Analysis of phase correcting and elliptical Fresnel-zone plate antennas

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Summary

The Fresnel-zone plate antenna is a promising device for use in direct broadcast satellite receiver systems. For this application it may become competitive to the commonly applied parabolic reflector antenna. The Fresnel-zone plate antenna is a planar structure having lens-like properties. These are established by diffraction and interference. Due to this structure, production costs are low and the zones of the antenna can be attached to a window. The borders of the zones are in general formed by ellipses. Only when the antenna axis is normal to the aperture plane, they reduce to circles (the axially symmetric Fresnel-zone plate antenna). There are two well known theories for analyzing the simplest design of the antenna. These are the Kirchhoff diffraction theory and the asymptotic high-frequency Uniform Theory of Diffraction (UTD).

The simplest design of the Fresnel-zone plate antenna consists of alternating transparent and absorbing zones. Due to the latter zones, which absorb the electromagnetic energy impinging on them, the efficiency is rather low. When replacing the absorbing zones by phase correcting ones, it is possible to increase the efficiency. In practice, some kind of dielectric will be used as phase correcting material. In literature it is always assumed that this dielectric can be modelled as though it is ideal phase correcting. This means that the properties of the dielectric with regard to reflection of energy are not considered. Only a constant phase shift is introduced, necessary for the correct functioning of this kind of Fresnel-zone plate antenna. This is a very simplistic approach. In this report the true properties of the dielectric are modelled in order to obtain a more realistic approach. The existing theories are extended and applied to the axially symmetric antenna. Efficiencies are calculated and compared to the case of absorbing/transparent and ideal phase correcting zones.

The elliptical Fresnel-zone plate antenna is a more practical device than the one with circular zones when the zones are mounted on a window. But it has not been studied until now in literature. In this report, expressions for the zones of the elliptical antenna are derived. Radiation patterns are calculated with the theories mentioned earlier. Only absorbing/transparent and ideal phase correcting zones are applied. The elliptical antenna shows beamsquint when it is illuminated by a circularly polarized feed, just as an offset parabolic reflector does. This can be an unwanted effect when frequency reuse techniques are used. This beamsquint is studied and numerical results are presented.
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1. Introduction

Communication by satellite has taken large proportions in the past few decades. Satellite links are set up for all kinds of services varying from ordinary telephone and distribution of television programs to the exchange of important data between multinationals and stock exchanges. However, the basic principle is the same for all services regardless of the information contained by the transmitted signals. A satellite link can be roughly divided into two parts, characterized by guided and unguided communication. Guided communication appears when signals are transported by coaxial transmission line or optical fibre, usually in the terrestrial network. Unguided communication occurs between a transmitter/receiver on earth and the satellite where signals are transported by means of electromagnetic waves propagating through space. The antenna is one of the key components of a satellite communication system because it forms the transition between guided and unguided communication.

The most popular and frequently applied groundstation antenna has undoubtedly been the parabolic reflector antenna. It has proven its value throughout the years with regard to antenna gain and sidelobe envelope. As a consequence, it has been studied thoroughly. Other antennas have never been a commercial success for several reasons. For example, the Fresnel-zone plate antenna has the disadvantage of low antenna gain and efficiency. Hence, very little research has been done on this antenna. But with the introduction of high-power satellite transmitters, these disadvantages have become of less importance. The Fresnel-zone plate antenna has therefore become a promising device for future applications in satellite communication. It has some mechanical properties making it competitive to the parabolic reflector antenna. First, it is a planar device consisting of, in its simplest design, alternating transparent and opaque zones. This makes it easy to fabricate resulting in low production costs. Second, the planar structure enables the antenna to be mounted on a window. So, it becomes an interesting device to be used for indoor satellite receivers. Because little research has been done on this antenna, there is some work to do for antenna engineers. This report will give a contribution to it.

Far-field radiation patterns of Fresnel-zone plates are usually calculated by applying the well-known Kirchhoff diffraction theory. Then, it is assumed that the opaque zones are made of perfectly absorbing material which do not influence the field incident on the aperture. When the opaque zones are considered to be made of perfectly conducting material, a high-frequency asymptotic method like the Uniform Theory of Diffraction (UTD) can be applied.

The low efficiency of the simplest design of the Fresnel-zone plate antenna can be increased by replacing the opaque zones by phase correcting ones. In practice the material of these zones will be some dielectric. Until now, the phase correcting material has been treated as though it is ideal. This means that it only introduces a phase shift and reflection of electromagnetic energy is not considered. It is obvious that this material does not exist. Therefore, one can doubt this theoretical approach. In this report dielectric material will be modelled in order to obtain a more realistic theory. This means that it is necessary to extend the existing theories.
In general the zones of a Fresnel-zone plate antenna are formed by ellipses. Only under a special condition, namely when the forward direction is at normal to the aperture, they reduce to circles. The elliptical Fresnel-zone plate antenna has not been studied until now, although it is a more practical device than the one with circular zones. A substantial part of this report will treat this antenna.

There are different kinds of Fresnel-zone plate antennas all being described in chapter 2 of this report. Chapter 3 gives a short review of the Kirchhoff diffraction theory and UTD. In chapter 4 the properties of lossless dielectric are modelled. The extended theory is described and applied to an antenna with circular zones. Chapter 5 treats the bandwidth and efficiency of this antenna. It also shows that a Fresnel-zone plate is scalable. In chapter 6 the Kirchhoff and UTD analysis of an elliptical Fresnel-zone plate antenna will be given for both the opaque/transparent and ideal phase correcting case. The effect of beamsquint is also studied. Conclusions and recommendations are given in chapter 7.
2. From Fresnel-zone plate to lens antenna

The Fresnel-zone plate antenna (FZPA) is a planar device which has lens-like properties. These are established by diffraction and interference, rather than refraction. Zone plates originate from optics where they are known for about one hundred years. Here they are used for example in X-ray microscopy and holography. Because for telecommunication services continuously higher frequency bands are exploited, zone plates have become of interest for microwave frequencies.

Like a lens, the FZPA has a focus which is a certain given point on the antenna-axis. Therefore, the position of the focus is a design parameter. The simplest FZPA consists of alternating transparent and opaque zones. In this chapter a summary of FZPA's will be given, where the antenna axis is normal to the aperture plane. For such a configuration the zones are circular and this type of antenna is called the axially symmetric FZPA throughout the report. A general expression for the radii of the zones can be derived in the following way. Consider a plane wave at normal incidence to the aperture plane. Then the radii of the various zones must be determined in such a way that the opaque zones reject the energy that would add destructively at the focus. In other words, going from the inside to the outside of the $m$-th zone, the distance to the focus must change by $\lambda/2$, as shown in figure 2.1.

![Fresnel-zone plate diagram](image)

Figure 2.1 Side-view of a Fresnel-zone plate where a plane wave is at normal incidence to the aperture plane.
Such a zone is called a Fresnel-zone. Then it is easy to see that the radius of the $m$-th zone, $r_m$, obeys the relation

$$r_m = \sqrt{\frac{\lambda m (F + \frac{\lambda m}{4})}{\lambda}}$$

(2.1)

where $F$ is the focal distance and $\lambda$ is the free space wavelength. From the latter equation it is obvious that the antenna is a narrow-band device, because the FZPA is designed for one single frequency. Deviations from the design frequency will lead to a disturbed interference pattern and thus to a worse performance. In the configuration of figure 2.1 the opaque zones are considered to be made of perfectly absorbing material. A rigorous derivation of far-field radiation patterns for this type of FZPA, using the Kirchhoff diffraction theory, has been given by Leyten [1]. In order to analyze the antenna with asymptotic high-frequency techniques, like the Uniform Theory of Diffraction (UTD), it must be assumed the opaque zones are made of perfectly conducting material. This is because for perfectly conducting wedges the diffraction coefficients, appearing in the formulas, are known. Analysis of a FZPA with metallic/transparent zones has been performed by Baggen [2].

A major disadvantage of the described FZPA is the low efficiency. That is because approximately one half of the energy of the incident wave can not pass the antenna. As a result, the antenna can only be used in high-power communication systems. It is useful to search for methods with which the efficiency can be increased. A first attempt to increase the received power at the focus, is to construct the opaque zones so that they behave as $\pi$ radian shifters. Then the contributions from these zones add constructively at the focus, because the phases of contributions from two adjacent zones differ by less than $\pi$. This type of FZPA is referred to as the half-wave or phase correcting FZPA [3],[4],[5],[6] (Wiltse). The effect of phase shift can be obtained by cutting grooves of depth $d$ into a plate of low-loss dielectric, as shown in figure 2.2.

In designing the phase correcting zone plate, it is assumed that all rays start their convergence towards the focus from the plane $z = 0$. The depth of a groove can now be derived by calculating the phase delay of the wave in the dielectric with respect to the wave in free space. Then the following relation must be valid
2. From Fresnel-zone plate to lens antenna

\[(k' - k)d = \pi\]  \hspace{1cm} (2.2)

With \(k' = k\sqrt{\varepsilon_r}\) the wavenumber in the dielectric and \(\varepsilon_r\) the relative dielectric constant of the material, the latter equation can be written as

\[k(\sqrt{\varepsilon_r} - 1)d = \pi\]  \hspace{1cm} (2.3)

The groove depth \(d\) is now given as

\[d = \frac{\lambda}{2(\sqrt{\varepsilon_r} - 1)}\]  \hspace{1cm} (2.4)

Wiltse analyses the far-field radiation pattern by applying the Kirchhoff integral over each zone. The grooved zones are treated as though they are transparent, for each ungrooved zone a constant phase shift of \(\pi\) is added in the formulas. Transmission and reflection of energy caused by the properties of the dielectric are neglected. So, the FZPA is modelled as an ideal phase correcting FZPA. This seems to be a very crude model.

The phase correcting FZPA presented in figure 2.2 suffers from some unwanted effects. In designing the depth of the grooves, it is assumed that all rays start their convergence to the focus from one plane. However, as soon as the rays leave the dielectric the convergence starts. This means there is a small phase error not accounted for as illustrated in figure 2.3.

![Figure 2.3 Deviations from the design approach of a phase correcting zone plate](image)

Figure 2.3 also shows the effect of shadowing which occurs when radiation emitted from a grooved zone must pass through an adjacent zone. Yet, Black and Wiltse [3] have shown that this is a minor effect. Finally there is scattering at the groove edges. All these effects cause slight errors in the analysis of the FZPA. Throughout this report they will not be considered.

An undesired mechanical property of the grooved structure of the phase correcting FZPA is that it can lead to accumulation of dirt or rain, which deteriorates the antenna performance. Therefore, it is more convenient to design a FZPA of uniform thickness. This leads to the necessity of using two different kinds of dielectrics, where the difference in relative dielectric constant causes the \(\pi\)
2. From Fresnel-zone plate to lens antenna

phase shift between two adjacent zones. This type of FZPA is referred to as the planar lens and is shown in figure 2.4.

\[ d = \frac{\lambda}{2(\sqrt{\varepsilon_{r2}} - \sqrt{\varepsilon_{r1}})} \quad \varepsilon_{r2} > \varepsilon_{r1} \]  

Figure 2.4 The planar lens of uniform thickness, made of two dielectrics with relative dielectric constants \( \varepsilon_{r1} \) and \( \varepsilon_{r2} \)

The thickness of the lens is given by

The flat front- and backsurface of the lens offer an advantage with regard to aerodynamics. In contrast to the grooved structure of figure 2.2, the absence of steps avoids the effects of shadowing and scattering. The phase error mentioned above is also eliminated. The planar lens is therefore a promising device, but until now it has not been studied thoroughly.

The efficiency of the phase correcting FZPA can be increased even more by stepping the plate in such a way that phase shifts of \( \pi/2 \) are introduced. This kind of antenna is known as the quarter period FZPA. Dielectric material can be used as in case of the phase correcting FZPA. But it is also possible to use printed resonators [7]. By still further decreasing the phase shifts, the FZPA asymptotically turns to the Fresnel lens. The efficiency however, is not significantly increased beyond the quarter period FZPA [4]. The complex contour of the Fresnel lens is hard to fabricate, making it a relatively expensive device. Then it is more convenient to use a FZPA. The Fresnel lens is in fact a simplified version of the well known hyperbolic lens. Every time the optical path of a ray in the hyperbolic lens exceeds one wavelength, the lens is stepped, resulting in the Fresnel lens. This way the weight of the lens is lowered and absorption loss due to the dielectric is reduced.

After this review on different designs of FZPA's, the question arises how to analyze far-field radiation patterns of phase correcting FZPA's. Dielectric lens antennas are usually treated by a combination of ray-optics and aperture integration [8],[9],[10]. First, in transmitting situation the field distribution at the backface of the lens antenna is determined. Then this field is integrated to obtain the far-field. But aperture integration techniques take a lot of CPU-time, so it is more convenient to look for other methods. One of these methods could be a high-frequency asymptotic technique, such as UTD. The problem is that until now there is no rigorous solution to the
2. From Fresnel-zone plate to lens antenna

canonical problem of diffraction by a dielectric wedge, though many authors have tried to solve it. Usually this leads to very extensive and complicated computations, or conditions are formulated which lead to very little practical application [11],[12],[13],[14],[15],[16]. Yet a useful method is given by Burnside and Burgener [17]. They proposed a simple heuristic UTD approach for diffraction by a lossless dielectric half-plane. The validity of this method is shown, because it yields results which are equal to those they obtained by applying the method of moments (MM). However, there are some restrictions. It was found that the thickness of the dielectric layer must be less than about $\lambda/4$ and the angle between the incident ray and the half-plane normal must not be much greater than $60^\circ$. Otherwise, the results do not agree with the MM-results. The FZPA on which the method can be best applied is the planar lens, where one "material" is chosen to be air. It can not be applied to the quarter period zone plate, because the formulas are not valid for stepped surfaces. The zones will be mounted on a thin supporting sheet, as shown in figure 2.5, which is assumed to have no influence on the antenna performance.

![Figure 2.5 Phase correcting zone plate with dielectric zones mounted on a thin supporting sheet](image)

The dielectric zones can be considered to be part of a half-plane as long as the width of a zone is large compared to the thickness. This thickness is given by Eq. (2.4) and is plotted in figure 2.6 against the relative dielectric constant in terms of the free space wavelength. It can be seen that, to meet the requirement of Burnside and Burgener, material with minimal $\varepsilon_r = 9$ has to be taken. Some practical values of the dielectric constant are given in table 2.1. [18],[19]. From figure 2.6 it can be seen that the thickness of the lens becomes in the order of one wavelength, for a number of materials in table 2.1. Then the requirement is not met and the zones can not be modelled as part of a half-plane. The zones should have to be modelled as block-shaped obstacles, but it is not carried out in this report. A practical and commercially available material is the RT/duroid family of microwave laminates. The loss tangent of this material is very small, so it can be considered as a lossless dielectric. It is a good material to be used in future measurements. This report gives only the theoretical analysis of the phase correcting FZPA. In chapter 4 the UTD method will be described and applied to the lens of figure 2.5.
2. From Fresnel-zone plate to lens antenna

Figure 2.6 Thickness of the planar lens in terms of free space wavelength $\lambda$

Table 2.1 Dielectric materials with $\varepsilon_r$ values

<table>
<thead>
<tr>
<th>material</th>
<th>$\varepsilon_r$</th>
<th>tan $\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>paraffin</td>
<td>2.1</td>
<td>-</td>
</tr>
<tr>
<td>polyethylene</td>
<td>2.2</td>
<td>-</td>
</tr>
<tr>
<td>plexiglas</td>
<td>2.6</td>
<td>-</td>
</tr>
<tr>
<td>polystyrene</td>
<td>2.5</td>
<td>-</td>
</tr>
<tr>
<td>flintglass</td>
<td>7</td>
<td>-</td>
</tr>
<tr>
<td>polycrystalline (TiO$_2$)</td>
<td>4 - 16</td>
<td>-</td>
</tr>
<tr>
<td>rutile (TiO$_2$) *</td>
<td>85 - 170</td>
<td>-</td>
</tr>
<tr>
<td>RT/duroid 5870 **</td>
<td>2.33</td>
<td>0.0012</td>
</tr>
<tr>
<td>RT/duroid 5880</td>
<td>2.20</td>
<td>0.0009</td>
</tr>
<tr>
<td>RT/duroid 6002</td>
<td>2.94</td>
<td>0.0012</td>
</tr>
<tr>
<td>RT/duroid 6006</td>
<td>6.15</td>
<td>0.0019</td>
</tr>
<tr>
<td>RT/duroid 6010</td>
<td>10.2</td>
<td>0.0024</td>
</tr>
<tr>
<td>RT/duroid 6010</td>
<td>10.5</td>
<td>0.0024</td>
</tr>
<tr>
<td>RT/duroid 6010</td>
<td>10.8</td>
<td>0.0024</td>
</tr>
</tbody>
</table>

*) depends on the orientation of the crystal with respect to the field

**) RT/duroid is a ceramic polytetrafluoroethylene (PTFE) composite, dielectric constants given at 10 GHz
3. Basic antenna analysis methods

This chapter gives an introduction to the Kirchhoff diffraction theory and UTD for calculating far-field radiation patterns of aperture antennas. Formulas are given and symbols used throughout this report are introduced. They are necessary when analyzing the planar lens and the elliptical FZPA.

3.1 The Kirchhoff diffraction theory

An aperture antenna can be considered as an electromagnetic source in a diffraction system. This general diffraction system is some volume \( V \) enclosing the antenna. The boundary of \( V \) is determined by the surface \( A \). Figure 3.1 shows the configuration.

The origin of \( V \) is denoted by \( O \). On \( A \) there is an electric field \( E(r') \), where \( r' \) is the vector from the origin of the system to a point on \( A \). \( \hat{r} \) is the unit vector in the direction of the observation point and \( R \) is the distance from a point on \( A \) towards this same point. Finally, \( \hat{n} \) is the unit vector normal to \( A \) and is directed outward. The vectorial electric field \( E(r) \) in a point outside the volume \( V \) is given by the Kirchhoff diffraction integral [20].
where \( k = \frac{2\pi}{\lambda} \) is the wavenumber in free space. In the far-field the following approximation can be made

\[
\frac{1}{R} \approx \frac{1}{r} \quad e^{-jkR} \approx e^{-jk(r-r')} \quad (3.2)
\]

The far-field formula then becomes

\[
E(r) = \frac{jke^{-jkr}}{2\pi r} \hat{r} \times \int_A (\hat{n} \times E(r')) e^{jk\hat{r} \cdot \hat{r}'} dA \quad (3.3)
\]

The surface \( A \) can be considered as the aperture of the antenna. Essential in this theory is that the field from the source, incident to the aperture is not influenced by the edges of the antenna system. However, in practice there will always be some interaction between the edge and the incident field. One can imagine that this assumption applies better when the aperture is large in terms of wavelength. In case of the FZPA, the field in the integral of Eq. (3.3) equals the field transmitted by the feedhorn.

**3.2 The Uniform Theory of Diffraction**

Aperture integration techniques, based on the Kirchhoff diffraction theory, have the disadvantage that the far-field expressions usually have to be evaluated numerically. This is because no closed formulas are available. For observation points outside the mainlobe of the antenna the phase function of the integrand oscillates rapidly over the integration interval. Then calculations take a lot of CPU-time. Hence, it is more convenient to use other techniques for this far-field angle region. Asymptotic high-frequency techniques are appropriate tools for such calculations.

In the early sixties, Keller [21] introduced the Geometrical Theory of Diffraction (GTD) which is based on ray optics and includes diffraction effects. GTD provides the field diffracted by an edge in a perfectly conducting screen. When this diffracted field is summed to the geometric optics (GO) field, a good approximation is obtained of the total field. Because no integral appears in the GTD formulas, this method will use much less CPU-time than the Kirchhoff method. The opaque zones of the simplest FZPA can be modelled as perfectly conducting rings, so GTD is a powerful tool for calculating far-field radiation patterns of this antenna. GTD assumes that only discrete points on the antenna edge, so called diffraction points, contribute to the far-field in the observation point. A diffraction point can be found by applying the stationary phase method. Then the pathlength from the feed of the antenna system, via the zone edge to the far-field observation point must reach a minimum or maximum. This definition of diffraction points implies that GTD can not be used in boresight of an antenna. In this special case, the mentioned pathlength is equal for all points on the edge, leading to an infinite number of stationary points. This is one of the main disadvantages of GTD and therefore it can only be used for wide-angle far-field calculations.

