Distributed Control of Light Networks

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Abstract

Wireless networks are integrated into many of today’s technologies. A new wireless communication system is visible light communication. In the first part of this thesis, mesh networks with a linear structure are analysed, using the ALOHA and Carrier Sense Multiple Access protocol. After that, we discuss grid topology mesh networks, involving visible light communication. By simulation we obtain results for the performance of these networks, and pose some conjectures. Last, some conclusions are drawn concerning the optimal values of parameters we are free to choose, optimising two performance measures.
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Chapter 1

Introduction

Nowadays, wireless communication systems are used in more and more technologies. A new wireless communication system, currently researched by PHILIPS, uses visible light to exchange data. The main problem proposed by PHILIPS is the following. Consider a room in which a number of LEDs are installed, positioned on a grid. Whenever nobody is in the room, all of these LEDs are turned off, but when someone enters, all LEDs should be turned on as quickly as possible. However, the system does not have a central entity that notifies all of the LEDs at the same time. Reasons for this may be that, for large rooms, a totally connected network is impractical or expensive. Instead, the LED closest to the entering person picks up a signal and sends a message, saying that someone has entered, forward to its surrounding LEDs. The transmission of this message is done using Visible Light Communication (VLC). In the type of VLC considered in this thesis, LEDs are connected to a sensor, which can detect visible light fluctuations. The LED itself is able to blink very rapidly. When this blinking is done in certain frequencies, messages can be coded using light. The sensors of other LEDs are (within a certain range) able to receive this coded light and translate the message back into its original form. Typically, the first LED, picking up the signal of an entering person, cannot reach each LED in the room. Therefore, the LEDs that did receive the message from the first LED and that are already turned on, have to pass on the message by sending it to their surrounding inactive LEDs. This goes on until all LEDs have been reached, and thus the whole room is lit. The bottleneck of this process is the fact that not all transmissions will be successful due to interference. Our main focus is to minimise the time it takes to light up the whole room, which will be referred to as the hitting time. The process as described above is a typical case of wireless mesh networking. For these mesh networks, several protocols have been proposed, of which two important ones will be discussed in Chapter 2. In Chapter 3, the concept of mesh networking is described more extensively and to develop some intuition on the subject, we also analyse linear mesh networks.
here. Next, the exact algorithm for the process is given in Chapter 4, after which simulation results for this algorithm will be discussed in Chapter 5. Finally, we draw some conclusions from the analyses we have done.
Chapter 2

Wireless Networking Protocols

2.1 Introduction and Terminology

A wireless network is a special type of computer network which, as the name suggests, does not require cable connections. There are several types of communication methods that can replace the function of cables. Among these are radio waves, which are used for connecting devices such as laptops and routers, or mobile telephones. An emerging technology is to use visible light to distribute information. We return to visible light technology in Chapter 4. Although there are several types of wireless networks, they share a large set of features. First, a wireless network typically consists of many users. Examples of users are laptops, mobile telephones or, as in this thesis, light bulbs. For generality reasons, the users of the network will from now on be referred to as nodes.

Second, two types of nodes can be considered, namely transmitter nodes and receiver nodes. Transmitter nodes are the ones that send packets of information, while the receiver nodes are the nodes that receive these packets. Every transmitter node has a transmission range, denoted by $\tau$. A transmitter node can only transmit a packet to a receiver node when the distance between the two nodes is smaller than $\tau$. Furthermore, it is possible for a node to be both a transmitter and a receiver. However, a node cannot transmit and receive at the same time. Therefore, the node has to switch modes whenever necessary. A connection between a transmitter node and a receiver node is called a link.

Third, the nodes of a wireless network share a medium, through which the packets are exchanged. This feature poses a problem. If multiple nodes within a certain range transmit at the same time, transmissions may fail. This phenomenon is known as interference. Our modelling assumption regarding interference is the following: whenever a receiver node senses signals from more than one transmitter node, that is, when there are multiple transmitter nodes active within the interference range $\eta$ of the receiver node, a collision occurs. A collision implies that all
current transmissions to this receiver node fail and the failed transmissions need to be repeated completely. To illustrate how these three features come together, we will look at an example. Figure 2.1 shows a wireless network consisting of four transmitter nodes $T_i$, and corresponding receiver nodes $R_i$, where $i = 1, 2, 3, 4$. The arrows between nodes represent the links. Also, the interference range of each receiver node is depicted. To model the interference between the links one can construct a so-called conflict graph. The vertices of a conflict graph represent the links, while the edges represent the interference constraints. This means that whenever two vertices in the conflict graph (links) share an edge, transmissions cannot occur over these links simultaneously, otherwise both transmissions will fail, due to a collision. The conflict graph corresponding to the wireless network in Figure 2.1 is shown in Figure 2.2. It can be seen that links 1 and 3 can be active at the same time without causing any problems, but a collision occurs if, for instance, links 1 and 2 are both active.

Because collisions in networks lead to a loss of capacity, collisions are undesirable. To prevent such collisions from happening, or minimise their occurrences, many network protocols have been introduced. These protocols attempt to guide the initiation of transmissions between nodes, by subjecting the links and nodes to certain rules. These rules are meant to regulate the occurrences of a collision. Two of such wireless networking protocols that are widely used are known under...
the names of ALOHA and *Carrier Sense Multiple Access* (CSMA). ALOHA represents the most basic protocol used, while CSMA describes a more sophisticated method to reduce the number of failed transmissions. The properties and variants of both protocols will be discussed in the next sections.

### 2.2 ALOHA

The ALOHA protocol was one of the first computer networking protocols, developed in the 1970s. Using a new method of medium access, called random access, ALOHA led the way to a new generation of network protocols. The original protocol (pure ALOHA) is considered simple, because it consists of only two basic rules. For every node in the network, these are:

- If a packet is available for transmission, send it.
- If a collision occurs, try sending the packet again later.

Now the question arises what is meant by ‘later’. When a node restarts its transmission immediately after the failed transmission ended, it can be easily understood that all future transmissions from this node will fail. Because a collision always involves two or more transmissions, these other nodes will also initiate a transmission immediately after their failed transmissions, which will result in even more collisions. It is therefore obvious that nodes should ‘wait’ a certain period of time before repeating their failed transmissions. This waiting time between transmissions of a node is referred to as the *backoff period*. This backoff period may be node specific or predefined for the whole network. However, all of these backoff periods should be random and independent. In the ALOHA protocol used in this thesis, the backoff periods are considered to be exponentially distributed with mean $1/\nu$. The parameter $\nu$ is referred to as the *activation rate* of a node. Figure 2.3 shows how collisions can occur using the ALOHA protocol. In this situation, the network described in Figure 2.1 is considered, using fixed deterministic
transmission times. Each block in the figure represents a attempted transmission.

These are either coloured green or red, whenever the transmission was successful or not, respectively. It can be seen that whenever two or more links, which share an edge in the corresponding conflict graph, are active simultaneously (which is depicted as overlapping blocks in the Figure 2.3), all transmissions under consideration will fail. This is the case for the first transmission over links 1 and 2, which overlap and therefore fail. Of course, transmissions without any overlapping transmission from interfering links will be successful, which is the case with the four green blocks.

ALOHA works fine for systems with low traffic. However, for systems with higher traffic rates, the chances of the occurrence of collisions increases greatly, which causes substantial loss of capacity. For networks with many nodes or more active nodes, more complex protocols are needed to reduce the number of collisions and maintain a reasonable level of throughput. One of these methods will be discussed now.

2.3 CSMA/CA

The Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) protocol, unlike ALOHA, is a method that actively attempts to prevent collisions from happening. This is done by prohibiting the initiation of a transmission by a transmitter node when another transmitter node is active within a certain range from the transmitter node. This range, specified by the protocol, is known as the sensing range, denoted by $\beta$, of the transmitter node. For all transmitter nodes in the network, the rules of this protocol are as follows:

- If a packet is available for transmission and no transmitter node is active within distance $\beta$, send it.

- If a packet is available for transmission and there is an active transmitter node within distance $\beta$, start a backoff period and try sending the packet again after this period.
• If a collision occurs, start a backoff period and try sending the packet again after this period.

Here, the backoff periods are again assumed to be exponentially distributed with mean $1/\nu$. Let us now see how the protocol works for the example given in the introduction of this chapter. We use a sensing range $\beta$ as depicted in Figure 2.4. From this figure, we can see that $T_2$ and $T_4$, and $T_3$ and $T_4$ cannot be active simultaneously. This means that whenever $T_2$ is active, $T_3$ cannot activate, and vice versa. The same holds for $T_3$ and $T_4$. Comparing the time scheme using the CSMA/CA protocol in Figure 2.5 to the one using the ALOHA protocol in Figure 2.3, we see that the two schemes differ in the second transmission of link 2 (which is absent in the second scheme) and the first transmission of link 4. Using the CSMA/CA protocol, link 2 cannot start its second transmission, because link 4 is already active. The protocol prohibits the activation of link 2. We see that
the first transmission of link 4 is successful because of that, in contrast with the network using ALOHA.

Because the sensing range $\beta$ is part of the protocol, we are free to choose its value. We can prove that for some values of $\beta$, no collisions can occur in the network.

