systems, i.e. measurements of radio signals and thermal quantities are carried out simultaneously.

In this chapter we shall deal with the problem described above. Some relationships between measurable quantities, which may be used to test the theory, are given. In this section, we shall not, however, consider the effects of inaccuracies in the measured quantities (which undoubtedly will play a part in practical situations) on the feasibility of the experiments discussed.

5.1. Monochromatic signals

For monochromatic signals the modification along rain path \([z, z_0]\) is completely characterized by the evolution matrix \( \bar{F} \) given in section 3, at least for plane waves. The field received at a special \( z \) level is related to the incident one at \( z_0 \):

\[
\begin{bmatrix}
E_x(z, \omega) \\
E_y(z, \omega)
\end{bmatrix} =
\begin{bmatrix}
F_{xx}(z, z_0; \omega) & F_{xy}(z, z_0; \omega) \\
F_{yx}(z, z_0; \omega) & F_{yy}(z, z_0; \omega)
\end{bmatrix}
\begin{bmatrix}
E_x(z_0; \omega) \\
E_y(z_0; \omega)
\end{bmatrix}
\]  

(5.1.1.)

This equation shows that the trajectory \([z_0, z]\) of an idealized rainpath is sufficiently described by four complex quantities. In turn, the latter are related to the rain parameters \( \Gamma_0, \delta \Gamma_0, \phi_0 \) along the path \([z_0, z]\) by the equations given in section 3, so that we may determine average values of some of the rain-parameters from the knowledge of the quantities \( F_{xx}, F_{xy}, F_{yx}, F_{yy} \).

These four quantities may be determined experimentally by measuring the electric field strengths at \( z \) and \( z_0 \) in two different modi, e.g. when the incident field is polarized in the \( x \) or in the \( y \) direction.

If we know only the state of polarization of the incident field, the four matrix elements presented above can only be determined in relation to each other; no more than three complex numbers, viz.
\[ \rho_{xy} = \frac{F_{xy}}{F_{xx}}, \quad \rho_{yx} = \frac{F_{yx}}{F_{xx}}, \quad \rho_{yy} = \frac{F_{yy}}{F_{xx}}, \quad (5.1.2) \]

can then be derived from measurements of \( E_x(z,w) \) and \( E_y(z,w) \). Measurements in three different modi are then needed, as shown in [32].

For some rain paths the approximative relation

\[ F_{xy} \approx F_{yx} \quad (5.1.3) \]

holds (section 3). In this case only two complex quantities are to be determined, viz. \( \rho_{yy} \) and \( \rho_{xy} = \rho_{yx} \); this requires measurements in two modi only.

Let us consider a rain path in which only weak depolarization effects occur. According to section 3 the approximative relation (5.1.3) then holds, and the following approximations result for \( \rho_{xy} \) and \( \rho_{yy} \):

\[ \rho_{xy} = \frac{1}{2} \int \frac{\delta \Gamma_o \sin(2\phi_o')}{1 + \frac{1}{2} \int \frac{\delta \Gamma_o}{\delta \Gamma_I} \cos(2\phi_o') dz'} + \ldots, \approx \frac{1}{2} \int \delta \Gamma_o \sin(2\phi_o') dz', \quad (5.1.4) \]

\[ \rho_{yy} = \frac{1}{1 + \frac{1}{2} \int \frac{\delta \Gamma_o}{\delta \Gamma_I} \cos(2\phi_o') dz'} + \ldots, \approx 1 - \int \delta \Gamma_o \cos(2\phi_o') dz'. \]

The Eq. 5.1.4, and the explicit expression for \( \sin(2\phi_o') \) and \( \cos(2\phi_o') \) in terms of the real quantities \( \phi_R \) and \( \phi_I \), lead to the further relations:

\[ \frac{I_m}{R} \left\{ \rho_{xy} \right\} = \frac{\int [\delta \Gamma_R \cos \theta + \delta \Gamma_I \sin \theta] dz'}{\int [\delta \Gamma_I \cos \theta - \delta \Gamma_R \sin \theta] dz'}, \quad (5.1.5) \]

\[ \frac{R}{e} \left\{ 1 - \rho_{yy} \right\} = \frac{\int [\delta \Gamma_R \cos \theta + \delta \Gamma_I \sin \theta] dz'}{\int [\delta \Gamma_I \cos \theta - \delta \Gamma_R \sin \theta] dz'}. \]

We shall next assume that the ratio \( \eta_o = \frac{\delta \Gamma_I}{\delta \Gamma_R} \) is a constant along the rain path [14]. In this case Eq. 5.1.5 becomes

\[ \frac{I_m}{R} \left\{ \rho_{xy} \right\} = \frac{\int \delta \Gamma_R \cos \theta \; dz' + \eta_o \int \delta \Gamma_R \sin \theta \; dz'}{\int \delta \Gamma_I \cos \theta - \delta \Gamma_R \sin \theta \; dz'}, \quad (5.1.6) \]

\[ \frac{R}{e} \left\{ 1 - \rho_{yy} \right\} = \frac{\int \delta \Gamma_R \cos \theta \; dz' + \eta_o \int \delta \Gamma_R \sin \theta \; dz'}{\int \delta \Gamma_I \cos \theta - \delta \Gamma_R \sin \theta \; dz'}. \]
Eq. 5.1.6 shows that if $\phi_I$ vanishes, and consequently also the quantity $sh$, the above quantities reduce to

$$\frac{I_m\{\rho_{xy}\}}{R_e\{\rho_{xy}\}} = \eta_c$$  \hspace{1cm} (5.1.7)

$$\frac{I_m\{1-\rho_{yy}\}}{R_e\{1-\rho_{yy}\}} = \eta_c^{-1}$$  \hspace{1cm} (5.1.8)

These relationships should provide some insight into the complex nature of the effective average canting angle $\phi_o$.

5.2. The effective temperature

According to Eq. 4.2.13 the effective temperature $\overline{T_e}(z, \omega)$ at $z$ depends on the effective temperature incident at $z_o$, before it, on $T$ and on the evolution matrix $H$. Since $H$ is related to the elements of $\overline{F}$, we expect that measurements of the effective antenna temperatures may yield some information on the elements $F_{xx}$, $F_{yy}$, $F_{xy}$, and $F_{yx}$. The connection between $H$ and the elements of $\overline{F}$ is shown by the following representation which refers to the $xy$ coordinate system:

$$H \overline{F} = \begin{bmatrix} |F_{xx}|^2 & |F_{xy}|^2 & R_e\{F_{xx} F^*_{xy}\} & I_m\{F_{xx} F_{xy}\} \\ |F_{yx}|^2 & |F_{yy}|^2 & R_e\{F_{xy} F_{yy}\} & I_m\{F_{xy} F_{yy}\} \\ 2R_e\{F_{xx} F_{yx}\} & 2R_e\{F_{xy} F_{yy}\} & \{F F^*_{xx} F_{yy} F_{xy}\} & \{F_{xx} F_{yy} F_{xy} - F_{xy} F_{yy}\} \\ -2I_m\{F_{xx} F_{yx}\} & -2I_m\{F_{xy} F_{yy}\} & -I_m\{F_{xx} F_{yy} F_{xy}\} & R_e\{F_{xx} F_{yy} - F_{xy} F_{yy}\} \end{bmatrix}$$

$$(5.2.1)$$

This equation may be arrived at by substituting Eq. 3.2.13 in the definition for the Stokes spectral vector applied to monochromatic signals. With the aid of this expression and Eq. 4.2.13 we may now derive the following explicit values of the elements of the effective temperature vector $\overline{T_e}$:
Eq. 5.2.2 is not very useful in its most general form owing to the fact that its right hand side contains more unknown quantities (the four complex elements of $F$, together with $T$) than equations. However, Eq. 5.2.2 reduces to the following form when the simplifications introduced in subsections 3.2 and 5.1 hold, and second-order terms are neglected,

$$T_{ex} = T - |F_{xx}|^2 (T-T_{inc})$$

$$T_{ey} = T - |F_{xx}|^2 |p_{22}|^2 (T-T_{inc})$$

$$T_e (x) = 2R e (F_{xx} F_{yx}^* + F_{xy} F_{yy}^*) (T-T_{inc})$$

$$T_e (y) = 0$$

These relations can further be worked out to

$$T_{ex} = T - |F_{xx}|^2 (T-T_{inc})$$

$$T_{ey} = T - |F_{xx}|^2 [1-R e \{ \int 8 \Gamma e \cos(2\phi_o') dx' \}] (T-T_{inc})$$

$$T_e (x) = 2|F_{xx}|^2 R e \{ \int 8 \Gamma e \sin(2\phi_o') dx' \} (T-T_{inc})$$

$$T_e (y) = 0$$

The following rain parameters may be calculated from these equations (see section A6)

$$e^{-\beta} = \frac{T-T_{ex}}{|T-T_{inc}|} \frac{1}{[1-e^{-\theta e (T-T_{ex})} TI]}$$
5.3. Simultaneous measurements on a monochromatic signal and the effective temperatures (theory)

From subsection 5.2, we infer that the knowledge of the state of polarization of the incident monochromatic signals requires, in the case of weak depolarization effects, (necessary and sufficient) measurements in two different polarization modi in order to determine the quantities $\rho_{xy}$ and $\rho_{yy}$.

Performing temperature measurements, as proposed in subsection 5.2., it is only possible to determine the quantities $R_e\{\rho_{xy}\}$ and $R_e\{\rho_{yy}\}$, together with the attenuation factor $\beta$.