The diffracted rays follow Keller's law of edge diffraction and diverge according to GO-laws, which means that the angle of diffraction \( \beta_d \) is equal to the angle of incidence \( \beta_o \). Consequently, the diffracted rays emanating from the point of diffraction form a cone whose half-angle is \( \beta_o \) and
3. Basic antenna analysis methods

whose axis is the tangent to the edge. This is shown in figure 3.2.

Figure 3.2 Edge diffracted rays forming a cone

Later, GTD was extended to UTD to remove some singularities in Keller’s original formulas [22].

The UTD diffracted field $\tilde{E}^d(P)$ can be expressed as

$$
\tilde{E}^d = D \cdot \tilde{E}^i \cdot A \cdot e^{-jks^d}
$$

where $A$ is the caustic divergence factor, $D$ the dyadic diffraction coefficient, $\tilde{E}^i$ the incident field at the point of diffraction and $s^d$ the distance between the point of diffraction and the far-field observation point.

The caustic divergence factor in Eq. (3.4) describes how the amplitude of the diffracted field varies in the propagation direction and follows from the laws of energy conservation. For a spherical wave incident on a curved edge it takes the following form

$$
A(p_c, s^d) = \sqrt{\frac{p_c}{s^d(p_c + s^d)}}
$$

where

$$
\frac{1}{p_e} = \frac{1}{p_e^i} - \hat{n} \cdot (\hat{s}^i - \hat{s}^d) / \rho_e \sin^2 \beta_0
$$

with

- $p_c$ the distance between the caustic at the edge and the second caustic of the diffracted ray
- $p_e$ the radius of curvature of the edge at the diffraction point
- $p_e^i$ the radius of curvature of the incident wavefront at the edge, which is the distance from the source to the diffraction point in case of a spherical wave
- $\hat{n}$ the unit vector normal to the edge and directed away from the centre of curvature

In order to describe the diffraction process, two edge-fixed coordinate systems are introduced.
3. Basic antenna analysis methods

They are related to the so-called edge-fixed plane of incidence and diffraction. These planes contain the incident ray ($\hat{s}^i$) or the diffracted ray ($\hat{s}^d$) and the vector tangential to the edge ($\hat{t}$) respectively. Figure 3.3 shows these planes.

![Figure 3.3 Edge-fixed plane of incidence and diffraction with coordinate systems](image)

Here, $\hat{n}_r$ is the unit vector normal to the surface and $\hat{t}$ is the unit surface tangent related as $\hat{t} = \hat{e} \times \hat{n}_r$. The unit vectors forming the orthonormal basis of the system are perpendicular and parallel to these planes. They are given by

$$
\hat{\phi}' = \frac{-\hat{e} \times \hat{s}^i}{|\hat{e} \times \hat{s}^i|},
$$

$$
\hat{\beta}' = -\hat{\phi}' \times \hat{s}^i,
$$

$$
\hat{\phi} = \frac{\hat{e} \times \hat{s}^d}{|\hat{e} \times \hat{s}^d|},
$$

$$
\hat{\beta} = -\hat{\phi} \times \hat{s}^d
$$

The dyadic diffraction coefficient $\bar{D}$ is in general a 3x3 matrix. But when the incident and diffracted field are described in the edge-fixed coordinate systems, the dyadic diffraction coefficient reduces to the sum of 2 dyads. Then it can be written as

$$
\bar{D} = -D_s \hat{\beta} \hat{\beta}' - D_h \hat{\phi} \hat{\phi}'
$$

$D_s$ and $D_h$ are the three dimensional diffraction coefficients and correspond to the soft ($\hat{\beta}$) and
the hard ($\hat{\phi}$) component respectively. In case of a perfectly conducting half-plane they can be expressed as

$$D_{sh} = \frac{-1}{2\sqrt{2}k\pi \sin \beta_0} \left[ F \left[ kL^t \cos^2 \left( \frac{\gamma - \gamma'}{2} \right) \right] \cos \left( \frac{\gamma - \gamma'}{2} \right) - F \left[ kL^r \cos^2 \left( \frac{\gamma + \gamma'}{2} \right) \right] \cos \left( \frac{\gamma + \gamma'}{2} \right) \right]$$

(3.9)

$$= D(\gamma - \gamma') + D(\gamma + \gamma')$$

The function $F[..]$ in the latter equation is known as the modified Fresnel integral or transition function

$$F[a] = 2j\sqrt{a} e^{ja} \int_{-j\infty}^{+j\infty} e^{-j\tau^2} d\tau$$

(3.10)

The Fresnel integral in the diffraction coefficients is the previously mentioned extension to GTD resulting in UTD. It ensures continuity of the total field across shadow and reflection boundaries, where the geometric optics field is discontinuous. Figure 3.4 shows these optical boundaries.

In Eq. (3.9), $D(\gamma - \gamma')$ is associated with the incidence shadow boundary (SB), whereas $D(\gamma + \gamma')$ is associated with the reflection shadow boundary (RB). The $+$ sign originates from the polarization dependence of the reflection coefficient of the electric field. Crossing the SB or RB means that $D(\gamma - \gamma')$ or $D(\gamma + \gamma')$ is discontinuous. Adding the diffracted field to the discontinuous GO-field, the total field is continuous again. For small arguments, close to the SB or RB, the Fresnel integral can be approximated by
and for large arguments \((a>15)\), away from the boundaries,

\[
F[a] = 1 + \frac{j}{2a} - \frac{3}{4a^2} - \frac{j}{8a^3} + \frac{15}{16a^4}
\]  

(3.12)

The latter equation asymptotically becomes 1, which means that UTD reduces to GTD. The region where one has to apply the UTD formulas instead of the GTD formulas, is called the transition region.

As shown in figure 3.4 \(\gamma\) and \(\gamma'\) are the angles between the incident ray and the diffracted ray with the surface of the half-plane respectively. To determine these angles the following vectors are introduced

\[
\hat{s}' = \frac{-\hat{s}' + (\hat{s}' \cdot \hat{e}) \hat{e}}{|-\hat{s}' + (\hat{s}' \cdot \hat{e}) \hat{e}|}
\]  

(3.13)

and

\[
\hat{s}^d = \frac{-\hat{s}^d + (\hat{s}^d \cdot \hat{e}) \hat{e}}{|-\hat{s}^d + (\hat{s}^d \cdot \hat{e}) \hat{e}|}
\]  

(3.14)

In fact, these vectors are projections of \(\hat{s}'\) and \(\hat{s}^d\) onto a plane perpendicular to the edge tangent \(\hat{e}\). The angles \(\gamma\) and \(\gamma'\) are now defined by [23]

\[
\gamma' = \pi - \left[ \pi - \arccos (\hat{s}' \cdot \hat{e}) \right] \text{sgn}(\hat{s}' \cdot \hat{n},)
\]  

(3.15)

\[
\gamma = \pi - \left[ \pi - \arccos (\hat{s}^d \cdot \hat{e}) \right] \text{sgn}(\hat{s}^d \cdot \hat{n},)
\]

where the \(\text{sgn}\)-function is given by

\[
\text{sgn}(a) = \begin{cases} 
1 & \text{if } a \geq 0 \\
-1 & \text{if } a < 0 
\end{cases}
\]  

(3.16)

The Fresnel integrals in Eq. (3.9) contain the distance parameters \(L^i\) and \(L'^i\). They are a measure for the angular separation between the vector \(\hat{s}^d\) and the SB and RB respectively. They are defined as

\[
L^i = \frac{s^d(p^i + s^d) p^i_1 p^i_2 \sin^2 \beta_0}{\rho^i_1(p^i_1 + s^d)(p^i_2 + s^d)}
\]  

(3.17)

and

\[
L'^i = \frac{s^d(p'^i + s^d) p'^i_1 p'^i_2 \sin^2 \beta_0}{\rho^i_1(p^i_1 + s^d)(p'^i_2 + s^d)}
\]  

(3.18)
3. Basic antenna analysis methods

where

\[
\frac{1}{\rho_e^*} = \frac{1}{\rho_e^i} - \frac{2 (\hat{n}_r \cdot \hat{n}_i)(\hat{s}_i \cdot \hat{n}_e)}{\rho_g \sin^2 \beta_0}
\]  
(3.19)
4. The planar lens

The theories described in chapter 3 will be extended and used to analyze the axially symmetric planar lens. Formulas will be given and radiation patterns obtained by the two techniques will be presented. But first the antenna system is defined.

4.1. Configuration of the antenna system

The axially symmetric planar lens is placed in a rectangular coordinate system as shown in figure 4.1.

![Figure 4.1 Configuration of the axially symmetric Fresnel zone plate antenna system](image)

The spherical coordinates $(\rho, \psi, \xi)$ are used to describe points on the antenna surface, whereas $(r, \theta, \varphi)$ denote far-field observation points. The feed is placed at the focus and the aperture is in the $x$-$y$-plane. The centre of the aperture is at a distance $F$ from it. The power pattern of the feed is assumed to be given by
4. The planar lens

\[ G(\psi,n) = \begin{cases} 2(n+1)\cos^n(\psi), & 0 \leq \psi \leq \pi/2 \\ 0, & \psi \geq \pi/2 \end{cases} \]  

(4.1)

The polarization is chosen to be that of an x-polarized Huygens source. Then the feed polarization vector \( \hat{f} \) can be written as

\[ \hat{f} = -\cos \xi \hat{x} + \sin \xi \hat{\xi} \]  

(4.2)

The far-field of the feed is now given by

\[ \vec{E}_f(\rho,\psi,\xi,n) = A_0 \frac{e^{-jk\rho}}{\rho} G(\psi,n) \hat{f} \]  

(4.3)

with

\[ A_0 = \sqrt{\frac{\eta P_T}{2\pi}} \]  

(4.4)

In the latter equation \( P_T \) is the total power radiated by the feed and \( \eta \) is the intrinsic impedance of free space.

4.2 Diffraction by a dielectric half-plane

In chapter 3 it is explained that the Fresnel integrals in the diffraction coefficient ensure continuity of the total field across the incidence and reflection shadow boundary. The \( \pm 1 \) could be regarded as the reflection coefficient in case of a perfectly conducting half-plane. Now, let us consider a dielectric half-plane and look at the discontinuity in the GO-field across the SB and RB. The discontinuity across the SB is given by \( (1-T)E(P) \) whereas the discontinuity across the RB is given by \( RE(P) \). These discontinuities can be removed when the diffraction coefficients are extended to the form

\[ D_{\pm h} = (1-T)D(\gamma - \gamma') + RD(\gamma + \gamma') \]  

(4.5)

The transmission and reflection coefficients are defined in a ray-fixed coordinate system which is related to the plane of incidence. The plane of incidence is the plane containing the incident and reflected ray and the normal to the half-plane. This means that the transmission and reflection coefficients have to be transferred to the edge-fixed coordinate system before inserting them in the diffraction coefficient. The vectors forming the orthonormal basis of the ray-fixed coordinate system are related as follows

\[ \hat{u}_\perp = \hat{n}_x \times \hat{s}, \]
\[ \hat{u}_{\parallel} = \hat{s} \times \hat{u}_\perp, \]
\[ \hat{u}'_\perp = \hat{s'} \times \hat{u}_\perp, \]
\[ \hat{u}'_\parallel = \hat{s'} \times \hat{u}_\perp \]  

(4.6)

The configuration is shown in figure 4.2.
In case of the axially symmetric planar lens, the diffraction problem reduces to a 2-dimensional one. Incident, transmitted, reflected and diffracted rays are all in the same plane, because $\beta_0 = \pi/2$. The edge-fixed plane of incidence and the plane of incidence are now perpendicular to each other. Then it is a quite simple task to derive the diffraction coefficients for the dielectric half-plane. The transmission and reflection coefficients for perpendicular ($E$) polarization have to be inserted in the diffraction coefficient for the soft component. The transmission and reflection coefficients for parallel ($M$) polarization correspond to the hard component. The field diffracted by a dielectric half-plane can now be expressed as

$$
\begin{bmatrix}
E_{\phi}^a \\
E_{\psi}^a
\end{bmatrix} =
\begin{bmatrix}
-D_s & 0 \\
0 & -D_h
\end{bmatrix}
\begin{bmatrix}
E_{\phi}' \\
E_{\psi}'
\end{bmatrix}
A e^{-jks^a}
$$

(4.7)

with

$$
D_s = (1 - T_E)D(\gamma - \gamma') + R_ED(\gamma + \gamma'),
$$

$$
D_h = (1 - T_M)D(\gamma - \gamma') + R_MD(\gamma + \gamma'),
$$

(4.8)

Baggen has analyzed the ideal phase correcting FZPA by applying the Kirchhoff integral, where it is assumed that the phase correcting zones act as ideal $\pi$ radian shifters. With the just described method in mind, it seems that the phase shifter can be modelled in a UTD approach as well. Then the appropriate transmission and reflection coefficients have to be determined and substituted in Eq. (4.8). Calculating the Kirchhoff integral with a computer costs a lot of CPU-time. If the same results are obtained by the heuristic UTD approach, we have found a method which is more time-efficient.

We will start with deriving the diffraction coefficients for the phase shifter. The phase shifter causes no reflections, which means...
4. The planar lens

\[ R_{EIM} = 0 \]  

(4.9)

The transmission coefficient of the material is simply given by

\[ T_{EIM} = -1 \]  

(4.10)

Inserting the latter two relations in Eq. (4.8) gives the diffraction coefficients of the ideal phase shifter,

\[ D_s = 2D(\gamma - \gamma') \]
\[ D_h = 2D(\gamma - \gamma') \]  

(4.11)

The transmission and reflection coefficients for the dielectric half-plane will be derived in the next section.

4.3 Plane wave transmission and reflection by a lossless dielectric slab

Consider a plane wave incident on a lossless dielectric slab which is shown in figure 4.3.

\[ \varepsilon_0 \quad \varepsilon_r \]
\[ d \]
\[ E^{i} \quad E^{r1} \quad E^{r2} \quad E^{t1} \quad E^{t2} \]

Here, \( \varepsilon_0 \) is the permittivity of free space and \( \varepsilon_r \) is the relative permittivity of the dielectric, which is assumed to be frequency independent. Wave propagation will be determined by means of ray-optics, so we can speak of incident, transmitted and reflected rays. From the left the ray \( E^i \) is incident on the slab and part of it is reflected (denoted by \( E^{r1} \)). The other part, the transmitted ray, propagates into the dielectric. The transmitted ray reaches the dielectric/free space boundary, where it is again partly transmitted (\( E^{r2} \)) and reflected, and so on. The rays leaving the slab are all parallel and make the same angle with the slab normal as the incident ray. The total transmitted
and reflected field is now given an infinite number of field contributions due to the process of multiple internal reflection. It is possible to derive so called 'infinite' transmission and reflection coefficients which take into account all these contributions. In order to calculate the 'infinite' transmission and reflection coefficients, it is necessary first to make some definitions. Let's start with the single plane wave transmission and reflection coefficients, which are related to the electric field. The electric field is described in a ray-fixed coordinate system. This means that the field vectors can either be perpendicular or parallel to the plane of incidence. The transmission and reflection coefficients correspond to these unit vectors. The coefficients are polarization dependent, so the perpendicular and parallel case must be considered separately. The transmission and reflection coefficients which appear when the wave travels from free space to the dielectric are treated first. They are marked with the subscript 1. In case of perpendicular polarization, the transmission coefficient $T_{1E}$ is given by

$$T_{1E} = \frac{2\cos \theta_i}{\cos \theta_i + \sqrt{\epsilon_r - \sin^2 \theta_i}}$$

and the reflection coefficient $R_{1E}$ is

$$R_{1E} = \frac{\cos \theta_i - \sqrt{\epsilon_r - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{\epsilon_r - \sin^2 \theta_i}}$$

In case of parallel polarization, the transmission coefficient $T_{1M}$ is given by

$$T_{1M} = \frac{2\sqrt{\epsilon_r \cos \theta_i}}{\epsilon_r \cos \theta_i + \sqrt{\epsilon_r - \sin^2 \theta_i}}$$

and the reflection coefficient $R_{1M}$ is

$$R_{1M} = \frac{\epsilon_r \cos \theta_i - \sqrt{\epsilon_r - \sin^2 \theta_i}}{\epsilon_r \cos \theta_i + \sqrt{\epsilon_r - \sin^2 \theta_i}}$$

There is a simple relation between the latter coefficients and the reflection coefficients in case the wave travels from the dielectric to free space, marked with the subscript 2 (see Appendix A). It is given by

$$R_{2,IM} = -R_{1,EM}$$

From figure 4.3 it is obvious that there are phase delays between the different rays leaving the slab. Two kinds of phase delays are illustrated in figure 4.4.

First, there is a phase delay $P_d$ whenever a ray crosses the dielectric. It is given by

$$P_d = e^{-jk\sqrt{\epsilon_r}l}$$, \quad \text{with} \quad l = \frac{d}{\cos \theta_i}$$

Second, there is a path length difference between subsequent rays in the far-field observation point. This is due to the fact that rays leave the slab at different points. From figure 4.4 it can be seen that the associated phase delay $P_a$ is given by
Now the single plane wave reflection coefficients and all the phase delays have been defined, it is possible to construct the infinite reflection coefficients $R_{E,M}$. The subscript $E,M$ will be omitted further on. The series for $R$ becomes

$$ R = R_1 + T_1 T_2 (R_2 P^2_a P_a + R_2^3 P^4_a P_a + \ldots) $$

$$ = R_1 + T_1 T_2 \sum_{n=1}^{\infty} R_2^{2n-1} P_a^{2n} P_a $$

$$ = R_1 + \frac{T_1 T_2}{R_2} \sum_{n=1}^{\infty} (R_2^2 P_a^2 P_a)^n $$

$$ = R_1 - \frac{T_1 T_2}{R_1} \sum_{n=1}^{\infty} (R_1^2 P_a^2 P_a)^n $$

By recognizing that for the dielectric layer (see Appendix A)

$$ T_1 T_2 = 1 - R_1^2 $$

the latter expression converges to

$$ R = \frac{R_1 (1 - P_a^2 P_a)}{1 - R_1^2 P_a^2 P_a} $$

The derivation of the transmission coefficient $T_{E,M}$ can be performed in the same way, but with a
slight extension. This is because we want to describe the transmitted field as the product of the transmission coefficient and the far-field in absence of the dielectric slab. The coefficient then has to be referred to the front face of the slab, which means an additional phase factor \( P \) has to be included. It is shown in figure 4.5.

![Figure 4.5 Referring the transmission coefficient to the front face of the slab](image)

From figure 4.5 it can be seen that

\[
P_t = e^{jkl\cos(\theta_i - \theta_t)}
\]  

(4.22)

With this result

\[
T = P_t T_1 T_2 \sum_{n=1}^{\infty} P_d^{2n-1} R_2^{2n-2} P_a^{n-1}
\]

\[= P_t P_d T_1 T_2 \sum_{n=0}^{\infty} (P_d^2 R_2^2 P_a)^n\]

\[= \frac{P_t P_d (1 - R_1^2)}{1 - R_1^2 P_d^2 P_a} \]

(4.23)

Now the infinite transmission and reflection coefficients have been derived, one can express the transmitted and reflected field as follows

\[
\begin{bmatrix}
E_t'(P_1) \\
E_t'(P_1)
\end{bmatrix} = \begin{bmatrix} T_M & 0 \\ 0 & T_E \end{bmatrix} \cdot \begin{bmatrix}
E_t'(P_1) \\
E_t'(P_1)
\end{bmatrix}
\]

(4.24)

and
The planar lens

\[
\begin{bmatrix}
E_r'(P_2) \\
E_i'(P_2)
\end{bmatrix} = \begin{bmatrix}
R_M & 0 \\
0 & R_E
\end{bmatrix} \begin{bmatrix}
E_r'(Q_r) \\
E_i'(Q_r)
\end{bmatrix} \frac{\rho_0}{\rho_0 + \rho} e^{-jks'}
\]

where \(\rho_0\) is the distance from the source towards the point of reflection, \(Q_r\) is the point of reflection and \(s'\) is the distance from \(Q_r\) to the far-field observation point as shown in figure 4.6.