**Proposition 2.3.1.** If $\beta > \eta + \tau$, then no collisions can occur.

**Proof.** A collision occurs whenever a link between a transmitter node, say $T_1$, and a receiver node, say $R_1$, is active, and some other transmitter node $T_2$ activates. $T_2$ can only activate when $T_2$ is not within the sensing range of $T_1$, thus

$$d(T_1, T_2) > \beta > \eta + \tau,$$

where $d(T_1, T_2)$ defines the (Euclidean) distance between $T_1$ and $T_2$. By the triangle equality we can furthermore say that

$$d(T_1, T_2) \leq d(T_1, R_1) + d(R_1, T_2) \leq \tau + d(R_1, T_2).$$

Combining Equations (2.1) and (2.2) yields,

$$\eta + \tau < \tau + d(R_1, T_2) \Rightarrow d(R_1, T_2) > \eta.$$

The distance between an active receiver node and an active transmitter node, other than the one it is linked to, is always greater than the interference range of the receiver node. Therefore, collisions are ruled out entirely. \(\square\)

Proposition 2.3.1 tells us that a large $\beta$ has the favorable effect of the absence of collisions. However, the drawback of a large $\beta$ is that it might block too many transmitter nodes, by which links unnecessarily stay inactive, which causes loss of throughput. Hence, there is a trade-off: a small $\beta$ makes the protocol faster, but also causes more collisions. This trade-off suggests that there must be some optimal sensing range, $\beta_{opt}$, for each network. We may even argue that there exists an lower and upper bound for $\beta_{opt}$, namely

$$\beta_{opt} \in [\eta - \tau, \eta + \tau].$$

For $\tau = 1$, [3] researches this trade-off for throughput optimality. As we have seen in Proposition 2.3.1, collisions cannot occur when $\beta > \eta + \tau$. So by choosing $\beta = \eta = \tau$, we already ruled out all collisions. By increasing $\beta$ further, the number of collisions is not reduced, but the extra transmitter nodes that are blocked would not have caused a collision when activated. Thus, choosing $\beta$ larger than $\eta + \tau$ only reduces capacity, which is undesirable. Because of this $\beta_{opt} \leq \eta + \tau$. The lower bound can be justified as follows. When choosing $\beta < \eta - \tau$, each extra transmitter node that is allowed to activate, compared to the situation with $\beta = \eta + \tau$, will cause a collision. And thus, $\beta_{opt} \geq \eta - \tau$. This yields the interval in Equation (2.4).
Chapter 3

Linear Mesh Networks

3.1 Model

Wireless mesh networks are a relatively new subject in the field of communication networks. These networks are characterised by the fact that packets are routed from a source to a destination through the network by hopping between intermediate nodes. This routing technique is useful whenever the destination node is not within the transmission range of the source node. Furthermore, mesh networks do not have a centralised entity that controls packet transmissions between nodes, which implies that each node can only use information about neighbouring nodes in its direct local area. In this chapter we will investigate a special type of mesh network, namely a linear network with transmission range set to the (fixed) distance between two directly neighbouring nodes. In this situation the nodes, which will be labeled $i = 1, \ldots, n$, are positioned along a line, where node 1 is leftmost and node $n$ is the rightmost node. Node 1 functions as the source, while node $n$ functions as the destination. The distance between neighbouring nodes is considered to be fixed and equal to one. In general mesh networks, nodes can transmit packets over more than one neighbouring node. However, in the network structure discussed here, only communication between directly neighbouring nodes is considered (so $\tau = 1$). Moreover, we consider only one packet to be in the system at all times, which starts at node 1, and hops from node 1 to 2, 2 to 3, and so on until it reaches node $n$. Transmitting nodes only interfere with their direct neigh-

![Figure 3.1: Linear single-hop network with five nodes](image-url)
bours (so \( \eta = 1 \)) and nodes can only initiate a transmission if they have received the packet correctly. This means that no node on the right of the current position of the packet can be active. Also, receiving nodes do not send any feedback to the transmitting node about whether the transmission was successful or not. Nodes which have received the packet therefore do not stop initiating transmissions until the packet has reached its destination. An exponentially distributed backoff period with mean \( 1/\nu_i \), specific for each node \( i = 1, \ldots, n \), as described in the previous chapter, is used. Without loss of generality, transmission times are assumed to be exponentially distributed with mean \( \mu = 1 \). By scaling \( \nu_i \) with \( \mu \) results can be found for each value of \( \mu \). Moreover, all of these backoff periods and transmission times are assumed to be independent.

Our goal is to minimise the time it takes for the packet under consideration to reach node \( n \), starting at node 1. This hitting time can be manipulated by changing the values of \( \nu_i \), where \( i = 1, \ldots, n \). In the next two sections we analyse this hitting time under the two protocols discussed in the previous chapter, namely ALOHA and CSMA/CA.

### 3.2 ALOHA on a Line

As described in the previous chapter, the ALOHA protocol is very basic. Nodes are allowed to transmit a packet whenever they have one available and should 'wait' for an exponentially distributed backoff period between two transmissions. It was argued that ALOHA works well for networks with a low traffic rate, but will lead to many collisions when traffic rates increase. We now analyse the performance of ALOHA in the described linear network.

Because the backoff periods and transmission times are both assumed to be exponentially distributed, the evolution of the packet through the network can be described as a Markov process. The corresponding state space \( S \) is defined by all feasible configurations of the \( n \) nodes. At any given point in time, the situation of an individual node is defined by the following two properties relevant for the Markov process:

- The node is either transmitting or silent (active or inactive).
- The node has either received the packet or not.

This implies that each state of the Markov process is uniquely defined by these properties for all nodes. A state is only feasible if an active node has also received the packet. Let us now introduce some notation. Because the configuration of each node individually is uniquely defined by the two (binary) properties above, each state in the state space can be easily defined numerically by a \( 2n \)-dimensional, binary vector. The first \( n \) coordinates of this vector describe which nodes are
active, and the last \( n \) coordinates show which nodes have received the packet. For example, for a line network of 5 nodes, the vector

\[
X = (1, 0, 1, 0, 0, 1, 1, 0, 0)
\]  

defines the state in which nodes 1 and 3 are transmitting, and so far nodes 1, 2 and 3 have received the packet correctly. Let \( e_i \), where \( i = 1, \ldots, n \), be the vector in \( 2^n \) dimensions, that equals 0 for all \( 2^n \) coordinates, except for coordinate \( n \), which equals 1, and \( w_i \), where \( i = 1, \ldots, n \), be the vector that equals 0 for all \( 2^n \) coordinates, except for coordinate \( n + i \), which equals 1. This way we can define the state space \( S \) corresponding to the Markov process as

\[
S := \left\{ \sum_{i \in I} e_i + \sum_{j \in J} w_j \mid I \subseteq J \subseteq \{1, \ldots, n\} \right\}
\]

(3.2)

To keep things tractable, we will consider separately the Markov processes at different stages in the process, in which the packet has reached node 1, 2, 3, etcetera, and fit the results of these submodels together afterwards. This way we only have to take into account the activation states of each node. Later we will see that the process to be observed at any stage, can be reduced to two nodes.

Now let us take a look at the initialization of the process. The image below shows a linear network of \( n \) nodes, where node 1 has a packet to send. The grey colour of node 1 indicates that this node has received the packet.

Because no other node has received the packet, only node 1 can now start a transmission, and no collisions can occur. But as the protocol defines, first an exponentially distributed backoff period with mean \( 1/\nu_1 \) is started. When this period ends, the transmission of the packet from node 1 to 2 is started. The time elapsed between the start and end of this transmission is exponentially distributed with mean \( 1/\mu = 1 \), as assumed earlier. The directed graph picturing the state transitions corresponding to the Markov process at this stage is shown in Figure 3.3, where ‘1’ represents the state in which node 1 is active and transmitting, and ‘0’ represents the state in which node 1 is inactive. The numbers on the arrows indicate the transition intensities. The transmission from node 1 to 2 is completed if the process, starting in state ‘0’, hops to state ‘1’, and enters state ‘0’ again. The expected time it takes for node 1 to start a transmission is \( 1/\nu_1 \), and the expected time it takes to complete the transmission equals \( 1/\mu = 1 \). We
can therefore conclude that this stage of the process takes on average $1/\nu_1 + 1$ units of time. The notation $T_i$ will be used for the time it takes the packet to reach node $i$. So here,

$$E_T = \frac{1}{\nu_1} + 1. \quad (3.3)$$

After the packet has reached node 2 successfully, we can look at a more general stage of the process, which will be referred to as stage $i$, when the packet has reached node $i$ and should be sent from node $i$ to node $i + 1$. As assumed before, active nodes only interfere with their direct neighbours. However, because node $i + 1$ has not received the packet yet, this node cannot become active. This implies that only the activation states of the left neighbour of node $i$, which is node $i - 1$, and node $i$ itself have to be taken into account. Remember that a transmission from one node fails whenever a direct neighbour of that node is active at any point during the transmission. Consequently, the transmission from node $i$ to node $i + 1$ is successful when node $i - 1$ is silent during its whole transmission time. Given that only nodes $i - 1$ and $i$ need to be considered in the analysis of this stage of the transmission, the Markov process consists of four states, namely

- (0,0) Both nodes $i - 1$ and $i$ are inactive.
- (1,0) Node $i - 1$ is active and node $i$ is inactive.
- (0,1) Node $i - 1$ is inactive and node $i$ is active.
- (1,1) Both nodes $i - 1$ and $i$ are active.

The matching state transition graph is shown in Figure 3.4. To successfully transmit the packet from node $i$ to $i + 1$, the following path should be followed:

$$(0, 0) \rightarrow (0, 1) \rightarrow (0, 0). \quad (3.4)$$

The process always starts in state (0,0), since the transmission from node $i - 1$ to $i$ has just been completed and node $i$ did not have any possibility to activate yet. Whenever the process enters state (1,0) or (1,1), the process needs to go back into
state (0,0) first, before trying to initiate a new transmission from node $i$. Because these ‘wrong paths’ play a role in our calculations, we need to find out how long it takes, on average, to return to state (0,0) from both states (1,0) and (1,1). We will denote these average times, respectively, by $E_{S_{i},(1,0),(0,0)}$ and $E_{S_{i},(1,1),(0,0)}$. Of course, $E_{S_{i},(0,1),(0,0)}$ stands for the mean time to reach state (0,0) from state (0,1).