Let us next consider the case of simultaneous measurements, concerning monochromatic signals as well as effective temperatures. This combination is useful in view of the relations, shown below, between the $XPL$ parameters and the thermal emission temperatures.

Comparing Eq. 5.1.4 with 5.2.6 and 5.2.7 we expect the validity of the following relations:

$$R_e\{\rho_{xy}\} = \frac{1}{\theta} T_e (x) \{T - T_{ex}\}^{-1},$$

(5.3.1)

$$R_e\{\rho_{yy}^{-1}\} = \frac{1}{\theta} T_e (x) \{T - T_{ex}\}^{-1},$$

(5.3.2)

Analogous general relations for $I_m\{\rho_{xy}\}$ and $I_m\{\rho_{yy}^{-1}\}$ cannot be derived.

However, for special cases outlined below, it proves to be possible to find approximative relationships connecting the latter two quantities to some thermal emission parameters. We assume the validity of the relation

$$\delta T = \eta \delta T_R$$

(5.3.3)

and

$$\delta T_R = q \Gamma_R$$

(5.3.4)

where $\eta$ and $q$ must be independent of $z$ throughout the rain path. In reality $\eta$ and $q$ are slightly dependent on the rain intensity; see Figs. 12 and 13 showing the theoretical value of $\eta$ and $\eta$ respectively (taken from [14]).
The special circumstances admitting simplifications are the following:

1. If $\phi_I$ vanishes, we have (see Eqs. 5.1.7 and 5.1.8)

\[
I_m (\rho_{xy}) = \eta_c R e (\rho_{xy}) ,
\]

\[
I_m (\rho_{yy} - 1) = \eta_c^{-1} R e (\rho_{yy} - 1) ,
\]

while the following inequality also holds (see section A 7)

\[
(\Delta T_e)^2 + (T_e (x))^2 < q^2 B^2 (T - T_{ex})^2
\]

2. If $\phi_o$ is constant throughout the rain path, the following equations are applicable (see section A 8)

\[
I_m (\rho_{xy}) = \left(\frac{q^2 B \eta_c}{\Delta T e (x) + \Delta T e} \right) \sqrt{\frac{(\Delta T e)^2 + (T_e (x))^2 - q^2 B^2 (T - T_{ex})^2}{(\Delta T e)^2 + (T_e (x))^2}}.
\]

3. If $\phi_I$ vanishes while $\phi_R$ is constant throughout the rain path, we have

\[
(\Delta T_e)^2 + (T_e (x))^2 = (T - T_{ex})^2 q^2 B^2
\]

The above considerations are summarized in the validity of the equations listed below, provided that the rain path conditions symbolized in the left hand side are realized:

\[
V [Eqs. 5.2.5, 5.3.1, 5.3.2],
\]

(5.3.11)
\[ V_{\text{rp } \varepsilon \text{c}} \text{ [Eqs. 5.3.8, 5.3.9]}, \]

\[ V_{\text{rp } \varepsilon \text{N}_L} \text{ [Eqs. 5.3.5, 5.3.6, 5.3.7]}, \]

\[ V_{\text{rp } \varepsilon \text{ N}_L \cap C} \text{ [Eqs. 5.3.10]}, \]

\( \text{rp symbolizes a specific rain path with parameters } \phi_o(rp), \delta \Gamma_o(rp), \text{ etc.}; \)

\( V \) is the ensemble of all rain paths compatible with our model (axisymmetric raindrops, weak depolarizations, etc.);

\( C \) is the ensemble of those elements of \( V \) for which \( \phi_o \) is constant throughout the rain path.

\( N_L \) is the ensemble of those elements of \( V \) for which \( \phi_I \) vanishes.
6. Methods of measurements; a polarization modulator using a rotating phase-shifting plate in a circular waveguide

Instruments that measure the four Stokes parameters are called polarimeters. A sub-class of polarimeters, suited for high-frequency applications, are the set of these instruments using the principle of polarization modulation. In such a system a phase or amplitude modulation, which is different for different polarizations, is induced on to the incoming signal. A suitable polarization component of the thus modulated signal is then isolated by an analyzer. The low frequency power of this component should contain harmonics, with frequencies which are multiples of the modulation frequency $\nu_0$. The amplitudes of these harmonics are functions of the required Stokes parameters, and in some cases complete recovery of the latter may be obtained.

In this section we shall present some theoretical considerations concerning a polarimeter using a modulator with a rotating phase-shifting plate in a circular waveguide. For a more general theoretical analysis of polarization modulators the reader is referred to the work of G.B. Gel'freikh[27], while technical descriptions of some polarimeters have been presented by several authors[28]-[31].

6.1. The ideal situation

The situation is as illustrated by Fig.12. Here the receiver, e.g. a Dicke radiometer, measures the power of the output of the analyzer. The antenna converts the incident wave $E_{inc}$ into a wave $A_{0}E_{x}(\theta)\tilde{f}_{1}$ in the antenna feed, while $E_{inc}$ produces $A_{0}E_{y}(\theta)\tilde{f}_{2}$, $\tilde{f}_{1}$, $\tilde{f}_{2}$ are unit field modi. of the antenna feed viz.

$$<\tilde{f}_{1}^{*}\tilde{f}_{1^{*}}> = 1, \quad <\tilde{f}_{2}^{*}\tilde{f}_{2^{*}}> = 1,$$

(6.1.1)

where the brackets $<$ symbolizes integration over the transverse plane in the feed. For an ideal antenna we shall consider $\tilde{f}_{2}$ to be perpendicular
The total wave $\vec{v}$ delivered by the antenna feed is given by the expression:

$$\vec{v} = A_0 \{ E_x(0) \vec{F}_1 + E_y(0) \vec{F}_2 \}$$  \hspace{1cm} (6.1.3)

This wave is transformed linearly by the modulator to the wave $\vec{w}$, so as to yield

$$\vec{w} = \overline{M}\vec{v}$$  \hspace{1cm} (6.1.4)

where $\overline{M}$ represents a linear operator.

The modulator consists of a phase-shifting plate which is rotated at a frequency $\Psi_0$ in a circular waveguide. Ideally, the operator $\overline{M}$ referring to this modulator should have mutually orthogonal eigenvectors $\vec{f}_{n1}$, $\vec{f}_{n2}$, known as the normal modi[27]. With respect to these unit vectors as bases, $\overline{M}$ has the normal representation:

$$\overline{M} = \begin{bmatrix} M_{n1n1} & 0 \\ 0 & M_{n2n2} \end{bmatrix}$$  \hspace{1cm} (6.1.5)

where $M_{n1n1}$ and $M_{n2n2}$ are the usually complex quantities

$$M_{n1n1} = \exp\{-(\frac{1}{2} \tau_1 + j \phi_1)\} \hspace{1cm} \text{(6.1.6)}$$

$$M_{n2n2} = \exp\{-(\frac{1}{2} \tau_2 + j \phi_2)\} \hspace{1cm} \text{(6.1.7)}$$

with $\tau_1$, $\tau_2$, $\phi_1$ and $\phi_2$ real. The phase-shifting plate causes the quantity $\phi = \phi_1 - \phi_2$ to have a significant constant value. Ideally, $\Lambda = \Psi_0(\tau_1 - \tau_2)$, representing the difference of absorption of the $\vec{f}_{n1}$ and $\vec{f}_{n2}$ modi in the modulator, should vanish.

The rotary movement of the phase-shifting plate implies that $\vec{f}_{n1}$, $\vec{f}_{n2}$ should be connected to the base system $\vec{f}_1$, $\vec{f}_2$ by a rotary transform contain-
ing the angle $\chi = \psi_0 t$. We assume the following relation:

\[
\begin{bmatrix}
\tilde{f}_{n1} \\
\tilde{f}_{n2}
\end{bmatrix} =
\begin{bmatrix}
\cos \chi & \sin \chi \\
-\sin \chi & \cos \chi
\end{bmatrix}
\begin{bmatrix}
\tilde{f}_1 \\
\tilde{f}_2
\end{bmatrix}.
\] (6.1.8)

The representation of Eq. 6.1.4 in the $\tilde{f}_1, \tilde{f}_2$ base system may now be derived with the aid of the above relation. This results in the following expression:

\[
\begin{bmatrix}
w_1 \\
w_2
\end{bmatrix} =
\begin{bmatrix}
\cos \chi & -\sin \chi \\
\sin \chi & \cos \chi
\end{bmatrix}
\begin{bmatrix}
M_{n1n1} & 0 \\
0 & M_{n2n2}
\end{bmatrix}
\begin{bmatrix}
\cos \chi & \sin \chi \\
-\sin \chi & \cos \chi
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix}.
\] (6.1.9)

The analyzer, again ideally, transmits only the $w_1 \tilde{f}_1$ mode of the incoming $\tilde{w}$ wave to the receiver. For a certain bandwidth, the latter measures the power $P_0$ of $w_1$; its low-frequency component is given by the expression (cf section A9):

\[
P_0 = \sum_{n=0,1,2} \left\{ P_{c, 2n+1} \sin(2n\psi_0 t) + P_{c, 2n} \cos(2n\psi_0 t) \right\} e^{-\tau},
\] (6.1.10)

where $\tau = \frac{1}{2}(\tau_1 + \tau_2)$, while the other coefficients are as follows:

\[
P_{c, 0} = \frac{1}{2}V + \frac{1}{2}V \{ \cosh \Lambda + \cos \phi \},
\]

\[
P_{c, 2} = \frac{1}{2}V(1) \sinh \Lambda + \frac{1}{2}V(2) \sin \phi,
\]

\[
P_{c, 4} = \frac{1}{2}V(1) \{ \cosh \Lambda - \cos \phi \};
\]

\[
P_{c, 4} = \frac{1}{2}V(1) \{ \cosh \Lambda - \cos \phi \}.
\] (6.1.11)
\( \Delta V, V \) are given by
\[
V = V_1 + V_2, \quad \Delta V = V_1 - V_2,
\]
(6.1.12)

while \( V_1, V_2, V^{(1)}, V^{(2)} \) are the Stokes parameters associated with \( v_1, v_2 \). From the above equations we see that detection of the \( \cos 4\omega_0 t \) and the \( \sin 4\omega_0 t \) components, may lead to a determination of \( \Delta V \) and \( V^{(1)} \). The d.c. and \( \sin \omega_0 t \) components contain information on \( V \) and \( V^{(2)} \) respectively; however, for our cases of interests the latter is only of minor importance since it is expected to vanish (cf. section 4.2).