![Figure 4.6 The total transmitted and reflected field](image)

In figure 4.7 and 4.8 the magnitude and phase of the infinite transmission coefficient for E-polarization is plotted against the relative dielectric constant \(\varepsilon_r\) for normal incidence. When \(\varepsilon_r = 4\) the amplitude is exactly equal to unity and the phase is 180°. This means that the slab behaves as a pure phase shifter. For larger values of the dielectric constant the amplitude rapidly falls down. This will have an effect on the gain of the planar lens. The larger \(\varepsilon_r\) becomes, the more the dielectric zones will reflect electromagnetic energy, asymptotically resulting in a FZPA with metallic/transparent zones. From this point of view it is desirable that \(\varepsilon_r\) is close to 4. On the other hand, the constraint found by Burgener and Burnside tells that \(\varepsilon_r\) should be large for the described UTD method to be valid. Otherwise, the zones have to be modelled as block-shaped obstacles. Diffraction coefficients are not known for this situation. The block-shaped zones would also give rise to a stronger shadowing effect. From these considerations it is obvious that there has to be made a compromise in the choice of material.
Figure 4.7 Magnitude of the "infinite" transmission coefficient as function of the relative dielectric constant, for normal incidence

Figure 4.8 Phase of the "infinite" transmission coefficient as function of the relative dielectric constant, for normal incidence
4.4 Diffraction by a ring

The formulas derived in the previous section will now be applied to a circular ring. Because of the special geometry, explicit expressions for the diffracted field can be derived. Multiple diffraction, as treated by Baggen, is not considered because it can not be modelled with this theory. One single ring possesses four diffraction points. They will be treated separately. Therefore, the subscripts of all variables indicating which diffraction point is meant, can be omitted. Only the angles of diffraction will be marked with subscripts in order to avoid confusion with the far field observation angle $\theta$. The distance from the feed to the point of diffraction will be denoted by $\rho_0$.

4.4.1 The upper, outer diffraction point $Q_{uo}$

Figure 4.9 illustrates the configuration for the diffraction point under consideration.

![Figure 4.9 The upper, outer diffraction point $Q_{uo}$](image)

First, the caustic divergence factor will be calculated. With figure 4.9 it is easy to see that

\[
\rho_c = \rho_0 \\
\hat{n} \cdot \hat{s} = \sin \alpha \\
\hat{n} \cdot \hat{s}_d = \sin \theta_{uo} \\
\rho_g = \rho_0 \sin \alpha
\]  

With these results the caustic distance $\rho_c$ is

\[
\rho_c = \frac{r_{uo}}{\sin \theta_{uo}}
\]

The caustic divergence factor becomes
The angles $\gamma$ and $\gamma'$ are given by
\[
\begin{align*}
\gamma &= \pi/2 - \alpha \\
\gamma &= 3\pi/2 - \theta_{uo} \\
\gamma &= 2\pi - \alpha - \theta_{uo}
\end{align*}
\]
which leads to
\[
\begin{align*}
\cos\left(\frac{\gamma - \gamma'}{2}\right) &= \cos\left(\frac{\pi + \alpha - \theta_{uo}}{2}\right) = -\sin\left(\frac{\alpha - \theta_{uo}}{2}\right) \\
\cos\left(\frac{\gamma + \gamma'}{2}\right) &= \cos\left(\frac{\pi - \alpha + \theta_{uo}}{2}\right) = -\cos\left(\frac{\alpha + \theta_{uo}}{2}\right)
\end{align*}
\]

The distance parameters are given by $L^i = \rho_0$ and $L' = \rho_o$. Furthermore, in the far-field the following approximations can be made
\[
\frac{1}{s^d} \approx \frac{1}{r}
\]
\[
e^{-jks^d} = e^{-jk(r - \rho_0 \sin \alpha \sin \theta_{uo})}
\]
\[
\theta_{uo} = \theta
\]
The diffracted field can now be written as
\[
\begin{bmatrix} E^d_\rho \\ E^d_r \end{bmatrix} = \begin{bmatrix} D_s & 0 \\ 0 & -D_h \end{bmatrix} e^{-jkr} \sqrt{\frac{\cos(\alpha \sin \theta) \sin \theta}{\rho_0 \sin \theta}} e^{j k \rho_0 \sin \alpha \sin \theta} \begin{bmatrix} E^i_\rho \\ E^i_\phi \end{bmatrix}
\]
Together with
\[
\begin{bmatrix} E^d_\varphi \\ E^d_\theta \end{bmatrix} = \begin{bmatrix} E^i_\beta \\ E^i_\phi \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} E^i_\varphi \\ E^i_\theta \end{bmatrix} = \begin{bmatrix} -E^i_\rho \\ -E^i_\phi \end{bmatrix}
\]
the far-field approximation of the diffracted field becomes
\[
\begin{bmatrix} E^d_\varphi \\ E^d_\theta \end{bmatrix} = A_0 \frac{e^{-jkr}}{r} \sqrt{\frac{2(n+1) \cos \varphi \sin \alpha}{\rho_0 \sin \theta}} e^{j k \rho_0 (\sin \alpha \sin \theta - 1)} \begin{bmatrix} D_s \sin \phi \\ -D_h \cos \phi \end{bmatrix}
\]
4.4.2 The upper, inner diffraction point $Q_{ui}$

In figure 4.10 the configuration for the diffraction point under consideration is shown. The caustic divergence factor can be easily calculated from it.

\[ \Theta_{ui} \]

\[ \Phi \]

\[ \Theta_i \]

\[ \Phi_i \]

\[ \beta \]

\[ \beta' \]

\[ \gamma \]

\[ \gamma' \]

\[ \rho_e^i = \rho_0 \]

\[ \hat{n} \cdot \hat{s}^f = \sin \alpha \]

\[ \hat{n} \cdot \hat{s}^d = \sin \theta_{ui} \]

\[ \rho_g = \rho_0 \sin \alpha \]

With these results the caustic distance $\rho_c$ is

\[ \rho_c = \rho_0 \sin \alpha \sin \theta_{ui} \] (4.36)

The caustic divergence factor becomes

\[ A = \frac{1}{s^d} \sqrt{\rho_c} \quad (s^d - \infty) \]

\[ = \frac{1}{r} \sqrt{\frac{\rho_0 \sin \alpha}{\sin \theta_{ui}}} \] (4.37)

The angles $\gamma$ and $\gamma'$ are given by
\[
\begin{align*}
\gamma' &= \pi/2 + \alpha \\
\gamma &= 3\pi/2 + \theta_{ui}, \quad \theta_{ui} < \pi/2 \\
\gamma &= \theta_{ui} - \pi/2, \quad \theta_{ui} > \pi/2
\end{align*}
\]

which leads to
\[
\begin{align*}
\cos \left( \frac{\gamma - \gamma'}{2} \right) &= \cos \left( \frac{\pi + \theta_{ui} - \alpha}{2} \right) = \sin \left( \frac{\alpha - \theta_{ui}}{2} \right), \quad \theta_{ui} < \pi/2 \\
&= \cos \left( -\pi/2 + \frac{\theta_{ui} - \alpha}{2} \right) = -\sin \left( \frac{\alpha - \theta_{ui}}{2} \right), \quad \theta_{ui} > \pi/2 \\
\cos \left( \frac{\gamma + \gamma'}{2} \right) &= \cos \left( \pi + \theta_{ui} + \alpha \right) = -\cos \left( \frac{\alpha + \theta_{ui}}{2} \right), \quad \theta_{ui} < \pi/2 \\
&= \cos \left( \frac{\alpha + \theta_{ui}}{2} \right), \quad \theta_{ui} > \pi/2
\end{align*}
\]

The distance parameters are given by \( L' = \rho_{0} \) and \( L' = \rho_{e} \). Furthermore, in the far-field the following approximations can be made
\[
\begin{align*}
\frac{1}{s_{d}} &= \frac{1}{r} \\
e^{-jks_{d}} &= e^{-jk(r - \rho_{0}\sin\alpha\sin\theta_{ui})} \\
\theta_{ui} &= \theta
\end{align*}
\]

The diffracted field can now be written as
\[
\begin{bmatrix}
E_{\phi}^{d} \\
E_{\phi}^{d}
\end{bmatrix} = \begin{bmatrix}
-D_{s} & 0 \\
0 & -D_{h}
\end{bmatrix} e^{-jkr} \frac{\rho_{0}\sin\alpha}{\sin\theta} e^{jkr} \rho_{0}\sin\alpha\sin\theta \begin{bmatrix}
E_{\phi}^{i} \\
E_{\phi}^{i}
\end{bmatrix}
\]

Together with
\[
\begin{bmatrix}
E_{\phi}^{d} \\
E_{\phi}^{d}
\end{bmatrix} = \begin{bmatrix}
E_{\phi} \\
E_{\phi}
\end{bmatrix} \quad \text{and} \quad \begin{bmatrix}
E_{\phi}^{i} \\
E_{\phi}^{i}
\end{bmatrix} = \begin{bmatrix}
-E_{\phi}^{i} \\
-E_{\phi}^{i}
\end{bmatrix}
\]

the far-field approximation of the diffracted field becomes
\[
\begin{bmatrix}
E_{\phi}^{d} \\
E_{\phi}^{d}
\end{bmatrix} = A_{0} e^{-jkr} \sqrt{\frac{2(n+1)\cos^{2}\alpha\sin^{2}\phi}{\rho_{0}\sin\theta}} e^{jkr} \rho_{0}(\sin\alpha\sin\theta - 1) \begin{bmatrix}
D_{s}\sin\phi \\
-D_{h}\cos\phi
\end{bmatrix}
\]
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4.4.3 The lower, outer diffraction point Q_{lo}

Figure 4.11 illustrates the situation for the diffraction point under consideration.

First, the caustic divergence factor will be calculated. From figure 4.11 it is easy to see that

\[ \rho_e^i = \rho_0 \]
\[ \hat{n} \cdot \hat{s}^i = \sin \alpha \]
\[ \hat{n} \cdot \hat{s}^d = -\sin \theta_{lo} \]
\[ \rho_g = \rho_0 \sin \alpha \]

With these results the caustic distance \( \rho_c \)

\[ \rho_c = \frac{-\rho_0 \sin \alpha}{\sin \theta_{lo}} \]  \hspace{1cm} (4.45)

The caustic divergence factor becomes

\[ A = \frac{1}{s^d \sqrt{\rho_c}} (s^d \to \infty) \]
\[ = \frac{1}{r \sqrt{\rho_0 \sin \alpha}} \]  \hspace{1cm} (4.46)

The angles \( \gamma \) and \( \gamma' \) are given by

\[ \begin{align*}
\gamma' &= \pi/2 - \alpha \\
\gamma &= 3\pi/2 + \theta_{lo}, \quad \theta_{lo} < \pi/2 \\
\gamma &= \theta_{lo} - \pi/2, \quad \theta_{lo} > \pi/2
\end{align*} \]  \hspace{1cm} (4.47)

which leads to
\[
\begin{align*}
\cos \left( \frac{\gamma - \gamma'}{2} \right) &= \cos \left( \frac{\pi + \alpha + \theta_{lo}}{2} \right) = -\sin \left( \frac{\alpha + \theta_{lo}}{2} \right), \quad \theta_{lo} < \pi/2 \\
&= \cos \left( -\pi/2 + \frac{\alpha + \theta_{lo}}{2} \right) = \sin \left( \frac{\alpha + \theta_{lo}}{2} \right), \quad \theta_{lo} > \pi/2 \\
\cos \left( \frac{\gamma + \gamma'}{2} \right) &= \cos \left( \pi + \frac{\theta_{lo} - \alpha}{2} \right) = -\cos \left( \frac{\theta_{lo} - \alpha}{2} \right), \quad \theta_{lo} < \pi/2 \\
&= \cos \left( \frac{\theta_{lo} - \alpha}{2} \right), \quad \theta_{lo} > \pi/2
\end{align*}
\]

(4.48)

The distance parameters are given by \( L' = \rho_0 \) and \( L' = \rho_0 \). Furthermore, in the far-field the following approximations can be made

\[
\frac{1}{s^d} = \frac{1}{r} \quad e^{-jks^d} = e^{-jk(r + \rho_0 \sin \alpha \sin \theta_{lo})}
\]

(4.49)

\[
\theta_{lo} = \theta
\]

The diffracted field can now be written as

\[
\begin{bmatrix}
E^d_{\phi} \\
E^d_{\theta}
\end{bmatrix} = \begin{bmatrix}
-D_s & 0 \\
0 & -D_h
\end{bmatrix} e^{-jkr} r \sqrt{\frac{\rho_0 \sin \alpha}{\sin \theta}} e^{-jk\rho_0 \sin \alpha \sin \theta + j\pi/2} \begin{bmatrix}
E^t_{\phi} \\
E^t_{\theta}
\end{bmatrix}
\]

(4.50)

Together with

\[
\begin{bmatrix}
E^d_{\phi} \\
E^d_{\theta}
\end{bmatrix} = \begin{bmatrix}
-E_{\phi} \\
-E_{\theta}
\end{bmatrix}, \quad \begin{bmatrix}
E^t_{\phi} \\
E^t_{\theta}
\end{bmatrix} = \begin{bmatrix}
-E_{\phi} \\
-E_{\theta}
\end{bmatrix} \quad \text{and} \quad \varphi := \varphi + \pi
\]

(4.51)

the far-field approximation of the diffracted field becomes

\[
\begin{bmatrix}
E^d_{\phi} \\
E^d_{\theta}
\end{bmatrix} = A_0 e^{-jkr} r \sqrt{\frac{2(n+1)\cos^2 \alpha \sin \alpha}{\rho_0 \sin \theta}} e^{-jk\rho_0 (\sin \alpha \sin \theta + 1) + j\pi/2} \begin{bmatrix}
D_s \sin \varphi \\
-D_h \cos \varphi
\end{bmatrix}
\]

(4.52)

### 4.4.4 The lower, inner diffraction point \( Q_{li} \)

Figure 4.12 illustrates the situation for the diffraction point under consideration. First, the caustic divergence factor will be calculated. From figure 4.12 it is easy to see that
The planar lens

Figure 4.12 The lower, inner diffraction point $Q_{ii}$

\[ \rho_c^i = \rho_0 \]
\[ \hat{n} \cdot \hat{s}^i = \sin \alpha \]
\[ \hat{n} \cdot \hat{s}^d = -\sin \theta_{ii} \]
\[ \rho_g = \rho_0 \sin \alpha \]

With these results the caustic distance $\rho_c$ is

\[ \rho_c = \frac{-\rho_0 \sin \alpha}{\sin \theta_{ii}} \]

The caustic divergence factor becomes

\[ A = \frac{1}{s^d} \sqrt{\rho_c} \quad (s^d \to \infty) \]
\[ = \frac{i}{r} \sqrt{\frac{\rho_0 \sin \alpha}{\sin \theta_{ii}}} \]

The angles $\gamma$ and $\gamma'$ are given by

\[ \begin{align*}
\gamma' &= \pi/2 + \alpha \\
\gamma &= 3\pi/2 - \theta_{ii}
\end{align*} \]

which leads to
\[
\begin{align*}
\left\{ \begin{array}{l}
\cos \left( \frac{\gamma - \gamma'}{2} \right) = \cos \left( \frac{\pi - \alpha + \theta_{ii}}{2} \right) = \sin \left( \frac{\alpha + \theta_{ii}}{2} \right) \\
\cos \left( \frac{\gamma + \gamma'}{2} \right) = \cos \left( \pi + \alpha - \theta_{ii} \right) = -\cos \left( \frac{\alpha - \theta_{ii}}{2} \right)
\end{array} \right.
\end{align*}
\] (4.57)

The distance parameters are given by \( L^i = \rho_0 \) and \( L' = \rho_0' \). Furthermore, in the far-field the following approximations can be made

\[
\frac{1}{s^d} = \frac{1}{r}
\]

\[ e^{-jk s^d} = e^{-jk (r + \rho_0 \sin \alpha \sin \theta_{ii})} \] (4.58)

\[ \theta_{ii} = \theta \]

The diffracted field can now be written as

\[
\begin{bmatrix}
E_\varphi^d \\
E_\theta^d
\end{bmatrix} = 
\begin{bmatrix}
-D_s & 0 \\
0 & -D_h
\end{bmatrix}
\frac{e^{-jk r}}{r}
\sqrt{\frac{\rho_0 \sin \alpha}{\sin \theta}}
\left[ E_{\varphi}^i \\
E_{\theta}^i
\right] - \frac{\rho_0 \sin \alpha \sin \theta + j \pi/2}{\rho_0 \sin \theta}
\begin{bmatrix}
E_{\varphi}^i \\
E_{\theta}^i
\end{bmatrix}
\] (4.59)

Together with

\[
\begin{bmatrix}
E_\varphi^d \\
E_\theta^d
\end{bmatrix} = 
\begin{bmatrix}
-E_{\varphi}^i \\
-E_{\theta}^i
\end{bmatrix}, \quad
\begin{bmatrix}
E_{\varphi}^i \\
E_{\theta}^i
\end{bmatrix} = 
\begin{bmatrix}
-E_{\varphi}^i \\
-E_{\theta}^i
\end{bmatrix}\quad \text{and} \quad \varphi = \varphi + \pi
\] (4.60)

The far-field approximation of the diffracted field becomes

\[
\begin{bmatrix}
E_\varphi^d \\
E_\theta^d
\end{bmatrix} = A_0 \frac{e^{-jk r}}{r}
\sqrt{\frac{2(n + 1) \cos \alpha \sin \alpha}{\rho_0 \sin \theta}}
\left[ e^{-jk \rho_0 (\sin \alpha \sin \theta + 1) + j \pi/2} \begin{bmatrix}
D_s \sin \varphi \\
-D_h \cos \varphi
\end{bmatrix}
\right]
\] (4.61)

The total diffracted field is given by the summation of the contributions from the four diffraction points, whereas the complete field is the summation of the diffracted field and the direct contribution of the feed (the GO field).