To obtain these values, a system of equations can be constructed. For instance, whenever the process is in state (1,0), it takes on average $\frac{1}{\nu_{i} + 1}$ units of time to leave this state. Then, it either enters state (0,0) or (1,1), with probability $\frac{1}{\nu_{i} + 1}$ and $\frac{\nu_{i}}{\nu_{i} + 1}$, respectively. When entering (0,0), we are done (so remaining time is 0). When entering (1,1) however, it still takes $E_{S_{i},(1,1),(0,0)}$ on average to return to (0,0). This way, the following relationship between the states can be stated:

$$E_{S_{i},(1,0),(0,0)} = \frac{1}{\nu_{i} + 1} + \frac{1}{\nu_{i} + 1} \cdot 0 + \frac{\nu_{i}}{\nu_{i} + 1} \cdot E_{S_{i},(1,1),(0,0)}. \quad (3.5)$$

For states (0,1) and (1,1) the same reasoning can be applied, which leads to the following equations:

$$E_{S_{i},(0,1),(0,0)} = \frac{1}{\nu_{i-1} + 1} + \frac{1}{\nu_{i-1} + 1} \cdot 0 + \frac{1}{\nu_{i-1} + 1} \cdot E_{S_{i},(1,1),(0,0)}, \quad (3.6)$$

$$E_{S_{i},(1,1),(0,0)} = \frac{1}{2} + \frac{1}{2} \cdot E_{S_{i},(0,1),(0,0)} + \frac{1}{2} \cdot E_{S_{i},(1,0),(0,0)}. \quad (3.7)$$

Equations (2.5)-(2.7) form a linear system of equations, which can be solved, with
solution

\[
\begin{align*}
\mathbb{E}S_{i,(1,0),(0,0)} &= \frac{(2 + \nu_{i-1})(1 + \nu_i)}{2 + \nu_{i-1} + \nu_i}, \\
\mathbb{E}S_{i,(0,1),(0,0)} &= \frac{(1 + \nu_{i-1})(2 + \nu_i)}{2 + \nu_{i-1} + \nu_i}, \\
\mathbb{E}S_{i,(1,1),(0,0)} &= \frac{3 + 2\nu_i + \nu_{i-1}(2 + \nu_i)}{2 + \nu_{i-1} + \nu_i}.
\end{align*}
\] (3.8) (3.9) (3.10)

The average time it takes to transmit the packet from node \( i \) to \( i+1 \) correctly, will be denoted by \( \mathbb{E}S_i \). Nothing useful can be said about \( \mathbb{E}S_i \) yet, but we can look at some conditional expectations:

- \( \mathbb{E}[ S_i \mid (0,0) \rightarrow (1,0) \rightarrow ... ] \)
  
  The expected time it takes to complete the transmission, when hopping from state \((0,0)\) to \((1,0)\), return to \((0,0)\), etcetera. The probability of following this path is \( \frac{\nu_{i-1}\nu_i}{\nu_{i-1} + \nu_i} \).

- \( \mathbb{E}[ S_i \mid (0,0) \rightarrow (0,1) \rightarrow (1,1) \rightarrow ... ] \)
  
  The expected time it takes to complete the transmission, when hopping from state \((0,0)\) to \((0,1)\) to \((1,1)\), return to \((0,0)\), etcetera. The probability of following this path is \( \frac{\nu_{i-1}\nu_i}{(\nu_{i-1} + \nu_i)(1 + \nu_{i-1})} \).

- \( \mathbb{E}[ S_i \mid (0,0) \rightarrow (0,1) \rightarrow (0,0) ] \)
  
  The expected time it takes to complete the transmission, when hopping from state \((0,0)\) to \((0,1)\) and return to \((0,0)\), which is the correct path. The probability of following this path is \( \frac{\nu_i}{(\nu_{i-1} + \nu_i)(1 + \nu_{i-1})} \).

Because these three paths represent mutually exclusive paths and together form the whole sample space of paths starting from state \((0,0)\), we can say

\[
\mathbb{E}S_i = \frac{\nu_{i-1}\nu_i}{\nu_{i-1} + \nu_i} \cdot \mathbb{E}[ S_i \mid (0,0) \rightarrow (1,0) \rightarrow ... ] \\
+ \frac{\nu_{i-1}\nu_i}{(\nu_{i-1} + \nu_i)(1 + \nu_{i-1})} \cdot \mathbb{E}[ S_i \mid (0,0) \rightarrow (0,1) \rightarrow (1,1) \rightarrow ... ] \\
+ \frac{\nu_i}{(\nu_{i-1} + \nu_i)(1 + \nu_{i-1})} \cdot \mathbb{E}[ S_i \mid (0,0) \rightarrow (0,1) \rightarrow (0,0) ].
\] (3.11)

We now take a closer look at the first of the three conditional expectations, \( \mathbb{E}[ S_i \mid (0,0) \rightarrow (1,0) \rightarrow ... ] \). First, it takes the process on average \( 1/(\nu_{i-1} + \nu_i) \) to leave state \((0,0)\) and enter state \((1,0)\). Then, for the transmission to be successful, state \((0,0)\) has to be entered again, which takes on average \( \mathbb{E}S_{i,(1,0),(0,0)} \), before trying to follow the correct path again starting from state \((0,0)\), which takes on average \( \mathbb{E}S_i \). We therefore find that

\[
\begin{align*}
\mathbb{E}[ S_i \mid (0,0) \rightarrow (1,0) \rightarrow ... ] &= \frac{1}{\nu_{i-1} + \nu_i} + \mathbb{E}S_{i,(1,0),(0,0)} + \mathbb{E}S_i \\
&= \frac{1}{\nu_{i-1} + \nu_i} + \frac{(2 + \nu_{i-1})(1 + \nu_i)}{2 + \nu_{i-1} + \nu_i} + \mathbb{E}S_i.
\end{align*}
\] (3.12) (3.13)
A similar reasoning holds for the last two conditional expectations, which yields
\[
\mathbb{E}[S_i \mid (0,0) \to (0,1) \to (1,1) \to \ldots] = 1 + \frac{1}{\nu_{i-1} + \nu_i} + \mathbb{E}S_i(1,1),(0,0) + \mathbb{E}S_i,
\]
\begin{equation}
(3.14)
\end{equation}
and
\[
\mathbb{E}[S_i \mid (0,0) \to (0,1) \to (0,0)] = 1 + \frac{1}{\nu_{i-1} + \nu_i} + \frac{1}{1 + \nu_{i-1}}.
\]
\begin{equation}
(3.15)
\end{equation}
Substituting equations (3.13), (3.14) and (3.15) into equation (3.11) then gives us
\[
\mathbb{E}S_i = 1 + \frac{1}{\nu_{i-1}}(2 + \mathbb{E}S_i)(1 + \nu_i) + \nu_i^2(1 + \mathbb{E}S_i + \nu_i) + \nu_i(1 + \nu_i - 1)(\nu_i - 1 + \nu_i).
\]
\begin{equation}
(3.16)
\end{equation}
We now have gathered enough information to prove the following result.

**Proposition 3.2.1.** The mean time it takes the packet, starting at node 1, to reach node \( n \), is equal to
\[
\mathbb{E}T_n = 1 + \frac{1}{\nu_1} + \sum_{i=2}^{n-1} \frac{(1 + \nu_{i-1})^2(1 + \nu_i)}{\nu_i}.
\]
\begin{equation}
(3.17)
\end{equation}

**Proof.** By solving equation (3.16) for \( \mathbb{E}S_i \) algebraically, we obtain
\[
\mathbb{E}S_i = \frac{(1 + \nu_{i-1})^2(1 + \nu_i)}{\nu_i}.
\]
\begin{equation}
(3.18)
\end{equation}
Since this calculation is the same for all stages \( i = 2, 3, \ldots, n - 1 \), and knowing that \( \mathbb{E}S_1 = \mathbb{E}T_2 = 1 + \frac{1}{\nu_1} \) from equation (3.3), it can be concluded that
\[
\mathbb{E}T_n = \mathbb{E}S_1 + \sum_{i=2}^{n-1} \mathbb{E}S_i = 1 + \frac{1}{\nu_1} + \sum_{i=2}^{n-1} \frac{(1 + \nu_{i-1})^2(1 + \nu_i)}{\nu_i}.
\]
\begin{equation}
(3.19)
\end{equation}

As we want to minimise the time it takes for the packet to reach node \( n \), an investigation of the function \( \mathbb{E}T_n \) with \( \nu_1, \ldots, \nu_n \) as its variables, seems appropriate. However, when increasing \( n \), finding the optimal values for \( \nu_1, \ldots, \nu_n \), becomes virtually impossible. To be able to say anything about the function and its minimum, we assume that \( \nu_1 = \nu_2 = \ldots = \nu_n = \nu \), which gives
\[
\mathbb{E}T_n = \mathbb{E}T_n(\nu) = 1 + \frac{1 + (n - 2)(1 + \nu)^3}{\nu}.
\]
\begin{equation}
(3.20)
\end{equation}
In Figure 3.5 the graphs of \( \mathbb{E}T_n \) are plotted for some values of \( n \). These graphs suggest that a finite optimal value for \( \nu \) exists for \( n \geq 3 \). We prove this in the next proposition.
Proposition 3.2.2. The function $E T_n$ has a finite minimising value for all $n \geq 3$. This optimal value equals

$$\nu^* = \frac{1}{2} \left( \left( \frac{n - 2}{2\sqrt{n - 1} + n} \right)^{1/3} + \left( \frac{2\sqrt{n - 1} + n}{n - 2} \right)^{1/3} - 1 \right).$$

(3.21)

Furthermore, $\nu^*$ converges to $1/2$ as $n$ tends to infinity.