In actual practice the analyzer also admits a certain crosspolar component to the receiver, so in this case it is the power \( P'_o \) of
\[
\omega_1' = \omega_1 + 2\omega_2
\]
that is measured by the receiver; \( b \) is complex with \(|b| << 1\).

The power \( P'_o \) measured by the receiver is now given by the expression (cf. section A.9)
\[
P'_o = \sum_{n=0,1,2} \{ P'_{e,n} \sin(2n\omega_0 t) + P'_{c,n} \cos(2n\omega_0 t) \} e^{-T}
\]
(6.1.13)
where, neglecting of the unimportant second order terms \(|b|^2\), we have the following approximations
\[
P'_{c,0} = 0 \quad ,
\]
\[
P'_{c,2} = P_{c,2} - V^{(2)} \sin(\phi) b_R - \Delta V \sin(\phi) b_I ,
\]
\[
P'_{c,4} = P_{c,4} - \frac{1}{2} V^{(1)} (\cosh\lambda - \cos\phi) b_R ,
\]
\[
P'_{c,4} = P_{c,4} - \frac{1}{2} V^{(1)} (\cosh\lambda - \cos\phi) b_R
\]
where \( b_R = \text{Re}(b), b_I = \text{Im}(b) \).

\[
\text{neglecting of the unimportant second order terms} \quad |b|^2
\]
The amplitudes of the $\sin(\psi_0 t)$ and $\cos(\psi_0 t)$ components, are now linear combinations of $V$, $V^{(1)}$, so that the determination of them demands either the knowledge of the value of $b_R$, or that $b_R$ should be sufficiently small to admit of valid approximations, viz.

$$|b_R| \ll |\Delta V/V^{(1)}|, \quad |V^{(1)}/\Delta V|$$  \hspace{1cm} (6.1.15)

For the determination of $V$, the errors introduced by $b_R$ and $b_I$ should be negligible, since

$$|V|^2 \gg |\Delta V|^2 + |V^{(1)}|^2 + |V^{(2)}|^2$$  \hspace{1cm} (6.1.16)

in accordance with the assumed weakness of the polarization degree of the emission.

6.2. Non ideal-modulator

In the previous section it was assumed that a set of orthogonal modi may be found which diagonalizes the matrix operator $\tilde{M}$. When dealing with a practical phase-shifter we must assume that a certain degree of interaction between orthogonal modi will always be induced by it. This property is manifest in the non-diagonal character of the $\tilde{M}$ matrix for all orthogonal bases, in other word, the eigenvectors of $\tilde{M}$ are not mutually orthogonal. Therefore, Eq.6.1.9 will now be replaced by the following one:

$$\egin{align*}
\begin{bmatrix}
\nu_1 \\
\nu_2
\end{bmatrix} &=
\begin{bmatrix}
cos \chi & -sin \chi \\
sin \chi & cos \chi
\end{bmatrix}
\begin{bmatrix}
m_{n1n1} & m_{n1n2} \\
m_{n2n1} & m_{n2n2}
\end{bmatrix}
\begin{bmatrix}
\nu_1 \\
\nu_2
\end{bmatrix}
\end{align*}$$ \hspace{1cm} (6.2.1)

$$|m_{n1n2}/m_{n1n1}| \ll 1 , \quad |m_{n2n1}/m_{n2n2}| \ll 1$$ \hspace{1cm} (6.2.2)

However, we shall assume that the deviation from orthogonality of the phase-shifter, in the above sense, is small, viz.
Eq. 6.2.1 may be related to the ideal situation by expressing it in the following form:

\[
\begin{bmatrix}
\omega_1 \\
\omega_2
\end{bmatrix}
= \begin{bmatrix}
\cos x & -\sin x \\
\sin x & \cos x
\end{bmatrix}
\begin{bmatrix}
M_{n1n1} & 0 \\
0 & M_{n2n2}
\end{bmatrix}
\begin{bmatrix}
\cos x & \sin x \\
-\sin x & \cos x
\end{bmatrix}
\begin{bmatrix}
\omega_1 \\
\omega_2
\end{bmatrix},
\]  

(6.2.3)

where

\[
\begin{bmatrix}
\omega_1' \\
\omega_2'
\end{bmatrix}
= \begin{bmatrix}
1+g_{11} & g_{12} \\
g_{21} & 1-g_{11}
\end{bmatrix}
\begin{bmatrix}
\omega_1 \\
\omega_2
\end{bmatrix},
\]  

(6.2.4)

here the matrix elements are as follows:

\[
g_{11} = l_1 \sin(2x), \quad g_{12} = l_2 + l_1 \cos(2x),
\]  

(6.2.5)

where \( l_1 \) and \( l_2 \) are related to the \( M \)'s by the equations:

\[
l_1 = \frac{k}{4}(M_{n1n2}M_2^{n1} + M_{n2n1}M_2^{n2})
\]  

(6.2.6)

\[
l_2 = \frac{k}{4}(M_{n1n2}M_2^{n1} - M_{n2n1}M_2^{n2}),
\]  

(6.2.7)

From Eq. 6.2.3 we see that the non-ideal modulator may be considered to function as an ideal modulator provided that the input signal is changed as in Eq. 6.2.4. This means that referring to \( \omega_1', \omega_2' \), in Eqs. 6.1.10 and 6.1.11 we must replace the Stokes parameters there appearing by the Stokes parameters \( \bar{V}' \):

\[
\bar{V}' = \bar{V} + \bar{V} + \Delta \bar{V} + V^{(1)} \bar{C} + V^{(2)} \bar{D}
\]  

(6.2.7)

the Stokes vectors \( \bar{A}, \bar{B}, \bar{C}, \bar{D} \) are homogeneous in the \( g \)'s, and may be
delivered from Eq.6.2.4.

As observed in the previous sections the expected degree of polarization should be small, and since the absolute values of the $g'$s are also assumed to be small, the following approximation of $\overline{V}'$ should hold:

$$\overline{V}' = \overline{V} + \overline{V}\Delta A$$  \hspace{1cm} (6.2.8)

Some calculation (section A.10) shows that $\Delta A$ is given by the following representation referring to the $\overline{f}_{1}, \overline{f}_{2}$ base system:

$$\Delta A = \begin{bmatrix} A \\ A^{(1)} \\ A^{(2)} \end{bmatrix} = \begin{bmatrix} |l_{1}|^2 + |l_{2}|^2 \\ -2l_{1}R\sin(2\chi)+2(l_{1}l_{2}^{*})R\cos(2\chi) \\ +2l_{1}R\cos(2\chi)+2(l_{1}l_{2}^{*})R\sin(2\chi) \\ 2l_{2}R \end{bmatrix} ,$$  \hspace{1cm} (6.2.9)

where the indices $R$ and $I$ denote real and imaginary parts of a complex quantity.

By replacing the Stokes parameters, occurring in Eqs.6.1.13 and 6.1.14, by the Stokes parameters $\overline{V}'$ given in Eqs.6.2.8 and 6.2.9, the power $P_{0}^{\pm}$ of the receiver (including the effect of the analyzer cross-polarization) is obtained:

$$P_{0}^{\pm} = \sum_{n=0, \pm 2} \left\{ P_{s,2n}^{\pm} \sin(2n\Omega t) + P_{c,2n}^{\pm} \cos(2n\Omega t) \right\} e^{-\tau} \hspace{1cm} , (6.2.10)$$

where

$$P_{s,0}^{\pm} = 0$$

$$P_{s,\pm 2}^{\pm} = P_{s,\pm 2}^{\pm} + V \left\{ \frac{1}{2}( |l_{1}|^2 + |l_{2}|^2 ) \cosh\Lambda + (l_{1}l_{2})*R \sinh\Lambda \\ +2(l_{1}l_{2}^{*})R \cos\phi + (l_{1}l_{2}^{*})R \sin\phi \right\} , \hspace{1cm} (6.2.11)$$
The above equations show that the amplitudes of the \( \sin(\Psi_0 t) \) and the \( \cos(\Psi_0 t) \) components, are not affected by the non-orthogonality of the normal modi of the modulator. Furthermore, the errors introduced in the d.c. component are of second order in the \( l \)'s and \( b \)'s and are therefore unimportant.