### 4.5 The Kirchhoff diffraction integral

Leyten has derived far-field equations for an antenna system with absorbing/transparent zones. These formulas can easily be adjusted for antenna systems with ideal phase correcting zones. This can be done by including an additional phase factor \( j m \pi \), where \( m \) is the number of the Fresnel zone. Then the far-field equations are given by

\[
\tilde{E}(r, \theta) = E_\varphi(r, \theta) \hat{\varphi} + E_\theta(r, \theta) \hat{\theta}
\] (4.62)

with
4. The planar lens

\[ E_0(r,\psi) = \sum_{m=0}^{\psi_{\text{max}}} \pi \cos(\phi) C(r) \int_{\psi_m}^{\psi_{m+1}} P(m) O(\psi,\rho) e^{iM(\psi)} \]

\[ \left[ -(\cos(\psi) + 1)J_0(N(\theta,\psi)) + (\cos(\psi) - 1)J_2(N(\theta,\psi)) \right] d\psi \]

\[ E_0(r,\psi) = \sum_{m=0}^{\psi_{\text{max}}} \pi \cos(\theta) \sin(\phi) C(r) \int_{\psi_m}^{\psi_{m+1}} P(m) O(\psi,\rho) e^{iM(\psi)} \]

\[ \left[ (\cos(\psi) + 1)J_0(N(\theta,\psi)) + (\cos(\psi) - 1)J_2(N(\theta,\psi)) \right] d\psi \]

and

\[ C(r) = \frac{jke^{-jkr}}{2\pi r} \sqrt{\frac{P_T \rho}{2\pi}} \]

\[ M(\psi) = \frac{-jkF}{\cos(\psi)} \]

\[ N(\theta,\psi) = kF \sin(\theta) \tan(\psi) \]

\[ O(\psi,\rho) = \sqrt{G(\psi,\rho)} \frac{F \tan(\psi)}{\cos(\psi)} \]

It must be stated that \( \psi_0 = 0 \). In case of absorbing/transparent zones

\[ P(m) = 1, \quad m = 0, 2, 4... \]

\[ = 0, \quad m = 1, 3, 5... \]

and in case of ideal phase correcting zones

\[ P(m) = (-1)^m \]

When calculating the far-field of the planar lens, integration must be performed over two different aperture planes (figure 2.3). The formulas for the transparent zones are equal to those for the antenna with absorbing/transparent zones. For the dielectric zones, the field polarization vector describing the incident field, is assumed to be given by

\[ \hat{f} = -T_M \cos \xi \hat{\psi} + T_E \sin \xi \hat{\xi} \]

The formulas for the dielectric zones can now be written as
\[ E_\theta(r,n) = \sum_{m=1}^{3,5...} \pi \cos(\psi) C(r) \int_{\psi_m}^{\psi_{m+1}} O(\psi,n) e^{M(\psi)} \, d\psi, \quad (4.72) \]
\[ E_\phi(r,n) = \sum_{m=1}^{3,5...} \pi \cos(\psi) \sin(\psi) C(r) \int_{\psi_m}^{\psi_{m+1}} O(\psi,n) e^{M(\psi)} \, d\psi, \quad (4.73) \]

4.6 Comparison of Kirchhoff theory and UTD results

Power radiation patterns of the FZPA with ideal phase shifting and dielectric rings are calculated. This is done with the Kirchhoff diffraction theory as well as the heuristic UTD approach. In the Kirchhoff theory the incident field is only integrated over the antenna surface. Outside the antenna surface perfectly absorbing material is assumed reaching to infinity. When making a fair comparison with UTD, one has to define perfectly conducting material outside the antenna, in order to block off the GO-field. This means that the outer ring of the antenna has to be modelled as a perfectly conducting ring, which means that the relevant transmission and reflection coefficients are given by

\[ R_E = -1 \]
\[ R_M = 1 \]
\[ T_{EM} = 0 \]

(4.74)

The antenna systems used will be the same as those in the report of Baggen. He has optimized the system with ideal phase correcting zones. For all systems the operating frequency \( f \) will be 11.1 GHz and the antenna diameter \( D \) is exactly 1 m. For two different systems the focal distance and the feed factor are given by

System S1 : \( F = 0.581 \text{ m}, n = 7.4 \)
System S2 : \( F = 1.934 \text{ m}, n = 75.3 \)

The gain function of the antenna, which determines the power radiation pattern is defined as

\[ G(\theta,\phi) = 10\log \left( \frac{r^2|E|^2}{2\eta} \right) \quad (4.75) \]

The co- (CP) and cross-polarization (XP) patterns are given by

\[ CP = 10\log \left( \frac{2\pi r^2}{\eta P_T} |\cos(\phi) E_\theta - \sin(\phi) E_\phi|^2 \right) \quad (4.76) \]
4. The planar lens

\[ XP = 10 \log \left( \frac{2 \pi r^2}{\eta P_T} \left| -\sin(\varphi)E_\theta - \cos(\varphi)E_\phi \right|^2 \right) \]  

(4.77)

Figure 4.13 and 4.14 show radiation patterns for the antenna systems S1 and S2 with ideal phase correcting zones. For system S2 the results from Kirchhoff’s theory and UTD are almost the same. This implies that the heuristic UTD method can be used to obtain the same results as with the Kirchhoff diffraction integral. For system S1 the patterns differ more, but this is because the number of zones is larger and the widths of the zones are smaller. Then the aperture field will be more influenced by the edges and the assumption in the Kirchhoff theory becomes debatable. Figure 4.15 and 4.16 show radiation patterns when dielectric rings are applied. The relative dielectric constant is taken as \( \varepsilon_r = 4 \). For this value the infinite transmission coefficient, though complex, is very close to -1 for all relevant angles of incidence. Then one can expect the highest gain and the dielectric zones will behave almost as ideal phase shifters. Again, the patterns agree very well for system S2. When the patterns of figure 4.15 and 4.16 are compared with those of figure 4.13 and 4.14 respectively, it is found that they are practically the same. So when \( \varepsilon_r = 4 \), the dielectric can be modelled as an ideal phase shifter, as done in literature until now. It must be stated that strictly speaking the constraint given by Burnside and Burgener, that \( \varepsilon_r > 9 \), is not met.

Figure 4.17 and 4.18 show patterns for system S1 with a practical value of the relative dielectric constant of \( \varepsilon_r = 10.8 \), for the planes \( \varphi = 0^\circ \) and \( \varphi = 90^\circ \). The results obtained with Kirchhoff and UTD match very well, except for the region around \( \theta = 20^\circ \). Figure 4.19 and 4.20 show the results for system S2. These patterns match even better. When comparing figure 4.17 and 4.19 to figure 4.13 and 4.14 respectively, it can be concluded that it is not valid to model these FZPA’s as though they are ideal phase correcting. Antenna gain is clearly decreased, which is confirmed by figure 4.21 and 4.22. Here, the gain of the systems S1 and S2 is shown as a function of \( \varepsilon_r \). In the region \( \varepsilon_r = 4 \) to \( \varepsilon_r = 10 \) the gain rapidly falls down, approximately 2 to 2.5 dB. From the patterns shown it can be seen that in the mainlobe the results of both theories are equal. This means that the UTD patterns, which are calculated in a time-efficient way, can be extended in forward direction with the Kirchhoff results.

Figure 4.23 and 4.24 show co- and cross-polar patterns for S1 in the plane \( \varphi = 45^\circ \). Figure 4.25 and 4.26 show these patterns for S2. The cross-polar patterns calculated with Kirchhoff and UTD are clearly different. This is caused by the assumption in the Kirchhoff theory that the aperture field is not influenced by the edges. The patterns are not equal around \( \theta = 0^\circ \) as in case of the co-polar patterns. So, there is no theory available for calculating cross-polar in this region.
Figure 4.13 Gain patterns calculated with Kirchhoff's theory and UTD for system S1
\((F = 0.581\ m, n = 7.4, \varphi = 0^\circ)\) with ideal phase correcting zones

Figure 4.14 Gain patterns calculated with Kirchhoff's theory and UTD for system S2
\((F = 1.934\ m, n = 75.3, \varphi = 0^\circ)\) with ideal phase correcting zones
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Figure 4.15 Gain patterns calculated with Kirchhoff's theory and UTD for system S1 
\( F = 0.581 \text{ m}, n = 7.4, \varphi = 0^\circ \) with dielectric/transparent zones, \( \varepsilon_r = 4 \)

Figure 4.16 Gain patterns calculated with Kirchhoff's theory and UTD for system S2 
\( F = 1.934 \text{ m}, n = 75.3, \varphi = 0^\circ \) with dielectric/transparent zones, \( \varepsilon_r = 4 \)
Figure 4.17 Gain patterns calculated with Kirchhoff’s theory and UTD for system S1 
($F = 0.581$ m, $n = 7.4$, $\varphi = 0^\circ$) with dielectric/transparent zones,
$\varepsilon_r = 10.8$

Figure 4.18 Gain patterns calculated with Kirchhoff’s theory and UTD for system S1 
($F = 0.581$ m, $n = 7.4$, $\varphi = 90^\circ$) with dielectric/transparent zones,
$\varepsilon_r = 10.8$
Figure 4.19 Gain patterns calculated with Kirchhoff's theory and UTD for system S2 
($F = 1.934$ m, $n = 75.3$, $\varphi = 0^\circ$) with dielectric/transparent zones, 
$\varepsilon_r = 10.8$

Figure 4.20 Gain patterns calculated with Kirchhoff's theory and UTD for system S2 
($F = 1.934$ m, $n = 75.3$, $\varphi = 90^\circ$) with dielectric/transparent zones, 
$\varepsilon_r = 10.8$
Figure 4.21 Gain of system S1 as a function of $\varepsilon$, $(F = 0.581 \text{ m}, n = 7.4)$

Figure 4.22 Gain of system S2 as a function of $\varepsilon$, $(F = 1.934 \text{ m}, n = 75.3)$
4. The planar lens

Figure 4.23 Co-polar patterns calculated with Kirchhoff's theory and UTD for system S1 \((F = 0.581 \text{ m}, n = 7.4, \varphi = 45^\circ)\) with dielectric/transparent zones, \(\varepsilon_r = 10.8\)

Figure 4.24 Cross-polar patterns calculated with Kirchhoff's theory and UTD for system S1 \((F = 0.581 \text{ m}, n = 7.4, \varphi = 45^\circ)\) with dielectric/transparent zones, \(\varepsilon_r = 10.8\)
Figure 4.25 Co-polar patterns calculated with Kirchhoff's theory and UTD for system S2 ($F = 1.934\ m$, $n = 75.3$, $\phi = 45^\circ$) with dielectric/transparent zones, $\varepsilon_r = 10.8$

Figure 4.26 Cross-polar patterns calculated with Kirchhoff’s theory and UTD for system S2 ($F = 1.934\ m$, $n = 75.3$, $\phi = 45^\circ$) with dielectric/transparent zones, $\varepsilon_r = 10.8$
5. Bandwidth, efficiency and scaling

The bandwidth of an antenna system is an important parameter. It determines how frequency selective the system is. This is important because communication implies operation within a non-zero frequency band around a carrier frequency. Therefore, part of this chapter is about the bandwidth of an axially symmetric FZPA. Another important parameter is the efficiency, which determines how efficient the aperture plane of the antenna is used. One should expect that the efficiency of a FZPA is maximum when it is operated with the design parameters $F$ and $\lambda$. Every deviation from these parameters would lead to a lower efficiency because of the disturbed interference pattern. For the FZPA’s considered in this report this is not true. The gain function of the antenna feed is such that the contributions of the various zones are not equal (a non-isotropic radiating source). Then one can not predict on forehand where the maximum efficiency will occur. In order to analyze how the efficiency varies when displacing the feed along the antenna axis or changing the frequency, the overall efficiency $\eta$ has to be defined. It is a product of three separate efficiencies, namely

$$\eta = \eta_s \cdot \eta_p \cdot \eta_i$$  \hspace{1cm} (5.1)

where

- $\eta_s$ is the spillover efficiency
- $\eta_p$ is the phase efficiency
- $\eta_i$ is the illumination efficiency

The gain of the antenna can then be written as

$$G = \eta \left( \frac{\pi D}{\lambda} \right)^2$$  \hspace{1cm} (5.2)

In the next sections explicit formulas for the efficiencies will be given.

5.1 Spillover efficiency

The spillover efficiency is defined as the ratio of the power radiated by the feed, which reaches the aperture plane, and the total power radiated by the feed. It is given by
5. Bandwidth, efficiency and scaling

\[ \eta_s = \frac{\psi_0}{\pi} \int_0^\infty G(\psi, n) d\psi \]

where \( \psi_0 \) is the half subtended angle of the antenna. For the gain function of the feed under consideration, Eq. (5.3) can be simplified to

\[ \eta_s = 1 - \cos^{n+1}(\psi_0) \]  

(5.4)

### 5.2 Phase efficiency

The performance of a FZPA is based on interfering wavefronts. That is why the phase efficiency is an important quantity. It defines the ratio of the power radiated in forward direction and the radiated power on condition that the phase in the aperture is constant. The power radiated in forward direction can be calculated from Eq. (4.63), by setting \( \theta = 0^\circ \). This means automatically \( \varphi = 0^\circ \). Then the phase efficiency for an antenna system with absorbing/transparent zones is given by

\[ \eta_p = \left( \sum_{m=0}^{2,4,...} \int_{\psi_m}^{\psi_{m+1}} (\cos(\psi) + 1) \sqrt{2(n+1)\cos^2(\psi)} \frac{\sin(\psi)}{\cos(\psi)} F e^{-jkF/\cos(\psi)} d\psi \right)^2 \]

(5.5)

In case of ideal phase correcting zones the total radiated power is the integral over the complete antenna aperture when the phase in the aperture is constant. The phase efficiency then becomes

\[ \eta_p = \frac{2,4,... \psi_{m+1}}{1,2,... \psi_m} \left[ \int_{\psi_m}^{\psi_{m+1}} (\cos(\psi) + 1) \sqrt{2(n+1)\cos^2(\psi)} \frac{\sin(\psi)}{\cos(\psi)} F e^{-jkF/\cos(\psi) + jm\pi} d\psi \right]^2 \]

(5.6)

In case of dielectric/transparent zones
5. Bandwidth, efficiency and scaling

\[
\eta_p = \frac{1,2,... \sum_{m=0}^{\psi_{m+1}} \int_{\psi_m} (T_M(\psi,m) \cos(\psi) + T_E(\psi,m)) \sqrt{2(n+1) \cos^n(\psi)} \sin(\psi) F e^{-jkF / \cos(\psi)} d\psi}{\sum_{m=0}^{\psi_{m-1}} \int_{\psi_m} |T_M(\psi,m) \cos(\psi) + T_E(\psi,m)| \sqrt{2(n+1) \cos^n(\psi)} \sin(\psi) F d\psi)^2}
\]

(5.7)

with

\[
T_{IM}(\psi,m) = 1, \quad m = 0,2,4...
\]

(5.8)

= \ T_{IM}, \quad m = 1,3,5...

5.3 Illumination efficiency

The illumination efficiency is governed by the strength and the distribution of the aperture field. It is defined in such a way that it completes the product of the three separate efficiencies in order to obtain the known formula of the overall efficiency. In case of absorbing/transparent zones it is given by

\[
\eta_l = \frac{2,4,... \psi_{m+1}}{\sum_{m=0}^{\psi_{m-1}} \int_{\psi_m} (\cos(\psi) + 1) \sqrt{2(n+1) \cos^n(\psi)} \sin(\psi) F d\psi}{D^2(1 - \cos^{n+1}(\psi_0))^{2}}
\]

(5.9)

In case of ideal phase correcting zones it is given by

\[
\eta_l = \frac{\int_{0}^{\psi_0} (\cos(\psi) + 1) \sqrt{2(n+1) \cos^n(\psi)} \sin(\psi) F d\psi}{D^2(1 - \cos^{n+1}(\psi_0))^{2}}
\]

(5.10)

and in case of dielectric/transparent zones by

\[
\eta_l = \frac{1,2,... \psi_{m+1}}{\sum_{m=0}^{\psi_{m-1}} \int_{\psi_m} |T_M(\psi,m) \cos(\psi) + T_E(\psi,m)| \sqrt{2(n+1) \cos^n(\psi)} \sin(\psi) F d\psi}{D^2(1 - \cos^{n+1}(\psi_0))^{2}}
\]

(5.11)
5.4 Displacing the feed along the antenna axis

Efficiencies will be evaluated when the feed is displaced along the antenna axis. This means that the operating frequency and the zone radii are kept constant. Maximum antenna gain will occur when the overall efficiency is maximum, because in Eq. (5.2) $D$ and $\lambda$ are constant. In order to get an idea of how the efficiencies vary, only the first transparent zone of an antenna will be considered (a hole in a perfectly absorbing screen). The antenna system with $F = 0.581$ m, $f = 11.1$ GHz, $n = 7.4$ is chosen. When the feed is moved away from the aperture plane, the half subtended angle becomes smaller. As a consequence the spillover efficiency is reduced. The phase distribution over the aperture will become more uniform, because the phase-front better resembles that of a plane wave. The phase efficiency will therefore increase. The effect of displacing the feed on the illumination efficiency can also be predicted. The amplitude of the field distribution becomes more uniform when moving the feed away from the aperture. The illumination efficiency will therefore increase. Figure 5.1 shows the efficiencies. It can be seen that the spillover efficiency decreases when the feed is moved away and the phase efficiency increases. These effects cancel each other, so that the overall efficiency remains approximately constant over the region shown. For this situation the illumination efficiency is almost equal to 1.

Let's now consider an antenna with a number of absorbing/transparent zones. When the feed is moved away from the aperture plane, the spillover and illumination efficiency will vary in the same way as in case of one transparent zone. The spillover efficiency will decrease and the illumination efficiency will increase. However, the curve of the phase efficiency will have a completely different shape. Though the phase distribution over the zones becomes more uniform, the transparent zones now partly contribute destructively in forward direction. Due to the growing phase efficiency of the first transparent zone, the total phase efficiency will increase in first instance. Eventually it decreases due to the mentioned destructive interference. Figure 5.2 shows the efficiencies for a system with $D = 1$ m. Here one can see that the phase efficiency completely determines the shape of the curve of the overall efficiency and therefore it is the most important one for a FZPA. One can clearly see that a FZPA is based on interference, because the efficiency reaches zero for certain values of the feedposition. Figure 5.3 shows a plot for the same system with ideal phase correcting zones. The illumination efficiency gives rise to the increase in overall efficiency, while the phase efficiency remains approximately constant with that in figure 5.2. Finally figure 5.4 shows a plot for the system with dielectric/transparent zones designed with $e_r = 10.8$. The decrease in overall efficiency is due to the decrease in the illumination efficiency, which is influenced by the transmission coefficient of the dielectric. From figure 5.2 to 5.4 it can be seen that maximum efficiency is not reached when the feed is at the focus of the antenna. It has to be moved away from the aperture plane to reach the maximum.

In Appendix D corresponding plots are shown for system S2 with the long focal distance ($F = 1.934$ m, $n = 75.3$ and $D = 1$ m). The efficiencies vary in the same way as for the system discussed above.
5. Bandwidth, efficiency and scaling

Figure 5.1 Efficiencies plotted against the feedposition for the first transparent zone 
\((F = 0.581 \text{ m and } n = 7.4)\)

Figure 5.2 Efficiencies plotted against the feedposition for an antenna with absorbing/transparent zones 
\((F = 0.581 \text{ m and } n = 7.4)\)
5. Bandwidth, efficiency and scaling

Figure 5.3 Efficiencies plotted against the feed position for an antenna with ideal phase correcting zones ($F = 0.581 \text{ m and } n = 7.4$)

Figure 5.4 Efficiencies plotted against the feed position for an antenna with dielectric/transparent zones ($F = 0.581 \text{ m and } n = 7.4$) and $\varepsilon_r = 10.8$
5. Bandwidth, efficiency and scaling

5.5 Changing the operating wavelength

Changing the wavelength of the feedhorn also has an effect on the efficiency of a FZPA. This is examined when the feedposition and the radii of the zones are kept constant. All systems are designed with $F = 0.581$ m, $n = 7.4$ and $f = 11.1$ GHz ($\lambda = 2.7$ cm). As in the previous section we start with analyzing the efficiencies for the first transparent zone. When it is assumed that the feed factor $n$ is frequency independent, the spillover and illumination efficiency will be independent of the wavelength. The phase efficiency however will increase with increasing $\lambda$, because the phase distribution over the first zone becomes more uniform. Figure 5.5 shows the efficiencies. The phase efficiency increases continuously and thus the overall efficiency.

In case of an antenna with a number of absorbing/transparent zones ($D = 1$ m), the phase efficiency will reach a maximum value for the same reason as in the previous section. With increasing $\lambda$ it will increase in first instance, but due to the destructive contributions from the other zones it will eventually fall down. Figure 5.6 shows the efficiencies for this case. When ideal phase correcting zones are applied, the illumination efficiency increases drastically, while the phase efficiency remains approximately constant with respect to the system with absorbing/transparent zones. Figure 5.7 shows a plot for a system with ideal phase correcting zones. Finally figure 5.8 shows the efficiencies for a system with dielectric/transparent zones and $\varepsilon_r = 10.8$. The phase efficiency has not changed. The illumination efficiency is smaller than in case of ideal phase correcting zones and has become frequency dependent. This is caused by the dielectric. Maximum efficiency does not occur at the design wavelength, but for a larger one. This does not automatically imply that the antenna gain is maximum at maximum efficiency, because the term $(\pi D/\lambda)^3$ also varies. Figure 5.9 shows the antenna gain for the system with dielectric/transparent zones. The gain is maximum for a larger wavelength than the design wavelength. In Appendix D plots are shown for system S2.

![Figure 5.5](image_url)  
*Figure 5.5 Efficiencies plotted against the wavelength for the first transparent zone  
($F = 0.581$ m, $n = 7.4$, $\lambda = 2.7$ cm)*
5. Bandwidth, efficiency and scaling

Figure 5.6 Efficiencies plotted against the wavelength for an antenna with absorbing/transparent zones ($F = 0.581$, $n = 7.4$, $\lambda = 2.7$ cm)

Figure 5.7 Efficiencies plotted against the wavelength for an antenna with ideal phase correcting zones ($F = 0.581$, $n = 7.4$, $\lambda = 2.7$ cm)
5. Bandwidth, efficiency and scaling

Figure 5.8 Efficiencies plotted against the wavelength for an antenna with dielectric/transparent zones \( (F = 0.581, n = 7.4, \lambda = 2.7 \text{ cm}) \) and \( \varepsilon_r = 10.8 \)

Figure 5.9 Gain plotted against the wavelength for an antenna with dielectric/transparent zones \( (F = 0.581, n = 7.4, \lambda = 2.7 \text{ cm}) \) and \( \varepsilon_r = 10.8 \)
5.6 Scaling the zone plate for bandwidth purposes

From the previous section it is known that maximum efficiency and antenna gain do not have to occur at the design wavelength. When this is desired anyway, the solution to this problem can be found in scaling the FZP. Assume that the design wavelength is \( \lambda \) and maximum gain (or efficiency) is reached when \( \lambda = q\lambda_0 \). The gain of a FZPA with absorbing/transparent zones is then

\[
G = \left| \frac{\pi}{\lambda} \sum_{m=0}^{\psi_{m+1}} \int_{\psi_m} \frac{\psi_{m+1}}{\cos^2 \psi} F e^{-jkF/\cos \psi} \right|^2
\]

(5.12)

This gain can also be reached at the design wavelength \( \lambda \), provided that the other parameters of the FZPA are scaled with \( q \). This means

\[
F := F/q \\
r_m := r_m/q \\
D := D/q
\]

(5.13)

and in case of dielectric zones the thickness of the zones has also to be scaled to

\[
d := d/q
\]

(5.14)

So, scaling a FZPA this way means that the efficiency or gain curves are shifted parallel to the horizontal axis (figure 5.9). We will use the method of scaling for calculating the 3 dB bandwidth of a FZPA, so that maximum gain occurs at the design wavelength.