Proof. The value $\nu^*$ minimising $E T_n$ can be found by equating the partial derivative of $E T_n$, with respect to $\nu$, to zero, and solving this equation. We take a partial derivative, because $E T_n(\nu)$ has two variables, namely $n$ and $\nu$. So,

$$\frac{\partial E T_n}{\partial \nu} = -\frac{1}{\nu^2} + (n - 2) \left( 2\nu + 3 - \frac{1}{\nu^2} \right) = 0.$$  (3.22)

Solving this equation yields,

$$\nu^* = \frac{1}{2} \left( \left( \frac{n - 2}{2\sqrt{n - 1} + n} \right)^{1/3} + \left( \frac{2\sqrt{n - 1} + n}{n - 2} \right)^{1/3} - 1 \right).$$  (3.23)

From Equation (3.23) it is evident that $E T_2$ does not have a finite optimal value for $\nu$, because $\nu^*$ has a singularity at $n = 2$. To verify that $\nu^*$ indeed defines a minimum, we look at the second partial derivative of $E T_n$ with respect to $\nu$,

$$\frac{\partial^2 E T_n}{\partial \nu^2} = \frac{2}{\nu^3} + (n - 2) \left( \frac{2}{\nu^3} + 2 \right),$$  (3.24)
which is positive for all \( \nu > 0 \) and \( n \geq 2 \). This implies that this is indeed a minimum.

Finally, to prove that \( \nu^* \) converges to \( 1/2 \) as \( n \) tends to infinity, we introduce the function

\[
f(n) = \frac{n - 2}{2\sqrt{n - 1} + n}.
\]

Then,

\[
\nu^* = \frac{1}{2} \left( (f(n))^{1/3} + (f(n))^{-1/3} - 1 \right).
\]

Finding the limit of \( f \) at infinity is fairly simple, since

\[
f(n) = \frac{n - 2}{2\sqrt{n - 1} + n} = \frac{1 - 2/n}{2\sqrt{\frac{1}{n}(1 - \frac{1}{n})} + 1} \quad \text{as} \quad n \to \infty
\]

\[
= \frac{1 - 0}{2\sqrt{0(1 - 0)} + 1} = 1.
\]

And thus,

\[
\lim_{n \to \infty} \nu^* = \lim_{n \to \infty} \frac{1}{2} \left( (f(n))^{1/3} + (f(n))^{-1/3} - 1 \right)
\]

\[
= \frac{1}{2} \left( \left( \lim_{n \to \infty} f(n) \right)^{1/3} + \left( \lim_{n \to \infty} f(n) \right)^{-1/3} - 1 \right)
\]

\[
= \frac{1}{2} \left( 1^{1/3} + 1^{-1/3} - 1 \right) = \frac{1}{2}.
\]

(3.26)

Proposition 3.2.2 states that for large \( n \), the optimal backoff period is approximately twice the transmission time. This is quite remarkable. However, we have not been able to find an intuitive explanation for this result and leave this as an open problem to the reader.

### 3.3 CSMA/CA on a Line

As we have gathered some results about the linear network using the ALOHA protocol, we will now investigate how the CSMA/CA protocol performs in these network configurations. In this thesis, only the case where the sensing range is equal to the distance between two neighboring nodes (\( \beta = 1 \)) is considered. Larger values for \( \beta \) can be considered using the same kind of analysis as in this section. Because only exponentially distributed backoff periods and transmission times are
used, the evolution of packets through the network can again be described by a Markov process.

Now let us look at the initialization of the process. The packet starts its journey at node 1, and at this point, no other node can activate. This first stage thus takes

\[ ET_2 = 1 + \frac{1}{\nu_1}. \]

(3.29)

After entering the second stage, both nodes 1 and 2 are able to activate. However, whenever one of the two nodes is active, the other one has to stay silent, due to the sensing range constraint. From Figure 3.4 we see that state (1,1) does not exist in the process discussed here. Hence, the transition diagram of this stage is as pictured in Figure 3.6, where the three states are the same as described in Section 3.2.

Figure 3.6: Transition diagram for the Markov process for the second stage using CSMA/CA

Again, the transmission of the packet from node 2 to 3 is successful when the process follows the path (0,0) \(\rightarrow\) (0,1) \(\rightarrow\) (0,0). But because state (0,0) can only be entered when leaving state (1,0) or (0,1), no collisions can occur. When the process has entered state (0,1), the transmission will always be successful, for node 1 cannot cause any interference. Consequently, it is sufficient to calculate how long it takes the process, starting at state (0,0), to enter (0,1), when we want to know the expected time it takes to send the packet from node 2 to 3. This expected time then equals this calculated time to enter (0,1) plus the mean transmission time, which is \(1/\mu = 1\). With the help of the transition diagram in Figure 3.6, some equations can be written down in the same manner as we did for ALOHA. If we denote the expected time it takes to enter (0,1) starting from (0,0) by \(E_{S(0,0),(0,1)}\), and the expected time it takes to enter (0,1) starting from
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Figure 3.7: Transition diagram for the Markov process for the third stage using CSMA/CA

\begin{align*}
(1,0) \text{ by } ES_{(1,0),(0,1)} \text{, we can state that} \\
ES_{(0,0),(0,1)} &= \frac{1}{\nu_1 + \nu_2} + \frac{\nu_1}{\nu_1 + \nu_2} \cdot ES_{(1,0),(0,1)} + \frac{\nu_2}{\nu_1 + \nu_2} \cdot 0, \quad (3.30) \\
ES_{(1,0),(0,1)} &= 1 + ES_{(0,0),(0,1)}. \quad (3.31)
\end{align*}

It follows that

\[ ES_{(0,0),(0,1)} = \frac{1 + \nu_1}{\nu_2}. \quad (3.32) \]

The total expected time it takes the packet to be transmitted from node 2 to node 3 is thus

\[ \frac{1 + \nu_1}{\nu_2} + 1. \quad (3.33) \]

Furthermore,

\[ ET_3 = ET_2 + \frac{1 + \nu_1}{\nu_2} + 1 = 2 + \frac{1}{\nu_1} + \frac{1 + \nu_1}{\nu_2}. \quad (3.34) \]

When the second stage is completed, the packet has to be transmitted to node 4. In contrast with the case in which ALOHA is used, the process to be observed cannot be reduced to two nodes. That is because the activation of, for instance, node \( i \) depends on the activation state of nodes \( i - 1 \) and \( i + 1 \). Inductively, the activation states of all nodes that have received the packet, are dependent and therefore cannot be analysed separately. In this light, we will look at the third stage of process. The process consists of five possible states, namely \((0,0,0)\), \((1,0,0)\), \((0,1,0)\), \((0,0,1)\) and \((1,0,1)\). Here \((1,0,0)\) represents the state in which node 1 is active and nodes 2 and 3 are inactive, etcetera. The possible transitions are displayed in Figure 3.7.
In this case, the transmission will be successful when the process has entered either state (0,0,1) or (1,0,1), for node 3 has to send the packet to node 4, and collisions cannot occur. We will thus again calculate the expected time it takes to reach state (0,0,1) or (1,0,1), starting from state (0,0,0), and increment this value by 1, which is the mean transmission time. The following equations will give us this value.

\[ E_S(0,0,0) = \frac{1}{\nu_1 + \nu_2 + \nu_3} + \frac{\nu_1}{\nu_1 + \nu_2 + \nu_3} \cdot E_S(1,0,0) + \frac{\nu_2}{\nu_1 + \nu_2 + \nu_3} \cdot E_S(0,1,0) \]  \quad (3.35)

\[ E_S(1,0,0) = \frac{1}{1 + \nu_3} + \frac{1}{1 + \nu_3} \cdot E_S(0,0,0) + \frac{1}{1 + \nu_3} \cdot 0 \]  \quad (3.36)

\[ E_S(0,1,0) = 1 + E_S(0,0,0). \]  \quad (3.37)

Here \( E_{S_a} \) represents the expected time it takes to reach either state (0,0,1) or (1,0,1) from state \( a \). Solving this linear system of equations yields

\[ E_{S_{(0,0,0)}} = \frac{1 + \nu_1 + \nu_2 + \nu_3 + \nu_2 \nu_3}{\nu_3(1 + \nu_1 + \nu_3)} \]  \quad (3.39)

\[ E_{S_{(1,0,0)}} = \frac{1 + \nu_1 + \nu_2 + \nu_3}{\nu_3(1 + \nu_1 + \nu_3)} \]  \quad (3.40)

\[ E_{S_{(0,1,0)}} = 1 + \frac{1 + \nu_1 + \nu_2 + \nu_3 + \nu_2 \nu_3}{\nu_3(1 + \nu_1 + \nu_3)} \]  \quad (3.41)

This gives us

\[ ET_4 = ET_3 + 1 + \frac{1 + \nu_1 + \nu_2 + \nu_3 + \nu_2 \nu_3}{\nu_3(1 + \nu_1 + \nu_3)} + 1 \]

\[ = 3 + \frac{1}{\nu_1} + \frac{1 + \nu_1}{\nu_2} + \frac{1 + \nu_1 + \nu_2 + \nu_3 + \nu_2 \nu_3}{\nu_3(1 + \nu_1 + \nu_3)}. \]  \quad (3.42)