6.3. Effect of antenna cross-polarization

In section 6.1 it was assumed that \( E_x \) and \( E_y \) induce mutually orthogonal fields in the antenna feed. In reality, owing to the asymmetric structure of the antenna reflector, (the supports remain present), orthogonality will be lost. The relation between \( v_1, v_2 \) and \( E_x, E_y \) should now be of the following type:

\[
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix} = A_o \begin{bmatrix}
1+a_{11} & a_{12} \\
a_{21} & 1-a_{11}
\end{bmatrix} \begin{bmatrix}
E_x(0) \\
E_y(0)
\end{bmatrix} ;
\]

(6.3.1)

the coefficients \( a_{11}, a_{12}, a_{21} \) should be such that the eigenvectors of the above matrix are not mutually orthogonal. We shall assume, however, slight departures from orthogonality, i.e.

\[
|a_{11}|, |a_{12}|, |a_{21}| < 1
\]

(6.3.2)

The Stokes vector \( \vec{V} \) is now no longer proportional to the Stokes vector \( \vec{I} \) associated with the incident wave \( E_x, E_y \). From Eq.6.3.1 we can show that \( \vec{V} \) is related to \( \vec{I} \) through an expression similar to Eq.6.2.7.

As in the previous section, we arrive at the following approximation (neglecting the factor \( |A_o|^2 \))

\[
\vec{V} = \vec{I} + \vec{Q} \]

(6.3.3)
where \( \bar{q} \) is given by the following representation referring to the \( f_1, f_2 \) base system:

\[
\begin{bmatrix}
  Q \\
  \Delta Q \\
  Q^{(1)} \\
  Q^{(2)}
\end{bmatrix}
= \begin{bmatrix}
  |a_{11}|^2 + \frac{1}{2}(|a_{12}|^2 + |a_{21}|^2) \\
  2 \text{Re}a_{11} + \frac{1}{2}(|a_{12}|^2 - |a_{21}|^2) \\
  \text{Re}\{(1+\alpha_{11})a_{21}^* + (1-\alpha_{11})a_{12}\} \\
  -\text{Im}\{(1+\alpha_{11})a_{21}^* + (1-\alpha_{11})a_{12}\}
\end{bmatrix}.
\]

(6.3.4)

From the above equations we see that the values of \( \Delta V \) and \( V^{(1)} \) may be significantly different from those of \( \Delta I \) and \( I^{(1)} \) respectively,

\[
\Delta V = \Delta I + I(2 \text{Re}a_{11} + \frac{1}{2}(|a_{12}|^2 - |a_{21}|^2)) ,
\]

\[
V^{(1)} = I^{(1)} + I \text{ Re}\{(1+\alpha_{11})a_{21}^* + (1-\alpha_{11})a_{12}\} ;
\]

(6.3.5)

the second terms on the right hand side of the above equations represent the errors obtained if \( \Delta V \) and \( V^{(1)} \) are used as measures of \( \Delta I \) and \( I^{(1)} \) respectively.

To facilitate further analysis we shall ignore the second order terms in Eq.6.3.4. The errors obtained in Eq.6.3.5 are much smaller than the parameters looked for if

\[
|2 \text{Re}a_{11}| \ll |\Delta I/I| \quad (6.3.6)
\]

in the case of measuring \( \Delta I \), and

\[
|\text{Re}(a_{12}^*a_{21})| \ll |I^{(1)}/I| \quad (6.3.7)
\]

in the case of measuring \( I^{(1)} \).

The factors \( 2 \text{Re}a_{11} \) and \( \text{Re}(a_{12}^*a_{21}) \) can be expressed approximately as functions of the gains for \( x \) and \( y \) polarizations, viz. \( G_x, G_y \), and the gains for linear polarizations in the \( \pi/4 \) and \( 3\pi/4 \) directions, viz. \( G_{\pi/4}, G_{3\pi/4} \), respectively as follows

\[
G_x^{-1} G_y^{-1} = 1+4 \text{Re}a_{11} \quad ,
\]

(6.3.8)

\[
G_{\pi/4}^{-1} G_{3\pi/4}^{-1} = 1+2 \text{Re}(a_{12}^*a_{21}) \quad .
\]
By substituting these expressions in Eqs. 6.3.6 and 6.3.7, we obtain the following constraints on the antenna characteristics:

\[ |G_x G_y^{-1} - 1| < 2 |\Delta I/I| \]  \hspace{1cm} (6.3.9)

and

\[ \left| G_{\pi} \frac{G_{3\pi}}{d} - 1 \right| < 2 \left| I^{(2)} / I \right| \]  \hspace{1cm} (6.3.10)

From the analysis in section 4 we may expect \(|\Delta I/I|\) to have the numerical value of about 4%, while from a consideration given by [14] we are led to a value of about 0.5% for \(|I^{(2)}/I|\), so that Eqs. 6.3.9 and 6.3.10 lead to the numerical limits:

\[ |G_x G_y^{-1} - 1| < 8 \times 10^{-2} \]  \hspace{1cm} (6.3.11)

and

\[ \left| G_{\pi} \frac{G_{3\pi}}{d} - 1 \right| < 10^{-2} \]  \hspace{1cm} (6.3.12)

From the above considerations it follows that one way to secure sufficient measurement accuracy is by using an antenna that is compatible with the constraints Eq. 6.3.11 and 6.3.12. However, since the factors \(\Delta Q\) and \(Q^{(1)}\) are expected to be statically dependent on the physical antenna configurations, we may from a knowledge of their numerical values and the value of \(I\) find the necessary corrections of the values of \(V\) and \(V^{(1)}\) to obtain \(\Delta I\) and \(I^{(1)}\). The inverse of Eq. 6.3.5 delivers

\[ \Delta I = \Delta V - \Delta Q I \]  \hspace{1cm} (6.3.13)

\[ I^{(1)} = V^{(1)} - Q^{(1)} I \]  \hspace{1cm} (6.3.14)
7. Conclusions

The average effective canting angle $\phi_0$, introduced in this paper, proves to be an important quantity in the theory concerning depolarization by rain. A dependence of the raindrop orientations on their sizes leads in general to a complex value of the quantity $\phi_0$. The occurrence of raindrops with different shapes may also produce this effect. The determination of the average effective canting angle $\phi_0$ is useful since the cross-polarization parameter depends on it. However, because of lack of information on the statistics of raindrop orientations it is not yet possible to obtain a reliable value of quantity $\phi_0$. The statistics of $\phi_0$ along the rainpath are needed as well. Thus Eq. 32.18. shows that $XPL$ will be reduced considerably, compared to the case of homogeneous rain $XPL$, when $\phi_0$ is known to alternate randomly between positive and negative values (while $\delta \Gamma_0$ is constant). In this context it is worth mentioning that a model explaining the canting of the individual raindrops has recently been put forward by Brussaard [9]. In this model the gradient of wind velocity has been shown to be important to the development of the canting in question.

The effect of thermal emission has been described by the Stokes spectral parameters. It was shown that the propagation of these parameters through the rain medium is governed by an equation constituting an extension of the classical transfer equation for the non-scattering media. According to the solution of this equation the thermal emission due to rain may be slightly polarized. Further, it appears that measurements of thermal emission may be used to calculate the values of the cross-polarization parameter, at least when some special conditions are satisfied.
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A.1. Derivation of the eigenvectors $\bar{M}_A, \bar{M}_B$ and of the eigenvalues $\Gamma_A, \Gamma_B$

To obtain the eigenvalues and eigenvectors of the matrix operator $\tilde{\Gamma}$, we give the eigenvalue equations for $\tilde{\Gamma}$ with respect to the bases $\bar{U}_x, \bar{U}_y$

$$
\begin{bmatrix}
\Gamma_{xx} - \Gamma & \Gamma_{xy} \\
\Gamma_{xy} & \Gamma_{yy} - \Gamma
\end{bmatrix}
\begin{bmatrix}
M_x \\
M_y
\end{bmatrix} =
\begin{bmatrix}
0 \\
0
\end{bmatrix}
$$

(A.1.1)

A necessary condition for the consistency of the two equations contained in Eq. A.1.1 is the vanishing of their determinant, that is

$$
\Gamma_B^2 - (\Gamma_{xx} + \Gamma_{yy})\Gamma + \Gamma_{xx}\Gamma_{yy} - \Gamma_{xy}\Gamma_{yx} = 0.
$$

(A.1.2)

The solution of this equation yields two eigenvalues $\Gamma_A$ and $\Gamma_B$, viz.

$$
\Gamma_A = \frac{1}{2}(\Gamma_{xx} + \Gamma_{yy}) + \sqrt{\frac{1}{4} + \mu^2}, \quad \Gamma_B = \frac{1}{2}(\Gamma_{xx} + \Gamma_{yy}) - \sqrt{\frac{1}{4} + \mu^2},
$$

(A.1.3)

in which

$$
\mu = \frac{2\Gamma_{xy}}{\Gamma_{xx} - \Gamma_{yy}}, \quad (\Gamma_{xx} \neq \Gamma_{yy})
$$

(A.1.4)

The eigenvectors are next obtained by substituting the values of $\Gamma_A$ and $\Gamma_B$ in Eq. A.1.1, and by subsequently solving $\bar{M}_x$ and $\bar{M}_y$ from

$$
\begin{bmatrix}
1 + \sqrt{1 + \mu^2} & \mu \\
\mu & -1 + \sqrt{1 + \mu^2}
\end{bmatrix}
\begin{bmatrix}
(M_A)_x \\
(M_A)_y
\end{bmatrix} =
\begin{bmatrix}
0 \\
0
\end{bmatrix}
$$