Bandwidth formulas of zone plates originate from optics. Young [24] has derived formulas for the 3 dB bandwidth in case of absorbing/transparent zones and ideal phase correcting zones. They are given by

\[
\frac{1}{0} I = \frac{2}{3 \text{dB} L}
\]

(5.15)

(5.16)

with \( l \) the number of transparent zones in case of absorbing/transparent zones,

\[
\Delta f_{3 \text{db}} = \frac{2f_0}{L}
\]

(5.16)

with \( L \) the total number of zones in case of ideal phase correcting zones. These formulas do not account for the gain function of the feed. For the two systems used throughout this report with absorbing/transparent, ideal phase correcting and dielectric/transparent zones, the bandwidth is calculated with Kirchhoff’s integral and Eq. (5.15) and Eq. (5.16). The results are given in table 5.1 to 5.3.
5. Bandwidth, efficiency and scaling

Table 5.1 Bandwidth for systems with absorbing/transparent zones

<table>
<thead>
<tr>
<th>F (m)</th>
<th>n</th>
<th>l</th>
<th>Δf_{3dB} (GHz)</th>
<th>Δf_{3dB} (GHz) Young</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.581</td>
<td>7.4</td>
<td>7</td>
<td>1.46</td>
<td>1.59</td>
</tr>
<tr>
<td>1.934</td>
<td>75.3</td>
<td>3</td>
<td>3.4</td>
<td>3.7</td>
</tr>
</tbody>
</table>

Table 5.2 Bandwidth for systems with ideal phase correcting zones

<table>
<thead>
<tr>
<th>F (m)</th>
<th>n</th>
<th>L</th>
<th>Δf_{3dB} (GHz)</th>
<th>Δf_{3dB} (GHz) Young</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.581</td>
<td>7.4</td>
<td>13</td>
<td>1.46</td>
<td>1.71</td>
</tr>
<tr>
<td>1.934</td>
<td>75.3</td>
<td>5</td>
<td>3.88</td>
<td>4.44</td>
</tr>
</tbody>
</table>

Table 5.3 Bandwidth for systems with dielectric/transparent zones and $\varepsilon_r = 10.8$

<table>
<thead>
<tr>
<th>F (m)</th>
<th>n</th>
<th>L</th>
<th>Δf_{3dB} (GHz)</th>
<th>Δf_{3dB} (GHz) Young</th>
</tr>
</thead>
<tbody>
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<td>1.934</td>
<td>75.3</td>
<td>5</td>
<td>4.17</td>
<td>4.44</td>
</tr>
</tbody>
</table>

These tables show that bandwidth is increased in case of phase correcting zones. The formulas given by Young are a useful tool for quickly estimating the bandwidth, especially for a system with large focal distance. Then the predicted bandwidth is quite close to the results found by the Kirchhoff integral.
6. The elliptical Fresnel-zone plate antenna

In the previous chapters the axially symmetric FZPA has been analyzed. The forward direction of this antenna, where a maximum in the power radiation pattern occurs, is perpendicular to the aperture plane. The zones are circular, which is characteristic for the axially symmetric FZPA. In a practical application, a FZPA can be used in an office environment to receive a signal from a satellite. When the zones are mounted on a window, which is in most situations perpendicular to the earth's surface, then the forward direction would be parallel to the earth. However, a satellite is always viewed at a certain elevation and azimuth angle with respect to the earth. This means that the position of the satellite does not coincide with the forward direction of the antenna. The signal will be received in some sidelobe, which can be 20 dB lower or even more than the antenna gain. Hence, it is required to design a FZPA where the position of the maximum in the power radiation pattern is in the direction of the satellite. The kind of antenna having this property is an elliptical FZPA. As the name already implies, the zones of the FZPA are ellipses. This can be explained when using the definition of Fresnel-zones [25]:

The boundary of the n-the Fresnel-zone is given by the points where the distance from transmitter to receiver via these points is \( n \lambda / 2 \) larger than the direct distance between transmitter and receiver.

From this definition Fresnel-zones are ellipsoids where transmitter and receiver are situated in the foci of the ellipses. The line between transmitter and receiver is the symmetry axis of the ellipsoids. When an ellipsoid is projected on a plane perpendicular to this line, the projection will be a circle, leading to the axially symmetric FZPA. But when the ellipsoid is projected on a plane which is oblique to this line, the projection will be an ellipse resulting in the elliptical FZPA. The office window can be considered as the plane containing the projection, so the aperture plane is formed by elliptical zones. In the next sections expressions for the zones of the elliptical FZPA will be derived and radiation patterns will be calculated by means of UTD and the Kirchhoff diffraction integral.

6.1 Configuration of the antenna system

The aperture plane is assumed to be in the x-y plane of the rectangular coordinate system \((x,y,z)\). The spherical coordinates \((r,\theta,\phi)\) are used to describe the far-field observation point \(P\). The forward direction of the antenna is given by \((\theta_0,\phi_0)\) and the focal distance \(F\) is defined as the distance from the origin of the rectangular coordinate system to the feed. The feed at its turn is
pointed in the forward direction. The electric field from the feed is described in a feed-fixed coordinate system. It is given by the rectangular coordinate system \((x',y',z')\) and the accompanying spherical coordinate system \((\rho,\psi,\xi)\). Figure 6.1 shows the configuration.

The position of the feed in \((x,y,z)\)-coordinates is now given by

\[
\begin{align*}
x_F &= -F\sin\theta_0 \cos\varphi_0 \\
y_F &= -F\sin\theta_0 \sin\varphi_0 \\
z_F &= -F\cos\theta_0
\end{align*}
\] (6.1)

Calculations in this report are made for \(\varphi_0 = 0^\circ\) for convenience which means that the \(y\)- and \(y'\)-axis are parallel. When the forward direction of the antenna has to be defined at \(\varphi_0 \neq 0^\circ\), then one can design the antenna system for \(\varphi_0 = 0^\circ\) and rotate the complete system over the desired angle in the aperture plane.

### 6.2 Designing the elliptical zones

Consider a plane wave incident to the antenna from the forward direction \(\theta_0\). The boundaries of the zones can be calculated by determining the pathlength difference between two rays reaching the feed. The first one is going through the origin of the rectangular coordinate system \((x,y,z)\), while the second one is parallel to the first and passing the aperture plane in an arbitrary point. Therefore, we start with calculating the pathlength difference of the two rays when they reach the aperture plane. Figure 6.2 shows the configuration. Here, \(\rho_m\) is the distance from the origin of the coordinate system to a point in the aperture, \(\varphi'\) is the angle with respect to the positive \(x\)-axis and \(\Delta l\) is the pathlength difference.
The following geometrical relations hold:

\[
\begin{align*}
\rho_m^2 &= (\Delta l)^2 + q^2 \\
p^2 &= \rho_m^2 + (\Delta l \sin \theta_0)^2 - 2 \rho_m \Delta l \sin \theta_0 \cos \varphi' \\
q^2 &= p^2 + (\Delta l \cos \theta_0)^2
\end{align*}
\]

From the latter equation it can be found that the pathlength difference \(\Delta l\) is given by

\[
\Delta l = \rho_m \sin \theta_0 \cos \varphi'
\]

The rays leave the aperture plane and travel towards the focus. The distance \(d_m\) from a point in the aperture plane to the focus can be written as

\[
d_m = \sqrt{(\rho_m \cos \varphi' - x_F)^2 + (\rho_m \sin \varphi')^2 + z_F^2}
\]

The principle of a zone plate is to block off that part of the incoming plane wave which adds destructively at the focus. In other words, going from the inside to the outside of the \(m\)-th zone, the phase at the focus must change \(\lambda/2\). For the elliptical FZPA the following equation has to be satisfied

\[
d_m = F + \frac{m\lambda}{2} + \Delta l
\]

In figure 6.3 this is illustrated for the 2-dimensional situation \(y = 0\).

The task is to calculate \(\rho_m\) when \(F\), \(\theta_0\), \(\lambda\) and \(\varphi'\) are given. Combining Eq. (6.3), (6.4) and (6.5) gives

\[
(\rho_m \cos \varphi' - x_F)^2 + (\rho_m \sin \varphi')^2 + z_F^2 = (F + \frac{m\lambda}{2} + \rho_m \sin \theta_0 \cos \varphi')^2
\]
It can be rewritten as
\[ \rho_{m}^2 - 2 \rho_{m} \cos \varphi' x_p + x_p^2 + z_p^2 = F^2 + \left( \frac{m \lambda}{2} \right)^2 + m \lambda F + \rho_{m}^2 \sin^2 \theta_0 \cos^2 \varphi' + 2 \rho_{m} \left( F + \frac{m \lambda}{2} \right) \sin \theta_0 \cos \varphi' \]
(6.7)

This is a quadratic equation of the form
\[ \alpha \rho_{m}^2 + \beta \rho_{m} + \gamma = 0 \]
(6.9)

with the solution
\[ \rho_{m} = \frac{-\beta + \sqrt{\beta^2 - 4 \alpha \gamma}}{2 \alpha} \]
(6.10)

The coefficients \( \alpha, \beta \) and \( \gamma \) are given by
\[ \alpha = 1 - \sin^2 \theta_0 \cos^2 \varphi' \]
\[ \beta = -m \lambda \sin \theta_0 \cos \varphi' \]
\[ \gamma = -\frac{m^2 \lambda^2}{4} - m \lambda F \]
(6.11)

With these formulas the ellipse can now be described in polar coordinates \((\rho_{m}, \varphi')\).

It is also possible to describe the ellipse in \((x, y)\) coordinates. The general formula of an ellipse in rectangular coordinates is
where $a_m$ and $b_m$ are the semi axes of the ellipse and $c_m$ is the displacement along the x-axis, as shown in figure 6.4.

\[
\left(\frac{x-c_m}{a_m}\right)^2 + \left(\frac{y}{b_m}\right)^2 = 1
\]  

\[ (6.12) \]

Explicit formulas for $a_m$, $b_m$ and $c_m$ can be found by using the expression for $\rho_m$, given by Eq. (6.9) and (6.10), and the configuration of figure 6.5. It shows the distance from the origin of the $(x,y)$-coordinate system to a point on the ellipse for certain values of $\varphi'$. Then $\rho_{m1}$ can be found by setting $\varphi' = 0^\circ$, resulting in

\[
\rho_{m1} = \frac{m \lambda \sin \theta_0 + \sqrt{4 m \lambda (F - F \sin^2 \theta_0 + \frac{m \lambda}{4})}}{2(1 - \sin^2 \theta_0)}
\]  

\[ (6.13) \]

Similarly, $\rho_{m2}$ can be found by setting $\varphi' = 180^\circ$, giving

\[
\rho_{m2} = \frac{-m \lambda \sin \theta_0 + \sqrt{4 m \lambda (F - F \sin^2 \theta_0 + \frac{m \lambda}{4})}}{2(1 - \sin^2 \theta_0)}
\]  

\[ (6.14) \]

Now $a_m$ and $c_m$ can be written as

\[
a_m = \frac{\rho_{m1} + \rho_{m2}}{2} = \frac{\sqrt{m \lambda (F \cos^2 \theta_0 + \frac{m \lambda}{4})}}{\cos^2 \theta_0}
\]  

\[ (6.15) \]
The only parameter left to calculate is $b_m$. This can be done by using the latter two equations and by setting $\varphi' = 90^\circ$. Then it is found that

$$\rho_m = \sqrt{m \lambda (F + \frac{m \lambda}{4})}$$

Substituting $y = \rho_m$ and $x = 0$ in Eq. (6.12) gives

$$\left(\frac{c_m}{a_m}\right)^2 + \left(\frac{\rho_m}{b_m}\right)^2 = 1$$

leading to

$$b_m = \frac{\rho_m}{\sqrt{1 - \left(\frac{c_m}{a_m}\right)^2}}$$

With

$$\left(\frac{c_m}{a_m}\right)^2 = \frac{1}{4} \left(\frac{m \lambda \sin \theta_0}{F \cos^2 \theta_0 + \frac{m \lambda}{4}}\right)^2$$

$b_m$ becomes
From the previous calculations it can be concluded that the displacement \( c_m \) of the ellipse is linear with the zone number \( m \). This means that the zones are mutually shifted, as shown in figure 6.6.

Furthermore, there is a simple relation between the semi axes of the ellipse given by

\[
\frac{b_m}{a_m} = |\cos \theta_0| \tag{6.22}
\]

The latter equation shows that the ratio of the semi axis only depends on the desired forward direction of the elliptical FZPA, and is independent of the focal distance or the wavelength.

### 6.3 UTD analysis of the elliptical Fresnel-zone plate antenna

The axially symmetric FZPA has the property that the UTD analysis turns out to be a 2-dimensional one. Incident, reflected and diffracted rays all lie in the same plane, because the angle \( \beta_0 \) between the incident ray \( \hat{s}^i \) and the edge tangent \( \hat{e} \) is given by \( \beta_0 = \pi/2 \). As a consequence, the positions of the diffraction points on the edge of each zone are exactly known. They are lying in the same \( \varphi \) -plane as the far-field observation point. For the elliptical FZPA this special condition does not apply. Hence, the UTD analysis becomes a 3-dimensional one which will be given in the next subsections.
6. The elliptical Fresnel-zone plate antenna

6.3.1 Location of the diffraction points

Given the antenna system and some far-field observation point \( P \), the locations of the diffraction points \( Q_i \) on the edge of each zone have to be determined. This can be done by using Keller’s law of edge diffraction which says that the angles \( \beta_0 \) and \( \beta_d \) are equal (chapter 3). Locating the diffraction points means searching points on the ellipse where this condition is satisfied for a given \( P \). Therefore, it is required to define the edge tangent \( \hat{e} \) of an ellipse. From Eq. (6.12) it can be found that

\[
\frac{dx}{dy} = \frac{-y}{x-c_m} \left( \frac{a_m}{b_m} \right)^2
\]  

(6.23)

The tangent \( \vec{T} \) can then be written as

\[
\vec{T} = \left( \frac{-y}{x-c_m} \left( \frac{a_m}{b_m} \right)^2, 1, 0 \right)
\]  

(6.24)

and the unit tangent \( \hat{e} \) is

\[
\hat{e} = \pm \vec{T} / |\vec{T}|
\]  

(6.25)

where the + sign applies for \( x - c_m > 0 \) and the - sign for \( x - c_m < 0 \). This formulation ensures that \( \hat{e} \) rotates clockwise along the ellipse. An expression for \( \beta_0 \) is given by

\[
\cos \beta_0 = \hat{s}_i \cdot \hat{e}
\]  

(6.26)

After diffraction the ray travels from the edge towards the far-field observation point. An expression for \( \beta_d \) is

\[
\cos \beta_d = \hat{s}_d \cdot \hat{e}
\]  

(6.27)

where the unit vector \( \hat{s}_d \) can be approximated by

\[
\hat{s}_d = (\sin \theta \sin \varphi, \sin \theta \cos \varphi, \cos \theta)
\]  

(6.28)

In fact this is the unit vector \( \hat{r} \) because in the far-field \( \hat{r} \) and \( \hat{s}_d \) are parallel. For \( Q \) to be a diffraction point the following equation must be satisfied

\[
(\hat{s}_i - \hat{s}_d) \cdot \hat{e} = 0
\]  

(6.29)

This equation will be solved numerically with a zero-searching procedure. Then it is found that on an ellipse there are two diffraction points which lie symmetrical around the ellipse axes. Figure 6.7 shows the configuration.
6.3.2 The caustic divergence factor

When calculating the caustic divergence factor $A$, defined by Eq. (3.5), expressions are required for the parameters $\rho^2, \rho_g$ and the vector $\hat{n}$. In case of a spherical wave, $\rho^2$ is given by the distance $\rho_0$ between the feed and the point of diffraction. Using Appendix C the radius of curvature $\rho_g$ of an ellipse is

$$\rho_g = \frac{\left(a_m^4 - a_m^2(x - c_m)^2 + b_m^2(x - c_m)^2\right)^{3/2}}{a_m^4 b_m} \quad (6.30)$$

The unit vector $\hat{n}$ is normal to the ellipse tangent, so it can be written as

$$\hat{n} = \hat{e} \times \hat{z} \quad (6.31)$$

Because $\hat{e}$ rotates clockwise over the ellipse, it is assured that $\hat{n}$ always points away from the origin of the ellipse. The caustic distance $\rho_c$ is given by Eq. (3.6) and with the far-field approximation the caustic divergence factor becomes

$$A = \frac{1}{r \sqrt{\rho_c}} \quad (6.32)$$

6.3.3 The diffraction coefficient

In this section the 3-dimensional diffraction coefficient will be derived in the general case of a
half-plane where an electromagnetic wave can pass partly through the material. As in chapter 4 we need to obtain the discontinuities across the incidence and reflection shadow boundary. Because the plane of incidence and the edge-fixed plane of incidence are not perpendicular any more, as shown in figure 6.8, a transformation has to be performed.

\[ E_{\beta'}^i = E_1^i \cos \alpha + E_2^i \sin \alpha \]
\[ E_{\phi'}^i = -E_1^i \sin \alpha + E_2^i \cos \alpha \]  

or in matrix notation

\[
\begin{pmatrix}
E_{\beta'}^i \\
E_{\phi'}^i
\end{pmatrix} =
\begin{pmatrix}
\cos \alpha & \sin \alpha \\
-\sin \alpha & \cos \alpha
\end{pmatrix}
\begin{pmatrix}
E_1^i \\
E_2^i
\end{pmatrix}
\]

where the transformation matrix \( S^i \) is

\[
S^i =
\begin{pmatrix}
\cos \alpha & \sin \alpha \\
-\sin \alpha & \cos \alpha
\end{pmatrix}
\]

Figure 6.9 shows the configuration for the reflected field.
The reflected field can be written as

\[ E_\phi' = -E_\phi' \cos \alpha + E_\perp' \sin \alpha \]
\[ E_\phi' = -E_\phi' \sin \alpha - E_\perp' \cos \alpha \]

or in matrix notation

\[
\begin{pmatrix}
E_\phi' \\
E_\phi'
\end{pmatrix} =
\begin{pmatrix}
-\cos \alpha & \sin \alpha \\
-\sin \alpha & -\cos \alpha
\end{pmatrix}
\begin{pmatrix}
E_1' \\
E_1'
\end{pmatrix}
\]

(6.37)

The transformation matrix \( S' \) is given by

\[
S' =
\begin{pmatrix}
-\cos \alpha & \sin \alpha \\
-\sin \alpha & -\cos \alpha
\end{pmatrix}
\]

(6.38)

The discontinuity across the reflection shadow boundary is equal to the reflected field just described. It can be rewritten as a function of the incident field,

\[
E_{\phi,\phi}' = S'E_{\phi,\phi} = S'R_{M,E}E_{\phi,\phi} = S'R_{M,E}(S^i)^{-1}E_{i,\phi}'
\]

(6.39)

Performing the matrix products leads to
6. The elliptical Fresnel-zone plate antenna

\[
\begin{pmatrix}
E'_\rho \\
E'_\phi
\end{pmatrix} =
\begin{pmatrix}
-\cos\alpha & \sin\alpha \\
-\sin\alpha & -\cos\alpha
\end{pmatrix}
\begin{pmatrix}
R_M & 0 \\
0 & R_E
\end{pmatrix}
\begin{pmatrix}
\cos\alpha & -\sin\alpha \\
\sin\alpha & \cos\alpha
\end{pmatrix}
\begin{pmatrix}
E'_i \\
E'_i
\end{pmatrix}