This method can be continued for stage four. However, because there were two states in the third stage in which transmission from node 3 to 4 was possible, the process could have been in either one of these states when the third stage was completed, namely state (0,0,0) (coming from (0,0,1)) or (1,0,0) (coming from (1,0,1)). Since this information is not known from the third stage of the process, we do not know from which state in the fourth stage should be started. This can either be (0,0,0,0) or (1,0,0,0). However, we can approximate the probabilities of the third stage ending in either state (0,0,0) or (1,0,0), and the fourth stage thus starting in respectively (0,0,0,0) or (1,0,0,0), by calculating the limiting probabilities of the Markov process describing the first stage. The corresponding balance
equations are, with the help of Figure 3.7,

\[(\nu_1 + \nu_2 + \nu_3)\pi(0,0,0) = \pi(1,0,0) + \pi(0,1,0) + \pi(0,0,1),\]  

(3.43)

\[(\nu_3 + 1)\pi(1,0,0) = \nu_1 \pi(0,0,0) + \pi(1,0,1)\]  

(3.44)

\[\pi(0,1,0) = \nu_2 \pi(0,0,0),\]  

(3.45)

\[(\nu_1 + 1)\pi(0,0,1) = \nu_3 \pi(0,0,0) + \pi(1,0,1),\]  

(3.46)

\[2\pi(1,0,1) = \nu_3 \pi(1,0,0) + \nu_1 \pi(0,0,1).\]  

(3.47)

Here \(\pi_a\) denotes the limiting probability of state \(a\). Together with the fact that

\[\pi(0,0,0) + \pi(1,0,0) + \pi(0,1,0) + \pi(0,0,1) + \pi(1,0,1) = 1,\]  

(3.48)

we obtain the solution

\[
\begin{align*}
\pi(0,0,0) & = \frac{1}{1 + \nu_1 + \nu_2 + \nu_3 + \nu_1 \nu_3}, \\
\pi(1,0,0) & = \frac{\nu_1}{1 + \nu_1 + \nu_2 + \nu_3 + \nu_1 \nu_3}, \\
\pi(0,1,0) & = \frac{\nu_2}{1 + \nu_1 + \nu_2 + \nu_3 + \nu_1 \nu_3}, \\
\pi(0,0,1) & = \frac{\nu_3}{1 + \nu_1 + \nu_2 + \nu_3 + \nu_1 \nu_3}, \\
\pi(1,0,1) & = \frac{\nu_1 \nu_3}{1 + \nu_1 + \nu_2 + \nu_3 + \nu_1 \nu_3}.
\end{align*}
\]

(3.49)

Conditioning on the fact that in the last phase of the third stage the process is in either state (0,0,1) or (1,0,1), the probabilities of ending in state (0,0,0) or (1,0,0) are \(1/(1 + \nu_3)\) and \(\nu_3/(1 + \nu_3)\), respectively.

Figure 3.8: Transition diagram for the Markov process for the fourth stage using CSMA/CA
For the fourth stage, the next task is to find the expected time it takes to reach one of the states $(0,0,0,1), (1,0,0,1)$ or $(0,1,0,1)$, starting from state $(0,0,0,0)$ and $(1,0,0,0)$. Using the same method as before, these values are

\[
\mathbb{E}S_{(0,0,0,0)} = \frac{\nu_2^2(1+\nu_3)(1+\nu_4)+(2+\nu_3+2\nu_4)(1+\nu_2+\nu_3+\nu_4+\nu_2\nu_3) + \nu_1(1+\nu_3)(\nu_2+1+\nu_3)(3+\nu_3+\nu_4)}{\nu_4(\nu_2^2+1+\nu_3)(1+\nu_4)(3+\nu_2+\nu_3+\nu_4) + (1+\nu_2+\nu_3)(2+\nu_3+2\nu_4)},
\]

\[
\mathbb{E}S_{(1,0,0,0)} = \frac{(2+\nu_1+\nu_3)(1+\nu_2+\nu_3+\nu_2\nu_3) + (1+\nu_3)(4+2\nu_2+\nu_3)(4+\nu_1+\nu_2+\nu_3) + (2+\nu_1)(1+\nu_3)\nu_2^2}{\nu_4(\nu_2^2+1+\nu_3)(3+\nu_2+\nu_3+\nu_4) + (1+\nu_2+\nu_3)(2+\nu_3+2\nu_4)}.
\]

The expected time in this fourth stage is therefore

\[
\frac{1}{1+\nu_3} \cdot \mathbb{E}S_{(0,0,0,0)} + \frac{\nu_3}{1+\nu_3} \cdot \mathbb{E}S_{(1,0,0,0)} + 1.
\]

And furthermore,

\[
\mathbb{E}T_5 \approx 4 + \frac{1}{\nu_1} + \frac{1 + \nu_1}{\nu_2} + \frac{1 + \nu_1 + \nu_2 + \nu_3 + \nu_2\nu_3}{\nu_3(1 + \nu_1 + \nu_3)} + \frac{1}{1+\nu_3} \cdot \mathbb{E}S_{(0,0,0,0)} + \frac{\nu_3}{1+\nu_3} \cdot \mathbb{E}S_{(1,0,0,0)}.
\]

As one may notice, state spaces and complexity of the Markov processes increase enormously when the number of nodes under consideration increases, yielding a curse of dimensionality. Finding good approximations for $\mathbb{E}T_n$ for $n > 5$ therefore becomes difficult. However, noting that in the fifth stage, the influence of node 1 on the activation state of node 5 is relatively small, we could assume those to be independent. Under this assumption, the fifth stage of the process is the same as the fourth stage, but shifted, approximations for $\mathbb{E}T_n$ with $n > 5$ can be found. Namely,

\[
\mathbb{E}T_n \approx 3 + \frac{1}{\nu_1} + \frac{1 + \nu_1}{\nu_2} + \frac{1 + \nu_1 + \nu_2 + \nu_3 + \nu_2\nu_3}{\nu_3(1 + \nu_1 + \nu_3)} + \\
+ \sum_{i=4}^{n-1} \left(1 + \frac{1}{1+\nu_{i-1}} \cdot \mathbb{E}S_{i,(0,0,0,0)} + \frac{\nu_{i-1}}{1+\nu_{i-1}} \cdot \mathbb{E}S_{i,(1,0,0,0)} \right).
\]

Where,

\[
\mathbb{E}S_{i,(0,0,0,0)} = \frac{\nu_2^2(1+\nu_{i-1})(1+\nu_3)+(2+\nu_{i-1}+2\nu_4)(1+\nu_{i-2}+\nu_{i-1}+\nu_i+\nu_{i-1}\nu_i)+\nu_{i-3}(1+\nu_i)(\nu_{i-2}+1+\nu_{i-1})(3+\nu_{i-1}+\nu_i)}{\nu_i(\nu_{i-3}^2+1+\nu_{i-1})(3+\nu_{i-2}+\nu_{i-1}+\nu_i)+(1+\nu_{i-2}+\nu_i)(2+\nu_{i-1}+2\nu_i)},
\]

\[
\mathbb{E}S_{i,(1,0,0,0)} = \frac{(2+\nu_{i-3}+\nu_{i-1})(1+\nu_{i-2}+\nu_{i-1}+\nu_{i-3}+\nu_{i-2}+\nu_{i-1})+(1+\nu_{i-1})(4+2\nu_{i-2}+\nu_{i-1}+\nu_{i-3}+\nu_{i-2}+\nu_{i-1})\nu_i+(2+\nu_i)(1+\nu_{i-1})\nu_2^2}{\nu_i(\nu_{i-3}^2+1+\nu_{i-1})(3+\nu_{i-2}+\nu_{i-1}+\nu_i)+(1+\nu_{i-2}+\nu_i)(2+\nu_{i-1}+2\nu_i)}.
\]

As may be noticed, this formula is quite intricate and therefore difficult to analyse.
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However, assuming $\nu_1 = \nu_2 = ... = \nu_n = \nu$, leads to

$$\mathbb{E}T_\nu = \mathbb{E}T_n(\nu) \approx 4 + \frac{2}{\nu} + \frac{1 + 3\nu + \nu^2}{\nu(1 + 2\nu)} +$$

$$(n - 4) \frac{2 + 16\nu + 45\nu^2 + 57\nu^3 + 35\nu^4 + 8\nu^5}{2\nu + 12\nu^2 + 23\nu^3 + 17\nu^4 + 4\nu^5}.$$  \hspace{1cm} (3.55)

for $n \geq 4$. We will use this approximating function many times and therefore give a formal definition.

**Definition 3.3.1.** The function $\mathbb{E}\tilde{T}_n$ is defined as

$$\mathbb{E}\tilde{T}_n(\nu) = 4 + \frac{2}{\nu} + \frac{1 + 3\nu + \nu^2}{\nu(1 + 2\nu)} + (n - 4) \frac{2 + 16\nu + 45\nu^2 + 57\nu^3 + 35\nu^4 + 8\nu^5}{2\nu + 12\nu^2 + 23\nu^3 + 17\nu^4 + 4\nu^5}.$$  \hspace{1cm} (3.56)

$\mathbb{E}\tilde{T}_n$ approximates $\mathbb{E}T_n$.

The image in Figure 3.9 displays the graphs corresponding to $\mathbb{E}\tilde{T}_n(\nu)$ for some values of $n$. These graphs suggest that the function $\mathbb{E}\tilde{T}_n$ is strictly decreasing and converges to a limit when $\nu$ tends to infinity for every $n$. This will be proved in the following proposition.