(A.1.5)

Eq. A.1.5 leads to eigenvectors proportional to:

$$
\bar{M}_A \equiv \left\{ \frac{\mu}{\sqrt{1 + \mu^2}}, \frac{1}{\sqrt{1 + \mu^2}} \right\}
$$

(A.1.6)

$$
\bar{M}_B \equiv \left\{ -1 + \frac{1}{\sqrt{1 + \mu^2}}, \frac{\mu}{\sqrt{1 + \mu^2}} \right\}
$$

Next, we define $\phi_0$ by the equation
\[ \tan(2\phi_0) = u, \quad (A.1.7) \]

the substituting of which in Eq. A.1.6 results in

\[ \bar{M}_A \hat{=} 2 \sin\phi_0 \{ \cos\phi_0, \sin\phi_0 \}, \quad (A.1.8) \]

\[ \bar{M}_B \hat{=} 2 \sin\phi_0 \{-\sin\phi_0, \cos\phi_0 \}. \]

We proceed to the corresponding normalized vectors, viz.

\[ \bar{M}_A \hat{=} D^{-k} \{ \cos\phi_0, \sin\phi_0 \}, \quad (A.1.9) \]

\[ \bar{M}_B \hat{=} D^{-k} \{-\sin\phi_0, \cos\phi_0 \}. \]

where

\[ D = |\cos\phi_0|^2 + |\sin\phi_0|^2 \quad (A.1.10) \]

The explicit expressions for \( \Gamma_A \) and \( \Gamma_B \) are obtained by passing from the representation of the \( \Gamma \) matrix to the \( U_x, U_y \) bases to that with respect to the bases \( \bar{M}_A, \bar{M}_B \). Hence:

\[
\begin{bmatrix}
\Gamma_A \\
\Gamma_B
\end{bmatrix}
= \begin{bmatrix}
\cos\phi_0 & \sin\phi_0 \\
-\sin\phi_0 & \cos\phi_0
\end{bmatrix}
\begin{bmatrix}
\Gamma_{xx} & \Gamma_{xy} \\
\Gamma_{yx} & \Gamma_{yy}
\end{bmatrix}
\begin{bmatrix}
\cos\phi_0 & -\sin\phi_0 \\
\sin\phi_0 & \cos\phi_0
\end{bmatrix},
\]

\[
(A.1.11)
\]

giving

\[ \Gamma_A = \Gamma_{xx} \cos^2\phi_0 + \Gamma_{yy} \sin^2\phi_0 + \Gamma_{xy} \sin2\phi_0, \quad (A.1.12) \]

\[ \Gamma_B = \Gamma_{xx} \sin^2\phi_0 + \Gamma_{yy} \cos^2\phi_0 - \Gamma_{xy} \sin2\phi_0. \]

The combination of this result with the Eq. 3.1.32 yields the Eq. 3.1.35.
A.2. Dipole approximation of the raindrop scattering mechanisms

The basic idea underlying this approximation is the assumption that the primary field induces in each raindrop a dipole moment; that moment generates the secondary field.

The relation between the dipole moment \( \vec{a} \) and the primary incident field \( \vec{E}^{\text{inc}} \) should be linear according to

\[
\vec{a} = \vec{q} \vec{E}^{\text{inc}}
\]  

(A.2.1)

\( \vec{q} \) being a 3 by 3 matrix operator. \( \vec{q} \) may be given by the following representation

\[
\begin{bmatrix}
\alpha_l \\
\alpha_m \\
\alpha_n
\end{bmatrix} =
\begin{bmatrix}
\lambda_l & 0 & 0 \\
0 & \lambda_m & 0 \\
0 & 0 & \lambda_n
\end{bmatrix}
\begin{bmatrix}
\vec{E}^{\text{inc}}_l \\
\vec{E}^{\text{inc}}_m \\
\vec{E}^{\text{inc}}_n
\end{bmatrix}
\]  

(A.2.2)

referring to its own eigenvectors, say, \( \vec{l}, \vec{m}, \vec{n} \) (with their corresponding eigenvalues \( \lambda_l, \lambda_m, \lambda_n \)).

In view of the axial symmetry of the raindrops, symmetry considerations involve that two of the eigenvalues, say, \( \lambda_l, \lambda_m \) are equal, while the third \( \lambda_n \), corresponds to an eigenvector \( \vec{n} \) directed along the symmetry axis. The two other eigenvectors \( \vec{l} \) and \( \vec{m} \) are situated in a plane perpendicular to the symmetry axis, and may be taken orthogonal to each other (see Fig. A.2.1).

By introducing the transform which connects the \( \vec{l}, \vec{m}, \vec{n} \) bases with the \( \vec{U}_x, \vec{U}_y, \vec{U}_z \) bases, we obtain from relation A.2.1 the following representation referring to the \( xyz \) coordinate system:

\[
\begin{bmatrix}
\alpha_x \\
\alpha_y \\
\alpha_z
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \theta & \sin \theta \\
0 & -\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
\lambda_l & 0 & 0 \\
0 & \lambda_l & 0 \\
0 & 0 & \lambda_n
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \theta & -\sin \theta \\
0 & \sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
\vec{E}^{\text{inc}}_x \\
\vec{E}^{\text{inc}}_y \\
\vec{E}^{\text{inc}}_z
\end{bmatrix}
\]  

(A.2.3)
This can be worked out to
\[
\begin{bmatrix}
\alpha_x \\
\alpha_y \\
\alpha_z
\end{bmatrix} = \begin{bmatrix}
\lambda_l & 0 & 0 \\
0 & \lambda_l \cos^2 \theta + \lambda_n \sin^2 \theta & (\lambda_n - \lambda_l) \frac{\sin 2\theta}{2} \\
0 & (\lambda_n - \lambda_l) \frac{\sin 2\theta}{2} & \lambda_l \sin^2 \theta + \lambda_n \cos^2 \theta
\end{bmatrix} \begin{bmatrix}
E_{\text{inc}}^x \\
E_{\text{inc}}^y \\
E_{\text{inc}}^z
\end{bmatrix}
\]
(A.2.4)

Let us now consider a primary TE field \(E_z = 0\), propagating in the \(z\)-direction incident upon the axisymmetric raindrop situated as in Fig. A.2.1. According to the classical theory (cf. [15]) the forward scattered field generated by the dipole is given by
\[
\begin{bmatrix}
E_{\text{scat}}^x \\
E_{\text{scat}}^y \\
0
\end{bmatrix} = C \begin{bmatrix}
\alpha_x \\
\alpha_y \\
0
\end{bmatrix} = C \begin{bmatrix}
\lambda_l E_{\text{inc}}^x \\
\{\lambda_l \cos^2 \theta + \lambda_n \sin^2 \theta\} E_{\text{inc}}^y \\
0
\end{bmatrix}
\]
(A.2.5)

where the proportionality constant \(C\) is to be determined by the following considerations. We shall require that for \(\theta = \frac{\pi}{2}\) Eq. A.2.5 should give the same field as Eq. 2.12. We thus obtain the equation
\[
\begin{bmatrix}
C \lambda_l E_{\text{inc}}^x \\
C \lambda_n E_{\text{inc}}^y
\end{bmatrix} = \frac{k_0^2}{2\pi R} e^{-jk_0 R} \begin{bmatrix}
S_{/\text{f}} (i) (r, \frac{\pi}{2}, \omega) & 0 \\
0 & S_{/\text{f}} (i) (r, \frac{\pi}{2}, \omega)
\end{bmatrix} \begin{bmatrix}
E_{\text{inc}}^x \\
E_{\text{inc}}^y
\end{bmatrix}
\]
(A.2.6)

This relation can only hold for all values of \(E_{\text{inc}}^x\) and \(E_{\text{inc}}^y\) if we take
\[
C \lambda_l = S_{/\text{f}} (i) (r, \frac{\pi}{2}, \omega) k_0^2 \frac{e^{-jk_0 R}}{2\pi R}
\]
(A.2.7)

\[
C \lambda_n = S_{/\text{f}} (i) (r, \frac{\pi}{2}, \omega) k_0^2 \frac{e^{-jk_0 R}}{2\pi R}
\]
(A.2.8)

Substitution of these formulas in Eq. A.2.5 yields the following approximations of \(S_{/\text{f}} (i) (r, \theta, \omega)\) and \(S_{/\text{f}} (i) (r, \theta, \omega)\)
Using these expressions we then obtain the phasors given by Eqs. 3.1.39 and 3.1.40.

\[ S^{(i)}_{\perp}(r, \theta, \omega) = S^{(i)}_{\perp}(r, \frac{\pi}{2}, \omega), \]  
\[ S^{(i)}_{\parallel}(r, \theta, \omega) = S^{(i)}_{\parallel}(r, \frac{\pi}{2}, \omega) \sin^2 \theta + S^{(i)}_{\perp}(r, \frac{\pi}{2}, \omega) \cos^2 \theta. \]
A.3. Derivation of the $\tilde{\mathbf{W}}$ matrix

The representation of $\mathbf{r}$ with respect to the bases $\mathbf{\bar{U}}$, $\mathbf{\bar{U}}'$ may be related to its diagonal representation with respect to $\mathbf{\bar{U}}$, $\mathbf{\bar{U}}'$ through the complex rotation given by Eq. 3.1.33, viz.

$$
\mathbf{\bar{r}} = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \Gamma_A & 0 \\ 0 & \Gamma_B \end{bmatrix} \begin{bmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{bmatrix}
$$