= \begin{pmatrix}
-R_M\cos\alpha & R_E\sin\alpha \\
-R_M\sin\alpha & -R_E\cos\alpha
\end{pmatrix}
\begin{pmatrix}
\cos\alpha & -\sin\alpha \\
\sin\alpha & \cos\alpha
\end{pmatrix}
\begin{pmatrix}
E'_i \\
E'_i
\end{pmatrix}
(6.40)

\[
= \begin{pmatrix}
-R_M\cos^2\alpha + R_E\sin^2\alpha & (R_M + R_E)\sin\alpha\cos\alpha \\
-(R_M + R_E)\sin\alpha\cos\alpha & R_M\sin^2\alpha - R_E\cos^2\alpha
\end{pmatrix}
\begin{pmatrix}
E'_i \\
E'_i
\end{pmatrix}
\]

In a similar way the discontinuity across the incidence shadow boundary can be derived. It is the difference between the incident field and the transmitted field

\[
\begin{pmatrix}
E'_\rho \\
E'_\phi
\end{pmatrix} = \begin{pmatrix}
E'_i \\
E'_i
\end{pmatrix}
(6.41)
\]

The transmitted field can be written as

\[
E'_{\rho,\phi} = S'E'_{L,\perp} = S'T_{M,E}E'_{L,\perp} = S'T_{M,E}(S')^{-1}E'_{\rho,\phi'}
\]

Performing the matrix products leads to

\[
\begin{pmatrix}
E'_\rho \\
E'_\phi
\end{pmatrix} = \begin{pmatrix}
\cos\alpha & \sin\alpha \\
-\sin\alpha & \cos\alpha
\end{pmatrix}
\begin{pmatrix}
T_M & 0 \\
0 & T_E
\end{pmatrix}
\begin{pmatrix}
\cos\alpha & -\sin\alpha \\
\sin\alpha & \cos\alpha
\end{pmatrix}
\begin{pmatrix}
E'_i \\
E'_i
\end{pmatrix}

= \begin{pmatrix}
T_M\cos\alpha & T_E\sin\alpha \\
-T_M\sin\alpha & T_E\cos\alpha
\end{pmatrix}
\begin{pmatrix}
\cos\alpha & -\sin\alpha \\
\sin\alpha & \cos\alpha
\end{pmatrix}
\begin{pmatrix}
E'_i \\
E'_i
\end{pmatrix}
(6.43)

\[
= \begin{pmatrix}
T_M\cos^2\alpha + T_E\sin^2\alpha & (-T_M + T_E)\sin\alpha\cos\alpha \\
(-T_M + T_E)\sin\alpha\cos\alpha & T_M\sin^2\alpha + T_E\cos^2\alpha
\end{pmatrix}
\begin{pmatrix}
E'_i \\
E'_i
\end{pmatrix}
\]

Then the discontinuity is given by

\[
\begin{pmatrix}
E'_\rho \\
E'_\phi
\end{pmatrix} = \begin{pmatrix}
T_M\cos^2\alpha + T_E\sin^2\alpha & (-T_M + T_E)\sin\alpha\cos\alpha \\
(-T_M + T_E)\sin\alpha\cos\alpha & T_M\sin^2\alpha + T_E\cos^2\alpha
\end{pmatrix}
\begin{pmatrix}
E'_i \\
E'_i
\end{pmatrix}
(6.44)
\]

The diffraction coefficient \(\bar{D}\) ensures continuity of the fields across the shadow boundaries. This implies that \(\bar{D}\) must have the form
The elliptical Fresnel-zone plate antenna (6.45)

\[ \overrightarrow{D} = \begin{pmatrix} -D_a & -D_b \\ -D_c & -D_d \end{pmatrix} \]

with

\[
D_a = (1 - T_M \cos^2 \alpha - T_E \sin^2 \alpha) D(\gamma - \gamma') + (-R_M \cos^2 \alpha + R_E \sin^2 \alpha) D(\gamma + \gamma')
\]

\[
D_b = (T_M - T_E) \sin \alpha \cos \alpha D(\gamma - \gamma') + (R_M + R_E) \sin \alpha \cos \alpha D(\gamma + \gamma')
\]

\[
D_c = (T_M - T_E) \sin \alpha \cos \alpha D(\gamma - \gamma') - (R_M + R_E) \sin \alpha \cos \alpha D(\gamma + \gamma')
\]

\[
D_d = (1 - T_M \sin^2 \alpha - T_E \cos^2 \alpha) D(\gamma - \gamma') + (R_M \sin^2 \alpha - R_E \cos^2 \alpha) D(\gamma + \gamma')
\]

The diffraction coefficient just given contains the distance parameters \( L_i \) and \( L'_i \), for which simplified expressions can be derived. The radii of curvature of the incident field are given by \( \rho_1 = \rho_0 \) and \( \rho_2 = \rho_0 \). Then Eq. (3.17) reduces in the far-field to

\[
L_i = \rho_0 \sin^2 \beta_0
\]

The radii of curvature of the reflected field are given by \( \rho'_1 = \rho_0 \) and \( \rho'_2 = \rho_0 \). Furthermore, \( \rho'_3 = \rho_0 \) and Eq. (3.18) reduces to

\[
L'_r = \rho_0 \sin^2 \beta_0
\]

The angles \( \gamma \) and \( \gamma' \) can not be simplified as in case of the axially symmetric FZPA.

### 6.3.4 The diffracted field

The diffracted field due to one single diffraction point is given by Eq. (3.4). The phase-factor related to the distance \( s^d \) from the point of diffraction \( Q_i \) towards the far-field observation point, can be approximated in the following way. In the far-field the vectors \( \hat{r} \) and \( \hat{s}^d \) are parallel. The pathlength difference \( u \) is then given by

\[
u = \vec{r}_Q \cdot \vec{r}
\]

where \( \vec{r}_Q \) defines the vector from the origin of the \((x, y, z)\)-coordinate system towards the point of diffraction \( Q \). The distance \( s^d \) can now be written as

\[
s^d = r - u
\]

and

\[
e^{-jks^d} = e^{-jkr} e^{jku}
\]
The elliptical Fresnel-zone plate antenna

Together with the results from the previous sections, the total diffracted field becomes

\[
\begin{pmatrix}
E^d_	heta \\
E^d_\phi
\end{pmatrix} = \sqrt{p_c/2} \frac{e^{-jkr}}{r} e^{jku} \begin{pmatrix}
-D_a & -D_b \\
-D_c & -D_d
\end{pmatrix} \begin{pmatrix}
E^i_	heta \\
E^i_\phi
\end{pmatrix}
\]

(6.52)

The incident and diffracted fields in the latter equation are defined in edge-fixed coordinate systems. The field of the feed and the far-field observation point are defined in coordinate systems which do not coincide with these edge-fixed systems. This means that the incident field in the feed-fixed coordinate system \((p,\psi,\xi)\) has to be transformed to the coordinate system related to the edge-fixed plane of incidence. Then the diffracted field is known in the coordinate system related to the edge-fixed plane of diffraction. Finally, the diffracted field has to be transformed to the \((r,\theta,\phi)\) coordinate system. We start with projecting the unit vectors \(\hat{\psi}\) and \(\hat{\xi}\) onto \(\hat{\psi}'\) and \(\hat{\xi}'\). The latter two vectors are known in \((x,y,z)\) coordinates. These vectors are perpendicular to \(\hat{s}'\), but so are \(\hat{\psi}\) and \(\hat{\xi}\). This means that \(\hat{\psi}\), \(\hat{\xi}\), \(\hat{\psi}'\) and \(\hat{\xi}'\) are all in the same plane. For the projection we need the transformation matrix \(S\) with which a vector in the \((x,y,z)\) system can be transformed to the \((x',y',z')\) system. From figure 6.1 it can be seen that the \((x',y',z')\) system is rotated over an angle \(\theta_0\) around the \(y\)-axis. Then \(S\) is given by

\[
S = \begin{pmatrix}
\cos \theta_0 & 0 & -\sin \theta_0 \\
0 & 1 & 0 \\
\sin \theta_0 & 0 & \cos \theta_0
\end{pmatrix}
\]

(6.53)

and the inverse matrix \(S^{-1}\) can be written as

\[
S^{-1} = \begin{pmatrix}
\cos \theta_0 & 0 & \sin \theta_0 \\
0 & 1 & 0 \\
-\sin \theta_0 & 0 & \cos \theta_0
\end{pmatrix}
\]

(6.54)

If the vectors \(\hat{\psi}\) and \(\hat{\xi}\) are known in \((x',y',z')\) coordinates the projection can be performed. This is possible via Eq. (B.1) in Appendix B. Then we need to know the angles \(\psi\) and \(\xi\). An expression for \(\psi\) is simply given by

\[
\cos \psi = \hat{\xi}' \cdot \hat{s}'
\]

(6.55)

Finding \(\xi\) is a bit more complicated. The angle \(\xi\) is measured in the \(x'-y'\) plane with respect to the positive \(x'-axis. The vector \(\hat{s}', which is given in the \((x,y,z)\) system, has to be projected onto this plane. So, first the transformation \(S\) should be performed and then the projection \(P\) given by

\[
P = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{pmatrix}
\]

(6.56)

Then \(\xi\) is given by
\[ \xi = \arccos \left( \frac{\hat{\xi}' \cdot P(S(\hat{s}'))}{|P(S(\hat{s}'))|} \right) \text{sign } P_y(S(\hat{s}')) \] \hspace{1cm} (6.57)

The vectors \( \psi \) and \( \hat{\xi}' \) can now be expressed in \((x',y',z')\) coordinates. The transformation to \( \psi' \) and \( \hat{\xi}' \) in \((x,y,z)\) coordinates can simply be performed via \( S^{-1} \) leading to

\[
\begin{pmatrix} \psi' \\ \hat{\xi}' \end{pmatrix} = S^{-1} \begin{pmatrix} \psi \\ \hat{\xi} \end{pmatrix}
\] \hspace{1cm} (6.58)

The vectors are written in the same coordinate system making it quite easy to project \( \psi' \) and \( \hat{\xi}' \) onto \( \hat{\beta}' \) and \( \hat{\phi}' \). The incident field can then be written as

\[
E^i_{\psi'} = (\psi' \cdot \hat{\beta}') E^i_\psi + (\hat{\xi}' \cdot \hat{\beta}') E^i_\xi \\
E^i_{\phi'} = (\psi' \cdot \hat{\phi}') E^i_\psi + (\hat{\xi}' \cdot \hat{\phi}') E^i_\xi
\] \hspace{1cm} (6.59)

The diffracted field is expressed in the coordinate system which is related to the edge-fixed plane of diffraction. The vectors \( \hat{\beta}' \) and \( \hat{\phi} \) have to be projected onto \( \hat{\theta} \) and \( \hat{\phi} \). In the far-field \( \hat{r} \) and \( \hat{s}^d \) are parallel. \( \hat{\theta} \) and \( \hat{\phi} \) are perpendicular to \( \hat{r} \) and \( \hat{\beta}' \) and \( \hat{\phi} \) are perpendicular to \( \hat{s}^d \). This means that \( \hat{\theta}, \hat{\phi}, \hat{\beta}' \) and \( \hat{\phi} \) are all in the same plane. All vectors are known in \((x,y,z)\) coordinates, so the expressions for the far-field vector components become

\[
E^d_{\theta} = (\hat{\beta} \cdot \hat{\theta}) E^d_\beta + (\hat{\phi} \cdot \hat{\theta}) E^d_\phi \\
E^d_{\phi} = (\hat{\beta} \cdot \hat{\phi}) E^d_\beta + (\hat{\phi} \cdot \hat{\phi}) E^d_\phi
\] \hspace{1cm} (6.60)
The elliptical FZPA will be analyzed by means of the Kirchhoff diffraction integral. It is assumed that the zones are absorbing/transparent or ideally phase correcting/transparent. The surface $A$ over which the incident field has to be integrated, is the aperture in the $(x,y)$-plane. Because the zones are formed by ellipses, shown in figure 6.6, the integration will be performed in polar coordinates $p'$ and $\phi'$. The field in the aperture plane has to be written as a function of these coordinates. The feed polarization vector $\hat{f}$ is described in the spherical feed-fixed coordinate system $(p,\psi,\xi)$

$$\hat{f} = -\cos \xi \psi + \sin \xi \hat{\xi} \hspace{1cm} (6.61)$$

Using Eq. (B.1) $\hat{f}$ can be transformed to the $(x',y',z')$ system. When applying the transformation matrix $S^{-1}$ the unit vectors $\psi$ and $\hat{\xi}$ in $(x,y,z)$ coordinates are given by

$$\psi = \begin{bmatrix} \cos \psi \cos \xi \cos \theta_0 - \sin \psi \sin \theta_0 \\ \cos \psi \sin \xi \\ -\cos \psi \cos \xi \sin \theta_0 - \sin \psi \cos \theta_0 \end{bmatrix} \hspace{1cm} (6.62)$$

$$\hat{\xi} = \begin{bmatrix} -\sin \xi \cos \theta_0 \\ \cos \xi \\ \sin \xi \sin \theta_0 \end{bmatrix} \hspace{1cm} (6.63)$$

and $\hat{f}$ becomes

$$\hat{f} = \begin{bmatrix} -\cos \xi (\cos \psi \cos \xi \cos \theta_0 - \sin \psi \sin \theta_0) - \sin^2 \xi \cos \theta_0 \\ -\cos \xi \cos \psi \sin \xi + \sin \xi \cos \xi \\ \cos \xi (\cos \psi \cos \xi \sin \theta_0 + \sin \psi \cos \theta_0) + \sin^2 \xi \sin \theta_0 \end{bmatrix} \hspace{1cm} (6.64)$$

With this result the crossproduct in the integral of Eq. (3.3) becomes

$$\hat{n} \times \hat{f} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} \sin \xi \cos \xi (\cos \psi - 1) \\ -\cos^2 \xi \cos \psi \cos \theta_0 - \cos \xi \sin \psi \sin \theta_0 - \sin^2 \xi \cos \theta_0 \\ 0 \end{bmatrix} \hspace{1cm} (6.65)$$

The angles $\psi$ and $\xi$ and the distance $\rho$ from the feed to the point in the aperture plane are all functions of $(\rho',\phi')$, so

$$\psi = \psi (\rho',\phi') \hspace{1cm} (6.66)$$

$$\xi = \xi (\rho',\phi')$$

$$\rho = \rho (\rho',\phi')$$

These angles can be found by using the same procedure as in the previous section. Furthermore
The electric field can now be written in rectangular coordinates as

\[
\vec{E}(r) = \left[ \frac{P_T \eta j k e^{-j k r}}{2 \pi} \hat{r} \times \int \sqrt{2(n+1)} \cos^n \psi \left( \frac{e^{-j k \rho}}{\rho} \right) e^{j k \rho' \sin \theta \cos(\varphi - \phi')} \sin \xi \cos \xi (\cos \psi - 1) \right. \\
\left. \begin{array}{c}
- \cos^2 \xi \cos \psi \cos \theta_0 - \cos \xi \sin \psi \sin \theta_0 - \sin^2 \xi \cos \theta_0 \\
\cos \theta \sin \xi \cos \xi (\cos \psi - 1) \\
0
\end{array} \right] \rho' \, d \rho' \, d \phi' 
\]

These formulas give the field due to one zone. Because the borders of the zones are formed by ellipses, the integration limits of \( \rho' \) depends on \( \phi' \). This means that the \( \rho' \) integration has to be performed over \( \rho_m(\phi') \) to \( \rho_{m+1}(\phi') \) and the \( \phi' \) integration over 0 to \( 2 \pi \). The total far-field is the sum of the individual contributions of the various zones.
6. The elliptical Fresnel-zone plate antenna

6.5 Computed results

Radiation patterns have been calculated of an elliptical FZPA with metallic/transparent zones by means of UTD. The transmission and reflection coefficients to be incorporated in Eq. (6.45) are $T_E = T_M = 0$, $R_E = -1$ and $R_M = 1$. The antenna is designed in such a way that it always ends with an entire transparent zone. Hence, the outer dimensions of the antenna are not constant when varying the angle $\theta_0$, the focal distance $F$ or the wavelength $\lambda$. As a guide it is assumed that $2a_{\text{max}} = 1$ m. In all calculations the focal distance $F = 1$ m and the feed factor $n = 3$. Figure 6.10 shows radiation patterns in the plane $\phi = 0^\circ$ for five different values of $\theta_0$, namely $0^\circ$, $5^\circ$, $10^\circ$, $15^\circ$, and $30^\circ$. For these values the outer dimensions of the antenna are

- $\theta_0 = 0^\circ \quad \rightarrow \quad 2a_{\text{max}} = 1.0159$ m
- $\theta_0 = 5^\circ \quad \rightarrow \quad 2a_{\text{max}} = 1.0200$ m
- $\theta_0 = 10^\circ \quad \rightarrow \quad 2a_{\text{max}} = 1.0325$ m
- $\theta_0 = 15^\circ \quad \rightarrow \quad 2a_{\text{max}} = 1.0539$ m
- $\theta_0 = 30^\circ \quad \rightarrow \quad 2a_{\text{max}} = 1.0357$ m

The angle $\theta_0 = 30^\circ$ is a practical value of the elevation angle of a satellite for the Eindhoven University of Technology. From figure 6.10 it can be seen that the shape of the patterns remains the same when varying $\theta_0$. For $\theta_0 = 0^\circ$ through $\theta_0 = 15^\circ$ the positions of the sidelobe levels remain fixed with respect to the forward direction. The patterns are shifted along the horizontal axis, where only the level of the sidelobes differ. For $\theta_0 = 30^\circ$ the pattern is somewhat stretched on the $\theta$-scale; the sidelobes move away from the forward direction. The UTD-analysis fails in forward direction, which is to be expected, because this direction is a caustic. Figure 6.11 shows the patterns obtained by the Kirchhoff diffraction theory. Here, it can be seen that neither the antenna gain nor the beamwidth is affected for the first four values of $\theta_0$. The 3 dB beamwidth is approximately $1.6^\circ$ and the gain is $28.0$ dB. When the angle $\theta_0$ becomes greater and the antenna diameter remains fixed at approximately 1 m, the effective aperture becomes smaller. This will lead to a greater 3 dB beamwidth and a lower gain. This is observed for $\theta_0 = 30^\circ$ where the beamwidth is $1.8^\circ$ and the gain is $26.3$ dB. The Kirchhoff method, as well as UTD, shows that the sidelobe levels remain fixed with respect to the forward direction. The sidelobe levels of these systems are approximately 20 dB lower than the antenna gain. Figure 6.12-6.16 compare the results obtained by the two methods for one single antenna system. It can be seen that patterns match very well. Only the values of the levels differ around the mainlobe.