**Proposition 3.3.1.** The function $\mathbb{E}\tilde{T}_n(\nu)$ is strictly decreasing with

$$\lim_{\nu \to \infty} \mathbb{E}\tilde{T}_n(\nu) = 2n - \frac{7}{2}.$$  \hspace{1cm} (3.57)

Figure 3.9: The function $\mathbb{E}\tilde{T}_n$ for $n = 2$ (red), 5 (blue), 10 (green) and 20 (magenta) using CSMA/CA

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Proof. The decreasing nature of $E \bar{T}_n(\nu)$ can be verified by analysing the terms of Equation (3.56) separately. Of course, the term ‘4’ is constant and therefore convergent. The second term, $2/\nu$ is strictly decreasing for $\nu > 0$, and converges to 0. Differentiating the third term yields

$$\frac{d}{d\nu} \left( \frac{1 + 3\nu + \nu^2}{\nu(1 + 2\nu)} \right) = \frac{1 + 4\nu + 5\nu^2}{\nu^2(1 + 2\nu)^2} < 0, \quad \forall \nu > 0,$$

which indicates that this term is also strictly decreasing. Furthermore, it converges to $1/2$. The same differentiation can be done for the last term, which yields

$$\frac{d}{d\nu} \left( \frac{(n-4)\left(2 + 16\nu + 45\nu^2 + 57\nu^3 + 35\nu^4 + 8\nu^5\right)}{\nu^2(2 + 12\nu^2 + 23\nu^3 + 17\nu^4 + 4\nu^5)^2} \right) < 0 \quad \forall \nu > 0,$$

which is why this term is also strictly decreasing and has $2(n-4)$ as its limit at infinity. Adding four strictly decreasing functions together again forms a strictly decreasing function. Therefore $E \bar{T}_n$ is a strictly decreasing function and because all of its terms converge,

$$\lim_{\nu \to \infty} E \bar{T}_n(\nu) = 4 + 0 + \frac{1}{2} + 2(n-4) = 2n - \frac{7}{2} \quad n \geq 4.$$

The fact that $E \bar{T}_n$ is a strictly decreasing function implies that the optimal value of $\nu$, minimising $E \bar{T}_n$, equals infinity. This suggests that, also for minimising the exact mean hitting time (without approximations), it is optimal to maintain no backoff period, and try to initiate a transmission all the time. This implies that the network will never be idle. Hence, a blocked node will immediately start a transmission when the blocking node stops transmitting. This idea is formalised in the following conjecture.

**Conjecture 3.3.1.** If $\nu_1, \nu_2 > 0$ such that $\nu_1 < \nu_2$, then $E T_n(\nu_1) > E T_n(\nu_2)$.

This means that, informally speaking, the optimal value of $\nu$, minimising $E T_n$, is equal to infinity. The conjecture can be justified by looking at the situation where the packet has just reached the $m^{th}$ node, and $k \leq m/2$ nodes are active. Here, the next ‘right’ event is the activation of node $m$, because then the packet will be transmitted to node $m + 1$. Due to the memoryless property of the exponential distribution of the backoff periods, the probability of node $m$ activating before the other $m - k - 1$ inactive nodes is

$$\frac{\nu}{\nu(m - k)} = \frac{1}{m - k},$$

(3.62)
which does not depend on $\nu$. However, the time before any activation happens in this situation is $1/(\nu(m - k))$. Now, because the probability of entering the ‘right’ state is not dependent on $\nu$, but the time before this happens is, the only function to minimise is the expected time before any activation happens, which is $1/(\nu(m - k))$. This implies that the optimal value for $\nu$ should be infinity. Simulation shows that the mean hitting time keeps decreasing as $\nu$ is increased, for all $n \geq 2$ which supports Conjecture 3.3.1.

### 3.4 Performance Comparison

Having obtained results of the performance of protocols ALOHA and CSMA/CA under prescribed network conditions, we can compare the two and find which performs best. We will only look at the situation with equal activation rates, so $\nu$ is the only parameter. In Figure 3.10 the graphs of $E_T$ for ALOHA and CSMA/CA are pictured. From this figure we can see that CSMA/CA achieves smaller hitting times than ALOHA for all $\nu$. The following proposition states that this is the case for all $n$, when using the approximating function of the hitting time using CSMA/CA.

**Proposition 3.4.1.**

$$\mathbb{E}T_{n,CSMA}(\nu) \leq \mathbb{E}T_{n,ALOHA}(\nu) \quad \forall n \geq 2, \nu > 0,$$

(3.63)
where $E_{n,CSMA}(\nu)$ is as in (3.56) and $E_{n,ALOHA}(\nu)$ as in (3.20).

**Proof.** For $n = 2$,

$$E_{2,ALOHA}(\nu) = E_{2,CSMA}(\nu) = 1 + 1/\nu,$$

and thus (3.63) is true.

For $n = 3$,

$$E_{3,ALOHA}(\nu) - E_{3,CSMA}(\nu) = \left(1 + \frac{1}{\nu} + \frac{(1+\nu)^3}{\nu}\right) - \left(3 + \frac{2}{\nu}\right) = 1 + 3\nu + \nu^2 > 0.$$

Thus (3.63) is also true for $n = 3$. For $n \geq 4$ we can used the whole formula in (3.55). Then,

$$E_{n,ALOHA}(\nu) - E_{n,CSMA}(\nu) = \left(1 + \frac{1+(n-2)(1+\nu)^3}{\nu}\right) - \left(4 + \frac{2}{\nu} + \frac{1+3\nu+\nu^2}{\nu(1+2\nu)} + (n-4)\frac{2+16\nu+45\nu^2+57\nu^3+35\nu^4+8\nu^5}{2\nu+12\nu^2+23\nu^3+17\nu^4+4\nu^5}\right).$$

Some simplification of this equation yields,

$$\frac{4 + 50\nu + 222\nu^2 + 477\nu^3 + 555\nu^4 + 366\nu^5 + 124\nu^6 + 16\nu^7}{2 + 16\nu + 47\nu^2 + 63\nu^3 + 38\nu^4 + 8\nu^5} + \frac{2 + 20\nu + 67\nu^2 + 101\nu^3 + 78\nu^4 + 29\nu^5 + 4\nu^6}{2 + 12\nu + 23\nu^2 + 17\nu^3 + 4\nu^4}.\tag{3.68}$$

For $n = 4$, this formula equals

$$\frac{2 + 11\nu + 14\nu^2 + 4\nu^3}{1 + 2\nu},$$

which is positive for all $\nu > 0$. Because the term

$$\frac{2 + 20\nu + 67\nu^2 + 101\nu^3 + 78\nu^4 + 29\nu^5 + 4\nu^6}{2 + 12\nu + 23\nu^2 + 17\nu^3 + 4\nu^4}\tag{3.70}$$

is positive for all $\nu > 0$, we can say that Equation (3.68) is positive for all $n \geq 4$. This proves the proposition.

We can conclude that CSMA/CA performs better than ALOHA for all possible activation rates. The guidance of the activation of nodes by the CSMA/CA protocol is therefore proved to be useful in this linear network configuration.
Chapter 4

Algorithm for General Topology

4.1 Visible Light Communication

We now apply wireless mesh networks to the special case of visible light communication networks. These networks use visible light as a medium to transmit packets of data. Here, the transmitter nodes as discussed earlier are LEDs. These LEDs can blink very rapidly with different frequencies. This way packets can be coded, and sent to the receiver. The receiver nodes are sensors in this case, which can pick up these light signals. Furthermore, a LED and a sensor can be connected, which makes the LED both a transmitter and receiver. This way, visible light communication networks can be used as mesh networks. The LEDs can pass through packets by receiving them and sending them further onwards. The transmission range $\tau$ corresponding to these networks is restricted to the sensitivity of the sensors and the brightness of the LEDs. The more sensitive the sensors are, the greater the transmission range. However, also the interference range $\eta$ is defined by the sensitivity of the sensors. We will use the concept of visible light communication to model the situation posed in the next section.

4.2 Model

For the algorithm we are about to discuss, we consider a room of specific size. In this room, a number of LEDs with receiving sensors are installed. All of these LEDs are turned off when nobody is in the room. However, when a person enters the room, the LEDs closest to this person picks up a signal and turns itself on. Then, the LED starts broadcasting a message (a small packet), saying that a person has entered, to all LEDs within its transmission range. When this transmission is done, the just informed LEDs turn themselves on and start
transmitting the packet to all LEDs within their transmission range. This process goes on until all LEDs in the room have received the packet and are thus turned on. The initiation of transmissions is guided by a protocol very similar to the CSMA/CA protocol described in Section 2.3. However, in this case we assume the transmission times to be deterministic and equal to 1 unit time. This is realistic, because the packet length is the same for all transmissions (it contains the same information). Furthermore, we restrict the number of times a LED can activate and start a transmission, which can also be infinity. We also use a fixed activation rate for all LEDs. A precise description of the protocol can be found in the next section. Our goal is to minimise the time it takes for all the LEDs to be turned on. The parameter we want to tune is the sensing range $\beta$ used. In Chapter 5, we analyse whether there is an optimal $\beta$, and if so, how this $\beta$ depends on the other characteristics of the network.

### 4.3 Protocol Specification

The network should operate according to the following rules. Here, a node is a LED together with a sensor and $d(i,j)$ denotes the (Euclidean distance) between node $i$ and $j$.

1. A room $R$ is defined by a collection of nodes, so

$$R = \{1, \ldots, N\}. \quad (4.1)$$

2. At each moment a node is in exactly one of the following states:

   (a) Receiving (the node is busy receiving the packet and this transmission has not been interrupted due to interference).