(A.3.1)

or worked out

$$
\mathbf{\bar{r}} = \begin{bmatrix} \Gamma_A \cos^2 \phi + \Gamma_B \sin^2 \phi \\ \{\Gamma_A - \Gamma_B\} \frac{\sin \phi}{2} \\ \{\Gamma_A - \Gamma_B\} \frac{\cos \phi}{2} \\ \Gamma_A \sin^2 \phi + \Gamma_B \cos^2 \phi \end{bmatrix}
$$

(A.3.2)

Introducing the quantities

$$
\Gamma_0 \equiv \frac{1}{2} \{\Gamma_A + \Gamma_B\},
$$

$$
\delta \Gamma_0 \equiv \Gamma_B - \Gamma_A,
$$

we may put Eq. 3.2. in the form

$$
\tilde{\mathbf{W}} = \Gamma_0 \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} \delta \Gamma_0 & 1 \end{bmatrix} \begin{bmatrix} \cos \{2 \phi_0\} & +\sin \{2 \phi_0\} \\ \sin \{2 \phi_0\} & -\cos \{2 \phi_0\} \end{bmatrix}
$$

(A.3.3)

By comparing this form with that of Eq. 3.2.2 we arrive at Eq. 3.2.4 for $\tilde{\mathbf{W}}$. 
A.4. An estimation of the complexity of $\Phi_0$

We shall consider a special situation characterized by the following circumstances:

(i) propagation and wind direction are both horizontal and perpendicular to each other; they can thus be taken along the $z$ and $y$ axis respectively (Fig. A.4.1).

Moreover the windflow $U$ is assumed to be linear, increasing with the vertical coordinate $x$, with a constant gradient $\alpha$, i.e.

$$U = \alpha x \quad (x > 0) \quad (A.4.1)$$

(ii) The drop distribution (referring to $r$) is given by the Marshall-Palmer expression (cf.[1]):

$$N = N_0 e^{-2\lambda r} \quad (A.4.2)$$

with

$$\lambda = 4.1 \times p^{-0.21} \text{mm}^{-1} \quad (A.4.3)$$

($p$ in mm/hr, $r$ in mm, $N_0 = 8000 \text{m}^{-3}\text{mm}^{-1}$).

(iii) The terminal speed $V$ (in vertical direction) of the falling drops is given by

$$V(r) = V_0 \{ 1 - e^{-\alpha r} \} \quad (A.4.4)$$

Referring to a figure given by Medhurst [4] we assume the representative parameter values

$$V_0 = 9.14 \text{ m/s}, \quad \alpha = 0.4 \text{ mm}^{-1} \quad (A.4.5)$$
The canting angle $\phi_M$ of each raindrop is given by the Brussaard formula [19]:

$$\tan \phi_M = \frac{aV}{g}$$  \hspace{1cm} (A.4.6)

where $g$ is the gravitational acceleration.

We then have

$$\sin(2\phi_M) = \frac{2aV/g}{1+(2aV/g)^2}$$  \hspace{1cm} (A.4.7)

$$\cos(2\phi_M) = \frac{1}{1+(2aV/g)^2}$$  \hspace{1cm} (A.4.8)

According to these data the density distribution has the form

$$N_f(r,\phi,\theta) = N_0 e^{-2\Lambda r} \delta(\phi-\phi_M(r)) \delta(\theta-\pi/2) .$$  \hspace{1cm} (A.4.9)

Substituting this expression in Eq. 3.1.36 leads to the following expression for $\tan(2\phi_0)$:

$$\tan(2\phi_0) = \frac{\int_0^\infty dr \ N_0 e^{-2\Lambda r} \sin(2\phi_M(r)) \Delta S(r,\pi/2)}{\int_0^\infty dr \ N_0 e^{-2\Lambda r} \cos(2\phi_M(r)) \Delta S(r,\pi/2)}$$  \hspace{1cm} (A.4.10)

In special cases $|2aV/g| \ll 1$ holds, so that with the aid of Eqs. A.4.4, A.4.7 and A.4.8 we arrive at

$$\tan(2\phi_0) = \frac{2aV/g}{\int_0^\infty dr \ N_0 e^{-2\Lambda r} \Delta S(r,\pi/2)}$$  \hspace{1cm} (A.4.11)

The quantity $\int_0^\infty dr \ N_0 e^{-2\Lambda r} \Delta S(r,\pi/2)$ represents the complex differential-attenuation coefficient in the case of equioriented drops, i.e.

$$\delta \Gamma_0(p) = \int_0^\infty dr \ N_0 e^{-2\Lambda(r)p} \Delta S(r,\pi/2)$$  \hspace{1cm} (A.4.12)

The related quantity $\int_0^\infty dr \ N_0 e^{-2\Lambda(r)p} \Delta S(r,\pi/2)$ may be written as
where the effective rain intensity \( p' \) is defined by

\[
4.1 \ p' - 0.21 = 4.1 \ p - 0.21 + \frac{a}{\epsilon} \tag{A.4.14}
\]

or by

\[
p' = \frac{p}{1 + \frac{1}{4.1} p^{0.21}}. \tag{A.4.15}
\]

This leads to the relation:

\[
\tan(2\phi_0) = 2aV_0/g\left[1 - \frac{\delta\Gamma_\infty[p']}{\delta\Gamma_\infty[p]}\right] \tag{A.4.16}
\]

As an example we consider the case when \( p = 50 \text{ mm/h} \), then from Eq. A.4.15 we obtain the value \( p' = 25 \text{ mm/h} \). For \( \delta\Gamma_\infty \) we use the figures given by Chu [14]

\[
\delta\Gamma_\infty[50] = 0.078 + j \ 0.105, \tag{A.4.17}
\]

\[
\delta\Gamma_\infty[25] = 0.032 + j \ 0.050 \tag{A.4.18}
\]

In defining \( 2aV_0/g = \phi_\infty \), we obtain

\[
\tan(2\phi_0) = \phi_\infty[1 - \frac{0.032 + j \ 0.050}{0.078 + j \ 0.105}] \tag{A.4.19}
\]

or

\[
\tan(2\phi_0) = \phi_\infty[0.47 - j \ 0.08] \tag{A.4.20}
\]

Since \( \phi_\infty \) is generally small, the imaginary part of \( \tan(2\phi_0) \) is not very important. We may presume, therefore, that \( \phi_0 \) is real.
A.5. The expression $\frac{\partial \bar{E}_{T'}^r}{\partial z} = -K \bar{I}_{T'}^r$

The components of Eq. 3.1.29 with respect to the $\bar{U}_p, \bar{U}_q$ base read

$$\frac{\partial \bar{E}_{T'}^r}{\partial z} = -\Gamma_{pp} \bar{E}_{T'}^r - \Gamma_{pq} \bar{E}_{T'}^q$$  \hspace{1cm} (A.5.1)

$$\frac{\partial \bar{E}_{T'}^q}{\partial z} = -\Gamma_{qp} \bar{E}_{T'}^r - \Gamma_{qq} \bar{E}_{T'}^q$$  \hspace{1cm} (A.5.2)

Substituting the above equations in the derivatives

$$\frac{\partial}{\partial z} (E_{T'}^p E_{T'}^p)^* = (\frac{\partial E_{T'}^p}{\partial z})^{*} E_{T'}^p + \tilde{E}_{T'}^p (\frac{\partial E_{T'}^p}{\partial z})$$ \hspace{1cm} (A.5.3)

$$\frac{\partial}{\partial z} (E_{T'}^q E_{T'}^q)^* = (\frac{\partial E_{T'}^q}{\partial z})^{*} E_{T'}^q + \tilde{E}_{T'}^q (\frac{\partial E_{T'}^q}{\partial z})$$ \hspace{1cm} (A.5.4)

gives

$$-\frac{2}{\partial z} (\bar{E}_{T'}^r \bar{E}_{T'}^r)^* = 2\text{Re}(\Gamma_{pp}) \bar{E}_{T'}^r \bar{E}_{T'}^r + \text{Re}(\Gamma_{pq}^*) 2\text{Re}(\bar{E}_{T'}^r \bar{E}_{T'}^q)^* - I_m(\Gamma_{pq}^*) \tilde{E}_{T'}^r \tilde{E}_{T'}^q$$  \hspace{1cm} (A.5.5)

$$-\frac{2}{\partial z} (\bar{E}_{T'}^q \bar{E}_{T'}^q)^* = \Gamma_{qp}^* \bar{E}_{T'}^r \bar{E}_{T'}^q + \text{Re}(\Gamma_{pq}^*) \bar{E}_{T'}^r \bar{E}_{T'}^q + (\Gamma_{pp}^* + \Gamma_{pq}^*) \bar{E}_{T'}^r \bar{E}_{T'}^q$$ \hspace{1cm} (A.5.6)