Radiation patterns have also been calculated in case of ideal phase correcting zones. For the UTD method this means that in Eq. (6.45) $T_{E,M} = -1$ and $R_{E,M} = 0$ have to be inserted. Figure 6.17 shows the patterns for the five different antenna systems. Antenna gain is clearly increased, but in order to obtain the exact increase in antenna gain, it is necessary to use Kirchhoff’s theory. Figure 6.18 shows the radiation patterns. The antenna gain is approximately increased by $4.9$ dB. Sidelobe levels are again approximately 20 dB below the antenna gain and they remain fixed with respect to the forward direction. Figure 6.19-6.23 compare the results obtained by UTD and Kirchhoff for one single antenna system. The patterns match very well. Away from boresight it is recommended to use UTD. It is known that the assumption in Kirchhoff, that the aperture field is not influenced by the edges, is invalid in this angle region.
Figure 6.10 Gain patterns calculated with UTD for five different antenna systems with metallic/transparent zones, $\theta_0 = 0^\circ$, $5^\circ$, $10^\circ$, $15^\circ$ and $30^\circ$ ($F = 1 \text{ m}$, $n = 3$, $\varphi = 0^\circ$)

Figure 6.11 Gain patterns calculated with Kirchhoff's theory for five antenna systems with absorbing/transparent zones, $\theta_0 = 0^\circ$, $5^\circ$, $10^\circ$, $15^\circ$ and $30^\circ$ ($F = 1 \text{ m}$, $n = 3$, $\varphi = 0^\circ$)
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![Graph](image)

**Figure 6.12** Gain patterns calculated with Kirchhoff’s theory and UTD for an antenna system with $\theta_0 = 0^\circ$ ($F = 1$ m, $n = 3$, $\varphi = 0^\circ$)

![Graph](image)

**Figure 6.13** Gain patterns calculated with Kirchhoff’s theory and UTD for an antenna system with $\theta_0 = 5^\circ$ ($F = 1$ m, $n = 3$, $\varphi = 0^\circ$)
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**Figure 6.14** Gain patterns calculated with Kirchhoff’s theory and UTD for an antenna system with $\theta_0 = 10^\circ$ ($F = 1$ m, $n = 3$, $\varphi = 0^\circ$)

**Figure 6.15** Gain patterns calculated with Kirchhoff’s theory and UTD for an antenna system with $\theta_0 = 15^\circ$ ($F = 1$ m, $n = 3$, $\varphi = 0^\circ$)
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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.16}
\caption{Gain patterns calculated with Kirchhoff’s theory and UTD for an antenna system with $\theta_0 = 30^\circ$ ($F = 1\ m$, $n = 3$, $\varphi = 0^\circ$)}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.17}
\caption{Gain patterns calculated with UTD for five different antenna systems with ideal phase correcting zones, $\theta_0 = 0^\circ$, $5^\circ$, $10^\circ$, $15^\circ$ and $30^\circ$ ($F = 1\ m$, $n = 3$, $\varphi = 0^\circ$)}
\end{figure}
Figure 6.18 Gain patterns calculated with Kirchhoff's theory for five antenna systems with ideal phase correcting zones, $\theta_0 = 0^\circ, 5^\circ, 10^\circ, 15^\circ$ and $30^\circ$ ($F = 1 \text{ m}, n = 3, \varphi = 0^\circ$)

Figure 6.19 Gain patterns calculated with Kirchhoff's theory and UTD for an antenna system with ideal phase correcting zones, $\theta_0 = 0^\circ$ ($F = 1 \text{ m}, n = 3, \varphi = 0^\circ$)
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Figure 6.20 Gain patterns calculated with Kirchhoff’s theory and UTD for an antenna system with ideal phase correcting zones, $\theta_0 = 5^\circ$ ($F = 1 \text{ m}, n = 3$, $\varphi = 0^\circ$)

Figure 6.21 Gain patterns calculated with Kirchhoff’s theory and UTD for an antenna system with ideal phase correcting zones, $\theta_0 = 10^\circ$ ($F = 1 \text{ m}, n = 3$, $\varphi = 0^\circ$)
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![Graph 1](image1)  
**Figure 6.22** Gain patterns calculated with Kirchhoff's theory and UTD for an antenna system with ideal phase correcting zones $\theta_0 = 15^\circ$ ($F = 1 \text{ m}, n = 3, \varphi = 0^\circ$)

![Graph 2](image2)  
**Figure 6.23** Gain patterns calculated with Kirchhoff's theory and UTD for an antenna system with ideal phase correcting zones $\theta_0 = 30^\circ$ ($F = 1 \text{ m}, n = 3, \varphi = 0^\circ$)
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The presented radiation patterns are all calculated in the \((r,\theta,\phi)\) coordinate system which is fixed to the aperture plane. When calculating co- and cross-polarization patterns, it is necessary to do this in the \((\rho,\psi,\xi)\) coordinate system. This is because the \(x'\)-component is the desired component. Co-polar (CP) and cross-polar (XP) are now defined as

\[
\text{CP: } 10\log \left[ \left( \frac{r}{A_0} \right)^2 | \cos \xi E_\psi - \sin \xi E_\xi |^2 \right] \\
\text{XP: } 10\log \left[ \left( \frac{r}{A_0} \right)^2 | -\sin \xi E_\psi - \cos \xi E_\xi |^2 \right]
\]

(6.71)

Maximum cross-polar occurs in the plane \(\xi = 90^\circ\). In this plane the radiation patterns are symmetric with respect to \(\psi = 0^\circ\). Figure 6.24 shows the co-polar patterns for the known antenna systems obtained with UTD. For \(\theta_0 = 0^\circ\) there is no cross-polar, so it is not calculated. There is no visible difference between the patterns for the systems with \(\theta_0 = 5^\circ, 10^\circ, 15^\circ\). Figure 6.25 shows the cross-polar patterns. With increasing \(\theta_0\) the cross-polar component in the far-field becomes greater. This can be explained by the fact that the zones become more elliptic; the ratio of the semi axes of the ellipses is decreasing. Figure 6.26 and 6.27 show the patterns obtained by the Kirchhoff formulas. From the co-polar patterns it can be found that the 3 dB beamwidth is equal to that in the perpendicular plane. This means that the mainlobe is approximately circular.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure6.24}
\caption{Co-polar patterns calculated with UTD for four different antenna systems with absorbing/transparent zones, \(\theta_0 = 5^\circ\) (solid), 10\(^\circ\) (dashed), 15\(^\circ\) (dotted) and 30\(^\circ\) (dash-dotted) \((F = 1\ m, n = 3, \phi = 90^\circ)\)}
\end{figure}
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**Figure 6.25** Cross-polar patterns calculated with UTD for four different antenna systems with absorbing/transparent zones, $\theta_0 = 5^\circ$ (solid), $10^\circ$ (dashed), $15^\circ$ (dotted) and $30^\circ$ (dash-dotted) ($F = 1$ m, $n = 3, \phi = 90^\circ$)

**Figure 6.26** Co-polar patterns calculated with Kirchhoff’s theory for four different antenna systems with absorbing/transparent zones, $\theta_0 = 5^\circ$ (solid), $10^\circ$ (dashed), $15^\circ$ (dotted) and $30^\circ$ (dash-dotted) ($F = 1$ m, $n = 3, \phi = 90^\circ$)
Figure 6.27 Cross-polar patterns calculated with Kirchhoff’s theory for four different antenna systems with absorbing/transparent zones, $\theta_0 = 5^\circ$ (solid), $10^\circ$ (dashed), $15^\circ$ (dotted) and $30^\circ$ (dash-dotted) ($F = 1$ m, $n = 3, \phi = 90^\circ$)
6.6 Practical and optimized antenna systems

Until now, antenna systems are considered, where outside the antenna absorbing (or metallic) material applied. Of course, this is not a practical situation. In practice there is transparent material outside the aperture. This will have an effect on the sidelobes in the radiation pattern. The mainlobe will not be affected, because the contributions in the forward direction are determined by the inner zones. Figure 6.28 shows the radiation patterns of the system with $\theta_0 = 30^\circ$. There is indeed a change in the sidelobe levels.

![Radiation patterns calculated with UTD for antenna systems with metallic (solid) and transparent material (dashed) outside the antenna ($F = 1$ m, $n = 3$, $\varphi = 0^\circ$)](image)

Figure 6.28 Radiation patterns calculated with UTD for antenna systems with metallic (solid) and transparent material (dashed) outside the antenna ($F = 1$ m, $n = 3$, $\varphi = 0^\circ$)

Radiation patterns presented in section 6.5 are all calculated with $n = 3$. The edge illumination which goes with this value of $n$ does not cause optimum antenna gain. In table 6.1 the optimum values are given, together with the gain.

<table>
<thead>
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<th>$\theta_0$</th>
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<th>$G$ (dB)</th>
<th>$n$</th>
<th>$G$ (dB)</th>
</tr>
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<td>20.3</td>
<td>36.54</td>
</tr>
<tr>
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<td>22</td>
<td>31.55</td>
<td>20.3</td>
<td>36.54</td>
</tr>
<tr>
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<td>31.57</td>
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<td>36.55</td>
</tr>
<tr>
<td>15</td>
<td>22.2</td>
<td>31.58</td>
<td>20.5</td>
<td>36.57</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>30.82</td>
<td>27.2</td>
<td>35.8</td>
</tr>
</tbody>
</table>
6.7 Beamsquint

The beamsquint phenomenon is observed when offset antennas are illuminated by a circularly polarized feed [26],[27]. The IEEE definition of circular polarization is such that when a wave travels away from the observer and the electric field rotates clockwise, the polarization is defined as right-hand circular. The elliptical FZPA can be regarded as an offset antenna. Hence, it is to be expected that beamsquint also occurs for this kind of FZPA. Beamsquint means that the maximum in the far-field radiation pattern does not occur in forward direction. This can be an important effect when frequency reuse techniques are employed. In this section beamsquint will be analyzed for the elliptical FZPA.

Beamsquint can be explained when the circularly polarized field is decomposed in two orthogonal, linear polarizations. Until now an x-polarized feed has been considered with the feed polarization vector \( \hat{f}_x \) given by

\[
\hat{f}_x = -\cos \xi \psi + \sin \xi \xi
\]

Now, an y-polarized field vector \( \hat{f}_y \) is introduced which is given by

\[
\hat{f}_y = \sin \xi \psi + \cos \xi \xi
\]

For circular polarization these vectors must be added with a phase shift of \( \pi/2 \). The resulting normalized feed polarization vector \( \hat{f}_{\text{circ}} \), giving a circularly polarized field, can be written as

\[
\hat{f}_{\text{circ}} = \frac{1}{\sqrt{2}} (\hat{f}_x + j\hat{f}_y)
\]

where the + sign corresponds to the right-hand case and the - sign to the left-hand case. Each of the linearly polarizations causes a co- and cross-polarized field. Figure 6.29 shows the field vectors in the aperture plane. In the upper part the vectors are due to the x-polarized field, whereas in the lower part the vectors are due to the y-polarized field.

It is easy to see that in the plane \( \xi = 0^\circ \) and for every angle \( \psi \) the cross-polar components of the linear polarizations are cancelled. Then there will be no circular cross-polar. In the plane \( \xi = 90^\circ \) the crosspolar components are only cancelled in forward direction. It has already been mentioned that the individual far-field radiation pattern of the x-polarized feed is symmetric in this plane and so is the pattern of the y-polarized feed. But this is not true for the circularly polarized feed which can be explained in the following way. The x-polarized field causes a complex far-field \( \vec{E}_x \)

\[
\vec{E}_x = \begin{cases} 
E_{\psi x} \psi + E_{\xi x} \xi & \psi < 0 \\
E_{\psi x} \psi - E_{\xi x} \xi & \psi > 0
\end{cases}
\]

Strictly speaking, the angle \( \psi \) can not become negative, but it is meant that the angle \( \xi \) becomes \( \xi + \pi \).
Similarly, the y-polarized field causes a complex far-field $\vec{E}_y$:

$$\vec{E}_y = \begin{cases} E_{\psi y} \psi + E_{\xi y} \xi & \psi < 0 \\ -E_{\psi y} \psi + E_{\xi y} \xi & \psi > 0 \end{cases} \quad (6.76)$$

Consider right-hand circular polarization. The circularly polarized field $\vec{E}_{circ,r}$ is then given by

$$\vec{E}_{circ,r} = \begin{cases} (E_{\psi x} + jE_{\psi y}) \psi + (E_{\xi x} + jE_{\xi y}) \xi & \psi < 0 \\ (E_{\psi x} - jE_{\psi y}) \psi + (-E_{\xi x} + jE_{\xi y}) \xi & \psi > 0 \end{cases} \quad (6.77)$$

The gain function is proportional to

$$|\vec{E}_{circ,r}|^2 = \begin{cases} (E_{\psi x}^{re} - E_{\psi y}^{im})^2 + (E_{\psi x}^{im} + E_{\psi y}^{re})^2 + (E_{\xi x}^{re} - E_{\xi y}^{im})^2 + (E_{\xi x}^{im} + E_{\xi y}^{re})^2 & \psi < 0 \\ (E_{\psi x}^{re} + E_{\psi y}^{im})^2 + (E_{\psi x}^{im} - E_{\psi y}^{re})^2 + (E_{\xi x}^{re} + E_{\xi y}^{im})^2 + (-E_{\xi x}^{im} + E_{\xi y}^{re})^2 & \psi > 0 \end{cases} \quad (6.78)$$

where the superscripts $re$ and $im$ denote the real and imaginary part respectively. What can be concluded from the latter equation? The difference for $\psi < 0$ and $\psi > 0$ can be written as

$$4(-E_{\psi x}^{re}E_{\psi y}^{im} + E_{\psi x}^{im}E_{\psi y}^{re} - E_{\xi x}^{re}E_{\xi y}^{im} + E_{\xi x}^{im}E_{\xi y}^{re}) \neq 0 \quad (6.79)$$

This means that the symmetry is lost with respect to the direction $\psi = 0^\circ$. This results in a squinting of the main lobe. In fact, the phenomenon is caused by the asymmetric cross-polar components. The same analysis can be applied to a left-hand circularly polarized feed. Then, the
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The complex far-field $\vec{E}_{\text{circ,l}}$ is

$$\vec{E}_{\text{circ,l}} = \begin{cases} (E_{\psi x} - jE_{\psi y})\psi + (E_{\xi x} - jE_{\xi y})\xi & \psi < 0 \\ (E_{\psi x} + jE_{\psi y})\psi + (E_{\xi x} - jE_{\xi y})\xi & \psi > 0 \end{cases}$$

The gain function is proportional to

$$|\vec{E}_{\text{circ,l}}|^2 = \begin{cases} (E_{\psi x}^re + E_{\psi y}^im)^2 + (E_{\psi x}^im - E_{\psi y}^re)^2 + (E_{\xi x}^im + E_{\xi y}^re)^2 & \psi < 0 \\ (E_{\psi x}^im - E_{\psi y}^re)^2 + (E_{\psi x}^re + E_{\psi y}^im)^2 + (E_{\xi x}^im + E_{\xi y}^re)^2 & \psi > 0 \end{cases}$$

The difference for $\psi < 0$ and $\psi > 0$ can be written as

$$4(E_{\psi x}^re E_{\psi y}^im - E_{\psi x}^im E_{\psi y}^re + E_{\xi x}^re E_{\xi y}^im - E_{\xi x}^im E_{\xi y}^re) = 0$$

It can be seen from Eq. (6.79) and (6.82) that the field in $\psi < 0$ for left-hand polarization is equal to that in $\psi > 0$ for right-hand polarization. In $\psi = 0^\circ$ they are equal. This means that the fields are symmetric with respect to the direction $\psi = 0^\circ$. It is to be expected that the squint effect is stronger with increasing $\Theta_0$, because then the cross-polar components become greater.

In general, the fields $\vec{E}_{\text{circ,r}}$ and $\vec{E}_{\text{circ,l}}$ are elliptically polarized. They can be decomposed in a left-hand $E_l$ and right-hand $E_r$, circular polarization. The field components can be described by

$$E_\psi = E_{\psi x} + jE_{\psi y} = |E_\psi|e^{j\Phi_\psi}$$
$$E_\xi = E_{\xi x} + jE_{\xi y} = |E_\xi|e^{j\Phi_\xi}$$

Decomposing the field gives

$$E_r = \frac{1}{2}(|E_\psi|e^{j\Phi_\psi} - |E_\xi|e^{j\Delta\Phi})\psi - \frac{1}{2}j(|E_\psi|e^{j\Phi_\psi} - |E_\xi|e^{j\Delta\Phi})\xi$$
$$E_l = \frac{1}{2}(|E_\psi|e^{j\Phi_\psi} + |E_\xi|e^{j\Delta\Phi})\psi + \frac{1}{2}j(|E_\psi|e^{j\Phi_\psi} + |E_\xi|e^{j\Delta\Phi})\xi$$

with

$$\Delta\Phi = \Phi_\xi - \pi/2$$

When calculating far-field radiation patterns it is necessary to derive the Kirchhoff formulas for a $y$-polarized feed. This is analogous to the method presented in section 6.4. Let’s start with describing the feed vector $\hat{f}_y$ in $(x,y,z)$-coordinates.
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\[
\hat{f}_y = \begin{bmatrix}
\sin \xi (\cos \psi \cos \xi \cos \theta_0 - \sin \psi \sin \theta_0) - \sin \xi \cos \xi \cos \theta_0 \\
\cos \psi \sin^2 \xi + \cos^2 \xi \\
- \sin \xi (\cos \psi \cos \xi \sin \theta_0 + \sin \psi \cos \theta_0) + \sin \xi \cos \xi \sin \theta_0
\end{bmatrix} (6.86)
\]

The crossproduct in the integral can be written in rectangular coordinates as

\[
\hat{n} \times \hat{f}_y = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} -\cos \psi \sin^2 \xi - \cos^2 \xi \\
\sin \xi (\cos \psi \cos \xi \cos \theta_0 - \sin \psi \sin \theta_0) - \sin \xi \cos \xi \cos \theta_0 \\
0 \end{bmatrix} \quad (6.87)
\]

The electric field is now given in rectangular coordinates by

\[
\bar{E}(r) = \frac{P_T \eta}{2\pi} \frac{j e^{-jkr}}{2\pi r} \hat{n} \times \int \int \sqrt{2(n+1)\cos^\psi} \frac{e^{-jk\rho}}{\rho} e^{jk\rho' \sin \theta \cos (\varphi - \phi')}.
\]

\[
\begin{bmatrix}
-\cos \psi \sin^2 \xi - \cos^2 \xi \\
\sin \xi (\cos \psi \cos \xi \cos \theta_0 - \sin \psi \sin \theta_0) - \sin \xi \cos \xi \cos \theta_0 \\
0
\end{bmatrix} \rho' \, d\rho' \, d\phi'
\]

(6.88)

The field can be transformed to spherical coordinates with Appendix B. The vector product can be rewritten leading to the final expressions for the far-field components.

\[
E_\theta(r) = \frac{P_T \eta}{2\pi} \frac{j e^{-jkr}}{2\pi r} \int \int \sqrt{2(n+1)\cos^\psi} \frac{e^{-jk\rho}}{\rho} e^{jk\rho' \sin \theta \cos (\varphi - \phi')}.
\]

\[
\begin{bmatrix}
-\cos \varphi (\sin \xi \cos \xi \cos \theta_0 (\cos \psi - 1) - \sin \xi \sin \psi \sin \theta_0) \\
\sin \varphi (\cos \psi \sin^2 \xi + \cos^2 \xi)
\end{bmatrix} \rho' \, d\rho' \, d\phi'
\]

(6.89)

\[
E_\varphi(r) = \frac{P_T \eta}{2\pi} \frac{j e^{-jkr}}{2\pi r} \int \int \sqrt{2(n+1)\cos^\psi} \frac{e^{-jk\rho}}{\rho} e^{jk\rho' \sin \theta \cos (\varphi - \phi')}.
\]

\[
\begin{bmatrix}
\cos \theta \sin \varphi (\sin \xi \cos \xi \cos \theta_0 (\cos \psi - 1) - \sin \xi \sin \psi \sin \theta_0) \\
\cos \theta \cos \varphi (\cos \psi \sin^2 \xi + \cos^2 \xi)
\end{bmatrix} \rho' \, d\rho' \, d\phi'
\]

(6.90)

These formulas give the field due to one zone. Because the borders of the zones are formed by ellipses, the integration limits of \(\rho'\) depends on \(\phi'\). This means that the \(\rho'\) integration has to be performed over \(\rho_m(\phi')\) to \(\rho_{m+1}(\phi')\) and the \(\phi'\) integration over 0 to 2\(\pi\). The total far-field is the sum of the individual contributions of the various zones.

Beamssquint is shown for an antenna system with absorbing/transparent zones, where the edge
illumination is optimized with respect to antenna gain. For a system with $F = 1 \text{ m}$, $\theta_0 = 30^\circ$ and $2a_{\text{max}} = 1.0357 \text{ m}$ the optimized factor $n$ is found to be 30. First, the cross-polar patterns caused by the individual linear polarizations is shown in figure 6.30. The $y$-polarization causes a much greater cross-polar than the $x$-polarization. This is due to the elliptical shape of the zones. Figure 6.31 shows the co-polar patterns of a right-hand and left-hand circularly polarized feed. It can be seen that these patterns are indeed mirrored with respect to $\psi = 0^\circ$. The beams are $0.1^\circ$ displaced and is towards the left for right-hand polarization and towards the right for left-hand polarization. Figure 6.32 shows the cross-polar patterns. In [27,Adatia] a simple formula for the squint angle $\psi_s$ of an offset parabolic reflector antenna is given,

$$\sin \psi_s = \frac{\sin \theta_0}{2Fk}$$

(6.91)

When applying this formula to the FZPA, this gives $\psi_s = 0.07^\circ$, which is not a very good approximation.