   (b) Sensing (the node has not received the packet yet and is either sensing signals from more than one transmitter node, or is sensing but not receiving from one transmitter node).

   (c) Informed (the node has received the packet, i.e. the LED is turned on).

   (d) Off (neither of the above applies).

3. A node can be either inactive or active (only possible when node is in state 'Informed').

4. The backoff period of node $i$ is exponentially distributed with mean $1/\nu_i$.

5. The transmission time of each node is equal to 1 time unit.

6. A node can activate at most $k$ times.
7. In the starting situation of the network, node 1 is ‘Informed’ and active. All other nodes are ‘Off’ and inactive.

8. A node $i$ has the possibility to activate when its backoff period has passed. This is only possible when no other node within its sensing range $\beta$ is active. That is, if the following statement is true:

$$d(i,j) \leq \beta \Rightarrow \text{node } j \text{ is inactive} \quad \forall j \in R.$$ (4.2)

If this is not possible, a new backoff period for node $i$ is started. If this constraint is met, a transmission time is started.

9. When a node $i$ activates, all nodes $j$ that satisfy the following conditions change to state ‘Receiving’:
   
   (a) Node $j$ lies within the transmission range of node $i$. That is,
   
   $$d(i,j) \leq \tau.$$ (4.3)

   (b) The number of nodes within the interference range $\eta$ of node $i$, excluding node $i$ itself, is equal to zero. That is,
   
   $$\sum_{l \in R \setminus \{i\}} 1_{\{d(j,l) \leq \eta\}} \cdot 1_{\{\text{node } l \text{ is active}\}} = 0.$$ (4.4)

   (c) Node $j$ is not in state ‘Informed’.

Furthermore, all nodes $l \in R$ that are not in state ‘Informed’ and satisfy the conditions $d(i,l) \leq \eta$ and

$$\sum_{m \in R \setminus \{i\}} 1_{\{d(l,m) \leq \eta\}} \cdot 1_{\{\text{node } m \text{ is active}\}} \geq 1.$$ (4.5)

change to state ‘Sensing’.

10. A node becomes inactive when its transmission time has passed. If the node has not activated $k$ times yet, a new backoff period is started.

11. When a node $i$ becomes inactive, all nodes $j$ that satisfy the following conditions change to state ‘Informed’:

   (a) Node $j$ is in state ‘Receiving’.

   (b) Node $j$ lies within the transmission range of node $i$. That is,

   $$d(i,j) \leq \tau.$$ (4.6)
For each of the nodes which has just changed to state ‘Informed’ a backoff period is started.

12. When a node $i$ becomes inactive, all nodes $j$ that satisfy the following conditions change to state ‘Off’:
   
   (a) Node $j$ is in state ‘Sensing’.
   (b) Node $j$ lies within the interference range of node $i$. That is,
   \[ d(i, j) \leq \eta. \]  
   (4.7)
   
   (c) \[ \sum_{m \in R \setminus \{i\}} 1_{\{d(j, m) \leq \eta\}} \cdot 1_{\{\text{node } m \text{ is active}\}} = 0. \]  
   (4.8)
Chapter 5

Simulation Results for Grid Topology

5.1 Simulation

The protocol in the previous chapter describes the algorithm for a general topology. In this chapter we will look at the special case of a grid topology. So in this situation, the LEDs in the room under consideration are positioned on a grid. Without loss of generality, the distance between two neighboring nodes is set to 1. Furthermore, we consider a rectangular room with the door in the upper left corner. The first node to activate is thus the most upper left node (which we called node 1 in the protocol in Section 4.3). Because in Section 3.3 we already saw that exact analysis of linear networks became impossible for even a small number of nodes, it is fair to assume that this is also the case for this grid topology. We therefore simulate the process, to examine the performance of the network. We implemented the protocol in Section 4.3 as a simulation program in Java. See Appendix A. For each run, the simulation of the process is stopped whenever either all LEDs in the room are lit or the nodes can no longer activate. This simulation allows us to investigate changes in hitting time and the percentage of nodes that stay inactive by changing parameters such as activation rate and room size. Because the transmission range $\tau$ and interference range $\eta$ are defined by the properties of the LEDs and their sensors, we cannot change these values. However, the sensing range $\beta$ is defined by the protocol, and we can thus choose this value ourselves. Hence, our main focus is on changing $\beta$. Because our goal is to minimise the hitting time, this will be the main objective function. We will thus determine the mean hitting time as a function of $\beta$ by simulation, and find the optimal value for $\beta$. To see how this optimal $\beta$ changes for different properties of the network, $\tau, \eta, \nu$, room size and the maximum number of times a node can activate are varied. After analysing what effect these changes have on the
optimal $\beta$, some conjectures will be posed. Another performance measure that will be analysed is the percentage of nodes that remain inactive. This measure is also important, because we want the total process to be successful, that is, we want all LEDs to be informed and thus the whole room to be lit.

5.2 Results

In this section, the simulated mean hitting times are plotted against the sensing range $\beta$. In each subsection, the value of some parameter of the network under consideration is varied, while the other parameters are fixed. This will clearly show us the effect of changing the value of these parameters. As argued in Section 2.3, the optimal $\beta$ lies in the interval $[\eta - \tau, \eta + \tau]$. Therefore, only simulations with $\beta$ lying within this interval are done. Moreover, we use a step size of 0.1 for $\beta$. So for a network with $\tau = 3$ and $\eta = 4$, we run the simulation for $\beta = 1.0, 1.1, 1.2, ..., 6.8, 6.9$ and 7.0. For each step, the simulation is run 2500 times. To be able to make a fair comparison, a ‘benchmark room’ will be used, which will function as the standard room for all analyses. This ‘benchmark room’, has the following properties:

- Dimensions: 12×12 LEDs.
- $\tau = 3$.
- $\eta = 4$.
- $\nu = 1$.
- $k = 1$.

A map of this room is depicted in Figure 5.1. In Figures 5.2 and 5.3, two later phases of the process are depicted. Because as $\beta$ increases, the number of nodes within this sensing range only changes at certain values, namely at points $\sqrt{a^2 + b^2}$, where $a, b \in \mathbb{N}$. We therefore expect the graphs of the simulation results not to be smooth. The step size for $\beta$ also has a part in this.

5.2.1 Transmission Range

In this subsection, we analyse the effect of the value of the transmission range $\tau$ on the hitting time. The room simulated is the benchmark room, but for different values of $\tau$, namely $\tau = 1, 2, 3, 4$. We cannot increase $\tau$ any further, because $\tau$ can never be greater than $\eta$, i.e. if a node cannot sense a signal from a transmitter node, it cannot receive the packet either. The results for the proposed values of $\tau$ are plotted in Figure 5.4. As we can see from this figure, the optimal value of $\beta$ is indeed contained in the interval $[\eta - \tau, \eta + \tau]$, for these values of $\tau$. Moreover, the
Figure 5.1: Benchmark room.

Figure 5.2: Benchmark room just after start-up, while first node transmits to all nodes in its transmission range. Big yellow nodes are currently transmitting. Receiving nodes are coloured green.
CHAPTER 5. SIMULATION RESULTS FOR GRID TOPOLOGY

Figure 5.3: Benchmark Room in an intermediate phase of the process. Big yellow nodes are currently transmitting. Nodes receiving from only one transmitter node are coloured green. Nodes sensing signals from multiple nodes are coloured red.

Figure 5.4: Mean hitting time plotted against $\beta$ for $\tau = 1$ (blue), 2 (green), 3 (red) and 4 (orange).
mean hitting time decreases when $\tau$ is increased. This makes sense, because if a transmitter node can send its packet to more receiver nodes at the same time, on average more nodes are informed per time unit, which reduces the hitting time. The optimal values for $\beta$, minimizing the hitting time, are approximately 4.3, 4.7, 5.1 and 5.7, for $\tau = 1, 2, 3, 4$ respectively. This pattern brings us to the following conjecture.

**Conjecture 5.2.1.** Let $\tau_1$ and $\tau_2$ be the transmission ranges of situation 1 and 2. If $\beta_1$ and $\beta_2$ are the corresponding optimal values for the sensing range, minimizing the mean hitting time, and all other properties are the same for the two situations, then

$$\tau_1 \leq \tau_2 \Rightarrow \beta_1 \leq \beta_2. \quad (5.1)$$

Looking at the mean percentage of the nodes that stay inactive after the process is finished in Figure 5.5, we see that this percentage is quite high for small $\beta$, but equals zero when $\beta$ is equal to its optimal value for all $\tau = 1, 2, 3, 4$. This is convenient, because setting $\beta$ to its optimal value thus also minimizes the number of nodes that are not informed.

### 5.2.2 Interference Range

Next, we study what effect changing the interference range $\eta$ has on the hitting time. We consider $\eta = 3, 4, 5$ and 6. Here, we cannot decrease $\eta$ any further.
for the same reason as we could not increase $\tau$ in the previous subsection. The results of the simulation are shown in Figure 5.6. We can see that the mean hitting time increases, for every value of $\beta$, when $\eta$ increases. This makes sense, because a larger interference range implies that more collisions are likely to occur, which increases the hitting time of the process. The corresponding optimal values for $\beta$, minimising the hitting time are approximately 4.3, 5.1, 6.5 and 8.3, for $\eta = 3, 4, 5, 6$ respectively. These optimal values seem to be steadily increasing with $\eta$. This leads us to the next conjecture.