Combining these relations with the definition for the "truncated" Stokes parameters we arrive at Eq. 4.1.16 with the following explicit expression:

$$\begin{bmatrix} I_{T'}^r(p) \\ I_{T'}^r(q) \end{bmatrix} = \begin{bmatrix} 2\text{Re}(\Gamma_{pp}) & 0 & \text{Re}(\Gamma_{pq}) & I_m(\Gamma_{pq}) \\ 0 & 2\text{Re}(\Gamma_{pq}) & \text{Re}(\Gamma_{pp} + \Gamma_{pq}) & I_m(\Gamma_{pp} + \Gamma_{pq}) \\ 0 & 0 & 2\text{I}_m(\Gamma_{pq}) & -2\text{I}_m(\Gamma_{pq}) \end{bmatrix} \begin{bmatrix} I_{T'}^r(p) \\ I_{T'}^r(q) \\ I_{T'}^r(r) \\ I_{T'}^r(q) \end{bmatrix}$$ \hspace{1cm} (A.5.7)
By taking \( p \) and \( q \) to refer to the components of \( \vec{E} \) with respect to the normalized eigenvectors \( \vec{M}_A, \vec{M}_B \) we see that Eqs. 4.1.17 and 4.1.18 follow. By substituting the presentation of \( \vec{E} \) as given by Eq.A.3.3 we may define the following expression for the extinction coefficient matrix:

\[
\vec{k} = K_O \vec{I} + \text{Re}\{\vec{\sigma}_c \cos(2\phi_0) + \vec{\sigma}_s \sin(2\phi_0)\};
\]

where the "average extinction coefficient" \( K_O \) equals the sum of the real parts of the complex attenuation coefficient for the two principal polarizations, i.e.

\[
K_O = \text{Re}\{\Gamma_A + \Gamma_B\}
\]

\( \vec{I} \) is the unit matrix, while the two matrices \( \vec{\sigma}_c \) and \( \vec{\sigma}_s \) have the following representation in the \( xy \) coordinate system:

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 0 & -i \\
0 & 0 & i & 0
\end{bmatrix}
\quad \begin{bmatrix}
0 & 0 & \frac{1}{2} & \frac{1}{2}i \\
0 & 0 & \frac{1}{2} & \frac{1}{2}i \\
1 & 1 & 0 & 0 \\
-i & i & 0 & 0
\end{bmatrix}
\]

(A.5.8)
A.6. Derivation of Eq. 5.2.5, 5.2.6 and 5.2.7

From Eq. 5.2.4 it follows that

\[ |F_{xx}|^2 = \frac{(T-T_{ax})}{(T-T_{inc})}, \]

and also that

\[ R_e \left\{ \int \delta \Gamma_o \cos(2\phi_o) dz' \right\} = \frac{i \frac{1}{2} (T_{ay} - T_{ax})}{\left[ |F_{xx}|^2 (T-T_{inc}) \right]} \]

\[ R_e \left\{ \int \delta \Gamma_o \sin(2\phi_o) dz' \right\} = \frac{i \frac{1}{2} T_a (x)}{\left[ |F_{xx}|^2 (T-T_{inc}) \right]} \]

By substitution of Eq. A.6.1 and Eq. A.6.2 we obtain Eqs. 5.2.6 and 5.2.7.

From subsection 3.2 we know that \( F_{xx} \) may be approximated by

\[ F_{xx} = e^{-\gamma} [1 + \frac{1}{2} \int \delta \Gamma_o \cos(2\phi_o) dz'] \]

This equation leads to the following approximation:

\[ |F_{xx}|^2 \approx e^{-\beta} [1 + R_e \left\{ \int \delta \Gamma_o \cos(2\phi_o) dz' \right\} + ...] \]

so that

\[ e^{-\beta} \approx |F_{xx}|^2 [1 - R_e \left\{ \int \delta \Gamma_o \cos(2\phi_o) dz' \right\} + ...] \]

By substituting in this expression Eqs. A.6.1 and 5.2.6 we obtain the form for \( e^{-\beta} \) as given by Eq. 5.2.5.
A.7. The inequalities Eq. 5.3.7 and 5.3.10

If $\phi_I$ is assumed to vanish, Eqs. 5.2.6 and 5.2.7 reduce to

$$\Delta T_a = 2(T-T_{ax}) \int_{z_0}^{z} \delta \Gamma_R(z') \cos 2\phi_R(z') dz'$$

$$T_a(x) = 2(T-T_{ax}) \int_{z_0}^{z} \delta \Gamma_R(z') \sin 2\phi_R(z') dz'$$

These two relations yield the equation

$$\frac{(\Delta T_a)^2 + (T_a'(x))^2}{4(T-T_{ax})^2} \int_{z_0}^{z} \int_{z_0}^{z} dz' dz'' \delta \Gamma_R(z') \cos 2[\phi_R(z') - \phi_R(z'')] \delta \Gamma_R(z'')$$

(A.7.1)

Since $\delta \Gamma_R(z') > 0$ and $|\cos \phi| < 1$, we may conclude that

$$\frac{(\Delta T_a)^2 + (T_a'(x))^2}{4(T-T_{ax})^2} \leq \frac{1}{4} \int_{z_0}^{z} dz' \delta \Gamma_R(z')$$

(A.7.2)

where the equal sign only holds if $\phi_R$ is constant throughout the integration path.

Finally, applying the theoretical relations (5.3.3), (5.3.4) and the definition for $\beta$, we arrive at Eq. 5.3.7 and Eq. 5.3.10.
A.8. Derivation of Eqs. 5.3.8 and 5.3.9

Let $\phi_o$ be constant throughout the rainpath. Further, the theoretical relations given by Eqs. 5.3.3 and 5.3.4, viz.,

\begin{align}
\delta \Gamma_1 &= \eta c \delta \Gamma_R, \quad \delta \Gamma_R = q \Gamma_R, \\
\end{align}

are assumed to hold.

$\Delta T_{a}$ and $T_{a}(x)$ may then be represented by

\begin{align}
\Delta T_a &= 2(T - T_{ax}) \{\omega c h + \eta c s i. s h\} \int \delta \Gamma_R \, dz', \\
T_{a}(x) &= 2(T - T_{ax}) \{s i. c h - \eta c c o s h\} \int \delta \Gamma_R \, dz',
\end{align}

as may be verified with the aid of the Eqs. 5.2.6 and 5.2.7.

The integral containing $\delta \Gamma_R$ may be related to $\beta = \int_0^z 2 \Gamma_R \, dz'$ through Eq. A.8.1, giving

\begin{align}
\int \delta \Gamma_R \, dz' &= \frac{1}{2} \beta q^2,
\end{align}

Let $D_1$ and $D_2$ be defined by:

\begin{align}
D_1 &= \Delta T_a / [q \beta (T - T_{ax})], \\
D_2 &= T_a(x) / [q \beta (T - T_{ax})],
\end{align}

so that the Eqs. A.8.2 and A.8.3 reduce to

\begin{align}
\omega c h + \eta c s i. s h &= D_1, \\
s i. c h - \eta c c o s h &= D_2,
\end{align}

These expressions together with the relations

\begin{align}
\omega^2 + s i^2 &= 1, \\
ch^2 - s h^2 &= 1,
\end{align}

yield the values of $\omega$, $s i$, $c h$ and $s h$, which in turn may be used to calculate
\( I \{ \rho_{xy} \} \) and \( I \{ 1-\rho_{yy} \} \).

We shall first determine \( \text{ch} \) and \( \text{sh} \). From Eq. A.8.7 we derive

\[
\begin{align*}
\omega_2 \sin \phi + 2 \eta_c \cos \phi + \cos^2 \sin \phi \sin^2 \phi &= D_1 \phi, \\
\sin^2 \phi + 2 \eta_c \cos \phi + \sin^2 \phi \sin \phi \cos \phi &= D_2 \phi,
\end{align*}
\]

which, when added, results in:

\[
\omega^2 + \eta_c^2 (\sin^2 \phi - 1) = D_1 \phi^2 + D_2 \phi.
\]

With the aid of Eq. A.8.8 we then find:

\[
\begin{align*}
\text{ch} &= + \sqrt{\frac{D_1 \phi^2 + D_2 \phi - \eta_c^2}{1 + \eta_c^2}}; \\
\text{sh} &= \sqrt{\frac{D_1 \phi^2 + D_2 \phi - 1}{1 + \eta_c^2}}.
\end{align*}
\quad \text{(A.8.9)}
\]

the plus sign of the square root should be used for \( \text{ch} \), while the sign of \( \text{sh} \) is left open.

To obtain next \( \omega \) and \( \sin \), we first multiply the first part of Eq. A.8.7 by \( \omega \) and the second by \( \sin \); adding then yields

\[
\omega = (D_1 + D_2) \frac{\omega \phi + D_2 \omega \phi}{1 - \eta_c \phi}, \quad \text{(A.8.10)}
\]

By solving Eqs. A.8.10 and A.8.11 we get the following expressions for \( \omega \) and \( \sin \):

\[
\begin{align*}
\omega &= (D_1 \phi^2 + D_2 \phi - 1)[D_1 \phi + D_2 \eta_c \phi] \\
\sin &= (D_1 \phi^2 + D_2 \phi - 1)[D_1 \phi + D_2 \eta_c \phi].
\end{align*}
\quad \text{(A.8.12)}
\]
$I_m\{\rho_{xy}\}$ and $I_m\{1-\rho_{yy}\}$ are connected to $co$, $si$, $ch$ and $sh$ by the relations:

$$I_m\{\rho_{xy}\} = \frac{4q}{\pi} \beta (co \, sh + \eta_c \, si \, sh),$$

$$I_m\{1-\rho_{yy}\} = \frac{4q}{\pi} \beta (\eta_c \, co \, sh - si \, sh),$$

which can be verified with the aid of the Eqs. 5.1.4 and A.8.4.