**Figure 6.30** Cross-polar patterns due to the linear polarizations, $x$-polarization (solid) and $y$-polarization (dashed)
The elliptical Fresnel-zone plate antenna

Figure 6.31 Co-polar patterns due to a right-hand (solid) and left-hand (dashed) circularly polarized feed ($\theta_0 = 30^\circ$, $F = 1$ m, $n = 30$, $\xi = 90^\circ$)

Figure 6.32 Cross-polar patterns due to a right-hand (solid) and left-hand (dashed) circularly polarized feed ($\theta_0 = 30^\circ$, $F = 1$ m, $n = 30$, $\xi = 90^\circ$)
7. Conclusions and recommendations

The heuristic UTD approach described in this report has first been applied to axially symmetric Fresnel-zone plate antenna systems with ideal phase correcting and dielectric/transparent zones. For large focal distances, the far-field radiation patterns calculated with UTD are equal to those obtained with the Kirchhoff diffraction theory. UTD results are always taken as the reference. The new method gives a substantial reduction in CPU-time, making it possible to calculate radiation patterns in a time efficient way. It was found that dielectric zones with relative dielectric constant \( \varepsilon_r = 4 \) behave like ideal phase shifters. Then the phase correcting Fresnel-zone plate antenna can be modelled in the way this is done until now in literature, by just adding a constant phase shift for the aperture field. When increasing the value of \( \varepsilon_r \), the antenna gain decreases, which is caused by the lower transmission coefficient of the dielectric. Hence, \( \varepsilon_r \) should be as low as possible. It is obvious that in general the reflection properties of the dielectric have to be considered instead of simply including the constant phase shift. A small value of \( \varepsilon_r \) will result in a larger thickness of the zones. The dielectric has been modelled as though it were part of a half-plane. When the thickness becomes larger, this model will be violated and diffraction by block shaped dielectric obstacles should be considered. Unfortunately, there is no rigorous solution to this problem yet.

Fresnel-zone plate antennas can be optimized with regard to antenna gain. Such optimized systems have been used throughout this report. But when the feed is displaced or its operating frequency is changed, a yet higher efficiency and gain can be reached. Due to the fact that a zone plate is scalable, it is possible to reach maximum gain at the design frequency provided that all the other parameters are scaled. This principle of scaling is used to define the 3 dB bandwidth. For a Fresnel-zone plate antenna the bandwidth is approximately inversely proportional to the number of transparent zones. When phase correcting zones are applied the bandwidth is increased.

Far-field radiation patterns of the elliptical Fresnel-zone plate antenna have been calculated with both UTD and Kirchhoff's diffraction theory. Antenna systems with absorbing/transparent and ideal phase correcting zones were considered. The patterns obtained by both theories match very well. For small offset angles the shape of the patterns remains the same. Only the cross-polarization component becomes clearly larger with increasing offset angle. With a circularly polarized feed the elliptical Fresnel-zone plate antenna causes beamsquint. This effect is stronger when the offset angle is greater.

The modelling of the dielectric has to be verified by future measurements. One should start with a dielectric strip and calculate the UTD diffracted field for this situation. When it is found that the theoretical and practical results agree, an axially symmetric Fresnel-zone plate antenna with dielectric/transparent zones can be built. In this report, dielectric zones are not applied in the case of the elliptical Fresnel-zone plate antenna. However, the first steps for the model are set; diffraction coefficients are given in the general, three dimensional case of dielectric material. It is to be expected that the properties of the dielectric have more influence on the antenna performance.
than in the case of the axially symmetric antenna. This is due to the increase in the angles of incidence of the waves propagating from the feed to the lens.
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Appendix A
Derivation of some relations involving Fresnel transmission and reflection coefficients

The Fresnel transmission and reflection coefficients for a plane wave incident to a lossless dielectric material are given by

\[ T_{IE} = \frac{2 \cos \theta_i}{\cos \theta_i + \sqrt{\varepsilon_r - \sin^2 \theta_i}} \]  \hspace{1cm} (A.1)

\[ R_{IE} = \frac{\cos \theta_i - \sqrt{\varepsilon_r - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{\varepsilon_r - \sin^2 \theta_i}} \]  \hspace{1cm} (A.2)

in case of perpendicular (E) polarization and

\[ T_{IM} = \frac{2 \sqrt{\varepsilon_r} \cos \theta_i}{\varepsilon_r \cos \theta_i + \sqrt{\varepsilon_r - \sin^2 \theta_i}} \]  \hspace{1cm} (A.3)

\[ R_{IM} = \frac{\varepsilon_r \cos \theta_i - \sqrt{\varepsilon_r - \sin^2 \theta_i}}{\varepsilon_r \cos \theta_i + \sqrt{\varepsilon_r - \sin^2 \theta_i}} \]  \hspace{1cm} (A.4)

in case of parallel (M) polarization.

When the wave travels from dielectric into free space, the coefficients are given by

\[ T_{2E} = \frac{2 \cos \theta_i'}{\cos \theta_i' + \sqrt{\frac{1}{\varepsilon_r} - \sin^2 \theta_i'}} \]  \hspace{1cm} (A.5)
Appendix A Derivation of some relations involving Fresnel transmission and ...

\[ R_{2E} = \frac{\cos \theta'_i - \sqrt{\frac{1}{\varepsilon_r} - \sin^2 \theta'_i}}{\cos \theta'_i + \sqrt{\frac{1}{\varepsilon_r} - \sin^2 \theta'_i}} \]  
(A.6)

\[ T_{2M} = \frac{2 \frac{1}{\sqrt{\varepsilon_r}} \cos \theta'_i}{\frac{1}{\varepsilon_r} \cos \theta'_i + \frac{1}{\varepsilon_r} - \sin^2 \theta'_i} \]  
(A.7)

\[ R_{2M} = \frac{\frac{1}{\varepsilon_r} \cos \theta'_i - \frac{1}{\varepsilon_r} - \sin^2 \theta'_i}{\frac{1}{\varepsilon_r} \cos \theta'_i + \frac{1}{\varepsilon_r} - \sin^2 \theta'_i} \]  
(A.8)

We are considering a dielectric slab, to which a plane wave is incident. The wave travels through the dielectric and reaches the dielectric/free space boundary. In this configuration the angle of incidence \( \theta'_i \) is related to \( \theta \) via Snell's law,

\[ \sin \theta_i = \sqrt{\varepsilon_r} \sin \theta'_i \]  
(A.9)

With the latter equation, Eq. (A.6) and (A.8) can now be rewritten as

\[ R_{2E} = \frac{\cos \theta'_i - \frac{1}{\varepsilon_r} - \sqrt{\frac{1}{\varepsilon_r} - \sin^2 \theta'_i}}{\cos \theta'_i + \frac{1}{\varepsilon_r} - \sin^2 \theta'_i} = \frac{\sqrt{\varepsilon_r \cos \theta'_i} - \sqrt{\frac{1}{\varepsilon_r} - \varepsilon_r \sin^2 \theta'_i}}{\sqrt{\varepsilon_r \cos \theta'_i} + \sqrt{\frac{1}{\varepsilon_r} - \varepsilon_r \sin^2 \theta'_i}} = \frac{\sqrt{\varepsilon_r \cos \theta'_i - \cos \theta_i}}{\sqrt{\varepsilon_r \cos \theta'_i + \cos \theta_i}} \]

\[ (A.10) \]

\[ = \frac{\sqrt{\varepsilon_r - \varepsilon_r \sin^2 \theta'_i - \cos \theta_i}}{\sqrt{\varepsilon_r - \varepsilon_r \sin^2 \theta'_i + \cos \theta_i}} \]

and
Appendix A Derivation of some relations involving Fresnel transmission and ...

\[ R_{2M} = \frac{1}{\varepsilon_r} \cos \theta'_i - \frac{1}{\varepsilon_r} \sin^2 \theta'_i = \frac{1}{\sqrt{\varepsilon_r}} \cos \theta'_i - \sqrt{1 - \varepsilon_r \sin^2 \theta'_i} = \frac{1}{\sqrt{\varepsilon_r}} \cos \theta'_i - \cos \theta_i \]

\[ \frac{1}{\varepsilon_r} \cos \theta'_i + \frac{1}{\varepsilon_r} \sin^2 \theta'_i = \frac{1}{\sqrt{\varepsilon_r}} \cos \theta'_i + \sqrt{1 - \varepsilon_r \sin^2 \theta'_i} = \frac{1}{\sqrt{\varepsilon_r}} \cos \theta'_i + \cos \theta_i \]

\( (A.11) \)

\[ \sqrt{\varepsilon_r \cos \theta'_i - \varepsilon_r \cos \theta_i} = \sqrt{\varepsilon_r - \varepsilon_r \sin^2 \theta'_i - \varepsilon_r \cos \theta_i} = \sqrt{\varepsilon_r - \sin^2 \theta'_i - \varepsilon_r \cos \theta_i} \]

\[ \sqrt{\varepsilon_r \cos \theta'_i + \varepsilon_r \cos \theta_i} = \sqrt{\varepsilon_r - \varepsilon_r \sin^2 \theta'_i + \varepsilon_r \cos \theta_i} = \sqrt{\varepsilon_r - \sin^2 \theta'_i + \varepsilon_r \cos \theta_i} \]

\[ = -R_{1M} \]

So, \( R_2 = -R_1 \) for both polarizations. The transmission coefficients of Eq. (A.5) and (A.7) become

\[ T_{2E} = \frac{2 \cos \theta'_i}{\cos \theta'_i + \frac{1}{\sqrt{\varepsilon_r}} \sin \theta'_i} = \frac{2 \sqrt{\varepsilon_r} \sqrt{1 - \sin^2 \theta'_i}}{\sqrt{\varepsilon_r \cos \theta'_i} + \sqrt{1 - \varepsilon_r \sin^2 \theta'_i}} \]

\( (A.12) \)

\[ T_{2M} = \frac{2 \frac{1}{\sqrt{\varepsilon_r}} \cos \theta'_i}{\frac{1}{\varepsilon_r} \cos \theta'_i + \frac{1}{\sqrt{\varepsilon_r}} \sin^2 \theta'_i} = \frac{2 \cos \theta'_i}{\sqrt{\varepsilon_r} \cos \theta'_i + \sqrt{1 - \varepsilon_r \sin^2 \theta'_i}} = \frac{2 \cos \theta'_i}{\frac{1}{\varepsilon_r} \cos \theta'_i + \cos \theta_i} \]

\( (A.13) \)

The products \( T_{1E}T_{2E} \) and \( T_{1M}T_{2M} \) are then given by

\[ T_{1E}T_{2E} = \frac{4 \cos \theta'_i \sqrt{\varepsilon_r - \sin^2 \theta_i}}{\{ \cos \theta_i + \sqrt{\varepsilon_r - \sin^2 \theta_i} \}^2} \]

\( (A.14) \)

\[ T_{1M}T_{2M} = \frac{4 \varepsilon_r \cos \theta'_i \sqrt{\varepsilon_r - \sin^2 \theta_i}}{\{ \varepsilon_r \cos \theta_i + \sqrt{\varepsilon_r - \sin^2 \theta_i} \}^2} \]

\( (A.15) \)

The following relations also apply
Appendix A Derivation of some relations involving Fresnel transmission and ...

\[
1 - R_{1E}^2 = 1 - \frac{\cos^2 \theta_i - 2 \cos \theta_i \sqrt{\varepsilon_r - \sin^2 \theta_i} + \varepsilon_r - \sin^2 \theta_i}{\{\cos \theta_i + \sqrt{\varepsilon_r - \sin^2 \theta_i}\}^2} \\
= \frac{4 \cos \theta_i \sqrt{\varepsilon_r - \sin^2 \theta_i}}{\{\cos \theta_i + \sqrt{\varepsilon_r - \sin^2 \theta_i}\}^2} 
\quad \text{(A.16)}
\]

\[
1 - R_{1M}^2 = 1 - \frac{\varepsilon_r \cos^2 \theta_i - 2 \varepsilon_r \cos \theta_i \sqrt{\varepsilon_r - \sin^2 \theta_i} + \varepsilon_r - \sin^2 \theta_i}{\{\varepsilon_r \cos \theta_i + \sqrt{\varepsilon_r - \sin^2 \theta_i}\}^2} \\
= \frac{4 \varepsilon_r \cos \theta_i \sqrt{\varepsilon_r - \sin^2 \theta_i}}{\{\varepsilon_r \cos \theta_i + \sqrt{\varepsilon_r - \sin^2 \theta_i}\}^2} 
\quad \text{(A.17)}
\]

As a consequence

\[
T_1 T_2 = 1 - R_1^2 
\quad \text{(A.18)}
\]

for both polarizations.
Appendix B  
Transformation between coordinate systems  

Figure B.1 shows the rectangular coordinate system \((x,y,z)\) and the accompanying spherical coordinate system \((r,\theta,\phi)\).

![Figure B.1 Relation between rectangular an spherical coordinate system](image)

The spherical system \((r,\theta,\phi)\) can be transformed to the rectangular system \((x,y,z)\) via

\[
\begin{align*}
\hat{r} &= \sin\theta \sin\phi \hat{x} + \sin\theta \cos\phi \hat{y} + \cos\theta \hat{z} \\
\hat{\theta} &= \cos\theta \cos\phi \hat{x} + \cos\theta \sin\phi \hat{y} - \sin\theta \hat{z} \\
\hat{\phi} &= -\sin\phi \hat{x} + \cos\phi \hat{y}
\end{align*}
\]  

(B.1)

Similarly, the rectangular system \((x,y,z)\) can be transformed to the spherical system \((r,\theta,\phi)\) via
Appendix B Transformation of coordinate systems

\[
\begin{align*}
\dot{x} &= \sin \theta \cos \varphi \dot{r} + \cos \theta \cos \varphi \dot{\theta} - \sin \varphi \dot{\phi} \\
\dot{y} &= \sin \theta \sin \varphi \dot{r} + \cos \theta \sin \varphi \dot{\theta} - \cos \varphi \dot{\phi} \\
\dot{z} &= \cos \theta \dot{r} - \sin \theta \dot{\theta}
\end{align*}
\] (B.2)
Appendix C
The radius of curvature of an ellipse

The curvature $k(\sigma)$ of an arbitrary curve is defined as the limit of the ratio between the change in direction $\Delta \alpha(\sigma)$ and the arclength $\Delta \sigma$ [28],

$$ k(\sigma) = \lim_{\Delta \sigma \to 0} \frac{\Delta \alpha(\sigma)}{\Delta \sigma} = \frac{d\alpha}{d\sigma} \quad (C.1) $$

The radius of curvature $R(\sigma)$ is then given by

$$ R(\sigma) = \frac{1}{|k(\sigma)|} = \left| \frac{d\sigma}{d\alpha} \right| \quad (C.2) $$

Assume the curve is given in the parameter expression $x = X(t)$, $y = Y(t)$. The arclength can be taken as

$$ \sigma = s(t) \quad \text{and} \quad t = s^{-1}(\sigma) \quad (C.3) $$

resulting in

$$ \alpha(t) = \alpha[s^{-1}(\sigma)] = \alpha^*(\sigma) \quad (C.4) $$

The curvature is now given by

$$ k(\sigma) = \frac{d\alpha^*}{d\sigma} = \frac{d\alpha}{dt} \cdot \frac{ds^{-1}}{d\sigma} = \frac{d\alpha}{dt} \cdot \frac{ds}{d\sigma} = k^*(t) \quad (C.5) $$

Using the relation

$$ \tan \alpha(t) = \frac{\dot{Y}(t)}{\dot{X}(t)} \quad (C.6) $$

and differentiating it, the following is valid

$$ \frac{\ddot{\alpha}(t)}{\cos^2 \alpha(t)} = \frac{\dot{X}(t)\dot{Y}(t) - \dot{Y}(t)\dot{X}(t)}{[\dot{X}(t)]^2} \quad (C.7) $$

giving
The radius of curvature of an ellipse

\begin{align*}
\frac{\dot{a}(t)}{\dot{X}(t)} &= \frac{\dot{X}(t) \ddot{Y}(t) - \ddot{X}(t) \dot{X}(t)}{\dot{X}(t)^2} \frac{1}{1 + \tan^2 \alpha(t)} \\
&= \frac{\dot{X}(t) \ddot{Y}(t) - \ddot{X}(t) \dot{X}(t)}{\dot{X}(t)^2} \frac{1}{1 + \left(\frac{\ddot{X}(t)}{\dot{X}(t)}\right)^2} \\
&= \frac{\dot{X}(t) \ddot{Y}(t) - \ddot{X}(t) \dot{X}(t)}{\dot{X}(t)^2 + \dot{Y}(t)^2} \\
\end{align*}

Furthermore

\[ \frac{ds}{dt} = \pm \sqrt{\dot{X}(t)^2 + \dot{Y}(t)^2} \]  

Now the curvature can be written as

\[ k^*(t) = \pm \frac{\dot{X}(t) \ddot{Y}(t) - \ddot{X}(t) \dot{X}(t)}{(\dot{X}(t)^2 + \dot{Y}(t)^2)^{3/2}} \]  

The radius of curvature is then given by

\[ R^*(t) = \frac{(\dot{X}(t)^2 + \dot{Y}(t)^2)^{3/2}}{|\dot{X}(t) \ddot{Y}(t) - \ddot{X}(t) \dot{X}(t)|} \]  

Assume that some curve is given by the expression \( y = f(x) \). So it is permitted to set

\[ X(x) = x \]  

\[ Y(x) = f(x) \]  

resulting in \( \dot{X}(x) = 1, \dot{X}(x) = 0, \ddot{Y}(x) = f'(x), \ddot{Y}(x) = f''(x) \). Eq. (C.11) then reduces to

\[ R(x) = \frac{(1 + |f'(x)|^2)^{3/2}}{|f''(x)|} \]  

The ellipse of an FZPA can be expressed as

\[ \left( \frac{x-c}{a} \right)^2 + \left( \frac{y}{b} \right)^2 = 1 \]  

which can be rewritten for \( y > 0 \) as
The first and second derivative of \( f(x) \) are

\[
f'(x) = -\frac{b}{a} \frac{x-c}{\sqrt{a^2-(x-c)^2}}, \quad x \neq a+c \tag{C.16}
\]

and

\[
f''(x) = \frac{-ba}{(a^2-(x-c)^2)^{3/2}}, \quad x \neq a+c \tag{C.17}
\]

Substituting the latter two equations in Eq. (C.13) gives the radius of curvature of the ellipse,

\[
R(x) = \frac{\sqrt{a^4-a^2(x-c)^2+b^2(x-c)^2}}{a^4 b} \tag{C.18}
\]

For \( y < 0 \) the same expression is found.
Appendix D
Efficiencies of a Fresnel-zone plate antenna with a large focal distance

In chapter 5, efficiencies of a FZPA with a short focal distance \( F = 0.581 \) m are evaluated. This was done when moving the feed along the antenna axis or changing the operating frequency. This appendix shows plots for the system with \( F = 1.934 \) m and \( n = 75.3 \). The same considerations are can be made as in chapter 5. First, the efficiencies are shown when the feed is moved. This is shown in figure D.1 to D.4. The dependence on the wavelength is shown in figure D.5 to D.9.

Figure D.1 Efficiencies plotted against the feedposition for the first transparent zone \( (F = 1.934 \) and \( n = 75.3) \)
Appendix D Efficiencies of a Fresnel-zone plate antenna with ...

Figure D.2 Efficiencies plotted against the feedposition for an antenna with absorbing/transparent zones ($F = 1.934 \text{ m}$ and $n = 75.3$)

Figure D.3 Efficiencies plotted against the feedposition for an antenna with ideal phase correcting zones ($F = 1.934 \text{ m}$ and $n = 75.3$)
Appendix D  Efficiencies of a Fresnel-zone plate antenna with ...

Figure D.4 Efficiencies plotted against the feedposition for an antenna with dielectric/transparent zones \((F = 1.934 \text{ m and } n = 75.3)\) and \(\varepsilon_r = 10.8\)

Figure D.5 Efficiencies plotted against the wavelength for the first transparent zone \((F = 1.934 \text{ m, } n = 75.3, \lambda = 2.7 \text{ cm})\)
Appendix D Efficiencies of a Fresnel-zone plate antenna with ...
Appendix D Efficiencies of a Fresnel-zone plate antenna with ...

Figure D.8 Efficiencies plotted against the wavelength for an antenna with dielectric/translucent zones ($F = 1.934$ m, $n = 75.3$, $\lambda = 2.7$ cm) and $\varepsilon_r = 10.8$

Figure D.9 Gain plotted against the wavelength for an antenna with dielectric/translucent zones ($F = 1.934$ m, $n = 75.3$, $\lambda = 2.7$ cm) and $\varepsilon_r = 10.8$