**Conjecture 5.2.2.** Let $\eta_1$ and $\eta_2$ be the interference ranges of situation 1 and 2. If $\beta_1$ and $\beta_2$ are the corresponding optimal values for the sensing range, minimizing the mean hitting time, and all other properties are the same for the two situations, then

$$\eta_1 \leq \eta_2 \Rightarrow \beta_1 \leq \beta_2.$$  \hspace{1cm} (5.2)

The idea behind this statement is simple. When the interference range $\eta$ increases, nodes become more sensitive to interference, and more collisions will occur, increasing the mean hitting time. However, by increasing the sensing range $\beta$ with $\eta$, more nodes attempting to activate will be blocked, and collisions will be prevented. This obviously reduces hitting time. The results of simulations considering the mean percentage of inactive nodes are shown in Figure 5.7. As in the previous subsection, we see that the mean percentage of inactive node is equal to zero for the optimal $\beta$'s, which implies the optimal $\beta$’s noted before are optimal for both performance measures.
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Figure 5.7: Mean percentage of nodes inactive plotted against $\beta$ for $\eta = 3$ (blue), 4 (red), 5 (green) and 6 (orange).

5.2.3 Activation Rate

Changes in the activation rate $\nu$ may also have consequences for the mean hitting time. In this subsection, we consider the benchmark room, but vary the value of $\nu$. The process is simulated for $\nu = 0.2, 0.5, 1.0$ and 2.0. The results or the simulation are shown in Figure 5.8. We can see that for $\beta < 3$, the graphs of the different simulations are nicely ordered in a decreasing fashion as $\nu$ increases. Then the graphs intersect in the interval between $\beta = 3$ and $\beta = 4.5$, but for $\beta > 4.5$ the graphs are ordered again. This suggests that the mean hitting time decreases when $\nu$ is increased for most $\beta$. From Figure 5.8 it can be seen that the optimal values for $\beta$ are approximately 4.9, 5.0, 5.1 and 5.2, for $\nu = 0.2, 0.5, 1.0$ and 2.0 respectively. Here, we again see a shift in these optimal values, which is formalized in the following conjecture.

**Conjecture 5.2.3.** Let $\nu_1$ and $\nu_2$ be the activation rates of situations 1 and 2, respectively. If $\beta_1$ and $\beta_2$ are the corresponding optimal values for the sensing range, minimising the mean hitting time, and all other properties are the same for the two situations, then

$$\nu_1 \leq \nu_2 \Rightarrow \beta_1 \leq \beta_2.$$  \hspace{1cm} (5.3)

Furthermore, if $\nu_1 \leq \nu_2$, then the mean hitting time of situation 1 is greater than or equal to the mean hitting time of situation 2.
The last statement of this conjecture implies that the mean hitting time should be minimised when setting \( \nu \) to infinity. We reached the same conclusion earlier for linear networks, in Section 3.3. The same reasoning applies here. Because the probability of an attempt to activate is the same for each informed node and independent of \( \nu \), one would want the time that the channel is available but idle to be as short as possible. This is done by choosing \( \nu \) large. Now looking at the mean percentage of inactive nodes in Figure 5.9, we see that this percentage is zero for \( \beta > 4 \) for all \( \nu \)'s considered, and thus also for the corresponding optimal values for \( \beta \). Note that the percentage for \( \beta < 4 \) is smaller for lower \( \nu \). This is logical, because a lower activation rate decreases the occurrence of collisions and therefore the number of nodes that stay inactive.

### 5.2.4 Room Size

To see whether the size of the room has consequences for the optimal \( \beta \), we simulate rooms of a few different sizes, namely \( 5 \times 5 \), \( 12 \times 12 \), \( 15 \times 15 \) and \( 20 \times 20 \). The results are shown in Figure 5.10. It can be seen that the mean hitting time increases as the room becomes larger, which is evident, because larger rooms consist of more nodes, which take more time to reach. Looking at the simulation results for the \( 5 \times 5 \) room (blue graph), the mean hitting time levels for values of \( \beta \) greater than 4.5. This is the case because when choosing \( \beta = 4.5 \), (almost) all
Figure 5.9: Mean percentage of nodes inactive plotted against $\beta$ for $\nu = 0.2$ (blue), 0.5 (green), 1.0 (red) and 2.0 (orange).

Figure 5.10: Mean hitting time plotted against $\beta$ for room sizes $5 \times 5$ (blue), $12 \times 12$ (red), $15 \times 15$ (green) and $20 \times 20$ (orange).
other nodes are blocked whenever one node activates. Increasing $\beta$ further does not block more nodes, for all nodes in the room were already blocked. That is why increasing $\beta$ does not change the mean hitting time anymore. For the other room sizes, the optimal value of $\beta$ is approximately 5.1 for all room sizes discussed. This suggests the next conjecture.

**Conjecture 5.2.4.** The optimal value of $\beta$, minimising the mean hitting time, is not dependent on the room size.

In Figure 5.11 it shows that the percentage of inactive nodes is zero for $\beta > 4$, and as a consequence this is also the case for the optimal value of $\beta \approx 5.1$.

### 5.2.5 Maximum Number of Times Active

Finally, we investigate whether the maximum number of times a node is allowed to activate $k$ has an influence on the mean hitting time and the corresponding optimal value of $\beta$. For the benchmark room $k = 1$. Also running simulations for $k = 2, 3$ and 4 gives us the results shown in Figure 5.12. In this figure, we see that increasing $k$ also yields an increase of the mean hitting time. However, near the optimal value for $\beta$, which is approximately 5.1 for all $k$ considered, the mean hitting times for $k = 2, 3, 4$ are almost equal. For $k = 1$ this hitting time is...
slightly smaller. This small difference in the hitting time can be understood by noting that whenever nodes are allowed to activate more times, traffic rates will be higher, and more collisions can occur. By this, more transmissions can fail and the hitting time is decreased. However, increasing $k$ does lower the percentage of nodes that stay inactive, due to the fact that there are more possibilities for the nodes to receive the packet. This can also be seen in Figure 5.13.

5.3 Conclusion

To conclude this chapter, we here summarise our main findings. We saw that increasing the transmission range $\tau$ reduces the mean hitting time. The corresponding optimal sensing range $\beta$ increases along with $\tau$. Increasing the interference range $\eta$ yields a larger hitting time, while the optimal $\beta$ also increases. Furthermore, minimising the backoff period, that is increasing the activation rate $\nu$, reduces the hitting time and increases the value of the optimal $\beta$. As expected, larger rooms have a greater mean hitting time, but the optimal value of $\beta$ does not change for different room sizes. At last, setting $k$ to 2, instead of $k = 1$ increases the mean hitting time a little bit. Increasing $k$ any further however does not change the mean hitting time significantly. For all the simulations done, we saw that the mean percentage of nodes that stay inactive is zero whenever the optimal $\beta$, minimising the mean hitting time, was chosen. Moreover, all optimal $\beta$’s did lie in the interval $[\eta - \tau, \eta + \tau]$, as was argued in Section 2.3.
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Figure 5.13: Mean percentage of nodes inactive plotted against $\beta$ for $k = 1$ (red), 2 (blue), 3 (green) and 4 (orange).
Appendix A

User Manual Simulation Program

To visualize the process described in Chapter 4, we made a user interface along with the simulation program. In this application, the sequence of events taking place in the process are pictured. Here, we will describe how to use the user interface.

When the program is started, the user interface looks as in Figure A.1. Now, the protocol (ALOHA or CSMA) can be chosen and parameters can be set. By clicking ‘Generate Room’ the room is shown as in Figure A.2. Each circle represents a LED. The light blue colour of a circle indicates that the LED is off and not receiving (state ‘Off’ in Section 4.3). The upper left circle is yellow, indicating that this LED is on (state ‘Informed’ in Section 4.3). Now, either ‘Step Forward’ or ‘Start Animation’ can be clicked, as you want the animation to be played step by step or automatically. Clicking ‘Start Animation’ plays the animation of the process. A snapshot of the animation is given in Figure A.3. Here, the big yellow circles represent LEDs that are on and broadcasting. The green circles represent receiving LEDs, whose transmission has not been interrupted (state ‘Receiving’ in Section 4.3). The red circles represent LEDs that either sense but not receive a signal from a transmitting LED, or receive signals from multiple transmitting LEDs (state ‘Sensing’ in Section 4.3). In the right lower corner, the current time of the process can be found. The animation speed can be adjusted at any time by shifting the slider next to the text ‘Animation Speed’. The animation can be stopped by clicking ‘Stop Animation’. A new animation of the process can be started by clicking ‘Generate Room’ again. In this application, results of simulations with multiple runs can be found. To do this, insert the parameters the same way as was done for the animation, fill in the number of times you want to run the simulation in the text field next to ‘Number of Runs’ and click
Figure A.1: User interface on start up

Figure A.2: User interface with generated room
Figure A.3: User interface during animation

'Run Simulation'. After the simulation is run, the expected values and confidence intervals will be given concerning the hitting time, percentage of inactive LEDs and number of collisions.
Appendix B

Tables Simulation Results

Here, the numerical simulation results as presented in Chapter 5, are given. However, only the relevant numbers are given, that is, the mean hitting times for the $\beta$’s around $\beta_{opt}$.

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Table B.1: Simulation results for the mean hitting time for different transmission ranges.
### APPENDIX B. TABLES SIMULATION RESULTS

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Table B.2: Simulation results for the mean hitting time for different interference ranges.
### APPENDIX B. TABLES SIMULATION RESULTS

#### Table B.3: Simulation results for the mean hitting time for different activation rates.

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#### Table B.4: Simulation results for the mean hitting time for different room sizes.

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### Table B.5: Simulation results for the mean hitting time for different maximum times a node can activate.

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