By substituting Eqs. A.8.9 and A.8.12 in the above expressions for $I_m\{\rho_{xy}\}$ and $I_m\{1-\rho_{yy}\}$ we obtain:

$$I_m\{\rho_{xy}\} = \frac{4q}{\pi} \beta (D_1^2 + D_2^2)^{-1} [\eta_c D_2 + D_1 \sqrt{(D_1^2 - 1)(D_1^2 + D_2^2 + \eta_c^2)}]$$

$$I_m\{1-\rho_{yy}\} = \frac{4q}{\pi} \beta (D_1^2 + D_2^2)^{-1} [\eta_c D_1 - D_2 \sqrt{(D_1^2 + D_2^2 - 1)(D_1^2 + D_2^2 + \eta_c^2)}]$$

Finally, with the aid of the definitions for $D_1$ and $D_2$ Eqs. 5.3.8 and 5.3.9 result.
A.9. The power $P_o$

The Stokes spectral vector $\mathbf{\tilde{V}}$ of the $\tilde{v}$ wave is given by the following representation referring to $\mathbf{\tilde{f}_1}$, $\mathbf{\tilde{f}_2}$ modi

$$\mathbf{\tilde{V}} = \begin{bmatrix} V \\ \Delta V \\ V^{(1)} \\ V^{(2)} \end{bmatrix},$$  

(A.9.1)

where $V = V_1 + V_2$, $\Delta V = V_1 - V_2$, while $V_1$, $V_2$, $V^{(1)}$, $V^{(2)}$ are defined as in section 4 for the $I$'s.

Likewise, we have

$$\mathbf{\tilde{W}} = \begin{bmatrix} W \\ \Delta W \\ W^{(1)} \\ W^{(2)} \end{bmatrix}$$  

(A.9.2)

as the representation of the Stokes spectral vector $\mathbf{\tilde{W}}$ of the corresponding $\tilde{w}$ wave. A straightforward analysis shows that Eq. 6.1.9 leads to the following relation between the Stokes spectral vectors $\mathbf{\tilde{V}}$ and $\mathbf{\tilde{W}}$, expressed in $\mathbf{\tilde{f}_1}$, $\mathbf{\tilde{f}_2}$ representation:

$$\begin{bmatrix} \Delta W \\ W^{(1)} \\ W^{(2)} \end{bmatrix} = e^{-t} [RT]^{-1} \begin{bmatrix} \cosh \Delta & \sinh \Delta & 0 & 0 \\ -\sin \Delta & \cosh \Delta & 0 & 0 \\ 0 & 0 & \cos \phi & -\sin \phi \\ 0 & 0 & \sin \phi & \cos \phi \end{bmatrix} [RT] \begin{bmatrix} V \\ \Delta V \\ W^{(1)} \\ W^{(2)} \end{bmatrix}$$  

(A.9.3)
where the matrix $[RT]$ representing a rotational transform is as follows:

$$
[RT] = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos(2\chi) & \sin(2\chi) & 0 \\
0 & -\sin(2\chi) & \cos(2\chi) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

(A.9.4.)

By working out the matrix multiplications on the right hand side of Eq. A.9.3 we obtain the following 4 by 4 matrix:

$$
\begin{bmatrix}
coshA & \sinhA\cos(2\chi) & \sinhA\sin(2\chi) & 0 \\
-sinhA\cos(2\chi) \coshA\cos^2(2\chi) + \frac{1}{2}\coshA\sin(4\chi) & -\sinhA\sin(2\chi) \\
+\cosA\sin^2(2\chi) - \frac{1}{2}\cosA\sin(4\chi) & \coshA\cos^2(2\chi) + \sinA\cos(2\chi) \\
-sinhA\sin(2\chi) & \coshA\sin(4\chi) - \cosA\sin(4\chi) & \coshA\sin(4\chi) + \sinA\cos(2\chi) & \cosA
\end{bmatrix}
$$

(A.9.5.)

The power $P_o$ of $W_1$ given by the expression $P_o = \frac{1}{2}(\dot{W} + \Delta W)$ then readily follows, and is given by Eqs. 6.1.10 and 6.1.11.
A.10. The explicit form of the Stokes vector $\mathbf{\overline{A}}$

From Eq. 6.2.4 we may show that $\mathbf{\overline{A}}$ is given by the following representation, referring to $\mathbf{\tilde{f}}_1$, $\mathbf{\tilde{f}}_2$:

$$
\begin{align*}
\mathbf{\overline{A}} &= \begin{bmatrix}
A \\
\Delta A \\
A^{(1)} \\
A^{(2)}
\end{bmatrix} = \\
&= \begin{bmatrix}
|g_{11}|^2 + \frac{1}{2}(|g_{12}|^2 + |g_{21}|^2) \\
2\text{Re}g_n + \frac{1}{2}(|g_{12}|^2 - |g_{21}|^2) \\
\text{Re}[(1+g_{11})g_{21}^* + (1-g_{11}^*)g_{12}^*] \\
\text{Im}[(1+g_n)g_{21}^* + (1-g_n^*)g_{12}^*]
\end{bmatrix}
\end{align*}
$$

By substituting Eq. 6.2.5, here, we obtain Eq. 6.2.9.
Fig. 1 An axisymmetric raindrop, with the unit vector $\vec{n}$.

Fig. 2 The orthogonal $xyz$ coordinate system showing the canting angle $\phi$ and the incident angle $\theta$. 
Fig. 3 The incident versus the scattered field in the \( \ell / \parallel z \) coordinate system.

Fig. 4 A coordinate transform in the \( xy \) plane.
Fig. 5 |sin(2φ₀)/sin(2φᵣ)|² as a function of φᵢ with φᵣ as parameter.
Fig. 6 ΔT as a function of the rain intensity p for different frequencies.
Fig. 7 The rain medium extending indefinitely in the horizontal plane between cloud and ground.

Fig. 8 $\gamma$ as a function of $\eta$. 
Fig. 9.

Fig. 10. An 8-cm map of the S-component on April 19, 1958, derived from eclipse observations. (Tanaka [34,])
The spectra of different components of solar radio emission. The quiet sun component corresponds to sunspot minimum, whereas the slowly varying component (statistically determined) corresponds to sunspot maximum (the IGY period). The spectra of bursts and storms correspond to their maximum values and are plotted in terms of flux density (after Smerd 1964a).

Fig. 11
Fig. 12 \((A_{II} - A_{I})/A_{I} \approx q\) as a function of the frequency and rain intensity \(p\) (taken from [14]).

Fig. 13 \((\phi_{II} - \phi_{I})/A_{I} \approx qn_{c}\) as a function of the frequency and rain intensity (taken from [14]).
Frequency: 11 GHz.

<table>
<thead>
<tr>
<th>$p$ (mm/h)</th>
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<th>$T_v$</th>
<th>$\Delta T$</th>
<th>$T_h$</th>
<th>$T_v$</th>
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Table 1. $\Delta T$ as a function of the rain intensity $p$, the frequency, and the rain path length $L$. 
Frequency: 11 GHz.

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<th>$T_v$ (°C)</th>
<th>$\Delta T$ (°C)</th>
<th>$T_h$ (°C)</th>
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Table 1. (continuation)
Frequency: 11 GHz.

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Frequency: 11GHz.

<p>| $p$ (mm/hr) | $L=5.0km$ | | | | $L=5.5km$ | | | | $L=6.0km$ | | |
|-------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
|             | $T_h$    | $T_v$    | $\Delta T$ | $T_h$    | $T_v$    | $\Delta T$ | $T_h$    | $T_v$    | $\Delta T$ |
| 0.000       | 26.000   | 26.000   | 0.000     | 26.000   | 26.000   | 0.000     | 26.00     | 26.00     | 0.00      |
| 0.250       | 26.797   | 26.751   | 0.047     | 26.877   | 26.825   | 0.052     | 26.957    | 26.900    | 0.056     |
| 1.250       | 31.356   | 30.920   | 0.437     | 31.886   | 31.406   | 0.479     | 32.414    | 31.892    | 0.522     |
| 2.500       | 38.728   | 37.588   | 1.140     | 39.967   | 38.718   | 1.248     | 41.199    | 39.844    | 1.356     |
| 5.000       | 55.995   | 53.181   | 2.815     | 58.800   | 55.739   | 3.061     | 61.571    | 58.844    | 3.301     |
| 50.000      | 259.508  | 250.860  | 8.648     | 265.392  | 257.628  | 7.764     | 270.135   | 263.221   | 6.913     |
| 100.000     | 287.866  | 286.219  | 1.648     | 288.669  | 287.516  | 1.153     | 289.168   | 288.367   | 0.801     |
| 150.000     | 289.851  | 289.644  | 0.207     | 289.927  | 289.814  | 0.113     | 289.964   | 289.903   | 0.061     |</p>
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Table 1. $\Delta T$ as a function of the rain intensity $p$, the frequency, and the rainpath-length $L$. 

#1.5

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<th>$T_p$</th>
<th>$\Delta T$</th>
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<td>0.000</td>
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Table 1. $\Delta T$ as a function of the rain intensity $p$, the frequency, and the rainpath-length $L$. 

#2.5

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Table 1. $\Delta T$ as a function of the rain intensity $p$, the frequency, and the rainpath-length $L$. 

#3

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Table 1. $\Delta T$ as a function of the rain intensity $p$, the frequency, and the rainpath-length $L$. 

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<th>TV(P)</th>
<th>TP(P)</th>
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</table>
Table 1. \( \Delta T \) as a function of the rain intensity \( P \), the frequency, and the rain-path length \( L \).
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L = 3.50 km

Frequency: 30 GHz.